0.1 Question 1(c) B

0.1.1 Assumption

For Question 1(c), the following assumptions are made:

1. Only the cost of maintenance and the cost of installation of air conditioners are considered.
2. All the budget are not separated, and can be used flexibly after the durable years is determined.
3. After the durable time is pre-determined, the practical duration for using and maintenance these air conditioners should be equal to the pre-determined durable time.

0.1.2 Analysis

According to assumption 2, part of the maintenance budget can also be used to install air conditioners after the durable years has been determined.

Comparing with Question 1(c) A, the target keeps the same, but the constraints is changed. For N years, the cost for installation and maintenance during N years should be considered together as shown in the following equation.

$$\sum_{n=1}^{N} 3000x + 1500y + C_x(n)x + C_y(n)y \leq 24000 + 300E_1 + (4000 + 100E_1)N \tag{1}$$

$C_x(n)$ is the maintenance cost of year n for X air conditioners, $C_y(n)$ is the maintenance cost of year n for Y air conditioners.

0.1.3 Model

According to analysis 0.1.2, the model of Question 1(c) can be made as following (for N years).

$$\begin{gathered} \min_{x,y} -(4x + 2.5y) \\ s.t. \quad x + y \leq 12 \\ \sum_{n=1}^{N} 3000x + 1500y + C_x(n)x + C_y(n)y \leq 24000 + 300E_1 + (4000 + 100E_1)N \\ x, y \geq 0 \end{gathered} \tag{2}$$

0.1.4 Solution

Different durations lead to different amount of maintenance budget can be used as installation budget, so the analysis to simplify the calculation in Question 1(c) A cannot be done anymore. We calculated the maximal value for different durable years separately and find the optimal one from them.

Based on equation in 0.1.2, the corresponding parameters in MATLAB function `linprog` are shown as follows:

$$f = \begin{bmatrix} -4 & -2.5 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 \\ 3000 + \sum_{n=1}^{N} C_x(n) & 1500 + \sum_{n=1}^{N} C_y(n) \end{bmatrix}$$

$$b = \begin{bmatrix} 12 & 24000 + 300E_1 + (4000 + 100E_1)N \end{bmatrix}^T$$

$$lb = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$$

For years ranging from 1 to 10, the installation plan and the maximum power are found as shown in

duration	1	2	3	4	5
max power	40.6212	42.9515	44.8285	45.6772	46.4063
X amount	7.0808	8.6344	9.8857	10.4515	10.9375
y amount	4.9192	3.3656	2.1143	1.5485	1.0625
duration	6	7	8	9	10
max power	46.3235	45.6208	44.4924	43.1050	41.5873
X amount	10.8824	10.4139	9.6616	8.7367	7.7249
y amount	1.1176	1.5861	2.3384	3.2633	4.2751

Table 1: Results for different duration time (non-integrization)

Without regard to integration, The optimal choice for duration time is 5 year, with 10.9376 X type, 1.0625 Y type and maximum power 46.4064 (kW). The next step is adjusting this result to integer and checking whether it is still the optimal answer.

Considering the duration time is 5 years, calculate the power of $(x, y) = (10, 1)$ $(x, y) = (10, 2)$ and $(x, y) = (11, 1)$. $(x, y) = (10, 2)$ leads to the maximal power 45 (kW). However, this value is lower than the maximum power of 4,6,7 years in the table 1. That means extra test for integer solution of 4,6,7 is needed to find out the best solution. The maximal power for durable years 4,5,6,7 is showed in the following table.

year	4			5			6			7		
amount x	10	10	11	10	10	11	10	10	11	10	10	11
amount y	1	2	1	1	2	1	1	2	1	1	2	1
power	42.5	45	46.5	42.5	45	46.5	42.5	45	46.5	42.5	45	46.5
whether feasible	Y	Y	N	Y	Y	N	Y	Y	N	Y	Y	N

Table 2: Results for integer points

It comes out that for duration time ranging from 4 to 7, the maximum power keep the same as 45(kW) with 10 X air conditioner and 2 Y air conditioner.

0.1.5 Answer

1. The problem is hard to transform to a sinlge LP problem.
2. The durable years can be chosen as 4,5,6,7, 10 X air conditioner and 2 Y air conditioner should be chosen and the maximum available power is 45kW.