An N-Body Code for Solving the Cosmological Vlasov-Poisson Equations in 2D

APC 523 Final Project

https://github.com/EBerzin/NBody

Elizabeth Berzin

Background

- Dark matter can be modeled as a collisionless fluid, governed by the Vlasov-Poisson equations:

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} - \frac{\partial \Phi}{\partial \mathbf{r}}\frac{\partial f}{\partial \mathbf{v}} = 0$$

$$\nabla^2 \Phi = 4\pi G m \int f(\mathbf{r}, \mathbf{v}, t) d\mathbf{v}$$

Outline

1. Generate appropriate initial conditions.
2. Calculate the mass density of particles on a fine mesh.
3. Solve the Poisson equation under the above mass density to find the potential on the mesh.
4. Calculate the acceleration on the mesh, by taking the gradient of the potential.
5. Interpolate the acceleration on the mesh back to the particles.
6. Update particle velocity and position using the appropriate equations of motion.

Coordinates

We use the following non-dimensional coordinates:

$$\tilde{\mathrm{x}} = \frac{\mathrm{x}}{L} \qquad \tilde{\mathrm{p}} = \frac{\mathrm{p}}{LH_0} \qquad \tilde{\phi} = \frac{\phi}{L^2 H_0^2} \qquad \tilde{\rho} = a^3 \frac{\rho}{\rho_0} \qquad \rho_0 = \frac{3H_0^2 \Omega_m}{8\pi G}$$

$$\boxed{\nabla^2 \tilde{\phi} = \frac{3\Omega_m}{2a}(\tilde{\rho} - 1)}$$

$$\boxed{\dot{\tilde{\mathrm{x}}} = H_0 \frac{\tilde{\mathrm{p}}}{\dot{a} a^2}} \qquad \boxed{\dot{\tilde{\mathrm{p}}} = -H_0 \frac{\nabla \tilde{\phi}}{\dot{a}}}$$

Mass Assignment

CIC

Fraction of mass assigned to a given cell is weighted by the distance between a particle and its neighboring cells.

