A rant about friction factors called 'f'

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Abstract

In the fluid mechanics field there is an important concept (frictional pressure drop of fluid flowing in a duct) that uses an empirical term (friction factor).
This document discusses three symbols (c_f, λ and f) that have been used to represent two different types of friction factor (Fanning friction factor and Darcy friction factor) and argues that using the symbol f is a bad idea in tunnel ventilation design.

1 What's wrong with these equations?[1]

Many engineers reading this will be familiar with the following equation for frictional pressure drop,

$$\Delta P = \frac{fL}{D_h} 1/2 \rho U^2. \tag{1}$$

Unfortunately many other engineers reading this are familiar with a slightly different equation that is disturbingly easy to confuse with (1). It is

$$\Delta P = \frac{4fL}{D_h} 1/2 \rho U^2. \tag{2}$$

(1) and (2) both use f to represent their friction factor, but their values of f differ by a factor of four (for the same air path).

These days, f in (1) is usually known as Darcy friction factor and f in (2) is usually known as Fanning friction factor.

When Darcy friction factor f is used in (1) and Fanning friction factor f is used in (2) everything is fine with your engineering calculations.

Alas, engineers commonly take values of friction factor f from a paper that has experimental results in it, assume that the paper uses the friction factor they

[1] with apologies to Prof. N D Mermin.

are familiar with and plug it into whichever of (1) or (2) they were taught to use. If they are wrong, their frictional pressure drop calculations will either be too high by a factor of four or too low by a factor of four.

When I was a young engineer I frequently mistook one for the other. Now I'm (mostly) wise to it. But I've seen a **lot** of colleagues fall into the same trap over the years, so I decided to write a rant about why we engineers ought to get our act together.

f in (1) and f in (2) both have long histories in the literature. The earliest instance of Darcy friction factor f in (1) I could find was 1942 (in the USA) and the earliest instance of Fanning friction factor f in (2) I could find was 1917 (also in the USA).

Friction factors that differ by a factor of four have been around since the 19th century: in his 1878 book [1], T J Fanning took Weisbach's data (Weisbach used the symbol ζ for Darcy friction factor) and converted it to Fanning friction factor k by dividing ζ by four.
Engineers in the 19th century were used to handling friction factors that differed by a factor of four, but had enough common sense to use different symbols for them (in this case friction factor ζ and friction factor k).

Unlike us! For at least 80 years the engineering literature has been rife with papers using f to represent Fanning friction factor while other papers used f to represent Darcy friction factor. These days, every time I read a paper that uses f to represent its friction factor, I look closely at it to figure out whether the authors mean Fanning or Darcy friction factor.

2 Some history, and a few things to be wary of

How did our profession get into this mess? The factor of four problem seems to have originated in the difference between hydraulic mean depth in open-channel flow and hydraulic diameter in pipe flow.

Serious fluid friction research started in the 1700s with open-channel flow in rivers and canals. Those researchers developed the concept of the hydraulic mean depth of a river (river flow area divided by riverbed perimeter).

When pipe flow developed in the 1800s, their successors tried renaming hydraulic mean depth as hydraulic radius, but quickly realized it was too confusing (as the hydraulic radius of a circular pipe is half of the pipe's physical radius). The pipe flow practitioners multiplied hydraulic mean depth by four and started calling it hydraulic diameter. Much saner, as the hydraulic diameter of a circular pipe is the same as its physical diameter.

Having created hydraulic diameter, it was an easy step to create a friction factor that was four times higher than the friction factor used in open channel flow. It let you remove that factor of four in the numerator of your circular pipe friction equation.

So in the late 19th century, different symbols were used for the two friction factors (for example, the ζ and k that Fanning quotes in his 1878 book, where $\zeta = 4k$).

As long as engineers working in pipe flow used different symbols to engineers working in open channel flow, Nothing Could Go Wrong.[2] Alas, someone started using f for Fanning's k and someone else started using f for Weisbach's ζ.

Both definitions of f became popular in different disciplines. From the early 20th century to the present, fluid mechanics literature became the minefield that I have to walk through whenever I pick up a paper I haven't read before. Because I see friction factor f and ask myself "OK, how do I figure out which f this paper means?"

