$$y(k) = c(n_{u,d}(k) + n_{o1,d}(k))$$

$$n_{u,d}(k+1) = n_{u,d}(k) + c(\alpha^{enter}_{u,d}(k) - \alpha^{leave}_{u,d}(k))$$

$\alpha^{enter}_{u,d}(k)$: known

$$\alpha^{leave}_{u,d}(k) = \sum \alpha^{leave}_{u,d,o}(k)$$

$$\alpha^{leave}_{u,d,o}(k) = \min(\mu_{u,d,o}\, g_d(k)/C,\ q_{u,d,o}(k)/C + \alpha^{arrive}_{u,d,o}(k),\ C_{d,o}(k)/c)$$

- $\mu_{u,d,o}$: saturation flow (known)
- $g_d(k)$: green light duration $u(k)$
- $q_{u,d,o}(k)/C$: capacity of flow
- $\alpha^{arrive}_{u,d,o}(k)$: cars waiting/arriving
- $C_{d,o}(k)/c$: available capacity next link → storage capacity

queue length

$$q_{u,d,o}(k+1) = q_{u,d,o}(k) + (\alpha^{arrive}_{u,d,o}(k) - \alpha^{leave}_{u,d,o}(k)) \cdot C$$

$$q_{u,d}(k) = \sum q_{u,d,o}(k)$$

$$\alpha^{arrive}_{u,d,o}(k) = \beta_{u,d,o}(k) \cdot \alpha^{arrive}_{u,d}(k)$$

- $\beta_{u,d,o}(k)$: known — fraction of traffic flow

$$\alpha^{arrive}_{u,d}(k) = \frac{C - r(k)}{C} \alpha^{enter}_{u,d}(k - T(k)) + \frac{r(k)}{C} \alpha^{enter}_{u,d}(k - T(k) - 1)$$

$$T(k) = \text{floor}\left\{ \frac{(C_{u,d}(k) - q_{u,d}(k)) \cdot l_{veh}}{N^{lane}_{u,d} \cdot v^{free}_{u,d} \cdot c} \right\}$$

(dynamic)

$$C_{u,d}(k) = N^{lane}_{u,d} \cdot l_{u,d}/l_{veh} \quad \text{(known)}$$

Others: known

$$x = [n_1, n_2, \alpha_{11}, \alpha_{12}, q_2, q_3, q_4, q_6, q_7, q_8] \qquad u(k) = g_d(k)$$

$$= [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}]$$

$x_1 \to n_1 \to n_{u,d}(k)$

$x_2 \to n_2 \to n_{o1,d}(k)$

$x_3 \to \alpha_{11} \to \alpha^{arrive}_{u,d}(k)$

$x_4 \to \alpha_{12} \to \alpha^{arrive}_{o1,d}(k)$

$x_5 \to q_2 \to q_{u,d,o1}(k)$

$x_6 \to q_3 \to q_{u,d,o2}(k)$

$x_7 \to q_4 \to q_{u,d,o3}(k)$

$x_8 \to q_6 \to q_{o1,d,u}(k)$

$x_9 \to q_7 \to q_{o1,d,o2}(k)$

$x_{10} \to q_8 \to q_{o1,d,o3}(k)$

known: $z = [\alpha_1, \alpha_6, C_1, C_2, C_3, C_4]$

$= [z_1, z_2, z_3, z_4, z_5, z_6]$

$z_1 \to \alpha_1 \to \alpha^{enter}_{u,d}(k)$

$z_2 \to \alpha_6 \to \alpha^{enter}_{o1,d}(k)$

$z_3 \to C_1 \to C_{d,u}(k)$

$z_4 \to C_2 \to C_{d,o1}(k)$

$z_5 \to C_3 \to C_{d,o2}(k)$

$z_6 \to C_4 \to C_{d,o3}(k)$

$I = [\alpha_2, \alpha_7, q_1, q_5] = [\dot{z}_1, \dot{z}_2, \dot{z}_3, \dot{z}_4]$

$\dot{z}_1 \to \alpha_2 \to \alpha^{leave}_{u,d}(k) = \alpha_3 + \alpha_4 + \alpha_5 = \min + \min + \min$

