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Combinatorial Coalition Formation for multi-item group-buying with heterogeneous customers

Cuihong Li [a,*], Katia Sycara [b], Alan Scheller-Wolf [c]

[a] School of Business, University of Connecticut, United States
[b] Robotics Institute, Carnegie Mellon University, United States
[c] Tepper School of Business, Carnegie Mellon University, United States

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ABSTRACT

A group-buying market may offer multiple items with non-additive values (i.e., items may be complementary or substitutable), to buyers who are often heterogeneous in their item valuations. In such a situation, the formation of buying groups should concentrate buyers for common items while taking into consideration buyers' heterogeneous preferences over item bundles. Also, it should permit non-uniform cost sharing among buyers in the same group, which benefits all buyers by drawing more group-buying participants. We introduce the concept of Combinatorial Coalition Formation (CCF), which allows buyers to announce reserve prices for combinations of items. These reserve prices, along with the sellers' price-quantity curves for each item, are used to determine the formation of buying groups for each item. Moreover, buyers in the same group may not necessarily all pay the same price. The objective of CCF is to maximize buyers' total surplus.

Determining the optimal coalition configuration in CCF is NP-hard, and the stability of such a configuration relies on the cost sharing rule within each group. We thus propose a heuristic algorithm for CCF based on augmented greedy selections, along with a cost sharing rule satisfying certain stability properties. Simulation results show that our approximate algorithm generates fairly good solutions compared to the optimal results, and is greatly superior to a simpler distributed approach. Furthermore, our algorithm's performance is enhanced when items are complementary or strongly substitutable, especially in settings when the prices decrease either rapidly or slowly with the quantities. Evaluations of the sellers' revenue under CCF demonstrate that sellers should offer a more gradually decreasing price-quantity curve for complementary or strongly substitutable items, and a more abruptly decreasing curve for weakly substitutable items. In addition, sellers may benefit from greater sales generated by simpler price-quantity curves with fewer steps.

1. Introduction

1.1. Motivation

Group-buying has been in vogue for many years in various industries [14,20]. Demand aggregation in group-buying benefits sellers, offering lower marketing costs and coordinated distribution channels, as well as buyers, who enjoy lower costs for product purchases [11]. The Internet provides a powerful tool for demand aggregation and hence is a natural platform to facilitate group-buying. It is thus not surprising that online group-buying was perceived as one of the most innovative business models of e-commerce, and has been employed by many companies. Although some early pioneers failed, online group-buying has been reviving in Europe, North America and Asia in recent years [7], in part thanks to the increasing connection of people in online forums and social networks [27]. In fact, web-facilitated group-buying is now a major phenomenon in China, under the name "tuangou" [2] (see [7] for a recent list of the successful tuangou websites). Since group-buying is driven by the demand of buyers, a central issue within a group-buying mechanism is how to improve the satisfaction of buyers. But this is not always easy.

As noted by [16], the early group-buying firms faced the dilemma of choosing between developing a wide product selection and mandating a large buying group: Consumers may have diverse preferences over products, and therefore offering a wide product selection can fragment the customer base. Thus, in order to form large-size buying groups, the early group-buying firms often limited product selection within each category [16,27]. But such limited choice of products detracted from consumers' experience, reducing their surplus. In contrast, the successful tuangou service providers in China offer a great variety of products and services to their members [27].

Buyers' surplus depends on both their valuation of the purchased items and the purchasing costs they pay. To maximize buyers' surplus

* Corresponding author.
E-mail addresses: Cuihong.Li@business.uconn.edu (C. Li), katia@cs.cmu.edu (K. Sycara), awolf@andrew.cmu.edu (A. Scheller-Wolf).

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is particularly complex in multi-item group-buying environments, as when buyers are offered a variety of products, their product selections determine not only the values they receive from the purchased items, but also influence the sizes of the buying groups for each item and thus their purchasing costs. We discuss the values and costs in greater details below.

The value of an item to a buyer is measured by the buyer's willingness-to-pay (reserve price) for the item. It is common that product values may not be simply additive: Items may be *substitutes*; that is, the total value of a combination of the items may be less than the sum of the values of individual items (items are *perfect substitutes* if the value of the combination is equal to the maximum of the single-item values). Items may also be *complements*; that is, the total value may be more than the sum of individual values (items are *perfect complements* if each single item alone has no value). For example, two cars of different models are typically substitutes, while a couch and a coffee table may be complements. Due to such non-additive values, the marginal value of an item to a buyer depends on the *other* items acquired by the buyer, by her participation in their buying groups. We refer to this effect as the *externality among items*. Due to this effect, a buyer's choice of buying groups should be made considering the values of combinations of multiple items.

In a group-buying market, the price of an item depends on the aggregated demand of buyers in the buying group for that item. Thus, whether or not a buyer should join a group depends on the number of participants in that group. We refer to this effect as the *externality among buyers*. Due to this effect, a buyer may not join the groups of her most preferred items. For example, a car buyer choosing between two car models may join the group for her less valued model if the aggregated demand for that model brings the price to a sufficiently low level. Or, a furniture buyer that is interested in both a couch and a coffee table may end up buying neither, even when a good deal is available for the coffee table, if not enough buyers join the group for the couch. These examples illustrate that buyers should be coordinated in their choices of buying groups.

Another critical issue faced by many group-buying markets is how to improve customer participation [16]. This is important because buyers' surplus from group-buying is driven by the aggregated demand of customers. A buyer will not join a group if she expects to pay a price higher than her willingness-to-pay. Thus lowering the prices for some buyers that would otherwise not be able to participate may be beneficial for all buyers. This can be seen in the following example.

Example 1. *Buyer 1 and 2 both want to buy a digital camera. Buyer 1 does not want to pay more than $350, and Buyer 2 can pay at most $300. The unit price provided by the seller is $350 for one camera and $310 for two.*

In Example 1, if the two buyers purchase alone, Buyer 1 would purchase at a price $350 and Buyer 2 would not be able to purchase, both having zero surplus. But if they form a group, then they can enjoy a surplus of $30 together. However, for Buyer 2 to join the group, she cannot pay more than $300, and thus Buyer 1 has to pay at least $320; for example, Buyer 1 pays $335 and Buyer 2 pays $285. With such non-uniform cost sharing, although Buyer 1 has to subsidize Buyer 2, she is better off than she would be purchasing alone.

The above example shows that customers should not be ignored in a group-buying market, even if their willingness-to-pay is lower than the group-buying price: Under non-uniform cost sharing, these low-value buyers may be able to join a group and drive down the price, benefitting all buyers. Such a benefit can be significant in a group-buying market for two reasons. First, group-buying markets have a heavy focus on the categories of expensive durable goods, such as home appliances, home remodeling materials, cars, and electronics [16,27]. For these products, the heterogeneity of customer values is usually high, due to different tastes on the design, quality, function, or brand of the product, as well as different budget constraints among customers. Second, customers interested in group-buying tend to be price-sensitive buyers. Thus there is likely a considerable portion of low-value customers in a group-buying market.

Motivated by the above considerations of multiple items with non-additive values and customer heterogeneity in item preferences and willingness-to-pay, we propose an innovative group-buying mechanism, *Combinatorial Coalition Formation* (CCF). CCF assigns buyers to buying groups of multiple items based on their reserve prices for combinations of items, with buyers in the same group not necessarily paying the same price. The objective of CCF is to maximize the total surplus of buyers. CCF is different from traditional group-buying mechanisms, which are usually based on a single item and uniform cost sharing: CCF balances the need of aggregating buyers for common items and the need of incorporating buyers' heterogeneous preferences of items, improving customer participation by allowing non-uniform cost sharing among buyers. Though CCF does not currently exist in practice, we believe that it can be implemented for real group-buying markets. Its implementability is discussed in the following subsection.

1.2. Implementability and difference from combinatorial auctions

CCF can be implemented in a centralized group-buying market in which a coordinator exists to configure the buying groups based on buyers' reports of their preferences. (This contrasts with a distributed mechanism in which buyers separately make their own decisions of which items to buy, based on their observations of groups' sizes and price-quantity curves.) Some existing group-buying markets exhibit features of a centralized mechanism. For example, in a typical purchasing process at UniStar (www.unistarllc.com), the firm first collects a buyer's information on spending and target prices. Then the firm analyzes the buyer's information and its own offering to establish a purchasing solution for the buyer. The latter stage usually takes a few weeks, which may allow the firm to aggregate demand from several buyers. A typical tuangou process in China also takes a centralized form: After buyers express their preferences and interests in joining a team, a leader or a committee is usually elected to conduct the negotiation with vendors and organize the buying groups. The solution often takes into consideration customer preferences over multiple items (although it is unlikely a result of CCF). For example, [2] reports a tuangou team of buyers that were interested in the Chevrolet Aveo, a new General Motors subcompact. The car had two models that were preferred by different buyers. In the end, some buyers in the team received the less expensive model and some bought the more expensive one. In another case reported by [2], a team of cabinet buyers reached deals for both cabinets and hinges, two complementary items.

Non-uniform cost sharing may currently be rare in group-buying markets; traditionally, buyers in a group pay the same price. Nevertheless, we believe that non-uniform cost sharing can be implemented in a straightforward manner in a centralized nontransparent market such as UniStar. (In a nontransparent market, buyers do not know each other but only interact with the coordinator.) Since a buyer is not aware of the other buyers' prices, it is relatively easy to charge different prices to buyers. However, implementing non-uniform cost sharing can be challenging in a transparent market, in which buyers know each other and are informed of each buyer's payment. Tuangou may be considered as such an example: In a tuangou event buyers gather together and meet in person during their visits to vendors. To implement non-uniform cost sharing in a transparent market, it is important to ensure that the cost sharing solution is fair and is in the best interest of all buyers. Otherwise, some buyers may not accept the solution but search for an alternative that would bring them better results. We consider this issue in our design of cost sharing rules.

CCF can be implemented as a two-stage process: In the first stage, customer preferences are expressed, and then in the second stage, buying groups are formed and cost shares are determined. Such a process is similar to a combinatorial auction, which allocates multiple items among a set of buyers based on the buyers' bids on combinations (packages) of items. In fact, the first stage of preference elicitation in CCF is no different from the one in combinatorial auctions. In this stage, the market coordinator has to capture the preference information of each buyer for all possible item combinations of interest. Although such a process may involve tremendous communication complexity, some advanced techniques of preference elicitation have been developed and used in practice, in the context of combinatorial auctions [10,22]. These techniques can be used equally well for CCF. The second stage of CCF uses buyers' preferences over combinations of items to determine the configuration of buying groups and the payments of each buyer. This is analogous to the winner determination problem in combinatorial auctions, namely determining the item allocation and bidders' payments based on the package bids.

However, CCF and combinatorial auctions are essentially different for two reasons: First, combinatorial auctions are concerned with how to efficiently allocate limited resources among bidders so that the total utility is maximized. But in CCF the resource constraint is not an essential factor—the sellers (retailers or manufacturers) often have abundant supply or capacity compared to buyers' demands, although the asking price varies by the total quantity acquired by buyers. Instead, CCF is more concerned with how to allocate buyers to different buying groups so as to maximize the total surplus of buyers, given the price-quantity curves of items. Second, in combinatorial auctions the transfer prices are determined by the bids of buyers (although they may be constrained by the reserve prices set by the auctioneer). But in CCF, the price is based on a price-quantity curve pre-specified by the seller. Although the payment of a buyer may be influenced by other buyers' bids under non-uniform cost sharing, the total cost of a group is independent of the bids. Due to these differences, the techniques for winner determination in combinatorial auctions cannot be applied to CCF; an independent solution technique needs to be developed to determine coalition configuration and cost sharing in CCF. This is the focus of our paper.

