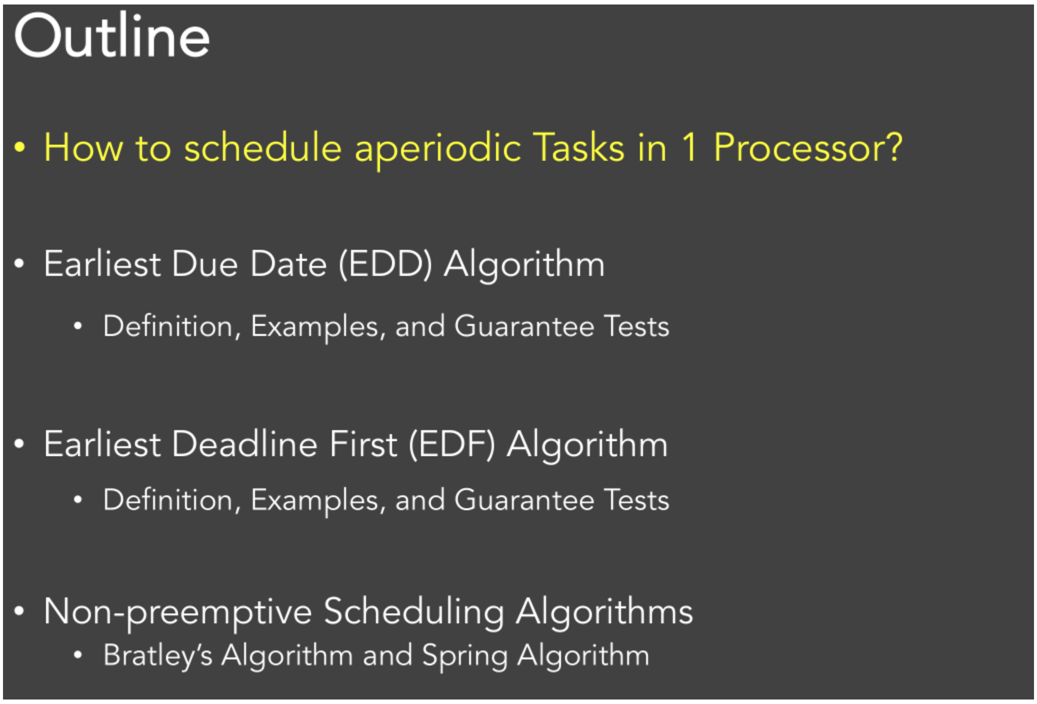
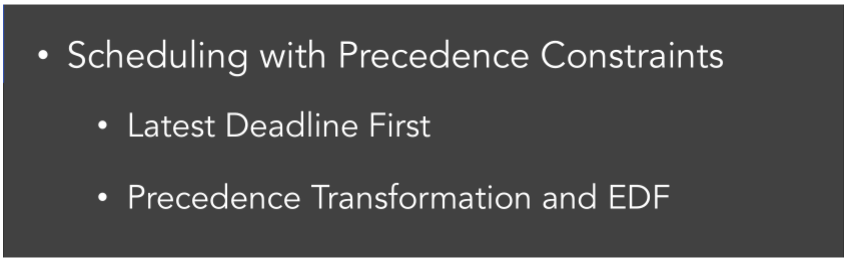
04_Aperiodic Task Scheduling