The current situation seems to be "Darcy friction factor f is used in north America, Fanning friction factor f is used in chemical engineering worldwide, and Fanning friction factor is used by some tunnel ventilation professionals in Europe and Japan (especially in high speed rail tunnels)".

But there are counterexamples:

- Harding and Willard's 1917 HVAC book [2] and Singstad's 1929 paper on the Holland Tunnel [3] both use Fanning friction factor f in conjunction with hydraulic radius. Both were written in the US.
- The US-sponsored translation of Nikuradse's 1933 sand roughness experiments (by NACA in 1950 [4]) uses Darcy friction factor λ.
- Shapiro's 1950s books on compressible fluid flow [5] (written in the US) use f to represent Fanning friction factor.
- D. S. Miller's 1978 book on the resistance of ducts [6] uses f for Darcy friction factor despite having being written in the UK.

A subtle source of confusion is that in older papers when the word "Darcy" is used it may not match present-day terminology. For example, Fox's 1968 ICE paper on the method of characteristics in waterhammer calculations [7] states "Darcy f", but a little investigation makes it clear that the friction factor he uses is the one that we nowadays call Fanning friction factor. So even if a paper states "Darcy friction factor f" in the text, it may not be the friction factor that you expect from the terminology used in more recent years. Be on your guard!

Worse, some journals switched symbols over time. Compare the typesetting in these three papers from the Proceedings of the Institution of Civil Engineers:

- Anderson's 1936 paper on the Mersey Tunnel [8] used f to represent Fanning friction factor,

[2] Narrator: it did.

- Colebrook's 1939 paper on pipe friction [9] used λ to represent Darcy friction factor,
- Colebrook's 1958 paper on hydropower tunnel friction [10] used f to represent Darcy friction factor.

I have no idea why the journal's editors thought that using f in [10] would not lead to confusion when reading [10] alongside [8]. I mean, tunnel friction is about as niche a field as you can get: both [8] and [10] are papers that have full-scale test data in them—they're going to be read side-by-side more than once!

I see the whole "*which friction factor is represented by* f" catching out a lot of engineers venturing into the high speed rail tunnel field for the first time.

Japanese and European high speed rail tunnel designs mostly use f in the sense of (2). Most of the engineers I've worked with from North America use f in the sense of (1) and don't learn about (2) until a reviewer points out their mistake on their first high speed rail project (getting called out for mistaking c_f for λ seems to be a rite of passage for tunnel ventilation engineers from North America when they first start working on overseas projects).

To suggest "OK, why don't we just agree to use a consistent set of symbols in future?" is *not* a solution. I reckon that is probably what started it: my guess is that some time between 1920 and 1940 a major engineering body in the USA decided "**From now on let's use symbol f for Darcy friction factor and stop using Fanning friction factor entirely. Everyone else will be sure to follow our lead.**" Everyone else did not.

And it would not solve the problem if every journal in the world agreed to use a consistent symbol and a consistent equation from now on (good though that would be). All those older published papers—many of which have genuinely useful full-scale test data—are still out there, ready to be misinterpreted by each new generation of graduates whose lecturers don't teach their students to ask "is the f in this paper Fanning friction factor c_f or Darcy friction factor λ?" every time they pick up a new paper.

3 How the program handles friction

If you've managed to make it this far, you probably understand why I wrote this rant. I'm writing tunnel ventilation software. I want to avoid confusing my users, and I want to educate those users who have no idea that there is a trap that they can fall into if they don't know their f in (1) from their f in (2).[3]

If I make f in (1) or f in (2) the default, I risk pushing unwary users into a trap that I've fallen into many times and that most of my colleagues have also fallen into.

My solution is this:

- I don't use the symbol f (except when raving about how much I hate f).
- I use the symbol c_f for Fanning friction factor and the symbol λ for Darcy friction factor.
- I don't set a default type of friction factor. Instead, I force the user to make a choice between Fanning and Darcy friction factor.
- I prepared a Moody chart that has c_f and λ on the ordinate axis (see the last page in this document).
- I wrote this rant about the difference between Fanning friction factor f and Darcy friction factor f.

