02_01_MRAC (Model Reference Adaptive Control)

> From: https://www.youtube.com/watch?v=GBBXZXmb8UE
> From: https://zhuanlan.zhihu.com/p/462662983

1. Overview

An **adaptive controller** adapts to **variations** in the process dynamics.

- environment variations
- **uncertain** dynamics
- **changing** dynamics

Other Solutions: Robust Control

Prepare enough **stability margin**, so that a single controller that can works well across all variations

- difficult with **large** uncertainty ranges

Other Solutions: Gain Scheduling

Change controller gains as system moves between states

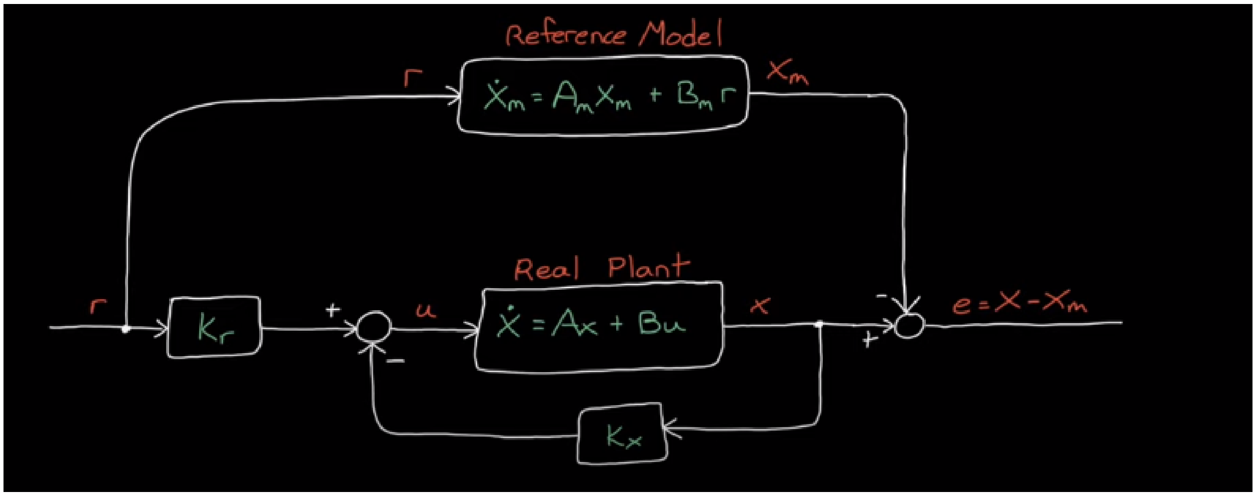
- Gain sets and states must be **known ahead** of time

