06_Jitter Analysis

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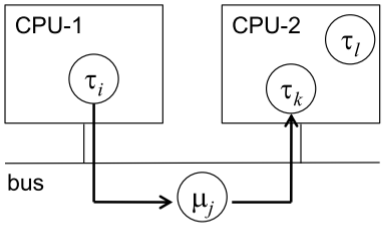
1. Background

- Motivation
  - Typically for multi-processor systems, but also applies to single-processor systems
  - *Finalization* (or *end*) *jitter* FJ: variation in finalization times
  - *Activation* (or *release*) *jitter* AJ: variation in activation times (e.g. output of one task triggers a next task)
  - Example: multimedia in a networked environment

1.1. An Example Multiprocessor Scenario

- Example multiprocessor



1. a strictly periodic *system* event activates a task τ_i
2. task τ_i sends message μ_j
   - FJ_i causes AJ_j
3. message μ_j triggers task τ_k
   - FJ_j causes AJ_k
4. task τ_k generates a *system* response

AJ_k influences *system* response and response times of task τ_l with a lower priority

1.2. Goal Jitter Analysis

- Determine schedulability in the context of jitter, hence, response times
- Determine **end-to-end response times**

1.3. How to understand Jitter

一个系统里面因为各个环节的因素，所产生的一个可以大概的判断上界的每次随机的额外时延

- **Latency** = Delay between an event happening in the real world and code responding to the event.
- **Jitter** = Differences in **Latencies** between two or more events.

2. Preliminary

2.1. Fixed-Priority Preemptive Scheduling (FPPS)

Nothing to explain, just as the name

2.2. Finalization Jitter

variation in finalization times

2.3. Activation Jitter

variation in activation times (e.g. output of one task triggers a next task)

3. Response Time

3.1. Schedulability Condition

$$\underset{i,k,\varphi}{\forall} BD_i \leq R_{i,k}(\varphi) \leq WD_i$$

3.2. Worst Case Response Time

Assumption

Based on RM algorithm

Condition

Worst Case happens when under the **Critical Instant** (maximum number of higher priority preemptions)

Calculation

Recursive Equation

$$x = C_i + \sum_{j<i} \left\lceil \frac{x}{T_j} \right\rceil C_j$$

- $\lceil \frac{x}{T_j} \rceil$ denotes the **maximum number of pre-emptions of task τ_i in an interval $[0, x)$ by task τ_j**
- LHS: amount of time available (or provided) in $[0, x)$
- RHS: max. amount of time requested by higher priority tasks in $[0, x)$

3.3. Best Case Response Time

Formalization

- Best-case Response time BR_i of a periodic task τ_i

$$BR_i = \inf_{\varphi,k} R_{i,k}(\varphi)$$

$$\text{where} \phi \text{ is the phasing of the task set}$$

- Hence

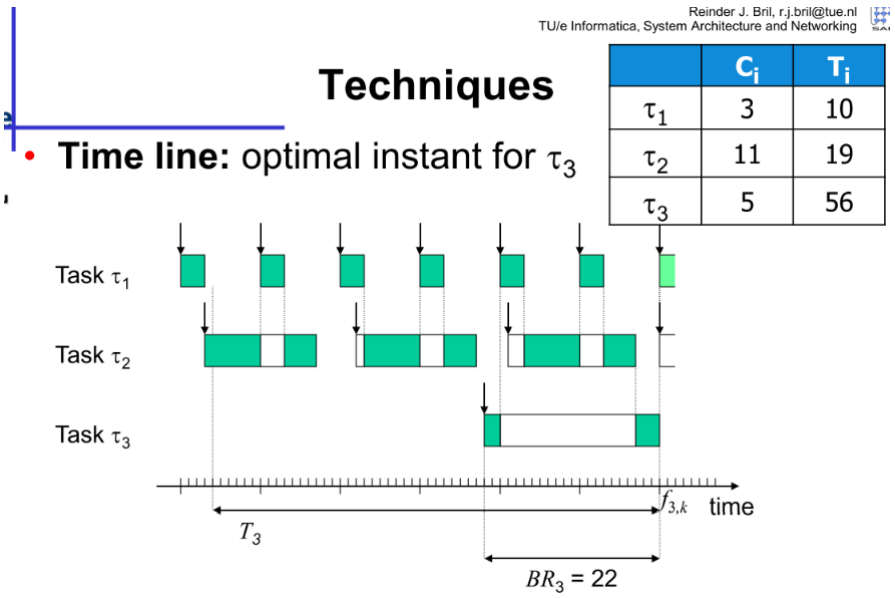
$$\forall_{i,k,\varphi} BR_i \leq R_{i,k}(\varphi)$$