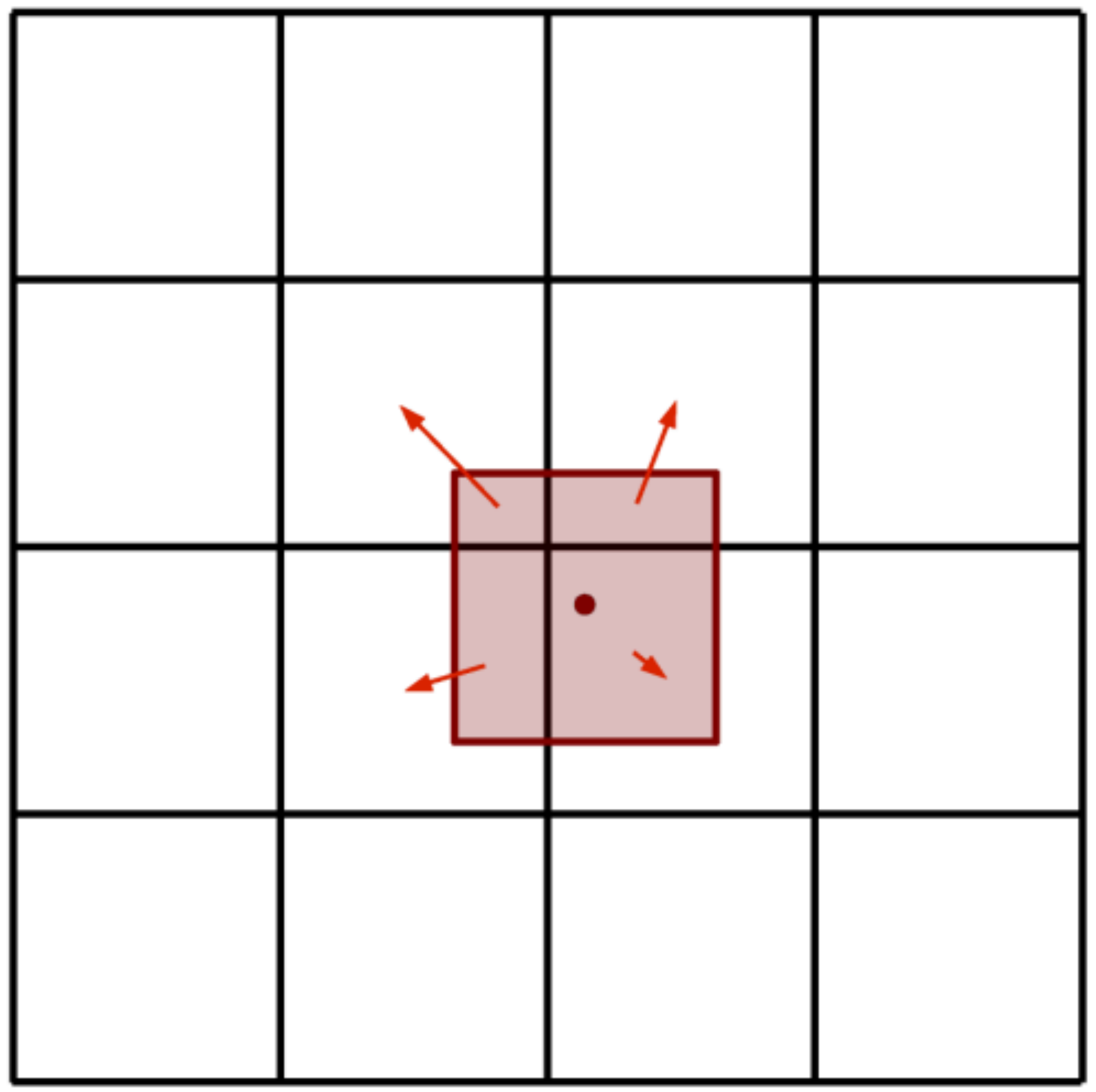
