Multi-robot Localization

Application to Area Coverage Optimization

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Outline

Introduction

What is Localization?

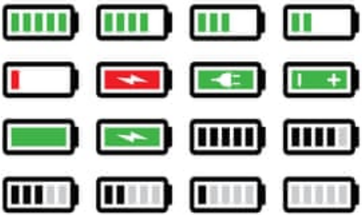
- For robot navigation, it is important to have a means of determining the location of the robot. Localization allows for accurate determination of a robots position in an In-Door or Out-Door environment.
- The main focus of this project is using In-Door Localization to determine the position of the robot. This will be accomplished by using the position of three beacons with known positions to the robot and the line of sight distance from each beacon to the robot.

Figure: Location Markers

- The goal for In-Door Localization, in this case, is to use Radio Frequency IDentification (RFID) system to determine the Radio Signal Strength (RSS). By using the RSS and the RFID system, the line of sight distance can be determined.
- By using the method of trilateration, the known position of the beacons, and the line of sight distance; the position of the robot can be determined.

Interoceptive Sensors are sensors that pertain to the robot's on-board functional components. Example of Interoceptive Sensors would be:

- Wheel Encoders
- Heading Angle
- Battery Status



Sensor Types

Exteroceptive Sensors

Exteroceptive Sensors are sensors that read data from the robot's work space/environment. Examples of Exteroceptive Sensors would be:

- Range measurement to object
- Light intensity
- Images captured by camera
- Microphone Data

Sensors in Localization

- Mainly exteroceptive sensors will be used for localization. RSS will be used to determine line of sight distance.
- Interoceptive sensors will be used to help determine where the robot moves based on the position determined from localization.

Other Localization Techniques

- External Kalmen Filter (EFK)
- Unscented Kalmen Filter (UFK)

Localization through Trilateration

Trilateration is the method of determining position by using the known positions of three beacons and the line of sight distance to each beacon.

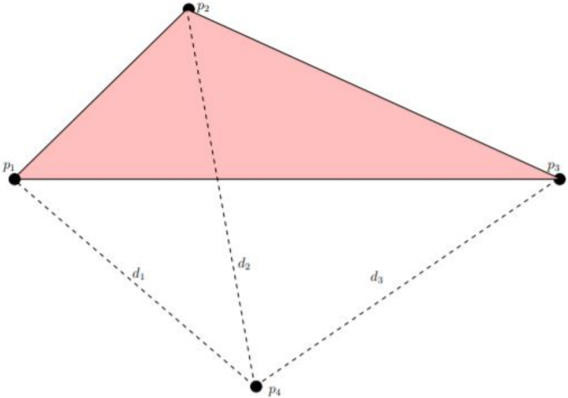


Figure: Trilateration Diagram

What is the Caley-Meger Determinant?

For trilateration, the equation found for the robot is:

Suppose that $\mathbf{p}_1$, $\mathbf{p}_2$, and $\mathbf{p}_3$ are the three dimensional (known) positions of three RF (radio frequency) beacons are placed in an indoor workspace of a mobile robot. The position vectors of these beacons are given as:

$$\mathbf{p}_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}, \quad \mathbf{p}_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}, \quad \text{and,} \quad \mathbf{p}_3 = \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix},$$

for $[x_i, y_i, z_i]^T \in \mathbb{R}^3$ and $i = 1, \ldots, 3$. The robot is placed on the ground and its three dimensional position $\mathbf{p}_4 = [x, y, z]^T$ is unknown. This problem can be solved using a trilateration setup shown in Figure 4.4, where the robot receives line-of-slight Euclidean distances, d_i, $i = 1, 2, 3$, from all three beacons in its operating range. The 3D position of the robot, $\mathbf{p}_4$ can be determined using the model

$$\mathbf{p}_4 = \mathbf{p}_1 + \frac{1}{D(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)} \cdot \Big(- D(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3; \mathbf{p}_1, \mathbf{p}_3, \mathbf{p}_4) \cdot \mathbf{v}_1 + D(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3; \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_4) \cdot \mathbf{v}_2 \pm \sqrt{D(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4)} \cdot (\mathbf{v}_1 \times \mathbf{v}_2) \Big) \qquad (4.4)$$

where $\mathbf{v}_1$ and $\mathbf{v}_2$ are the two known vectors defined by

$$\mathbf{v}_1 = \mathbf{p}_2 - \mathbf{p}_1, \quad \mathbf{v}_2 = \mathbf{p}_3 - \mathbf{p}_1,$$

What is the Caley-Meger Determinant?

