02_Modeling of RTS & Schedule Introduction

1. Basic Modeling Concepts and Definitions

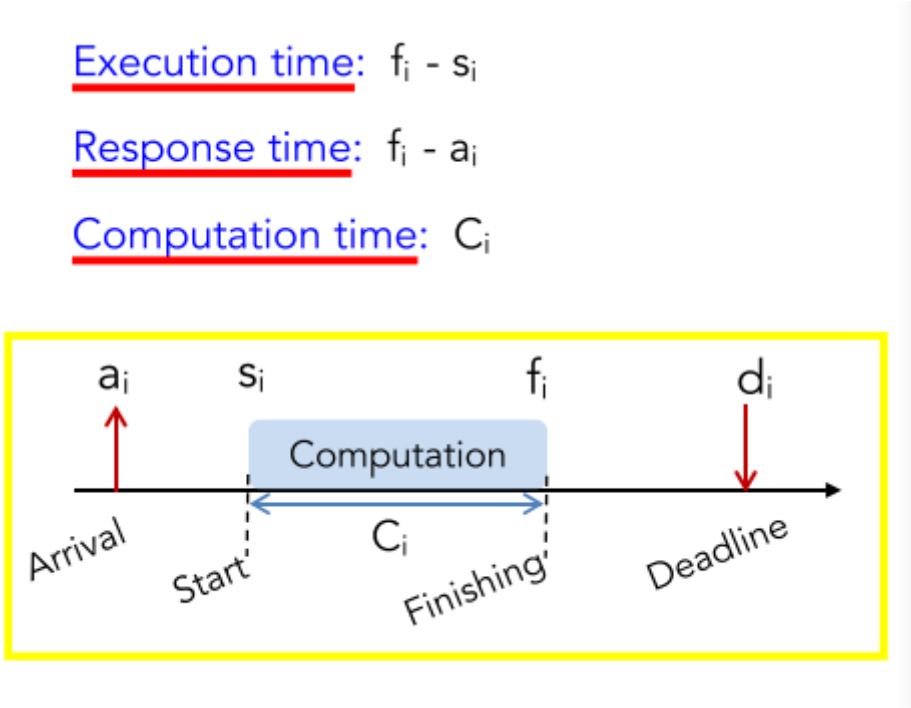
1.1. Tasks and Jobs

Task

- **sequence of instructions** that, in the absence of other activities, is **continuously executed by the processor until completion**.
- a.k.a. “process” or “thread”

Always:

- **Upper-capital** for relative
- **Lower-capital** for absolute



Jobs

obs are **“instances”** of Tasks, i.e., a Task **“releases”** Jobs.

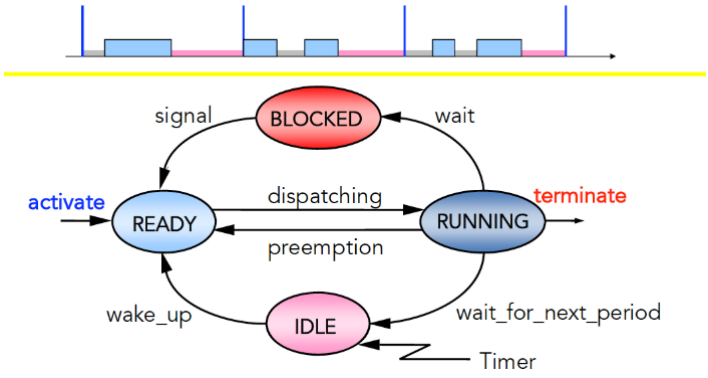
Possible Task States

Blocked: always means waiting for resources

Running: after being dispatched

Ready: Exist job that can be executed, just not his turn

IDLE: after running, wait for job release



Real-Time Task

characterized by a timing constraint on its response time, called deadline

Feasible & Schedulable

Single Task Feasible

an RT task τ_i is called **feasible**, if it completes before its absolute deadline, i.e., if:

$$f_i \leq d_i \text{ or, equivalently } R_i \leq D_i$$

A Schedule Feasible

a schedule is **feasible** if all tasks can be completed while satisfying certain predetermined constraints

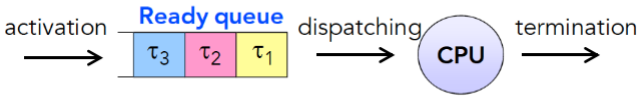
Task Set Schedulable

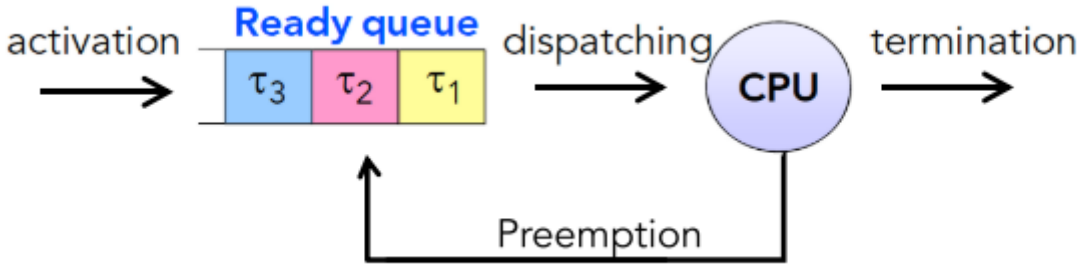
a set of tasks is **schedulable** if there **exists an algorithm** that produces a feasible schedule.

1.2. Ready Queue & Scheduling Policy

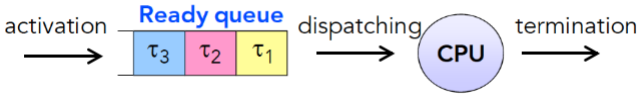
In a concurrent system with one server (processor) :

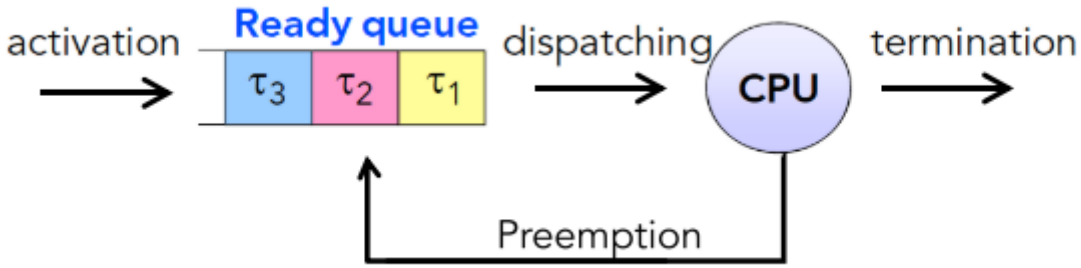
- Several tasks can be simultaneously active; but
- only one can be in execution (running).