1.3. Summary of results

We show that CCF is an NP-hard problem, and then present a computationally efficient heuristic algorithm for CCF based on augmented greedy selections. In addition, we propose a cost sharing rule such that for each coalition, no buyers can obtain a higher surplus by deviating from the group and forming a coalition by themselves (i.e., the cost sharing is in the core of each coalition). Through numerical experiments, we evaluate the performance of CCF based on our heuristic algorithm and compare it to the distributed approach that applies uniform cost sharing with no coordination among buyers. Since CCF is driven by the externality among items and among buyers, in the experiments we vary the level of substitutability/complementarity among items, as a measure of the item externality, and the rate of price decrease in the item price-quantity curves, as a measure of the buyer externality. We find that our algorithm generates fairly good solutions compared to the optimal result (the coalition configuration that maximizes buyers' total surplus), and is greatly superior to the distributed approach. The performance of our heuristic algorithm is enhanced when items are complementary or strongly substitutable, especially in settings with high or low price decrease rates.

Our mechanism focuses on buyers' surplus, not sellers' revenue, by assuming that price curves are given. However, anticipating the group-buying outcome, a seller could be expected to rationalize the price curves in order to maximize her own revenue. Therefore, leveraging our algorithm, we proceed to evaluate the seller's revenue, and generate managerial insights into the seller's pricing strategies for a group-buying market under CCF. We find that low rates of price decrease may be used for complementary or strongly substitutable items, while high rates of price decrease are suitable for weakly substitutable items. Surprisingly, due to the externality among buyers, total sales may *increase* with slower price decreases or fewer price-decrease steps. Therefore, a simpler price curve with fewer steps may allow sellers to achieve greater revenue.

The rest of the paper is organized as follows. We review the relevant literature in Section 2. In Section 3 we formulate the problem. Then in Section 4 we present a heuristic algorithm for coalition formation, and a cost sharing rule given the coalitions. Experimental results evaluating the performance of the heuristic algorithm and the seller's pricing decisions appear in Section 5. Section 6 concludes and discusses some additional issues.

2. Related work

There are several group-buying papers based on empirical study. [16,17] are among the earliest papers that examine group-buying as a dynamic pricing mechanism. Through an investigation of the early group-buying websites, [17] study buyers' behaviors, and [16] assess the business models of group-buying. They offer valuable insights for distributed group-buying mechanisms under uniform cost sharing. Recently, [18] and [15] use online experiments to study customer participation in group-buying, the former focusing on the effect of textual comments and existing orders, and the latter investigating different incentive mechanisms that overcome startup participation inertia. All these papers study distributed group-buying mechanisms in which buyers separately make their purchasing decisions. We propose and analyze CCF, a centralized mechanism that coordinate buyers to maximize their total surplus.

There also exists theoretical work on group-buying. [1] analyze the value of group-buying and the optimal price curve from a seller's perspective. They assume atomic buyers, whose individual participation in a buying group has no influence on the price. Considering limited supply of an item and private information of buyers, [5] analyze buyers' bidding strategies in a group-buying auction. Based on this analysis, [6] further compare group-buying with fixed pricing mechanisms, [8] examine the effect of uncertain customer values, and [7] analyze the benefits of buyer collusion. These papers are based on uniform cost sharing among group members, though non-uniform cost sharing may improve the total surplus of buyers. [19] study different cost sharing mechanisms with information asymmetry, considering both buyers' surplus and the coalition stability based on buyers' strategic responses to the mechanisms. Note that all the above theoretical papers consider a single item, making coalition formation trivial. Different from these, we focus on the efficient outcome of coalition configuration for multiple items with non-additive values, without considering strategic buyers (i.e., assuming buyers' reserve prices are public information).

If buyer's reserve prices are public information, the cost sharing mechanism is typically concerned with the stability or some other axiomatic properties of the coalition. Various solution concepts of stability have been proposed in cooperative game theory [21]. We adopt the concept of the *core*. [30] presents a dynamic approach to divide the surplus of a coalition among members that converges to a division in the core, if the core is non-empty. In group-buying, a buyer's surplus is divisible into the value and cost, and buyers' total cost is determined by the aggregated quantity and the price curve. Based on such a specific structure, we provide a cost sharing vector that can be easily calculated and is guaranteed to be in the core of a coalition. Our cost sharing adopts a threshold rule—the buyers with reserve prices lower than a threshold pay their reserve prices, while the buyers with reserve prices above the threshold pay at the threshold level. [23] also proposes a threshold rule (among other rules) for auctions. Although similar in form, these two payment rules are essentially different, due to the difference between group-buying and auctions, as discussed in Section 1.

Given a set of agents and constant values of each agent coalition, coalition structure generation determines the partition of agents into disjoint coalitions in order to maximize the sum of coalition values. [24] prove that exponentially many coalition structures with respect to the number of agents have to be searched to guarantee the solution within a bound of the optimum. Thus to generate an optimal coalition structure is computationally difficult. While CCF is also concerned with allocating buyers into disjoint coalitions for item bundles, the value of a coalition (the total surplus of buyers in the coalition) is not constant—it depends on the configuration of other coalitions that acquire some common items. Thus CCF is even more difficult than coalition structure generation.

Our work is an extension and generalization of [31], who consider the situation in which items are perfectly substitutable (i.e., the value of a bundle is equal to the maximum value of a single item in the bundle), and therefore, a buyer purchases at most one item. For that case, they propose a greedy heuristic algorithm to construct the coalition for each item, and a cost sharing rule in the core of each coalition. In this paper, we provide a solution under a general preference pattern of items (i.e., there is no restriction on the value of a bundle given the values of single items). In this general case, a buyer may acquire multiple items, resulting in coalition formation for item bundles. We thus adapt the cost sharing rule in [31] as well as their greedy heuristic algorithm for bundle coalitions. Furthermore, even in their setting with perfectly substitutable items, our algorithm improves over theirs by testing different limits of the number of buyers added into a coalition in each step. In addition, our work generates new insights about the effects of substitutability/complementarity of items and price curve design on the algorithm performance and on the seller's revenue.

Finally, the design of quantity discounts has been studied considering either aggregation over customers with fixed unit demand [1,8], or considering consumption choices of customers with flexible demand but without demand aggregation [28,29]. Our (numerical) evaluation of the seller's revenue in CCF provides insights on the design of volume discounts in the presence of demand aggregation among buyers with flexible demand.

3. Formulation

In this section, we provide a mathematical formulation of the CCF problem. We first present the inputs and formulate the optimal coalition formation problem in Section 1, and then present the cost sharing problem in Section 2. This is followed by an example of CCF in Section 3. Table 6 and Table 7 in Appendix A summarize the notation and key terms.

3.1. Problem setting

Items and bundles: There are M items available in a category in a group-buying market. Denote by $I = \{g_1, g_2, \ldots, g_M\}$ the collection of items. We assume that a buyer is interested in at most one unit of each item,[1] although a buyer may purchase multiple items at the same time. A *bundle*, g_Γ, is defined as a nonempty set of items $\{g_m\}_{m \in \Gamma}$, where Γ is a subset of $\{1, 2, \ldots, M\}$. Denote by G the set, and $K \doteq |G|$ the number, of bundles. For each item m, there is a discrete non-increasing price-quantity curve, $p_m(k)$, where p_m is the unit price, and k is the aggregate quantity of a buying group.

Buyers: There are N buyers interested in the category of items offered in the market. Denote by $B = \{b_1, b_2, \ldots, b_N\}$ the set of buyers. For each bundle g_Γ, a buyer b_n holds a reserve price $r_{n,\Gamma}$, the highest price she is willing to pay for g_Γ.

Coalitions: Buyers submit their reserve prices to a mediator (such as a group-buying intermediary or coordinator), who forms coalitions for buyers. The *item coalition* $\tilde{C}_m$, for an item g_m, is the entire set of buyers who purchase g_m; the total demand of $\tilde{C}_m$ for the item g_m is equal to $|\tilde{C}_m|$. Thus the unit price charged to $\tilde{C}_m$ is $p_m(|\tilde{C}_m|)$ based on the price curve $p_m(\cdot)$. A buyer in $\tilde{C}_m$ may purchase the item as a part of a bundle. We define a *bundle coalition* C_Γ, as the group of buyers each of whom purchases the bundle g_Γ. Given the coalitions C_Γ for each bundle g_Γ, the coalition for an item m can be obtained by combining the coalitions for all bundles that contain the item g_m: $\tilde{C}_m = \cup_{\Gamma \ni m} C_\Gamma$.

Coalition configuration: A coalition configuration, denoted by $\mathcal{C}$, is the set of bundle coalitions $\{C_\Gamma\}_{g_\Gamma \in G}$ (or the equivalent item coalitions). Given a coalition configuration, the unit price p_Γ of a bundle g_Γ is uniquely determined as the sum of unit prices of all included items: $p_\Gamma(\mathcal{C}) = \sum_{m \in \Gamma} p_m(|\tilde{C}_m|)$. Note that the unit price of an item depends on the sizes of all bundle coalitions acquiring that item. Denote by $t_\Gamma(\mathcal{C}) \doteq p_\Gamma(\mathcal{C})|C_\Gamma|$ the total cost of a bundle coalition C_Γ given the coalition configuration C. The *surplus of a coalition configuration* $\mathcal{C}$, $v(\mathcal{C})$, is defined as the difference between the total reserve prices and costs of the coalitions:

$$v(\mathcal{C}) = \sum_{g_\Gamma \in G} \left[\sum_{b_n \in C_\Gamma} r_{n,\Gamma} - t_\Gamma(\mathcal{C}) \right]. \tag{1}$$

The problem of optimal Combinatorial Coalition Formation (CCF) is to find a configuration $\mathcal{C}^*$ such that $v(\mathcal{C})$ is maximized at $\mathcal{C} = \mathcal{C}^*$, subject to the constraints that a buyer belongs to at most one bundle coalition and a bundle has at most one coalition.

CCF shares some similarity in structure with the weighted set packing problem (WSPP). In WSPP, one is given a set of elements, and a collection of subsets of the elements, each subset with a constant non-negative value (weight). The goal of WSPP is to find the largest value collection of subsets that are pairwise disjoint in their contained elements [3,13]. In CCF, both buyers and bundles can be considered as the elements, and each possible bundle coalition is a subset of the elements. The goal of CCF is to find the largest surplus set of bundle coalitions in which no two coalitions share the same buyer or are associated with the same bundle. Fig. 1 illustrates the similarity of CCF to set packing. The left plot shows the collection of bundle coalitions: Each closed shape represents a bundle coalition, encompassing a subset of buyers and a bundle. The right plot shows a valid coalition configuration.

Despite such structural similarity, CCF is different from WSPP in that the value (surplus) of a coalition is not constant—since the unit price of an item depends on the total number of buyers involved in all the bundle coalitions acquiring that item, generally, the surplus of a

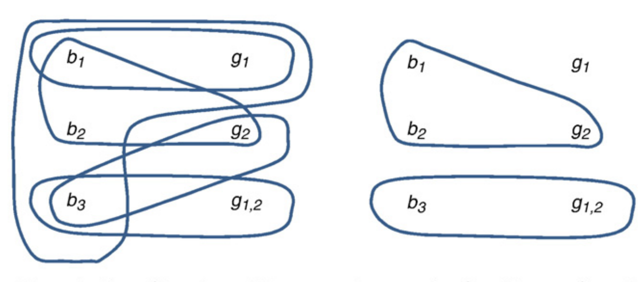

Fig. 1. An illustration of coalition configuration in CCF.

