1 Question 1

1.1 Quesiont1.(a)

1.1.1 Assumption

A maximum linear programming problem was described in this question. We assume that:

1. We will install central air conditioner X, the number of which is x, and split-type air conditioner Y, the number of which is y.
2. All air conditioners are installed at one time.
3. Although x and y are integer, we consider them as non-negative real number first.

1.1.2 Analysis

Apparently, there are two constrains, namely, budget constrains and amount constrains.

1. We can not install more than 12 air conditioners or install a negative number of air conditioners.
2. Total budget for all installation is €$24000 + 300E1$, where $E1 = 9$.

1.1.3 Model

According to analysis above, we can formalize this optimization problem, we have:

$$\begin{aligned} \max_{x,y} \quad & 4x + 2.5y \\ s.t. \quad & x + y \leq 12 \\ 3000x + 1500y &\leq 24000 + 300E1 \\ x, y &\geq 0 \end{aligned} \tag{1}$$

Obviously, model above isn't a standard form of linear programming problem. The corresponding standard form is as follows:

$$\begin{aligned} -\min_{x,y,s_1,s_2} \quad & -4x - 2.5y \\ s.t. \quad & x + y + s_1 = 12 \\ 3000x + 1500y + s_2 &= 24000 + 300E1 \\ x,\ y,\ s_1,\ s_2 &= 0 \end{aligned} \tag{2}$$

Formula (2) is a standard form of LP problem described in Question1.(a).

1.2 Question1.(b)

When solving problem in Question1.(a) by using MATLAB, we will use following function:

$$[x, val, flag] = linprog(c, A, b, A_{eq}, b_{eq}, lb, ub, options) \tag{3}$$

Formula (3) represent solution of following questions:

$$\begin{aligned} \min_{x} \quad & fX \\ AX &\leq b \\ A_{eq}X &= b_{eq} \\ lb \leq X &\leq ub \end{aligned} \tag{4}$$

In formula (3) x represent the optimization of independent variable, val represent the optimized outcome, $flag$ is a sign of whether the problem has a solution. When the $flag$ is 1, it means that the problem has an optimal

solution. If the $flag$ is 0, the problem has no optimal solution.
We can easily tell that, in Question1.(b):

$$f = \begin{bmatrix} -4 & -2.5 \end{bmatrix}$$
$$A = \begin{bmatrix} 1 & 1 \\ 3000 & 1500 \end{bmatrix}$$
$$b = \begin{bmatrix} 12 \\ 24000 + 300E1 \end{bmatrix}$$
$$lb = \begin{bmatrix} 0 & 0 \end{bmatrix}$$
$$ub = \begin{bmatrix} inf & inf \end{bmatrix}$$
$$A_{eq} = b_{eq} = [\,] \tag{5}$$

1.2.1 Solution

MATLAB code, which is not shown here, will be uploaded as an attachment in the form of .m file. Without considering that the result should be integer, we should install 5.8 air conditioners X, and 6.2 air conditioners Y.

Taking integral constraints into consideration, installation plan $(x, y) = (5, 7), (x, y) = (6, 5), (x, y) = (6, 6)$ should be calculated. $(x, y) = (5, 7)$ obtain maximum power 37.5 kW, $(x, y) = (6, 5)$ obtain power 36.5 kW and they all meet the other constraints. But $(x, y) = (6, 6)$ break the budget constraints.

1.2.2 Answer

The final result is that we should install 5 air conditioners X, and 7 air conditioner Y, leading to a maximum power, which is 37.5 kW.

1.3 Question 1(c)

1.3.1 Assumption

For Question 1 (c), make the following assumptions:

1. Only the cost of maintenance and the cost of installation of air conditioners are considered.
2. All budgets are not separated, and can be used flexibly after determining the service life.
3. After the durable time is pre-determined, the practical duration for using and maintenance these air conditioners should be equal to the pre-determined durable time.

1.3.2 Analysis

According to assumption 2, after the service life is determined, part of the maintenance budget can also be used to install air conditioners. For N years, the cost for installation and maintenance during N years should be considered together as shown in the following formula.

$$\sum_{n=1}^{N} 3000x + 1500y + C_x(n)x + C_y(n)y \leq 24000 + 300E_1 + (4000 + 100E_1)N \tag{6}$$

$C_x(n)$ is the maintenance cost of year n for air conditioner X, $C_y(n)$ is the maintenance cost of year n for air conditioner Y.

1.3.3 Model

According to analysis 1.3.2, the model of Question 1(c) can be made as following (for N years).

$$
\begin{gathered}
\min_{x,y} -(4x+2.5y) \\
s.t. \quad x+y \le 12 \\
\sum_{n=1}^{N} 3000x + 1500y + C_x(n)x + C_y(n)y \le 24000 + 300E_1 + (4000+100E_1)N \\
x, y \ge 0
\end{gathered} \tag{7}
$$

1.3.4 Solution

Different durations lead to different amount of maintenance budget can be used as installation budget. We calculated the maximal value of different durable years separately and find the best one from them.

Based on formula in 1.3.2, the corresponding parameters in MATLAB function `linprog` are shown as follows:

$$
\begin{gathered}
f = \begin{bmatrix} -4 & -2.5 \end{bmatrix} \\
A = \begin{bmatrix} 1 & 1 \\ 3000 + \sum_{n=1}^{N} C_x(n) & 1500 + \sum_{n=1}^{N} C_y(n) \end{bmatrix} \\
b = \begin{bmatrix} 12 & 24000 + 300E_1 + (4000+100E_1)N \end{bmatrix}^T \\
lb = \begin{bmatrix} 0 & 0 \end{bmatrix}^T
\end{gathered}
$$

For years ranging from 1 to 10, the installation plan and the maximum power are found as shown in the following table.

duration	1	2	3	4	5
max power	41.7069	44.0789	45.8932	46.5493	47.0043
X amount	7.8046	9.3860	10.5955	11.0329	11.3362
y amount	4.1954	2.6140	1.4045	0.9671	0.6638
duration	6	7	8	9	10
max power	46.6992	45.8421	44.6238	43.1983	41.6791
X amount	11.1328	10.5614	9.7492	8.7989	7.7861
y amount	0.8672	1.4386	2.2508	3.2011	4.2139

Table 1: Results for different duration time (non-integrization)

Without considering the integral, The optimal choice duration is 5 year, with 11.3362 X type, 0.6638 Y type and maximum power 47.0043 (kW). The next step is to adjust this result to integer and check whether it is still the best plan.

Considering that the duration time is 5 years, calculate the power of $(x, y) = (11, 0)$ $(x, y) = (11, 1)$ and $(x, y) = (12, 0)$. gives a maximal power 46.5 (kW). However, this value is lower than the maximum power of 4 and 6 years in the table 1. This means that the integer solution of 6 need to be tested extra to find the optimal solution. See the following table for the maximal power for durable years 4,5,6.

year	4			5			6		
amount x	11	11	12	11	11	12	11	11	12
amount y	0	1	0	0	1	0	0	1	0
power	44	46.5	48	44	46.5	48	44	46.5	48
whether feasible	Y	Y	N	Y	Y	N	Y	Y	N

Table 2: Results for integer points

The results show that the maximum power of 11 air conditioner X and 1 air conditioner Y remains unchanged, which is 46.5 kW for durable years 4,5,6.

1.3.5 Answer

1. The problem is hard to transform to a sinlge LP problem.
2. The durable years can be selected as 4,5,6, 11 air conditioner X and 1 air conditioner Y should be chosen and the maximum available power is 46.5kW.