1. Earliest Due Date (EDD) Algorithm

$$1 \mid \text{sync} \mid L_{\max}$$

1.1. Rules

Rules:

select the task with the earliest **relative deadline**

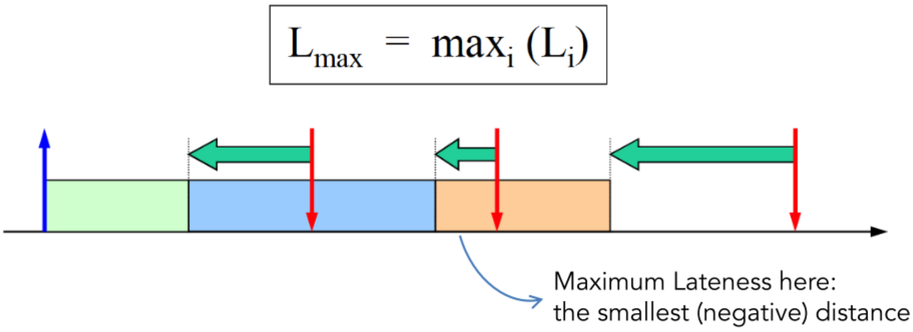
Assumptions:

- All tasks are activated **simultaneously** (like a "**batch**")
- Preemption is not used (since tasks arrive simultaneously);
- Static policy

1.2. Performance

Lateness

- If L_{max} is non-positive, then no task misses its deadline.
- **EDD minimizes** L_{max} (has the maximum slack time)



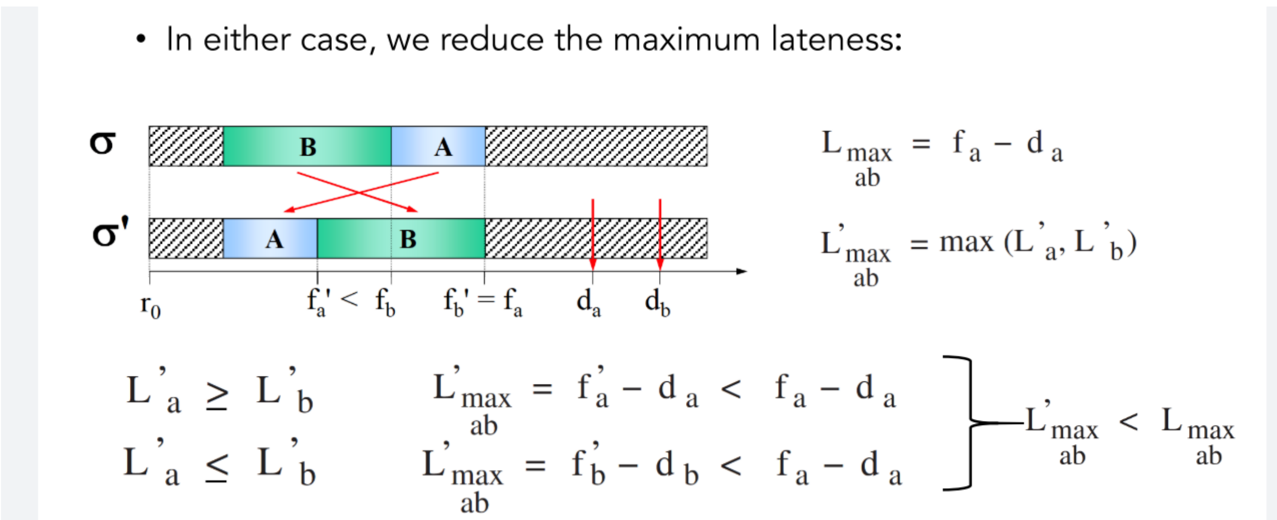
Jackson's Theorem

Given a set of n independent tasks, any algorithm that executes the tasks in order of increasing deadlines **is optimal w.r.t. minimizing** L_{max}

Proof:

Proof with two steps:

- optimal in one step



- recursively optimizing (like bubble sort)

If we continue doing these swaps, we will end up with the optimal, in terms of L_{max}, EDD policy.

$$\sigma \longrightarrow \sigma' \longrightarrow \sigma'' \longrightarrow \dots \longrightarrow \sigma^*$$

$$L_{\max}(\sigma) \geq L_{\max}(\sigma') \geq L_{\max}(\sigma'') \dots \geq L_{\max}(\sigma^*)$$