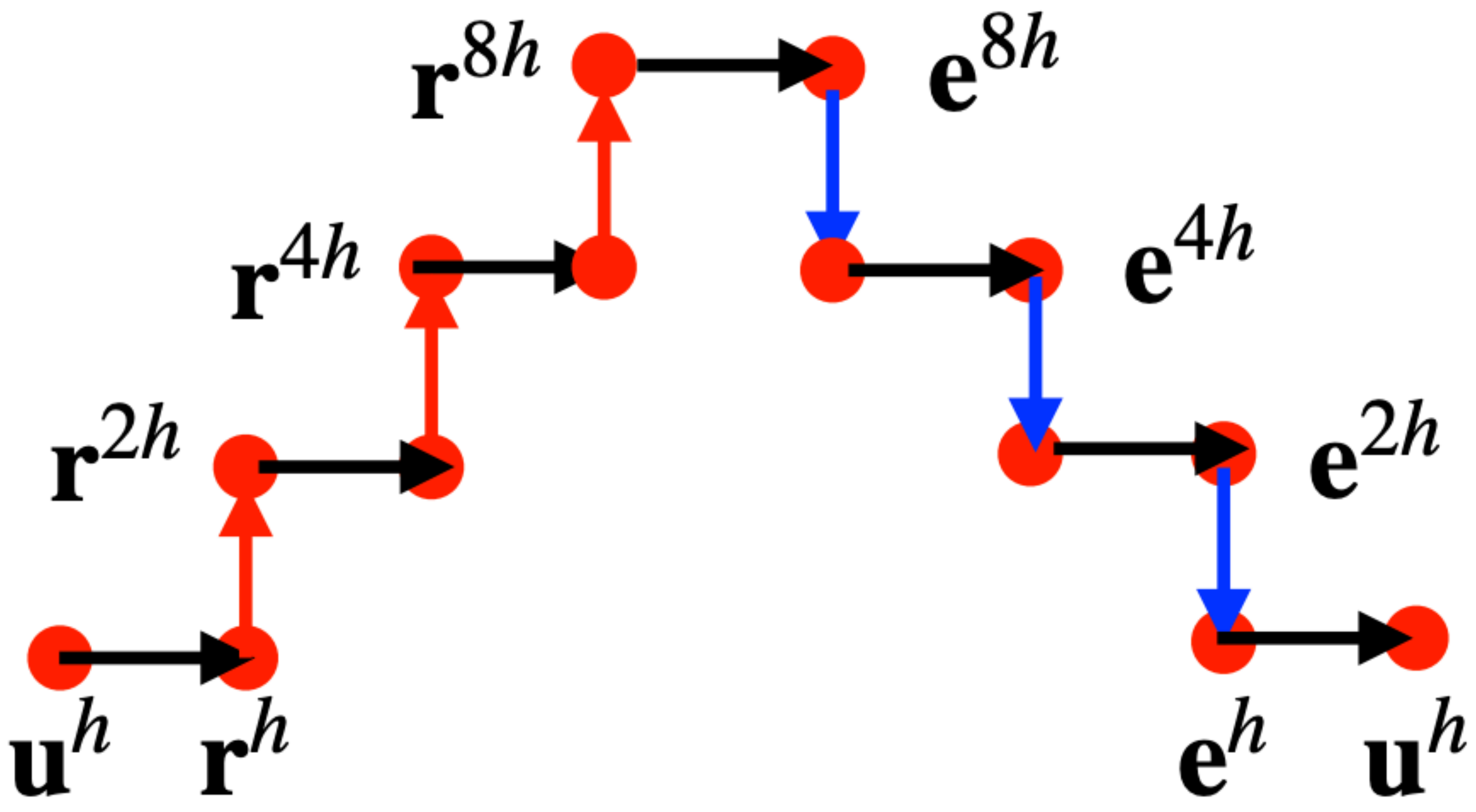


