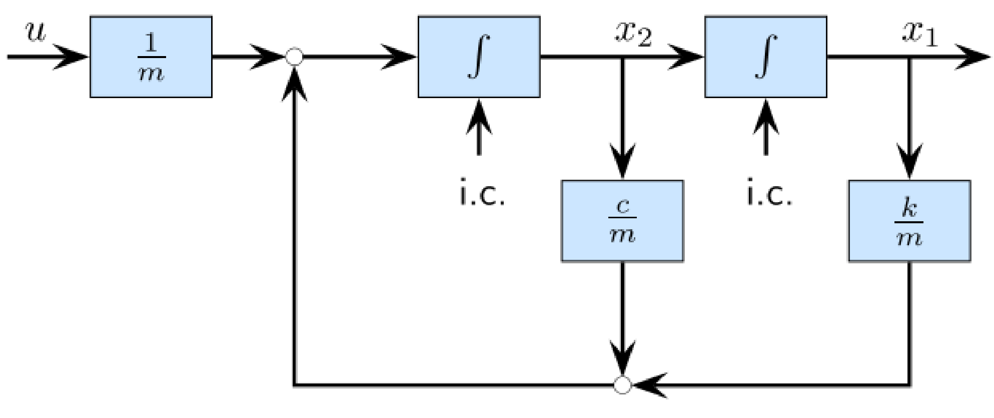
01_System Modelling

1. Mathematical Models in Control System Design

Three Methods

- **physics-based** model (conservation laws, physical geometry)
- models based on **known interactions and properties** (e.g.: energy-based models, stoichiometric models)
- Models from **experiments** (data driven)

State-Space Models



states: $x_1(t) = q(t)$ and $x_2(t) = \dot{q}(t)$

- State-Space Models are always ODE

2. Solvability Differential Equations

Mainly focus on **Existence and uniqueness**

Lipschitz Continuity

for some fixed $c \in \mathbb{R}$, $\|F(x) - F(y)\| < c\|x - y\|$ for all x, y

<u>A sufficient Condition for Lipschitz Continuity</u>

A sufficient condition for a function to be Lipschitz is that the Jacobian $\partial F / \partial x$ is **uniformly bounded** for all x

Picard–Lindelöf theorem

若已知 $y(t)$ 有界, f 符合Lipschitcz Continuity, 则微分方程问题 $y'(t) = f(t, y(t)), \quad y(t_0) = y_0$ 刚好有一个解。

- 在应用上, t通常属于一有界闭区间，于是 $y(t)$ 必有界，故 y 有唯一解

3. System Properties

Memoryless vs Dynamics

<u>Definition: Memoryless</u>

A system is said to be **<u>memoryless</u>** if its output at a given time is **dependent only on the input at that same time**

Casual

<u>Definition: Casual</u>

A system is **<u>casual</u>** if the output at any time depends only on values of the input **at that the present time and in the past**

- All **physical systems** in the real world are causal because they cannot anticipate on the future
- In **signal/image processing** we can sometimes use non-causal filters.

Linear

<u>Definition: Linear</u>

Let $y_1(t)$ be the output of the system to the input $u_1(t)$, and let $y_2(t)$ be the output to the input $u_2(t)$. A system is said to be **<u>linear</u>** if it satisfies the following properties:

1. the input $u_1(t) + u_2(t)$ will give an output $y_1(t) + y_2(t)$.
2. the input $\alpha u_1(t)$ will give an output $\alpha y_1(t)$ for any (complex) constant α.

Time-Invariant

<u>Definition: Time-Invariant</u>

A system is said to be **time-invariant** if the behavior and characteristics are fixed over time.

An input-output system is time-invariant if a time-shift τ in the input leads to the same time-shift in the output

$$u(t) \Longrightarrow y(t), t \in \mathbf{R} \quad \Longrightarrow \quad u(t-\tau) \Longrightarrow y(t-\tau), t \in \mathbf{R}$$

Summary

- State-Space Model
- Solvability Differential Equations:
    - Lipschitz Continuity
    - sufficient condition of Lipschitz Continuity: The Jacobian uniformly bounded
    - Picard-Lindelof Theorem for solution existence
- System Properties
    - Memoryless: only dependent on current input
    - Casual: for all practical system
    - Linear
    - Time-invariant