Delft University of Technology

Quantitative Evaluation of Embedded Systems

IN4390

Lab 2 (Mandatory Part)

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1 Global Clarification and Acknowledge

1.1 Some Statements

Several global configuration that are used while dealing with these 4 questions should be clarified:

1. Question 6 and 8 are implemented in the same computer, while Question 5 and 7 are implemented in another computer. Question6's remote communication is done with Group50.

2. Same as previous experiment, during each test, abnormal data can be found in the beginning or in the middle. Reasons for unexpected value in the beginning may be that the build up of communication need time during the early stage of communication, it can be partly illustrated by "cold start" phenomenon. Reasons for unexpected value in the middle may be that some turbulence(e.g. some unrelated programs or tasks) occurred during the experiment. Method to eliminate the influence of these unexpected value can be discarding or substituting them with other data. Details will be mentioned in each chapter.

1.2 Acknowledgements

We would like to express our appreciation to all those who provided us the possibility to complete this experiment. They are Tianrui Mao and Yuan Fu from Group 50. Together with them, we completed the experiment 6, discussed the results, and solved some problems.

2 Question 1

For Question 1, we will first find out a sequence of transitions that lead to a marking that causes the deadlock. Then, we will point out one of the smallest modifications to avoid the deadlock. Finally, the mathematical proof will be given to show the correctness of our modification.

2.1 Deadlock Marking

The Petri net model shown in Fig.1 in Document does have a deadlock. A sequence that will lead to this deadlock is:

$$\begin{aligned}&\text{register order} \cdot \text{check availability} \cdot \text{in stock} \cdot \text{deliver} \cdot \text{send bill}\\ &\cdot \text{send payment reminder} \cdot \text{send bill} \cdot \text{receive payment}\end{aligned} \tag{1}$$

After this sequence of transition, place p8 will have the only and last one token in this system, then the marking $(0, 0, 0, 0, 0, 0, 0, 0, 1, 0)$ will cause the deadlock.

2.2 modification

This Petri Net describes a buyer-seller model. Above deadlock marking is due to the token in p7 is consumed by transition *send payment reminder*. Then transition *archive* is disabled because of the lacking of a token in p7. So our modification is to add an arc from transition *send payment reminder* to place p7 as shown in Figure 1. Then the transition "send payment reminder" will not cause the reduction of the token in p7. The mathematical proof of this modification is in the next subsection.

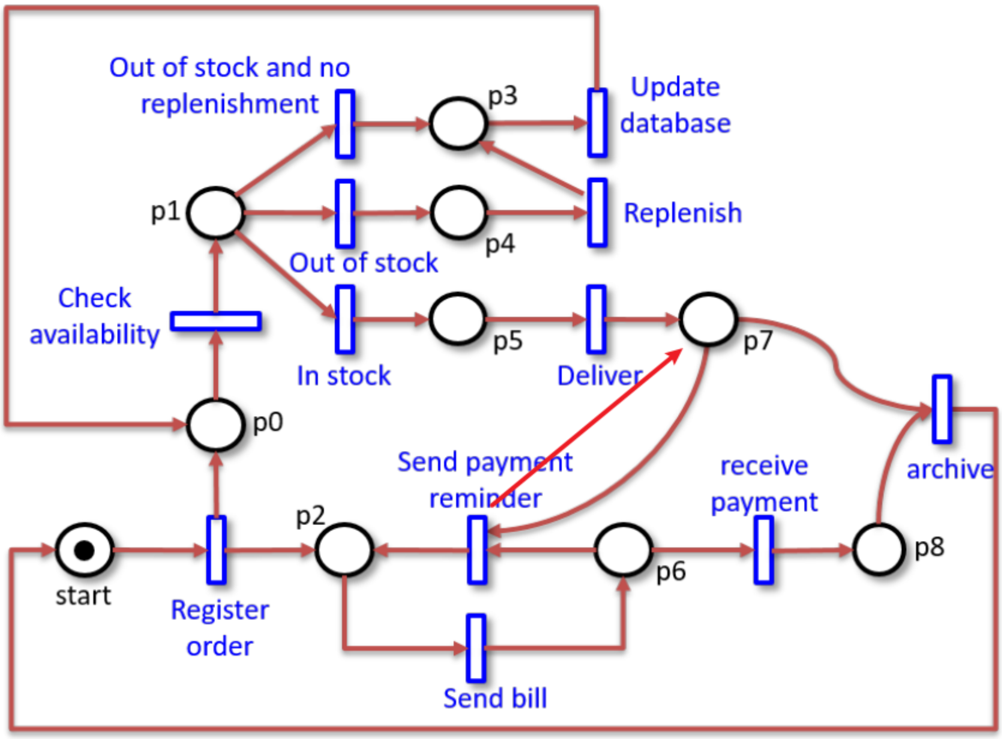


Figure 1: Q1 modification

2.3 Mathematical Proof

There are two markings that will cause a deadlock in this Petri net.

