PRESSURE TRANSIENTS IN TUNNELS

EXTENSION OF THEORY TO IRREVERSIBLE, NON-ADIABATIC FLOW

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Introduction

In previous work on the problems of unsteady flow in train tunnels carried out at Leeds University three basic equations have been used to describe the flow. There are (a) conservation of mass, (b) conservation of momentum, and (c) a pressure-density relationship. Equation (c) is that

$$\frac{p}{\rho^{\gamma}} = \text{constant}, \qquad (1)$$

this equation is true for any reversible, adiabatic process of a gas that obeys Boyles Law and for which γ is a constant. If the air in a train tunnel is assumed to be a perfect gas the last two conditions are satisfied, at normal temperatures and pressures this is a good assumption. An adiabatic process is one in which no heat is transferred between the gas and its surroundings. In regions remote from trains the flow will approximate to adiabatic, in some cases a better assumption may be that the wall temperature is constant, i.e. the thermal capacity of the tunnel lining is large in relation to the total heat transferred, the distinction is of little practical importance. In the region around a train engine there is considerable local heat transfer to the gas, the deviation from adiabatic conditions is significant. A reversible process is an ideal, never strictly achieved, it is one in which no mechanical energy is dissipated as heat due to friction, and in which gas temperature gradients are small. In the tunnel situation friction and temperature gradients are of significance only near to trains. It is possible that inaccuracies due to the crude representation of friction in previous work are greater than those due to irreversibility of the gas process.

Inaccuracies in the present theories due to the use of equation (1) will be mainly due to the heating effect of the engine. Extension of the theory on the grounds that friction ensures irreversibility is not justified, any extension should be justified primarily on the grounds that heat injection and consequent temperature gradients about the train are of significance. Application of the extended theory to a single train/tunnel configuration, to test the sensitivity of the solution to the use of equation (1) is recommended.

Notation

The notation used throughout is the same as used by Vardy (1973), additional symbols are defined where they first appear.

Equations of Motion

In the extended theory the equations of continuity and momentum may be derived in the same way as Vardy (1973). Those are

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} + \frac{\rho v}{a}\frac{da}{dx} = \frac{\rho' q}{a}, \tag{2}$$

and

$$\frac{\partial p}{\partial x} + \rho v \beta \frac{\partial v}{\partial x} + v(1-\beta)\frac{\partial \rho}{\partial t} + \rho \frac{\partial v}{\partial t} = \frac{\rho' q}{a}(\bar{v} - v\beta) - \frac{F}{a} - \rho v^2 \frac{\partial \beta}{\partial x} \tag{3}$$

respectively, where ρ' is the density of any spatial inflow and $\bar{v}$ is the axial component of its velocity ($\bar{v} = v$ for the outflow case). β is the Coriolis coefficient of momentum defined by

$$\beta = \frac{\int_{\text{cross section}} v_\ell^2 \, da}{a v^2},$$

where v_ℓ is the local velocity at any point in the cross section. a is not a function of time. The equation that replaces the pressure-density relationship(1) is the First Law of Thermodynamics.

Consider an element of duct, length δx, and define this as the control volume, as shown in fig.(1). Consider that system of particles that occupies the control volume at time t plus that matter which flows into it through the duct walls in time Δt.

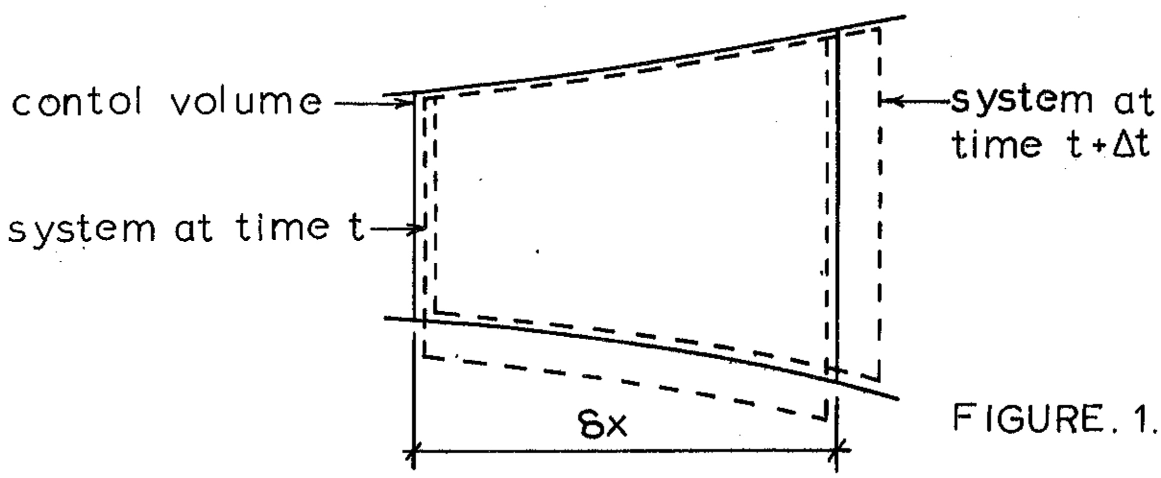

FIGURE. 1.