Image Credit: [2]

Force Calculation

Poisson Solver

Multigrid with Jacobi base iteration



Acceleration Calculation

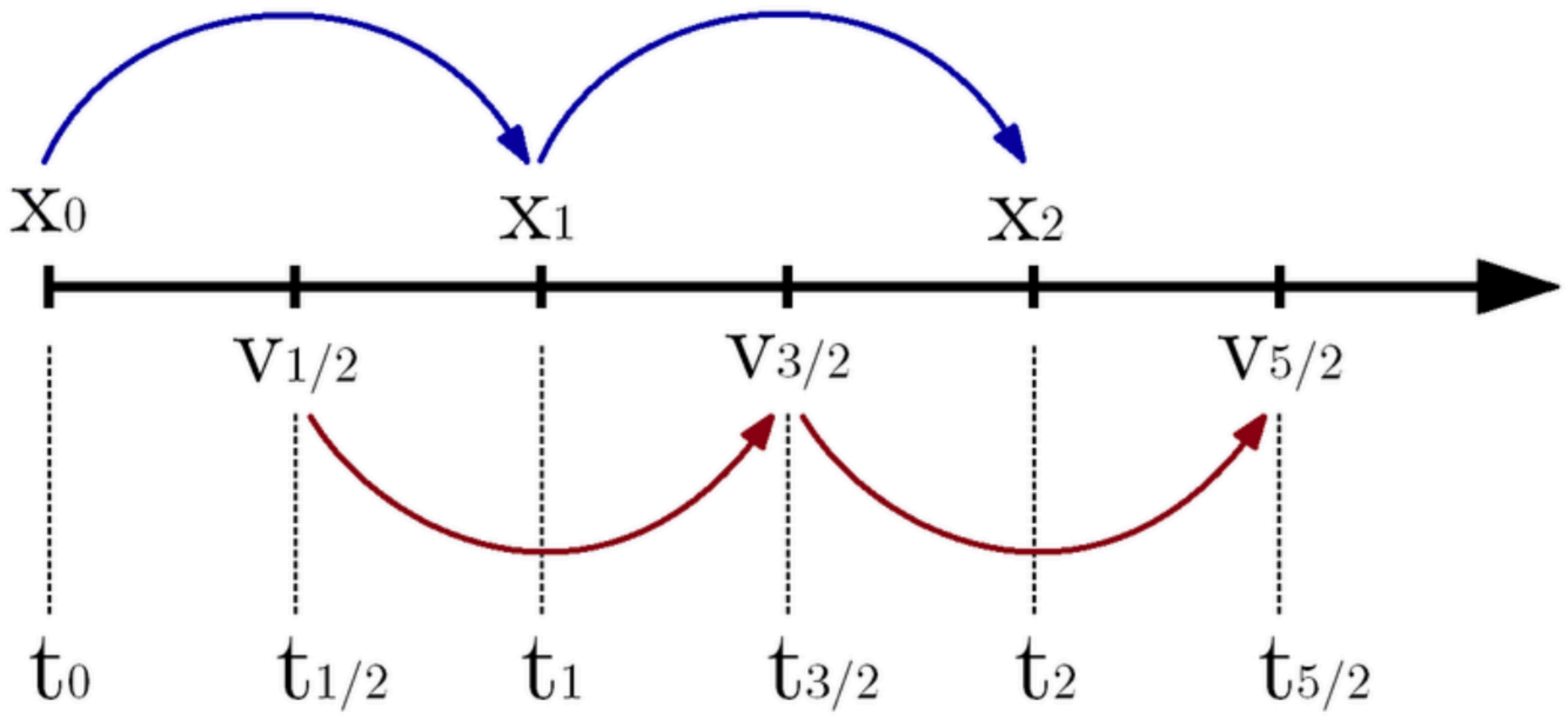
Fourth-order central difference scheme

$$\tilde{\phi}'(x_i) = \frac{-\tilde{\phi}_{i-2} + 8\tilde{\phi}_{i-1} - 8\tilde{\phi}_{i+1} + \tilde{\phi}_{i+2}}{12h}$$