[1] This assumption is reasonable for expensive durable goods, such as cars, home appliance, electronics, etc., which are the focus categories in group-buying markets. This assumption allows us to focus on the externality among distinct items, and to limit the number of bundles. Without this assumption, a bundle would be defined not only by the items included, but also by the quantity of each item. Thus relaxing this assumption may greatly increase the number of bundles, but does not essentially change the solution.

bundle coalition depends on the entire coalition configuration. Only in the special case when the considered bundles consist of disjoint items, is the surplus of a bundle coalition independent of other coalitions, and CCF is equivalent to WSPP. This is the case if the items are perfectly substitutable, and thus each bundle is comprised of a single item. Thus WSPP is a special case of the CCF problem. Since WSPP is NP-complete, to find an optimal solution of CCF is NP-hard.

Proposition 2. *CCF is NP-hard.*

Proof. All proofs are in Appendix A. □

In Section 1, we propose a heuristic algorithm to find an approximate solution for CCF, inspired by the heuristic of greedy selection for WSPP.

3.2. Cost sharing

In this section, we formulate the cost sharing rule for a bundle coalition. Recall that given the coalition configuration $\mathcal{C}$, $p_\Gamma(\mathcal{C})$ is the unit price for a bundle g_Γ, and $t_\Gamma(\mathcal{C})$ is the total cost of the bundle coalition C_Γ. A *cost sharing* vector of a bundle coalition C_Γ, $(x_{n,\Gamma})_{n \in C_\Gamma}$, specifies the cost $x_{n,\Gamma}$ to be paid by each buyer b_n in the coalition. A cost sharing vector is feasible if it satisfies 1) budget balance (the total payment is no less than the total cost: $\sum_{b_n \in C_\Gamma} x_{n,\Gamma} \geq t_\Gamma(\mathcal{C})$, and 2) individual rationality (all individual payments are no greater than the corresponding reserve prices: $x_{n,\Gamma} \leq r_{n,\Gamma}$ for all $b_n \in C_\Gamma$).

To ensure the stability of coalitions, we further impose the following condition on the cost sharing vector for each bundle coalition C_Γ:

$$\text{for all}\, \Omega \subseteq C_\Gamma : \sum_{b_n \in \Omega} x_{n,\Gamma} \leq p_\Gamma(\mathcal{C})|\Omega|. \tag{2}$$

If a set of buyers Ω in the bundle coalition C_Γ deviate from the coalition and form a coalition for the bundle g_Γ by themselves, then for them the unit price of the bundle will be at least $p_\Gamma(\mathcal{C})$. Eq. (2) says that such a deviation would never benefit these buyers, as their total payment, if they remain in the coalition, is no greater than if they deviate. A cost sharing vector of a coalition satisfying Eq. (2) is said to be in the *core* of the coalition [12].

A cost sharing vector satisfying Eq. (2) prevents “local” deviations—deviations limited to be within each bundle coalition, without changing the items acquired by each buyer. A stronger property would be for the cost sharing of coalitions to be immune to any deviation, including those in which buyers could change the items they acquire by joining coalitions for other bundles. Cost sharing satisfying such a property is said to be in the coalition structure core [4].[2] We adopt the weaker concept of the core of each coalition because the coalition structure core can only be defined for the optimal coalition configuration—if the configuration is not optimal, buyers can always improve their surplus by all deviating to the optimal configuration. But as shown in Proposition 2, finding the optimal coalition configuration is NP-hard. In Section 6 we discuss how our cost sharing rule compares to the Shapley value, another well-known cost sharing solution for coalition games.

[2] A *coalition structure* for N players is a partition of N. Consider a coalition game $(N, v, \mathcal{R})$, in which N is the set of players, $v(\beta)$ associates a value with each subset β of N, and $\mathcal{R}$ is the coalition structure. A payoff vector $\mathbf{x}$ determines the payoff x_n for each player n. Given $\mathbf{x}$, let $x(R)$ be the sum of payoffs of players in a coalition $R \in \mathcal{R}$. A payoff vector $\mathbf{x}$ is feasible for $(N, v, \mathcal{R})$ if $x(R) \leq v(R)$ for every $R \in \mathcal{R}$, i.e., for each coalition, the total payoff of players is no greater than the value of the coalition. A payoff vector is in the *coalition structure core* if it is feasible, and satisfies $x(\beta) \geq v(\beta)$ for all $\beta \subset N$, i.e., for any subset of players, their total payoff is no less than the value they can achieve by forming a coalition by themselves.

3.3. Example

We now provide an illustrative example of Combinatorial Coalition Formation. Assume four buyers are offered three items: g_1 and g_2 are two cameras of different brands, and g_3 is a flash memory card. The two cameras are considered substitutes, while either camera or the memory card is the complement. Table 1 shows the reserve prices of the four buyers. (Since g_1 and g_2 may not be perfectly substitutable, $g_{1,2}$ and $g_{1,2,3}$ are included in the bundles.).

A price-quantity curve is provided for each item. It specifies the unit price of an item as a non-increasing discrete function of the (aggregated) quantity, with the total cost increasing in the quantity. An example of price curves is shown in Table 2.

Assume that the bundle coalitions are configured as follows: $C_{2,3} = \{a, b\}$ for $g_{2,3}$, and $C_2 = \{c, d\}$ for g_2. This is equivalent to the set of item coalitions: $\tilde{C}_2 = \{a, b, c, d\}$ for g_2, and $\tilde{C}_3 = \{a, b\}$ for g_3. Given this configuration, no buying group is formed for g_1, the group-buying price for g_2 is 350, and that for g_3 is 40. Thus the unit price of the bundle $g_{2,3}$ is 390. Then the total cost of $C_{2,3}$ is $2 \cdot 390 = 780$, and the cost of C_2 is $2 \cdot 350 = 700$. The surplus of the bundle coalition $C_{2,3}$ is $385 + 405 - 780 = 10$, and the surplus of C_2 is $355 + 360 - 700 = 15$. Then the total surplus of the coalition configuration is $10 + 15 = 25$. A feasible cost sharing vector for $C_{2,3}$ is: $x_a = 385$, $x_b = 395$, and for C_2 is: $x_c = x_d = 350$.

In view of the NP-hardness of CCF, we present a heuristic algorithm to find a solution for coalition configuration in Section 1. Then, given a coalition configuration, we present a cost sharing rule in Section 2 that generates cost sharing vectors in the core of each bundle coalition. The performance of the heuristic algorithm is evaluated in Section 5.

4. Algorithm

4.1. Coalition formation

We adapt the widely used heuristic of greedy selection for weighted set packing problems (WSPP) to CCF. Given the value of each set of elements in a WSPP, the greedy selection adds to the current solution a set of elements with the maximum value from the remaining sets, remove all sets that intersect it from the remaining sets, and repeats until all sets have been removed [13]. In CCF, a coalition configuration is a collection of bundle coalitions that are disjoint in buyers and bundles. Therefore, each bundle coalition can be regarded as a set. The volume discounts of items have two implications for this heuristic algorithm: 1) The value of a set should be defined as the *marginal value* that the set *adds* to the current solution, and 2) removing the intersecting sets means to remove only the buyers, but not the bundle, involved in the set (bundle coalition),

Table 1
Example: buyers' reserve prices.

Buyer	g_1	g_2	g_3	$g_{1,3}$	$g_{2,3}$	$g_{1,2}$	$g_{1,2,3}$
a	350	350	20	380	385	500	545
b	380	365	25	400	405	600	650
c	0	355	0	0	360	355	360
d	360	360	0	365	370	510	525

Table 2
Example: price-quantity curves.

Item	Volume (units)				
	1	2	3	4	≥5
g_1	340	340	320	320	320
g_2	365	365	365	350	340
g_3	40	40	38	38	35

from remaining consideration. The bundle should not be removed because as the process continues, more buyers may join coalitions for some bundles that share common items with the original bundle. This may lower this bundle's price, improving some remaining buyers' marginal values to this bundle coalition.

A greedy selection of buyers that together generates the maximum marginal value for a bundle coalition tends to create a large coalition of many buyers, because more buyers drive down the price with their aggregated demand. But such a greedy allocation may hurt buyers' surplus. This is due to the heterogeneity of buyers' preferences over multiple items: Since buyers may prefer different bundles, the total surplus of a few subgroups, each for a different bundle, may be higher than that of the entire group for one bundle. Balancing the considerations of aggregating buyers for common items and heterogeneity of buyers' preferences, we augment the greedy algorithm with a size limit on the set added to a bundle coalition each time: Given a size limit $L \le N$, each added set cannot contain more than L buyers. The size limit can vary from 1 to N. For each value of L, we apply the algorithm and obtain a coalition configuration. Among all these L solutions, we choose the one that generates the largest total surplus for buyers.

The coalition formation algorithm for a given size limit L is described in Table 3.

It can be shown that this algorithm requires polynomial time with respect to the number of buyers, N, and the number of bundles, K.

Proposition 3. *The computational complexity of the coalition formation algorithm as described in* Table 3 *is* $O(N^2 K \log N)$.

In the worst case, the number of bundles, K, is exponential in the number of items, M (there are total $2^M - 1$ combinations of items). In that case, the computational complexity of the algorithm is manageable if the number of items is small, which is reasonable in group-buying settings where the primary complexity typically comes from the large number of buyers. If the number of items is large, some pre-processing can be performed to reduce the number of bids. We discuss this below.

To generate a positive marginal value to a coalition, a buyer's reserve price has to be greater than the marginal cost incurred by his joining. Such a marginal cost is equal to the difference of the coalition's total cost with and without the buyer's participation. If a buyer's reserve price for a bundle is less than the lowest marginal cost of the bundle, the buyer's bid for that bundle can be pruned. The lowest marginal cost of a bundle is equal to the sum of the lowest marginal prices of each item in the bundle, and the lowest marginal price of an item m can be calculated by finding the lowest value of $kp_m(k) - (k-1)p_m(k-1)$ among $k = 1, 2, \ldots, N$. In the example presented in Section 3.3, the lowest marginal price for camera g_1 is \$300, *for camera* g_2 it is \$305, and for the memory card g_3 it is \$36. Thus the lowest marginal cost for the bundle of the two cameras $g_{1,2}$ is \$605, and for the bundle of all three items $g_{1,2,3}$ is \$641. It can be seen that all bids for the memory card alone, for the bundle of the two cameras, and for the bundle of all three items, are lower than their lowest marginal costs. Thus all these bids can be pruned.

The above example of bid pruning represents a plausible and arguably typical situation in reality. When buyers purchase high-value durable goods, there is usually a primary good and some accessory goods, such as a digital camera and a memory card, or a couch and a coffee table. Due to the high price and low demand of the primary good, different choices of the primary good are usually strongly substitutable. In other words, a buyer's willingness-to-pay for a second primary good is typically low compared to the price. Thus it may be common that the bids for a bundle of multiple primary goods are too low to cover the marginal cost, and such bids are likely to be pruned. On the other hand, an accessory good is often not purchased unless bundled with a primary good. Thus a bid for an accessory good alone can be very low and likely be pruned as well. There may also be cases in which the goods under consideration are comparable in price and complementary on function, such as a desk and a chair for customers interested in office furniture. In these cases, the bids for each individual item are also likely to be pruned.

Table 3
Algorithm of coalition formation with a size limit L.

