Panel Regression Analysis: Micro application to production function

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Example 2. Micro-level application of panel regression analysis `RiceFarms`

- Are larger farms more productive if compared to the smaller once?
- This example:
  - explores relationship between farm size and productivity.
  - guides through the process of panel regression analysis.

Part 1. Theoretical basis

We employ the Cobb-Douglas Production function:

$$\ln y = \ln \beta_0 + \sum_{n=1}^{N} \beta_n \ln x_n + \sum_{k=1}^{K} \gamma_k \delta_k + \epsilon$$

where,

- y is the output and x_n are the inputs all in physical mass (or monetary value);
- N is the number of independent variables;
- δ_k are the shift parameters of additional dummy variables;
- β_0 , β_n , γ_n are the estimated coefficients;

Estimation strategy. Part 1.

Pooled OLS production function:

$$\begin{aligned}\ln \text{output}_{it} = A_0 &+ \beta_1 \cdot \ln \text{land}_{it} + \beta_2 \cdot \ln \text{labor}_{it} \\ &+ \beta_3 \cdot \ln \text{seed}_{it} + \beta_4 \cdot \ln \text{urea}_{it} \\ &+ \beta_5 \cdot \ln \text{pesticide}_{it} + e_{it}\end{aligned}$$

- What are the ex-ante expectations about the regression coefficients?
  - Ideas? …
  - Probably all β should be positive.

What about OVB?

$$
\begin{aligned}
\ln \text{output}_{it} = A_0 &+ \beta_1 \cdot \ln \text{land}_{it} + \beta_2 \cdot \ln \text{labor}_{it} \\
&+ \beta_3 \cdot \ln \text{seed}_{it} + \beta_4 \cdot \ln \text{urea}_{it} \\
&+ \beta_5 \cdot \ln \text{pesticide}_{it} + e_{it}
\end{aligned}
$$

What omitted variables could cause bias of our estimates?

- Any!? …
- Any!? …
- Any OVB!? …
- Capital, Ability, Climate, Geography…

Using the OVB formula

- Let us make an educated guess about the effect of OVB on the estimates of land-related coefficient β_1?

- Short: • $\text{output}_{it} = A_0^s + \beta_1^s \cdot \text{land}_{it} + \beta_2^s \cdot \text{labor}_{it} + \beta_3^s \cdot \text{seed}_{it} + \beta_4^s \cdot \text{urea}_{it} + \beta_5^s \cdot \text{pesticide}_{it} + \epsilon$
- Long: • $\text{output}_{it} = A_0 + \beta_1 \cdot \text{land}_{it} + \beta_2 \cdot \text{labor}_{it} + \beta_3 \cdot \text{seed}_{it} + \beta_4 \cdot \text{urea}_{it} + \beta_5 \cdot \text{pesticide}_{it} + \gamma$
- Aux. • $\text{Ability}_i = \pi_0 + \pi_1 \cdot \text{land}_{it} + \pi_2 \cdot \text{labor}_{it} + \pi_3 \cdot \text{seed}_{it} + \pi_4 \cdot \text{urea}_{it} + \pi_5 \cdot \text{pesticide}_{it} + e_i$
- OVB: • $\text{OVB}_{\text{land}} = \beta_1^s - \beta_1 = \pi_1 * \gamma$

- Educated guess about the bias of the estimates.
- What are the signs of π_1 and γ?
  - $\pi_1 > 0$
  - $\gamma > 0$
  - $\text{OVB} = (+) \times (+) > 0$
- Not controlling for the OV ability_i may cause overestimation of the effect of the farm size β_1.
- ability_i does not vary over time for each farm!

How to resolve such OVB?

- Introduce a proxy variable for ability?
  - No such data.
- Rely on the panel structure of the data.
  - Use the individual fixed effect model, for example.

Estimation strategy. Part 2.

Individual fixed effect panel regression production function.

$$
\begin{aligned}
\ln \text{output}_{it} = & A_0 + \beta_1 \cdot \ln \text{land}_{it} + \beta_2 \cdot \ln \text{labor}_{it} \\
& + \beta_3 \cdot \ln \text{seed}_{it} + \beta_4 \cdot \ln \text{urea}_{it} \\
& + \beta_5 \cdot \ln \text{pesticide}_{it} \\
& + \color{red}{\alpha_i} + e_{it}
\end{aligned}
$$

- What are the ex-ante expectations about the regression coefficients?
  - Ideas? …
  - Probably the same as before: all β should be positive.

Estimation strategy. Part 3. Return to scale (1)

Are larger farms more productive?

To understand this, we need to calculate how joint increase of all inputs change the output.

- If increase of all inputs by 1% increases output also by **the same 1%**, we have a **constant return to scale**.
- If increase of all inputs by 1% increases output also by **more than 1%**, we have an **increasing return to scale**.
- If increase of all inputs by 1% increases output also by **less than 1%**, we have an **decreasing return to scale**.

Estimation strategy. Part 3. Return to scale (2)

From the main equation,

$$\begin{aligned}\ln \text{output}_{it} = & A_0 + \beta_1 \cdot \ln \text{land}_{it} + \beta_2 \cdot \ln \text{labor}_{it} \\ & + \beta_3 \cdot \ln \text{seed}_{it} + \beta_4 \cdot \ln \text{urea}_{it} \\ & + \beta_5 \cdot \ln \text{pesticide}_{it} \\ & + \color{red}{\alpha_i} + e_{it}\end{aligned}$$

- return to scale can be estimated as a sum of all coefficients:

$$\text{Rreturn to scale} = \beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5$$

- To perform a hypothesis testing about the return to scale, we need to employ:
  - HT about a linear combination of parameters, and
  - **delta method** for estimating standard errors.

Estimation strategy. Part 3. Return to scale (3)

HT about a linear combination of parameters:

- H0 - $\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 = 1$ (also can be $= 0$ or any number)
- H1 - $\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 \neq 1$ (also can be $\neq 0$ or any number)

We compute standard errors using **delta method** (`car::deltaMethod()`).

