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## SHADOW PRICES, MARKET WAGES, AND LABOR SUPPLY

BY JAMES HECKMAN<sup>1</sup>

IN RECENT RESEARCH on married women in the labor market, a common set of variables is used to explain their wage rates, hours of work, and decision to work. Typically, these topics are treated separately, although Kalachek and Raines [14] and Boskin [7] estimate an expected hours of work equation for all women by separately estimating an hours of work relation for working women, and an equation determining the probability that a woman works. In other research, observations on non-working women are assumed to lie on the same hours of work function as observations for working women, with a particular value of zero for their hours of work (Leibowitz [15 and 16]). No theoretical justification is provided for this procedure. Moreover, a "missing variable" problem arises in regressing hours worked on wage rates since wages are not reported for non-working women. One solution to this problem is to estimate a wage function on a subsample of working women to predict missing wages. However, this procedure can lead to biased parameter estimates for wage functions and hence for labor supply functions (Aigner [1], Gronau [11]).

In this paper, we derive a common set of parameters which underlie the functions determining the probability that a woman works, her hours of work, her observed wage rate, and her asking wage or shadow price of time. We rely on two behavioral schedules: the function determining the wage a woman faces in the market (the "offered wage"), and the function determining the value a woman places on her time (the "asking wage"). If a woman works, her hours of work adjust to equate these wages if she has freedom to set her working hours. If a woman does not work, no offered wage matches her asking wage. If we estimate both wage schedules, we can use the estimated parameters to determine the probability that a woman works, her actual hours of work given that she works, the potential market wage rates facing nonworking women, and the implicit value of time for non-working women.

We develop a statistical procedure which yields estimates of this common parameter set. This procedure extends Tobit (Tobin [23]) to a simultaneous equations system, and differs from it by allowing different parameters to affect the probability that a woman works, and her hours of work. The method allows us to utilize an entire sample of observations on women, whether or not they work, to estimate the functions determining their hours of work, wage rates, and probabilities

<sup>1</sup> Reuben Gronau made valuable comments at an early stage of this research. I am indebted to Jacob Mincer, Marc Nerlove, Paul Schulz, members of the Columbia University Labor Workshop, members of the research staff of the Rand Corporation, Santa Monica, California, and members of the Labor Workshop of the University of Chicago for their comments on a previous version of this paper. Ralph Shnelvar was exceptionally skillful and patient in programming the function optimizations. I assume responsibility for all errors in the paper. This research was sponsored by National Science Foundation and Office of Economic Opportunity grants to the National Bureau of Economic Research.

of working. However, we utilize the sample information in a way distinct from previous studies.

In Section 1, we derive the shadow price or asking wage function, and present the general model. In Section 2, we discuss statistical issues, and in Section 3 we present parameter estimates derived from the procedure presented in Section 2.

### 1. SHADOW PRICES AND MARKET WAGES

We assume that households maximize a well behaved twice differentiable utility function subject to time, wealth, and other constraints. Proceeding in the usual fashion, we may derive the conventional demand relations for goods and leisure as functions of parametric prices and wages, non-labor income, and other constraints. However, as Hotelling [13], Samuelson [21], and Pollak [20] have shown, these are not the only functions associated with a constrained maximum, nor are they necessarily the most convenient for theoretical or empirical purposes. Assuming interior solutions, it is possible to express a subset of prices and wages as functions of their associated quantities and the remaining prices and wages, non-labor income, and other constraints.<sup>2</sup> More importantly, it is possible to give a different interpretation to these functions which makes them useful in cases where corner solutions exist, and the usual demand functions are not defined. In particular, we may interpret these prices as shadow prices or marginal values. It is well known that if positive quantities of a market good are purchased, a necessary equilibrium condition is that its price equals its marginal value, while if a good is not purchased, its price exceeds the marginal valuation at zero quantities of the good. For labor supply or the demand for leisure, a similar condition applies except that now there are two possible corners: given a fixed amount of time in the decision period, an individual may work no more than that amount of time, and cannot work less than zero hours. For equilibrium at the first type of corner, the marginal valuation at zero quantities of *leisure* is less than the market wage while at the second corner, the marginal valuation at the maximal quantity of leisure exceeds the market wage.

Working with the shadow price functions, it is possible to characterize both interior and corner solutions within a common theoretical framework because the shadow price functions are defined at corners where demand functions are not defined. In this paper, we apply this insight to develop a unified econometric methodology for estimating the parameters of both the hours of work and decision to work functions for married women. The problem of corner solutions is particularly pronounced in analyzing labor supply data for married women, and for this reason we focus on this group in this paper. Nonetheless, the methodology developed below is generally applicable to any situation where corner solutions are of practical importance.