1.3. Feasibility Check

Feasibility must be **checked offline**

- for each task should hold:

$$\forall i = 1, \dots, n \quad f_i \leq d_i$$

- Assume task indices are ordered in increasing deadlines, then, worst-case finishing time of each task is

$$f_i = \sum_{k=1}^{i} C_k$$

- feasibility requires to satisfy the following constraints

$$\forall i = 1, \dots, n \quad \sum_{k=1}^{i} C_k \leq d_i$$

1.4. Complexity

$$O(nlog(n))$$

2. Earliest Deadline First (EDF) Algorithm

2.1. Rules

execute the task with the earliest **absolute deadline**

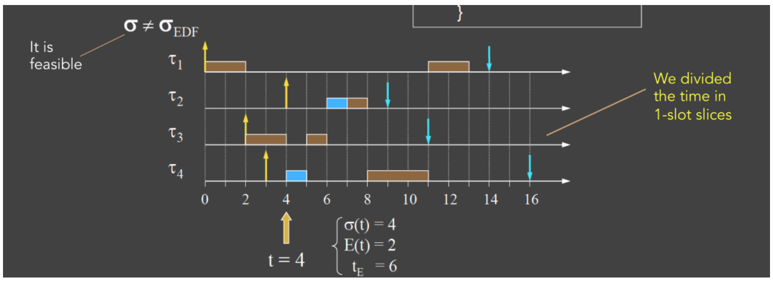
Assumption

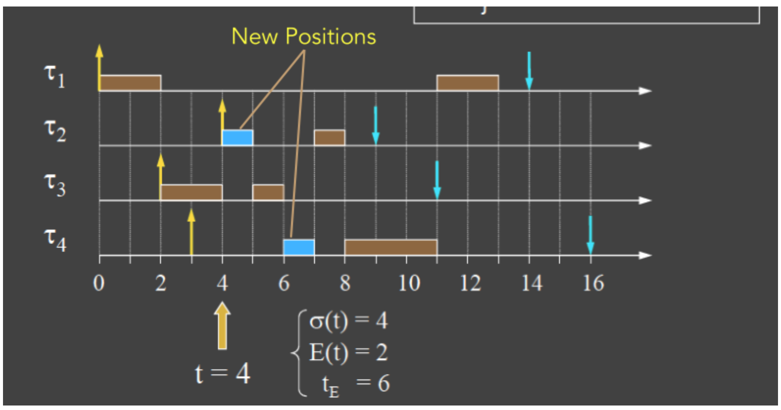
- tasks may arrive at any time **(asynchronous)**;
- **Preemption** is used;
- **Dynamic** priority; (d_i depends on arrival)

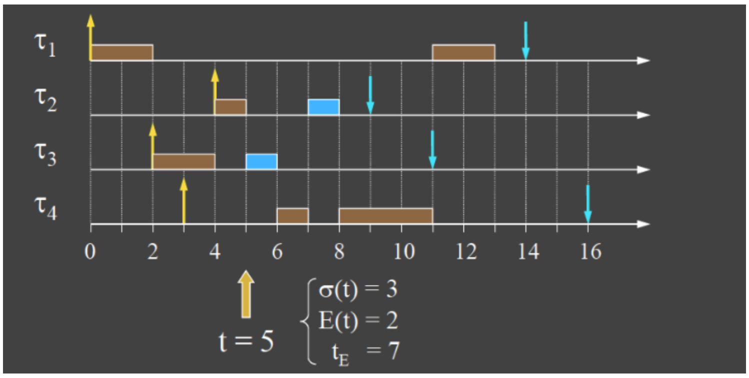
2.2. Performance (M.Dertouzos Theorem)