1. the only place that has token is p7
2. the only place that has token is p8

When one of these two conditions happens, transitions start from p7 and p8 are disabled because these transitions need tokens not only from p7 or p8, but also from another place. Meanwhile, all other places have possible transitions that need only one token. So if another place has at least one token, the marking will not be a deadlock.

Then, we will prove the deadlock situation 1 is not reachable. The index matrix of this Petri net, the initial marking, and the possible deadlock marking is shown in equations 3, equations 2. The lines in index matrix presents transactions in a certain sequence: "Register order","Send bill","Send payment reminder","Receive payment","archive","Check availability","In stock","Deliver","Out of stock","Replenish","Out of stock and no replenishment", "Update database"

$$
\begin{aligned}
marking &\triangleq [p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, start, p_0] \\
m_0^T &= [0,0,0,0,0,0,0,0,1,0] \\
m_{d7}^T &= [0,0,0,0,0,0,1,0,0,0] \\
m_{d8}^T &= [0,0,0,0,0,0,0,1,0,0]
\end{aligned} \tag{2}
$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

And the necessary condition that a marking is reachable is that, if a marking is reachable, the equation 4 will have at least one available non-negative solution.

$$\Delta m = A^T x \tag{4}$$

So if the solution of the equation does not exist, then a marking is not reachable. To judge whether the solution of equation 4 exist, we use MATLAB to calculate rank of matrix of each situation.

$$\begin{aligned} &\mathrm{rank}(A^T) = 9; \\ &\mathrm{rank}(\big[A^T|(m_{d7}^T - m_0^T)\big]) = 10; \\ &\mathrm{rank}(\big[A^T|(m_{d8}^T - m_0^T)\big]) = 9 \end{aligned} \tag{5}$$

The above result shows the deadlock marking in which only p7 has a token cannot be reached. The result is not sufficient to say the deadlock marking in which only p8 has a token can be reached, but the sequences we figured out before is a sufficient reason.

After the modification, the index matrix is changed, the new index matrix is shown in 6, and the new rank of each part is shown in 7

$$A' = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{6}$$

$$\begin{aligned} &\mathrm{rank}((A')^T) = 9; \\ &\mathrm{rank}(\big[(A')^T|(m_{d7}^T - m_0^T)\big]) = 10; \\ &\mathrm{rank}(\big[(A')^T|(m_{d8}^T - m_0^T)\big]) = 10 \end{aligned} \tag{7}$$

The result shows that after the modification, the previous deadlock marking is not reachable, which means the modification is correct. Because we only add one arc in the system, so it is one of the smallest solutions to solve the deadlock problem.

3 Question 2

2

3.1 Draw the Coverability Tree

In this part, a coverability graph of this given question should be presented. The figure is shown as follow.

