Nonlinear Programming Assignment

SC42056 Optimization for Systems and Control

E_1, E_2 and E_3 are parameters changing from 0 to 18 for each group according to the sum of the last three numbers of the student IDs:

$$E_1 = D_{a,1} + D_{b,1}, \quad E_2 = D_{a,2} + D_{b,2}, \quad E_3 = D_{a,3} + D_{b,3}$$

with $D_{a,3}$ the right-most digit of the student ID of the first student, $D_{b,3}$ the right-most digit of the student ID of the other student, $D_{a,2}$ the one but last digit of the student ID of the first student, etc.

Important: Please note that all questions regarding this assignment should be asked via the Brightspace Discussion forum.

Note: The texts in blue are clarifications and corrections that have already been provided in the Brightspace Discussion forum.

In recent years, the number of vehicles has grown larger and larger, and even though large and sound traffic networks (freeways and roads) have already been constructed, traffic congestion still cannot be avoided efficiently. Moreover, it is often too time- and money-consuming to build new transportation infrastructures or to reconstruct the ones that already exist. Therefore, traffic jams occur frequently and have a severe impact, when a lot of people need to use the traffic infrastructures with limited capacity at the same time, especially during rush hours. Traffic congestion can give rise to traffic delays, economic losses, traffic pollution, and so on. To reduce traffic jams and to promote efficiency in traveling, effective traffic control methods are necessary. In this context, traffic control strategies are one of the most efficient and also effective methods to solve the problem.

Here we consider traffic signal control, which is the main control measure used in cities. In this assignment, we consider a network that consists of two intersections, as shown in Fig. 1.

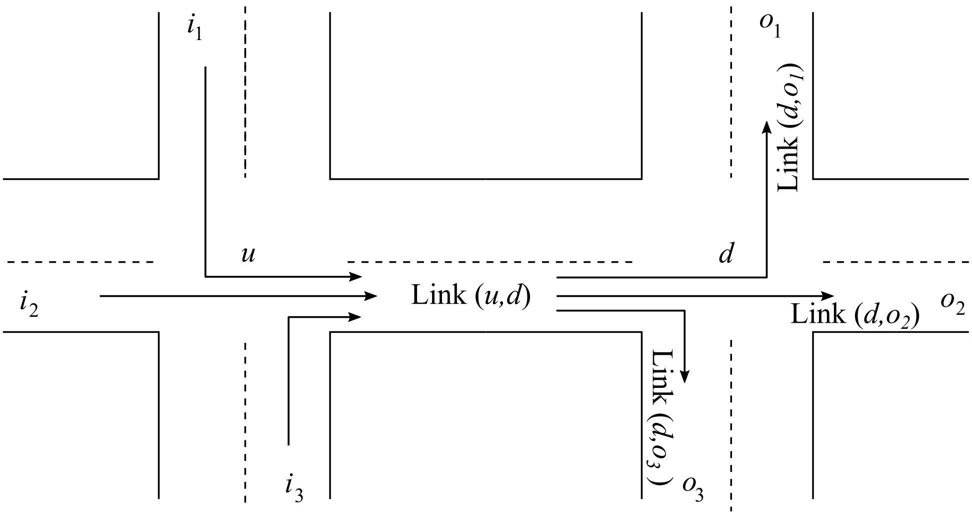

Figure 1: Network with two intersections.

In order to describe the urban traffic model [Reference: S. Lin, Efficient model predictive control for large-scale urban traffic networks, TRAIL Thesis Series, the Netherlands TRAIL Research School, 2011], we define J as the set of nodes (intersections), and L as the set of links (roads) in the urban traffic network. Link (u, d) is marked by its upstream node u ($u \in J$) and downstream node d ($d \in J$). The sets of the upstream nodes of input links and downstream nodes of output links for link (u, d) are denoted as $I_{u,d} \subset J$ and $O_{u,d} \subset J$. In our situation, we have $I_{u,d} = \{i_1, i_2, i_3\}$ and $O_{u,d} = \{o_1, o_2, o_3\}$, and each link has three lanes where vehicles can turn left, go straight, and turn right respectively.

In this assignment we consider the traffic signal control of node d, while the traffic flow that crosses node u to link (u, d) is given a priori as an input signal. The cycle of the traffic light in node d is divided into 2 phases: the green-light phase when the vehicles in link (u, d) can cross the intersection, and red-light phase when the vehicles in link (u, d) are not allowed to cross the intersection. In this particular case, the vehicles that turn right are not influenced by the traffic signal, as shown in Fig. 2.