The shadow price function for the wife's time may be written as

$$(1) \quad W^* = g(h, W_m, P, A, Z)$$

<sup>2</sup> This requires the usual assumption of the non-vanishing of the appropriate Jacobian.

where  $W^*$  is the shadow price,  $h$  is the hours of work, or alternatively, the amount of time the wife does not have available for her nonmarket activities,  $W_m$  is the wage of the husband,  $P$  is a vector of goods prices,  $A$  is the asset income of the household, and  $Z$  is a vector of constraints which arise from previous economic choices or chance events, such as the number of children, the education of the family members, and the state of household technology.  $W^*$  is the value the household places on marginal units of the wife's time in production and consumption.

The formal derivation of this function is relegated to Appendix 1. There we establish that if the ordinary labor supply function is a positive monotonic function of wage rates, equation (1) may be derived in a straightforward fashion, with the range of that function constituting the domain of the marginal valuation function. We further establish that equation (1) possesses a continuous partial derivative with respect to  $h$  at  $h = 0$  if the household preference function is defined for quantities of leisure in excess of the total time currently available to the wife. Historical time-series and cross-section studies suggest that there is a monotonic *positive* relationship between wage rates and labor supply for married women (Mincer [18], Ashenfelter and Heckman [3]) so that excluding the "backward bending" case is not objectionable, at least for an analysis of this demographic group. However, in the empirical work presented below, the hypothesis of a positive relationship is tested rather than directly imposed on the data.

While there may be strong intuitive feelings about the direction of the relationship between  $W^*$  and other variables, the assumption of utility maximization yields no information on these signs. Nonetheless, previous empirical analysis suggests that children tend to increase  $W^*$  and that this effect is more pronounced the younger their ages. If leisure is a normal good, it is easy to show that increments in net worth raise  $W^*$ . Michael [17] argues that the education of the wife raises the wife's efficiency in producing domestic services. Thus, education might affect  $W^*$  but the direction of that effect is uncertain.

The determinants of the market wage rate ( $W$ ) are better known. Education and years of labor force experience are expected to increase the wage (Mincer [19]). The market wage function may thus be written as

$$(2) \quad W = B(E, S)$$

with  $S$  defined as the number of years of schooling, and  $E$  defined as the extent of labor market experience. Previous research suggests  $B_E > 0$  and  $B_S > 0$ .

If a woman is free to adjust her working hours, a working woman will have  $W = W^*$  as an equilibrium condition.<sup>3</sup> If she does not work, and hours of work cannot become negative,  $W^* \geq W$ . In this analysis, hours of work play the role of a slack variable in nonlinear programming and the basic condition  $h(W^* - W) = 0$  applies to all women free to choose their working hours.

If a woman works, equations (1) and (2) become a recursive system determining hours worked. Just as in the Marshallian model of market demand where quantity adjusts to equate demand and supply prices, hours adjust in this model to equate

<sup>3</sup> Note, we dismiss the possibility of a corner solution where a woman works all available hours since it is empirically uninteresting.

offered and asking wages. Since the offered wage is assumed to be independent of hours worked, and the asking wage is assumed to increase with hours worked, a necessary condition for equilibrium to occur is that at zero hours of work, offered wages should exceed asking wages.

## 2. ESTIMATION

In order to estimate equations (1) and (2), we must specify their functional form and associated stochastic structure. Assuming that there exists a suitable monotonic transformation of the dependent variables so that each equation may be expressed as a linear function of its independent variables, and letting  $l(\cdot)$  be that transformation, equations (1) and (2) for the  $i$ th observation may be written as

$$(3) \quad l(W_i^*) = \beta_0 + \beta_1 h_i + \beta_2 (W_m)_i + \beta_3 P_i + \beta_4 A_i + \beta_5 Z_i + \varepsilon_i,$$

$$(4) \quad l(W_i) = b_0 + b_1 S_i + b_2 E_i + u_i.$$

We assume that  $\varepsilon_i$  and  $u_i$  are jointly normally distributed, each with mean zero, and correlation between these disturbances is allowed. The disturbances for each observation are assumed to be independent of the other disturbances, and the right-hand side variables. These disturbances reflect variations in functions known to the appropriate individual, but not known to the economist. Our results are not directly applicable to the case of labor market search where the wage is unknown to the individual before she enters the market.

The linearity of equation (3) does not necessarily imply that an exogenous wage increase causes a working woman to increase her working hours at a rate independent of the level of the wage rate. Only if  $l(W_i) = W_i$  does the substitution effect remain constant for all values of wages and associated hours of work.

In applying the model, it is important to make a distinction between observed and hypothetical values for the variables. Thus, in equations (3) and (4), we assume that the disturbances are uncorrelated with the regressors. However, *observed* hours of work will depend on those disturbances.

To see this, consider a woman with  $l(W) > l(W^*)$  at the zero hours of work position. If this condition holds for individual  $i$ , i.e., if

$$(5) \quad b_0 - \beta_0 + b_1 S_i + b_2 E_i - \beta_2 (W_m)_i - \beta_3 P_i - \beta_4 A_i - \beta_5 Z_i > \varepsilon_i - u_i,$$

hours of work adjust so that  $W_i^* = W_i$ . Then equations (3) and (4) become a recursive system determining working hours, and the particular adjustment of hours depends, in part, on the magnitude of the discrepancy  $\varepsilon_i - u_i$ .

