Model Predictive Control : Exercise 6

Prob 1 | Solving Explicit MPC using parametric LCPs

Consider the discrete-time linear time-invariant system defined by the dynamics

$$x^+ = 2x + u - 1$$

with constraints

$$U = \{u \,|\, 0 \leq u \leq 2\}$$

We formulate the following MPC problem with horizon $N = 1$:

$$\begin{aligned} f(x) = \min\ & x^2 + u^2 + (x^+)^2 \\ \text{s.t.}\ & x^+ = 2x + u - 1 \\ & 0 \leq u \leq 2 \end{aligned}$$

Your goal is to calculate the explicit solution $f^\star(x)$ of the parametric program and the corresponding explicit control policy $u^\star(x)$.

Tasks:

- To simplify the problem, eliminate the decision variable x^+.
- Write down the Lagrangian function $\mathcal{L}(x, u, \lambda, \nu)$ where λ corresponds to the constraint $0 \leq u$ and ν corresponds to the constraint $u \leq 2$.
- Write down the KKT conditions (stationarity, primal/dual feasibility, complementarity).
- Give matrices M, Q and vector q such that the optimal solution of the problem is a linear transform of the solution $y(x)$ to the following parametric LCP:

$$w - My = Qx + q \qquad\qquad w, y \geq 0 \qquad\qquad w^T y = 0$$

- Draw the complementarity cones of the pLCP.
- Compute the optimal value function $f^\star(x)$ and its corresponding control policy $u^\star(x)$.
- Use Matlab to estimate $u^\star(x)$ and $f^\star(x)$ by solving the optimization problem for a number of different values of x and compare this to your parametric solution.

Prob 2 | Implement explicit MPC using MPT3

We revisit the MPC problem from exercise 4, where we considered the discrete-time linear time-invariant system defined by

$$x^+ = \begin{bmatrix} 0.9752 & 1.4544 \\ -0.0327 & 0.9315 \end{bmatrix} x + \begin{bmatrix} 0.0248 \\ 0.0327 \end{bmatrix} u$$

with constraints

$$X = \{x \mid |x_1| \leq 5, |x_2| \leq 0.2\} \qquad\qquad U = \{u \mid |u| \leq 1.75\}$$

This is a second-order system with a natural frequency of $0.15r/s$, a damping ratio of $\zeta = 0.1$ which has been discretized at $1.5r/s$. The first state is the position, and the second is velocity.

Your goal is to implement an explicit MPC controller for this system with a horizon of $N = 10$ and a stage cost given by $l(x, u) := 10x^T x + u^T u$ using the MPT3 toolbox.

Tasks:

- Define your MPC problem using MPT3. You can proceed as follows:
  - Define the system `sys = LTISystem('A',A,'B',B)`
  - Define the constraints on the signals by setting the values `sys.x.max = ...`, `sys.x.min = ...`, etc
  - Define the stage costs by setting the penalty terms for x and u, e.g., `sys.x.penalty = QuadFunction(Q)`
  - Extract desired sets and weights with `sys.LQRGain`, `sys.LQRPenalty.weight` and `sys.LQRSet`,
  - Set the terminal cost and terminal set with `model.x.with('terminalPenalty')`, `model.x.terminalPenalty = QuadFunction(Qf)` and `model.x.with('terminalSet')`, `model.x.terminalSet = Xf`,
  - Define the MPC controller with `controller = MPCController(sys, N)`.
- Generate the explicit MPC with `empc = controller.toExplicit()`.
- Plot the generated solution, including regions, with `empc.feedback.fplot()`.
- Simulate the closed-loop system starting from the state $x = \begin{bmatrix} 3 & 0 \end{bmatrix}^T$.
  Confirm that your constraints are met. Reuse the simulation code from exercise 4. You can evaluate the explicit controller with `empc.evaluate(x)`.

Prob 3 | Compare explicit MPC with YALMIP implementation

We now compare the explicit MPC with the YALMIP implementation from exercise 4.

Tasks:

- If (for some reason) you skipped exercise 4, implement the controller using YALMIP.
- Plot the position, velocity, and input of the system using the YALMIP controller. Confirm that your solution is the same as for the explicit MPC case.
- Compare the solve times of the explicit MPC against the YALMIP implementation. What do you notice, is it as expected?