Week 15 (B): Game Theory(part2: Game Theory)

1. Game Theory Domain

1.1. Type of Game

Zero-sum game

the payoffs in **each cell** of the payoff matrix **sum to zero**

Constant-sum game

the payoffs in **each cell** of the payoff matrix sum to **a constant c**

Coordination Game

game in which players need to **communicate**

1.2. Strategies

Pure Strategies

- A **pure strategy** provides a complete definition of how a player will play a game.
- Pure strategy can be thought about as **a plan subject to the observations** he makes during the course of the game of play.
- In particular, it determines the certain (specified) move player will make for any situation they could face

Mixed Stratagies

- Randomized policy that selects actions according to a probability distribution over actions
- $[p_1 : a_1, p_2 : a_2, ..., p_n : a_n]$

1.3. Dominant Strategy

Strongly dominates

Strategy s1 **strongly dominates** s2 if the outcome for s1 is **better than** the outcome for s2, for **every cohice** of strategies by the other players

Weakly dominates

Strategy s1 **weakly dominates** s2 if the outcome for s1 is better than the outcome for s2, for **at least one** hoice of strategies by the other players and **no worse** for any other

- Dominates Compare by **fixed own choice, change other's choice**
- "**No matter what the others choose, this one will be better for me**"
- A dominant stragegy may not exist

1.4. Pareto Optimal Outcomes

- An outcome is **Pareto optimal** if there is no outcome preferred by all players
- Pareto efficiency or Pareto optimality is a situation where no individual or preference criterion can be better off **without making** at least one individual or preference criterion worse off or without any loss thereof.
- In the state space, if all chase max, the right up side will be the pareto optimal part for a state, states in this area is more better than current 1

1.5. Nash Equilibrium

(s, s’) is a **Nash equilibrium** if no player, after knowing the strategies of all players, can benefit **by switching strategies**, given that every other player **sticks with** its current strategy

- An equilibrium is a **local optimum** in the space of policies
- Nash Equilibrium compares by **fixed others, change my own choice**
- "**what is my best choose if another choose that**"

Existence

if **mixed strategies** (where a player chooses probabilities of using various pure strategies) are allowed, then every game with **a finite number of players** in which each player can choose from **finitely many pure strategies** has **at least one** Nash equilibrium, which might be a **pure strategy** for each player or might be a **probability distribution** over strategies for each player.

1.5. Dominant Strategy equilibrium

When each player p_i has a dominant strategy s_i, then $(s_i), i > 1$ is called a **dominant strategy equilibrium**.

- the outcome **may not be** pareto optimal
- **not all games** have dominant strategies
- A dominant strategy equilibrium **is a** Nash Equilibrium

1.6. Pareto versus Nash

- Choose the unique **Pareto-optimal Nash Equilibrium** if one exists

- Every game has **at least one** Pareto-Optimal solution
- Pareto-optimal Solutions don't have to be Nash equilibrium

1.7. Payoff

The value associated with a possible outcome of a game

2. Ways towards Nash Equilibria

2.1. Von-Neumanns's Maximin Technique

Every Nash equilibrium in a zero-sum game is a **maximin** for both players

Example 1：2-finger Morra

Solution 1:

1. Force E to reveal its strategy first, followed by O
2. Force O to reveal its strategy first, followed by E

Process 1:

1. Suppose E plays (one: x, two: 1-x) against O. Force O to play first:

$u_E((one : x, two : 1 - x), one) = 2x – 3(1 - x) = 5x - 3$

$u_E((one : x, two : 1 - x), two) = -3x + 4(1 - x) = -7x + 4$

- How can E maximize its worst-case payoff?

Both expressions should yield the same utility, (otherwise when playing for real, O would choose the action that makes it best for O and thus worst for E.)

$5x – 3 = -7x + 4 \Leftrightarrow x = 7/12$

- So E's maximum strategy is (one:7/12,two 5/12), maximum value is -1/12

2. Do the same for O

$u_O((one : y, two : 1 - y), one) = -2y - +3(1 - y) = -5y + 3$

$uO((one : y, two : 1 - y), two) = 3y - 4(1 - y) = 7y - 4$

- To maximize O’s worst-case payoff:

5y + 3 = 7y - 4 iff y = 7/12

- So O’s maximin strategy is (one: 7/12, two: 5/12).
- O’s maximin value is +1/12, so O is **better off** than E.