Input: Price schedules $p_m(\cdot)$ for all items $g_m \in I$, reserve prices $r_{n,\Gamma}$ for all buyers $b_n \in B$ and bundles $g_\Gamma \in G$, size limit L.
Output: A coalition configuration $\mathcal{C} = \{C_\Gamma\}_{\Gamma \in G}$.

1. At initialization, let each bundle coalition C_Γ be $\emptyset$, and the set of remaining buyers Φ be the collection of all buyers B.
2. For each bundle Γ, select a set Ω^*_Γ of no more than L buyers from the set of remaining buyers Φ that has the largest marginal value to $\mathcal{C}$ upon joining the coalition for the bundle g_Γ:

   $\Omega^*_\Gamma \doteq \arg\max_{\Omega \subseteq \Phi, |\Omega| \le L} (v(\cup_\Gamma(\mathcal{C}, \Omega)) - v(\mathcal{C}))$,

   where $\cup_\Gamma(\mathcal{C}, \Omega)$ denotes the coalition configuration after Ω is added to the bundle coalition C_Γ in the configuration $\mathcal{C}$: $C_\Gamma \Leftarrow C_\Gamma \cup \Omega$. Call Ω^*_Γ the candidate set for bundle g_Γ.
3. If the candidate sets are empty for all bundles, stop and return the configuration $\mathcal{C}$. Otherwise go to Step 4.
4. From the candidate sets for all bundles, select the one with the largest marginal value to C. If there is a tie, break the tie by selecting the set with the most buyers. Assume the selected set is Ω^*_Γ for the bundle g_Γ. Add the selected set Ω^*_Γ to the configuration $\mathcal{C}$: $C_\Gamma \Leftarrow C_\Gamma \cup \Omega^*_\Gamma$. Update the set of remaining buyers Φ by removing the buyers newly added to the coalitions: $\Phi = \Phi \setminus \Omega^*_\Gamma$.
5. If there are no more remaining buyers, i.e., $\Phi = \emptyset$, stop and return the configuration $\mathcal{C}$. Otherwise go to Step 2, continuing adding sets of buyers with positive marginal values.

4.2. Cost sharing

We now describe our cost sharing rule for a coalition configuration $\mathcal{C}$. For each bundle coalition C_Γ, the payment of a buyer b_n in the coalition, $x_{n,\Gamma}$, is determined by a threshold level h_Γ: $x_{n,\Gamma} = \min(r_{n,\Gamma}, h_\Gamma)$. In other words, all buyers with reserve prices higher than h_Γ pay a price equal to h_Γ, and the rest of the buyers pay their reserve prices. The threshold level h_Γ is calculated so that the total payment of buyers exactly covers the total cost of the coalition: $h_\Gamma |\bar{C}_\Gamma| + \sum_{b_n \in C_\Gamma \setminus \bar{C}_\Gamma} r_{n,\Gamma} = t_\Gamma(\mathcal{C})$, where $\bar{C}_\Gamma \subseteq C_\Gamma$ is the set of buyers in C_Γ with $r_{n,\Gamma} \ge h_\Gamma$. It can be proved that such a cost sharing rule satisfies Eq. (2), and thus is in the core of each bundle coalition.

Proposition 4. *The threshold cost sharing rule specified above is in the core of each bundle coalition; that is, it satisfies Eq. (2).*

This cost sharing rule is not limited to our heuristic algorithm. It can be used in general, after coalitions are formed, and generate a cost sharing vector in the core of each bundle coalition.

5. Numerical evaluation

Through numerical experiments, we compare the buyers' surplus generated with our *approximate* algorithm for CCF by comparing it to the *optimal* result, and the result based on a *distributed* approach. The optimal result refers to the coalition configuration that maximizes the total surplus of buyers. This is obtained by calculating the surplus from all possible coalition configurations and then choosing the one that generates the highest surplus. Due to the NP-hardness of CCF (Proposition 2), we calculate the optimal result only for small-scale problems.

In the distributed approach, there is no coordination among buyers in the coalition formation, and all buyers in a coalition pay the same price: When a buyer arrives at the market, she observes the price-quantity curves and the prices currently reached for each item. The buyer then chooses the coalitions to join by selecting the bundle that brings her the largest surplus. In other words, given the prices ρ_m for each item m upon her joining the coalitions of the items, the buyer will place an order for the bundle g_Γ if $r_{n,\Gamma} - \sum_{m \in \Gamma} \rho_m$ is non-negative, and is the highest among all bundles. If the current prices are too high for a buyer to order, the buyer will return and place an order later if the price of a bundle drops to a level lower than her reserve price after more buyers join the buying groups. This coalition formation process terminates when all buyers have arrived and none of the unassigned buyers wants to join any coalition. It thus does not exclude any buyer that would ultimately benefit from group-buying based on the emerging buying groups.

The rest of this section is organized as follows. We first describe the experimental setting in Section 1. Then our approximate result is compared with the optimal solution and that of the distributed approach in Section 2. In Section 3, we extend the numerical study to sellers' revenue, deriving managerial insights related to a seller's pricing decision.

5.1. Experimental setting

The inputs to a CCF problem include the price-quantity curves of all items and the buyers' reserve prices for each bundle (i.e., each combination of items). As CCF is driven by the externality among items and among buyers, we design our experiments to capture these two main factors.

Reserve prices: The value of a bundle for a buyer is generated randomly. We assume symmetric items in the sense that the distribution of the value of a bundle does not depend on which items, but only on how many items, are included in the bundle. For a bundle with κ items, a buyer's reserve price is drawn from a uniform distribution with the lower bound $\underline{p} \cdot \kappa^{\alpha}$ and upper bound $\bar{p} \cdot \kappa^{\alpha}$, where $\underline{p}$ and $\bar{p}$ are the lower and upper bounds of the reserve prices for a single item, and $\alpha > 0$ indicates if the items in a bundle are substitutes or complements. With $\alpha = 1$, the value of a bundle is stochastically equivalent to the sum of the values of the single items in the bundle. Thus $\alpha = 1$ represents a situation when the total value of items does not change with them forming a bundle. For $\alpha < 1$ (> 1), the value of a bundle is stochastically lower (higher) than the sum of the item values. Thus items tend to be viewed as substitutes when $\alpha < 1$, and complements when $\alpha > 1$.

Price-quantity curves: We assume that items have identical price curves, consistent with the symmetric item assumption. The unit price of an item lies between a ceiling price p_h and a floor price p_l. The price drops from p_h to p_l regularly, each drop coming after the addition of the same number of units in the order quantity and resulting equal price reductions. Thus, for given p_h and p_l, a price curve can be characterized by two parameters: the *number of steps* (price drops) denoted by R, and the *price decrease rate* (PDR) defined as the ratio of price reduction to the quantity increase over each step. The assumption of a constant PDR over all steps is a simplification; in reality, the PDR may become smaller as the quantity increases [25]. This simplification allows us to evaluate the impact of price curves by focusing on the overall price decrease rate and the number of steps along the entire curve, without concerning each specific step of price changes. We believe that the PDR and the number of steps have first-order effects on the performance of the group-buying mechanism. The PDR determines how steep the quantity discounts are: The higher the PDR, the smaller the quantity required for the price to drop to a certain level, and the lower the price for a buying group. Thus, with a higher PDR, more buyers are able to purchase in a buying group. The number of steps on the price curve determines the frequency of the price drops as the order quantity increases. The more the steps, the smaller the quantity required for each (smaller) price drop. Thus, with more steps on the price curve, the formation of buying groups is easier as even a small increase in the order quantity is able to drive down the price. By varying the PDR and the number of steps in the numerical study, we carry out extensive experiments to evaluate our mechanism, with a focus on the impacts of these two factors. Fig. 2 shows two sample price-quantity curves with PDR equal to 0.5 and 0.2, and the number of steps equal to 4.

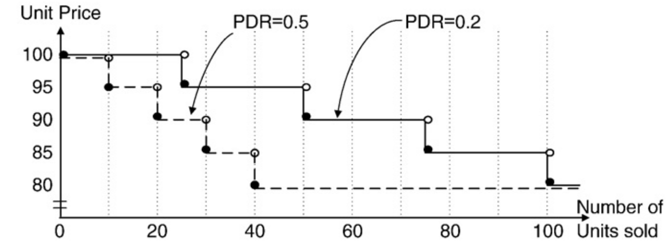

Fig. 2. Examples of the price-quantity curve.

Parameter values: In the experiments the number of buyers varies with $N = 4, 6, 8, 10, 50, 80, 100$, and the number of items varies with $M = 3, 4, 5$. We assume any items can form a bundle, and thus the number of bundles is $K = 7, 15, 31$. For the price curves, we keep the ceiling price $p_h = 100$ and floor price $p_l = 80$, with the number of steps $R = 4, 5, 10$. Considering customer variance in reserve prices, we let the lower bound of a buyer's single-item reserve price be lower than p_l: $\underline{p} = 70$, and the upper bound be (weakly) higher than p_h: $\underline{p} = 100, 110$, with the two values representing two levels of the purchasing power of an individual buyer. For smaller problems ($N \leq 10$), we compare all three mechanisms–the optimal, approximate, and distributed–using the average performance of 100 randomly generated instances for each parameter value set. For larger problems ($N \geq 50$), calculating the optimal solution is difficult because the computational time is exponential in problem scale. For these problems, we compare the approximate algorithm with the distributed approach only, and generate 1000 random instances for each parameter value set. We summarize the overall findings from the experiments, but demonstrate only representative results due to the space limitations.

5.2. Comparison of the approximate, optimal, and distributed solutions

In the evaluation of our algorithm, we focus particularly on the impact of the price decrease rate (PDR) and the preference pattern (α) of items. The PDR determines the significance of benefits for buyers in forming coalitions; it can be regarded as a measure of the externality among *buyers*. The value of α characterizes the non-additiveness of item values, thus governing the externality among *items*. In Section 1 we present the results with different PDRs and $\alpha = 1$ (representing the case when bundling does not change the total value of items) as the base case, focusing on the impact of PDR. Then in Section 2 we compare the results with different α, discussing the joint impact of PDR and the substitutability/complementarity of items.

5.2.1. The base case: $\alpha = 1$

Fig. 3 demonstrates the buyers' surplus generated with the approximate algorithm and with the distributed approach as percentages of the optimal, with $N = 8$, $M = 3$ ($K = 7$), $\alpha = 1$, $R = 4$, $\underline{p} = 70$, and $\bar{p} = 110$.