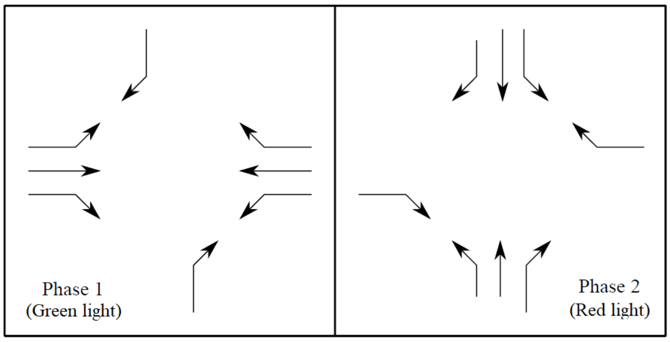

Figure 2: Signal phases of traffic line control intersections.

Taking c as the length of the simulation time interval and k as the corresponding time step counter, the number of the vehicles in any link is updated according to the input and output average flow rate over the entire cycle of length c at every time step k by

$$n_{u,d}(k+1) = n_{u,d}(k) + c\left(\alpha_{u,d}^{\text{enter}}(k) - \alpha_{u,d}^{\text{leave}}(k)\right) \quad . \tag{1}$$

In the remainder we take link (u, d) for example and build the model of traffic flow. The leaving average flow rate $\alpha_{u,d}^{\text{leave}}(k)$ is the sum of the leaving flow rates turning to each output link:

$$\alpha_{u,d}^{\text{leave}}(k) = \sum_{o \in O_{u,d}} \alpha_{u,d,o}^{\text{leave}}(k) \quad . \tag{2}$$

The leaving average flow rate over cycle time c is determined by the capacity of the intersection, the number of cars waiting and/or arriving, and the available space in the output link. In particular, $\alpha_{u,d,o}^{\text{leave}}(k)$ is expressed as:

$$\alpha_{u,d,o}^{\text{leave}}(k) = \min\left(\mu_{u,d,o} \cdot g_d(k)/c,\ q_{u,d,o}(k)/c + \alpha_{u,d,o}^{\text{arriv}}(k),\ C_{d,o}(k)/c\right) \quad , \tag{3}$$

where $C_{d,o}(k)$ is the (possibly time-varying) available space in the corresponding downstream link (d, o). Note that the lanes turning right is not influenced by the traffic light; so e.g., $g_{u,d,o_3}(k) = c$ for all k.

The number of vehicles waiting in the queue turning to o is updated as

$$q_{u,d,o}(k+1) = q_{u,d,o}(k) + \left(\alpha_{u,d,o}^{\text{arriv}}(k) - \alpha_{u,d,o}^{\text{leave}}(k)\right) \cdot c \quad . \tag{4}$$

Then, the number of waiting vehicles in link (u,d) is

$$q_{u,d}(k) = \sum_{o \in O_{u,d}} q_{u,d,o}(k) \quad . \tag{5}$$

The vehicles that entered link (u,d) will arrive at the tail of the queues after a time delay $\tau(k) \cdot c + \gamma(k)$, i.e.,

$$\alpha^{\text{arriv}}_{u,d}(k) = \frac{c-\gamma(k)}{c} \cdot \alpha^{\text{enter}}_{u,d}(k-\tau(k)) + \frac{\gamma(k)}{c} \cdot \alpha^{\text{enter}}_{u,d}(k-\tau(k)-1), \tag{6}$$

where

$$\tau(k) = \text{floor}\left\{ \frac{(C_{u,d}(k) - q_{u,d}(k)) \cdot l_{\text{veh}}}{N^{\text{lane}}_{u,d} \cdot v^{\text{free}}_{u,d} \cdot c} \right\} \quad , \tag{7}$$

$$\gamma(k) = \text{rem}\left\{ \frac{(C_{u,d}(k) - q_{u,d}(k)) \cdot l_{\text{veh}}}{N^{\text{lane}}_{u,d} \cdot v^{\text{free}}_{u,d}}, c \right\} \quad , \tag{8}$$

and floor$\{x\}$ refers to the largest integer smaller than or equal to x, and rem$\{x,y\}$ is the remainder of the division of x by y.
For link (u,d) the maximum capacity $C_{u,d}(k)$ is constant in this assignment; it can be calculated using the parameters of the link:

$$C_{u,d}(k) = N^{\text{lane}}_{u,d} \cdot l_{u,d}/l_{\text{veh}} \qquad \text{for all } k \quad .$$

