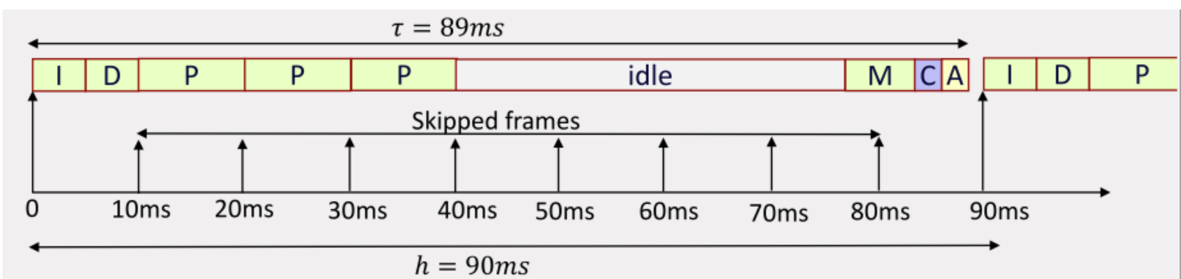
2_6_Workload_Variation_Switched_System_and_Stability

For workload variation case, we will still use the example IBC system used in last part. But now, we assume we assume the workload of the RolMerging step will vary during execution.

1. Timing Analysis

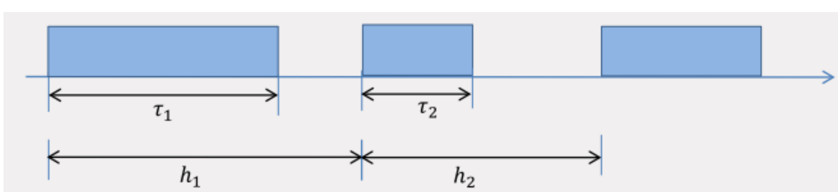
Constant Delay Case



Simplest, we can just use the worst-case model and set a constant sensor-to-actuator delay, which means, the system will have some **idle time**

Variable Delay

If we do not use worst-case model, then we will introduce variable delay case.

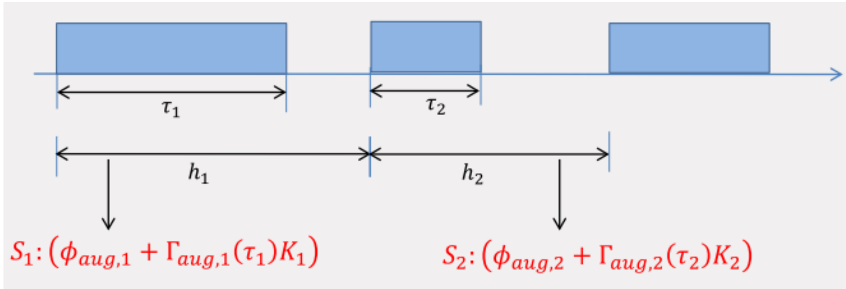


System closed-loop dynamic keeps switching between different (sampling period and delay) combinations:

$$(h_1, \tau_1) \to (h_2, \tau_2) \to (h_3, \tau_3) \to (h_4, \tau_4) \to (h_5, \tau_5) \dots$$

