0.1 Analysis

The model of the problem has been clearly stated in the problem document, so the key point is to transform the model into a matrix available in MATLAB. Define the variable vector as follows:

$$x = \begin{bmatrix} \dot{q_{ac,1}} & \dot{q_{ac,2}} & \cdots & \dot{q_{ac,2160}} & T_{b,2} & T_{b,3} & \cdots & T_{b,2160} \end{bmatrix}$$

Considering the unit of Φ_k is €/kWh and the unit of $\dot{q_{ac,k}}$ is kW, so ΔT in $\sum_{k=1}^{N} \Phi_k \dot{q_{ac,k}}$ should be $\Delta t = 1(h)$.

There are two ways for finding the corresponding matrices H and C in the standard form of quadratic optimization problem.

1. Measure 1: from equation to matrices

$$\begin{aligned}
\sum_{k=1}^{N} \Phi_k \dot{q_{ac,k}} \Delta t + (0.1 + E_2/10)(T_{B,k} - T_{ref})^2 &= \sum_{k=1}^{N} \Phi_k \dot{q_{ac,k}} + \sum_{k=1}^{N} 1.4(T_k^2 + T_{ref}^2 - 2T_r T_k) \\
&= \sum_{k=1}^{N} \Phi_k \dot{q_{ac,k}} - 2.8T_r T_k + \sum_{k=1}^{N} 1.4T_{ref}^2 + \sum_{k=1}^{N} 1.4T_k^2 \\
&= \sum_{k=1}^{N} 1.4T_{ref}^2 + C^T x + \frac{1}{2} x^T H x
\end{aligned} \tag{1}$$

so,

$$C = [\underbrace{\Phi_1 \ \cdots \ \cdots \ \Phi_{2160}}_{N} \ \ \underbrace{-2.8T_{ref} \ \cdots \ \cdots \ -2.8T_{ref}}_{N-1}]^T$$

$$H = \begin{array}{c} \begin{array}{cc} \overbrace{}^{N} & \overbrace{}^{N-1} \end{array} \\ \left[\begin{array}{ccc|ccc} 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ \hline 0 & \cdots & 0 & 2.8 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 2.8 \end{array}\right] \end{array} \begin{array}{l} \left.\vphantom{\begin{array}{c}0\\ \vdots\\ 0\end{array}}\right\} N \\ \left.\vphantom{\begin{array}{c}0\\ \vdots\\ 0\end{array}}\right\} N-1 \end{array} \tag{2}$$

2. Measure 2: directly from matrices operation

$$\begin{aligned}
\sum_{k=1}^{N} \Phi_k \dot{q_{ac,k}} \Delta t + (0.1 + E_2/10)(T_{B,k} - T_{ref})^2 &= \sum_{k=1}^{N} \Phi_k \dot{q_{ac,k}} + \sum_{k=1}^{N} 1.4(T_k^2 + T_{ref}^2 - 2T_r T_k) \\
&= (\sum_{k=1}^{N} \Phi_k \dot{q_{ac,k}}) - 1.4(Ax - T_{mref})^T (Ax - T_{mref}) \\
&= (\sum_{k=1}^{N} \Phi_k \dot{q_{ac,k}}) - 1.4x^T A^T Ax - 1.4T_{mref}^T T_{mref} + 2.8T_{mref}^T Ax \\
&= -2.8T_{mref}^T T_{mref} + \Phi x + 2.8T_{mref}^T Ax - 1.4xA^T Ax
\end{aligned} \tag{3}$$

where:

$$\Phi = [\underbrace{\Phi_1 \ \cdots \ \cdots \ \Phi_{2160}}_{N} \ \underbrace{0 \ \cdots \ \cdots \ 0}_{N-1}]$$

$$T_{mref} = [\underbrace{0 \ \cdots \ \cdots \ a0}_{N} \ \underbrace{T_{ref} \ \cdots \ \cdots \ T_{ref}}_{N-1}]$$

$$A = \begin{array}{c} \begin{array}{cc} \overbrace{\qquad\qquad\qquad}^{N} & \overbrace{\qquad\qquad\qquad}^{N-1} \end{array} \\ \left[\begin{array}{ccc|ccc} 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ \hline 0 & \cdots & 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 1 \end{array}\right] \begin{array}{l} \left.\vphantom{\begin{array}{c}0\\ \vdots\\ 0\end{array}}\right\} N \\ \left.\vphantom{\begin{array}{c}0\\ \vdots\\ 0\end{array}}\right\} N-1 \end{array} \end{array} \tag{4}$$

so,

$$C = \Phi + 2T_{mref}^T = [\underbrace{\Phi_1 \ \cdots \ \cdots \ \Phi_{2160}}_{N} \ \underbrace{-2.8T_{ref} \ \cdots \ \cdots \ -2.8T_{ref}}_{N-1}]^T$$

$$H = 2A^TA = \begin{array}{c} \begin{array}{cc} \overbrace{\qquad\qquad\qquad}^{N} & \overbrace{\qquad\qquad\qquad}^{N-1} \end{array} \\ \left[\begin{array}{ccc|ccc} 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ \hline 0 & \cdots & 0 & 2.8 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 2.8 \end{array}\right] \begin{array}{l} \left.\vphantom{\begin{array}{c}0\\ \vdots\\ 0\end{array}}\right\} N \\ \left.\vphantom{\begin{array}{c}0\\ \vdots\\ 0\end{array}}\right\} N-1 \end{array} \end{array} \tag{5}$$