$$\forall_i BD_i \leq BR_i$$

Optimal Instant

- Optimal Instant incurs **the lowest amount of pre-emption** by higher priority tasks
- An optimal instant (not only one)
    - Job $\tau_{i,k}$ **ends simultaneously with the release of all tasks with a higher priority**, and $\tau_{i,k'}$s release time is equal to its start time
    - Specific for each task

Timeline Caculation



Calculation

$$x = C_i + \sum_{j<i} \left(\lceil \frac{x}{T_j} \rceil - 1 \right) C_j$$

- $\lceil \frac{x}{T_j} \rceil - 1$ denotes **the minimal number of pre-emptions of** task τ_i in an interval $[0, x)$ by task τ_j
- LHS: amount of time available (or provided) in $[0, x)$
- RHS: min. amount of time requested by higher priority tasks in $[0, x)$
- stopped when
    - the same value is found for two successive iterations
    - the deadline BD_i is exceeded (hence not schedulable)
- All intermediate values are at least equal to BR_i
- We always start

Notes

Why $\lceil \frac{x}{T_j} \rceil - 1$

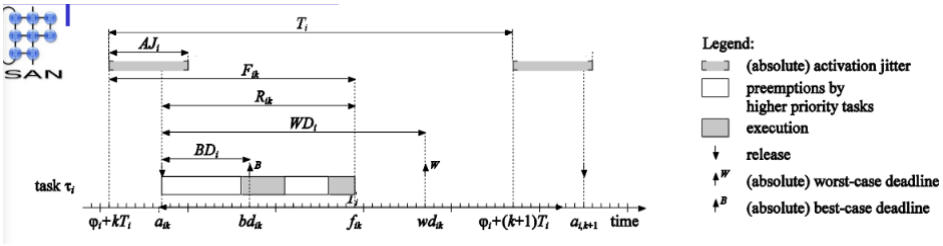
- if it $\lceil \frac{x}{T_j} \rceil$ is an integer. That means, if it is a integer, then because preemptive, we can start later,

- For example in above picture, if count task 3 we initially set x to 20, then an integer appear, we can move the release of task 3 after the finish of task 1.

Many Fixed Points

- **There maybe many fixed point**, the smallest positive one always equal to computation time.
- In order to get correct fixed point, **we always start from WR or T_i**

4. Jitter Analysis



Assumption

The analysis **does not hold** for deadlines plus jitter larger than periods, i.e. for $WD_i + AJ_i > T_i$

4.1. Types of Jitter

Activation (release) jitter AJ

variation in activation times (e.g. output of one task triggers a next task)