And perform HT using F statistics.

Part 2. Exploratory data analysis and data description

We operate a farm-level data with following variables:

- `output` - gross output of rice in kg
- `land` - the total area cultivated with rice, measured in hectares
- `seed` - seed in kilogram
- `urea` - urea in kilogram
- `pesticide` - urea in kilogram
- `labor` - total labor in hours (excluding harvest labor)

Data preparation

Load data and calculate rice yields and logs of all variables.

```r
library(tidyverse)
library(plm)
library(modelsummary)
library(performance)
library(lmtest)

farm_dta <- read_csv("farm_panel.csv")

glimpse(farm_dta)
```

```
Rows: 1,026
Columns: 15
$ id         <dbl> 101001, 101001, 101001, 101001, 101001, 101001, 101017, 101…
$ time       <dbl> 1999, 2000, 2001, 2002, 2003, 2004, 1999, 2000, 2001, 2002,…
$ output     <dbl> 7980, 4083, 2650, 4500, 16300, 17424, 3840, 2800, 950, 240,…
$ land       <dbl> 3.000, 2.000, 1.000, 2.000, 3.572, 3.572, 1.420, 1.420, 0.4…
$ labor      <dbl> 2915, 2155, 1075, 2091, 3889, 3519, 810, 855, 460, 109, 230…
$ hiredlabor <dbl> 2875, 2110, 980, 2081, 3889, 3519, 670, 805, 380, 40, 210, …
$ famlabor   <dbl> 40, 45, 95, 10, 1, 1, 140, 50, 80, 69, 20, 1, 108, 63, 57, …
$ seed       <dbl> 90, 40, 100, 60, 105, 105, 50, 20, 15, 7, 15, 15, 5, 10, 10…
$ urea       <dbl> 900, 600, 700, 600, 400, 400, 120, 100, 150, 50, 100, 100, …
$ pest       <dbl> 6000, 3000, 5000, 5000, 10200, 10200, 0, 0, 900, 0, 2000, 2…
$ varieties  <chr> "mixed", "trad", "high", "high", "high", "high", "trad", "h…
$ status     <chr> "owner", "owner", "owner", "owner", "share", "share", "mixe…
$ bimas      <chr> "mixed", "mixed", "mixed", "mixed", "no", "no", "mixed", "m…
```

Computing yields and logs

```
farm_dta_log <- farm_dta %>%
  mutate(
    l_output = log(output),
    l_land = log(land),
    l_seed = log(seed),
    l_urea = log(urea),
    l_pest = log(pest),
    l_labor = log(labor),
    yields_mt_ha = output / land / 1000
  )
```

Summary statistics

```r
n_inf <- function(x) sum(is.infinite(x))
n_missing <- function(x) sum(is.na(x)|is.nan(x))
datasummary(
  l_output + l_land + l_seed + l_urea + l_pest + l_labor + yields_mt_ha ~
    N + n_missing + n_inf + Mean + SD + Median + Min + Max,
  data = farm_dta_log)
```

	N	n_missing	n_inf	Mean	SD	Median	Min	Max
l_output	1026	0.00	0.00	6.73	0.99	6.79	3.74	9.95
l_land	1026	0.00	0.00	−1.30	0.95	−1.25	−4.61	1.67
l_seed	1026	0.00	0.00	2.37	0.94	2.30	0.00	7.13
l_urea	1026	0.00	0.00	3.98	1.16	4.09	0.00	7.13
l_pest	1026	0.00	713.00					11.04
l_labor	1026	0.00	0.00	5.56	0.85	5.53	2.83	8.47
yields_mt_ha	1026	0.00	0.00	3.38	1.68	3.16	0.40	27.50

- Any problems with data?

Any problems with data?

- Any? …
- `pest`, when transformed with logs, produces `-Inf` values.
  - Why is that so?
  - Any? …
  - Because there are zero values of pesticides application $\ln 0 = -\infty$.
- How to resolve the $\ln 0$ problem?

`-Infinity` in logs: lazy solution

- Before log transformation, substitute any zero with a small value, for example `0.0001`;

```r
farm_dta_log <- farm_dta_log %>%
  mutate(l_pest_lazy = ifelse(is.infinite(l_pest), log(0.0001), l_pest))
```

`-Infinity` in logs: smart solution

- Introduce reverse dummy variables for each variable with log of zero, see: Battese (1997);
- Substitute negative infinity with zero. . . .

```r
farm_dta_log <- farm_dta_log %>%
  mutate(pest_revdum = ifelse(is.infinite(l_pest), 1, 0),
         l_pest_smart = ifelse(is.infinite(l_pest), 0, l_pest))
```

