02_a-b-c Filter

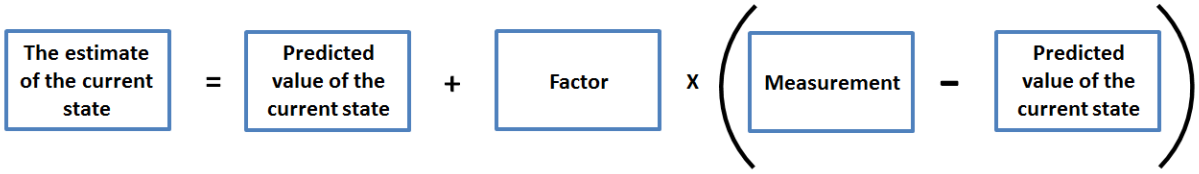
> https://www.kalmanfilter.net/alphabeta.html

1. α-Filter

Using α-Filter including two steps:

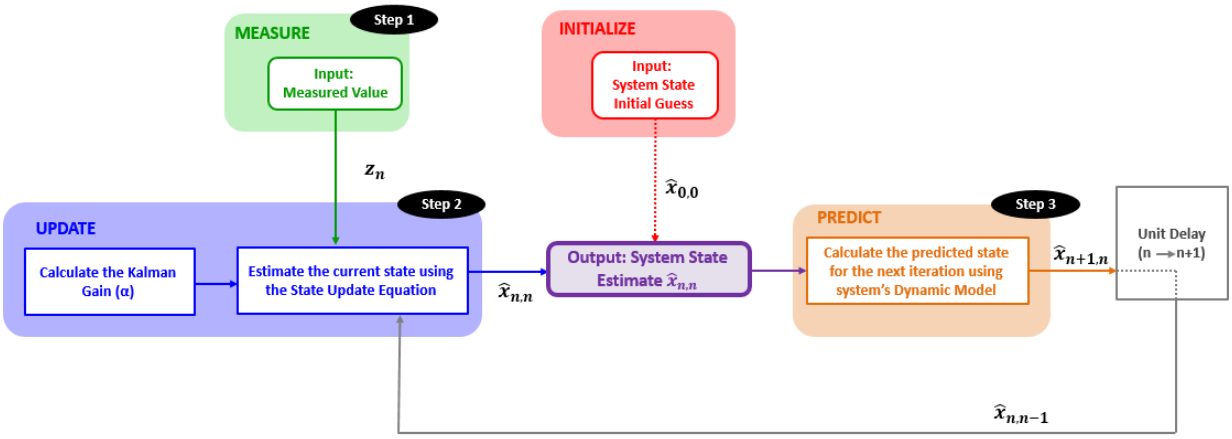
1. **Prediction**: Predict next state based on dynamic model
2. **Updating**: Update state based on real measurement

$$\hat{x}_{n,n} = \hat{x}_{n,n-1} + \alpha_n \left(z_n - \hat{x}_{n,n-1}\right)$$



The term $\left(z_n - x_{n,n-1}\right)$ is the “measurement” residual, also called the **innovation**

- In our example, $\frac{1}{N}$ decreases as N increases. This means that
  - in the beginning, we don't have enough information about the current state, thus we base the estimation on the measurements.
  - As we continue, each successive measurement has less weight in the estimation process, since $\frac{1}{N}$ decreases.
  - At some point, the contribution of the new measurements would become negligible.



2. α- β- Filter

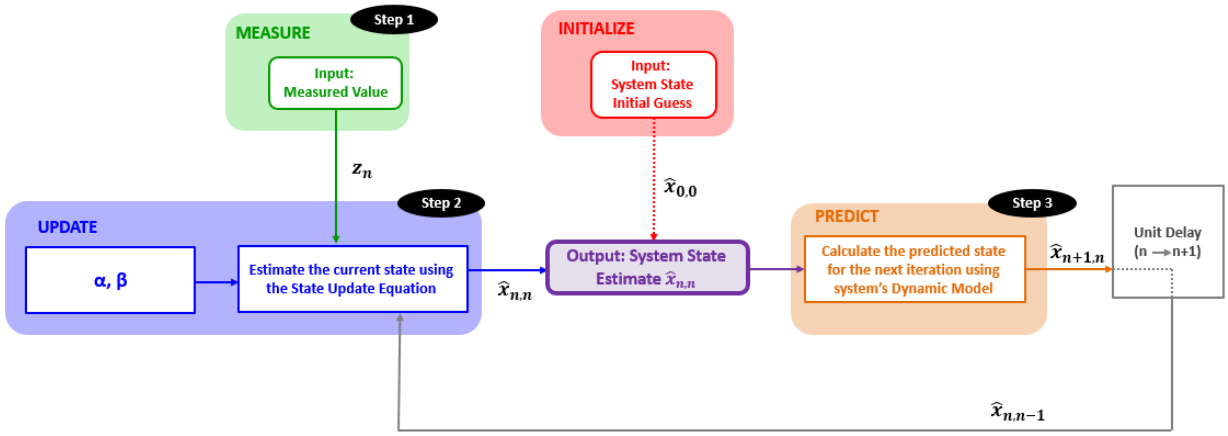
After prediction step, in α- β - Filter, the update has two steps:

- Update State (like position):

$$\hat{x}_{n,n} = \hat{x}_{n,n-1} + \alpha\left(z_n - \hat{x}_{n,n-1}\right)$$

- Update State Derivative (like velocity)

$$\hat{\dot{x}}_{n,n} = \hat{\dot{x}}_{n,n-1} + \beta\left(\frac{z_n - \hat{x}_{n,n-1}}{\Delta t}\right)$$



3. α- β- γ-Filter

After prediction, in α- β- γ-Filter, the updating has three parts:

- Update normal state (like position)

$$\hat{x}_{n,n} = \hat{x}_{n,n-1} + \alpha\left(z_n - \hat{x}_{n,n-1}\right)$$

- Update the first-order Derivative State (like velocity)

$$\hat{\dot{x}}_{n+1,n} = \hat{\dot{x}}_{n,n} + \hat{\ddot{x}}_{n,n}\Delta t$$

- Update the second-order Derivative State (like acceleration)

$$\hat{\ddot{x}}_{n,n} = \hat{\ddot{x}}_{n,n-1} + \gamma \left(\frac{z_n - \hat{x}_{n,n-1}}{0.5\Delta t^2} \right)$$

Summary

We introduced α- β- γ Filters. We used because in reality, the measurement may not be exact, so we want to combine our prediction based on model with the data from measurement. So, in these filters, after prediction:

- α Filter: tries to update directly the state
- β Filter: tries to update state and state derivative
- γ Filter: tries to update state, first-order derivative and second-order derivative