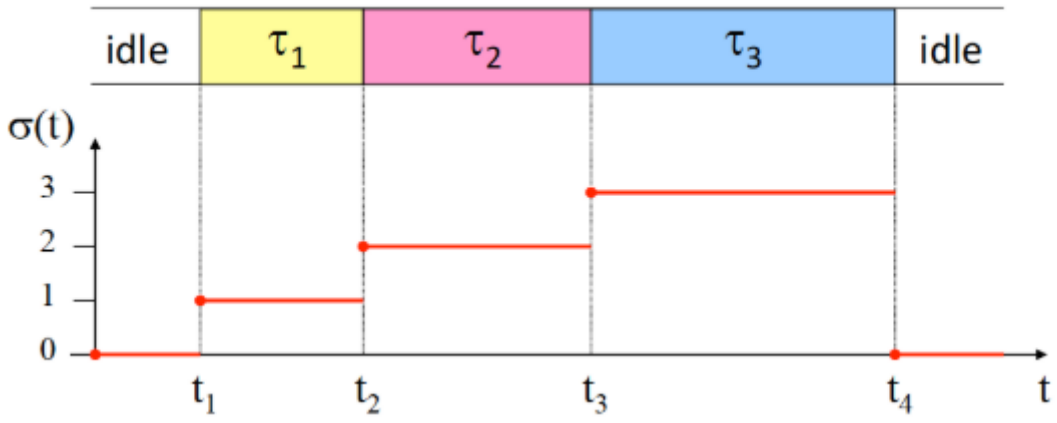




1.3. Preemption







Preemption is a **Kernel mechanism**, that enables us to:

- Suspend the execution of the running task;
- Place it back in the ready queue;
- And execute instead another more important task.

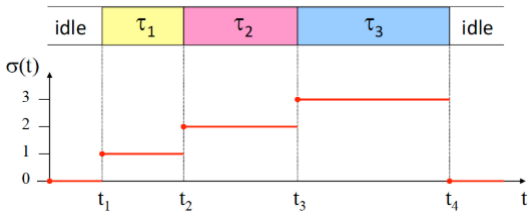
1.4. Schedule

given the tasks $\Gamma = \tau_1, \ldots, \tau_n$, a schedule is a function:

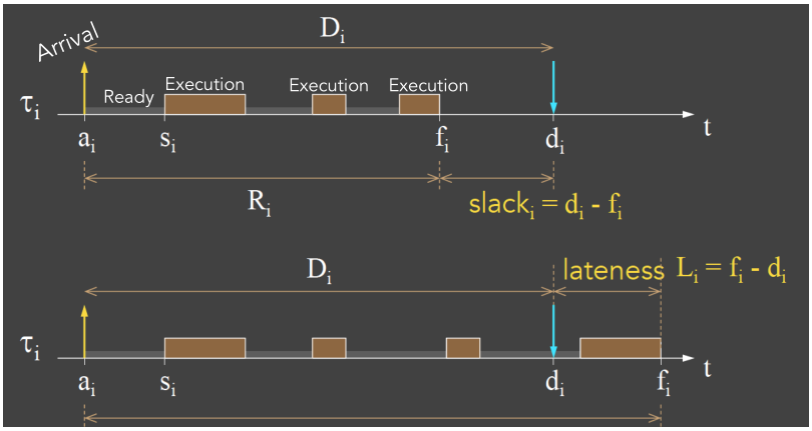
$$\sigma : R^+ \mapsto k \in N$$

which associates an integer k to each time slice such that:

- if k=0, then CPU is idle in that time slice (or, slot);
- else, CPU executes task τk in that time slice



1.5. Slack and Lateness



Lateness: $f_i - d_i$

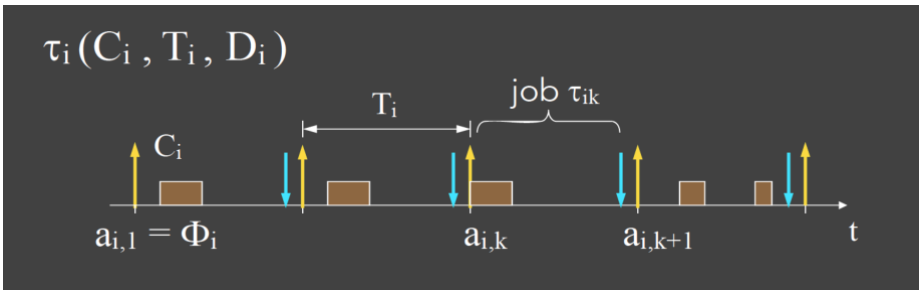
Tardiness(Exceeding time): $max(0, f_i - d_i)$

Laxity(Slack Time): $(d_i - a_i) - C_i$

2. Types of Tasks

2.1. Time Driven: Periodic

Tasks that are automatically activated (e.g., by the operating system) at predefined regular time instants.



$$a_{i,k} = \Phi_i + (k-1)T_i$$
$$d_{i,k} = a_{i,k} + D_i$$

Single-Rate

all tasks in the system have **the same period**

Multi-Rate

tasks have **different periods**

Phase

the release time of its first instance

Utilization Factor

$$U_i = \frac{C_i}{T_i}$$

2.2. Event Driven: Aperiodic

Tasks that are activated due to an external event, an interruption, or as result of the output of another Task, e.g., by an explicit system call.