Solution 2:

compute the **maxmin value of E** is to directly compute the maxmin value:

$$
\begin{aligned}
s_E &= \operatorname{argmax}_{s_E} \min_{s_0} \quad \text{ut}_E(\,(\text{one }: x,\ \text{two }: 1-x),\ (\text{one}: y,\ \text{two }: 1-y)\,) \\
&= \operatorname{argmax}_{s_E} \min_{s_0} \quad 2xy - 3(1-x)y - 3x(1-y) + 4(1-x)(1-y)
\end{aligned}
$$

Example 2: Battle of Sexes

		Wife	
		Theatre	Restaurant
Husband	Theatre	(4,3)	(2,2)
	Restaurant	(1,1)	(3,4)

Question: There are already two nash equilibria in the game, how can we choose now?

Solution:

Use the same way, we will find a mixed solution: ((T:3/4,R:1/4),(T:1/4.R:3/4)), and the expected utility is (5/2,5/2).

So there are actually three mixed strategies of the game:

- $((T : \frac{3}{4}, R : \frac{1}{4}), (T : \frac{1}{4}, R : \frac{3}{4}))$
- $((T : 1, R : 0), (T : 1, R : 0))$
- $((T : 0, R : 1), (T : 0, R : 1))$

3. Some Classical Game Theory Problems

3.1. Prisoner's dilemma

	A: testify	A: refuse
B: testify	A=-5, B=-5	A=-10, B=0
B: refuse	A=0, B=-10	A=-1, B=-1

Dominant Strategy

- for B, testify is **strongly dominates** A

Considering strategy testify for B:

1. if A choose testify, compare column 1, B(testify)=-5>B(refuse)=-10;
2. if A choose refuse, compare column 2, B(testify)=0 >B(refuse)=-1

Dominant Strategy Equilibrium

(testify, testify) is dominant strategy equilibrium

Pareto Optimality

(-1,-1) ,(-10,0),(0,-10) are the pareto optimality points

Nash equilibrium

(testify, testify) is a **Nash equilibrium**

3.2. Manufacturing of video games

	H: DVD	H: CD
S: DVD	H=9, S=9	H=-4, S=-1
S: CD	H=-3, S=-1	H=5, S=5

Dominant Strategy:

None

Nash Equilibria

yes, there are two: (DVD,DVD),(CD,CD)

Pareto Optimal

(DVD,DVD)

3.3. 2-finger Morra

	O: one	O: two
E: one	E=2, O=-2	E=-3, O=3
E: two	E=-3, O=3	E=4, O=-4

Nash Equilibria:

see section 2

Knowing about the other's strategy:

- E can use a **random number generator** or E can decide for either of the pure strategies:
  one or two.
- **Unilaterally** choosing a particular action does not harm one’s expected payoff, but if the other **knows about it**, then he can adjust his strategy and thus influence the payoff

4. Iterated Games

4.1. Basic Introduction

4.2. Iterated Prisoner's Dilemma

4.2.1. Situation 1:

Assumption:

- the number of rounds is uncertain, neither knows for sure which round will be the last round

Solution:

(refuse,refuse) will be the optimal strategy

The objective of the iterated version of the Prisoner’s Dilemma is to maximize your score at the end of a number of rounds of playing. The number of rounds is unknown and so to maximize the score, it is best that both players refuse to testify and receive a 1 year prison sentence in each round.

4.2.2. Situation 2

Assumption:

the players know when the iteration will end

Solution:

the strategy will change becuase it becomes a standlone game

4.3. Vickrey auction protocol

Vickrey auction protocol

1. Auctioneer presents item.
2. Each bidder propose a bid, sealed in an envelope, for the item.
3. Outcome: bidder with highest bid wins.
4. Winner has to pay the amount of the second highest bid.

5. Auction

5.1. Types

First-price open cry (English,japanese auction)

- as usual
- The auctioneer opens the auction by announcing a suggested opening bid, a starting price or reserve for the item on sale.
- Then, the auctioneer accepts increasingly higher bids from the floor, consisting of buyers with an interest in the item. The auctioneer usually determines the minimum increment of bids, often raising it when bidding goes high.

First-price sealed bid (blind auction)

- In this type of auction, all bidders **simultaneously** submit **sealed bids** so that no bidder knows the bid of any other participant. The highest bidder pays the price that was submitted
- bidding without knowing the other bids

Dutch Auction (descending aution)

the seller **lowers** the price until it is taken

Second-price sealed bid (Vickrey auction)

Highest bidder wins, but the price is the second highest bid.