In summary: in my software Fanning friction factor is represented by c_f (a popular symbol for it in the aeronautics field) and Darcy friction factor is represented by λ (a popular symbol for it in the pipe flow field). The friction equations then become

$$\begin{aligned} \Delta P &= \frac{c_f S L}{A} 1/2 \rho U^2, && (3) \\ &= \frac{4 c_f L}{D_h} 1/2 \rho U^2, && (4) \\ &= \frac{\lambda L}{D_h} 1/2 \rho U^2, && (5) \\ &= \frac{c_f L}{R_h} 1/2 \rho U^2 && (6) \end{aligned}$$

where

$$\begin{aligned} \Delta P &= \text{pressure drop } (Pa), \\ c_f &= \text{Fanning friction factor } (-), \\ S &= \text{tunnel perimeter } (m), \\ A &= \text{tunnel area } (m^2), \\ L &= \text{tunnel length } (m), \end{aligned}$$

[3] For any Scots reading this: there might be a Matt McGinn skit in "Effen 1 and Effen 2".

$$\begin{aligned}
\lambda &= \text{Darcy friction factor } (-), \\
D_h &= \text{hydraulic diameter} \equiv \frac{4A}{S} \ (m), \\
R_h &= \text{hydraulic mean depth/hydraulic radius} \equiv \frac{A}{S} \ (m), \\
\rho &= \text{fluid density } (kg/m^3), \\
U &= \text{fluid velocity } (m/s).
\end{aligned}$$

If you only know about one friction factor f and don't know if your f is c_f in (4) or λ in (5), then the Moody chart at the end of this document is probably the best place to start.

That Moody chart has two ordinate axes; one for c_f, the other for λ. Compare them to the ordinate axis on a Moody chart in a textbook that you trust. You should be able to figure out if the f you were taught about is c_f or λ.

One final point. If you cast your net wide enough you will encounter other definitions of friction factor. I know of four. Here are their names, symbols and their relationships with Fanning friction factor c_f:

- c_f, skin friction coefficient or Fanning friction factor (dimensionless) [11]
- λ, Darcy friction factor (dimensionless) [12], $\lambda = 4c_f$
- k, Atkinson friction factor (units kg/m^3) [13], [14], $k = \frac{1}{2}\rho c_f$
- α, friction factor from mine ventilation in the USSR (units kg-s^2/m^4) [12], [15], $\alpha = \frac{1}{2}\rho c_f/g$

where ρ is standard air density and g is the acceleration of gravity.

k and α are included here because I have often stumbled across useful experimental data in mine ventilation literature. This document is a good place to store their definitions for future reference.

4 Friction factor approximations

When C F Colebrook defined the Colebrook–White function for friction factor in his 1939 ICE paper [9] he used an implicit approximation,

$$\frac{1}{\sqrt{\lambda}} = -2\log_{10}\left(\frac{\varepsilon}{3.7D_h} + \frac{2.51}{Re\sqrt{\lambda}}\right). \tag{7}$$

Later in the paper Colebrook noted that the implicit formulation was inconvenient and gave an explicit term that could replace the last term in (7) with little loss of accuracy (his equation 14). He stated that

$$-2\log_{10}\frac{2.51}{Re\sqrt{\lambda}} \approx -1.8\log_{10}\frac{Re}{7}. \tag{8}$$

Equations (7) and (8) were useful when engineers used log tables and slide rules, but are less useful in an era of spreadsheets. When converted to a form that is more useful today, Colebrook's explicit approximation is

$$\lambda \approx \frac{0.25}{\left\{\log_{10}\left[\frac{\varepsilon}{3.7D_h} + \left(\frac{7}{Re}\right)^{0.9}\right]\right\}^2} \approx \frac{0.25}{\left[\log_{10}\left(\frac{\varepsilon}{3.7D_h} + \frac{5.76}{Re^{0.9}}\right)\right]^2}. \tag{9}$$

Colebrook's 1939 explicit approximation (9) is pretty darned accurate.

Constants of 0.9 & 7 (or 0.9 & 5.76 if you prefer them) are easier to remember than some of the multibracketed horrors I've seen in recent papers.

Alas, the presence of an adequate explicit approximation in the original paper did not stop everyone and their dog piling in to write another over the years. I'd guess there must be well over a hundred explicit approximations by now.