Aperiodic Tasks

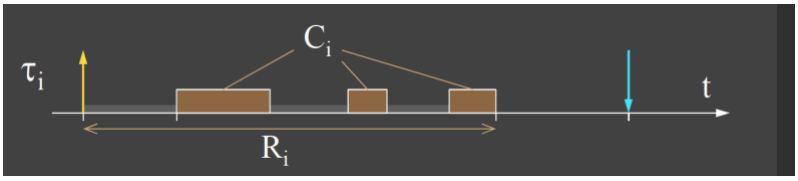
Unbounded arrival time, or event-based triggering

Sporadic Tasks

Arrival times vary, but there is a **minimum gap (inter-arrival time)**

3. Estimatin the Computation Time

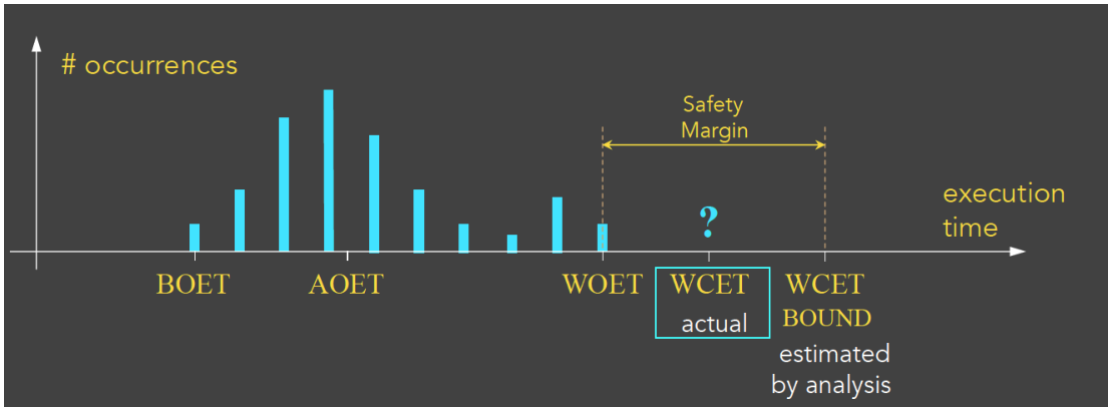
Computation Time



Computation time (or, Execution time) is the time the CPU needs to execute the task, **without considering any possible suspension time intervals.**

3.1. By Measurement

run the task with different input data, analyze the statistics.



BOET: best observed execution time

AOET: average observed execution time

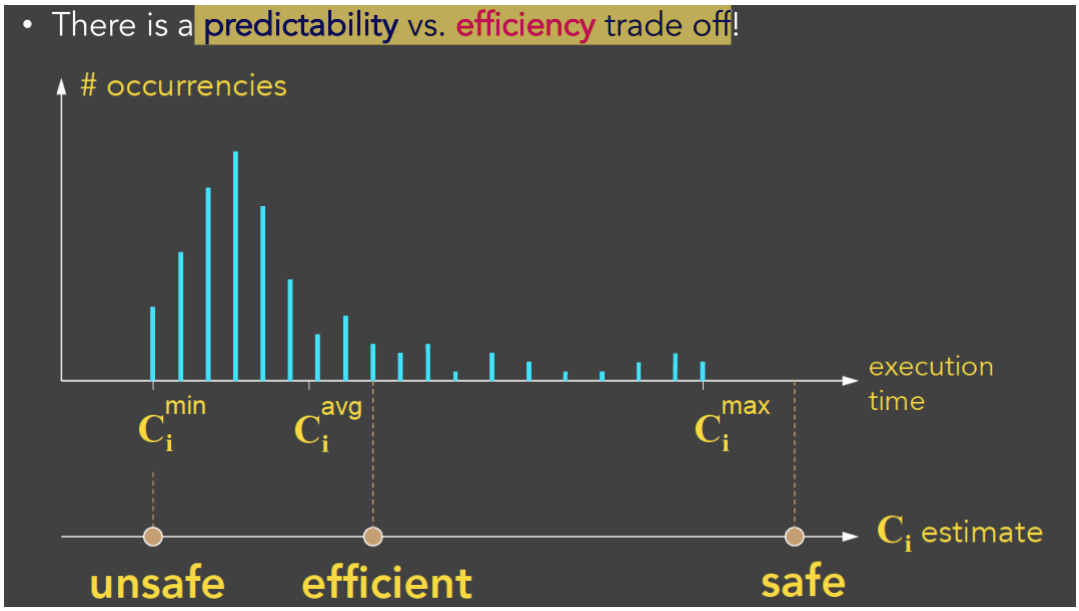
WOET: wors observed execution time

WCET: worst-case execution time

3.2. Analytical Methods

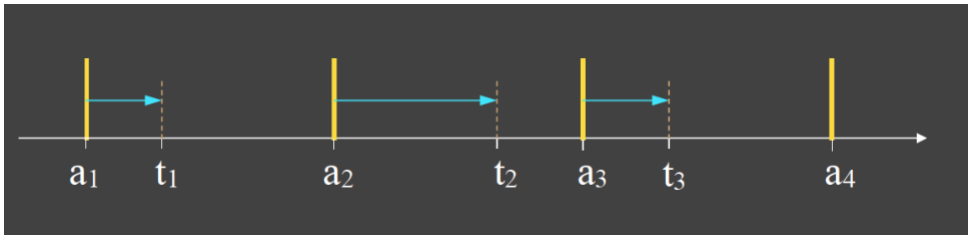
model the system and explicitly calculate the times.

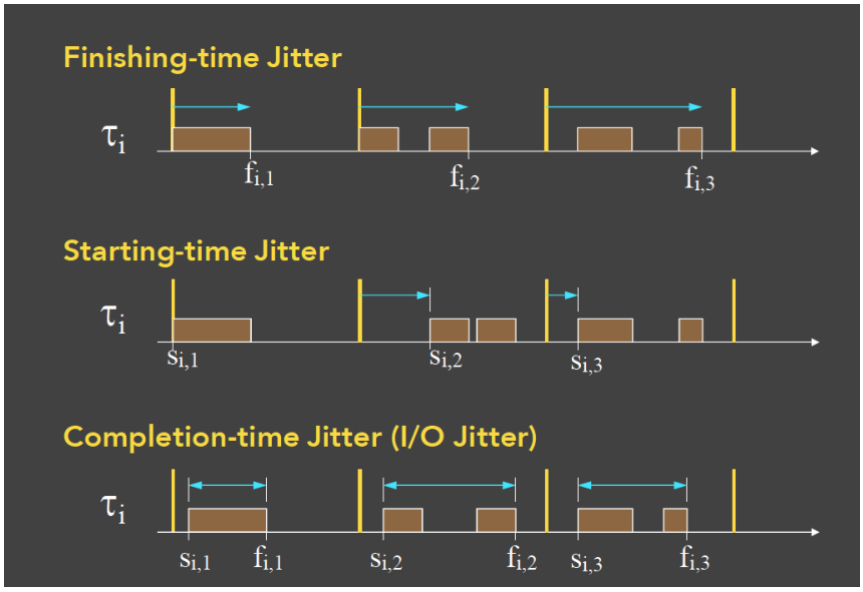
- **upper-bound** the loop cycles
- **compute** the longest path
- **upper-bound** the cache misses
- **Upper-bound** the I/O interruptions
- **Compute** the execution time for each instruction



3.3. Jitter

Measures the **time variation** of a **periodic event**





Absolute Jitter

$$\max_k \{t_k - a_k\} - \min_k \{t_k - a_k\}$$

Relative Jitter

$$\max_k |(t_k - a_k) - (t_{k-1} - a_{k-1})|$$

4. Task Constraints

4.1. Timing Constraints

Explicit Timing Constraints

included in the specs.