For constraints $A_{eq}x = b_{eq}$, the matrices are shown

$$A_{eq} = \begin{array}{c} \left[\begin{array}{cccc|c|ccccc} a & 0 & \cdots & 0 & 0 & 1 & 0 & \cdots & & 0 \\ 0 & a & \ddots & \vdots & \vdots & -1 & \ddots & \ddots & & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots & 0 & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & a & 0 & \vdots & \ddots & \ddots & \ddots & 0 \\ & & & & & 0 & \cdots & 0 & -1 & 1 \end{array}\right] \\ \begin{array}{ccc} \underbrace{\qquad\qquad\qquad\quad}_{N-1} & \underbrace{\quad}_{1} & \underbrace{\qquad\qquad\qquad\qquad}_{N-1} \end{array} \end{array} \tag{6}$$

$$b_{eq} = \begin{bmatrix} (a_1 \dot{q_{solar,1}} + a_2 \dot{q_{occ,1}} t - a_2 \dot{q_{vent,1}} + a_3 T_{amb,1})\Delta t + T_{b,1} \\ \vdots \\ (a_1 \dot{q_{solar,N}} + a_2 \dot{q_{occ,N}} - a_2 \dot{q_{vent,N}} + a_3 T_{amb,N})\Delta t \end{bmatrix}$$

0.2 Model

The model has been clearly explained in the problem document, and the model is explained by using matrices here. Because $1.4T_{ref}^2$ is a constant in this problem, it was ignored in the standard form, it will be added after optimization with MATLAB.

0.3 Solution

Based on equations, the corresponding parameters in MATLAB function `quadprog` are shown as follows:

$$H = \begin{array}{c} \begin{array}{cc} \overbrace{\hphantom{0 \quad \cdots \quad 0}}^{N} & \overbrace{\hphantom{2.8 \quad \cdots \quad 0}}^{N-1} \end{array} \\ \left[\begin{array}{ccc|ccc} 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ \hline 0 & \cdots & 0 & 2.8 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 2.8 \end{array}\right] \begin{array}{l} \left.\vphantom{\begin{array}{c} 0 \\ \vdots \\ 0 \end{array}}\right\} N \\ \left.\vphantom{\begin{array}{c} 0 \\ \vdots \\ 0 \end{array}}\right\} N-1 \end{array} \end{array}$$

$$f = [\underbrace{\Phi_1 \ \cdots \ \cdots \ \Phi_{2160}}_{N} \ \underbrace{-2.8T_{ref} \ \cdots \ \cdots \ -2.8T_{ref}}_{N-1}]^T$$

$$A_{eq} = \begin{array}{c} \left[\begin{array}{cccc|c|ccccc} & & & & & 1 & 0 & \cdots & & 0 \\ a & 0 & \cdots & 0 & 0 & -1 & \ddots & \ddots & & \vdots \\ 0 & a & \ddots & \vdots & \vdots & 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 & 0 & \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & a & & 0 & \cdots & 0 & -1 & 1 \end{array}\right] \\ \begin{array}{ccc} \underbrace{\hphantom{a \quad 0 \quad \cdots \quad 0}}_{N-1} & \underbrace{\hphantom{0}}_{1} & \underbrace{\hphantom{1 \quad 0 \quad \cdots \quad -1 \quad 1}}_{N-1} \end{array} \end{array}$$

$$b_{eq} = \begin{bmatrix} (a_1\dot{q}_{solar,1} + a_2\dot{q}_{occ,1}t - a_2\dot{q}_{vent,1} + a_3T_{amb,1})\Delta t + T_{b,1} \\ \vdots \\ (a_1\dot{q}_{solar,N} + a_2\dot{q}_{occ,N} - a_2\dot{q}_{vent,N} + a_3T_{amb,N})\Delta t \end{bmatrix}$$

$$lb(i) = \begin{cases} 0 & \text{i=1,...,N} \\ 15 & \text{i} > 2160 \text{ and } \dot{q}_{occ,(i-2160)} > 0 \\ -\inf & (others) \end{cases}$$

$$ub(i) = \begin{cases} \dot{q}_{ac,max} & \text{i=1,...,N} \\ 28 & \text{i} > 2160 \text{ and } \dot{q}_{occ,(i-2160)} > 0 \\ \inf & (others) \end{cases}$$

Finally, the quadprog got the optimal answer, which obtained the minimal cost of €20533.

0.4 Answer

1. The optimal cost for air-conditioning along the horizon of N(2160) steps is €20533.