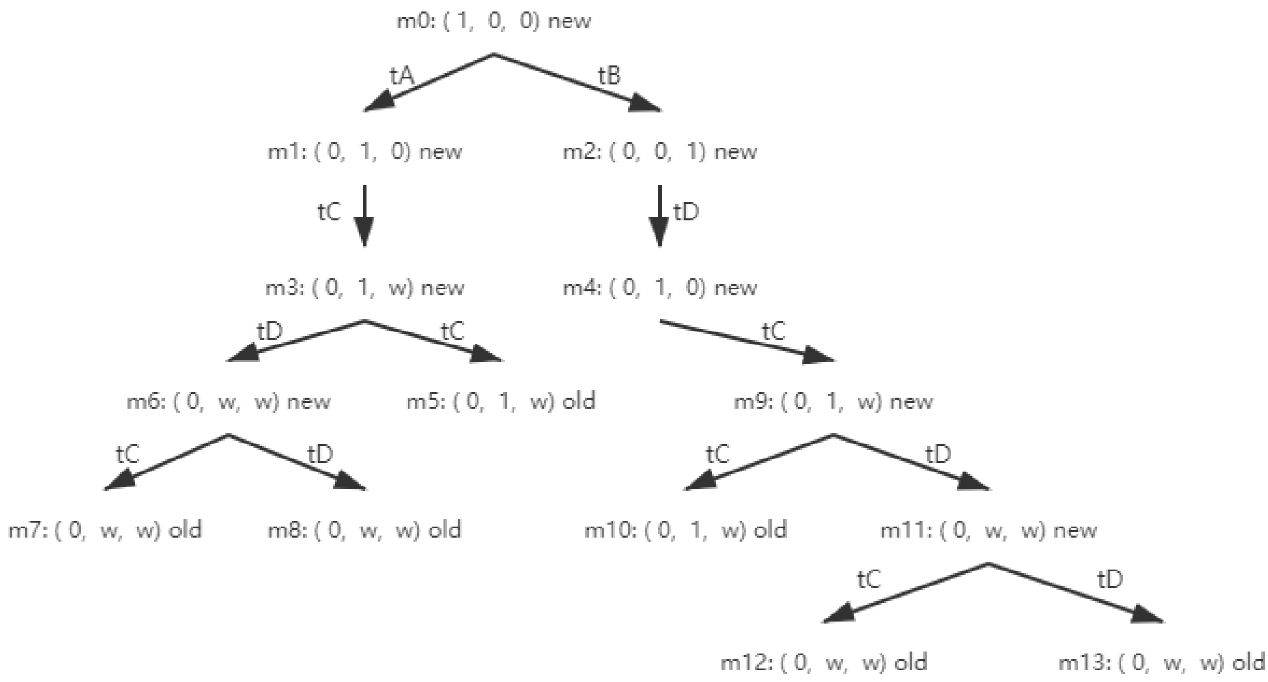


Figure 2: converbility tree for Question 2

3.2 Answer for Whether Holding Property 1 or Not

In this question, we need to find a counterexample to this proposition to prove that this property does not hold for this Petri net.

Using an incidence matrix is a nice way to find a combination of transitions that may lead to the counterexample. Because this proposition is a necessary, but not sufficient condition for reachability, the combination of transitions may violate the timing of transitions. As a result, a certain firing sequence should be found according to the combination we obtained.

The incidence matrix is defined here:

$$A = \begin{pmatrix} -1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & -1 \end{pmatrix}$$

The initial marking m_0 and a possible counterexample m_1 are:

$$m_0 = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}^T$$

$$m_1 = \begin{pmatrix} 0 & 2 & 1 \end{pmatrix}^T$$

Solving the equation, two solutions can be obtained:

$$x_1 = \begin{pmatrix} 1 & 0 & 2 & 1 \end{pmatrix}^T$$

$$x_2 = \begin{pmatrix} 0 & 1 & 2 & 2 \end{pmatrix}^T$$

According to this solution, a specific counterexample can be $t_A.t_C.t_C.t_D$, therefore, property 1 doesn't hold for this Petri net.

3.3 Answer for Whether Holding Property 2 or Not

The conclusion is that Property 2 doesn't hold for this Petri net. As displayed in the Figure2, only when t_D never been fired, token in P_2 will remain at 1. So, if Property 2 is true, the firing sequence should exclude t_D.

When a firing sequence exclude t_D, this sequence only consists of t_C. But in that case, token in P_3 will increase infinitely, therefore, marking $(0 \quad 1 \quad 1000)$ will appear once, which is opposite to statement in Property 2.
Besides, it can also be provide way. if we set:

$$m_0 = \begin{pmatrix} 0 & 1 & 1000 \end{pmatrix}^T$$

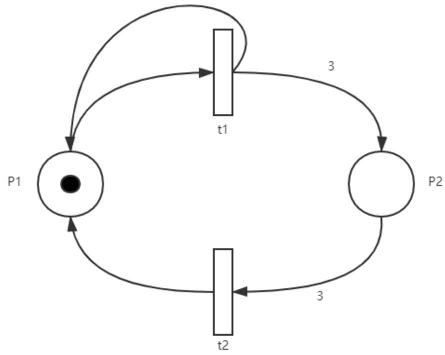
$$m_1 = \begin{pmatrix} 0 & 1 & 1000 \end{pmatrix}^T$$

then the only solution for $Ax = m_1 - m_0$ is the zero solution, that means from $(0 \quad 1 \quad 1000)$, there is no other way back to this marking anymore.

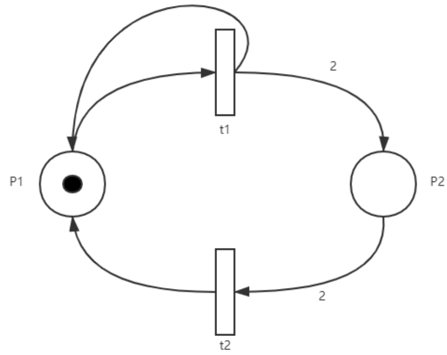
4 Question 3

4.1 Two Examples

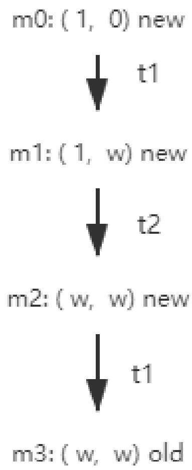
The difference may appear at the w representation, we explored some possible marking around marking with w, the result is displayed as follow:



(a) Example 1 with weigh 3



(b) Example 2 with weigh 2



(c) Coverability tree of these two examples

Figure 3: Example and coverability tree

4.2 Justification for these Two Examples

These two Petri nets have the same coverability graph, as shown in Figure 3, but have many different markings, (e.g. $(1 \quad 3)$ and $(1 \quad 2)$) . The phenomenon relates to the property of coverability tree, coverability method uses w, to assist us to represent the (infinite) increment of some tokens. The price of using this representation is we cannot know exactly every part of firing sequence, because $(1 \quad 3)$ and $(1 \quad 2)$ can all be expressed as $(1 \quad w)$. Due to this property, two Petri net can have the same coverability graph, but have difference in some specific marking.