Implicit Timing Constraints

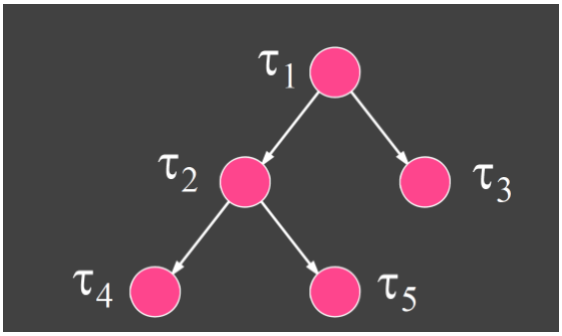
Not included in specs

Example 1

4.2. Precedence Constraints

tasks need to be executed with a certain order

Directed Acyclic Graph



Predecessor

$$\tau_1 \prec \tau_4$$

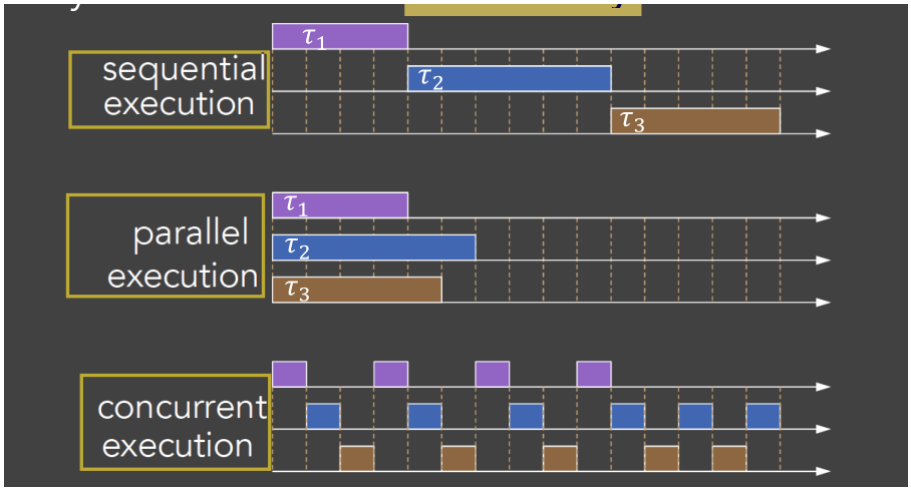
Immediate Predecessor

$$\tau_1 \rightarrow \tau_2$$

4.3. Resource Constraints

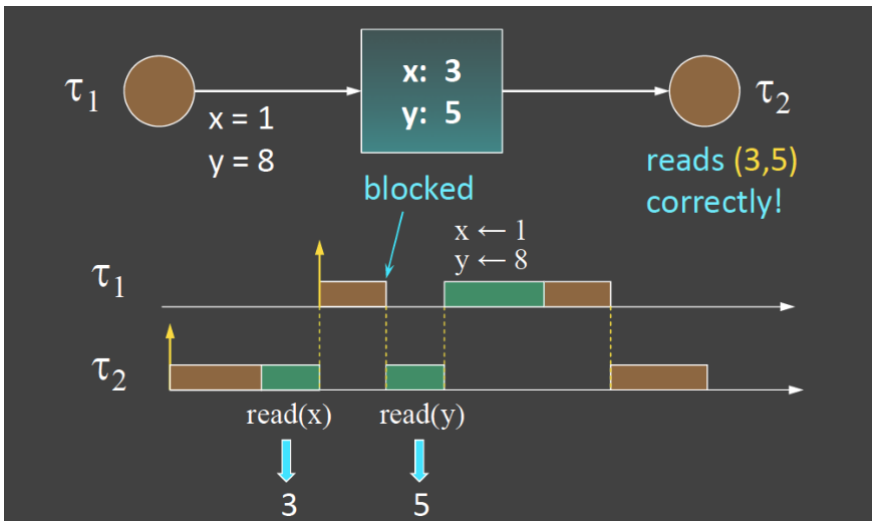
Concurrency

Sequential Execution, Parallel execution, Concurrent Execution



Mutual Exclusion

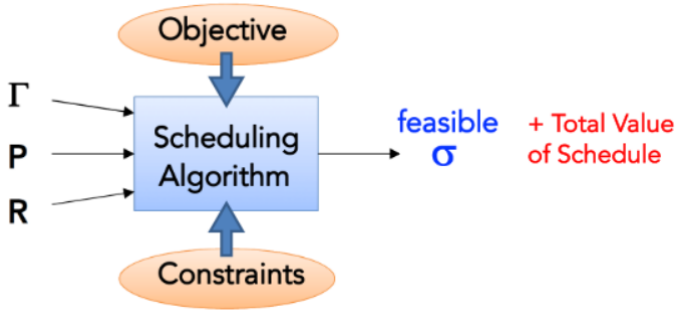
Inconsistent access of shared memory, we can use mutual exclusion to solve this problem



5. Introduction of Scheduling Problem

5.1. Definition of of Scheduling Problem

Given a **set Γ of n tasks**, a **set P of p processors**, and a **set R of r resources**, find an **assignment** of P and R to Γ that produces a feasible schedule under a set of constraints



