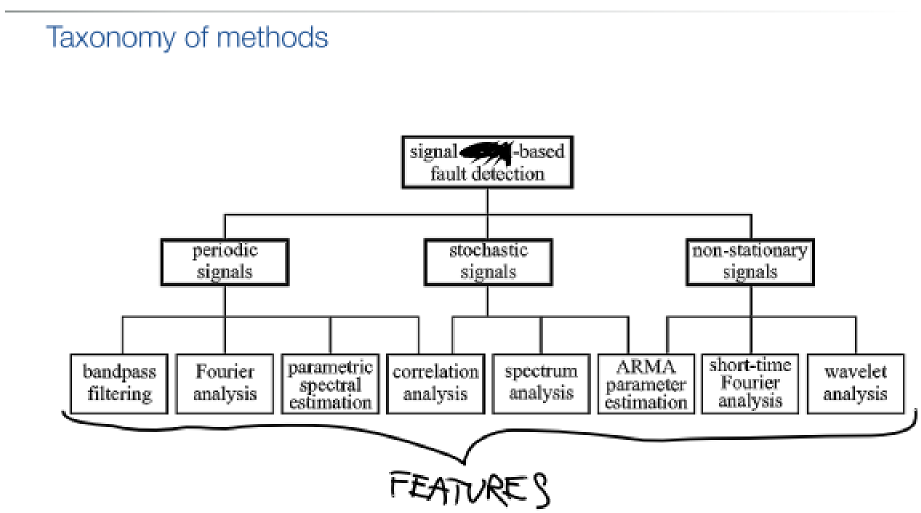
04_Signal Based Fault Diagnosis Method

1. Introduction

In this chapter, we mainly focused on signal-based fault diagnosis method. Which is used to do **feature extraction** and **symptoms generation** jobs.



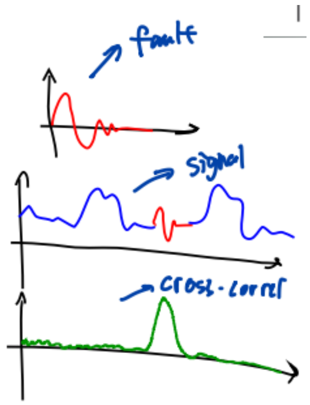
2. Time-Based Methods

There are several approaches to do time-based analysis, based on previous lectures, these approaches can be:

- applied to **instantaneous raw values**
- applied to some **evaluation function**

Some classical methods are:

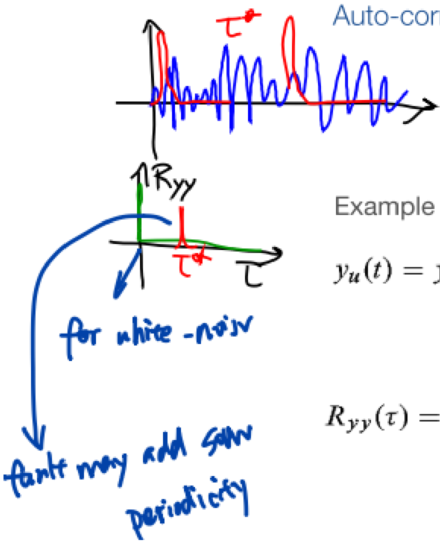
- **Band-pass filtering**: filtered out noise and then analysis
- **Cross-correlation**: known fault feature, do cross correlation



- **Auto-correlation**:

  always used in case where normally signal is white noise, and fault signal may **add some periodicity**

Auto-correlation



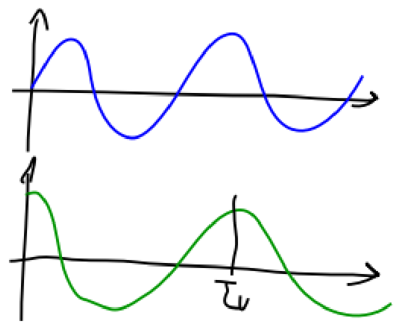
$$\boxed{R_{yy}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T y(t)\, y(t+\tau)\, dt}$$

