Part 1.

$$\hat{x}_k = \hat{x}_{k-1} + K_k \cdot (y_k - H_k \cdot \hat{x}_{k-1})$$

$$= (I - K_k \cdot H_k) \cdot \hat{x}_{k-1} + K_k \cdot \hat{y}_k$$

$$\therefore \hat{P}_k = (I - K_k \cdot H_k) \cdot \hat{P}_{k-1} (I - K_k \cdot H_k)^T + K_k \cdot R_k \cdot K_k^T$$

$$= \hat{P}_{k-1} - K_k \cdot H_k \cdot \hat{P}_{k-1} + K_k H_k \cdot \hat{P}_{k-1} (K_k H_k)^T - \hat{P}_{k-1} (K_k H_k)^T$$
$$+ K_k \cdot R_k \cdot K_k^T$$

Target: minimize least square error:

$$\min \sum (\hat{x}_{k,t} - x_{k,t})^2 = \min Tr(\hat{P}_k)$$

$$Tr(\hat{P}_k) = Tr\left[(I - K_k H_k) \cdot \hat{P}_{k-1} (I - K_k H_k)^T\right] + Tr\left[K_k \cdot R_k \cdot K_k^T\right]$$

For $\min Tr(\hat{P}_k)$, we need $\frac{\partial Tr(\hat{P}_k)}{\partial K_k} = 0$

$$\frac{\partial Tr(\hat{P}_k)}{\partial K_k} = \frac{\partial Tr\left[(I - K_k H_k) \cdot \hat{P}_{k-1} (I - K_k H_k)^T\right]}{\partial K_k} + \frac{\partial Tr(K_k \cdot R_k \cdot K_k^T)}{\partial K_k}$$

$$= \frac{\partial Tr(\hat{P}_{k-1})}{\partial K_k} - 2 \frac{\partial Tr(K_k H_k \cdot \hat{P}_{k-1})}{\partial K_k}$$
$$+ \frac{\partial Tr(K_k H_k \cdot \hat{P}_{k-1} H_k^T K_k^T)}{\partial K_k} + \frac{\partial Tr(K_k \cdot R_k K_k^T)}{\partial K_k}$$

$$= -2 \cdot \hat{P}_{k-1}^T \cdot H_k^T + 2 \cdot K_k \cdot H_k \cdot \hat{P}_{k-1} H_k^T + 2 K_k \cdot R_k$$

$$\therefore \quad K_k = \hat{P}_{k-1} \cdot H_k^T \cdot (H_k \hat{P}_{k-1} H_k^T + R_k)^{-1}$$