Summary statistics after data cleaning

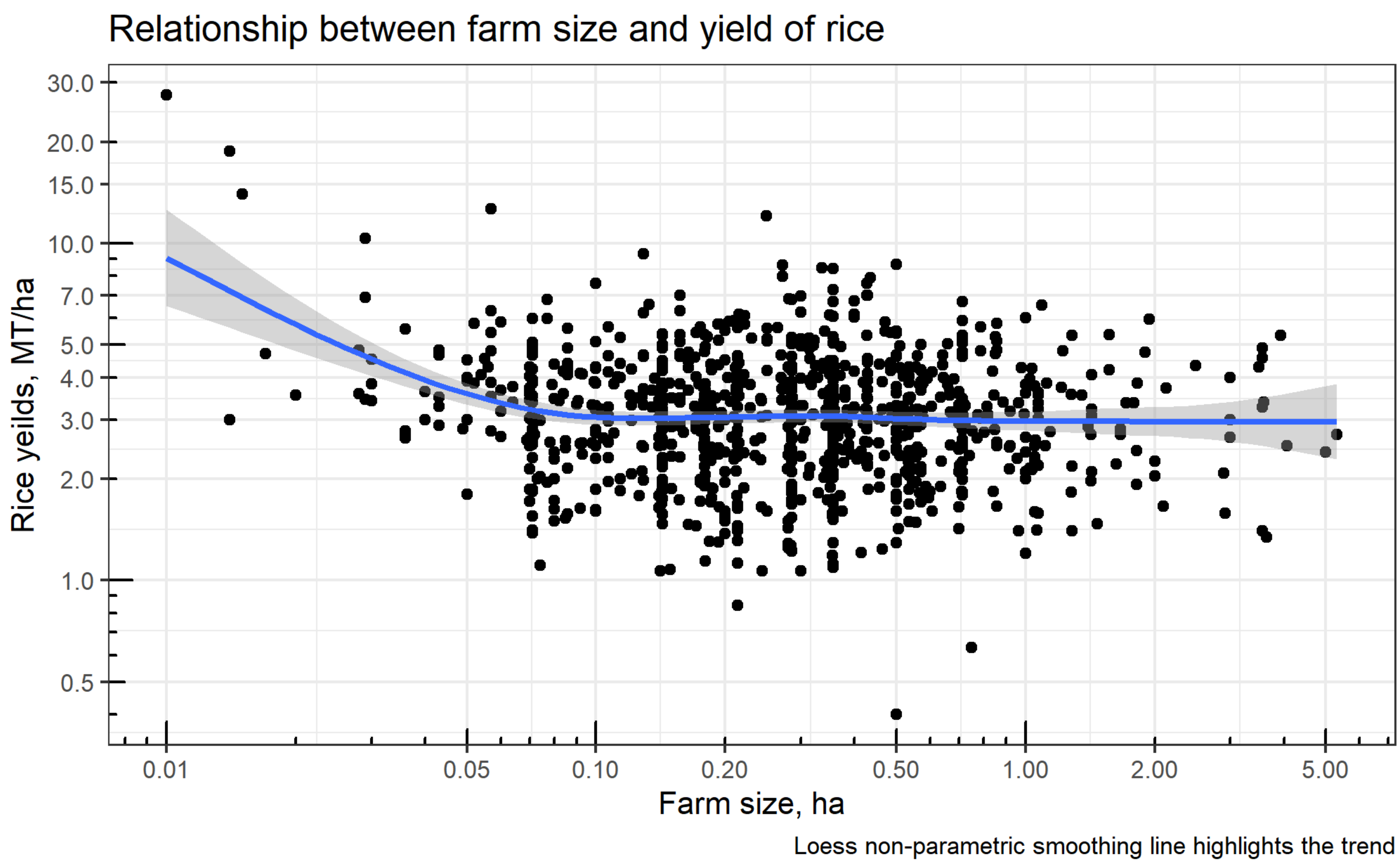
```
datasummary(l_pest + l_pest_lazy + l_pest_smart +  pest_revdum ~
    N + n_missing + n_inf + Mean + SD + Median + Min + Max,
    data = farm_dta_log)
```

	N	n_missing	n_inf	Mean	SD	Median	Min
l_pest	1026	0.00	713.00				
l_pest_lazy	1026	0.00	0.00	−4.37	7.35	−9.21	−9.21
l_pest_smart	1026	0.00	0.00	2.04	3.15	0.00	0.00
pest_revdum	1026	0.00	0.00	0.69	0.46	1.00	0.00

Farm size vs rice yields

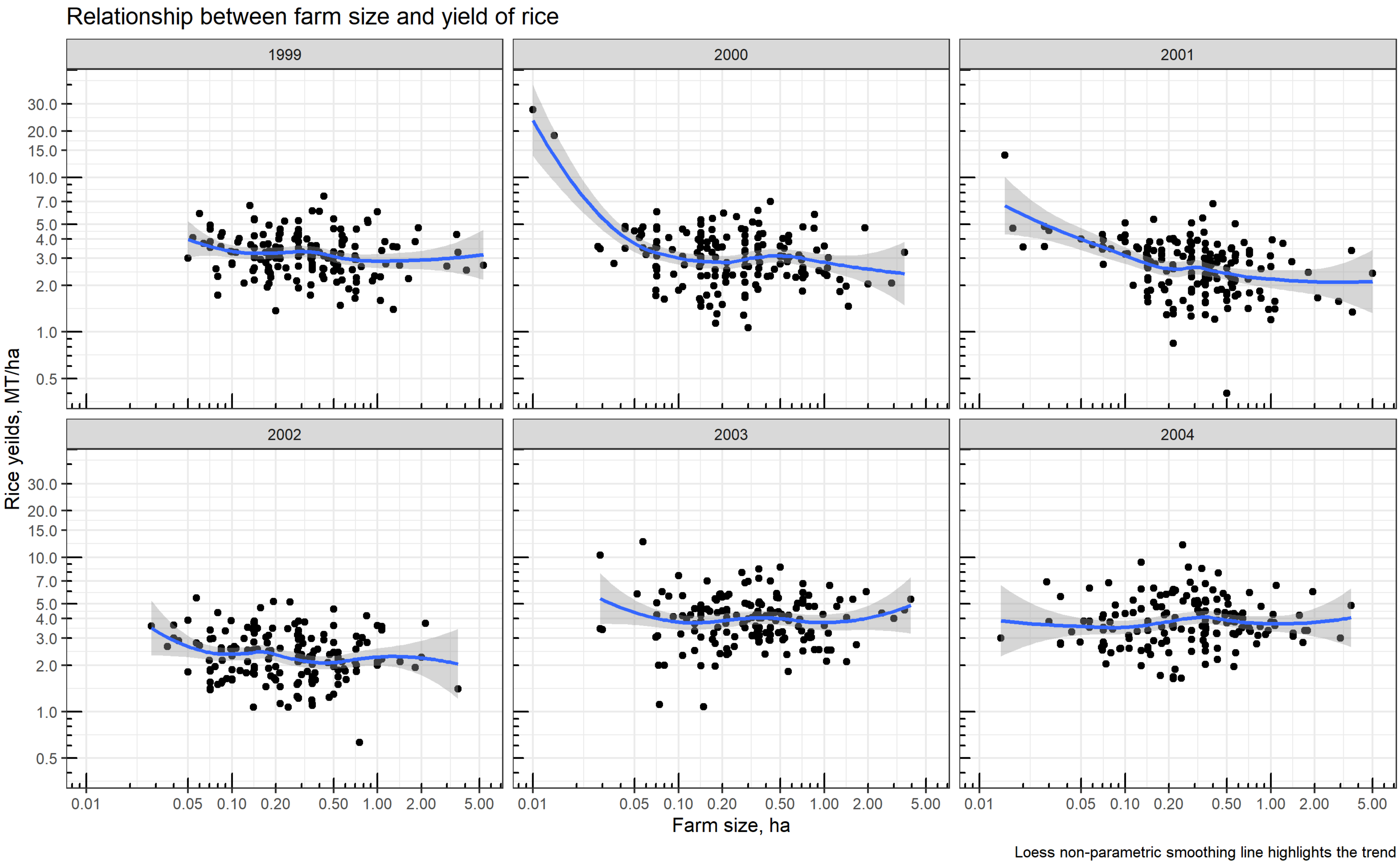
```r
yield_size <-
  farm_dta_log %>%
  ggplot() +
  aes(x = land, y = yields_mt_ha) +
  geom_point() +
  geom_smooth() +
  scale_x_log10("Farm size, ha",
                breaks = c(0.01, 0.05, 0.1, 0.2, 0.5, 1, 2, 5, 10)) +
  scale_y_log10("Rice yeilds, MT/ha",
                breaks = c(0.2, 0.5, 1, 2, 3, 4, 5, 7, 10, 15, 20, 30)) +
  annotation_logticks() +
  labs(title = "Relationship between farm size and yield of rice",
       caption = "Loess non-parametric smoothing line highlights the trend")
yield_size
```