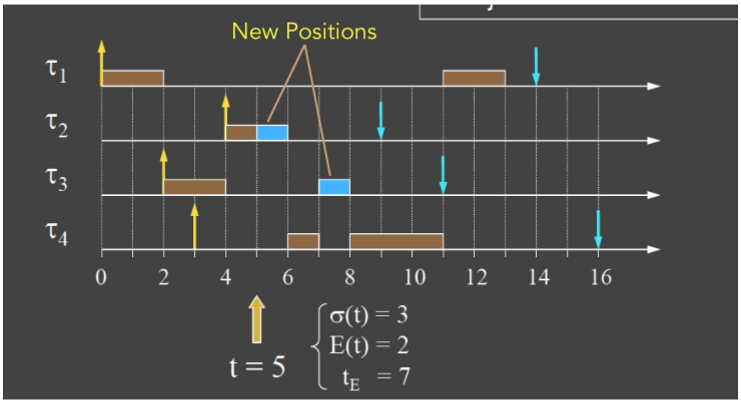
Given a set of n independent tasks **with arbitrary arrival times**, any algorithm that at any instant executes the task with the earliest absolute deadline among all ready tasks **is optimal w.r.t. minimizing the maximum lateness** L_{max}

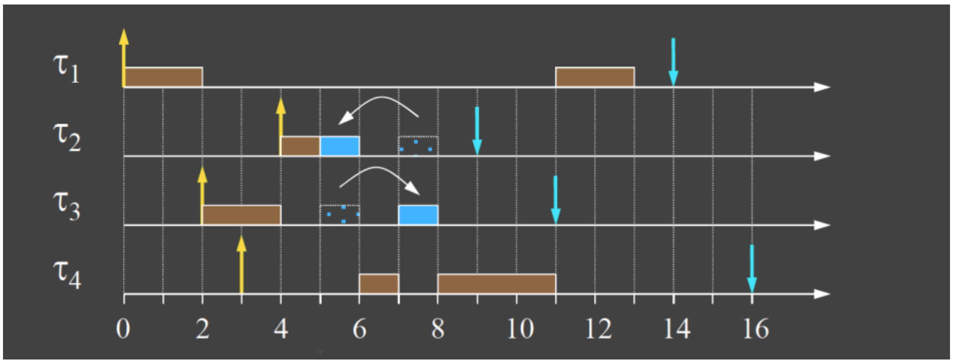
Proof











Feasibility is also guarantee, and the lateness is improve

- If the later deadline one's finish time is not change $\rightarrow$ absolutely hold
- If the later deadline one's finish time change (swap):
  - It can be easily prove the earlier deadline one is the L_{max} if the swap happen

Property

So, EDF is also optimal in the sense of feasibility. i.e. **If there exists a feasible schedule for this set of tasks, EDF will find it**

2.3. Feasibility Check

- The test runs upon arrival of every new task
- we perform **admission control** by checking if the new task set is schedulable
- check if **under worst-case scenario** the tasks can meet their deadlines.
- Tasks are preemptable, with C_i being the initial computation time and $c_i(t)$ the remaining comp time at t

$$f_i = \sum_{k=1}^{i} c_k(t)$$

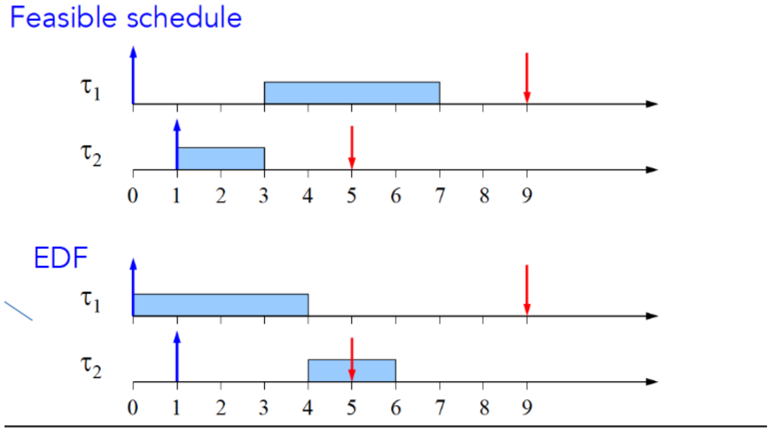
$$\forall i = 1, \ldots, n \quad \sum_{k=1}^{i} c_k(t) \leq d_i$$