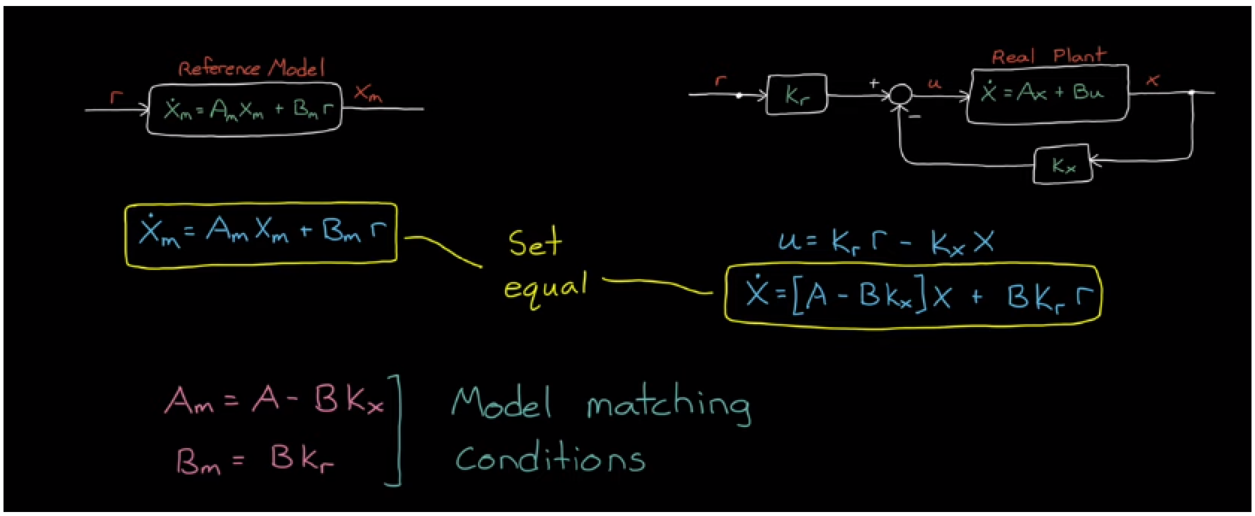
Adaptive Control

Constantly optimizes controller parameters to adapt to variations

2. Model Reference Adaptive Control

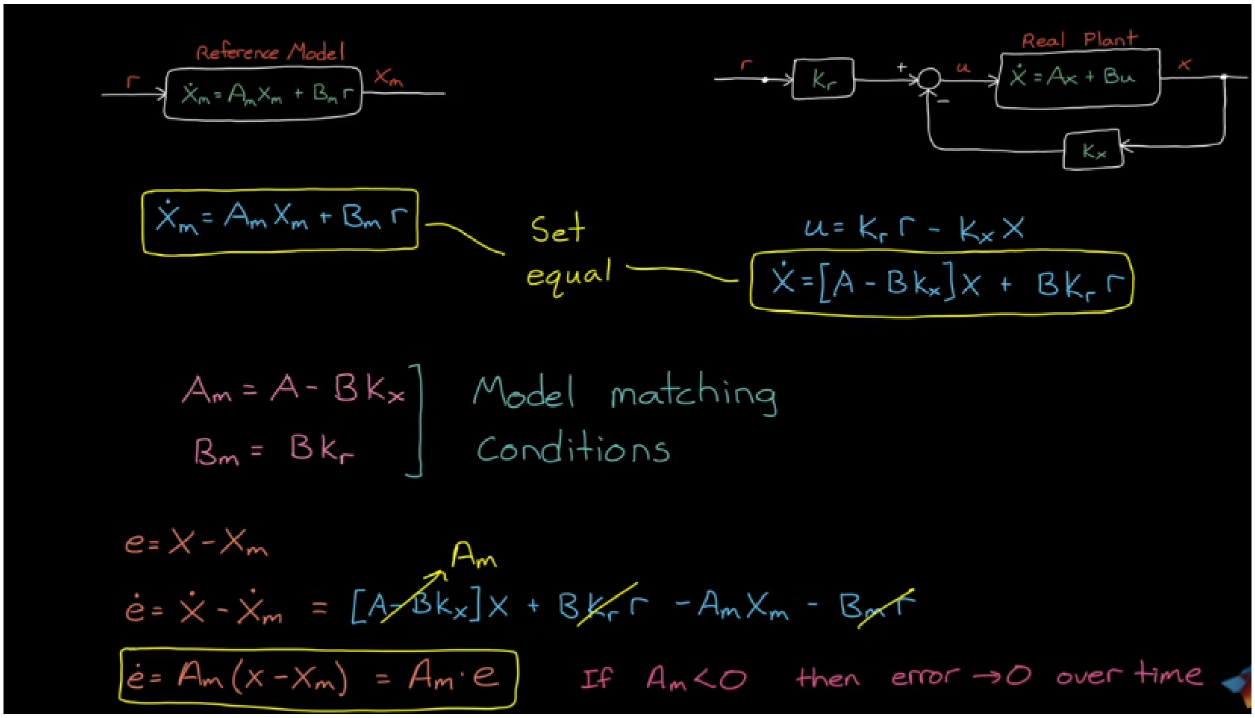
We specify a **reference model** we want the closed loop system to match