Rate of heat inflow to system	=	$Qs\delta x$
Rate of increase of stored energy	=	$\frac{\partial}{\partial t}\left\{\rho a\delta x\left(e + \beta\frac{v^2}{2}\right)\right\}$
Net rate of energy outflow axially	=	$\frac{\partial}{\partial x}\left\{\rho av\left(e + \alpha\frac{v^2}{2}\right)\right\}\delta x$
Net rate of energy inflow spatially	=	$\rho' q\left(e' + \frac{\bar{v}^2}{2}\right)\delta x$
Net rate of work done by system axially against normal stresses	=	$\frac{\partial}{\partial x}(pav)\delta x$
Net rate of work done by system spatially against normal stresses	=	$- p'q\delta x$

Q is the rate of heat inflow per unit wall area, the primes refer to properties of the spatially inflowing air. α is the Coriolis coefficient of kinetic energy, defined by

$$\alpha = \frac{\int_{\text{cross section}} v_\ell^3 da}{av^3} \quad ,$$

and e is the internal energy per unit mass of the system. Internal energy consists of the potential and kinetic energy of the individual gas molecules, the potential energy is not significant and e is a function of temperature only.

From the First Law of Thermodynamics -

Rate of heat inflow = Rate of increase of stored energy + Net rate of energy outflow (4)

+ Net rate of work done by system.

Thus

$$Qs\delta x = \frac{\partial}{\partial t}\left\{\rho a\delta x\left(e + \beta\frac{v^2}{2}\right)\right\} + \frac{\partial}{\partial x}\left\{\rho av\left(e + \alpha\frac{v^2}{2}\right)\right\}\delta x - \rho' q\left(e' + \frac{\bar{v}^2}{2}\right)\delta x$$

$$+ \frac{\partial}{\partial x}(pav)\delta x - p'q\delta x. \qquad (5)$$

For a perfect gas,

$$e = c_v T = \frac{1}{\gamma - 1}\frac{p}{\rho}$$

and $$e + \frac{p}{\rho} = c_p T = \frac{\gamma}{\gamma - 1}\frac{p}{\rho} \quad ,$$

where c_v and c_p are the specific heats at constant volume and pressure respectively, T is temperature.

Thus

$$Qs = \frac{\partial}{\partial t}\left\{\rho a\left(\frac{1}{\gamma-1}\frac{p}{\rho} + \frac{\beta v^2}{2}\right)\right\} + \frac{\partial}{\partial x}\left\{\rho a v\left(\frac{\gamma}{\gamma-1}\frac{p}{\rho} + \frac{\alpha v^2}{2}\right)\right\}$$

$$- \rho' q \left\{c_p T' + \frac{\bar{v}^2}{2}\right\} \qquad (6)$$

On expanding and using equations (2) and (3) a general energy equation may be written that involves the pressure derivative in a useful form, it is

$$\frac{1}{\gamma-1}\left\{\frac{\partial p}{\partial t} + \frac{v\partial p}{\partial x}\right\} + \rho v(\beta-1)\frac{\partial v}{\partial t} + \rho v^2(\alpha-\beta)\frac{\partial v}{\partial x} + \frac{\partial \rho}{\partial t}\left\{v^2\left(\frac{3\beta}{2} - 1 - \frac{\alpha}{2}\right) - \frac{\gamma}{\gamma-1}\frac{p}{\rho}\right\}$$

$$- \frac{v\partial\rho}{\partial x}\left\{\frac{\gamma}{\gamma-1}\frac{p}{\rho}\right\} = \frac{Qs}{a} + \frac{Fv}{a} + \rho' q\left\{c_p(T'-T) + \frac{\bar{v}^2}{2} - v\bar{v} + v^2\left(\beta - \frac{\alpha}{2}\right)\right\} - \rho\frac{v^2}{2}\frac{\partial\beta}{\partial t}$$

$$+ \rho v^3\left\{\frac{\partial\beta}{\partial x} - \frac{1}{2}\frac{\partial\alpha}{\partial x}\right\} \qquad (7)$$

The solution of equations (2), (3) and (7) is the general solution for one dimensional unsteady flow of a perfect gas in a duct. The effects of area variation with distance (but not time), heat and mass inflow, and velocity distribution (Coriolis coefficients) are included.

