0.1 Analysis

In order to transform model to a discrete-time model, we need to use the following approximation:

$$\dot{T_{b,k}} = \frac{dT_{b,k}}{dt} \approx \frac{T_{b,k+1} - T_{b,k}}{\Delta t} \tag{1}$$

where $T_{b,k}$ represents the indoor temperature in the building at time step k.

The target equation has derivative on both sides, but according to Question 3, the derivate of $q_{solar,k}, q_{occ,k}, q_{ac,k}, q_{vent,k}$ are able to observed directly. So only the left side of the equation needs to be discretized.

0.2 Solution

The process to obtain discrete-time model is shown as follows.

$$\begin{aligned} \frac{T_{b,k+1} - T_{b,k}}{\Delta t} &= a_1 \dot{q_{solar,k}} + a_2[\dot{q_{occ,k}} + \dot{q_{ac,k}} - \dot{q_{vent,k}}] + a_3[T_{amb,k} - T_{b,k}] \\ \Rightarrow T_{b,k+1} &= \{a_1 \dot{q_{solar,k}} + a_2[\dot{q_{occ,k}} + \dot{q_{ac,k}} - \dot{q_{vent,k}}] + a_3[T_{amb,k} - T_{b,k}]\}\Delta t + T_{b,k} \\ \Rightarrow T_{b,k+1} &= \{a_1 \dot{q_{solar,k}} + a_2[\dot{q_{occ,k}} + \dot{q_{ac,k}} - \dot{q_{vent,k}}] + a_3 T_{amb,k}\}\Delta t + (1 - a_3\Delta t)T_{b,k} \end{aligned} \tag{2}$$

so,

$$\begin{aligned} A &= 1 - a_3\Delta t \\ B &= \begin{bmatrix} a_1\Delta t & a_2\Delta t & a_2\Delta t & -a_2\Delta t & a_3\Delta t \end{bmatrix} \end{aligned} \tag{3}$$

0.3 Answer

From 0.2, the answer is:

$$\begin{aligned} A &= 1 - a_3\Delta t \\ B &= \begin{bmatrix} a_1\Delta t & a_2\Delta t & a_2\Delta t & -a_2\Delta t & a_3\Delta t \end{bmatrix} \end{aligned} \tag{4}$$