Given that condition (5) holds, the reduced form equations for observed wages and hours become

$$(6) \quad h_i = \frac{1}{\beta_1} (b_0 - \beta_0 + b_1 S_i + b_2 E_i - \beta_2 (W_m)_i - \beta_3 P_i - \beta_4 A_i - \beta_5 Z_i) + \frac{u_i - \varepsilon_i}{\beta_1},$$

$$(7) \quad l(W_i) = b_0 + b_1 S_i + b_2 E_i + u_i.$$

The crucial feature of the model is that we obtain observations on which to estimate equations (6) and (7) only if condition (5) holds. Thus, for a sample of working women, the distributions of the disturbances of equations (6) and (7) are conditional on inequality (5) and hence are conditional distributions. Since the same exogenous variables appear in condition (5) and equations (6) and (7), the mean and other characteristics of these conditional distributions, for a particular observation, depend on the values of the exogenous variables for the observation. Thus, it is not possible to obtain unbiased or consistent estimates of equations (6) and (7) using ordinary least squares since the regressors are correlated with the disturbances. The same remarks apply to any instrumental variable estimation technique such as two-stage least squares which uses the exogenous variables appearing in condition (5) as instruments.

However, it is possible to obtain consistent parameter estimates. Using the well known relationship between conditional and unconditional distributions, the joint distribution of *observed* hours and wages for the  $i$ th working women may be written as

$$(8) \quad j(h_i, l(W_i))(W_i^* < W_i)_{h=0} = \frac{n(h_i, l(W_i))}{\text{pr}([W_i > W_i^*]_{h=0})},$$

where  $n(h_i, l(W_i))$  is the unconditional distribution,  $\text{pr}([W_i > W_i^*]_{h=0})$  is the probability that the woman works, and  $j(\cdot)$  is the conditional distribution. Since  $\varepsilon_i$  and  $u_i$  are jointly normally distributed,  $n(\cdot)$  is a multivariate normal density, and  $\text{pr}(\cdot)$  is a univariate cumulative normal density function with many of the same parameters as  $n(\cdot)$ . These statements are proved in Appendix 2.

If a sample of  $T$  married women contains  $K$  who work, and  $T - K$  who do not, the likelihood function for the entire  $T$  observations may be written as

$$L = \prod_{i=1}^K j(h_i, l(W_i))(W_i > W_i^*)_{h=0} \cdot \text{pr}([W_i > W_i^*]_{h=0}) \\ \times \prod_{i=K+1}^T \text{pr}([W_i < W_i^*]_{h=0}).$$

Using equation (8), the likelihood function collapses to

$$(9) \quad L = \prod_{i=1}^K n(h_i, l(W_i)) \prod_{i=K+1}^T \text{pr}([W_i < W_i^*]_{h=0}).$$

Maximizing this function with respect to the parameters of the model, including the variances and covariances of the disturbances in equations (3) and (4), yields consistent, asymptotically unbiased, and efficient parameter estimates which are asymptotically normally distributed.<sup>4</sup>

The likelihood function in equation (9) differs from Tobin's [23] in that  $n(\cdot)$  is a multivariate normal density, while it is a univariate density in the Tobit model, and the parameter  $\beta_1$  which appears in  $n(h_i, l(W_i))$  does not appear in the cumulative density  $\text{pr}(W_i < W_i^*|_{h=0})$ . Further differences are explored in Appendix 2.

<sup>4</sup> The proof of these propositions follows from a straightforward extension of Amemiya's [2] valuable proof of the consistency and asymptotic normality of the Tobit estimator.

If all women work, so that  $T = K$ , maximizing function (9) is equivalent to the full information maximum likelihood method (FIML). But if  $K < T$ , FIML applied to a subsample of working women will not be a maximum likelihood method with desirable properties as long as *any* parameter affects both the distribution governing the work decision ( $\text{pr}(W_i > W_i^* | h=0)$ ) and the density for hours of work and wages,  $n(h_i, l(W_i))$ .

The usual rules for identification of the parameters apply to this model. In the present case, identification is assured since labor market experience is excluded from the shadow price equation, and the effects of children, the husband's wage, and net assets are excluded from the market wage equation. Moreover, hours of work are excluded from the market wage equation.

Our system of equations expands Gronau's [11] work in several ways. By introducing the extent of work into the analysis, rather than focusing solely on the work participation decision inequality (5), we utilize the further information that in equilibrium working women equate offered and asking wages. Since Gronau works exclusively with inequality condition (5) to estimate the determinants of asking and offered wages by probit analysis, he cannot identify the parameters of the wage functions associated with variables common to both wage equations since only the difference in these parameters enters inequality (5) and, indeed, equation (6). A further problem in utilizing probit analysis is that the coefficients in inequality (5) are estimated only up to a factor of proportionality.<sup>5</sup> By using Tobit analysis, which utilizes inequality (5) and equation (6), it is possible to estimate the factor of proportionality, but the problem of identifying parameters of variables common to both equations remains. Secondly, Gronau assumes that the disturbances across equations are uncorrelated, whereas we allow for correlation. Since the disturbances capture such omitted variables as ability, quality of schooling, and taste factors, it is plausible that they should be correlated and in fact the estimated correlation turns out to be quite high.