Image Credit: Lecture 11

Time Integration

Leapfrog



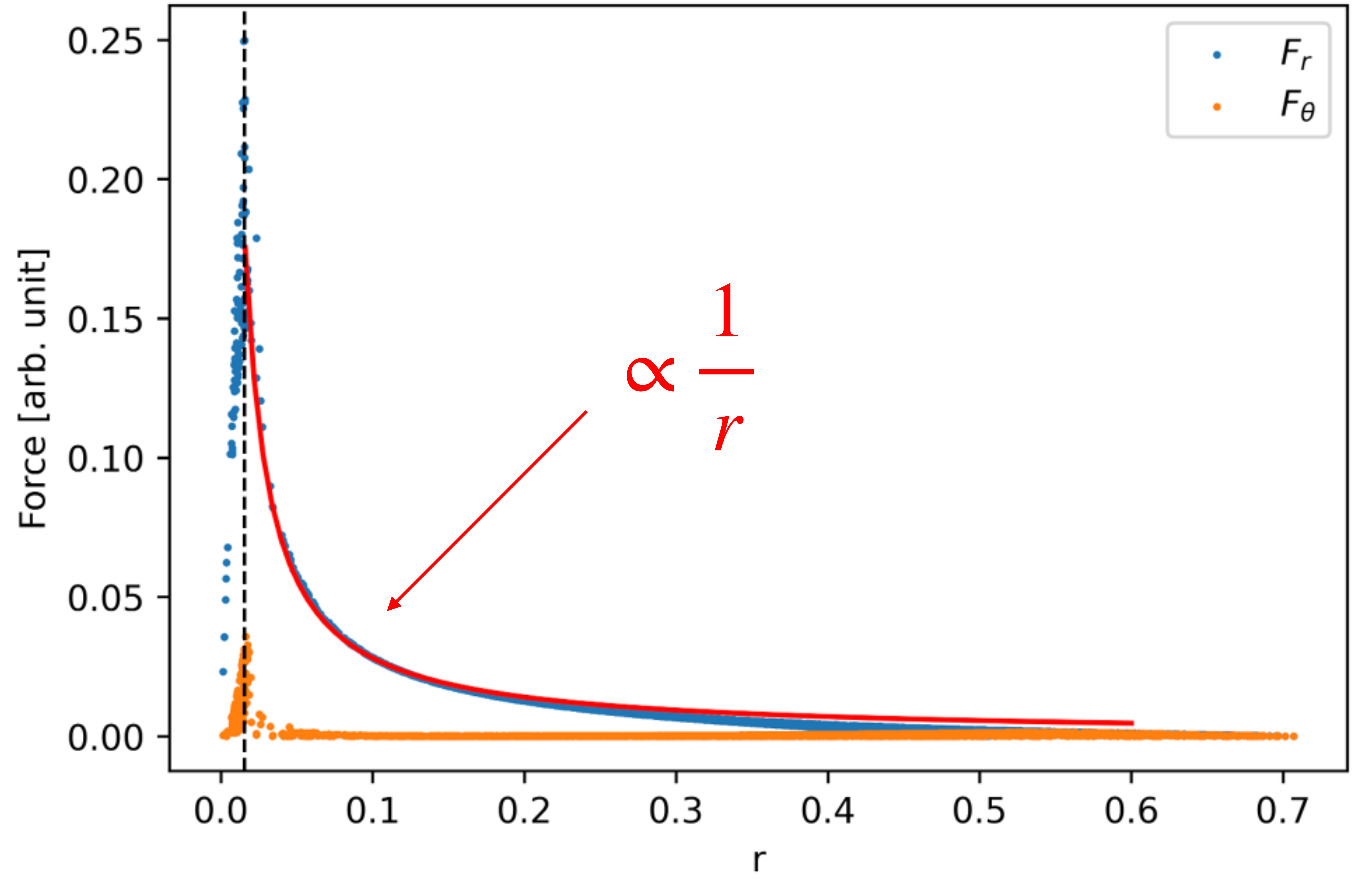
$$\tilde{\mathrm{p}}_{n+1/2} = \tilde{\mathrm{p}}_{n-1/2} - H_0 \frac{\nabla \tilde{\phi}_n}{\dot{a}_n} \Delta a$$

$$\tilde{\mathrm{x}}_{n+1} = \tilde{\mathrm{x}}_n + H_0 \frac{\tilde{\mathrm{p}}_{n+1/2}}{\dot{a}_{n+1/2} a^2_{n+1/2}} \frac{\Delta a}{2}$$