One that has been popular in the tunnel vent field was produced by L F Moody in 1947 [16]. It is

$$\lambda = 0.0055\left\{1 + \left(20{,}000\frac{\varepsilon}{D_h} + \frac{10^6}{Re}\right)^{1/3}\right\}. \tag{10}$$

Moody's approximation couldn't have been popular for its accuracy—it is woeful at high ε/D_h.[4] Perhaps it was popular because it was easy to work out on slide rules that could calculate cube roots in the late 1940s, while Colebrook's approximation forced engineers to consult log tables?

Various researchers have developed more complex explicit expressions that correlate ever better to the exact values. I suspect that few would have bothered if they had read Colebrook's paper thoroughly. That paper has a *lot* of rounding of empirically-derived values from papers by different researchers.

Early in the paper Colebrook derived the constant 3.7 in the denominator of the Colebrook–White function. He took 0.113 (a value that Colebrook and White

[4] It is 5% too low at $\varepsilon/D_h = 0.02$ and 26% too low at $\varepsilon/D_h = 0.1$. I reckon that Moody's approximation sucks.

had derived from experiments) and multiplied it by 33 (a value that Nikuradse had derived from experiments) to get 3.7.

The true product of those two numbers is 3.729. I did a quick test in a spreadsheet: if the 3.7 in the denominator of (7) is changed to 3.729 the calculated values of friction change by about a quarter of a percent at $\varepsilon/D_h = 0.008$. If that level of imprecision was accepted in the original paper then developing an explicit approximation that matches (7) to better than a quarter of a percent seems a bit pointless.

Developers of tunnel ventilation software have, by and large, chosen an approximation from the popular literature and stuck with it over the years.

Henson and Fox [17] used Moody's approximation (10) when developing the 1D method of characteristics that became the Mott MacDonald Aero program.

Those who have looked into the SES v4.1 source code (`Omega5.for`) will know that it uses Moody's approximation in the open tunnel and a modified version of Moody's approximation in the annulus around trains. It is

$$\lambda = 0.0055 \left\{ 1 + \left(\frac{\varepsilon}{D_h} \left[19000 + 16000 \left(\frac{\varepsilon}{0.05 D_h} \right)^{1.5} \right] + \frac{10^6}{Re} \right)^{1/3} \right\}. \tag{11}$$

The modified version improves the accuracy at high ε/D_h. It remains less accurate than Colebrook's 1939 approximation, though.

I don't know what is in SVS v6, but I have seen no indication that WSP have moved away from (11).

Two commonly-used tunnel vent programs can't be directly compared, as I don't know the details of how they calculate friction. They are

1. **Thermotun**. I'd guess that Thermotun started off using Moody's approximation (both Aero and Thermotun originated in Prof. Fox's research group at the University of Leeds). But Prof. Vardy mentioned approximations attributed to Schlichting and to Jain in his 1996 IMechE paper [18]. And he has published a number of papers in the field of transient friction factors, including an interesting one co-authored with Fox about transient friction factors in the annulus around trains [19]. So Thermotun probably has a sensible default and some esoteric options.

2. **IDA**. The friction factor calculation in IDA Tunnel/IDA RTV is unclear. Users of IDA will know that you set a fully-turbulent Darcy friction factor λ for each section. But if you delve into the subsegment properties after a calculation, it is clear that each subsegment's friction factor is being adjusted for Reynolds number somehow. As far as I'm aware the formula of the adjustment isn't stated in IDA's Theoretical Reference.

5 How the program handles friction

The Hobyah tunnel ventilation program has no default friction factor. You must include `frictiontype Darcy` or `frictiontype Fanning` in your input file's `settings` block to tell it which type you want it to use. This is to push you to understand the difference between Darcy and Fanning friction factor.

I did seriously consider making Darcy friction factor the default. It would be consistent with two widely used programs (SES and IDA Tunnel). But when I thought back on the engineers I've worked with who only knew about one friction factor, the majority of them only knew about Darcy friction factor f and had no idea that a Fanning friction factor with symbol f existed. These engineers are most at risk of falling into the trap when they encounter a source that uses Fanning friction factor f (most likely on a high speed rail project). Not setting a default forces them to stop, think, and (hopefully) avoid the trap.

If you set `frictiontype Darcy` in the `settings` block, the program will interpret your friction factors as λ. If you set `frictiontype Fanning` instead, it will interpret your friction factors as c_f instead.