2.4. Complexity

$$O(n)$$

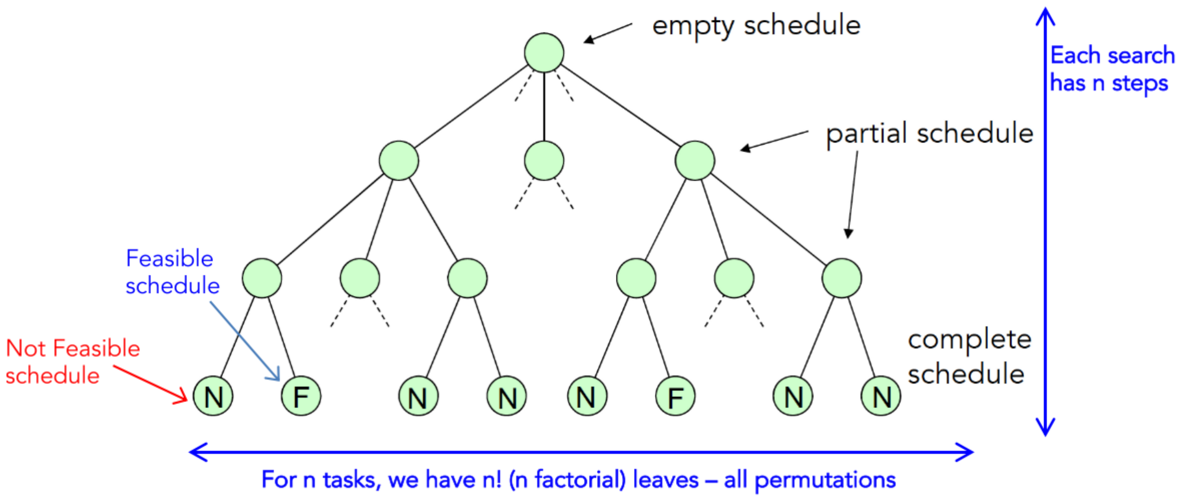
3. Non-preemptive Scheduling Algorithms

- EDF is **no longer optimal** under no preemption (with idle choice)
    - Results can be improved if we know the future, we can first let the CPU be idle for 1 slot until task 2 come
- EDF is still optimal if we focus on **non-idle** algorithms
    - Two algorithms in this secion consider **with-idle** situation



3.1. Model

Even if task arrivals are known, we must **search a tree** with $n!$ leaves and n depth; hence $O(nn!)$ worst-case complexity



3.2. Bratley's Algorithm

$$1|\text{no-preem}|L_{max}$$

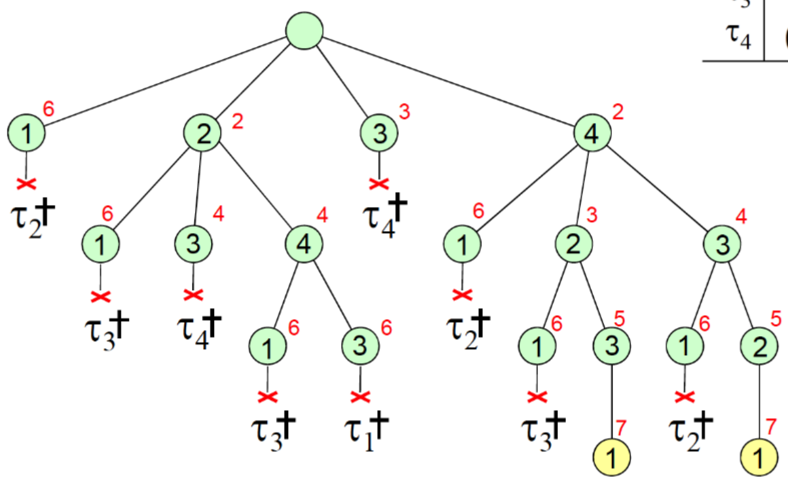
Pruning Rule

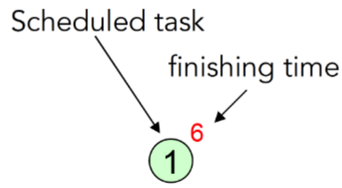
Stop searching a path:

- If a feasible schedule is **already found**; or
- If the addition of **any node** to current path causes a missed deadline

- A quick example

	a_i	C_i	d_i
τ_1	4	2	7
τ_2	1	1	5
τ_3	1	2	6
τ_4	0	2	4

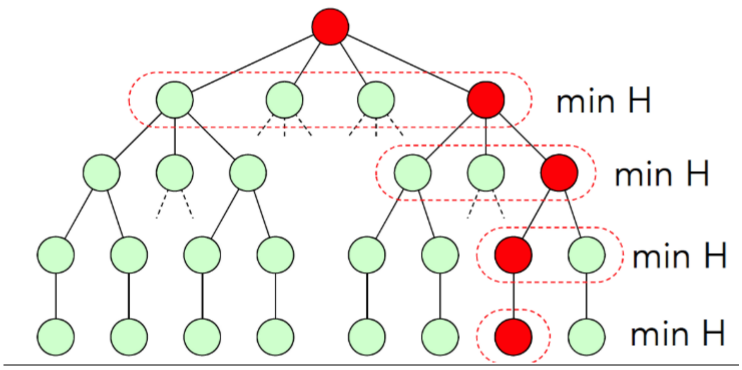




3.3. Spring Algorithm

Reduce complexity by sacrificing optimality: **Heuristics**

At each step select the task **minimizing a heuristic function H** (n steps)



- Optimizing one criterion:

$$H = r_i \Rightarrow \text{FCFS}$$
$$H = C_i \Rightarrow \text{SJF}$$
$$H = d_i \Rightarrow \text{EDF}$$

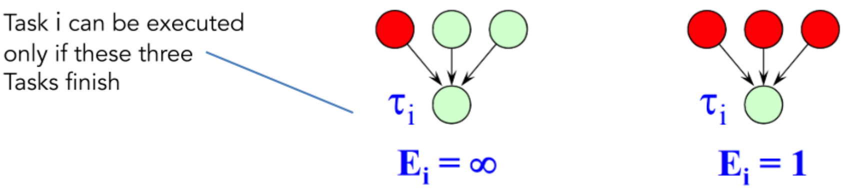
- Optimizing multiple criteria:
    - Pareto optimization through scalarization

$$H = w_1 r_i + w_2 D_i$$
$$H = w_1 C_i + w_2 d_i$$
$$H = w_1 V_i + w_2 d_i$$

Precedence Constraints

Use "Flag" parameter

$$H = E_i (w_1 r_i + w_2 D_i)$$
$$H = E_i (w_1 C_i + w_2 d_i)$$



4. Scheduling with Precedence Constraints

4.1. Latest Deadline First (LDF) Scheduling (1|Prec,Sync| L_{max})

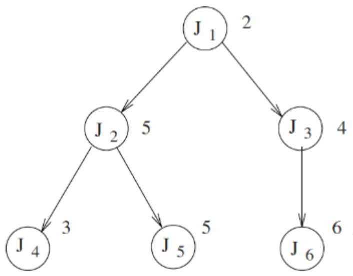
Assumptions

Synchronous Activation of Tasks

Rules

- Take as **input** the set of tasks and their **precedence graph (DAG)**;
- Add task from the end of the queue:
    - Construct the schedule starting **from the tail of the DAG**;
    - **Among tasks with no successors (or, already-selected successors), pick the task with the latest deadline to execute last**.
    - At runtime, select first tasks from the head of queue.