Farm size vs rice yields



Farm size vs rice yields by year

```
yield_size + facet_wrap(. ~ time)
```



Part 3. Estimating the models

Step 1. Pooled OLS (with `lm()` and `plm()` functions);

- Validate all assumptions (linearity, collinearity, homogeneity)

Step 2. Fixed Effect and Random Effect models (with `plm()`);

- Choose a consistent model (models) relying on: F-test (`pftest()`), Lagrange multiplier test (`plmtest()`), Hausman test (`phtest()`);

Step 3. Validate homogeneity assumption (cross-sectional dependency and autocorrelation)

- Wooldridge's test (`pwtest()`) and Lagrange-Multiplier tests (`pbsytest()`).

Step 4. Robust inference and results interpretation.

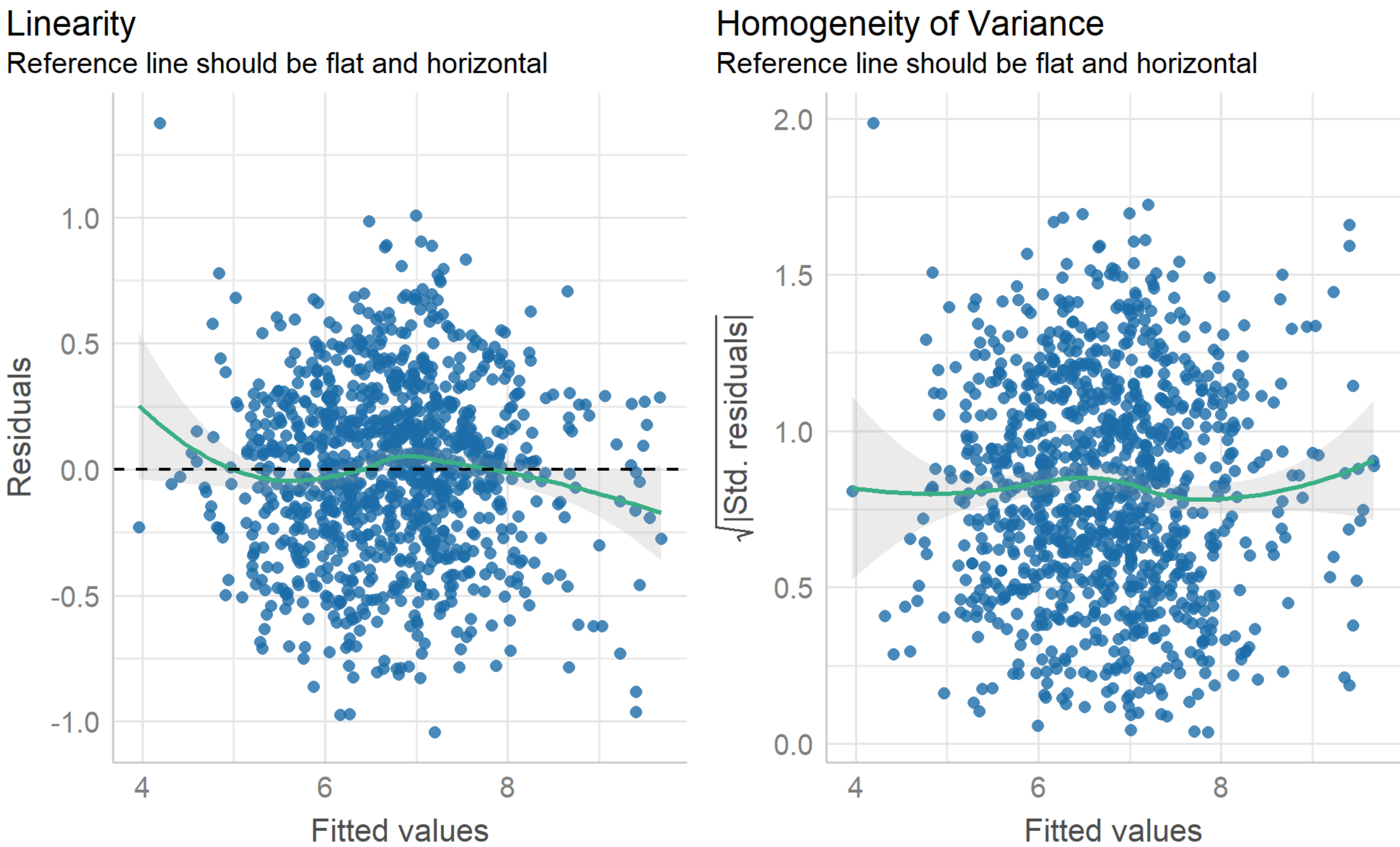
Step 1 Pooled OLS

```
rice_pooled <-
  plm(l_output ~ l_land + l_labor + l_seed + l_urea +
        l_pest_smart + pest_revdum,
      data = farm_dta_log, model = "pooling", index = c("id", "time"))

rice_pooled_2 <-
  lm(l_output ~ l_land + l_labor + l_seed + l_urea +
       l_pest_smart + pest_revdum,
      data = farm_dta_log)

rice_pooled
```

```
Model Formula: l_output ~ l_land + l_labor + l_seed + l_urea + l_pest_smart +
    pest_revdum

Coefficients:
 (Intercept)       l_land      l_labor       l_seed       l_urea l_pest_smart
    4.314437     0.434792     0.253482     0.148404     0.171475     0.091171
 pest_revdum
    0.512910
```