Solution of Equations

The inclusion of α and β makes the solution of the basic equations by the method of characteristics extremely complicated, the special case where $\alpha \equiv \beta \equiv 1$ will be considered. For this case -

$$\frac{\partial\rho}{\partial t} + \frac{\rho\partial v}{\partial t} + \frac{v\partial\rho}{\partial x} = \frac{\rho' q}{a} - \frac{\rho v}{a}\frac{da}{dx} \qquad (8)$$

$$\frac{\partial p}{\partial x} + \frac{\rho v\partial v}{\partial x} + \frac{\rho\partial v}{\partial t} = \frac{\rho' q}{a}(\bar{v}-v) - \frac{F}{a} \qquad (9)$$

$$\frac{1}{(\gamma-1)}\left\{\frac{\partial p}{\partial t} + \frac{v\partial p}{\partial x}\right\} - \frac{\gamma}{(\gamma-1)}\frac{p}{\rho}\left\{\frac{\partial\rho}{\partial t} + \frac{v\partial\rho}{\partial x}\right\} = \frac{Qs}{a} + \frac{Fv}{a} + \frac{\rho' q}{a}\left\{c_p(T'-T) + \frac{(\bar{v}-v)^2}{2}\right\}, \qquad (10)$$

where F and Q may not be functions of the derivatives of ρ, p or v with respect to x or t.

When dealing with two dependent variables (v and c) two characteristic directions may be defined in which characteristic conditions may be found. In this case as there are three dependent variables (p, ρ and v) three characteristic directions and conditions may be found. The characteristic directions may be shown to be -

$$\frac{dx}{dt} = v \pm c, \text{ and } \frac{dx}{dt} = v, \tag{11}$$

where $c^2 = \gamma p/\rho$

The condition that applies on $\frac{dx}{dt} = v$, the particle path, is equation (10). The conditions that apply in the other directions are derived from -

$$\text{Equation (8)} + \frac{1}{\frac{dx}{dt} - v} \cdot \text{Equation (9)} + \frac{1}{c^2} \cdot \text{Equation (10)} = 0. \tag{12}$$

With the appropriate substitutions for $\frac{dx}{dt}$ (from (11)) and simplifying, the solution equations are -

$$\begin{aligned}
\frac{dv}{dt} + \frac{c}{\gamma p}\frac{dp}{dt} &= E_I \quad \text{on} \quad \frac{dx}{dt} = (v+c), \\
\frac{dv}{dt} - \frac{c}{\gamma p}\frac{dp}{dt} &= E_{II} \quad \text{on} \quad \frac{dx}{dt} = (v-c), \\
\frac{dp}{dt} - c^2\frac{d\rho}{dt} &= E_{III} \quad \text{on} \quad \frac{dx}{dt} = v,
\end{aligned} \tag{13}$$

where

$$\begin{aligned}
E_I &= \frac{(\gamma-1)}{c}\frac{Q}{\rho h_m} + \frac{F}{\rho a}\left\{\frac{v}{c}(\gamma-1) - 1\right\} - \frac{vc}{a}\frac{da}{dx} \\
&\quad + \frac{j}{\rho a}\left\{\frac{(\gamma-1)}{c}\left[c_p(T'-T) + \frac{(\bar{v}-v)^2}{2}\right] + c + \bar{v}-v\right\}, \\
E_{II} &= -\frac{(\gamma-1)}{c}\frac{Q}{\rho h_m} - \frac{F}{\rho a}\left\{\frac{v}{c}(\gamma-1)+1\right\} + \frac{vc}{a}\frac{da}{dx} \\
&\quad + \frac{j}{\rho a}\left\{-\frac{(\gamma-1)}{c}\left[c_p(T'-T) + \frac{(\bar{v}-v)^2}{2}\right] - c + \bar{v}-v\right\}, \\
E_{III} &= (\gamma-1)\left\{\frac{Q}{h_m} + \frac{Fv}{a} + \frac{j}{a}\left\{c_p(T'-T) + \frac{(\bar{v}-v)^2}{2}\right\}\right\},
\end{aligned} \tag{14}$$

$h_m = a/s$ and $j = \rho' q$ (mass inflow rate/unit wall area)

