Model Predictive Control : Exercise 1

Consider the discrete-time LTI system defined by

$$\begin{aligned} x_{i+1} &= Ax_i + Bu_i \\ y_i &= Cx_i \end{aligned}$$

with

$$A = \begin{pmatrix} 4/3 & -2/3 \\ 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad C = \begin{pmatrix} -2/3 & 1 \end{pmatrix}$$

Prob 1 | Write a code that, given a horizon N, computes the optimal control law that minimizes the following cost

$$V = \sum_{i=0}^{N-1} (x_i' Q x_i + u_i' R u_i) + x_N' P_f x_N$$

with

$$Q = C'C + 0.001 I_{2\times 2} \qquad R = 0.001 \qquad P_f = Q$$

Use the discrete-time Bellman recursion.

Prob 2 | Receding horizon control

- Compute the closed-loop state trajectory in a receding horizon fashion from state $x = \begin{bmatrix} 10 & 10 \end{bmatrix}'$. Find the minimum horizon length N^* that stabilizes the system.
- Plotting the prediction, motivate why increasing the horizon stabilizes the closed loop system.
- Given a horizon length N^* that stabilizes the closed loop system, can you be sure that the system will be stable for $N > N^*$?

Prob 3 | Linear quadratic regulator

- Implement the infinite horizon LQR controller $u = K_\infty x$.
- Compute the infinite horizon cost for the system in closed loop with $u = K_\infty x$ and compare it with the infinite horizon cost for the system in closed loop with $u = K_{N^*} x$, where K_{N^*} is the control law computed in the Ex.2 bullet one. Which one gives a lower cost ?

Hints:

- Use `dlqr` Matlab function to compute the LQR controller
- You can approximate the infinite horizon cost for the closed loop system numerically using a long state and input trajectory:

$$V_\infty = \sum_{i=0}^{\infty} (x_i' Q x_i + x_i' K' R K x_i) \sim \sum_{i=0}^{1000} (x_i' Q x_i + x_i' K' R K x_i)\,.$$