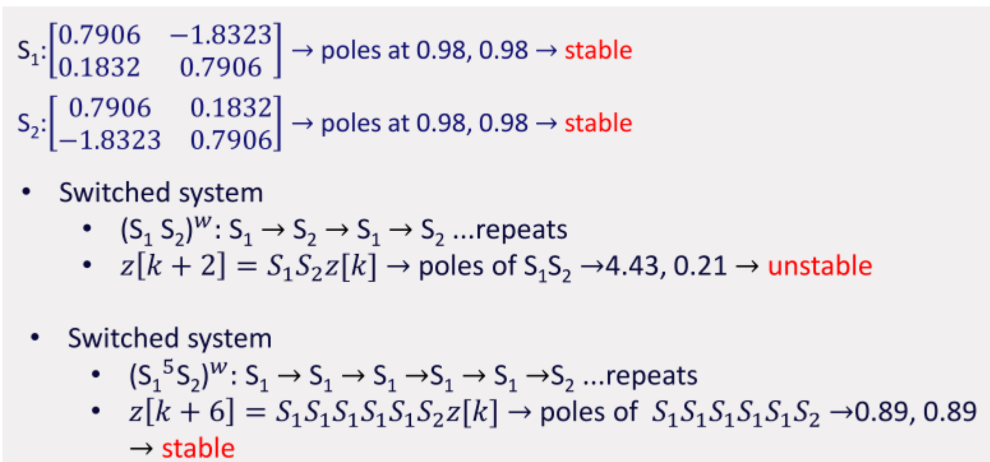
Switched System

Then we will have a switched system with switched state-space model



Problem of Switched System

A big problem of Switched System is that sometimes we cannot guarantee the stability easily, like shown in the following graph



From the two discussed cases, we can find that the stability of the such system depends on exact order of switching, even though each sub-system is stable

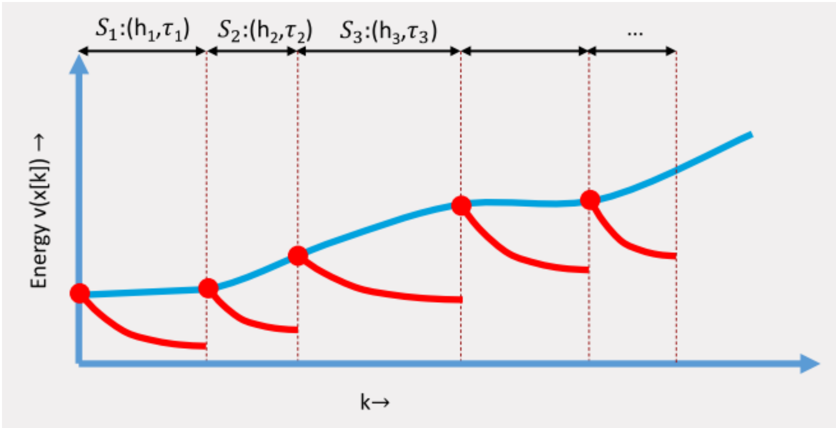
2. Stability Analysis

For switched system, we need analyze its stability:

- For switching pattern that is known at the design time
  - Compute System Poles and check for stability directly
- Switching pattern are unknown at the design time
  - We need to use some analysis tools:
    - Common Quadratic Lyapunov Function (CQLF)
    - Switched Lyapunov Function (SLF)
    - Multiple Lyapunov Function (MLF)

Instability due to Switching

From energy lypunov perspective, the instability due to switching may be illustrated in the following graph



In each sampling period, the energy decrease, but it increase at each sampling point

Common Quadratic Lyapunov Functions (CQLF) Analysis

Theorem: Common Quadratic Lyapunov Functions (CQLF)

Assume we have two systems:

- System A: $x[k+1] = A_1 x[k]$
  - Lyapunov Function for System A $V_A(x[k]) = x^T[k]P_1 x[k]$
- System B: $x[k+1] = A_2 x[k]$
  - Lyapunov Function for System B $V_B(x[k]) = x^T[k]P_2 x[k]$

If the system has a common P, i.e. $P_1 = P_2 = P$, which means $V_A(x[k]) = V_B(x[k]) = \mathrm{V}(x[k])$, then

There exists a $P = P^T > 0$ such that $V(x[k]) = x^T[k]Px[k]$ is Lyapunov function for both System A and B; this is called **CQLF**.

Calculation of CQLF: LMIs

The switched system is **stable for any arbitrary switching** between systems A_i **if** there exists solution P for the following Linear Matrix Inequalities (LMIs)

$$P = P^T > 0$$
$$P - A_i^T P A_i > 0 \quad \forall i$$

Summary

- If we do not use WCET model, we may have a switched system
  - hard to guarantee stability even if each sub-system is stable
- Stability Analysis of Switched System
  - CQLF Analysis