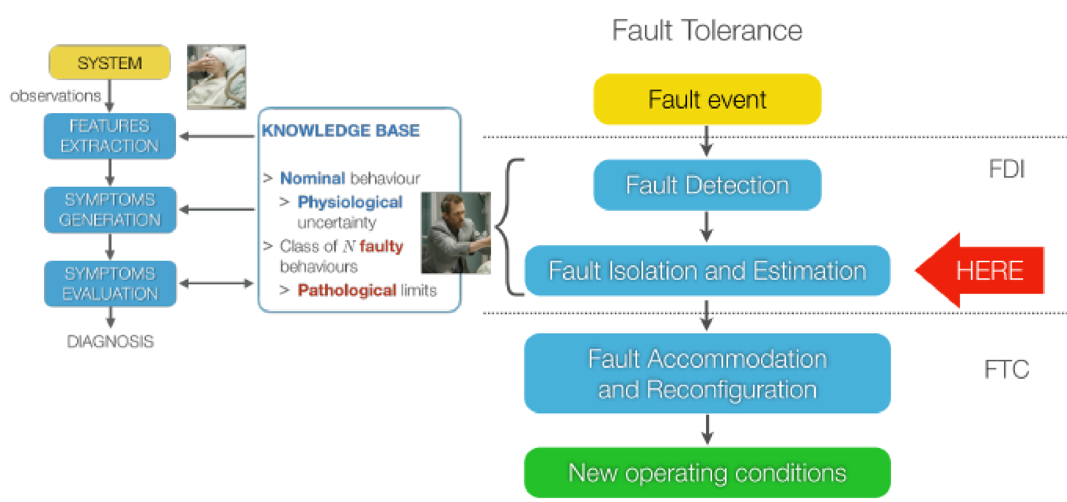
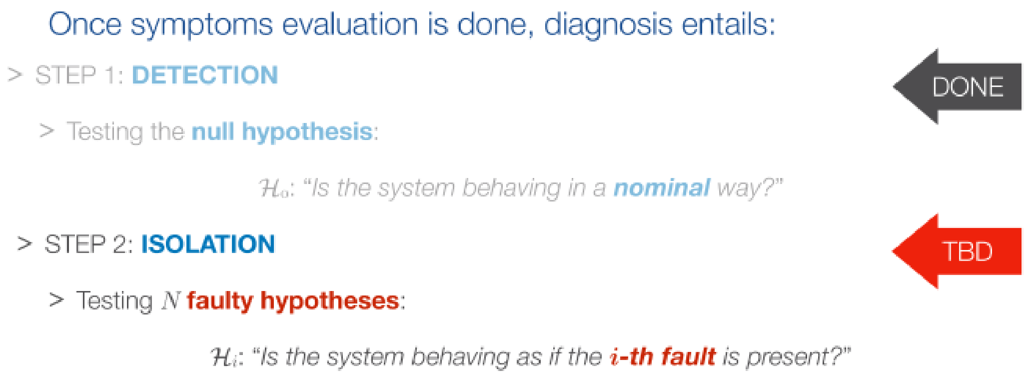
05_02_Model Based Fault Identification and Estimation