The program also accepts `frictiontype Atkinson`, which may be of interest to mine ventilation engineers.

Wherever a friction factor can be set, a roughness height can be set instead. When converting roughness heights into friction factors you can tell the program which friction factor approximation to use by an optional setting, `frictionapprox`. It has the following options:

- `frictionapprox Colebrook`—you set a roughness height, it uses Colebrook's 1939 explicit approximation (9).
- `frictionapprox Moody`—you set a roughness height, it uses Moody's 1947 explicit approximation (10).
- `frictionapprox SES`—you set a roughness height, it uses the explicit approximation from the SES source code (11).
- `frictionapprox Colebrook-White`—you set a roughness height, the program makes up to fifty iterations of the Colebrook-White function (7), stopping when the difference between successive estimates is less than 0.1%. I reckon that level of precision is pointless for engineering software, given how low the accuracy of all our other inputs are (e.g. train cross-sectional areas, draught relief shaft pressure loss factors). But it is there.

The heatmaps on the next page identify the absolute percentage difference between the three inbuilt explicit approximations (`Colebrook`, `Moody`, `SES`) and the exact (iterated) value of the Colebrook-White function for a range of relative roughnesses and Reynolds numbers.

The inbuilt approximations are all steady-state friction factor approximations. I have no plans to handle transient friction factors.

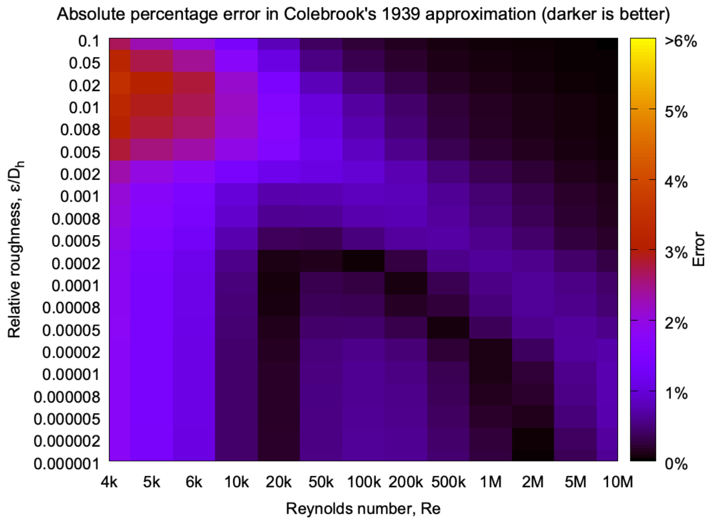
The default is Colebrook's 1939 explicit approximation (9), mostly on the basis of the heatmaps: it matches the Colebrook-White function much better than the others, especially at the high relative roughness figures we see all the time in tunnel ventilation air ducts.

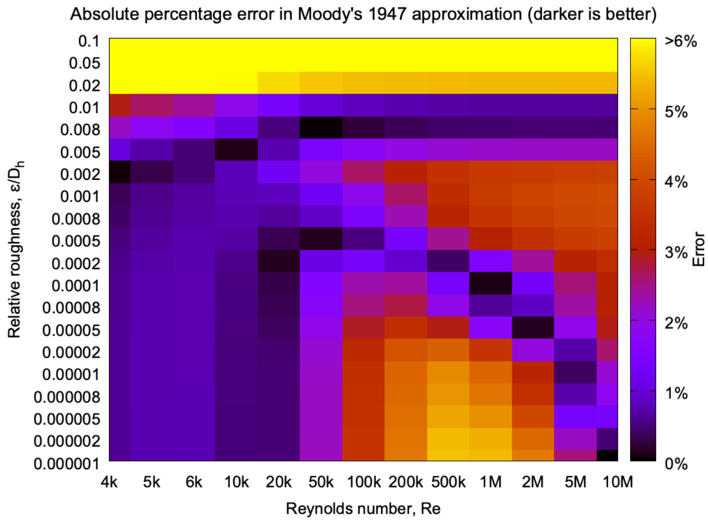
The other options are included so that runs can be set up that can be compared to the output of other tunnel ventilation programs (for verification) and to let you assess for yourself what the effects of the different approximations are.

