Model Predictive Control : Exercise 1 - Solutions

Consider the discrete-time LTI system defined by

$$
\begin{aligned}
x_{i+1} &= Ax_i + Bu_i \\
y_i &= Cx_i
\end{aligned}
$$

with

$$
A = \begin{pmatrix} 4/3 & -2/3 \\ 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad C = \begin{pmatrix} -2/3 & 1 \end{pmatrix}
$$

Prob 1 | Write a code that, given a horizon N, computes the optimal control law that minimizes the following cost

$$
V = \sum_{i=0}^{N-1} (x_i' Q x_i + u_i' R u_i) + x_N' P_f x_N
$$

with

$$
Q = C'C + 0.001 I_{2\times 2} \qquad R = 0.001 \qquad P_f = Q
$$

Use the discrete-time Bellman recursion.

> See code.

Prob 2 | Receding horizon control

- Compute the closed-loop state trajectory in a receding horizon fashion from state $x = \begin{bmatrix} 10 & 10 \end{bmatrix}'$. Find the minimum horizon length N^* that stabilizes the system.
- Plotting the prediction, motivate why increasing the horizon stabilizes the closed loop system.
- Given a horizon length N^* that stabilizes the closed loop system, can you be sure that the system will be stable for $N > N^*$?

> - $N^* = 7$, see code.
> - For short horizons, the balance between the cost of the state trajectory and the input trajectory comes down in favour of the input, since the state doesn't move "too much" over the short term if no input action is taken. Over a longer horizon, the cost of the state trajectory will dominate, causing the input to take action to decrease it.
> - No.

Prob 3 | Linear quadratic regulator

- Implement the infinite horizon LQR controller $u = K_\infty x$.

- Compute the infinite horizon cost for the system in closed loop with $u = K_\infty x$ and compare it with the infinite horizon cost for the system in closed loop with $u = K_{N^*} x$, where K_{N^*} is the control law computed in the Ex.2 bullet one. Which one gives a lower cost ?

Hints:

- Use `dlqr` Matlab function to compute the LQR controller
- You can approximate the infinite horizon cost for the closed loop system numerically using a long state and input trajectory:

$$V_\infty = \sum_{i=0}^{\infty} (x_i' Q x_i + x_i' K' R K x_i) \sim \sum_{i=0}^{1000} (x_i' Q x_i + x_i' K' R K x_i) \,.$$

> The LQR controller has lower cost. See code.