1. Overview





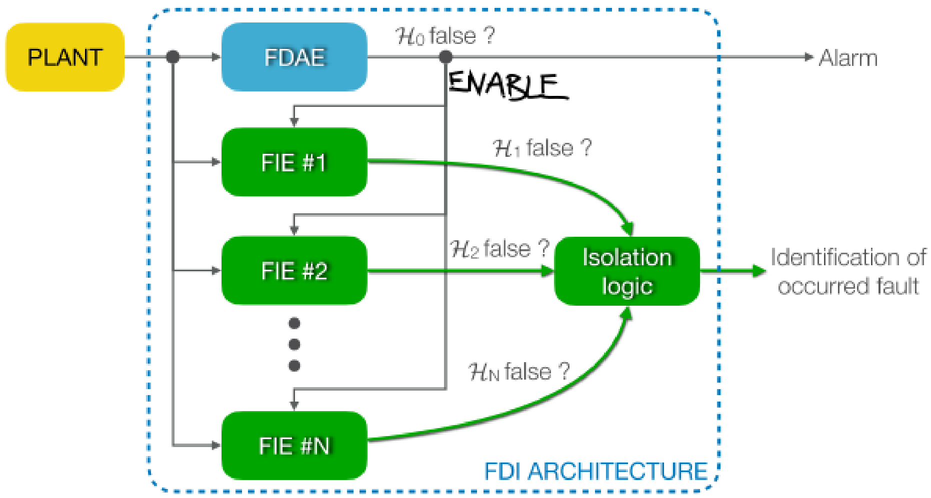
Intuition

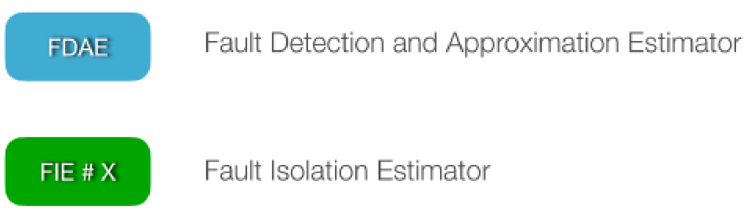
This is as "easy" as **replicating the detection block N times**

Two Approaches

- **Dedicated Observer Scheme**: each block says yes to his hypothesis, no to others (always **hard** to implement)
- **Generalized Observer Scheme**: each block says no to his hypothesis, yes to others

Generalized Observer Scheme





2. Generalized Observer for FDI

Apart from the same assumptions as for FD, we need

Assumptions

For isolation purposes, we assume that there are N types of possible nonlinear fault functions; specifically, $\phi(x, u)$ belongs to a finite set of functions given by

$$\mathscr{F} \triangleq \{\phi^1(x,u), \ldots, \phi^N(x,u)\}.$$

Each fault function in $\mathscr{F}$ is assumed to be in the form

$$\phi^l(x(k), u(k)) = \left[\left(\vartheta_1^l\right)^\top g_1^l(x(k), u(k)), \ldots, \left(\vartheta_n^l\right)^\top g_n^l(x(k), u(k))\right]^\top$$

where, for $i \in \{1, \ldots, n\}, l \in \{1, \ldots, N\}$, the **known functions** $g_i^l(x(k), u(k)) : \mathbb{R}^n \times \mathbb{R}^m \mapsto \mathbb{R}^n$ provide the "structure" of the fault, and the unknown parameter vectors $\vartheta_i^l \in \Theta_i^l \subset \mathbb{R}^{q_i^l}$ provide its "magnitude".

$$\phi = \alpha_1 \cos(x) + \beta_1 \sin(x)$$
$$+ \alpha_2 \cos(u) + \beta_2 \sin(u)$$
$$\theta = [\alpha_1 \; \alpha_2 \; \beta_1 \; \beta_2]^T$$
$$g = \begin{bmatrix} \cos(x) \\ \cos(u) \\ \sin(x) \\ \sin(u) \end{bmatrix}$$
$$\theta = [0 \; 1]$$
$$\tilde{u} = (1-\alpha)u$$
$$\theta = \alpha \quad g = -u$$

Residual Generation

$$\begin{cases} \hat{x}^l(k+1) & = f(y(k), u(k)) + \hat{\phi}^l\left(y(k), u(k), \hat{\vartheta}^l(k)\right) + \Lambda\left[\hat{y}^l(k) - y(k)\right] \\ \hat{y}^l(k) & = \hat{x}^l(k) \end{cases}$$

$$r^l(k) \triangleq y(k) - \hat{y}^l(k)$$

- **<u>Key idea</u>**: if the fault is **matched** (i.e. it is actually the l-th one) then the residual **will not exceed** the threshold and the l-th fault hypothesis will **not be falsified**

Residual Dynamics

<u>Residual Dynamics (with fault)</u>

$$x(k+1) = f(x(k), u(k)) + \eta(x(k), u(k), k) + \phi^l\left(x, u, \vartheta^\ell\right)$$
$$\hat{x}^l(k+1) = f(y(k), u(k)) + \hat{\phi}^l\left(y(k), u(k), \hat{\vartheta}^l(k)\right) + \Lambda\left[\hat{y}^l(k) - y(k)\right]$$