Step 1 Linearity and homoscedasticity

```
check_model(rice_pooled_2, check = c("linearity", "homogeneity"))
```



Step 1 Collinearity

```
check_collinearity(rice_pooled_2)
```

```
# Check for Multicollinearity

Low Correlation

    Term  VIF     VIF 95% CI Increased SE Tolerance Tolerance 95% CI
 l_labor 4.77 [ 4.28,  5.32]         2.18      0.21     [0.19, 0.23]
  l_urea 2.67 [ 2.42,  2.95]         1.63      0.37     [0.34, 0.41]

Moderate Correlation

   Term  VIF     VIF 95% CI Increased SE Tolerance Tolerance 95% CI
 l_land 6.94 [ 6.21,  7.78]         2.64      0.14     [0.13, 0.16]
 l_seed 5.07 [ 4.55,  5.66]         2.25      0.20     [0.18, 0.22]

High Correlation
```

Step 2 Fixed Effect

```r
rice_fe <-
  plm(l_output ~ l_land + l_labor + l_seed + l_urea +
        l_pest_smart + pest_revdum,
      data = farm_dta_log,
      model = "within",
      effect = "individual",
      index = c("id", "time"))
rice_fe
```

```
Model Formula: l_output ~ l_land + l_labor + l_seed + l_urea + l_pest_smart + 
    pest_revdum

Coefficients:
      l_land      l_labor       l_seed       l_urea l_pest_smart  pest_revdum 
     0.41984      0.27084      0.12381      0.16291      0.11029      0.63498 
```

Step 2 FE with lazy log(0)

```r
rice_fe_lazy <-
  plm(l_output ~ l_land + l_labor + l_seed + l_urea + l_pest_lazy ,
      data = farm_dta_log,
      model = "within",
      effect = "individual",
      index = c("id", "time"))
rice_fe_lazy
```

```
Model Formula: l_output ~ l_land + l_labor + l_seed + l_urea + l_pest_lazy

Coefficients:
   l_land   l_labor    l_seed    l_urea l_pest_lazy
 0.421042  0.263876  0.134373  0.174145    0.007703
```

Step 2.2 F test for individual effects

- Compares FE model to OLS. OLS is always consistent, when Gauss-Markov assumptions are satisfied.
  - H0: One model is inconsistent.
  - H1: Both models are equally consistent.

```
pFtest(rice_fe, rice_pooled)
```

```
	F test for individual effects

data:  l_output ~ l_land + l_labor + l_seed + l_urea + l_pest_smart +  ...
F = 1.4988, df1 = 170, df2 = 849, p-value = 0.0001704
alternative hypothesis: significant effects
```

Step 2.3 Lagrange Multiplier Tests

- Compares FE model to OLS. OLS is always consistent, when Gauss-Markov assumptions are satisfied.
  - H0: One model is inconsistent.
  - H1: Both models are equally consistent.

```
plmtest(rice_pooled, effect = "individual", type = "honda")
```

```
	Lagrange Multiplier Test - (Honda)

data:  l_output ~ l_land + l_labor + l_seed + l_urea + l_pest_smart +  ...
normal = 3.7129, p-value = 0.0001025
alternative hypothesis: significant effects
```

```
plmtest(rice_pooled, effect = "individual", type = "bp")
```

```
	Lagrange Multiplier Test - (Breusch-Pagan)

data:  l_output ~ l_land + l_labor + l_seed + l_urea + l_pest_smart +  ...
chisq = 13.785, df = 1, p-value = 0.0002049
alternative hypothesis: significant effects
```

Step 2 Random Effect

```r
rice_re <-
  plm(l_output ~ l_land + l_labor + l_seed + l_urea +
        l_pest_smart + pest_revdum,
      data = farm_dta_log,
      model = "random",
      effect = "individual",
      index = c("id", "time"))
rice_re
```

```

Model Formula: l_output ~ l_land + l_labor + l_seed + l_urea + l_pest_smart +
    pest_revdum

Coefficients:
 (Intercept)       l_land      l_labor       l_seed       l_urea l_pest_smart
    4.276601     0.433744     0.257738     0.142589     0.169423     0.096608
 pest_revdum
    0.546997
```

