1 Task1

1.1 Task1.(a)

1.1.1 Assumption

A maximum linear programming problem was described in this question. We assume that:

1. We will install central air conditioner K, the number of which is x, and split-type air conditioner Y, the number of which is y.
2. We only install air conditioners once.
3. Although x and y is integer, we consider them as non-negative real number first.

1.1.2 Analysis

We can easily find that there are two constrains, namely, budget constrains and amount constrains.

1. We can not install more than 12 air conditioners or install a negative number of air conditioners.
2. Total budget for all installation is $24000 + 300E1$ Euro, where $E1 = 9$.

1.1.3 Model

According to analysis above, we can formalize this optimization problem, we have:

$$\begin{gathered} max_{x,y} \quad 4*x + 2.5*y \\ s.t. \quad x + y \leq 12 \\ 3000*x + 1500*y \leq 24000 + 300*E1 \\ x, y \geq 0 \end{gathered} \tag{1}$$

Obviously, model above isn't a standard form of linear programming problem, we can transform it into standard form, and we have:

$$\begin{gathered} -min_{x,y,s_1,s_2} \quad -4*x - 2.5*y \\ s.t. \quad x + y + s_1 = 12 \\ 3000*x + 1500*y + s_2 = 24000 + 300*E1 \\ x,\ y,\ s_1,\ s_2 = 0 \end{gathered} \tag{2}$$

Formula (2) is a standard form of LP problem described in Task1.(a).

1.2 Task1.(b)

When solving problem in Task1.(a) by using MATLAB, we will use following function:

$$[x, val, flag] = linprog(c, A, b, Aeq, beq, lb, ub, options) \tag{3}$$

Formula (3) represent solution of following questions:

$$\begin{aligned} min_x \quad & f * X \\ A * X &= b \\ Aeq * X &= beq \\ lb \leq X &\leq ub \end{aligned} \tag{4}$$

In formula (3) x represent the optimization of independent variable, val represent the optimized outcome, $flag$ is a sign of whether the problem has a solution. When the $flag$ is 1, it means that the problem has an optimal solution. If the $flag$ is 0, the problem has no optimal solution.
We can easily tell that, in Task1.(b),:

$$f = \begin{bmatrix} -4 & -2.5 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 \\ 3000 & 1500 \end{bmatrix}$$

$$b = \begin{bmatrix} 12 \\ 24000 + 300 * E1 \end{bmatrix}$$

$$lb = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

$$ub = \begin{bmatrix} inf & inf \end{bmatrix}$$

$$Aeq = beq = [\,] \tag{5}$$

1.2.1 Solution

MATLAB code, which is not shown here, will be uploaded as an attachment in the form of .m file. The final result is we install 6 X air conditioners, and 6 Y air conditioner, which leads to a maximum power, which is $39kW$

2 Task3

2.1 Analysis

In order to have a more accurate indoor temperature discrete model, we want the finest coefficient of a_1, a_2, a_3. We can achieve that by minimizing the square

of the error between real $T_{b,k+1}$ and estimation.
From 2.1.2, we can rewrite the formula:

$$\sum_{i=1}^{N} \{[(T_{b,k+1} - T_{b,K}) - \Delta t * (a_1 * q_{\dot{solar},k} + a_2 * (q_{\dot{occ},k} + q_{\dot{ac},k} - q_{\dot{vent},k}) + a_3 * (T_{amb,k} - T_{b,k})]\} \tag{6}$$

We set:

$$\begin{aligned} T &= (T_{b,k+1} - T_{b,K}) \\ (q_{\dot{occ},k} + q_{\dot{ac},k} - q_{\dot{vent},k}) &= (q_{\dot{sigma},k}) \\ T_{amb,k} - T_{b,k} &= T_{sigma,k} \\ (a_1 * q_{\dot{solar},k} + a_2 * (q_{\dot{sigma},k}) + a_3 * (T_{sigma,k}) &= \begin{bmatrix} q_{\dot{solar},k} & q_{\dot{sigma},k} & T_{\dot{sigma},k} \end{bmatrix} * \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \\ \begin{bmatrix} q_{\dot{solar},k} & q_{\dot{sigma},k} & T_{\dot{sigma},k} \end{bmatrix} &= \begin{bmatrix} \dot{q_k} \end{bmatrix} \\ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} &= \begin{bmatrix} a \end{bmatrix} \end{aligned} \tag{7}$$

We can rewrite (6) to:

$$\begin{aligned} &\begin{bmatrix} T - [\dot{q}] * [a] \end{bmatrix}^T * \begin{bmatrix} T - [\dot{q}] * [a] \end{bmatrix} \\ &= a^T \dot{q}^T \dot{q} a - 2T^T \dot{q} a + T^T T \end{aligned} \tag{8}$$

2.2 Model

This is a quadratic programming problem, standard form of a quadratic programming is:

$$\begin{aligned} minimize_x \quad & \frac{1}{2} x^T H x + c^T x \\ Ax = b & \\ x \geq 0 & \end{aligned} \tag{9}$$

For formula (10), it is a standard form of quadratic programming problem, where:

$$\begin{aligned} H &= 2\dot{q}^T \dot{q} \\ c^T &= 2T^T \dot{q} \end{aligned} \tag{10}$$

2.3 solution

We can use function in MATLAB to solve it. Details of each matrix are store in MATLAB, as a result, we only show final result here:

$$
\begin{aligned}
a_1 &= 2.662 \times 10^{-6} \\
a_2 &= 1.056 \times 10^{-6} \\
a_3 &= 2.135 \times 10^{-6}
\end{aligned} \tag{11}
$$