$$
\begin{aligned}
r^l(k+1) &= y(k+1) - \hat{y}^l(k+1) \\
&= x(k+1) + \xi(k+1) - \hat{x}^l(k+1) \\
&= f(y-\xi, n) - f(y,u) + \eta(k) + \phi^l\left(y-\xi, u, \theta^l\right) - \phi^l\left(y, u, \hat{\theta}^l\right) + \Lambda r^l(k) + \xi(k+1) \\
&= \Delta f(y,n,\xi) + \eta(k) + \xi(k+1) + \Lambda r^l(k) + \phi^l\left(y,u,\theta^l\right) - \hat{\phi}^l\left(y,u,\hat{\theta}^l\right) + \Delta\phi^l\left(y,u,\xi,\theta^l\right)
\end{aligned}
$$

where $\phi^l\left(y,u,\theta^l\right) - \hat{\phi}^l\left(y,u,\hat{\theta}^l\right) = \left(\hat{\theta}^l - \hat{\theta}^l\right)^\top g^l(y,u)$, which is called **approximation error** (wrong argument, wrong parameter)

That is:

$$
\begin{gathered}
r^l(k+1) = \Lambda r^l(k) + \delta^l(k) \triangleq \Sigma\left(r^l(k), \delta^l(k)\right) \\
\delta^l(k) \triangleq f(y(k)-\xi(k), u(k)) - f(y(k), u(k)) + \eta(k) + \xi(k+1) + \\
\phi^l\left(y(k)-\xi(k), u(k), \vartheta^l(k)\right) - \hat{\phi}^l\left(y(k), u(k), \hat{\vartheta}^l(k)\right)
\end{gathered}
$$

Fault Parameter Estimation

The residual has a **dual role**. It is used both for **fault hypothesis falsification** and for **parameter estimation**

Problem Formulation

- Adaptive term:

$$
\hat{\phi}^l_{(i)}\left(y(k), u(k), \hat{\vartheta}^l(k)\right) = \left(\hat{\vartheta}^l_i\right)^\top g^l_i(y(k), u(k))
$$

- Constraint : $\hat{\vartheta}^l_i \in \Theta^l_i$
- Goal

$$
\begin{gathered}
r^l(k) \to 0 \\
\hat{\vartheta} i^l \to \vartheta i^l
\end{gathered}
$$

Solution

The l-th FIE **adapts the fault parameter** via this **learning law (gradient descent)**

$$
\begin{array}{ll}
\text{Learning law} & \hat{\vartheta}^l_i(k+1) = \mathcal{P}_{\Theta^l_i}\left(\hat{\vartheta}^l_i(k) + \gamma^l_i(k) g^l_i(k) \Delta r^l_i(k+1)\right) \\
\text{Error term} & \Delta r^l_i(k+1) \triangleq r^l_i(k+1) - \lambda r^l_i(k) \\
\text{Learning rate} & \gamma^l_i(k) \triangleq \frac{\mu^l_i}{\varepsilon^l_i + \left\|g^l_i(k)\right\|^2}, \varepsilon^l_i > 0, 0 < \mu^l_i < 2
\end{array}
$$

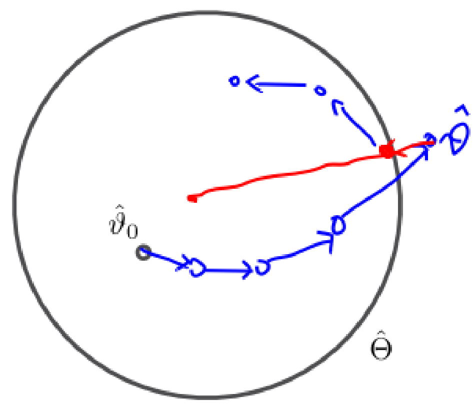
- There are three parts in the gradient part:
  - γ is the learning rate, step size
  - g is the gradient direction
  - Δr **is an auxiliary part of the step size: when far away it is large, when closer, it is small.**

- For error term: it can be seen that, the λr_i is a part in the residual dynamics (**which will cause a error but not mean we are not at optimal solution**) and other parts are some uncertainty, noise or fault part