5.2. Feasibility and Schedulable

Feasibility of a Schedule

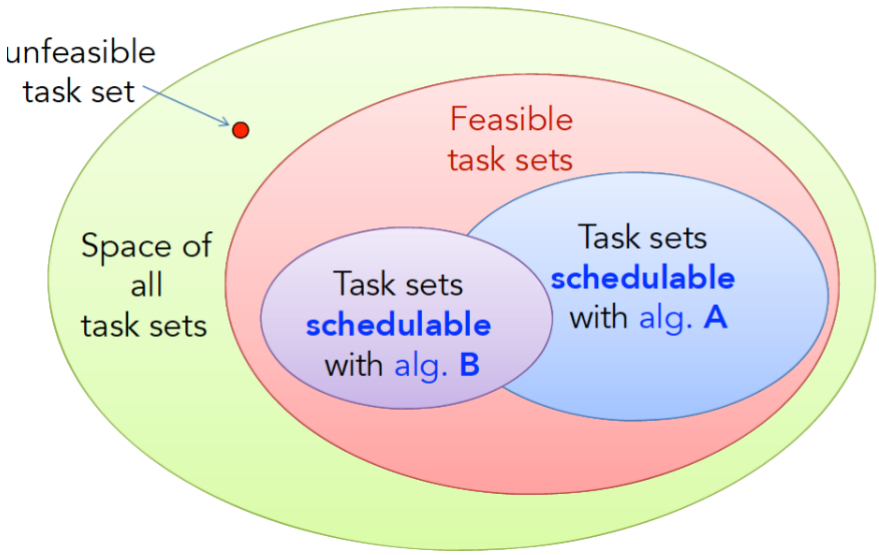
A schedule σ is **feasible** it if satisfies a predetermined set of constraints.

Feasibility of a set of tasks

A set Γ of tasks is said to be **feasible** if there exists an algorithm that generates a feasible schedule for these tasks.

Schedulable set of tasks

A set Γ of tasks is said to be **schedulable with an algorithm A** if A generates a feasible schedule.



5.3. Computational Complexity

Big-O Notation

$$f(x) = O(g(x)) \Rightarrow |f(x)| \leq Mg(x), \forall x \geq x_0$$

Computational Complexity

- Amount of time the algorithm needs to finish (and do its job).
- We are interested in the **worst-case execution** of large problem instances – hence, we use Big-O notation.

```
Algorithm A (int n) {
 int sum=0;
 int mat[100];
 for (int i=0; i<100; i++)
    sum+=mat[i];
 return sum;
}
```

$O(1)$

```
Algorithm B (mat[n]) {
 int sum=0;
 for (int i=0; i<n; i++)
    sum+=mat[i];
 return sum;
}
```

$O(n)$

```
Algorithm F (int n) {
 if n <= 1:
    return n;
 else
    return F(n-1)+F(n-2);
}
```

$O(2^n)$

```
Algorithm D (mat[n], W) {
 int sum=0;
 for (int i=0; i<n; i++)
    for (int j=0; j<W; i++)
      sum+=mat[n]+j;
 return sum;
}
```

$O(nW) = O(n \cdot 2^x)$

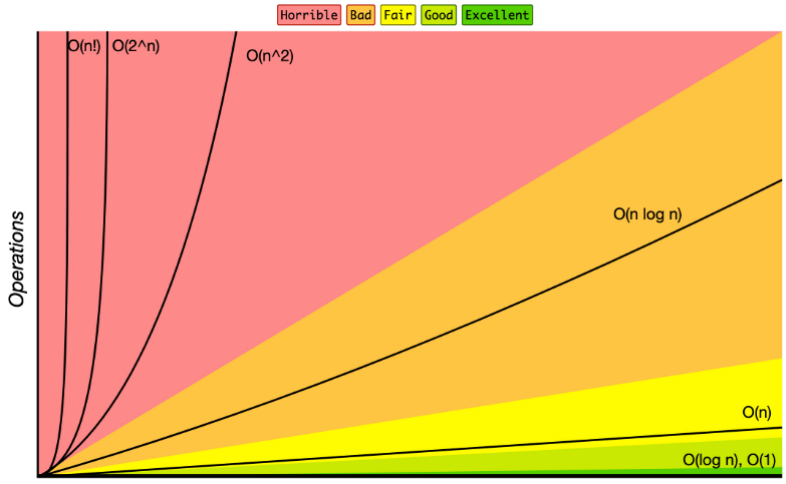
- the first situation, the complexity is independent of it input
- **the forth situation, we use the 2^x as part of the upperbound because actully, here the x is the input bit number of W, it is the largest number of computation, W can take in**

Polynomial vs Pseudo-polynomial

- **Polynomial** in the number of **bits** we need to represent/store the input

Using 2 bits → n=2, max iterations
Using 3 bits → n=4, max iterations
Using 4 bits → n=8, max iterations
Using 5 bits → n=16, max iterations

- **Pseudo-polynomial** if it is polynomial in the value of input (here: n) but not on number of bits



5.4. NP-hard Problems

NP problem

The complexity class NP can be defined in terms of NTIME as follows:

$$\mathrm{NP} = \bigcup_{k \in \mathbb{N}} \mathrm{NTIME}\left(n^k\right)$$

where is the set of decision problems that can be solved by a non-deterministic Turing machine in time $\mathcal{O}(n^k)$.

- NP is the set of decision problems for which the problem instances, where the answer is "yes", **have proofs verifiable in polynomial time by a deterministic Turing machine**.

NP-complete

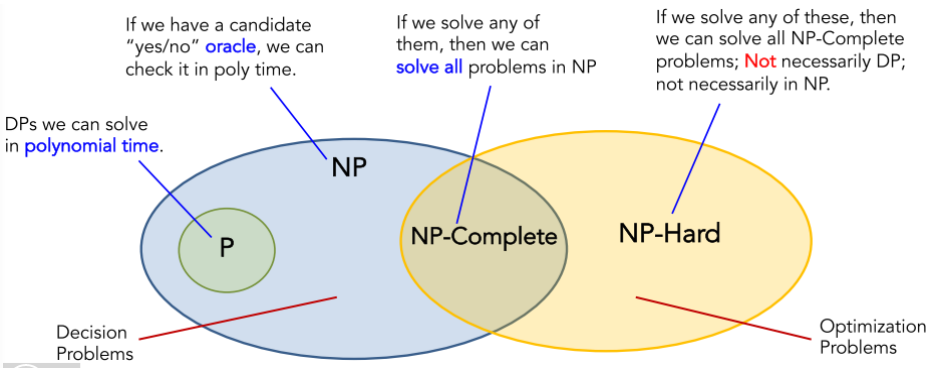
In computational complexity theory, a problem is **NP-complete** when:

1. A nondeterministic Turing machine can solve it in polynomial-time.
2. A deterministic Turing machine can solve it in large time complexity classes (e.g., EXPTIME, as is the case with brute force search algorithms) and can verify its solutions in polynomial time.
3. It can be used to simulate any other problem with similar solvability.

