

## TP 2 - Members in the US Electoral College

### General information

Population data is from 2016. We chose to use the eligible population as it is more representative of the 2016 Elections.

The US' rules about their Electoral College are rather complicated and the first one we should mention is specific to the 2016 Elections as Puerto Rico was not allowed to take part in the voting process as it is seen as a territory and not a state.

### Linear Programming problem

$$\begin{cases} \min u - v \\ v - \frac{\alpha_i}{x_i} \leq 0 \\ \frac{\alpha_i}{x_i} - u \leq 0 \\ \sum_i \alpha_i = N \end{cases} \quad (1)$$

This LP problem is acts a bit like a dichotomy. By minimizing our objective function  $u - v$  which are tool variables, while keeping  $u > v$ , will diminish the interval where  $\frac{\alpha_i}{x_i}$  can be as it is "sandwiching" it. This ratio between the members at the Electoral College and the eligible population for each state is important. Indeed, if for each state, having a small interval for those ratios, meaning that all ratios are nearby one another for every given state will insure that the number of members in the Electoral College is well and evenly spread accordingly to the population of that state.

The last constraint is also extremely important as it keeps the sum of those members to that given  $N = 538$  in the US constitution. The solutions  $\alpha_i$  of this LP problem will give us the best distribution of Electoral College seats while focusing **ONLY** on the number of eligible voters per state and the number of available seats.

Other discussions could be made about some other constraints such as adding a minimal number of seats per state (it is currently 3 by US Constitution) but we noticed that even without that constraint, no state has 0 seats. In order to make it as fair as possible and to avoid large discrepancies between states, we decided to consider to not take the current minimal 3 seats condition into account as this will create a larger variation in seats for states while having a small change in population.

### Results and optimization

With our problem as stated before, we have a rather disturbing problem as we can see that some states have fairly similar populations but sometimes have double the members in the electoral college. This is particularly obvious when we took at the lower end of the population spectrum where there isn't a lot of population but we might see. For example, with our first LP, Vermont had a ratio  $\frac{\alpha_{Vermont}}{x_{Vermont}} \times 10^6 = 4.041$  while California had  $\frac{\alpha_{California}}{x_{California}} \times 10^6 = 2.151$ . This is due to a matrix conditioning problem when working on the LP problem as the orders of magnitude vary greatly between  $10^{-6}$  ( $u, v$ ),  $10^0$  ( $\alpha_i$ ) and  $10^6$  ( $x_i$ ).

Thus, our problem has been slightly modified to :

$$\begin{cases} \min u - v \\ v - \frac{\alpha_i}{x_i} \times 10^6 \leq 0 \\ \frac{\alpha_i}{x_i} \times 10^6 - u \leq 0 \\ \sum_i \alpha_i = N \end{cases} \quad (2)$$

By doing this, we will do some kind of pre-conditioning which will make our results better. With this change, the number of members per state is given by :

State	Voting Population	Members in Electoral C.	$\frac{\alpha_i \times 10^6}{x_i}$
Alabama	3 609 447	9	2.493
Alaska	522 679	1	1.913
Arizona	4 740 310	12	2.531
Arkansas	2 140 097	5	2.336
California	25 104 844	49	1.952
Colorado	3 974 405	10	2.516
Connecticut	2 582 761	6	2.323
Delaware	691 720	1	1.446
DC	515 248	1	1.941
Florida	14 601 066	37	2.534
Georgia	6 959 963	17	2.443
Hawaii	1 012 860	2	1.975
Idaho	1 166 706	2	1.714
Illinois	8 985 443	22	2.448
Indiana	4 849 937	12	2.474
Iowa	2 288 536	5	2.185
Kansas	2 054 025	5	2.434
Kentucky	3 282 420	8	2.437
Louisiana	3 384 435	8	2.364
Maine	1 058 372	2	1.890
Maryland	4 189 616	10	2.387
Massachusetts	4 948 028	12	2.425
Michigan	7 420 628	18	2.426
Minnesota	3 973 204	10	2.517
Mississippi	2 191 241	5	2.282
Missouri	4 517 925	11	2.435
Montana	804 250	2	2.487
Nebraska	1 343 821	3	2.232
Nevada	1 961 587	4	2.039
New Hampshire	1 042 795	2	1.918
New Jersey	6 013 656	15	2.494
New Mexico	1 464 515	3	2.048
New York	13 604 645	34	2.499
North Carolina	7 352 501	18	2.448
North Dakota	566 783	1	1.764
Ohio	8 736 808	22	2.518
Oklahoma	2 778 219	7	2.520
Oregon	3 024 174	7	2.315
Pennsylvania	9 691 160	24	2.476
Rhode Island	786 012	2	2.554
South Carolina	3 709 283	9	2.426
South Dakota	631 173	1	1.584
Tennessee	4 909 426	12	2.444
Texas	17 448 910	44	2.522
Utah	1 991 885	5	2.510
Vermont	494 871	1	2.021
Virginia	6 027 152	15	2.489
Washington	5 123 020	13	2.538
West Virginia	1 423 031	3	2.108
Wisconsin	4 285 071	10	2.334
Wyoming	429 682	1	2.327