### 3. EMPIRICAL RESULTS

The model was estimated on a sample of 2,100 married white women age 30–44 from the 1967 National Longitudinal Survey. These data are described elsewhere in detail (Shea, et al. [22]). A novelty of these data is that information about retrospective labor market experience is asked of all women.

For empirical purposes, the class of monotonic transformations of the wage variables  $l(W)$  is restricted to the "power transformations" suggested by Box and Cox [6]. This class may be written as

$$l(W) = \frac{W^\lambda - 1}{\lambda},$$

<sup>5</sup> Gronau circumvents these difficulties by assuming that the market wage for working women is the appropriate market wage for nonworking women of similar demographic characteristics. This implies that we know the portion of inequality (5) that corresponds to the market wage for all women, whether or not they work, and the probit coefficient for the market wage yields an estimate of the factor of proportionality. This procedure is equivalent to estimating a wage function on the subsample of working women and utilizing these estimates in a subsequent analysis. As we show in the text, such a procedure leads to inconsistent parameter estimates.



and  $\lambda$  may be estimated from the data. If  $\lambda = 1$ , the wage variable enters linearly. If  $\lambda = 0$ , the natural logarithm of the wage is the appropriate transformation. In practice, values of  $\lambda$  near zero were found whether hourly or weekly wages were used. As an empirical strategy, it was then decided to assume that  $\lambda = 0$  to estimate wage functions more directly comparable to previous studies. In fact, Chiswick [8] and Mincer [19] have suggested theoretical reasons why natural logarithms should be used as dependent variables for the market wage equation. The natural logarithm has the additional desirable feature that the derived labor supply functions have an uncompensated substitution effect which varies depending on hours worked and wages.

Optimization of likelihood function (9) requires a numerical technique. In this paper, GRADX (Goldfeld and Quandt [23]) was used to perform the optimization. To test against the possibility of a local optimum a variety of initial values was used. In all cases, the function converged to the same general set of parameter values.<sup>6</sup>

The appropriate time unit for the empirical analysis is a matter of judgement. In this paper, a year is taken as the "current period." The appropriate measure of labor supply is also a matter for debate. It is possible that women can adjust their weeks worked much more freely than their hours per week. If this is so, weeks worked would be a more appropriate measure of labor supply than annual hours worked, since freedom to adjust is assumed in the model. In practice, both annual hours and weeks worked were used to quantify annual labor supply, and the results from each measure are presented below.

The number of children less than six was used to approximate the constraining effect of children. When additional children variables were introduced, such as the number of children 6–18, they were insignificant using asymptotic normal tests.<sup>7</sup> To measure unearned income, the net worth of the household was estimated by summing over all components of debt and assets. The measure of work experience was the number of years respondents had worked at least six months. The wage rate of the husband was obtained by dividing his estimated annual hours worked into his annual earnings. This variable exerts income and cross substitution effects on the wife's asking wage. The education of the wife is measured in years. With all these variables, there is the serious possibility of correlation with the disturbances of equations (5) and (6), since many of these variables may be the result of previous choices partly dependent on previous disturbances which may be highly correlated with current disturbances. In a world of perfect certainty and unchanging tastes, this correlation may be quite large. On the other hand, the correlation will be weakened by the occurrence of unforeseen events, and random changes in tastes, and the correlation would be expected to be weaker the greater the distance in time between the age when an "exogenous" variable was chosen and the current age. However, there is a possibility that our estimates are biased, but evaluating the importance of the bias is a non-trivial statistical and theoretical task.

<sup>6</sup> Readers interested in the results of all these iterations may obtain them from the author on request.

<sup>7</sup> This finding may be due to the unusual sample composition. Virtually all women in the sample had at least one child 6–18 years in age.



The empirical results are presented in Tables I and II. In Table I, annual hours are used as a measure of annual labor supply, while in Table II weeks worked are employed. The results in Table I are “cleaner” because the labor supply measure is derived independently of the wage rate, since separate questions were asked to derive these measures. The weeks estimates rely on a weekly wage obtained by dividing weeks worked into annual earnings.

In both tables, the estimated effect of one child less than six is to raise the asking wage by roughly fifteen per cent. Increases in net assets increase the asking wage in both equations but the effect is only statistically significant in the annual hours table. A one dollar increase in the husband's wage rate raises the wife's asking wage by five per cent. Unit increases in the wife's schooling raise the asking wage by five per cent for hours worked, but only four per cent for weeks worked. In both tables, the effect of education is to raise the offered wage more than the asking wage and the differences are significant.<sup>8</sup> This implies that *ceteris paribus* more educated women work more frequently, and work longer hours than less educated women. An additional unit of labor market experience raises the market wage by 4.5 per cent in both tables.