NP-Hard

In computational complexity theory, **NP-hardness** (non-deterministic polynomial-time hardness) is the defining property of a class of problems that are informally "**at least as hard as the hardest problems in NP**"

Relations



- The general scheduling problem is NP-Hard, because it is similiar to:
    - Knapsack problem
    - Travelling Salesman problem

6. Scheduling Algorithms

6.1. Taxonomy

Preemptive vs. Non-Preemptive

- Based on whether tasks can be **interrupted** or not.

Static vs. dynamic

- Scheduling is based on **fixed parameters**, assigned to tasks before their activation (like given priority); or on **dynamic parameters** that might change in runtime (deadline in a dynamic system).
- A randomized assignment policy can be static or dynamic
    - if total random, dynamic
    - if not, for example based on prior knolwedge, like "if C>5, 0.6 probability...", it based on static

- The key point is: the parameter is known before or during runtime:
    - For example, FCFS, the "first come" always is known when task arrive

Online vs. Offline

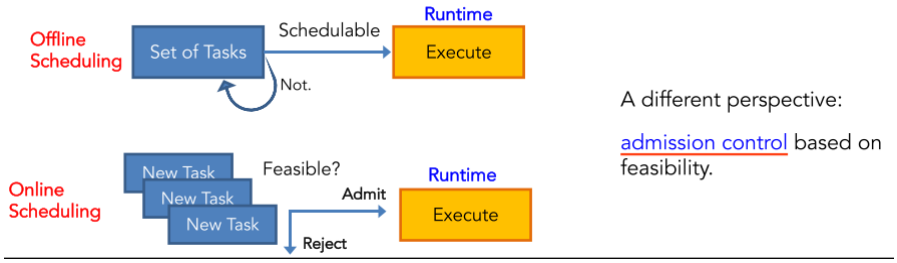
- The schedule is devised in the **beginning**; or **upon arrival** of each task.
- **The key point is: whether the "mapping" is established in the beginning**

Optimal vs. Heuristic

- Maximizes (or not) the value of the adopted performance criterion.

Guarantee-based vs Best-effort based

Need to decide, in advance, if a certain task set is feasible or not



6.2. Optimality Criteria

- Ensure Feasibility
    - Find a feasible schedule if there exists one.
- Minimize maximum lateness, or deadline misses
    - min-max formulation;
    - lexicographic optimization if there are different criteria
- Maximize the cumulative value of admitted tasks
    - Utility maximization approach; - can be used to achieve fairness, load-balancing, etc

7. Scheduling Anomalies

7.1. Graham Theorem

If a task set is optimally scheduled on a multiprocessor with:

- some priority assignment; a fixed number of processors; fixed execution times; and some precedence constraints,

Then:

- increasing the number of processors; reducing execution times; or weakening the precedence constraints

Can increase the schedule length!

Personal Understanding

We can divide the planner into 3 parts to explain Graham Theorem:

1. Optimizer:
    - If the optimizer try to maximum an objective function combining with several different criterions:

      It is easy to get Graham Theorem, beacuse the length will may not be the dominant factor
    - If the optimizer only try to optimize the schedule length, then there will definitely no deterioration
2. Fixed Strategy:
    - Totally can deteriorate

7.2. Generalized Schedule Problem

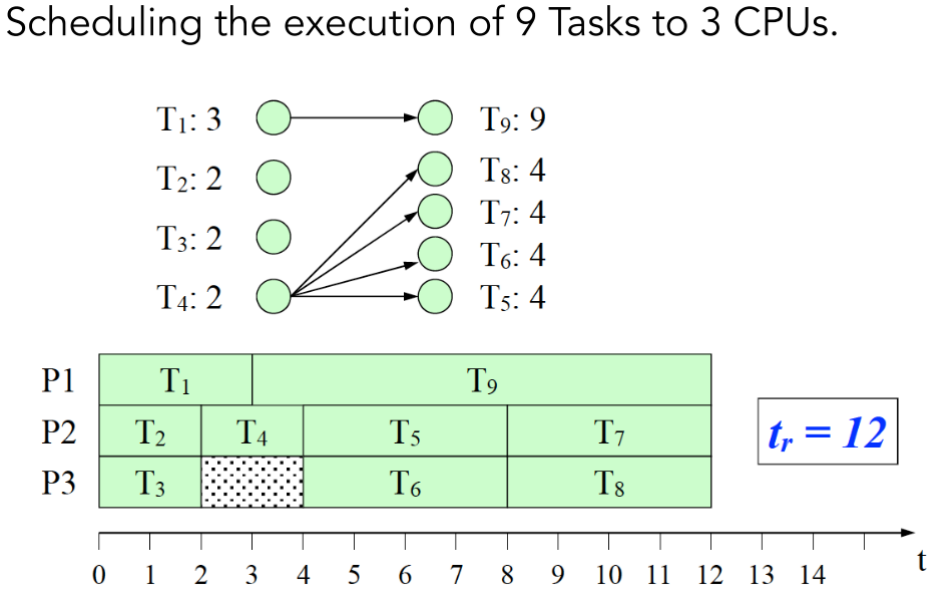
Maximize: Performance Criterion

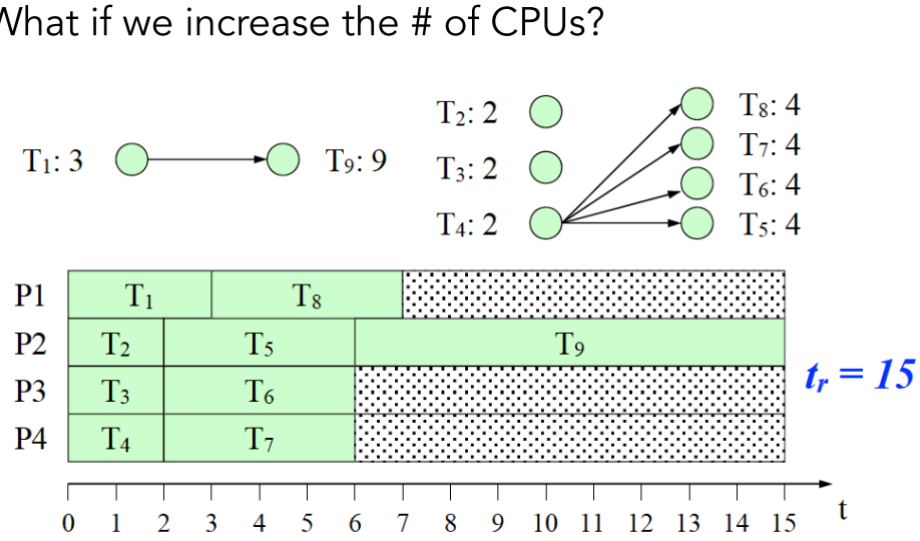
Subject to:

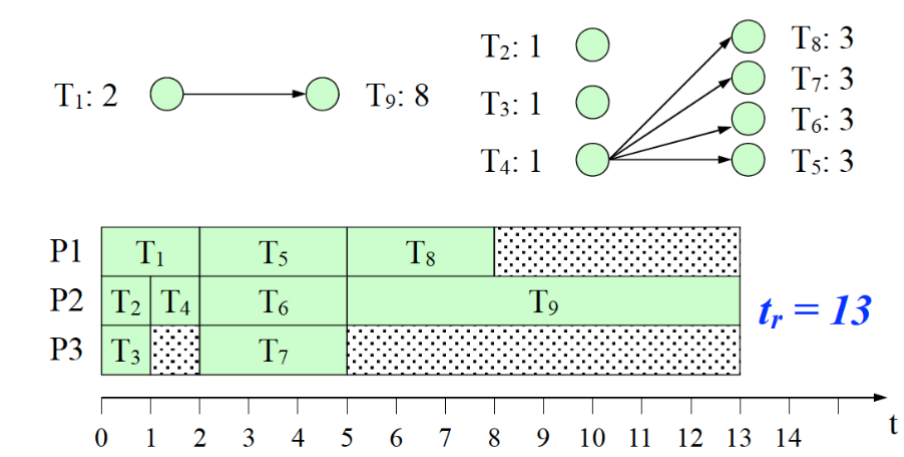
- Resource Constraints
- Timing Constraints
- Task Dependency Constraints

7.3. Anomalies: An example

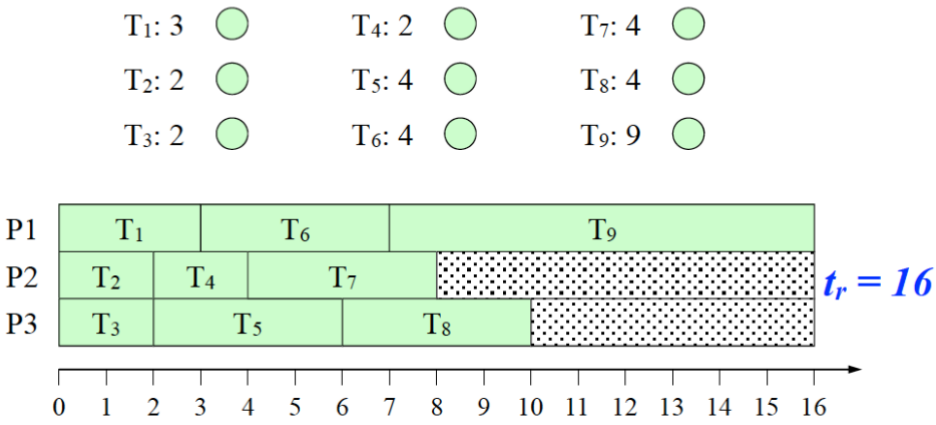
For example: without dependency, we always want to first deploy task with small number (highest priority)







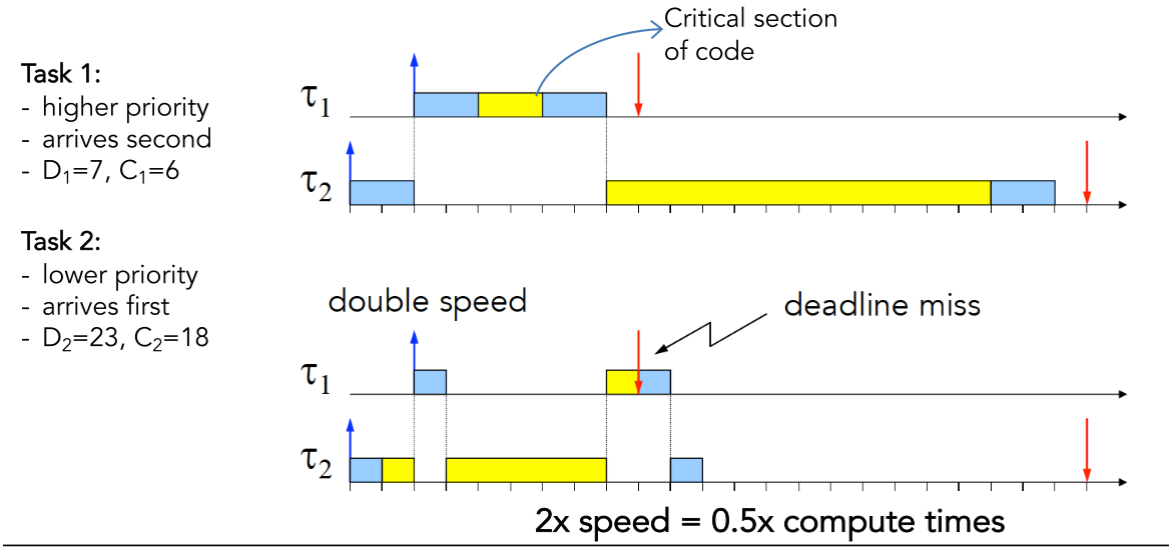
What if we remove Task dependencies?