Comparing the surplus generated in the different mechanisms, we find that the approximate algorithm is better than the distributed approach for all cases. In addition, the approximate algorithm achieves a performance close to (generally greater than 90% of) the optimal for most cases. Thus, overall, the approximate solution performs well compared to the optimal result. Under some special cases, however, the total surplus based on the approximate algorithm may be unsatisfactorily low (less than 80%) relative to the optimal. These are cases in which the highest possible customer valuation of a single item, $\bar{p}$, falls below (or equal to) the ceiling price p_h, the PDR is relatively low, *and* the number of buyers is small (e.g., when $\bar{p} = 100$, PDR = 1 or 2 with $N = 4$ or 6, and PDR = 1 with $N = 8$). Under these

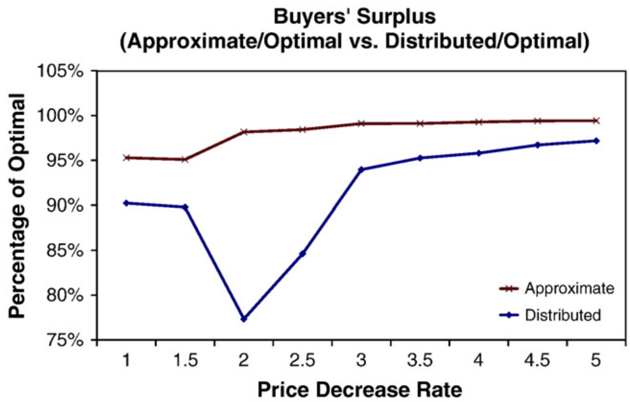

Fig. 3. Buyers' total surplus generated with the approximate algorithm and with the distributed approach as percentages of the optimal. $N=8$, $M=3$ ($K=7$), $\alpha=1$, $R=4$, $\underline{p}=70$, $\bar{p}=110$, each data point presents the average result of 100 random instances.

situations, coalition formation is difficult due to the lack of "seed" buyers: Recall that in our algorithm the coalitions are formed sequentially. In each step, some buyers are added to a bundle coalition, increasing buyers' total surplus. With the expansion of the coalitions, the prices of the items get lower, and more buyers are able to join the coalitions. In other words, some low-value buyers may not be able to join a coalition until the coalition reaches a size that brings the price to a sufficiently low level. Thus, it is important that there are enough high-value buyers that can form fairly large coalitions at the early stage of the process in order to draw low-value buyers. We call these high-value buyers the "seed buyers". If there are not enough seed buyers, then coalition formation is difficult because not many buyers are able to join the coalitions based on this sequential process. Though such difficult situations lead to extremely low buyers' surplus generated with our approximate algorithm, they are *less important* because under these situations even the optimal surplus of buyers is very small relative to their willingness-to-pay.[3] Furthermore, under these situations, the distributed approach is greatly inferior to the approximate algorithm, failing to form any coalitions even when the surplus achieved in the approximate algorithm is significant.

The relative performance of the mechanisms is significantly influenced by the PDR. We observe that the approximate algorithm achieves near-optimal results both when the PDR is very low (if the optimal surplus is still considerable) or very high, while for intermediate PDR values, its suboptimality is relatively significant. This behavior can be explained as follows: When PDR is very low, quantity discounts are difficult to achieve even with the aggregated demand of multiple buyers. Thus essentially all buyers face the ceiling prices, receiving no benefit from demand aggregation. When PDR is very high, the demand of very few buyers is enough to bring the price down to the floor. Thus the assignment of an individual buyer to the coalitions has little effect on the group-buying prices. In both conditions–with very low and high PDR values–the greedy heuristics in the approximate algorithm lead to results that are close to the optimal.

In our experiments, we assume a constant price decrease rate (PDR). A more realistic price curve may have the PDR decreasing with the increase of quantity. With such a curve, it may be relatively easy (requiring only a small number of buyers) to reach a price discount at the beginning of the coalition formation process based on our algorithm, but it may become increasingly difficult (requiring many more buyers) to reach further discounts. Compared to the case with a constant PDR, such a situation with a decreasing PDR may improve the relative performance of the approximate algorithm: Fewer "seed" buyers would be needed at the early stage of the coalition formation process, and thus the approximate algorithm would be more likely to form profitable buying groups in the early stage, which draw more buyers in the later stage. Therefore, the efficiency loss of the approximate algorithm may become lower with a decreasing PDR. Studying the effect of a decreasing PDR on CCF is deferred for future work.

5.2.2. Comparison for complementary ($\alpha>1$) and substitutable ($\alpha<1$) items

The substitutability or complementarity of items influences the reserve prices submitted by buyers, which, along with the price curves, determine the coalition formation results. In this subsection, we investigate the influence of the substitutability or complementarity of items under various price decrease rates of the price curves. Fig. 4 demonstrates a representative result of the surplus based on our approximate algorithm relative to the optimal, with $N=6$, $M=3$ ($K=7$), $R=4$, $\underline{p}=70$, and $\bar{p}=110$.[4] For clarity, we do not show the result of the distributed approach in the figure; its performance is always worse than the approximate algorithm.

We find that the performance of the approximate algorithm relative to the optimal is best for extremal α values—in the example shown in Fig. 4, it grows increasingly better for decreasing α below 0.85 or increasing α above 0.95. For $\alpha\leq 0.7$ or $\alpha\geq 1.0$, the surplus difference between the two mechanisms is nearly always within 7% of the optimal. Furthermore, for α high, the approximate algorithm performs especially well with high PDR values (PDR$\geq$3), while for α low, it favors low PDR (PDR$\leq$2.5). A possible explanation for the joint impact of α and PDR is as follows: First, with α *high*, buyers prefer large bundles, as the marginal value of an item increases with α. This includes the situation not only when items are complementary ($\alpha>1$), but also when items are only slightly substitutable (e.g., $\alpha\in[0.95, 1]$), since with volume discounts buyers tend to buy more items. As α increases, the greedy selection becomes more consistent with the optimal allocation, both turning to large bundles. This desire for large bundles is further enhanced by *high PDR*, as this results in low item prices. Second, when α is *low*, buyers tend to purchase single items in both mechanisms because the marginal value of additional items is low. In this situation, coalitions are formed for single items or small bundles, which have few overlapping items. This results in a low impact of bundle coalitions on each other with respect to the prices, reducing the efficiency loss of the approximate algorithm. Such an effect is strengthened with a *low PDR*, as then the cost of acquiring additional items will be high.

In summary, the approximate algorithm benefits from both strong complementarity and strong substitutability among items, and the benefit from strong complementarity or substitutability is enhanced by high or low price decrease rates, respectively.[5] Note that even outside these situations the algorithm is nearly always within 15%, and usually within 10% of the optimal.

5.3. Managerial insights on pricing

We have evaluated the approximate group-buying algorithm from the perspective of buyers, maximizing the total surplus of buyers with given price curves. Anticipating the group-buying results, sellers should set price curves to maximize their own profits. In this section, we leverage our approximate algorithm to examine *sellers'* pricing decisions.

[3] In addition, the PDR values in these cases would not be selected by a revenue-maximizing seller based on the study in Section 3. Under the PDR values preferred by the seller, the approximate algorithm achieves a surplus no less than 87.9% of the optimal.

[4] We exclude the consideration of PDR$<$0.9 for $N=6$. Under these situations, no discount can be achieved even with the entire buyer population, and hence the approximate and optimal solutions generate identical results.

[5] Since CCF is a centralized mechanism (i.e., the coalition formation is performed by a mediator given the reserve prices submitted by buyers), consumer behavior is irrelevant in our coalition formation process. Consumer behavior would be an important issue in a distributed mechanism in which individual buyers make their own decisions as to which coalitions to join.

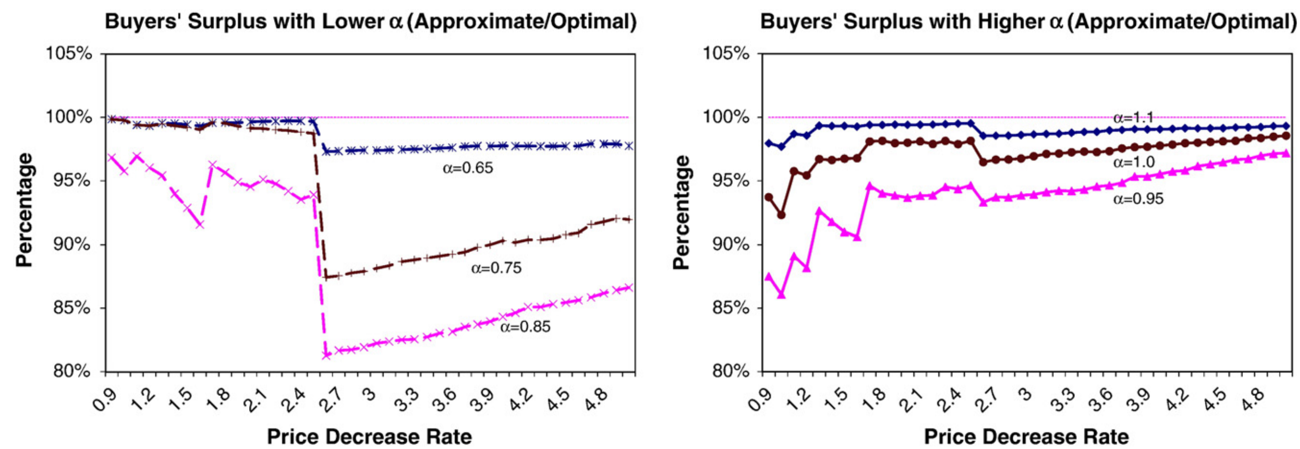

Fig. 4. Buyers' surplus generated with the approximate algorithm as percentages of the optimal. $N=6$, $M=3$ ($K=7$), $R=4$, $\underline{p}=70$, $\bar{p}=110$, each data point presents the average result of 100 random instances.

As explained in §5.1, we continue to assume a constant PDR over all steps in a price curve in order to focus on the impact of the number of steps and the overall price decrease rate, without concerning each specific step of price changes. Then a price curve includes four components: the ceiling and floor prices, the number of steps, and the price decrease rate (PDR). We posit that the ceiling and floor prices are relatively easy to determine—the ceiling price may be set referring to the outside market price without group-buying, and the floor price may be determined according to the minimum acceptable revenue. Therefore, our focus is on the PDR and the number of steps. It is worth noting that our investigation of the seller's pricing decision is based on a centralized group-buying mechanism in which coalitions are not formed until all buyers have submitted their bids. It ignores the dynamics of expectancy and consumer willingness to participate, that are typically a concern when coalitions are formed dynamically along with buyers' bidding [17]. Such issues may require some additional considerations in the price curve design in order to mitigate the startup inertia and encourage more customer orders [15]. This might also be interesting future work.

5.3.1. Impact of PDR

We calculate the total revenue from CCF under varying PDR, and select the optimal PDR that maximizes the revenue. Fig. 5 shows the optimal PDR value with different substitutabilities or complementarities among items (α values), based on the optimal and approximate algorithms, for $N=6$, $M=3$ ($K=7$), $R=4$, $\underline{p}=70$ and $\bar{p}=110$.

The results show similar patterns of the optimal PDR as a function of α for both the approximate and optimal algorithms: The optimal

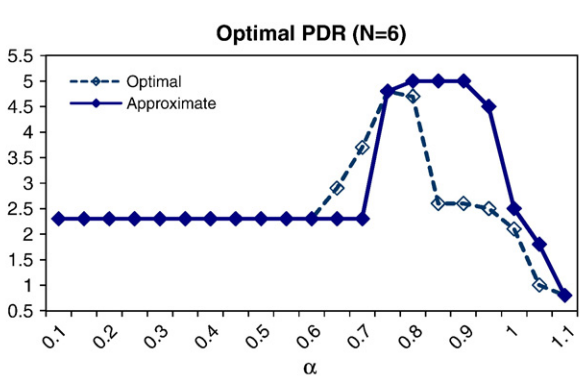

Fig. 5. Optimal selection of PDR based on the approximate and optimal algorithms. $N=6$, $M=3$ ($K=7$), $R=4$, $\underline{p}=70$, $\bar{p}=110$, each data point presents the average result of 100 random instances.

PDR is relatively low for high or low values of α, but high for intermediate α values. (Although the specific optimal PDR values may differ in the optimal and approximate solutions, they lead to very close expected revenues.) This result suggests that increasing PDR is most effective at boosting sales for intermediate values of α. The reason is explained below.