Step 2 Hausman Test for Panel Models

- Compares RE to FE model. FE is assumed to be consistent
  - H0: One model is inconsistent.
  - H1: Both models are equally consistent.

```
phtest(rice_fe, rice_re)
```

```
	Hausman Test

data:  l_output ~ l_land + l_labor + l_seed + l_urea + l_pest_smart +  ...
chisq = 7.285, df = 6, p-value = 0.2953
alternative hypothesis: one model is inconsistent
```

- Fixed Effect model is recommended

Step 3 Serial correlation and cross-sectional dependence

- Wooldridge's test for unobserved individual effects
  - H0: no unobserved effects
  - H1: some effects also dues to serial correlation

```
pwtest(rice_pooled, effect = "individual")
```

```
    Wooldridge's test for unobserved individual effects

data:  formula
z = 2.1603, p-value = 0.03075
alternative hypothesis: unobserved effect
```

```
pwtest(rice_pooled, effect = "time")
```

```
    Wooldridge's test for unobserved time effects

data:  formula
z = 1.6899, p-value = 0.09105
alternative hypothesis: unobserved effect
```

Step 3 lm tests for random effects and/or serial correlation

- H0: serial correlation is zero
- H1: some serial correlation

```
pbsytest(rice_pooled)
```

```
    Bera, Sosa-Escudero and Yoon locally robust test

data:  formula
chisq = 20.988, df = 1, p-value = 4.622e-06
alternative hypothesis: AR(1) errors sub random effects
```

Step 4. Robust inference

```r
library(lmtest)

rice_pooled_robust <- coeftest(
  rice_pooled,
  vcovHC(rice_pooled, method = "arellano", type = "HC3", cluster = "group")
)

rice_fe_robust <- coeftest(
  rice_fe,
  vcovHC(rice_fe, method = "arellano", type = "HC3", cluster = "group")
)

rice_felazy_robust <- coeftest(
  rice_fe_lazy,
  vcovHC(rice_fe_lazy, method = "arellano", type = "HC3", cluster = "group")
)
```

Step 4. Robust inference

```r
modelsummary(
  list(
    `Pooled` = rice_pooled_robust,
    `FE (rev. dum.)` = rice_fe_robust,
    `FE (lazy)` = rice_felazy_robust
    ),
  fmt = 4, statistic = NULL,
  estimate = "{estimate}{stars} ({std.error})",
  notes = "Robust standard errors clustered at the group level are reported in the brackets.")
```

	Pooled	FE (rev. dum.)	FE (lazy)
(Intercept)	4.3144*** (0.2691)		
l_land	0.4348*** (0.0418)	0.4198*** (0.0479)	0.4210*** (0.0486)
l_labor	0.2535*** (0.0333)	0.2708*** (0.0347)	0.2639*** (0.0348)
l_seed	0.1484*** (0.0344)	0.1238*** (0.0370)	0.1344*** (0.0386)
l_urea	0.1715*** (0.0228)	0.1629*** (0.0262)	0.1741*** (0.0269)
l_pest_smart	0.0912*** (0.0172)	0.1103*** (0.0185)	
pest_revdum	0.5129*** (0.1141)	0.6350*** (0.1290)	
l_pest_lazy			0.0077*** (0.0020)
Num.Obs.	1026	1026	1026
AIC	2796.9	2187.6	2222.6
BIC	7824.1	6376.1	6416.0

Robust standard errors clustered at the group level are reported in the brackets.

Step 4. Robust inference

```r
compare_performance(
  list(
    `Pooled` = rice_pooled,
    `FE with rev. dum` = rice_fe,
    `FE lazy` = rice_fe_lazy
  )
)
```

```
# Comparison of Model Performance Indices

Name             | Model | AIC (weights) | AICc (weights) | BIC (weights) |    R2 | R2 (adj.) |
RMSE | Sigma
-------------------------------------------------------------------------------------------------
-------------
Pooled           |   plm | 774.9 (<.001) |  775.1 (<.001) | 814.4 (<.001) | 0.876 |     0.875 |
0.350 | 0.351
FE with rev. dum |   plm | 503.6 (>.999) |  503.7 (>.999) | 538.2 (>.999) | 0.732 |     0.676 |
0.307 | 0.308
FE lazy          |   plm | 534.6 (<.001) |  534.6 (<.001) | 564.2 (<.001) | 0.723 |     0.666 |
0.312 | 0.313
```

Part 4. Return to scale (1)

```
rice_fe
```

```
Model Formula: l_output ~ l_land + l_labor + l_seed + l_urea + l_pest_smart +
    pest_revdum

Coefficients:
     l_land     l_labor      l_seed      l_urea l_pest_smart  pest_revdum
    0.41984     0.27084     0.12381     0.16291      0.11029      0.63498
```

Computing sum of the coefficients and robust SE:

```
library(car)
deltaMethod(
  rice_fe,
  "l_land + l_labor + l_seed + l_urea + l_pest_smart",
  vcov = vcovHC(rice_fe, method = "arellano", type = "HC3", cluster = "group")
)
```

```
                                                  Estimate       SE    2.5 %
l_land + l_labor + l_seed + l_urea + l_pest_smart 1.087685 0.031039 1.026850
                                                  97.5 %
l_land + l_labor + l_seed + l_urea + l_pest_smart 1.1485
```

Part 4. Return to scale (2)

HT about the sum of the coefficients:

- H_0 : return to scale $= 1$
- H_0 : return to scale $\neq 1$

```r
linearHypothesis(rice_fe,
  "l_land + l_labor + l_seed + l_urea + l_pest_smart = 1",
  vcov = vcovHC(rice_fe, method = "arellano", type = "HC3", cluster = "group"))
```

```
Linear hypothesis test

Hypothesis:
l_land  + l_labor  + l_seed  + l_urea  + l_pest_smart = 1

Model 1: restricted model
Model 2: l_output ~ l_land + l_labor + l_seed + l_urea + l_pest_smart + 
    pest_revdum

Note: Coefficient covariance matrix supplied.

  Res.Df Df  Chisq Pr(>Chisq)   
1    850                        
2    849  1 7.9806   0.004728 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Conclusions

Regression results

	Pooled	FE (rev. dum.)	FE (lazy)
(Intercept)	4.3144*** (0.2691)		
l_land	0.4348*** (0.0418)	0.4198*** (0.0479)	0.4210*** (0.0486)
l_labor	0.2535*** (0.0333)	0.2708*** (0.0347)	0.2639*** (0.0348)
l_seed	0.1484*** (0.0344)	0.1238*** (0.0370)	0.1344*** (0.0386)
l_urea	0.1715*** (0.0228)	0.1629*** (0.0262)	0.1741*** (0.0269)
l_pest_smart	0.0912*** (0.0172)	0.1103*** (0.0185)	
pest_revdum	0.5129*** (0.1141)	0.6350*** (0.1290)	
l_pest_lazy			0.0077*** (0.0020)
Num.Obs.	1026	1026	1026

Return to scale

```
library(car)
deltaMethod(rice_fe,
            "l_land + l_labor + l_seed + l_urea + l_pest_smart'
            vcov = vcovHC(rice_fe, method = "arellano", type =
```

```
                                                    Estimate
SE    2.5 %
l_land + l_labor + l_seed + l_urea + l_pest_smart 1.087685
0.031039 1.026850
                                                    97.5 %
l_land + l_labor + l_seed + l_urea + l_pest_smart 1.1485
```

```
linearHypothesis(rice_fe,
                 "l_land + l_labor + l_seed + l_urea + l_pest_s
                 vcov = vcovHC(rice_fe, method = "arellano", ty
```

```
Linear hypothesis test

Hypothesis:
l_land  + l_labor  + l_seed  + l_urea  + l_pest_smart = 1

Model 1: restricted model
Model 2: l_output ~ l_land + l_labor + l_seed + l_urea +
l_pest_smart +
    pest_revdum

Note: Coefficient covariance matrix supplied.

  Res.Df Df  Chisq Pr(>Chisq)
1    850
2    849  1 7.9806   0.004728 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Individual fixed effects

From the regression equation below,

$$\begin{aligned}\ln \text{output}_{it} &= A_0 + \beta_1 \cdot \ln \text{land}_{it} + \beta_2 \cdot \ln \text{labor}_{it} \\ &+ \beta_3 \cdot \ln \text{seed}_{it} + \beta_4 \cdot \ln \text{urea}_{it} \\ &+ \beta_5 \cdot \ln \text{pesticide}_{it} \\ &+ \alpha_i + e_{it}\end{aligned}$$

we know that α_i are the individual fixed effects.

- R calculates them and we can explore them.
- In fact, those individual fixed effects are the simplest possible measured of farms efficiency!

Extra on fixed effects

Individual fixed effects (extraction 1)

In the model:

```
rice_fe
```

```
Model Formula: l_output ~ l_land + l_labor + l_seed + l_urea + l_pest_smart +
    pest_revdum

Coefficients:
     l_land     l_labor      l_seed      l_urea l_pest_smart  pest_revdum
    0.41984     0.27084     0.12381     0.16291      0.11029      0.63498
```

Individual fixed effects can be extracted as:

```
fixef(rice_fe)
```

```
101001 101017 101026 101035 101056 101057 101067 101068 101069 101073 101089
3.8470 4.1040 4.2375 4.2631 4.5649 4.2051 4.2591 4.3471 4.3598 4.0599 4.0611
101094 102111 102113 102119 102126 102157 102194 102220 201001 201002 201003
4.0503 4.1591 4.1073 4.2289 3.9991 4.2420 3.9798 4.2032 4.3270 4.3719 4.2906
201009 202039 202061 202066 203079 203080 204096 204114 204116 204124 205132
4.1036 4.2036 4.1584 4.1318 4.2522 4.0977 4.2539 3.9338 4.3095 4.0783 3.9765
205136 205151 205153 206158 206169 207209 208225 209232 209241 209250 301004
3.9870 4.2319 4.2504 4.4004 4.0218 4.2235 4.2522 4.1496 4.0292 4.0267 4.0658
301010 301023 301038 301055 301058 301067 301070 301075 301084 301105 301110
3.7676 4.1062 4.1388 4.1066 4.1385 4.1110 4.0471 4.3944 3.8461 3.9620 3.9876
302116 302120 302131 302134 302137 302142 302143 302144 302146 302147 302151
4.1645 3.8708 4.0910 4.2774 4.1428 4.0946 4.0225 4.1853 4.1072 3.8265 4.1892
302153 302161 302163 302169 302182 302189 302192 302194 302195 302197 302199
3.9304 4.1563 4.1978 4.2176 4.1114 4.2771 4.3379 4.1691 4.3303 4.2350 4.0927
302205 302207 302209 401002 401006 401032 401034 401036 401037 401041 401043
```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Individual fixed effects (extraction 3)

Or, we can extract individual fixed effects with effect-specific standard errors:

```
fef_dta <-
  fixef(rice_fe) %>%
  summary() %>%
  as.data.frame() %>% rownames_to_column("id") %>%
  as_tibble() %>%
  mutate(id = as.double(id))
fef_dta
```

```
# A tibble: 171 × 5
       id Estimate `Std. Error` `t-value` `Pr(>|t|)`
    <dbl>    <dbl>        <dbl>     <dbl>      <dbl>
 1 101001     3.85        0.317      12.2   1.96e-31
 2 101017     4.10        0.282      14.6   5.20e-43
 3 101026     4.24        0.285      14.9   1.09e-44
 4 101035     4.26        0.279      15.3   1.11e-46
 5 101056     4.56        0.285      16.0   9.02e-51
 6 101057     4.21        0.292      14.4   3.46e-42
 7 101067     4.26        0.296      14.4   2.89e-42
 8 101068     4.35        0.283      15.4   3.87e-47
 9 101069     4.36        0.279      15.6   1.62e-48
10 101073     4.06        0.294      13.8   3.35e-39
# ℹ 161 more rows
```