As expected, increases in hours and weeks worked are associated with increases in the marginal value of remaining units of the wife's time used for consumption and home production. Since the sign of this coefficient is not imposed on the data by the estimation procedure, its positivity confirms the validity of our hypothesis about this sign made in Section 1. The estimated correlation between the disturbances of the two equations turns out to be quite large, and for both measures of labor supply is significantly different from zero and unity. The correlation is higher in Table II, possibly due to the way the market wage is derived for this measure of labor supply.

The estimated coefficients may be used to generate the probability that a woman works, and the actual hours worked for a working woman. An exogenous wage increase is equivalent to a shift in the intercept of the market wage equation. Using equation (6),

$$\frac{\partial h_i}{\partial b_0} = \frac{1}{\beta_1},$$

where  $b_0$  is the intercept in market wage equation (7) measured in “units” of natural logarithms of wages.

For the annual hours model, this partial is estimated to be 1,600. This implies that a unit increase in the natural logarithm of real hourly wages leads to 1,600 additional hours of work. This may seem to be disturbingly large until one recognizes that a unit change in the natural logarithm of wages represents almost a trebling of actual wages. A 10 per cent increase in real wages would be expected to increase work effort by 160 hours. For weeks worked the point estimate is 50, suggesting that a 10 per cent increase in the weekly wage rate raises weeks worked by 5. These coefficients are large, but not unreasonable.

<sup>8</sup> The estimated asymptotic standard errors for the difference in coefficients is .0052 for the hours case and .0055 in the weeks case.

TABLE I  
ANNUAL HOURS WORKED<sup>a</sup>

	Intercept	Number of Children Less Than Six	Net Assets	Wage Rate of Husband	Experience	Education	Labor Supply	Standard Deviation
In Asking Wage	-.623 (.088)	.179 (.019)	$.135 \times 10^{-5}$ (.055 $\times 10^{-5}$ )	.051 (.007)	—	.0534 (.007)	$.63 \times 10^{-3}$ (.05 $\times 10^{-3}$ )	.532 (.019)
In Offered Wage	-.982 (.11)	—	—	—	.048 (.004)	.0761 (.0075)	—	.452 (.0121)
The estimated correlation of disturbances across equations is .6541 (.046)								

<sup>a</sup> Asymptotic standard errors in parentheses.

TABLE II  
ANNUAL WEEKS WORKED<sup>a</sup>

	Intercept	Number of Children Less Than Six	Net Assets	Wage Rate of Husband	Experience	Education	Labor Supply	Standard Deviation
In Asking Wage	3.1 (1.36)	.149 (.022)	$.50 \times 10^{-6}$ (.55 $\times 10^{-6}$ )	.046 (.008)	—	.039 (.0124)	.02 (.003)	.671 (.021)
In Offered Wage	2.75 (1.62)	—	—	—	.0445 (.0062)	.061 (.010)	—	.677 (.018)
The estimated correlation of disturbances across equations is .83 (.043)								

<sup>a</sup> Asymptotic standard errors in parentheses. Data: National Longitudinal Survey of Work Experience for Women 30-44; 2,100 white married spouse present women.

To further assess the estimates of the model, we present the predicted probabilities of working for women with different characteristics. For a typical woman with four years of labor market experience whose husband makes \$2.50 per hour, with net assets of \$5,000, we present expected work participation rates, cross-classifying education with the number of children less than six. The estimates appear to be consistent with published statistics on labor force participation rates, and are also consistent with Bowen and Finegan’s finding that labor force participation rate differentials by education narrow with the presence of pre-school children (Bowen and Finegan [5, p. 123]).

TABLE III  
ESTIMATED PROBABILITIES OF WORKING

Number of Children Less Than Six	Years of Schooling				
	8	10	12	14	16
0	.30	.38	.47	.56	.66
1	.09	.13	.18	.25	.32
2	.013	.025	.04	.065	.09

Husband’s wage rate is \$2.50 per hour, net worth is \$5,000, and the woman has four years of experience.

To contrast our results with those derived from conventional methods, we present full information maximum likelihood estimates of equations (5) and (6) based on a subsample of 804 working white women. Although we have argued that this procedure is inappropriate since it does not account for sample censoring, it is of some interest to determine whether application of the proposed method leads to any important differences in the estimates.

Table IV corresponds to Table I while Table V corresponds to Table II. The most pronounced differences arise in estimates of the effects of pre-school children. In Tables IV and V, the coefficients on the variable “number of children less than six” are considerably reduced in size, and become statistically insignificant using asymptotic normal tests at five per cent significance levels. The estimates of the market wage function derived from the subsample of working women give much lower estimates of the response of weekly or hourly wages to years of labor market experience, and somewhat lower estimates of the effect of years of schooling on these wages. The slope coefficients for education in each set of equations for either labor supply variable become virtually identical, suggesting that education has little effect on labor supply for women in this age group.

While it is impossible to conclude from this evidence that the proposed method gives better results, the comparisons show that it does lead to differences in empirical estimates. These differences, especially in the case of the determinants of market wages and effect of children on labor supply, would seem to favor the proposed method.