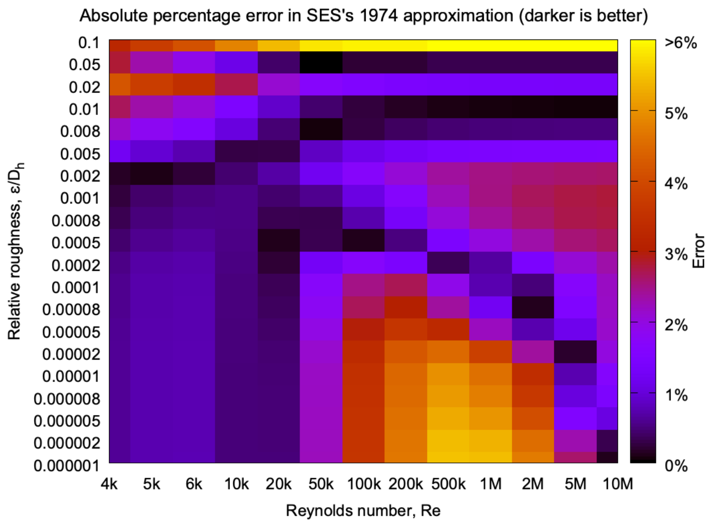
I have heard plausible arguments that Moody's approximation ought to be preferred because it was the approximation used to back-calculate train and tunnel roughnesses in many full-scale tests in the UK and Europe in the 1970s and 1980s. I have a lot of sympathy for that view, as I have test reports/papers that give both friction factors and roughness heights, all of which are related by Moody's approximation. If accurate approximations like Colebrook's are used more widely, it is less likely that such mismatches will occur again. So I decided to make the default the explicit approximation in Colebrook's original 1939 paper—it is easy to remember and (in my opinion) is more accurate than most of the approximations that came after it. I've provided alternate approximations that I know are used in other tunnel ventilation software. Competent programmers who want specific approximations should have no trouble adding them.

There are three other behaviours worth mentioning about the default friction factor routines in the program, all based on Fox's recommendations [20].

- At Reynolds number of 2300 and above the turbulent friction factor approximation is used.
- When the Reynolds number falls below 2300, the friction factor is estimated by the expression for laminar flow in a circular pipe ($c_f = 16/Re$, $\lambda = 64/Re$).
- If Reynolds number falls below 0.1 a fixed value of $c_f = 160$ is used.


Absolute percentage error in Colebrook's 1939 approximation (darker is better)
Relative roughness, ε/D_h
0.1
0.05
0.02
0.01
0.008
0.005
0.002
0.001
0.0008
0.0005
0.0002
0.0001
0.00008
0.00005
0.00002
0.00001
0.000008
0.000005
0.000002
0.000001
4k 5k 6k 10k 20k 50k 100k 200k 500k 1M 2M 5M 10M
Reynolds number, Re
>6%
5%
4%
3%
2%
1%
0%
Error


Absolute percentage error in Moody's 1947 approximation (darker is better)
Relative roughness, ε/D_h
0.1
0.05
0.02
0.01
0.008
0.005
0.002
0.001
0.0008
0.0005
0.0002
0.0001
0.00008
0.00005
0.00002
0.00001
0.000008
0.000005
0.000002
0.000001
4k 5k 6k 10k 20k 50k 100k 200k 500k 1M 2M 5M 10M
Reynolds number, Re
>6%
5%
4%
3%
2%
1%
0%
Error


Absolute percentage error in SES's 1974 approximation (darker is better)
Relative roughness, ε/D_h
0.1
0.05
0.02
0.01
0.008
0.005
0.002
0.001
0.0008
0.0005
0.0002
0.0001
0.00008
0.00005
0.00002
0.00001
0.000008
0.000005
0.000002
0.000001
4k 5k 6k 10k 20k 50k 100k 200k 500k 1M 2M 5M 10M
Reynolds number, Re
>6%
5%
4%
3%
2%
1%
0%
Error