Response jitter RJ

variations in response times

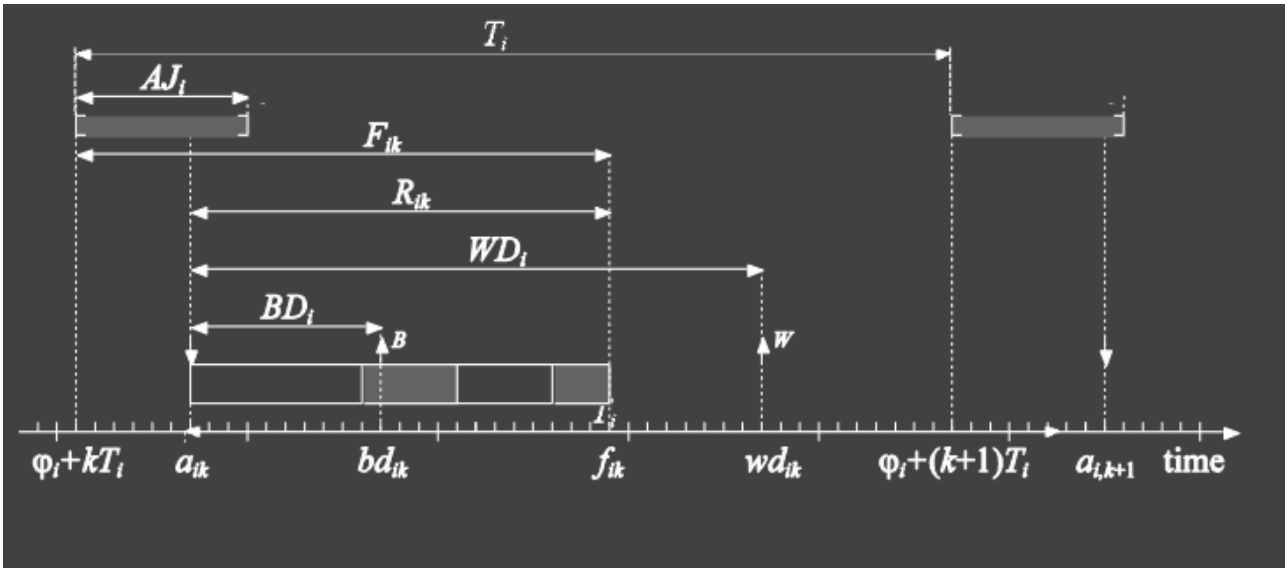
Finalization (or end) jitter FJ

variation in finalization times

4.2. Activation Jitter

$$\varphi_i + kT_i \leq a_{i,k} \leq \varphi_i + kT_i + AJ_i$$

whereφ_i denotes the start of the jitter interval



- We can regard Activation Jitter is **decided by "outside"**, than it leads to jitter of response jitter and finalization jitter, given the fixed computation time

4.3. Response Jitter

$$RJ_i = \sup_{\varphi,k,l} \left(R_{i,k}(\varphi) - R_{i,l}(\varphi)\right)$$

- Note when consider jitter, the Response time is finish time - actual release time
- A bound on response jitter

$$RJ_i \leq WR_i - BR_i$$

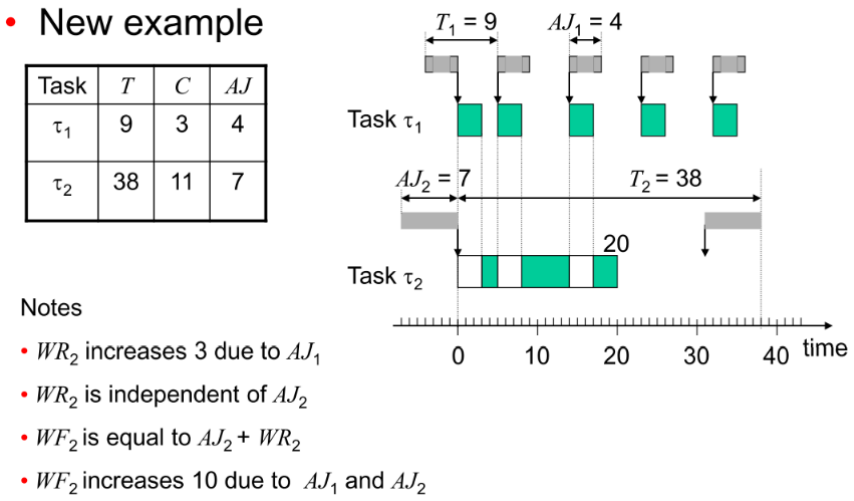
$\leq$ because WR_i and BR_i **are not necessarily assumed for the same phasing**

4.3.1. Worst Case Response Time

Condition

Critical Instant:

- Task τ_i is released simultaneously with all tasks with a higher priority and



- all tasks with a higher priority experience
    - a **maximal release delay** at that **simultaneous release,** and
    - a **minimal release delay** at **subsequent releases**

Calculation

$$x = C_i + \sum_{j<i} \lceil \frac{x + AJ_j}{T_j} \rceil C_j.$$

- WR_i is the **smallest positive solution** of the equation
- From point 1, we know we can start from C_i

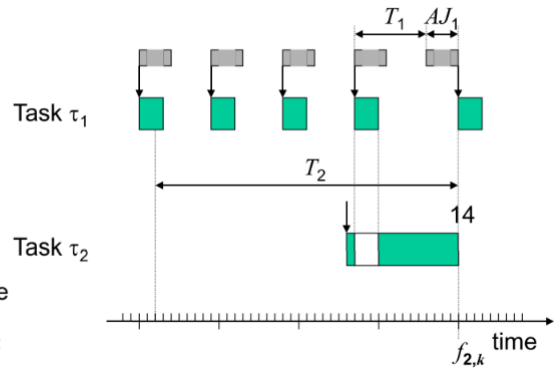
4.3.2. Best-case Response Time

Condition

Optimal instant

- Job $\tau_{i,k}$ ends simultaneously with the release of all tasks with a higher priority, and $\tau_{i,k'}$ release time is equal to its start time, and

Task	T	C	AJ
τ_1	9	3	4
τ_2	38	11	7



Notes

- BR_2 does not change here
- BR_2 is independent of AJ_2
- $BF_2 = BR_2$

- all tasks with a higher priority experience
    - a **maximal release delay** at that **simultaneous release**, and
    - a **minimal release delay** at **previous releases**
- BR_i is independen of AJ_i

Calculation

$$x = C_i + \sum_{j<i} \left(\lceil \frac{x - AJ_j}{T_j} \rceil - 1 \right)^+ C_j$$

$$= C_i + \sum_{j<i} \max \left(\lceil \frac{x - AJ_j}{T_j} \rceil - 1, 0 \right) C_j$$

4.4. Finalzation Jitter

Relative Finalization Time

$$F_{i,k} \overset{\text{def}}{=} f_{i,k} - (\varphi_i + kT_i)$$

- absolute finish time - absolute **release time without jitter** (actual release time)

Finalization Jitter

$$FJ_i = \sup_{\varphi,k,l} (F_{i,k}(\varphi) - F_{i,l}(\varphi))$$

Relation

$$WF_i = AJ_i + WR_i$$
$$BF_i = BR_i$$
$$WF_i - BF_i = AJ_i + WR_i - BR_i$$
$$FJ_i \leq AJ_i + WR_i - BR_i$$