Examples:



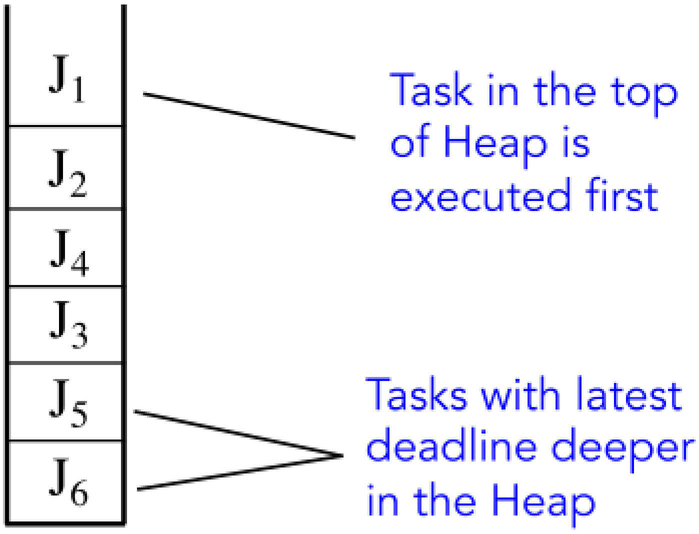
absolute deadlines

	J_1	J_2	J_3	J_4	J_5	J_6
C_i	1	1	1	1	1	1
d_i	2	5	4	3	5	6

- Start from the end of queue; select tasks with no "children" or already-selected children.

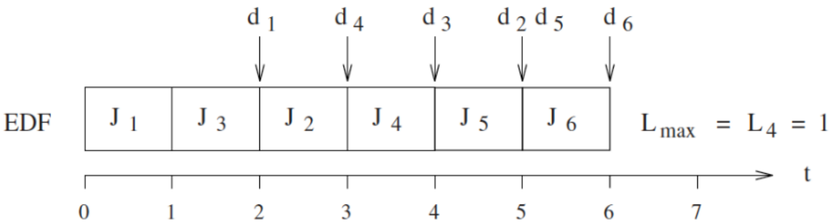
We begin at the end
from end build the queue

J_1	J_2	J_4	J_3	J_5	J_6



Notice:

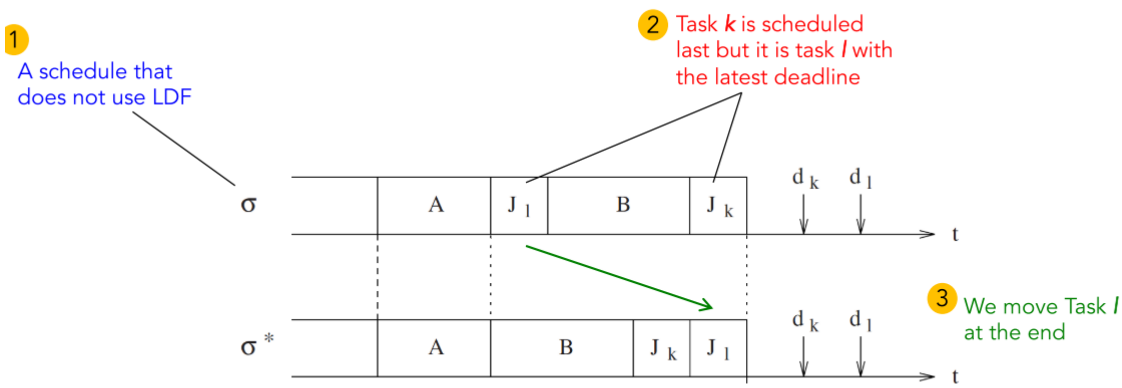
EDF would have produced an **infeasible** schedule: select the task with the **earliest deadline** among all **currently-eligible tasks**