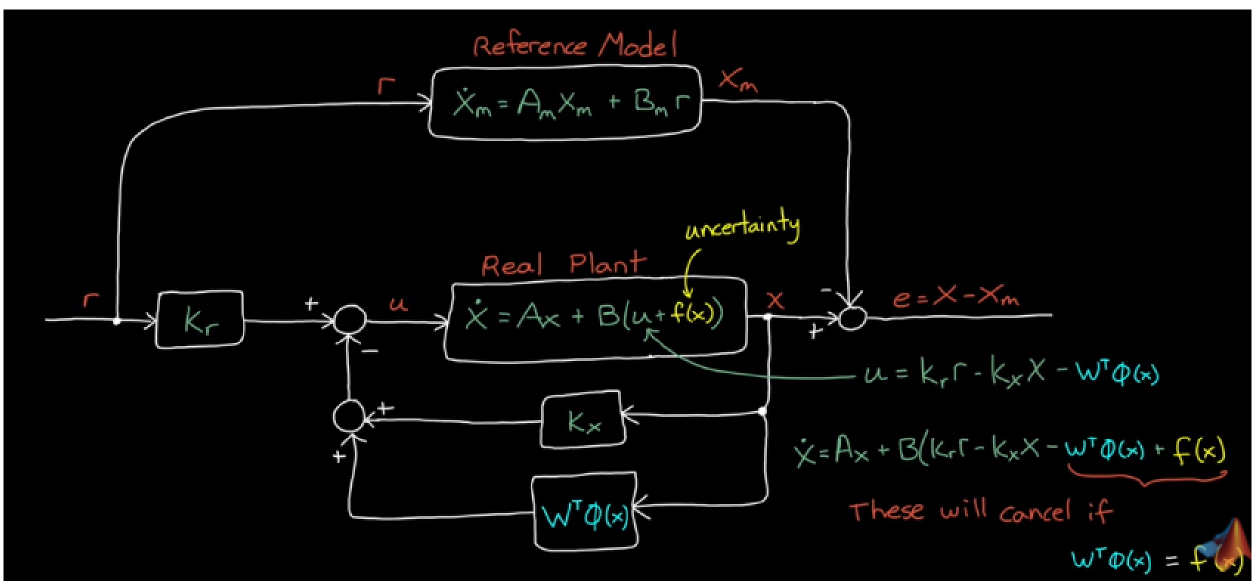
Perfect Knowledge Case

If we assume that we have the perfect knowledge of the system, then we can directly compute the setting of the controller.



Imperfect Knowledge Case

However, most times we cannot get a perfect knowledge of the system, like the example shown in the following figure.