Example

$$y_u(t) = y_{0v} \sin(\omega_v t + \varphi_v) + n(t)$$

$$R_{yy}(\tau) = \frac{y_{0v}^2}{2} \cos \omega_v \tau$$

$\omega_v = \frac{2\pi}{\tau_v}$

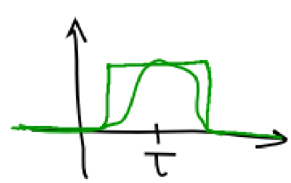


3. Frequency-Based Methods

There are several approaches to do Frequency Analysis

- **Fourier transform** $Y(f) = \mathcal{F}\{y(t)\}$
- **Short time Fourier Transform**

$$Y(f,\tau) = \mathcal{F}\{y(t)w(t-\tau)\}$$



- **Wavelet analysis [GR95]**

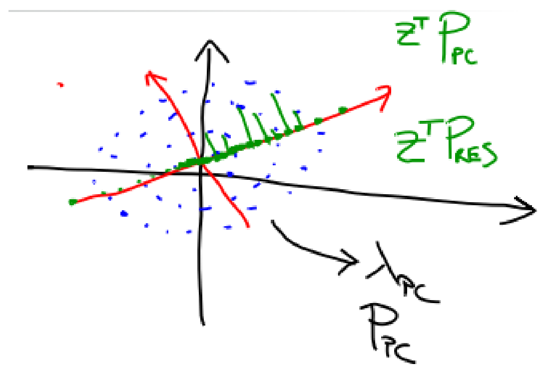
- Power Cepstrum

$$Y'(q) = \left|\mathcal{F}^{-1}\left\{\log\left(\left|\mathcal{F}\{y(t)\}\right|^2\right)\right\}\right|^2$$

  - it converts convoluted signals (e.g. input and filter impulse response) into sums of their cepstra, for linear separation

4. Compression-Based Methods: PCA

- Assume **huge set of measurements** from “slow” process (e.g. Chemical Reactor), besides, assume analysis is always in **stationary states**
- **Reduce dimensionality** of data
- Express data in terms of projection on a few direction of **“maximum variance**”



Process

Decomposition

1. Data set: N **normalised** samples for each of m sensors

$$Z^\top = [z_1, z_2, \dots z_N] \in \mathcal{R}^{m\times N}$$

2. Compute covariance matrix

$$C = \frac{1}{N-1} Z^\top Z$$

3. Compute singular value decomposition

$$C = P \Lambda P^\top$$

$$\Lambda = \operatorname{diag}(\lambda_1, \dots, \lambda_m), \quad \lambda_1 \geq \dots \geq \lambda_m > 0$$

4. Keep at most l components

$$\Lambda=\left[\begin{array}{cc}\Lambda_{pc} & 0 \\ 0 & \Lambda_{\text{res}}\end{array}\right],$$

$$\Lambda_{pc}=\operatorname{diag}\left(\lambda_1, \ldots, \lambda_l\right), \quad \Lambda_{\text{res}}=\operatorname{diag}\left(\lambda_{l+1}, \ldots, \lambda_m\right)$$

$$P=\left[\begin{array}{ll}P_{pc} & P_{\text{res}}\end{array}\right], \quad P_{pc} \in \mathcal{R}^{m \times l}, P_{\text{res}} \in \mathcal{R}^{m \times(m-l)}$$

__**Detection of Changes in New Data**__

- Compute the following limits

$$J_{th,SPE}=\theta_1\left(\frac{c_\alpha \sqrt{2 \theta_2 h_0^2}}{\theta_1}+1+\frac{\theta_2 h_0\left(h_0-1\right)}{\theta_1^2}\right)^{1 / h_0}, \quad h_0=1-\frac{2 \theta_1 \theta_3}{3 \theta_2^2}.$$
$$J_{th,T^2}=\frac{l\left(N^2-1\right)}{N(N-l)} F_\alpha(l, N-l) \quad \theta_i=\sum_{j=l+1}^m\left(\lambda_j\right)^i, \quad i=1,2,3$$

- **Normalize every new data sample** z and **compute** the following statistics

$$SPE(\text{Squared Prediction Error})=z^T P_{\text{res}} P_{\text{res}}^T z,] \quad T^2=z^T P_{pc} \Lambda_{pc}^{-1} P_{pc}^T z.$$

$SPE \leq J_{\text{th,SPE}}$ and $T^2 \leq J_{\text{th, } T^2} \Rightarrow$ fault free, otherwise faulty

Summary

- Signal-Based Method for Feature Extraction and Symptoms Generation
- Time-Based Method: auto-correlation, cross-correlation, band-filter
- Frequency-Based Method: Fourier,...
- Compression-Based Method: PCA
    - maximum variance dimension
    - detection change based on Q and T