TABLE IV  
ANNUAL HOURS WORKED: FULL INFORMATION MAXIMUM LIKELIHOOD  
APPLIED TO THE SUBSAMPLE OF WORKING WOMEN<sup>a</sup>

	Intercept	Number of Children Less Than Six	Net Assets	Wage Rate of Husband	Experience	Education	Labor Supply	Standard Deviation
In Asking Wage	-1.28 (.18)	.0703 (.09)	$.169 \times 10^{-5}$ ( $.78 \times 10^{-6}$ )	.0376 (.01)	—	.0623 (.008)	$.83 \times 10^{-3}$ ( $.95 \times 10^{-4}$ )	.469 (.012)
In Offered Wage	-.36 (.086)	—	—	—	.0195 (.0025)	.0681 (.007)	—	.507 (.035)
The estimated correlation of disturbances across equations is .591 (.09)								

<sup>a</sup> Asymptotic standard errors in parentheses.

TABLE V  
ANNUAL WEEKS WORKED: FULL INFORMATION MAXIMUM LIKELIHOOD  
APPLIED TO THE SUBSAMPLE OF WORKING WOMEN<sup>a</sup>

	Intercept	Number of Children Less Than Six	Net Assets	Wage Rate of Husband	Experience	Education	Labor Supply	Standard Deviation
In Asking Wage	1.55 (.31)	.046 (.03)	$-.36 \times 10^{-7}$ ( $1.0 \times 10^{-6}$ )	.022 (.012)	—	.0485 (.012)	.043 (.007)	.65 (.015)
In Offered Wage	3.13 (1.21)	—	—	—	.026 (.0035)	.0561 (.0098)	—	.78 (.05)
The estimated correlation of disturbances across equations is .697 (.07)								

<sup>a</sup> Asymptotic standard errors in parentheses. Data: National Longitudinal Survey of Work Experience for Women 30-44; 804 married spouse present women who worked in 1966.

## 4. SUMMARY AND CONCLUSIONS

In this paper, we develop a model which generates the probability that a woman works, her hours of work, her asking wage, and her offered wage from a common set of parameters. These parameters allow us to estimate the value of time for non-working women, and the wage rates they would face in the market. A method of estimating these parameters is proposed and applied. The model appears to give believable parameter estimates.

The method allows us to use an entire sample of women, whether or not they work, to estimate the hours of work equation. In this sense, our procedure is similar in spirit to the procedure of Leibowitz [15 and 16] and others. However, we have shown that observations on nonworking women enter the sample likelihood function in a different way than observations on working women, since the former group give information on many parameters of the asking and offered wage functions, but no information on the substitution effect ( $1/\beta_1$  in equation (6)). Moreover, by posing the problem in the suggested way, we convert a "missing variable" problem (i.e., the problem that we do not observe wages for nonworking women) into a source of information about more fundamental parameters which underlie the labor supply and work decision equations.

Several important qualifications are in order. Throughout this paper, we assume that the individual faces a parametric wage that does not depend on the number of hours worked. This assumption is both convenient and conventional, but might be in conflict with the facts. It is possible to relax this assumption within our framework as long as consumer equilibrium is characterized by marginal equality conditions. However, if wage rates for a standard work year are suitably high because employers have incentives to economize on heads for a given number of manhours, consumer work equilibrium might no longer be characterized by the marginal equality condition we have exploited in the text. This possibility, if empirically important, would require an alternative model of binary choice.

In deriving the estimates, we assume that the disturbances for wages and shadow prices are normally distributed. Clearly, the statistical approach is more general, and estimates for alternative multivariate densities are both possible and desirable. More crucially, we must admit that our dichotomy between work and non-work should be replaced by a trichotomy: work, looking for work, and out of the labor force. To make this extension, it is necessary to develop a more complete model of labor supply under uncertainty than currently exists.

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## APPENDIX 1

For convenience, we change the notation from that of Section 1. Without loss of generality, we neglect the  $Z$  restrictions which may easily be reintroduced into the analysis. The household is assumed to possess a twice-differentiable quasiconcave utility function,

$$(A1.1) \quad U(X_1, \dots, X_n),$$

which is assumed to have positive first partial derivatives for all its arguments. For convenience, let  $X_1$  be the wife's leisure. The argument presented below remains valid if there are many separate uses for the wife's leisure as discussed in Becker [4].  $A$  is asset income,  $P_i$  is the price of good  $i$ ,  $T$  is the amount of time available to the wife, and  $h$  is hours of work,  $T - X_1$ , with associated wage rate  $P_1$ . The household is assumed to maximize (A1.1) for a fixed  $h$  subject to

$$(A1.2) \quad \sum_{i=2}^n P_i X_i - A - P_1 h = 0$$

and

$$(A1.3) \quad T - X_1 - h = 0.$$

The LaGrangian may be written as

$$U(X_1, \dots, X_n) - \lambda \left( \sum_{i=2}^n P_i X_i - A - P_1 h \right) - \mu (X_1 + h - T)$$

where  $\lambda$  and  $\mu$  are LaGrange multipliers.