7.4. Two important Anomalies Condition

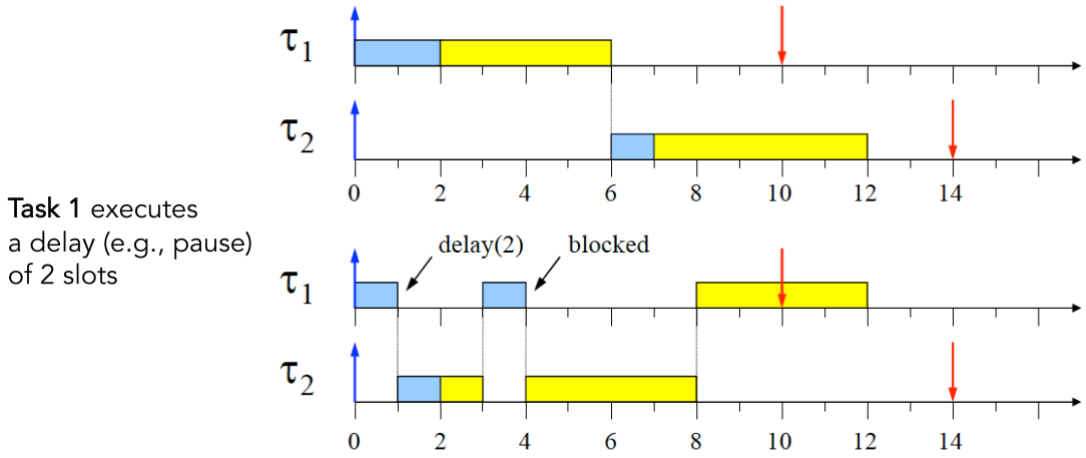
Faster Process

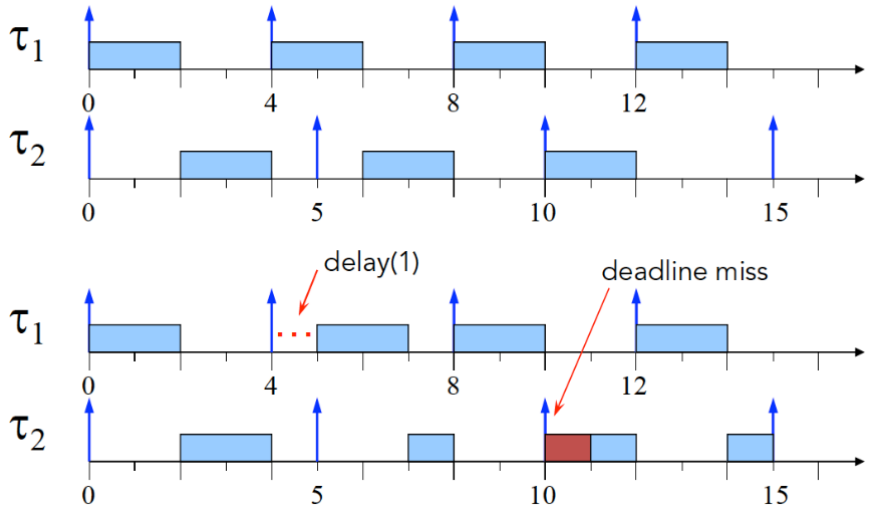
- Anomalies in uniprocessor systems.
- Doubling CPU speed induces deadline miss



Delay： Dangerous System Call

- Anomalies in uniprocessor systems.
- A delay(Δ) may cause a delay longer than Δ.
- A delay in a task may also increase the response of other tasks





7.5. How to achieve Predictability

- **Tests are not enough** for real-time systems
- Concurrency control must be enforced by:
    - appropriate scheduling algorithms;
    - appropriate synchronization protocols;
    - efficient communication mechanisms;
    - predictable interrupt handling

8. Fundamental Algorithms

In this Section we will introduce some scheduling policies **that do not work for Real-time Systems**

8.1. Graham's Notation

$$\alpha|\beta|\gamma$$

α : Number of Servers
β : Constraints on the Tasks
γ : Optimality Criterion

8.2. First Come First Served (FCFS)

Rule: serve tasks in the order of their arrival.

Property:

- very unpredictable result
- the response times depend on the arrivals

8.3. Shortest Job First

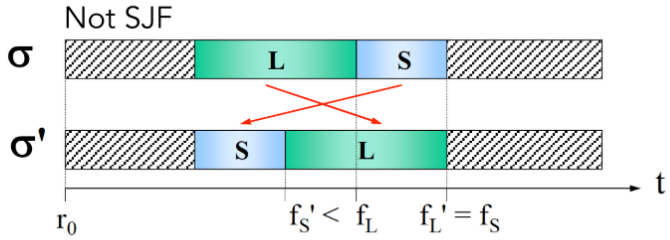
Rule:

serve the ready task with shortest compute time

Property:

- A **static policy**
- Can be **offline or online**
- Can be preemptive or non-preemptive
- minimize the average response time

- Proof of optimality:



$$f_S' + f_L' \leq f_S + f_L$$

The exchange has improved the response at least for one task

$$\overline{R}(\sigma') = \frac{1}{n}\sum_{i=1}^{n}(f'_i - r_i) \leq \frac{1}{n}\sum_{i=1}^{n}(f_i - r_i) = \overline{R}(\sigma)$$

Repeating the swaps:

$$\sigma \rightarrow \sigma' \rightarrow \sigma'' \rightarrow \ldots \rightarrow \sigma^*$$

$$\overline{R}(\sigma) \geq \overline{R}(\sigma') \geq \overline{R}(\sigma'') \ \ldots \ \geq \overline{R}(\sigma^*)$$

- Not good in achieving **feasibility**, because it **ignores key parameters such as deadline**

8.4. Priority Scheduling

Rules:

serve the task with the highest priority

- Each task is assigned a priority number P_i ;
- Apply FCFS for tasks with same priority;
- Preemptive; Online; Static or dynamic

Property:

- May have **Starvation**
    - Low priority tasks might stay in the queue forever.
    - Can be partially solved with aging counters (make it dynamic)

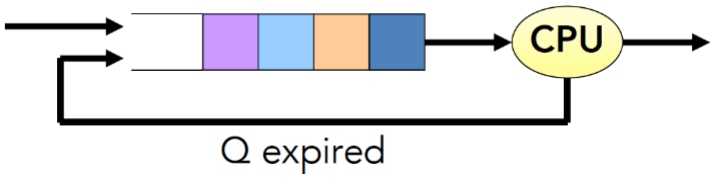
8.5. Round Robin

Rules:

FCFS + rotational assignment of time slices Q

- The ready queue is served with FCFS;
- but, each task is executed for up to Q slots;

- and then placed back in the beginning of queue



Property:

- If Q is larger than the computation time of any task, then we are **back to FCFS**
- the cost of **context swichting overhead**