Note that for other links we will consider a time-varying maximum capacity (see below) due to, e.g., traffic incidents that may temporarily block part of the road.
Before reaching the tail of the waiting queues in link (u,d), the flow rate of arriving vehicles needs to be divided according to the turning rates:

$$\alpha^{\text{arriv}}_{u,d,o}(k) = \beta_{u,d,o}(k) \cdot \alpha^{\text{arriv}}_{u,d}(k) \quad . \tag{9}$$

One should note that equations (1)–(9) were particularly formulated for link (u,d). Therefore, a similar system of equations should be formulated in order to model the traffic in link (o_1,d). Then the total time spent (TTS) by the drivers on the link (u,d) and (o_1,d) is taken as the output of the system:

$$y(k) = c(n_{u,d}(k) + n_{o_1,d}(k)) \quad . \tag{10}$$

The meaning of the symbols appearing in the models equations is:

c	:	simulation time interval for intersection d,
k	:	simulation step counter for intersection d,
$n_{u,d}(k)$(veh)	:	number of vehicles in link (u,d) at time step k,
$N^{\text{lane}}_{u,d}$	:	number of lanes in link (u,d),
$v^{\text{free}}_{u,d}$(km/h)	:	free flow vehicle speed in link (u,d),
l_{veh}(m/veh)	:	average vehicle length,
$l_{u,d}$(m)	:	length of link (u,d)
$q_{u,d}(k)$(veh)	:	queue length (expressed as the number of vehicles) at step k in link (u,d), $q_{u,d,o}$ is the queue length of the sub-stream turning to link (d,o),
$\alpha^{\text{leave}}_{u,d}(k)$ (veh/h)	:	average flow rate leaving link (u,d) at step k, $\alpha^{\text{leave}}_{u,d,o}(k)$ is the leaving average flow rate of the sub-stream going toward link (d,o),
$\alpha^{\text{arriv}}_{u,d}(k)$ (veh/h)	:	average flow rate arriving at the tail of the queue in link (u,d) at step k, $\alpha^{\text{arriv}}_{u,d,o}(k)$ is the arriving average flow rate of the sub-stream going toward link (d,o),

$\alpha_{u,d}^{\text{enter}}(k)$ (veh/h) : average flow rate entering link (u, d) at time step k,
$\beta_{u,d,o}(k)$: fraction of the traffic in link (u, d) anticipating to turn to link (d, o) at step k,
$\mu_{u,d,o}$(veh/h) : saturation flow rate leaving link (u, d) turning to link (d, o),
$C_{u,d}(k)$(veh) : the storage capacity of link (u, d),
$g_{u,d,o}(k)$(s) : green time length during time interval $[kc,\ (k+1)c]$ for the traffic stream towards link (d, o) in link (u, d).

Tasks:

1. Formulate the discrete-time state-space model that predicts the number of vehicles on link (u, d) and link (o_1, d) for the next simulation time step $k + 1$ as follows:

$$\begin{cases} x(k+1) = f(x(k, u(k))) \\ y(k) = g(x(k), u(k)) \end{cases} \tag{11}$$

where f and g are vector-valued, nonlinear functions.

Hint: Choose the states properly so that the output can be expressed by states; the input should be the time length of green light at every cycle time. All the parameters have to be expressed with the same units: h, km, veh, ...

The parameters of the link (u, d) are given in Table 1.

Table 1: Parameters of link (u, d)

c	$N_{u,d}^{\text{lane}}$	$v_{u,d}^{\text{free}}$	l_{veh}	$l_{u,d}$	β_{u,d,o_1}	β_{u,d,o_2}	β_{u,d,o_3}	μ_{u,d,o_1}	μ_{u,d,o_2}	μ_{u,d,o_3}
60 s	3	50 $\frac{\text{km}}{\text{h}}$	7 m	1000 m	0.33	0.34	0.33	1600 $\frac{\text{veh}}{\text{h}}$	1800 $\frac{\text{veh}}{\text{h}}$	1500 $\frac{\text{veh}}{\text{h}}$

The traffic flow coming from upstream link of link (u, d) is given as follows:

$$\alpha_{u,d}^{\text{enter}}(k) = \begin{cases} 1800 + 10 \cdot E_1 \text{ veh/h} & \text{if } k \leq 20 \\ 2100 + 10 \cdot E_2 \text{ veh/h} & \text{if } 20 < k \leq 40 \\ 2300 + 10 \cdot E_3 \text{ veh/h} & \text{if } k > 40 \end{cases}$$