The first order conditions are

$$U_1 - \mu = 0,$$

$$U_2 - \lambda P_i = 0 \quad (i = 2, \dots, n),$$

and (A1.2) and (A1.3). The assumption of an interior maximum is innocuous and is easily relaxed. In particular, (A1.3) will always hold if the marginal utility of leisure is positive.

From these conditions, a system of equations for  $X_2, \dots, X_n, \lambda$ , and  $\mu$  may be solved as functions of  $P_2, \dots, P_n$ , and  $P_1 h + A$ .

The shadow price of time may be defined as

$$\frac{U_1}{\lambda} = \frac{\mu}{\lambda}.$$

This is the money value the household places on marginal units of the wife's time  $X_1$ . Note that we assume that the utility function is defined for quantities of leisure in excess of the amount  $T$  currently available.<sup>9</sup> This condition will be met if the household can "imagine" having more of the wife's time available than it currently possesses, just as it is conventionally assumed to be able to evaluate baskets of market goods not currently attainable with its money budget constraint. This assumption is not equivalent to the assumption that the preference map is defined for negative quantities of time or goods since hours of work indicate the *absence* of time to be used for leisure or home production, and do not enter as a direct argument in the utility function.

For any arbitrary  $P_1$ , we may write  $U_1/\lambda = W^*$  as

$$(A1.4) \quad W^* = k(h, P_1 h + A, P_2, \dots, P_n).$$

If the wife's leisure is a normal good,  $k_2 > 0$ . If the price weighted sum of all other goods and leisure (treated as a composite commodity) is a normal good,  $k_1 > 0$ . Assuming twice differentiability of the preference function, and a non-vanishing Hessian, (A1.4) will have continuous first partial derivatives.

It is important to notice that (A1.4) is defined whether or not labor supply functions exist. For a particular configuration of  $h, P_2, \dots, P_n, A$  to be an equilibrium solution to the utility maximization problem with  $h$  voluntarily chosen, it is necessary that  $P_1 = W^*$ , i.e., that the income flow from the parametric wage  $P_1$ , given the value of  $A, P_2, \dots, P_n$ , yield a value of the shadow price equal to the parametric wage. The relationship between the equilibrium values of  $W^*$  and  $h$ , if one exists, defines the labor supply relationship. Over the domain of  $h$  where equilibrium values exist, the continuity of  $k$  implies the continuity in the labor supply function. Note that under our assumption about preferences we can always adjoin the value of  $W^*$  at  $h = 0$ , or for that matter, values of  $W^*$  for suitably chosen values of  $h < 0$ , to the conventional labor supply function, and that continuity of  $k$  assures us that "adjoined labor supply" is continuous and differentiable in equilibrium wages.

<sup>9</sup> It is possible to imagine a household augmenting the wife's time by purchasing perfect substitutes for her home production time and defining these input hours as negative work.

If a labor supply function exists,<sup>10</sup> an “adjointed labor supply function” also exists. Assuming that the former is a positive monotonic relationship, we can solve out the latter relationship for equilibrium values of  $W^*$ , and hence we reach equation (1) in the text.

## APPENDIX 2

### STATISTICAL MODELS

The joint distribution of  $\varepsilon_i$  and  $u_i$ ,  $M(\varepsilon_i, u_i)$  is assumed to be a bivariate normal fully characterized by

$$E(\varepsilon_i) = 0,$$

$$E(u_i) = 0,$$

$$E(u_i^2) = \sigma_u^2,$$

$$E(\varepsilon_i^2) = \sigma_\varepsilon^2,$$

$$E(u_i \varepsilon_i) = \sigma_{\varepsilon u} = \rho \sigma_u \sigma_\varepsilon,$$

where  $\rho$  is the population correlation coefficient between  $u_i$  and  $\varepsilon_i$ .

The joint density may be written as

$$(A2.1) \quad M(\varepsilon_i, u_i) = \frac{(\sigma_\varepsilon^2 \sigma_u^2 (1 - \rho^2))^{-\frac{1}{2}}}{2\pi} \exp - \frac{1}{2(1 - \rho^2)} \left\{ \frac{\varepsilon_i^2}{\sigma_\varepsilon^2} + \frac{u_i^2}{\sigma_u^2} - \frac{2\rho \varepsilon_i u_i}{\sigma_\varepsilon \sigma_u} \right\},$$

$$-\infty < u_i < \infty, -\infty < \varepsilon_i < \infty.$$

The probability associated with condition (5) in the text,  $\text{pr}([W_i > W_i^*]_{h=0})$  comes to

$$(A2.2) \quad \text{pr}(b_0 - \beta_0 + b_1 S_i - \beta_2 (W_m)_i + b_2 E_i - \beta_3 P_i - \beta_4 A_i - \beta_5 Z_i > \varepsilon_i - u_i).$$

The distribution of  $\varepsilon_i - u_i$  may be obtained from (A2.1) by letting  $t_i = \varepsilon_i - u_i$ , substituting  $u_i + t_i$  for  $\varepsilon_i$ , and integrating out  $u_i$ , or by noting that sums and differences of normal variates are normal variates. Thus,  $\varepsilon_i - u_i$  is a normal variate with mean zero and variance  $\sigma_\varepsilon^2 + \sigma_u^2 - 2\rho\sigma_\varepsilon\sigma_u$ .