6 References

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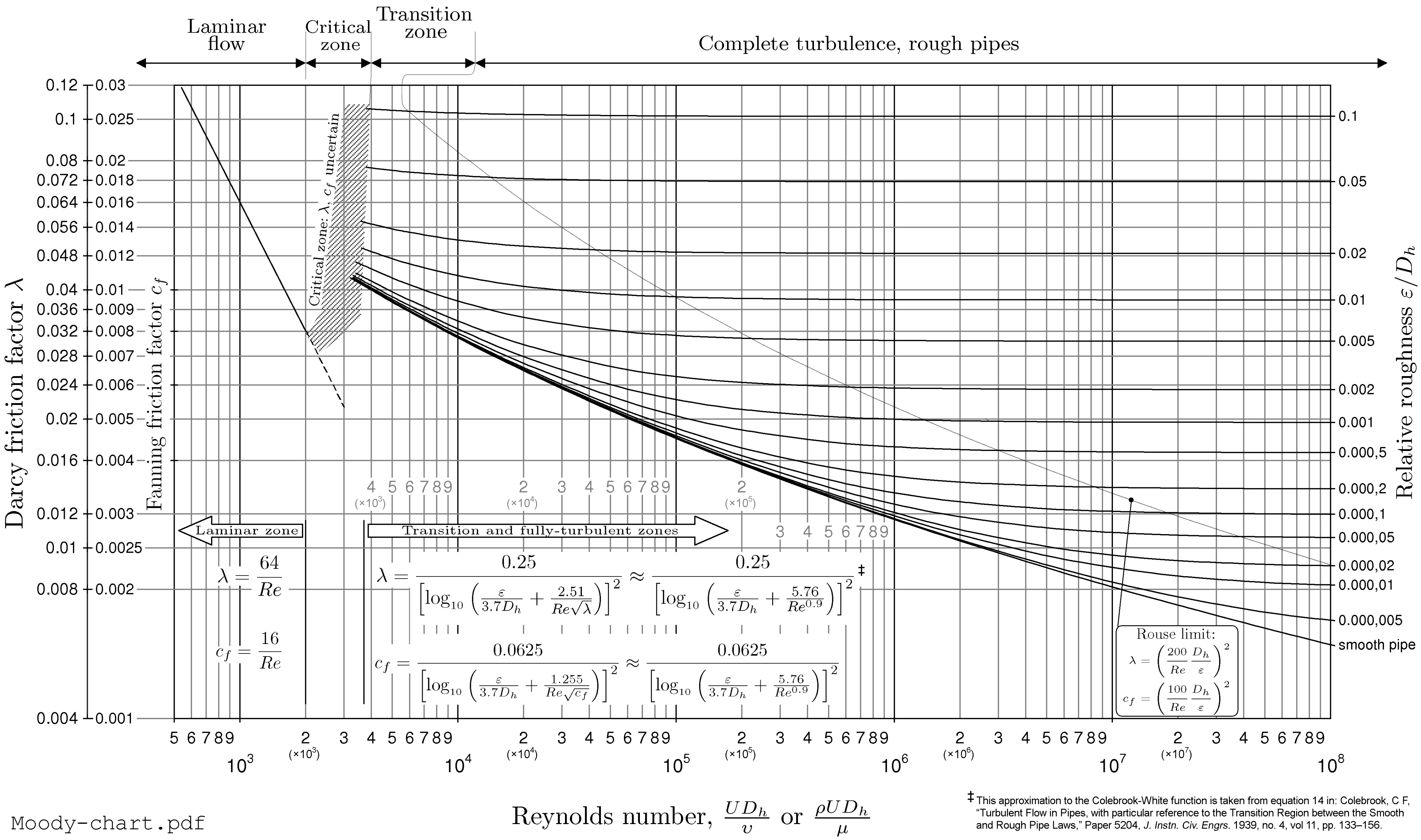
Moody chart for Darcy friction factor λ, used in $\Delta P = \frac{\lambda L}{D_h}\frac{1}{2}\rho U^2$

(also shows Fanning friction factor c_f, used in $\Delta P = \frac{4 c_f L}{D_h}\frac{1}{2}\rho U^2$)



‡ This approximation to the Colebrook-White function is taken from equation 14 in: Colebrook, C F, "Turbulent Flow in Pipes, with particular reference to the Transition Region between the Smooth and Rough Pipe Laws," Paper 5204, *J. Instn. Civ. Engrs.* 1939, no. 4, vol 11, pp. 133–156.