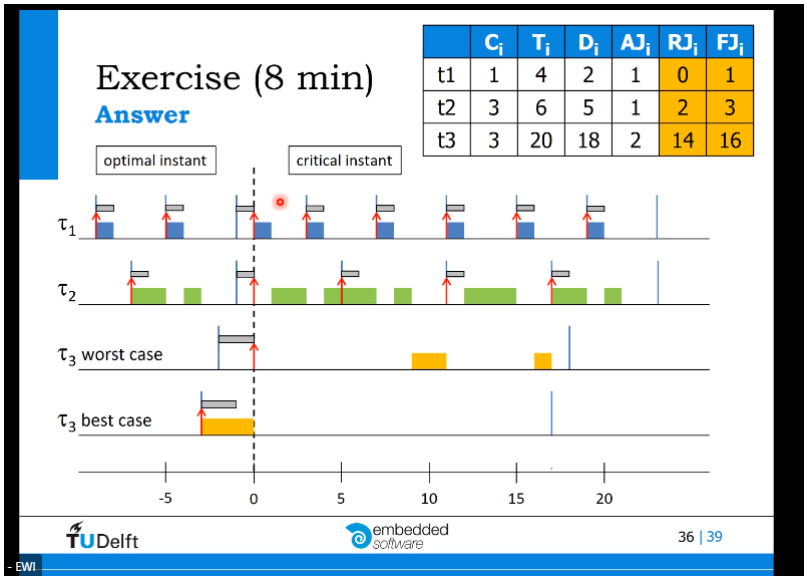
4.4.1. Worst-case Finalization Times

$$WF_i = AJ_i + WR_i$$

4.4.2. Best-case Finalization Times

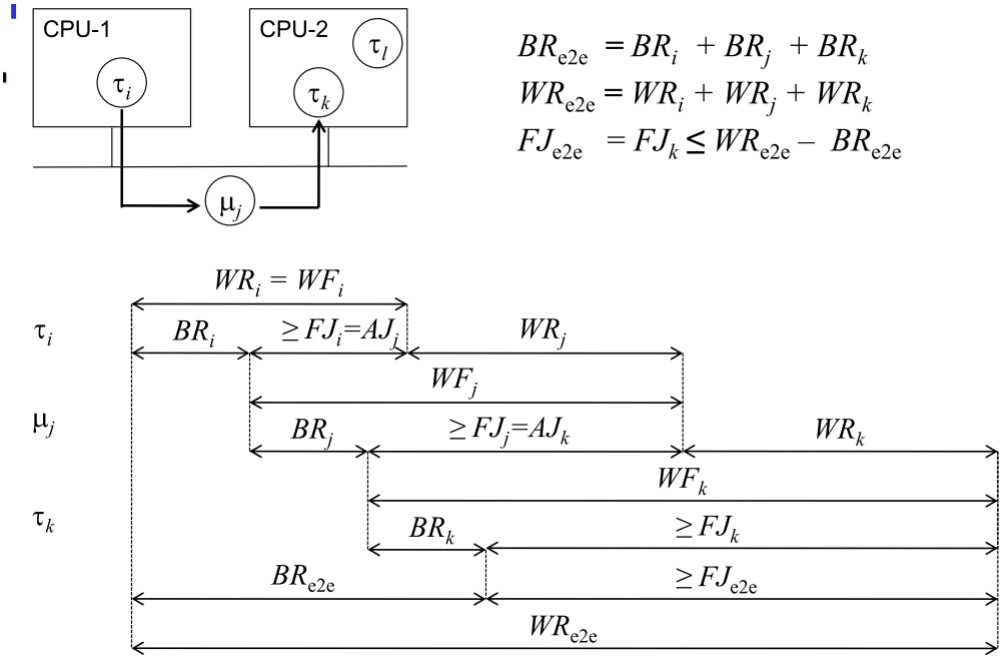
$$BF_i = BR_i$$

4.5. Example



	C_i	T_i	D_i	AJ_i	RJ_i	FJ_i
t1	1	4	2	1	0	1
t2	3	6	5	1	2	3
t3	3	20	18	2	14	16

5. An Multi-Processor Scenario



5.1. The actual process from external event to task activation

1. Extent event occurs
2. Detection by sensor
3. Interrupt generated by sensor
4. may **take some time** before the bus is free
5. Interrupt arrival at CPU
6. may **take some time** before the interrupt is handled
7. Immediate Interrupt Service
8. may be **pre-empted** by higher priority interrupts
9. Activation of the scheduler
10. may be delayed to the next clock-tick
11. Activation of the task

- hence, both a **latency** and **potential jitter** between the arrival of the external event and activation of the task
- Jitter can be **reduced by using buffers** (i.e. by changing precedence constraints into data dependency constraints). This may increase the latency, however

如果直接访问硬件，或者直接等待另一个处理器触发本处理器所属的任务，由于总线/硬件等各种东西的状态未知（可能被占用），以及硬件速度本身的不确定性，会导致有一个比较大的variation。那么，所做的方式就是，外设硬件，在总线空闲的时候，放到内存的一个queue里面，然后task以一个较高的频率/处理器以一个较高的detect这个queue，如果有数据，就执行，如果没有就跳过。这样的话，因为CPU+内存的访问jitter要小，而且速度快，相当于克服了总线速度慢+硬件variation大的问题