Property

- polynomial on number of tasks n
- **Optimal in terms of minimum** L_{max}

Optimal Proof



Lmax of the new schedule is reduced:

- For Tasks in set A, we do not have any change in L= {finish – deadline} times;
- For Tasks in set B, we have, actually, reduced (improve) the Lateness;
- For Task k we have reduced the Lateness;
- For Task l the lateness is no worse than the L_{max} of schedule σ since $f - d_l < f - d_k$

So for three of the part, they have better L, for one part, it has better L than one part of the original schedule

$$a > a'; b > b'; c > c'; c > d';$$
$$\max(a, b, c, d) \geq c;$$
$$\max(a', b', c', d') < c$$

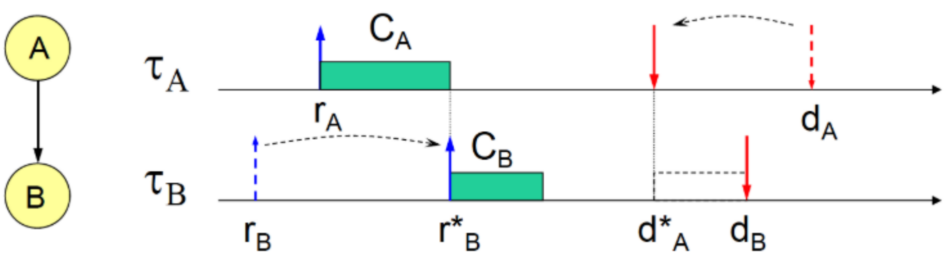
4.2. EDF* Methods (1|Prec,Preem| L_{max})

Assumptions

Arrival times must be known in advance

Basic Ideas

- Transform task set Γ to an equivalent set Γ with no Prec. constraints;
    - Postpone the release time of a successor;
    - Advance the deadline of a predecessor
    - E.G. Task A is a predecessor and in the new schedule Γ* should finish:



        - Before its original deadline: d_A
    - Before the maximum start time of Task B: $d_B - C_B$
- Apply EDF to the new task set Γ*

Rules

- Arrival Time Modification: (downward)

  a. For all root nodes, set: $r_i^* = r_i$

  b. Select a task i that:

     - its release time has not been modified;
     - the release times of all its immediate predecessors have been modified;

  c. Set: $r_i^* = \max\{r_i, \max_{\tau_k \to \tau_i}(r_k^* + C_k)\}$

  d. GoTo b.

- Deadline Modification: (upward)

  a. For all Leaf nodes, set: $d_i^* = d_i$

  b. Select a task i that:

     - its deadline has not been modified;
     - the deadlines of all its immediate successors have been modified

  c. Set: $\mathrm{d_i^*} = \min\{\mathrm{d_i}, \min_{\tau_\mathrm{i} \to \tau_\mathrm{k}}(\mathrm{d_k^*} - \mathrm{C_k})\}$

  d. GoTo b.

Property

- If Γ* is schedulable with EDF, then Γ is also schedulable with EDF
  - because in Γ^*, we have $r_i^* \geq r_i \quad d_i^* \leq d_i$
- The precedence constraints of Γ are satisfied in Γ*
  - if $\tau_i \to \tau_j$ then $d_i^* < d_j^*$ and $r_i^* < r_j^*$

Overview of Aperiodic Scheduling

	sync. activation	preemptive async. activation	non-preemptive async. activation
independent	**EDD** (Jackson '55) *O(n logn)* Optimal	**EDF** (Horn '74) $O(n^2)$ Optimal	***Tree search*** (Bratley '71) $O(n\ n!)$ Optimal
precedence constraints	**LDF** (Lawler '73) $O(n^2)$ Optimal	**EDF** * (Chetto et al. '90) $O(n^2)$ Optimal	**Spring** (Stankovic & Ramamritham '87) $O(n^2)$ Heuristic