Using the standard normal, probability (A2.2) may be written

$$\text{pr} \left( \frac{b_0 - \beta_0 + b_1 S_i - \beta_2 (W_m)_i + b_2 E_i - \beta_3 P_i - \beta_4 A_i - \beta_5 Z_i}{(\sigma_\varepsilon^2 + \sigma_u^2 - 2\rho\sigma_\varepsilon\sigma_u)^{\frac{1}{2}}} > \frac{\varepsilon_i - u_i}{(\sigma_\varepsilon^2 + \sigma_u^2 - 2\rho\sigma_\varepsilon\sigma_u)^{\frac{1}{2}}} \right)$$

so that

$$\text{pr}([W_i > W_i^*]_{h=0}) = \int_{-\infty}^{J_i} \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{1}{2} r^2 \right) dr$$

where

$$J_i = \frac{(b_0 - \beta_0 + b_1 S_i - \beta_2 (W_m)_i + b_2 E_i - \beta_3 P_i - \beta_4 A_i - \beta_5 Z_i)}{(\sigma_\varepsilon^2 + \sigma_u^2 - 2\rho\sigma_\varepsilon\sigma_u)^{\frac{1}{2}}}.$$

Similarly,

$$\text{pr}([W_i < W_i^*]_{h=0}) = \int_{J_i}^{\infty} \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{1}{2} r^2 \right) dr.$$

The derivation of the distribution for  $n(h_i, l[W_i])$  proceeds along similar lines. We may write equations (6) and (7) as

$$h_i - D_i = \frac{u_i - \varepsilon_i}{\beta_1},$$

$$l[W_i] - F_i = u_i,$$

<sup>10</sup> It is tedious, but straightforward, to establish sufficient conditions for a labor supply function to exist. Since  $U$  is assumed to have positive first partials,  $W^* > 0$  if  $P_1 = 0$ . If for all  $P_1, k_2 h \leq 0$  a unique solution  $\tilde{W}^* = P_1$  exists for each  $h$ . If for all  $P_1, k_2 h > 1$ , no solution exists. If  $0 < k_2 h < 1$  for all  $P_1$ , a solution might exist, and if it does, it is unique. A sufficient condition for a solution to exist is that  $k_{22} h^2 \leq 0$  (i.e., the marginal rate of substitution between goods and leisure increases at a decreasing rate for increasing levels of utility). Note that at  $h = 0$ , a solution always exists.



so that the distributions of the variables on the left are clearly linked to those on the right.

Now  $(u_i - \varepsilon_i)/\beta_1$  and  $u_i$  are jointly normally distributed:

$$\begin{aligned} E(u_i) &= 0 = E\left(\frac{u_i - \varepsilon_i}{\beta_1}\right), \\ E(u_i)^2 &= \sigma_u^2, \\ E\left(\frac{u_i - \varepsilon_i}{\beta_1}\right)^2 &= \frac{\sigma_u^2 + \sigma_\varepsilon^2 - 2\rho\sigma_\varepsilon\sigma_u}{\beta_1^2}, \\ \text{cov}\left(\frac{u_i - \varepsilon_i}{\beta_1}, u_i\right) &= \frac{\sigma_u^2 - \rho\sigma_\varepsilon\sigma_u}{\beta_1}. \end{aligned}$$

The joint density of  $h_i, l[W_i]$  may be written as

$$n(h_i, l[W_i]) = |\beta_1| \frac{(\sigma_\varepsilon^2 \sigma_u^2 (1 - \rho^2))^{-\frac{1}{2}}}{2\pi} \exp\left(-\frac{G}{2(1 - \rho^2)}\right)$$

where  $|\beta_1|$  is the absolute value of  $\beta_1$ , and where  $G$  is defined as

$$\begin{aligned} G &= (h_i - D_i)^2 \left(\frac{\beta_1^2}{\sigma_\varepsilon^2}\right) - 2(h_i - D_i)(l[W_i] - F_i) \left(\frac{1}{\sigma_\varepsilon^2} - \frac{\rho}{\sigma_\varepsilon\sigma_u}\right) \\ &\quad + (l[W_i] - F_i)^2 \left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_u^2} - \frac{2\rho}{\sigma_\varepsilon\sigma_u}\right). \end{aligned}$$

If we let  $l[W_i] = (W_i^\lambda - 1)/\lambda$  as Box and Cox [6] propose, then the joint density for  $h_i$  and  $W_i$  may be written as

$$q(h_i, W_i) = W_i^{\lambda-1} \cdot n(h_i, l[W_i]).$$

Using this transformation, we may estimate  $\lambda$  along with the other parameters of the model. Introducing this parameter creates another difference between our method and the Tobit method.

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