# US Maps

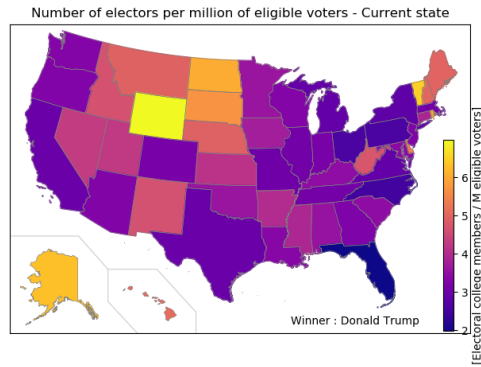


Figure 1: Current distribution :  $\frac{\alpha_{real,eligible_i}}{x_i}$

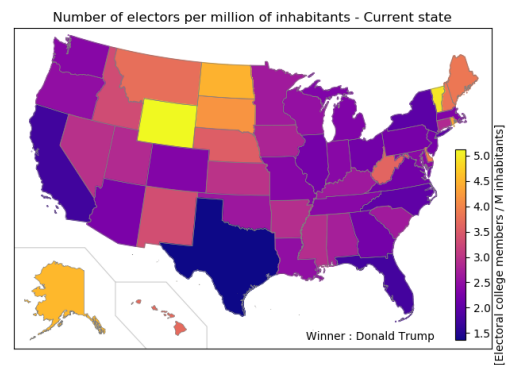


Figure 4: Current distribution :  $\frac{\alpha_{real,total_i}}{x_i}$

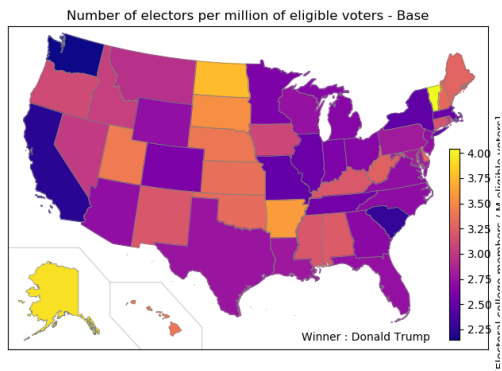


Figure 2: Distribution without conditioning  
- Eligible voters only

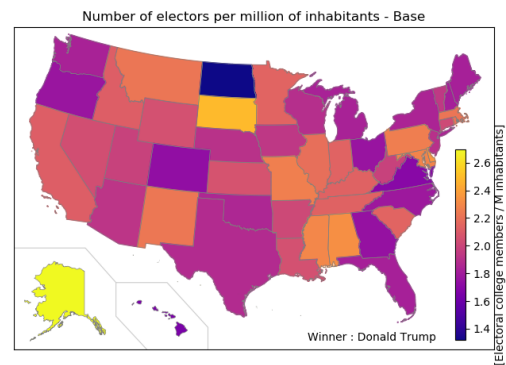


Figure 5: Distribution without conditioning  
- Whole population

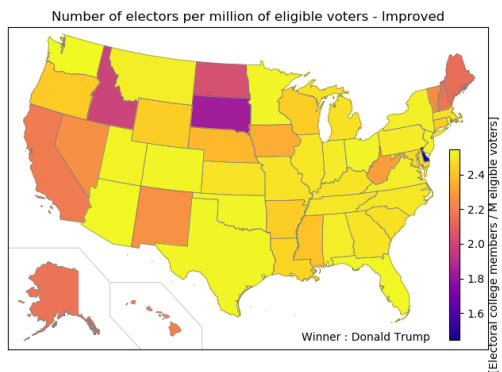


Figure 3: Distribution with conditioning -  
Eligible voters only

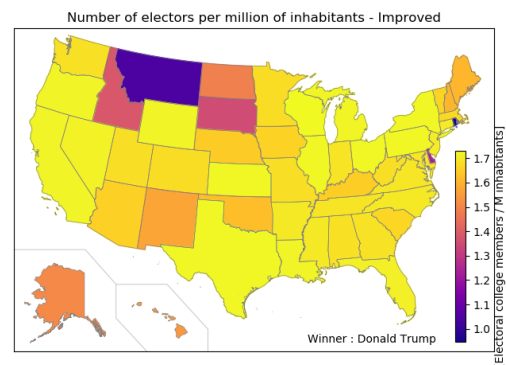


Figure 6: Distribution with conditioning -  
Whole population