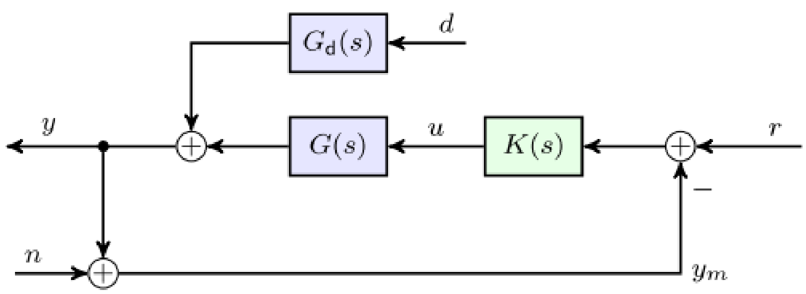
01_Closed_Loop_Feedback_Control

1. Basic Feedback Model



- d: disturbance
- r: reference
- n: noise

Objectives:

- closed-loop stability
- reference tracking
- disturbance rejection: d influence as small as possible
- noise response: as small as possible

Difficulties:

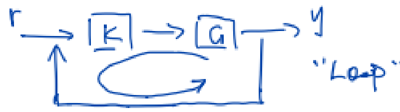
- model errors
- fundamental **limits on controllability** of $G(s)$: not only “yes” or “no”, but also **“hard” or “easy”**

- actuation constraints

Transfer Functions

- **Loop Transfer Function**

$$L(s) = G(s)K(s)$$



- **Sensitivity Function**

$$S(s) = \frac{1}{1 + G(s)K(s)} = \frac{1}{1 + L(s)}$$

  - **Sensitivity** is a criteria to describe how much the closed-loop system will change if the plant changes.

$$S(s) = \frac{\text{relative closed-loop response change}}{\text{relative open-loop response change}} = \frac{dT(s)/T(s)}{dG(s)/G(s)}$$

  - A change of α **percent in the open-loop plant** DC gain gives a magnitude **change of** $|\alpha S(0)|$ **percent in the closed-loop** DC gain.

- **Complementary Sensitivity Function**

$$T(s) = 1 - S(s) = \frac{G(s)K(s)}{1 + G(s)K(s)} = \frac{L(s)}{1 + L(s)}$$

Constraints

$$S(s) + T(s) = 1$$

System Response

- **Output Response**

$$y = T(s)r + S(s)G_d(s)d - T(s)n$$

- **Error Response**

$$e = r - y = S(s)r - S(s)G_d(s)d + T(s)n$$

Objectives

- Performance Tracking

$$T(s) \approx 1$$

- Noise Rejection

$$T(s) << 1$$

- Disturbance Rejection

$$S(s)G_d(s) << 1$$

- Low closed-loop plant sensitivity

$$S(s) << 1$$

These objectives are somehow conflicting because the constraints between T and S

Closed-Loop Performance

Closed-Loop Bandwidth ω_B

Frequency at which $|S(jw)| = -3dB = 1/\sqrt{2}$

Crossover Frequency ω_c

Frequency at which $L(jw)$ = 1

Maximum Control Frequency

Frequency where $|K(jw)|$ is still significant

Maximum Peak Criteria:

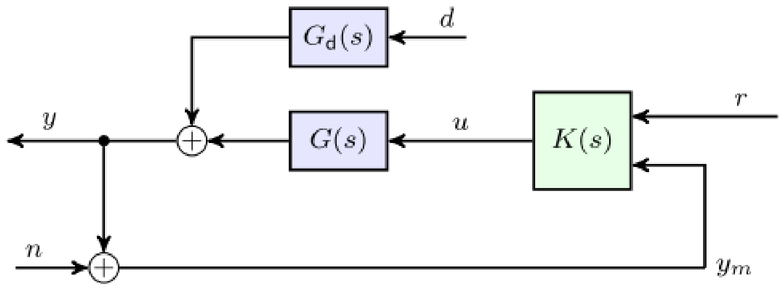
$$\begin{aligned} M_S &= \max_\omega |S(j\omega)| = \|S(s)\|_{\mathcal{H}_\infty} \\ \text{and} \quad M_T &= \max_\omega |T(j\omega)| = \|T(s)\|_{\mathcal{H}_\infty} \end{aligned}$$

Property:

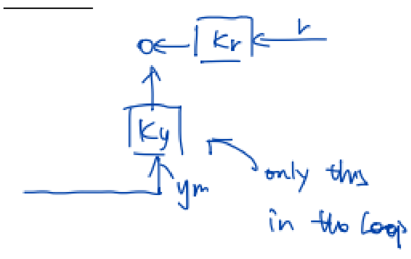
- If $\mathrm{PM} < 90^\circ$ then, $\omega_B < \omega_C < \omega_{BT}$
- $\mathrm{GM} \geq \frac{M_S}{M_S - 1}, \quad \mathrm{PM} \geq 2 \arcsin \left(\frac{1}{2M_S}\right) \geq \frac{1}{M_S} (\mathrm{rad})$
    - $M_s \rightarrow 1, GM$ larger
- Typical Specification:
    - $M_S \leq 2 \quad \text{and} \quad M_T < 1.25$

2.Alternative Structures

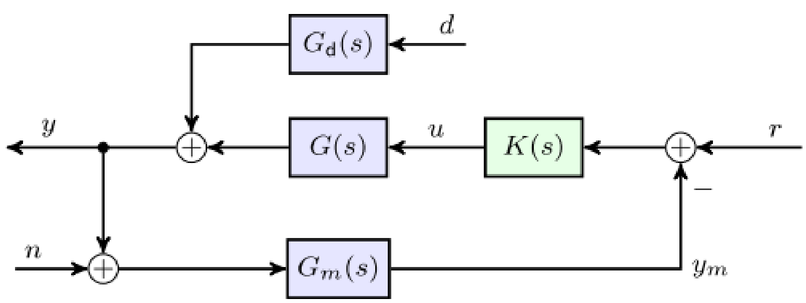
Two degrees-of-freedom structure



$$
\begin{aligned}
u &= K(s)\begin{bmatrix} r \\ y_m \end{bmatrix} = [K_r(s) \quad K_y(s)]\begin{bmatrix} r \\ y_m \end{bmatrix} \\
L(s) &= -G(s)K_y(s) \\
y &= S(s)G_d(s)d + S(s)G(s)K_r(s)r + S(s)G(s)K_y(s)n
\end{aligned}
$$

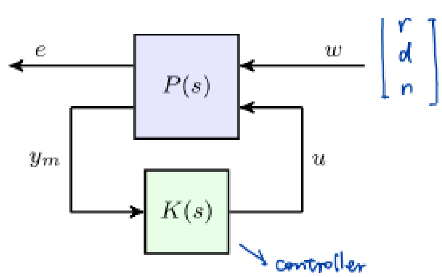


Additional Loop Dynamics



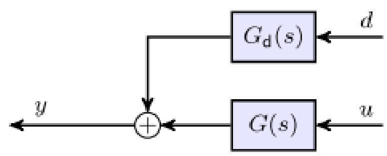
$$
\begin{aligned}
L(s) &= G(s)K(s)G_m(s) \\
y &= S(s)G_d(s)d + S(s)G(s)K(s)r - T(s)n
\end{aligned}
$$

A general Structure

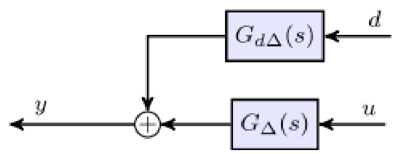


3. Perturbed System

Nominal case:



Perturbed case:



$$G_\Delta(s) = \{G(s) + \Delta(s) \mid \Delta(s) \in \text{ Set }\}$$
$$G_{d\Delta}(s) = \{G_d(s) + \Delta_d(s) \mid \Delta_d(s) \in \text{ Set }\}$$

Sources of Uncertainty

- Nonlinear dynamics
- Operating point variation
- Neglected dynamics in the model
- Non-repeatable dynamics

Robust Objectives

Nominal Stability (NS)

Closed-loop system **stable** with **no model uncertainty.**

Nominal Performance(NP)

Closed-loop system satisfies **the performance requirements** with n**o model uncertainty**

Robust Stability (RS)

Closed-loop system is **stable** for **all models** in a **prescribed set.**

Robust Performance(RP)

Closed-loop system satisfies **the performance requirements** for all models in a **prescribed set.**

Control Design

There are three main approaches:

- **Loop Shaping**:

  Design $K(s)$ so that the loop, $L(s)$, has the required properties (classical approach).

- **Signal-based optimal control**

  Design $K(s)$ to satisfy certain closed-loop system or signal objectives.

- **Numerical optimisation-based**

  Use **multi-objective optimisation** with closed-loop and robustness objectives

Summary

- Basic Feedback Model
  - **Sensitivity and Complementary Sensitivity**
  - Closed-Loop Performance
- Alternative Structure
  - Two DoF Structure
  - A general structure
- Perturbed Systems
  - Robust Objectives