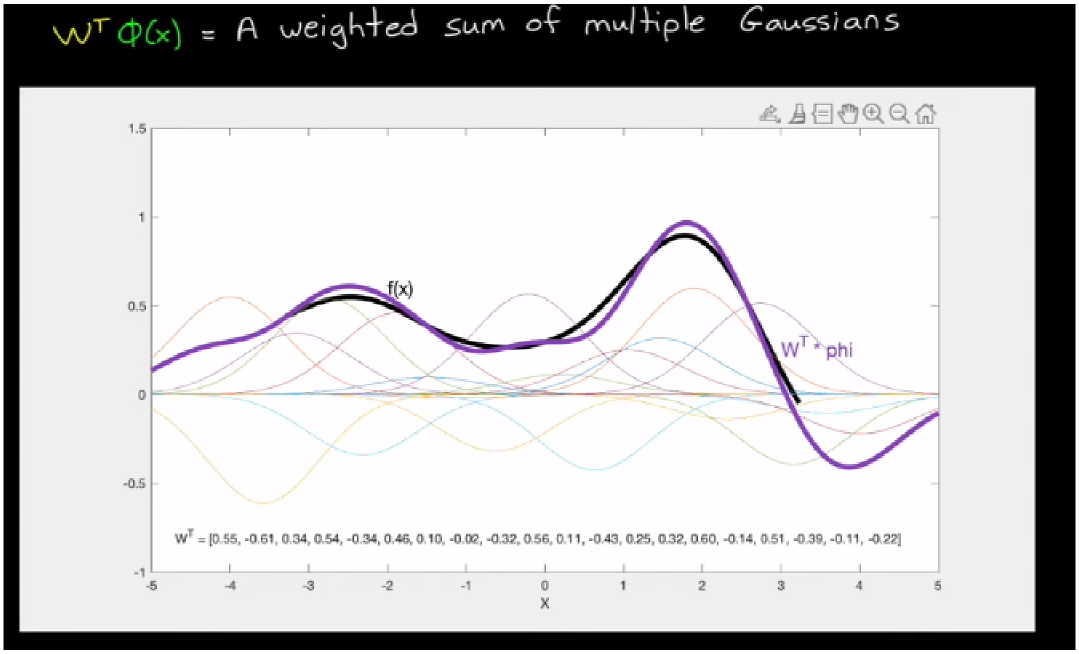


The solution is we somehow try to **learn** the system uncertainty part **online** and try to design the controller based on the learning process.

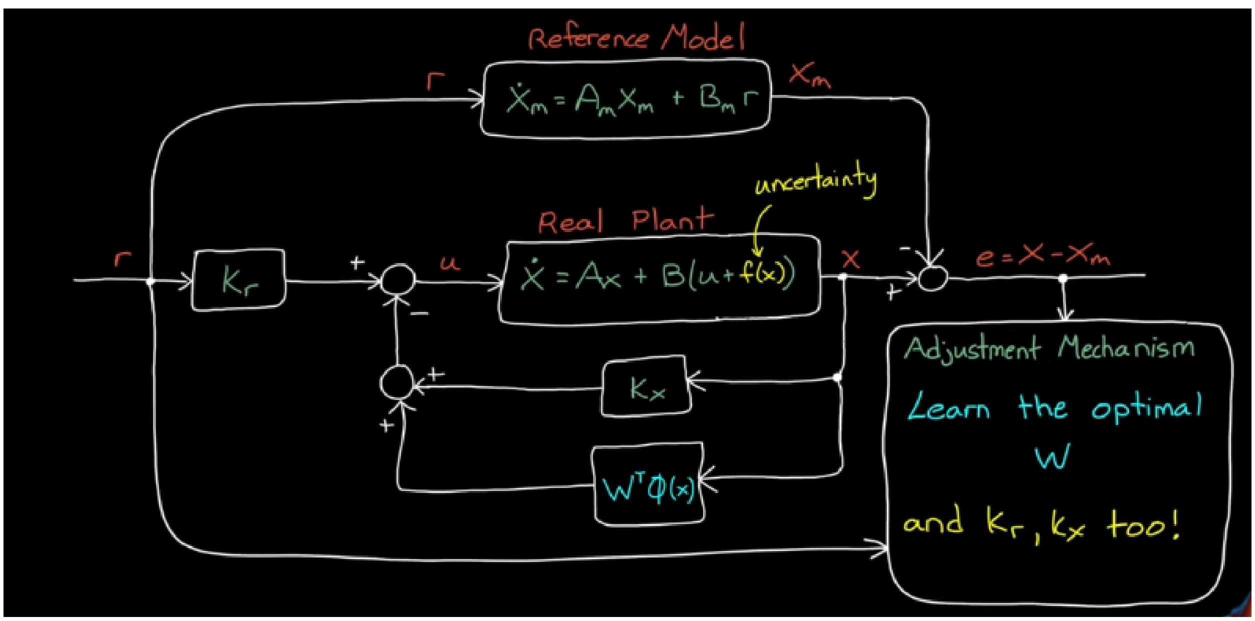
Basis Function Approximation

Radial Basis Functions

Use a set of Gaussian Functions as the basis functions.