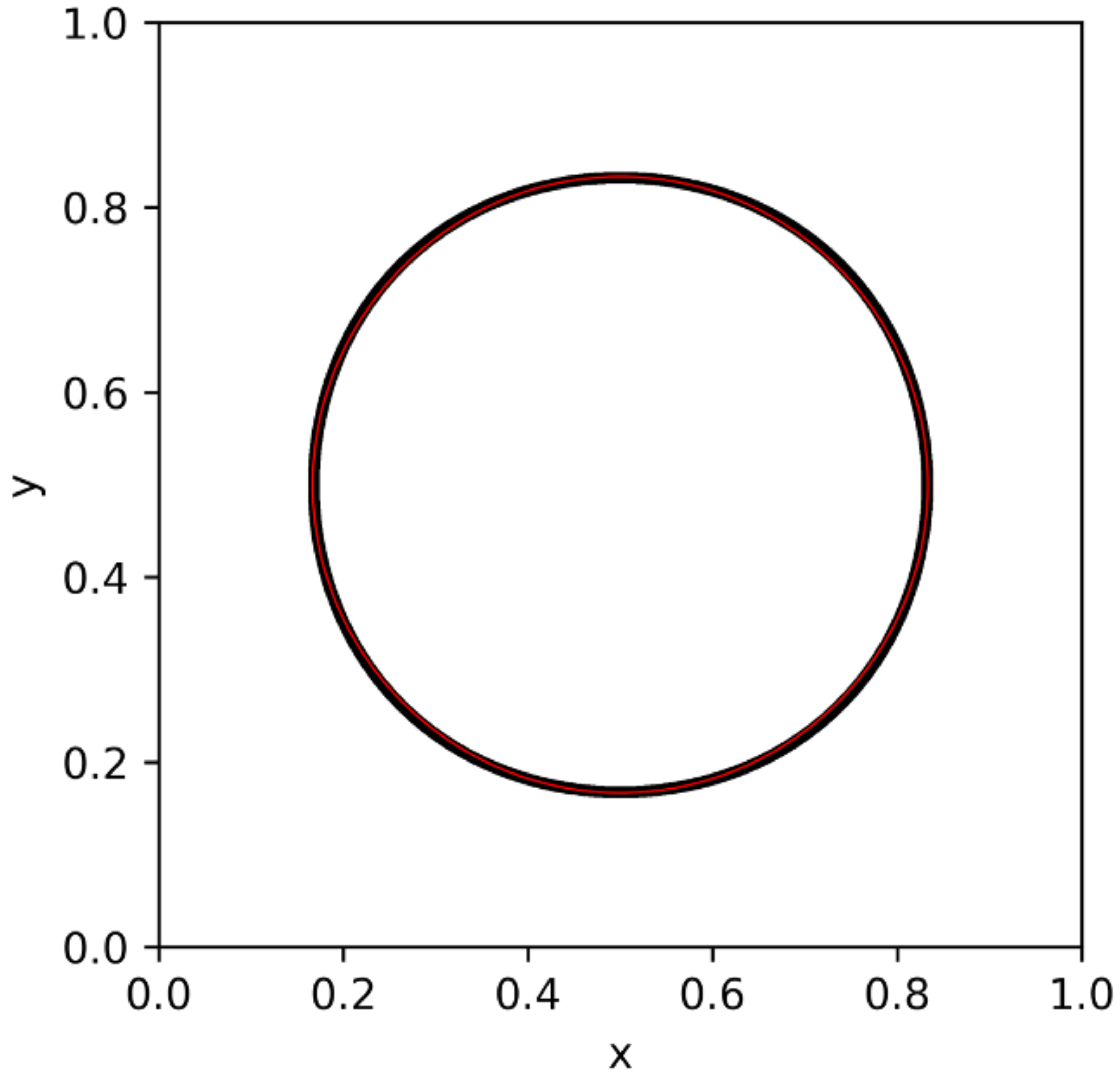
Image Credit: [2]