$\dot{z}_2 \to \alpha_7 \to \alpha^{leave}_{o1,d}(k) = \alpha_8 + \alpha_9 + \alpha_{10} = \min + \min + \min$

$\dot{z}_3 \to q_1 \to q_{u,d}(k) = q_2 + q_3 + q_4 = x_5 + x_6 + x_7$

$\dot{z}_4 \to q_5 \to q_{o1,d}(k) = q_6 + q_7 + q_8 = x_8 + x_9 + x_{10}$

$$x_1(k+1) = n_{u,d}(k+1) = x_1(k) + C \cdot [z_1(k) - \dot{z}_1(k)]$$

$$\dot{z}_1(k) = O_1 + O_2 + O_3 \quad (0.33, 0.34, 0.33) \quad (1600, 1800, 1500)$$

$$= \min\{1600 \cdot u(k)/C \mid x_5(k)/C + 0.33 \cdot x_3(k) \mid z_4(k)/C\}$$
$$+ \min\{1800 \cdot u(k)/C \mid x_6(k)/C + 0.34 \cdot x_3(k) \mid z_5(k)/C\}$$
$$+ \min\{1500 \mid x_7(k)/C + 0.33 \cdot x_3(k) \mid z_6(k)/C\}$$

$$x_5(k+1) = q_{u,d,o1}(k+1) = x_5(k) + (0.33 \cdot x_3(k) - \min_{u,d,o1}(k))$$

$$x_6(k+1) = q_{u,d,o2}(k+1) = x_6(k) + (0.34 \cdot x_3(k) - \min_{u,d,o2}(k))$$

$$x_7(k+1) = q_{u,d,o3}(k+1) = x_7(k) + (0.33 \cdot x_3(k) - \min_{u,d,o3}(k))$$

$$x_2(k+1) = n_{o1,d}(k+1) = x_2(k) + C \cdot [z_2(k) - \dot{z}_2(k)]$$

$$\dot{z}_2(k) = O_1 + O_2 + O_3 \quad (0.33, 0.34, 0.33) \quad (1600, 1800, 1500)$$

$$= \min\{1600 \cdot \mid x_8(k)/C + 0.33 \cdot x_4(k) \mid z_3(k)/C\}$$
$$+ \min\{1800 \cdot (C - u(k))/C \mid x_9(k)/C + 0.34 \cdot x_4(k) \mid z_5(k)/C\}$$
$$+ \min\{1500 \cdot (C - u(k))/C \mid x_{10}(k)/C + 0.33 \cdot x_4(k) \mid z_6(k)/C\}$$

$$x_8(k+1) = q_{o1,d,u}(k+1) = x_8(k) + (0.33 \cdot x_4(k) - \min_{o1,d,u}(k))$$

$$x_9(k+1) = q_{o1,d,o2}(k+1) = x_9(k) + (0.34 x_4(k) - \min_{o1,d,o2}(k))$$

$$x_{10}(k+1) = q_{o1,d,o3}(k+1) = x_{10}(k) + (0.33 x_4(k) - \min_{o1,d,o3}(k))$$

$$x_3(k+1) = \alpha^{arrive}_{u,d}(k+1) = \begin{cases} \frac{C - r(k)}{C} \cdot z_1(k-1) + \frac{r(k)}{C} \cdot z_1(k-2) & \dot{z}_3 \le \dot{z}_1 \\ \frac{C - r(k)}{C} \cdot z_2(k) + \frac{r(k)}{C} \cdot z_2(k-1) & \dot{z}_3 \ge \dot{z}_2 \end{cases}$$

$$r(k) = \text{rem}\left\{\frac{3000 - 7 \cdot \dot{z}_3}{2500}\right\}$$

$$x_4(k+1) = \alpha^{arrive}_{o1,d}(k+1) = \frac{C - r_2(k)}{C} \cdot z_2(k) + \frac{r_2(k)}{C} \cdot z_2(k-1)$$

$$r_2(k) = \text{rem}\left\{\frac{3000 - 7 \cdot \dot{z}_3}{3000}\right\} = (3000 - 7\dot{z}_3)$$