Model Predictive Control : Exercise 2

Your goal is to implement the barrier method to solve the quadratic program:

$$\begin{aligned} \min_z \quad & \frac{1}{2} z^T H z + q^T z \\ \text{s.t.} \quad & Gz \leq d \end{aligned}$$

The barrier method:

repeat $\{$*Outer loop*$\}$

  repeat $\{$*Inner centering loop*$\}$

    Compute search direction Δz

$$\left(H + \kappa \sum_{i=1}^{m} \frac{1}{(d_i - g_i z)^2} g_i^T g_i \right) \Delta z = -Hz - q - \kappa \sum_{i=1}^{m} \frac{1}{d_i - g_i z} g_i^T$$

    Line search: Choose $0 \leq t \leq 1$

    Update: $z := z + t\Delta z$

  until *stopping criterion is satisfied*

  Decrease barrier parameter: $\kappa := \mu\kappa$

until $\kappa < \epsilon$

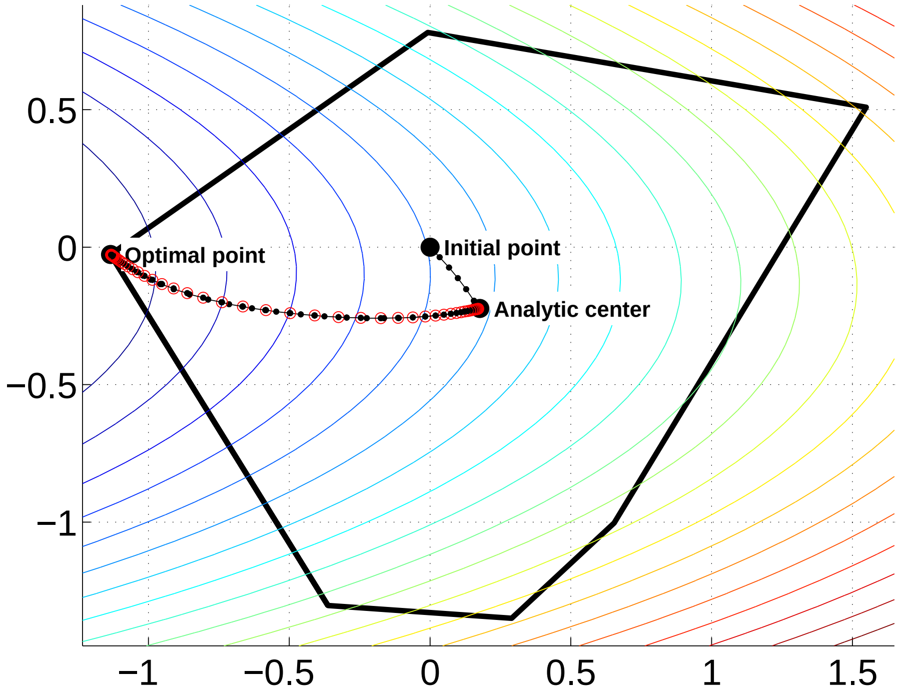
(Note that g_i is a *row vector* in the above pseudo-code)

Prob 1 | Fill in the missing code from the file `ex2.m` to compute the search direction.

Run the code and you should see something like the figure below (you get a random problem each time, so it will be a little different).

The black dots are the iterates of the inner loop (the centering step). Note the first phase where the solution moves from the initial point (0) to the central path (the analytic center is the point on the central path as κ goes to infinity).

The red circles are the outer iterates, and are on the central path.



Prob 2 | Change line 12 to read `speed = 'fast';`. This changes the tuning parameters (μ, β, α, etc) to 'normal' values. The algorithm will now converge so quickly that you won't be able to see the central path.

Your goal in this exercise is to compute the sensitivity of solve-time to the choice of the parameter μ. Generate a plot of μ vs the total number of Newton steps (i.e., the number of times the search direction is computed).

Consider the range of μ between `opt.epsilon` and 1. The command `logspace` may be useful.

Change the dimension (line 11) to 50. How does the number of iterations change?