Finite Difference Replacement

The derivatives in equations (13) may be replaced by backward differences in a fixed space-time grid, the characteristic directions are shown in fig.(2)

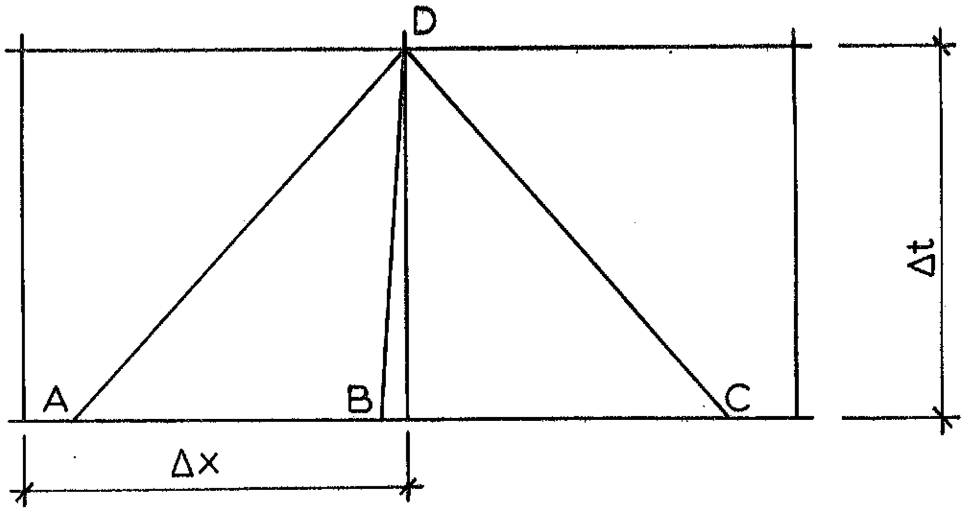

FIGURE. 2.

The dependent variables at points A,B and C are found by interpolation at time t. The finite difference equations are -

$$p_D = \frac{\gamma}{\left\{\frac{c_{AD}}{p_{AD}} + \frac{c_{CD}}{p_{CD}}\right\}} \left\{ (v_A - v_C) + \frac{1}{\gamma} \left(\frac{p_A c_{AD}}{p_{AD}} - \frac{p_C c_{CD}}{p_{CD}} \right) \right\} + (E_I - E_{II})\Delta t,$$

$$v_D = v_A + \frac{c_{AD}}{\gamma p_{AD}} (p_A - p_D) + E_I , \qquad (15)$$

$$\rho_D = \rho_B + \frac{1}{(c_{BD})^2} \left\{ (p_D - p_B) - E_{III}\,\Delta t \right\} ,$$

the subscripts refer to the relevant point. Where there are two subscripts, and for the E terms the values used refer to the relevant characteristic line. The actual values used here depend on computational considerations, e.g. the solution may be far less sensitive to an alteration from c_A to $\frac{1}{2}(c_A + c_D)$ than to grid size changes. For all points other than boundaries equations (15) are used to produce the solution.

Heat Transfer

The friction and lateral inflow terms in equations (15) may be treated in the same way as Vardy. The heat transfer Q (zero for adiabatic conditions) is used to define a film coefficient of heat transfer H.

$$Q = H(T_w - T_{aw}) \tag{16}$$

T_w is the wall temperature, T_{aw} is the adiabatic wall temperature. T_{aw} is defined by

$$T_{aw} = T_{free\ stream} + \frac{Rv^2}{2c_p} , \tag{17}$$

R is the recovery factor which for flow part an insulated flat plate may be taken as 0.88 as a mean value for subsonic flows. Reynolds Analogy between friction and heat transfer is accurate within a few percent for subsonic flow and is -

$$\frac{H}{\rho c_p v} = \frac{f}{2} , \tag{18}$$

where f is the coefficient of friction. If, as is most likely in practice where it is required to consider other than adiabatic conditions, the wall temperature is taken as the free stream temperature -

$$Q = - 0.22 \rho f v^3 , \tag{19}$$

the minus sign indicates heat outflow.

It is felt that at this stage assuming other than adiabatic conditions distant from trains is not justified. The above determination of Q is crude, many more details are available in Shapiro (1954). In the region close to trains the heating effect of the engine is of interest and values of Q should be directly available from engine efficiency considerations. An estimate of Q in the remainder of the annulus may be available from details of the carriage heating supply system. These values of Q should be used in equation (14) to calculate E values which may be used in equation (15) to solve the single train/tunnel test as recommended earlier.

References

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