The total sales depend not only on the number of buyers who purchase (i.e., who are assigned to the item coalitions), but also on the average number of items that a buyer purchases (i.e., the average number of item coalitions that a buyer is assigned to). When α is low, items are strongly substitutes and the reserve prices submitted by buyers for large bundles (i.e., bundles with many items) tend to be low relative to their prices. Therefore, even with a high PDR, the reserve prices may not be high enough to get a buyer assigned to coalitions of many items. This results in little power of PDR to boost the number of items purchased per buyer. (But increasing PDR can increase the number of assigned buyers, improving sales somewhat.) When α is high, the reserve prices for large bundles tend to be high. This results in buyers assigned to many item coalitions at the same time even under a low PDR, leaving very little "momentum" of PDR to increase sales appreciably. It is thus for medium α values that the number of items purchased by a buyer is sensitive to PDR; in such conditions, increasing PDR effectively improves sales. *This is the area, for slightly substitutable products, in which sellers should offer significant discounts.*

The above reasoning is verified in numerical experiments. In Fig. 6, we show the total sales and the average number of items purchased per buyer, following the same setting as for Fig. 6. From the left plots of the figure we can see that the positive effect of PDR on sales is most significant for intermediate α values (e.g., $\alpha\in[0.8, 0.9]$). Consistent with the change of sales, the right plots show that increasing PDR is most effective at driving up the number of items purchased per buyer for intermediate α values.

It is interesting to see from the figure that the total sales do *not* necessarily (weakly) increase with PDR. This is somewhat surprising because a higher PDR reduces the price faced by buyers. This result is due to the impact of PDR on the marginal value of buyers to a coalition. With a higher PDR, a price discount is easier to reach even with a smaller coalition, reducing the marginal value of additional buyers to a coalition. Thus increasing PDR may oust some low-value buyers from a coalition because they no longer generate positive value to the coalition. Increasing the number of steps in the price curve has similar effects. This is discussed in the next section.

5.3.2. Impact of the number of steps

Given the same PDR, more steps on a price curve imply that fewer additional units are needed to reduce the price to the next lower level.

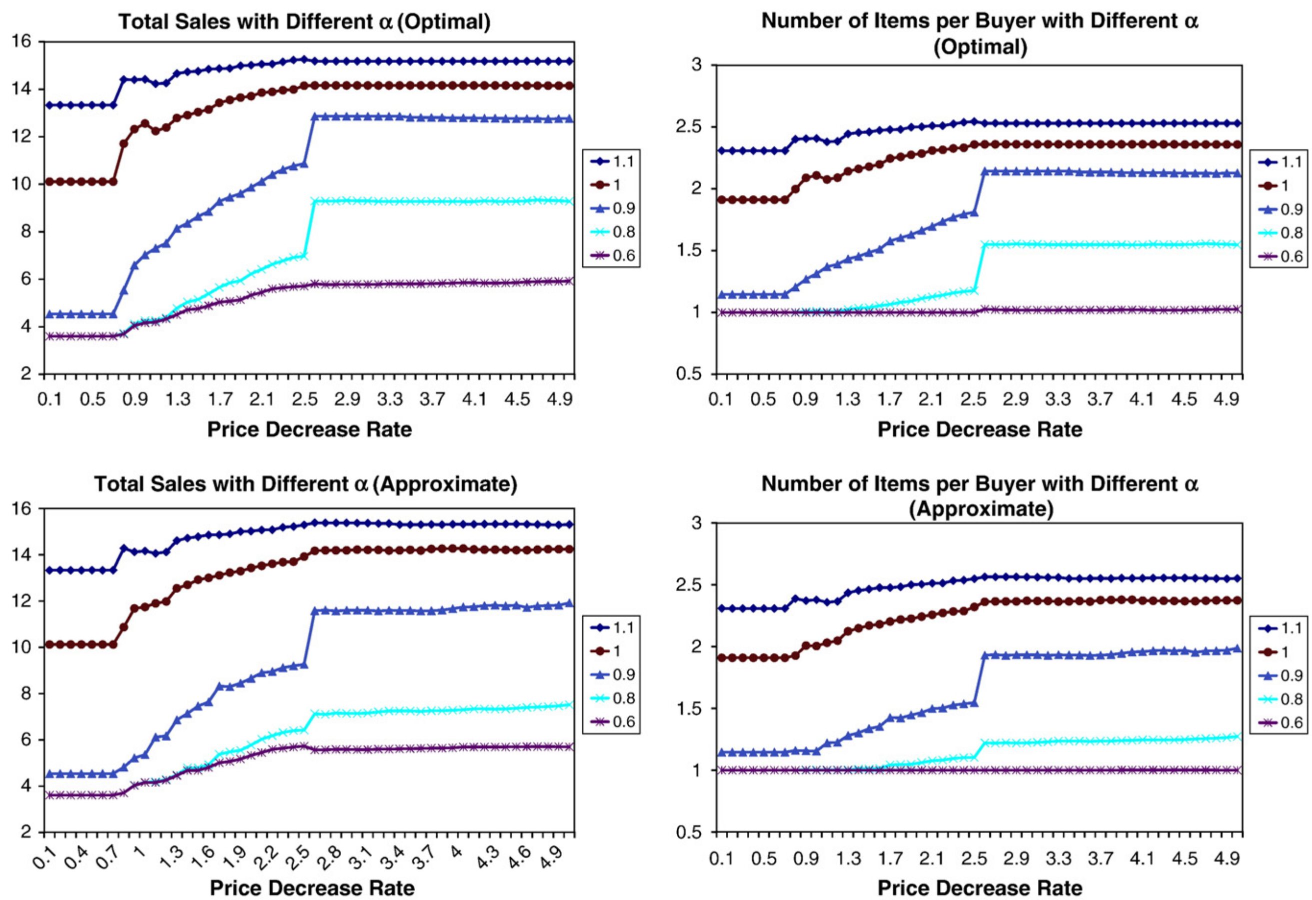

Fig. 6. Total sales and the average number of items purchased per buyer by the approximate and optimal algorithms. The legends indicate the values of α. $N=6$, $M=3$ ($K=7$), $R=4$, each data point presents the average result of 100 random instances.

Thus increasing the number of steps may decrease the marginal value of some buyers to a coalition, leading to lower purchasing quantities. Therefore, although a larger number of steps on the price curve allow the seller to achieve finer control of the price, it can *reduce* sales. This possibility is demonstrated in Table 4, which records the difference of sales achieved with 2 and 5 steps on the price curves for $N=6$, based on the optimal result, the approximate algorithm, and the distributed approach. Positive entries, in bold, indicate when more sales are achieved with the smaller number of steps on the price curve.

The results show that in the optimal and approximate algorithm, a price curve with fewer steps may bring higher sales (e.g., for $\alpha=1.1$, 0.8 in Table 4). However, this is rarely the case for the distributed approach, which generally results in lower sales if the price curve has fewer steps. This is because the distributed approach cannot aggregate buyers as well as the other algorithms, due to the lack of coordination among buyers.

Because of the possibility of generating greater sales with fewer steps on a price curve, sellers may achieve more revenue by using a

Table 4
Difference of total sales with $R=2$ and $R=5$, based on the optimal result (Optimal), approximate algorithm (Approximate), and distributed approach (Distributed). The numbers in bold indicate where greater sales are achieved with the smaller number of steps ($R=2$) on the price-quantity curves, given the mechanism and α value. $N=6$, $M=3$ ($K=7$), each data point presents the average result of 100 random instances.

		PDR							
		0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$\alpha=1.1$	Optimal	0	−0.85	**0.16**	**0.07**	−1.09	**0.19**	−0.44	−0.34
	Approximate	0	−0.73	**0.13**	−0.13	−1.03	**0.33**	−0.16	−0.14
	Distributed	0	−0.34	−0.51	−0.36	−0.47	−0.38	**0.04**	−0.02
$\alpha=1.0$	Optimal	0	−2.04	−0.3	−0.19	−1.36	−0.11	−0.5	−0.36
	Approximate	0	−1.67	−0.89	−0.49	−1.34	−0.01	−0.24	−0.33
	Distributed	0	−0.72	−1.35	−1.12	−1.09	−0.73	−0.20	−0.36
$\alpha=0.9$	Optimal	0	−2.58	−0.62	−0.44	−2.81	−0.68	**0.01**	−0.03
	Approximate	0	−1.34	−2.07	−1.37	−4.08	−1.10	−0.15	−0.27
	Distributed	0	−0.19	−1.82	−2.30	−3.07	−2.22	−0.75	−0.99
$\alpha=0.8$	Optimal	0	−0.50	0	−0.40	−2.20	−1.10	**1.01**	**0.91**
	Approximate	0	−0.40	−0.30	−0.50	−1.90	−0.10	**0.25**	**0.15**
	Distributed	0	−.10	−0.50	−0.70	−1.30	−1.50	−0.40	−0.40
$\alpha=0.6$	Optimal	0	−0.50	−0.20	−0.10	−0.80	**0.27**	**0.09**	**0.06**
	Approximate	0	−0.40	−0.20	−0.20	−0.80	**0.33**	−0.10	−0.10
	Distributed	0	−0.10	−0.50	−0.70	−1.10	−1.00	−0.10	−0.20

Table 5
Difference of revenue with $R=2$ and $R=5$, based on the optimal result (Optimal), approximate algorithm (Approximate), and distributed approach (Distributed). The numbers in bold indicate where more revenue is achieved with the smaller number of steps ($R=2$) on the price-quantity curves given the mechanism and α value. $N=6$, $M=3$ ($K=7$), each data point presents the average result of 100 random instances.

		PDR						
		0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$\alpha=1.1$	Optimal	0	−29.2	**20.1**	**16.9**	**56.4**	**37.5**	−38.7	−32.1
	Approximate	0	−19.0	**17.9**	**10.4**	**51.2**	**39.0**	−24.3	−19.1
	Distributed	0	**15.3**	**8.7**	−5.2	**75.3**	**37.5**	−8.1	−7.6
$\alpha=1.0$	Optimal	0	−158.8	−19.6	−8.0	**19.2**	**17.8**	−50.0	−40.5
	Approximate	0	−125.6	−75.5	−28.1	**6.2**	**11.6**	−34.7	−38.2
	Distributed	0	−41.0	−75.1	−65.4	−10.7	**0.9**	−30.7	−34.8
$\alpha=0.9$	Optimal	0	−235.1	−61.2	−34.7	−131.3	−30.2	−13.7	−16.8
	Approximate	0	−118.8	−190.6	−121.3	−261.8	−87.1	−24.8	−33.8
	Distributed	0	−15.0	−156.8	−192.6	−121.3	−161.1	−74.5	−86.5
$\alpha=0.8$	Optimal	0	−43.8	−5.8	−36.2	−123.4	−84.7	**68.1**	**60.5**
	Approximate	0	−35.0	−29.1	−41.1	−107.9	−15.1	**18.3**	**9.9**
	Distributed	0	−6.0	−41.8	−56.3	−91.5	−115.8	−41.4	−45.4
$\alpha=0.6$	Optimal	0	−37.2	−20.0	−12.2	−17.7	**13.0**	**9.8**	**6.5**
	Approximate	0	−32.3	−18.1	−16.2	−17.6	**19.1**	−6.5	−6.9
	Distributed	0	−6.0	−41.8	−56.3	−79.5	−75.3	−19.6	−23.2

simpler price curve with fewer steps. This is in addition to the benefit that fewer steps with the same PDR generally lead to (weakly) higher unit prices. In Table 5, we compare the revenue under different numbers of steps on the price curves, based on the same setting as for Table 4. We see that in the distributed mechanism, a smaller number of steps generates more revenue only when α is high (e.g., $\alpha=1.1$). This is when large coalitions are easy to form and sales are high, and hence it is profitable for sellers to induce higher prices by imposing fewer (and larger) steps on the price curves. But for the optimal and approximate mechanisms, the superiority of fewer steps is not limited to these high values of α: When a price curve with fewer steps generates more sales, e.g., for those cases indicated in Table 4, it also generally results in higher revenue. Thus for these cases simpler *is* better.

