:— title: “S3” author: “Manu” date: “20 February 2017” output: html_document —

S3 Group

The S3 group is the group of permutation on 3 elements. It has 6 elements. It is the group of all the automorphism on the set {1, 2, 3}.

Here is the full list of the elements of S3:

```
e <- as.cycle()

a <- as.cycle(c(1,2))
b <- as.cycle(c(1,3))
c <- as.cycle(c(2,3))

p <- as.cycle(c(1,2,3))
p2 <- as.cycle(c(1,3,2))
```

The group

= {e, p, p2} is normal in S3:

```
all(p^a == p2, p^b == p2, p^c == p2)
```

```
## [1] TRUE
```

The classes of

in S3 are:

{e, p, p2}, {a, b, c}

The action of a on

: (p p2)

Subgroups of S3

is isomorphic to Z/3Z and it has no non-trivial subgroups.

If G is a subgroup of S3 containing

then G*

is a subgroup of S3/

. S3/

is a group of order 2 so it is isomorphic to Z/2Z and it has no non trivial subgroup. The only subgroups containing p are {e} and S3.

If G is a sub-group not including

. Then inter(G,

) is {e}. because inter(G,

) is a subgroup of

and it cannot be

because G does no include

.

So G is included in {e, a, b, c}. G cannot contain more than 2 elements because any of products ab, bc, . . . will yield an element of

. So G is either {e, a}, {e, b}, {e, c}.

Conclusion: Here is the full list of the subgroups of S3: {e}, {e, a}, {e, b}, {e, c}, {e, p, p2}, {e, a, b, c, p, p2}

Action of the group

Act on the 'edges' of a triangle {1, 2, 3}: {{1, 2}, {1, 3}, {2, 3}}

```
## [1] TRUE
```

The action of a can also be defined: a({1, 2}) == {1, 2} a{1, 3} == {2, 3} a{2, 3} == {1, 3}

Since a and p generate S3 we deduce that S3 acts on the edges. And the action is faithfull.

The natural action of S3 on {1, 2, 3} is also faithfull.

The action of S3 on S3/

is not faithfull. S3/

= {{e, p, p2}, {a, b, c}} Any element of

is sent to the identity. Any other element is sent to ({e, p, p2} {a, b, c})

There is also an action of S3 on {a, b, c}.

And an action of S3 on

How to see if 2 actions are isomorphic?

An action is a group morphism phi1: $G \to Aut(S)$ where S is some set. Let phi2: $G \to Aut(S2)$ be another group action. 2 action are isomorphic if there exist a set isomorphism $f : S \to S2$ such that

$$\forall g \in G, f(phi1(g)(x)) = phi2(g)(f(x))$$

f(phi1(g1*g2)(x)) = f(phi1(g1)(phi1(g2)(x))) = phi2(g1)(f(phi1(g2)x)) = phi2(g1)(phi2(g2)(fx)) = phi2(g1g2)(fx)

Let's show that the action on {1, 2, 3} is isomorphic to the action on {{1, 2}, {1, 3}, {2, 3}} by taking f: 1 -> {2, 3} f: 2 -> {1, 3} f: 3 -> {1, 2}

f(p.1) = f(2) = {1, 3} p.f(1) = p.{2, 3} = {1, 3}

f(p.2) = f(3) = {1, 2} p.f(2) = p.{1, 3} = {1, 2}

f(a.1) = f(2) = {1, 3} a.f(1) = a.{2, 3} = {1, 3}

f(a.3) = f(3) = a.f(3) => f(3) = {1, 2}

f(b.2) = f(2) = b.f(2) => f(2) = {1, 3}

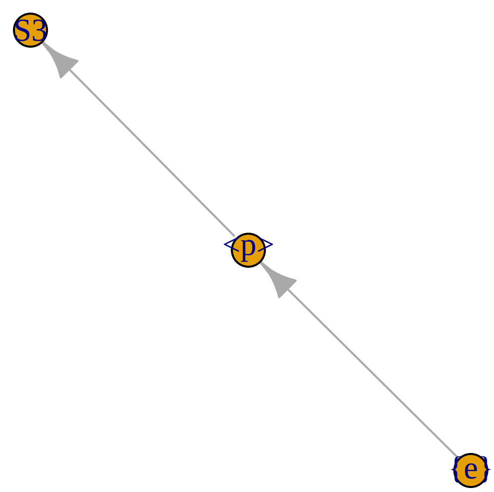
f(c.1) = f(1) = c.f(1) => f(1) = {2, 3}

So f is an isomorphism of actions.

Lattice of normal subgroups

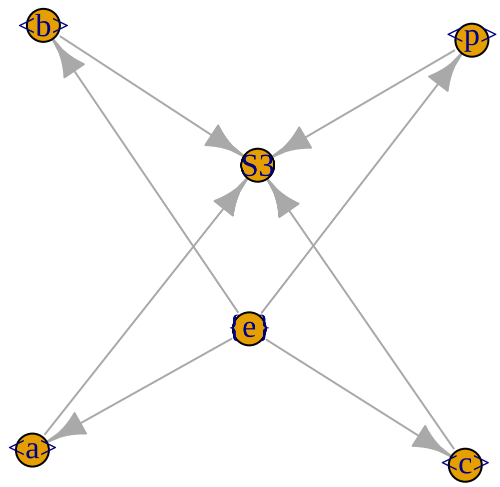
```
g <- make_empty_graph() +
vertex('S3') +
vertex('{e}') +
vertex('<p>') +
path('{e}', '<p>', 'S3')

plot(g)
```




```
g <- make_empty_graph() +
  vertex('S3') +
  vertex('{e}') +
  vertex('<p>') +
  vertex('<a>') +
  vertex('<b>') +
  vertex('<c>') +
  path('{e}', '<p>', 'S3') +
  path('{e}', '<a>', 'S3') +
  path('{e}', '<b>', 'S3') +
  path('{e}', '<c>', 'S3')

plot(g)
```


b
p
S3
e
a
c