Farm size and efficiency

Let us compute average farm size and plot fixed effects versus farm size:

```
farm_mean_size <- farm_dta_log %>% group_by(id) %>% summarise(mean_size = mean(land))
glimpse(farm_mean_size)
```

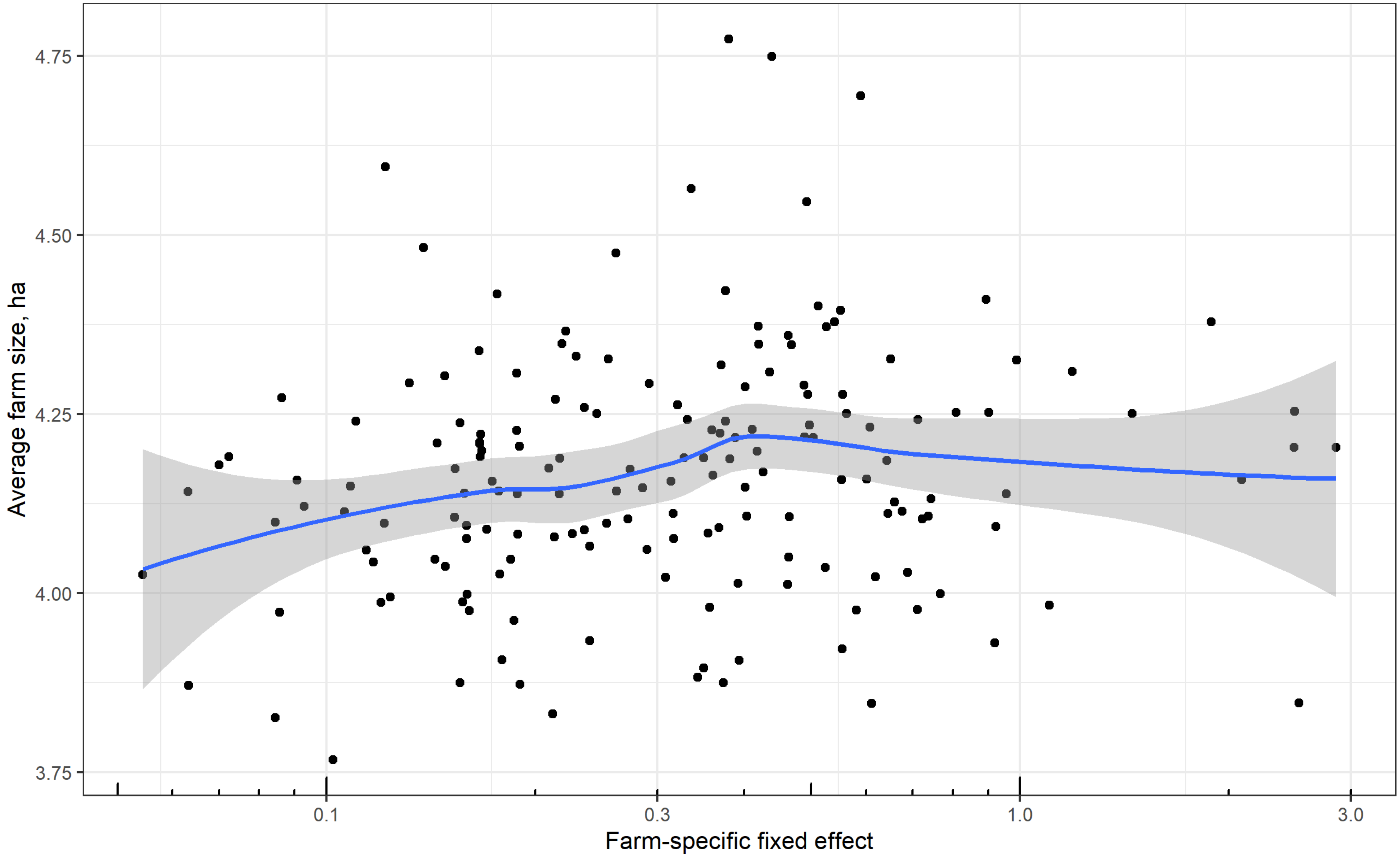
```
Rows: 171
Columns: 2
$ id        <dbl> 101001, 101017, 101026, 101035, 101056, 101057, 101067, 1010…
$ mean_size <dbl> 2.5240000, 0.7230000, 0.1558333, 0.3211667, 0.3360000, 0.189…
```

```
plot_dta <- farm_mean_size %>% left_join(fef_dta)
glimpse(plot_dta)
```

```
Rows: 171
Columns: 6
$ id           <dbl> 101001, 101017, 101026, 101035, 101056, 101057, 101067, 1…
$ mean_size    <dbl> 2.5240000, 0.7230000, 0.1558333, 0.3211667, 0.3360000, 0.…
$ Estimate     <dbl> 3.847036, 4.103960, 4.237548, 4.263063, 4.564856, 4.20514…
$ `Std. Error` <dbl> 0.3166160, 0.2819603, 0.2847204, 0.2792276, 0.2846006, 0.…
$ `t-value`    <dbl> 12.15048, 14.55510, 14.88319, 15.26734, 16.03951, 14.3927…
$ `Pr(>|t|)`   <dbl> 1.958956e-31, 5.202280e-43, 1.090622e-44, 1.109427e-46, 9…
```

```
plot_dta %>% ggplot() + aes(x = mean_size, y = Estimate) +
  geom_point() + geom_smooth() + scale_x_log10() +
  xlab("Farm-specific fixed effect") +
  ylab("Average farm size, ha") +
  annotation_logticks(side = "b")
```

Farm size and efficiency



References

Battese, G. E. (1997). A note on the estimation of cobb-douglas production functions when some explanatory variables have zero values. *Journal of Agricultural Economics*, *48*(1-3), 250–252. http://doi.org/10.1111/j.1477-9552.1997.tb01149.x