6. Conclusion and discussion

In a group-buying market that offers multiple selections of products, the total surplus of buyers depends on both the prices they pay for the products and their valuation of the products. While the costs of purchasing a product rely on the aggregated demand of buyers for the same product, the value of a product may vary among buyers and depend on the entire combination of items acquired by a buyer. Thus it is critical to concentrate buyers into buying groups for common products while taking into consideration buyers' heterogeneous preferences over combinations of items. In addition, the heterogeneity of buyers' willingness-to-pay implies that non-uniform cost sharing among buyers in the same group may benefit all buyers, drawing more buyers to drive down the price for everyone. Considering multiple items with non-additive values and heterogeneous customers, we propose Combinatorial Coalition Formation (CCF) as a group-buying mechanism that allows buyers to announce reserve prices for combinations of items, and implements non-uniform cost sharing among buyers in a buying group. We show that solving CCF is NP-hard, and therefore present a computationally efficient heuristic algorithm, along with a cost sharing rule that is in the core of each coalition formed for item bundles. Leveraging our algorithm, we also evaluate the seller's revenue and generate insights into the seller's pricing decision.

Our mechanism is potentially useful for the design of an efficient online group-buying market. First, the approximate algorithm allows real-time formation of buying groups that appropriately balances the group sizes and heterogeneous customer preferences over multiple items. Second, our non-uniform cost sharing mechanism encourages customer participation, by attracting the buyers that are able to make positive contributions to a coalition, even those having reserve prices below the group-buying price. This effect of non-uniform cost sharing is similar to bidder collusion in group-buying auctions, which, as shown in [7], may benefit both the seller and the buyers.

Third, by coordinating buyers to optimize their total surplus while taking into account individual preferences, our mechanism reduces the need of a buyer to wait-and-see when making the decision whether to place an order, thus motivating more early orders. This is critical because early orders draw more future orders, which eventually push down the price (a phenomenon called the positive participation externality effect by [17]). Otherwise, for example in a distributed mechanism without buyer coordination, buyers tend to delay placing orders until the aggregated demand approaches a price drop level or the time approaches the closure of the deal [17]. Such a behavior is harmful for a group-buying market. More early orders also effectively shorten the completion time of group-buying. This is also important as the typically long time to complete a transaction is regarded as another primary factor that contributed to the failure of some early online group-buying markets [2]. While different incentive mechanisms may be designed to mitigate such participation dynamics and thus encourage early orders [15], our mechanism minimizes such dynamic effects by coordinating buyers in a centralized form.

In this paper, we assume buyers are truthful in reporting their reserve prices. In reality, a buyer with private information of her reserve prices may behave strategically in order to reduce her share of the costs. The response of strategic buyers to our cost sharing rule merits some discussion. Recall that in our cost sharing rule, the buyers who bid below a threshold level pay their reserve prices, and the others pay at the threshold level. Thus a buyer may have an incentive to under-report her reserve price, as this will lower her payment if her report falls below the threshold level. But such an incentive to lie is weaker for a buyer with greater willingness-to-pay (who thus has more latitude to under-report). This occurs for two reasons: First, a buyer that highly values an item tends to bid a high price in order to reduce the risk of not being included in the coalition. Second, if a buyer bids a higher price, her payment is less dependent on her bid, because a higher bid is more likely to lie above the threshold level. Thus with our cost sharing rule, the incentive for under-bidding is skewed toward low-value buyers. Given that high-value buyers contribute more than low-value buyers to the total surplus, we believe that such a property leads to less efficiency loss caused by buyers' strategic behaviors. Indeed, in [19], it is shown by simulation (based on a single item) that our cost sharing rule generates higher total surplus than some other reasonable payment rules in the presence of strategic buyers. In ongoing work we further study coalition formation and cost sharing considering private information and strategic buyers in multi-unit group-buying [9].

One type of cost sharing mechanism that would help prevent under-reporting of reserve prices is the Shapley value. A Shapley value cost sharing rule allocates to each coalition member a payoff equal to a weighted average of her contributions to all sub-coalitions [26]. This sharing satisfies the monotonicity property: A buyer that reports a higher reserve price makes greater contributions, and hence receives greater surplus (based on the reported value). This monotonicity property weakens the incentive for a buyer to under-report her reserve price. This advantage is also shared by our cost sharing rule: Among buyers within the same coalition, a buyer who reports a higher reserve price always receives a larger surplus. In addition, our cost sharing rule is in the core of each bundle coalition. It thus ensures certain stability of the coalitions, which is not considered with the Shapley value. Finally, our cost sharing rule is easier to compute than the Shapley value, which requires calculation of the surplus of all possible sub-coalitions.

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Appendix A

Proof of Proposition 2. To see clearly the relationship between CCF and the weighted set packing problem, we can alternatively formulate CCF as follows: Let $\gamma(S, \Gamma)=1$ if the bundle coalition for Γ is made up of the buyer set $\Omega \subseteq B$ (i.e., $C_\Gamma=\Omega$), otherwise $\gamma(\Omega, \Gamma)=0$. Then a coalition configuration $\mathcal{C}$ can be specified by an assignment $\gamma \doteq \{\gamma(\Omega, \Gamma)\}_{\Gamma \in G}$. Thus the assignment γ fully determines the total cost of a bundle coalition $t_\Gamma(\gamma)$. Define $v(\Omega, \Gamma|\gamma)=\sum_{n \in \Omega} r_{n,\Gamma}-t_\Gamma(\gamma)$ as the surplus of the bundle coalition for Γ. Then the CCF problem can be formulated as follows:

$$\begin{aligned} \max_{\gamma} \; & \sum_{\Gamma \in G} \sum_{\Omega \subseteq B} \gamma(\Omega, \Gamma) v(\Omega, \Gamma | \gamma) \\ \text{s.t.} \; & \sum_{\Omega \ni n} \sum_{\Gamma \in G} \gamma(\Omega, \Gamma) \le 1, \ \forall n \in B; \\ & \sum_{\Omega \subseteq B} \gamma(\Omega, \Gamma) \le 1, \ \forall \Gamma \in G; \\ & \gamma(\Omega, \Gamma) = \{0, 1\}, \ \forall \Omega \subseteq B, \ \Gamma \in G. \end{aligned}$$

The first constraint assigns a buyer to at most one bundle coalition (this is not a modeling limitation, as discussed in Section 3.1). The second constraint ensures that at most one coalition is established for each bundle. In the objective function, $v(\Omega, \Gamma|\gamma)$ can be regarded as the "weight" associated with the set (Ω, Γ). Notice that the objective function is not linear—$v(\Omega, \Gamma|\gamma)$ depends on the assignment γ because the price of a bundle depends on the sizes of all bundle coalitions that acquire some common items as in that bundle. From the formulation, we see that the weighted set packing problem can be reduced to the CCF problem. Particularly, when the value of a set $v(\Omega, \Gamma|\gamma)$ is a constant (independent of the assignment γ), CCF is a weighted set packing problem. This is the case when the items are perfectly substitutable (i.e., the value of a bundle for a buyer is equal to the highest value of a single item in the bundle). In that situation, a buyer will purchase at most one single item; each of the "bundle" coalitions formed degenerates to an item coalition, whose total cost is independent of the other coalitions. Therefore, CCF is an NP-hard problem. □

Proof of Proposition 3. Calculating a candidate set for a bundle in each round requires $O(N\log N)$ computations (the complexity is dominated by the sorting process). For all bundles, this calculation costs $O(KN\log N)$. Since the algorithm requires at most N rounds of adding sets to $\mathcal{C}$, the complexity of the algorithm in Table 3 for a given size limit is $O(KN^2\log N)$. It follows that the total complexity is $O(KN^3\log N)$ for N different size limits. □

Proof of Proposition 4. Following the cost sharing rule, a buyer's payment is a non-decreasing function of her reserve price. Therefore, we can prove the core property by showing that for any integer $\kappa<|C_\Gamma|$, we have $\sum_{n \in \Omega(\kappa)} x_{n,\Gamma} \le t_\Gamma(\Omega(\kappa))$, where $\Omega(\kappa)$ is the set of buyers with the κ highest reserve prices in C_Γ. Denote by $\upsilon_\Gamma(\Omega)=\sum_{n \in \Omega} r_{n,\Gamma}-t_\Gamma(\Omega)$ the surplus of a coalition Ω for the bundle Γ. According to the coalition formation algorithm, we have $\upsilon_\Gamma(\Omega(\kappa)) \le \upsilon_\Gamma(C_\Gamma)$ for any κ. If $\Omega(\kappa)$ forms a stand-alone bundle coalition for Γ, let $\eta_\Gamma(\Omega(\kappa))$ be the threshold in the cost sharing rule that would be reached in the cost sharing of $\Omega(\kappa)$. It is equivalent to prove $\eta_\Gamma(\Omega(\kappa)) \ge h_\Gamma$.

If $\Omega(\kappa) \supseteq \bar{C}_\Gamma$, then $\eta_\Gamma(\Omega(\kappa)) \ge h_\Gamma$ because the buyers in $\bar{C}_\Gamma \backslash \Omega(\kappa)$ gain zero surplus, but $\upsilon_\Gamma(\Omega(\kappa)) \le \upsilon_\Gamma(C_\Gamma)$. If $\Omega(\kappa) \subset \bar{C}_\Gamma$, we prove $\eta_\Gamma(\Omega(\kappa)) \ge h_\Gamma$ by contradiction: Suppose $\eta_\Gamma(\Omega(\kappa))<h_\Gamma$, then $\eta_\Gamma(\Omega(\kappa))< r_{n,\Gamma}$ for all $n \in \Omega(\kappa)$, and hence $\eta_\Gamma(\Omega(\kappa))=p_\Gamma(\kappa)$, where $p_\Gamma(\kappa)=\sum_{m \in \Gamma} p_m(\kappa)$ is the unit price of bundle Γ when the volume is κ. Therefore, $\eta_\Gamma(\Omega(\kappa)) \ge p_\Gamma(\bar{C}_\Gamma)$ and $\upsilon_\Gamma(\bar{C}_\Gamma) \ge \sum_{n \in \bar{C}_\Gamma} r_{n,\Gamma}-\eta_\Gamma(\Omega(\kappa))|\bar{C}_\Gamma|$. Given $\upsilon_\Gamma(C_\Gamma)=\sum_{n \in \bar{C}_\Gamma} r_{n,\Gamma}-h_\Gamma|\bar{C}_{\Gamma}|$, it follows that the marginal value of $C_\Gamma \backslash \bar{C}_\Gamma$ to $\bar{C}_\Gamma$, $\upsilon_\Gamma(C_\Gamma)-\upsilon_\Gamma(\bar{C}_\Gamma)$, satisfies $\upsilon_\Gamma(C_\Gamma)-\upsilon_\Gamma(\bar{C}_\Gamma) \le \left(\sum_{n \in \bar{C}_\Gamma} r_{n,\Gamma}-h_\Gamma|\bar{C}_\Gamma|\right)-\left(\sum_{n \in \bar{C}_\Gamma} r_{n,\Gamma}-\eta_\Gamma(\Omega(\kappa))|\bar{C}_\Gamma|\right)$. The RHS is equal to $(\eta_\Gamma(\Omega(\kappa))-h_\Gamma)|\bar{C}_\Gamma|$, which is negative. This contradicts $\upsilon_\Gamma(C_\Gamma) \ge \upsilon_\Gamma(\bar{C}_\Gamma)$. Therefore, $\eta_\Gamma(\Omega(\kappa)) \ge h_\Gamma$. □

Table 6
Summary of notation.