MRAC



We can also extend it to adjust the K_r and K_x

3. Lyapunov Method For Model Reference Adaptive Control

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In this blog, there are a lot of examples of design an adaptive control controller. Here, we only summarize the simplest one to get an direct knowledge of the adaptive controller and its Lyapunov Stability Proof.

System Models

- **Original System**

$$\dot{y}(t) = a_p y(t) + k_p u(t)$$

- **Reference System**

$$\dot{y}_m(t) = a_m y_m(t) + k_m r(t)$$

- **Controller Structure:**

$$u(t) = \theta y(t) + k r(t)$$

- **Error**

$$e(t) = y(t) - y_m(t)$$

Adaptive Control Law

$$\begin{gathered} u(t) = \theta y(t) + k r(t) \\ \dot{\theta}(t) = -\operatorname{sgn}(k_p)\, \gamma_1 e(t) y(t) \\ \dot{k}(t) = -\operatorname{sgn}(k_p)\, \gamma_2 e(t) r(t) \end{gathered}$$

Lyapunov Stability Analysis

- Define the Lyapunov Stability Function:

$$\begin{gathered} V(e, \phi, \psi) = \frac{1}{2}\left[e^2 + |k_p|\left(\gamma_1^{-1}\phi^2 + \gamma_2 \psi^2\right)\right] = \frac{1}{2}\begin{bmatrix} e \\ \phi \\ \psi \end{bmatrix}^T \begin{bmatrix} 1 & 0 & 0 \\ 0 & |k_p|\,\gamma_1^{-1} & 0 \\ 0 & 0 & |k_p|\,\gamma_2^{-1} \end{bmatrix} \begin{bmatrix} e \\ \phi \\ \psi \end{bmatrix} \\ e(t) = y(t) - y_m(t), \phi = \theta - \theta^*, \quad \psi = k - k^* \end{gathered}$$

- Assume θ^* and k^* exists. If exists, they will be constant, then we will have:

$$\dot{\boldsymbol{\phi}} = \dot{\theta} - \dot{\theta}^* = \dot{\theta} = -\operatorname{sgn}(k_p)\, \gamma e(t) y(t), \dot{\boldsymbol{\psi}} = \dot{\boldsymbol{k}} - \dot{\boldsymbol{k}}^* = \dot{\boldsymbol{k}} = -\operatorname{sign}(\boldsymbol{k}_p)\, \gamma_2 e(t) r(t)$$

- Based on the Lyapunov Stability, we need to calculate the derivative of the Lyapunov Function. To do that, we need to know the $\dot{e}$, for that, we need $\dot{y}$

$$
\begin{aligned}
\dot{y} &= a_p y + k_p u = a_p y + k_p \theta y + k_p k r \\
&= a_p y + k_p \left(\phi + \theta^*\right) y + k_p \left(\psi + k^*\right) r \\
&= \left(a_p + k_p \theta^*\right) y + k_p \phi y + k_p \psi r + k_p k^* r
\end{aligned}
$$

then

$$
\dot{e} = \dot{y} - \dot{y}_m = a_m e + k_p \phi y + k_p \psi r
$$

- Then we can get the derivative of the Lyapunov Function, for a stable system, we should have $a_m \leq 0$

$$
\begin{aligned}
\dot{V} &= e\dot{e} + |k_p| \left|\gamma_1^{-1} \phi \dot{\phi} + \gamma_2^{-1} \psi \dot{\psi}\right| \\
&= a_m e^2 + k_p \phi e y + k_p \psi e r - |k_p| \left[\operatorname{sgn}\left(k_p\right) \phi e y + \operatorname{sgn}\left(k_p\right) \psi e r\right] \\
&= a_m e^2 \leq 0
\end{aligned}
$$

Then the error we gradually come to zero

Summary

- Adaptive Controller is used to deal with **variation**
- Model Reference Adaptive Control: perform like a reference model
  - perfect knowledge: direct computation
  - imperfect knowledge: **dynamically optimize to the controller and the system model**
    - basis function approximation
- Lyapunov Stability Method for MRAC Design