Assume the available space in the downstream link are all known, and capacity of downstream links of link (u, d) is given as follows:

$$C_{d,o_1}(k) = \begin{cases} 40 + E_1 \text{ veh} & \text{if } k \leq 20 \\ 40 + E_1 - 2 \cdot (k - 20) \text{ veh} & \text{if } 20 < k \leq 35 \\ 10 + E_1 \text{ veh} & \text{if } 35 < k \leq 45 \\ 10 + E_1 + 2 \cdot (k - 45) \text{ veh} & \text{if } k > 45 \end{cases}$$

$$C_{d,o_2}(k) = C_{d,o_1}(k) - E_2 \text{ veh} \quad \text{for all } k$$

$$C_{d,o_3}(k) = \begin{cases} 30 - E_3 \text{ veh} & \text{if } k \leq 30 \\ 30 + E_3 \text{ veh} & \text{if } k > 30 \end{cases} .$$

Note that $C_{d,o}(k)$ should have a nonnegative value; so make $C_{d,o}$ equal to 0 if it is negative for some k for your combination of E_1, E_2, E_3. The parameters of link (o_1, d) are given in Table 2.

Table 2: Parameters of link (o_1, d)

$N_{o_1,d}^{\text{lane}}$	$v_{o_1,d}^{\text{free}}$	l_{veh}	$l_{o_1,d}$	$\beta_{o_1,d,u}$	β_{o_1,d,o_3}	β_{o_1,d,o_2}	$\mu_{o_1,d,u}$	μ_{o_1,d,o_3}	μ_{o_1,d,o_2}
3	60 $\frac{\text{km}}{\text{h}}$	7 m	1000 m	0.33	0.34	0.33	1600 $\frac{\text{veh}}{\text{h}}$	1800 $\frac{\text{veh}}{\text{h}}$	1500 $\frac{\text{veh}}{\text{h}}$

In addition, the available space for downstream links of link (o_1, d) is given as: $C_{d,o_2}(k)$ and $C_{d,o_3}(k)$ are already specified above, $C_{d,u}(k)$ is:

$$C_{d,u}(k) = \begin{cases} 40 - E_3 \text{ veh} & \text{if } k \leq 30 \\ 40 + E_3 \text{ veh} & \text{if } k > 30 \end{cases} .$$

The entering flow of link (o_1, d) is :

$$\alpha_{o_1,d}^{\text{enter}}(k) = 2000 + 10{\cdot}E_1 \text{ veh/h} \quad \text{for all } k \ .$$

2. Use the model that you built in Task 1, and formulate the optimization problem to find the time length of the green light at node d for every cycle that minimizes the Total Time Spent (TTS) by the drivers on link (u, d) and (o_1, d) for the following hour (i.e., for the period $[0, 60c]$).

   Assume that at $k = 0$ there is no traffic, i.e., the numbers of vehicles in links (u, d) and (d, o) are equal to 0, and the numbers of vehicles waiting in the queues are 0 as well. For simplicity, we consider the green time as continuous variables that are constrained in [15s, 45s]. Select an appropriate optimization algorithm, and explain your choice.

   In addition, we assume that we perfectly know the future inputs and capacities of the traffic network, as they are specified above.

3. Use the chosen optimization algorithm to optimize the traffic flows by choosing two different starting points for the green time lengths: $g_d(k) = 15\text{s}\ \forall k$ and $g_d(k) = 45\text{s}\ \forall k$, respectively. Is there a substantial difference between the obtained solutions? Why? How can the solution be improved (if possible)? Can you prove that the solution obtained is the global optimum?

4. Plot the states and inputs of your simulation in Task 3 and compare them with the ones obtained for the no-control case ($g_d(k) = 30\text{s}\ \forall k$). Analyze the traffic situation for the considered simulation period of one hour, including the number of vehicles on the links and the queue length of each lane. Moreover, find the TTS values over the entire simulation period for the different cases (including the no-control case) and compare them. Explain the results obtained.

5. Now consider that the green light time is limited for the following discrete time set: {15s, 20s, 25s, 30s, 35s, 40s, 45s}. Formulate the problem of Task 2 and solve it by using an optimization approach that can directly deal with integer optimization variables. Repeat the analysis of Task 4 and compare the results. Also add a discussion to show/plot/analyze/compare the different algorithms/approaches.

The solutions of the assignment should be uploaded to Brightspace before Friday, October 30, 2020 at 17:00 as two separate files:

1. A written report on the practical exercise as a single .pdf file (no other formats allowed).

2. A single .m file with the Matlab code you used; please make sure that the code is error free.

After uploading, please verify the uploaded files so as make sure that you have uploaded the correct files and that they are not broken.
Please also note that you will lose 0.5 point from your grade for this assignment for each (started) day of delay in case you exceed the deadline.