Notation	Definition
Items	
M	The number of items
g_m	An item
I	The set of items
$p_m(k)$	The unit price of item m when the quantity is k
Bundles	
K	The number of bundles
g_Γ	A bundle
G	The set of bundles
Buyers	
N	The number of buyers
b_n	A buyer
B	The set of buyers
$r_{n,\Gamma}$	Buyer n's reserve price for bundle Γ
Coalitions	
$\tilde{C}_m$	The coalition for item m
C_Γ	The coalition for bundle Γ
$\mathcal{C}$	The coalition configuration
$\lvert C \rvert$	The number of elements in a set $\mathcal{C}$
Coalition configuration	
$p_\Gamma(\mathcal{C})$	The unit price for bundle Γ given the coalition configuration $\mathcal{C}$
$t_\Gamma(\mathcal{C})$	The total cost of the coalition for bundle Γ given the coalition configuration $\mathcal{C}$
$v(\mathcal{C})$	Buyers' total surplus given the coalition configuration $\mathcal{C}$
$x_{n,\Gamma}$	Buyer n's cost share for bundle Γ
Coalition formation algorithm	
L	The size limit of an added set
Φ	The set of buyers that have not been assigned to coalitions
Ω	A subset of buyers
Ω_Γ^*	The set of buyers that adds the largest value to the coalition for bundle Γ
Numerical experiments	
R	The number of steps on a price-quantity curve
PDR	The price decrease rate
α	Measure of complementarity/substitutability of items ($\alpha<1$: substitutable, $\alpha>1$: complementary)
$\underline{p}$ ($\bar{p}$)	The lower (upper) bound of the reserve price for a single item
p_l (p_h)	The floor (ceiling) price on a price-quantity curve

Table 7
Summary of key terms.

Terms	Definitions
Externality among items	The marginal value of an item to a buyer depends on the other items acquired by the buyer
Externality among buyers	The purchasing cost of a buyer depends on the participation of other buyers in the buying groups
Substitutes (complements)	The value of the items in combination is less (greater) than the value sum of the individual items
Perfect substitutes	The value of the items in combination is equal to the maximum of the single-item values
Perfect complements	Items that are complementary and have no value individually
Item coalition	The set of buyers who all purchase an item
Bundle coalition	The set of buyers who all purchase a bundle of items
Coalition configuration	The set of all bundle coalitions (or the equivalent item coalitions)
Surplus of a coalition configuration	The difference between the total reserve price and purchasing cost of the coalitions
Coalition core	A cost sharing vector for a bundle coalition by which no buyers in the coalition can be better off by deviating from the coalition and forming a coalition to purchase the same bundle by themselves
Coalition structure core	Cost sharing vectors for all bundle coalitions by which no buyers can be better off by deviating from the coalitions in any way

References

[1] K.S. Anand, R. Aron, Group buying on the web: a comparison of price-discovery mechanisms, Management Science 49 (11) (November 2003) 1546–1562.
[2] J.T. Areddy, Chinese Consumers Overwhelm Retailers with Team Tactics, The Wall Street Journal (February 28 2006).
[3] E.M. Arkin, R. Hassin, On local search for weighted k-set packing, Mathematics of Operations Research 23 (3) (1998) 640–648.
[4] R. Aumann, J. Dreze, Cooperative games with coalition structures, International Journal of Game Theory 3 (1974) 217–237.
[5] J. Chen, X. Chen, X. Song, Bidder's strategy under group-buying auction on the internet, IEEE Transactions on Systems, Man and Cybernetics — Part A: Systems and Humans 32 (6) (November 2002) 680–690.
[6] J. Chen, X. Chen, X. Song, Comparison of the group-buying auction and the fixed pricing mechanism, Decision Support System 43 (2) (March 2007) 445–459.
[7] J. Chen, X. Chen, R.J. Kauffman, X. Song, Should we collude? analyzing the benefits of bidder cooperation in online group-buying auctions, Electronic Commerce Research and Applications 8 (4) (July-August 2009) 191–202.
[8] J. Chen, R.J. Kauffman, Y. Liu, X. Song. Segmenting uncertain demand in group-buying auctions. Electronic Commerce Research and Applications (2009), doi:10.1016/j.elerap.2009.03.001.
[9] R. Chen, C. Li, R. Zhang, Group Buying Mechanisms under Quantity Discounts, 2008 Working paper.
[10] P. Cramton, Y. Shoham, R. Steinberg, Combinatorial Auctions, MIT Press, 2006.
[11] R. Dolan, Quantity discounts: managerial issues and research opportunities, Marketing Science 6 (1) (1987) 1–22.
[12] D. Gillies, in: A. Tucker, R. Luce (Eds.), Solutions to general non-zero sum games, Contributions to the Theory of Games, vol. IV, Princeton University Press, 1959, pp. 47–83.
[13] M.M. Halldórsson, B. Chandra, Greedy local improvement and weighted set packing approximation, Journal of Algorithms 39 (2) (2001) 223–240.
[14] D. Hannon. Big U.S. companies unite to make group buying work. Purchasing Magazine Online, August 23, August 23 2001.
[15] C.-T. Ho, R. J. Kauffman, and H. Lai. Incentive mechanisms, fairness and participation in online group-buying auctions. Electronic Commerce Research and Applications (2009), doi:10.1016/j.elerap.2008.11.009.
[16] R.J. Kauffman, B. Wang, Bid together, buy together: on the efficiency of group-buying business models in internet-based selling, in: P. Lowry, J. Cherrington, R. Watson (Eds.), Handbook of Electronic Commerce in Business and Society, CRC Press, 2002.
[17] R.J. Kauffman, B. Wang, New buyer's arrival under dynamic pricing market microstructure: the case of group-buying discounts on the internet, Journal of Management Information Systems 18 (2) (Fall 2001) 157–188.
[18] R.J. Kauffman, H. Lai, H.-C. Lin, Y.-S. Chang, Do textual comments and existing orders affect consumer participation in online group-buying? 42nd Hawaii International Conference on System Sciences, January 5–8 2009, pp. 1–10, Big Island, Hawaii.
[19] C. Li, S. Chawla, U. Rajan, K. Sycara, Mechanisms design for coalition formation and cost sharing in group-buying markets, Electronic Commerce Research and Applications 30 (4) (Winter 2004) 341–354.
[20] P. Mitchell, Internet-enabled Group Purchasing: Get a Buy with a Little Help from Your Friends, February 2002 Technical report, AMR Research.
[21] H. Moulin, H. Moulin, Axioms of Cooperative Decision Making, Cambridge University Press, 1988.
[22] N. Nisan, Bidding Languages for Combinatorial Auctions, MIT Press, 2006, pp. 215–232, chapter 9.
[23] D. Parkes, Achieving budget-balance with vickrey-based payment schemes in exchanges, Proceedings of 17th International Joint Conference on Artificial Intelligence, August 4–10 2001, pp. 1161–1168, Seattle, Washington, USA.
[24] T. Sandholm, K. Larson, M. Andersson, O. Shehory, F. Tohme, Coalition structure generation with worst case guarantees, Artificial Intelligence 111 (1–2) (1999) 209–238.
[25] F. Schotanus, J. Telgen, L. de Boer, Unraveling quantity discounts, Omega 37 (3) (June 2009) 510–521.
[26] L. Shapley, A value for n-person games, Annals of Mathematics Studies 280 (2) (1953) 307–317.
[27] C.S. Tang, United we may stand, The Wall Street Journal (May 12 2008).
[28] S. Viswanathan, Q. Wang, Discount pricing decisions in distribution channels with price-sensitive demand, European Journal of Operational Research 149 (3) (September 2003) 571–587.
[29] Q. Wang, Determination of suppliers' optimal quantity discount schedules with heterogeneous buyers, Naval Research Logistics 49 (1) (January 2002) 46–59.
[30] L. Wu, A dynamic theory for the class of games with nonempty cores, SIAM Journal of Applied Mathematics 32 (2) (1977) 328–338.
[31] J. Yamamoto, K. Sycara, A stable and efficient buyer coalition formation scheme for e-marketplaces, Proceedings of the 5th International Conference on Autonomous Agents, May 28–June 01 2001, Montreal, Canada.

Cuihong Li is an assistant professor of Operations and Information Management at the School of Business, University of Connecticut. Her research interests lie in the interfaces between operations, economics, and technology, in particular supplier chain management and online markets. Her current research focuses on sourcing strategies, procurement auctions, and group-buying mechanisms. Her research papers have appeared in or been accepted to *Manufacturing and Service Operations Management*, *Production and Operations Management*, *Naval Research Logistics*, and numerous other academic journals. She has taught courses on operations management and spreadsheet modeling for business analysis. Dr. Li holds a Ph.D. from the Tepper School of Business, Carnegie Mellon University, and B.S. and M.S. degrees from Tsinghua University, China.

Katia Sycara is a Professor in the School of Computer Science at Carnegie Mellon University (CMU) and holds the Sixth Century Chair (part time) in Computing Science at the University of Aberdeen in the U.K. She is also the Director of the Laboratory for Agents Technology and Semantic Web Technologies at CMU. She holds a B.S in Applied Mathematics from Brown University, M.S. in Electrical Engineering from the University of Wisconsin and PhD in Computer Science from Georgia Institute of Technology. She holds an Honorary Doctorate from the University of the Aegean (2004). She is a member of the Scientific Advisory Board of France Telecom, a member of the Scientific Advisory Board of the Greek National Center of Scientific Research “Demokritos” Information Technology Division. She is a Fellow of the Institute of Electrical and Electronic Engineers (IEEE), Fellow of the American Association for Artificial Intelligence (AAAI), and the recipient of the 2002 ACM/SIGART Agents Research Award.

Prof. Sycara has given numerous invited talks, has authored more than 350 technical papers dealing with Multiagent Systems, Negotiation, Software Agents, Agent Teams, Web Services, the Semantic Web, and Human–Agent–Robot Teams. She has led multimillion dollar research effort funded by DARPA, NASA, AFOSR, ONR, AFRL, NSF and industry. She is a founding member and member of the Board of Directors of the International Foundation of Multiagent Systems (IFMAS). She is a founding member of the Semantic Web Science Association, and serves as the US co-chair of the Semantic Web Services Initiative. She is a founding Editor-in-Chief of the journal “Autonomous Agents and Multiagent Systems” and is currently serving on the editorial board of 5 additional journals.

Alan Scheller-Wolf is a professor in Operations Management at the Graduate School of Industrial Administration of CMU. He did his doctoral studies in Operations Research at Columbia University, where he earned M.S., M. Phil., and Ph.D. degrees. Dr. Scheller-Wolf's research focuses on stochastic processes, and how they can be used to estimate and improve the performance of manufacturing and service systems. In addition to this, Dr. Scheller-Wolf is actively pursuing research on problems dealing with inventory systems and supply chains. His work has appeared in *Operations Research*, *Management Science*, *Queueing Systems*, and *EJOR*, as well as at Proceedings of *ICDCS*, *SPAA* and *Sigmetrics.* Professor Scheller-Wolf has or is currently working on operations consulting projects Caterpillar, the American Red Cross, John Deere, Intel, and Air Products International, and serves on the editorial boards of *Operation Research, Management Science*, *M&SOM*, *MMOR Letters*, *and SORMS.*