and $D(\cdot)$ is the Caley-Menger determinant or bideterminant. If $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$, and $\mathbf{q}_1$, $\mathbf{q}_2$, $\mathbf{q}_3$ are the two sequences of points, then the Caley-Menger bideterminant of these two sequences is defined as:

$$D(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3; \mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3) = 2\left(\frac{-1}{2}\right)^3 \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & D(\mathbf{p}_1, \mathbf{q}_1) & D(\mathbf{p}_1, \mathbf{q}_2) & D(\mathbf{p}_1, \mathbf{q}_3) \\ 1 & D(\mathbf{p}_2, \mathbf{q}_1) & D(\mathbf{p}_2, \mathbf{q}_2) & D(\mathbf{p}_2, \mathbf{q}_3) \\ 1 & D(\mathbf{p}_3, \mathbf{q}_1) & D(\mathbf{p}_3, \mathbf{q}_2) & D(\mathbf{p}_3, \mathbf{q}_3) \end{vmatrix}$$

where $D(\mathbf{p}_i, \mathbf{q}_j)$ with $1 \leq i, j \leq 3$ is the square of the Euclidean distance between $\mathbf{p}_i$ and $\mathbf{q}_j$, *i.e.*, $D(\mathbf{p}_i, \mathbf{q}_j) = \|\mathbf{p}_i - \mathbf{q}_j\|^2$. If the above two sequences of points are the same, then $D(\cdot)$ is simply called a Cayley-Menger determinant. As such, $D(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4)$ of the equation (4.4) can be written as:

$$D(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4) = \frac{1}{8} \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & D(\mathbf{p}_1, \mathbf{p}_2) & D(\mathbf{p}_1, \mathbf{p}_3) & D(\mathbf{p}_1, \mathbf{p}_4) \\ 1 & D(\mathbf{p}_2, \mathbf{p}_1) & 0 & D(\mathbf{p}_2, \mathbf{p}_3) & D(\mathbf{p}_2, \mathbf{p}_4) \\ 1 & D(\mathbf{p}_3, \mathbf{p}_1) & D(\mathbf{p}_3, \mathbf{p}_2) & 0 & D(\mathbf{p}_3, \mathbf{p}_4) \\ 1 & D(\mathbf{p}_4, \mathbf{p}_1) & D(\mathbf{p}_4, \mathbf{p}_2) & D(\mathbf{p}_4, \mathbf{p}_3) & 0 \end{vmatrix}$$

More details on Cayley-Menger determinants can be sought in [19]. Here, the point $\mathbf{p}_4$ is the estimated robot location on the ground. Note that the $D(\mathbf{p}_4, \mathbf{p}_1)$, $D(\mathbf{p}_4, \mathbf{p}_2)$, and $D(\mathbf{p}_4, \mathbf{p}_3)$ in equation (4.6) are the squares of distances d_1, d_2, and d_3 from the robot to the beacon locations $\mathbf{p}_1$, $\mathbf{p}_2$, and $\mathbf{p}_3$, which are assumed to be known to the robot

What is the Caley-Meger Determinant?

using beacon measurements. As such, equation (4.6) can be rewritten as:

$$D(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4) = \frac{1}{8} \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & D(\mathbf{p}_1, \mathbf{p}_2) & D(\mathbf{p}_1, \mathbf{p}_3) & d_1^2 \\ 1 & D(\mathbf{p}_2, \mathbf{p}_1) & 0 & D(\mathbf{p}_2, \mathbf{p}_3) & d_2^2 \\ 1 & D(\mathbf{p}_3, \mathbf{p}_1) & D(\mathbf{p}_3, \mathbf{p}_2) & 0 & d_3^2 \\ 1 & d_1^2 & d_2^2 & d_3^2 & 0 \end{vmatrix}.$$

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Caley-Meger Determinant Example

Example 4.6: Localization using trilateration

Suppose that an RF (radio frequency) beacon receiver mounted on an indoor wheeled robot receives RF signal strength measurements from three RF beacons with their 16-bit IDs, 0x$FFFA$, 0x$FFFB$, 0x$FFFC$, which are spatially placed at 3D positions $\mathbf{p}_1 = [x_1, y_1, z_1]^T = [5, 4, 3]^T$ m, $\mathbf{p}_2 = [x_2, y_2, z_2]^T = [3, 8, 3]^T$ m, and $\mathbf{p}_3 = [x_3, y_3, z_3]^T = [-3, 5, 3]^T$ m. Assume that the RF signal strength measurements correspond to noise-free line-of-sight distances (not a practical assumption though!) between the robot and three beacons given by $d_1 = \sqrt{17}$ [m], $d_2 = \sqrt{45}$ [m], and $d_3 = \sqrt{54}$ [m], respectively. Determine the robot's 2D position on the ground.

Solution. *Note that the vectors* $\mathbf{v}_1 = [-2, 4, 0]^T$ m *and* $\mathbf{v}_2 = [-8, 1, 0]^T$ m. *Therefore,* $\mathbf{v}_1 \times \mathbf{v}_2 = [0, 0, 30]^T$.

$D(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = 900$.

$D(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4) = 8.1 \times 10^3$.

$D(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3; \mathbf{p}_1, \mathbf{p}_3, \mathbf{p}_4) = 540$.

$D(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3; \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_4) = 360$.

$$\mathbf{p}_4 = \mathbf{p}_1 + \frac{1}{D(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)} \cdot \Big(-D(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3; \mathbf{p}_1, \mathbf{p}_3, \mathbf{p}_4) \cdot \mathbf{v}_1 + D(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3; \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_4) \cdot \mathbf{v}_2 - \sqrt{D(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4)} \cdot (\mathbf{v}_1 \times \mathbf{v}_2) \Big) = [3, 2, 0]^T$$

Therefore, the robot's 2D position $(x, y) = (3, 2)$ m.

Future Plans - Incoming Week

- To be determined

Questions?