- **Project Operator:**

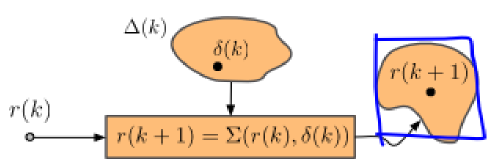
$$\mathcal{P}_{\hat{\Theta}}(\hat{\vartheta}) \triangleq \begin{cases} \hat{\vartheta} & \text{if } |\hat{\vartheta}| \leq M_{\hat{\Theta}} \\ \frac{M_{\hat{\Theta}}}{|\hat{\vartheta}|}, & \text{if } |\hat{\vartheta}| > M_{\hat{\Theta}} \end{cases}$$

drift may happen mainly because there will be period the fault not occur



Threshold Design

$$\bar{r}^l_{(i)}(k+1) = \lambda \bar{r}^l_{(i)}(k) + \bar{\delta}^l_{(i)}(k)$$
$$\bar{\delta}^l_{(i)} \triangleq \max_{\theta^l_i, \eta, \xi} \left|\delta^l_{(i)}\right|$$



- for $\bar{\delta}$ calculation, we do not use the error of the estimation, we use the whole area, that is because we do not know the true θ^l and we want to guarantee robustness, we try to maximize the error caused by inaccurate $\hat{\theta}$

Theorem: Isolability, simplified statement

Given a fault $\phi^s \in \mathscr{F}$, if for each $l \in \{1, \ldots, N\} \backslash s$ there exists some time instant $k_l > k_d$, where k_d is the **detection time**, and some $i_l \in \{1, \ldots, n\}$ such that

$$\sum_{h=k_d}^{k_l-1} \lambda^{k_l-1-h} \left|\Delta\phi^{s,l}_{i_l}(h)\right| > 2\bar{r}^l_{il} + \text{ approx. error}$$

where

$$\Delta\phi^{s,l}_{i_l}(k) \triangleq \left(1 - b^{-(k-k_0)}\right)\left(\vartheta^s_{i_l}\right)^\top g^s_{i_l}(k) - \left(\hat{\vartheta}^l_{il}\right)^\top g^l_{i_l}(k)$$

is the **fault mismatch function**, then the s-th fault will be isolated at time $\max_{l\in\{1,...,N\}\setminus s}(k_l)$.

3. Fault Learning

For fault learning, we mainly focus on:

> If $\mathcal{H}_0$ and every $\mathcal{H}_i$ are falsified ⇒ **identify** model of a new fault

Overview

We need some **general structure** to parametrize an **unknown fault**, such as:

- polynomials
- neural networks (RBF)
- wavelet networks
- etc

Parametrization and Learning

We can express a parametrized approximation for an unknown fault function as:

$$\hat{\phi}^0\left(x(k), u(k), \hat{\vartheta}^0(k)\right) \qquad \hat{\vartheta}^0(k) \in \hat{\Theta}^0 \subset \mathbb{R}^{q^0}$$

- This is **not generally linear** in the parameters
- We can anyway get the **gradient** with respect to parameters

$$Z(k) \triangleq \frac{\partial}{\partial \hat{\vartheta}^0} \hat{\phi}^0\left(x(k), u(k), \hat{\vartheta}^0(k)\right) \in \mathbb{R}^{n \times q^0}$$

Fault Detection and Approximation Estimator

- Observer:

$$\begin{cases} \hat{x}^0(k+1) & = f(y(k), u(k)) + \hat{\phi}^0\left(y(k), u(k), \hat{\vartheta}^0(k)\right) \\ \hat{y}^0(k) & = \hat{x}^0(k) \end{cases}$$

- Residual

$$r^0(k) \triangleq y(k) - \hat{y}^0(k)$$

Online Fault Learning

- Learning Law:

$$\hat{\vartheta}^0(k+1) = \mathcal{P}_{\hat{\Theta}^0}\left(\hat{\vartheta}^0(k) + \gamma^0(k) Z^\top \Delta r^0(k+1)\right)$$

- Error Term:

$$\Delta r^0(k+1) \triangleq r^0(k+1) - \lambda r^0(k)$$

- Learning Rate:

$$\gamma^0(k) \triangleq \frac{\mu^0}{\varepsilon^0 + \|Z\|_F^2}, \varepsilon^0 > 0, 0 < \mu^0 < 2$$

Summary

- Observer:
  - **Generalized Observer: Say no to its fault**
  - Dedicated Observer
- Generalized Observer for FDI
  - Scheme
  - Residual Generation: **consider fault approximation**
  - **Fault Parameter Estimation: Gradient Descend**
  - Threshold Design
- Fault Learning
  - General Structure for unknown fault
  - FDAE: learning by gradient descend