Validation

Force Calculation

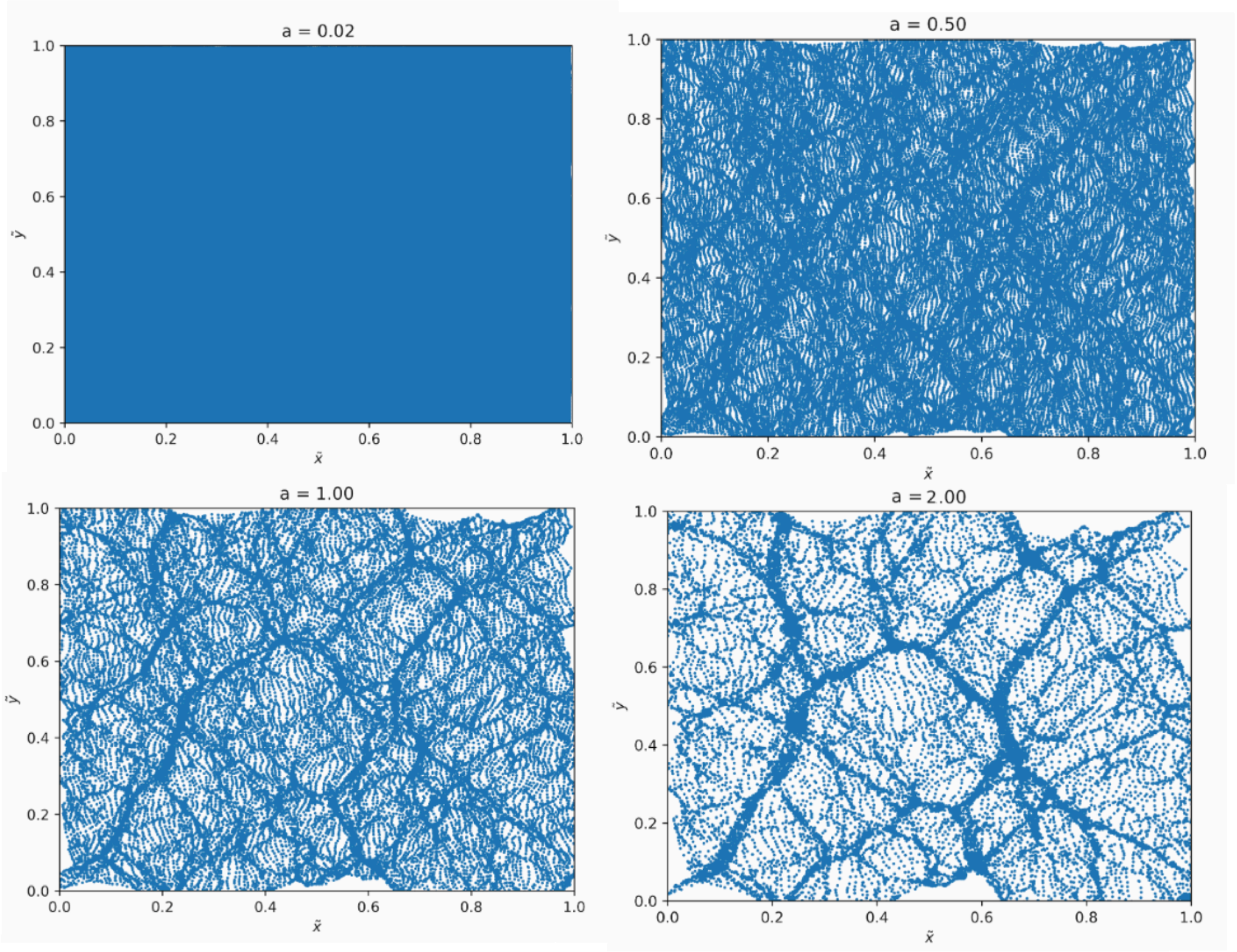


Time Integration



Cosmological Simulation

EdS Cosmology, Initial Conditions from MUSIC



$N_{particles} = 256^2$ $\quad$ $N_{grid} = 2^9$

References

[1] C. Rampf, *Cosmological Vlasov–Poisson equations for dark matter: Recent developments and connections to selected plasma problems*, Rev. Mod. Plasma Phys. **5**, 10 (2021), 2110.06265.

[2] V. Springel, *High Performance Computing and Numerical Modelling*, Saas-Fee Advanced Course **43**, 251 (2016), 1412.5187.

[3] B. Ryden, *Introduction to Cosmology* (Cambridge University Press, 2017).

[4] A. Klypin and J. Holtzman, PMCode: Particle-Mesh Code for Cosmological Simulations, Astrophysics Source Code Library, 1999, 9909.001.

[5] O. Hahn and T. Abel, *Multi-scale initial conditions for cosmological simulations*, Monthly Notices of the Royal Astronomical Society **415**, 2101 (2011).