# 05_Similarity Transformation and Kalman Decomposition

1. Similarity Transformation
2. Kalman Decomposition
Case 1: Distinct Eigenvalues
Case 2: General Case
Summary

## 1. Similarity Transformation

Similarity Transformation is used to bring system to:

- Diagonal Form
- Jordan Form
- Reachable Canonical Form
- Observable Canonical Form

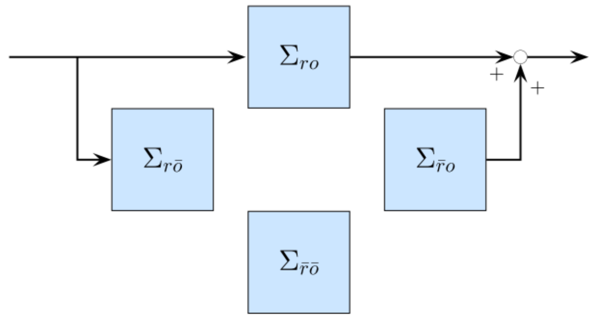
## 2. Kalman Decomposition

In control theory, a **Kalman decomposition** provides a mathematical means to convert a representation of **any linear time-invariant (LTI)** control system to a form in which the system can be **decomposed into a standard form** which makes clear the **observable and controllable(reachable)** components of the system.

### Case 1: Distinct Eigenvalues

Introduce similarity transformation such that the system becomes

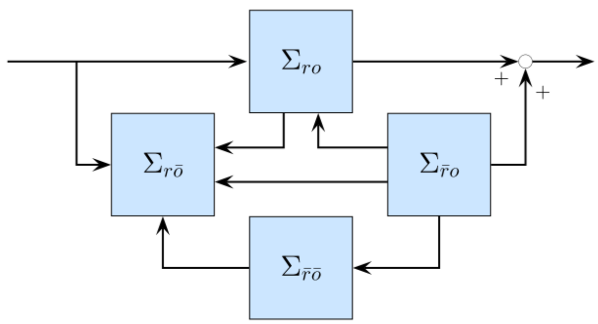
$$\dot{x}(t) = \begin{bmatrix} A_{ro} & 0 & 0 & 0 \\ 0 & A_{r\bar{o}} & 0 & 0 \\ 0 & 0 & A_{\bar{r}o} & 0 \\ 0 & 0 & 0 & A_{\overline{ro}} \end{bmatrix} x(t) + \begin{bmatrix} B_{ro} \\ B_{r\bar{o}} \\ 0 \\ 0 \end{bmatrix} u(t),$$
$$y(t) = \begin{bmatrix} C_{ro} & 0 & C_{\bar{r}o} & 0 \end{bmatrix} x(t) + Du(t),$$




## Case 2: General Case

Introduce similarity transformation such that the system becomes:

$$\dot{x}(t) = \begin{bmatrix} A_{ro} & 0 & * & 0 \\ * & A_{r\bar{o}} & * & * \\ 0 & 0 & A_{\bar{r}o} & 0 \\ 0 & 0 & * & A_{\overline{ro}} \end{bmatrix} x(t) + \begin{bmatrix} B_{ro} \\ B_{r\bar{o}} \\ 0 \\ 0 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} C_{ro} & 0 & C_{\bar{r}o} & 0 \end{bmatrix} x(t) + Du(t)$$




# Summary

- Similarity Transformation
- Kalman Decomposition: decompose subsystems based on controllability and observability