ECE 374 B - Fall 2023

Homework 1

- **Submit your solutions electronically on the course Gradescope site as PDF files.** If you plan to typeset your solutions, please use the LaTeX solution template on the course web site. If you must submit scanned handwritten solutions, please use a black pen on blank white paper and a high-quality scanner app (or an actual scanner, not just a phone camera). We will mark difficult to read solutions as incorrect and move on.
- **Every homework problem must be done *individually*.** Each problem needs to be submitted to Gradescope before 6AM of the due data which can be found on the course website: https://ecealgo.com/homeworks.html.
- For nearly every problem, **we have covered all the requisite knowledge required to complete a homework assignment prior to the "assigned" date.** This means that there is no reason not to begin a homework assignment as soon as it is assigned. Starting a problem the night before it is due a recipe for failure.

Policies to keep in mind

- **You may use any source at your disposal**—paper, electronic, or human—but you ***must*** cite ***every*** source that you use, and you ***must*** write everything yourself in your own words. See the academic integrity policies on the course web site for more details.
- **Being able to clearly and concisely explain your solution is a part of the grade you will receive.** Before submitting a solution ask yourself, if you were reading the solution without having seen it before, would you be able to understand it within two minutes? If not, you need to edit. Images and flow-charts are very useful for concisely explain difficult concepts.

See the course web site (https://ecealgo.com) for more information.

If you have any questions about these policies,
please don't hesitate to ask in class, in office hours, or on Piazza.

1. Give the recursive definition of the following languages. For both of these you should concisely explain why your solution is correct.

   (a) A language L_A that contains all palindrome strings using some arbitrary alphabet Σ.

   (b) A language L_B that does not contain either three 0's or three 1's in a row. E.g., $001101 \in L_B$ but 10001 is not in L_B.

2. For each of the following problems:

   i. Formulate the problem as a *regular* language (give an example of the problem instances and how they are encoded, you don't have to write every problem instance).

   ii. Describe the regular expression that describes the expression

   Note that how you encode the language matters for the regular expression you end up with.

   a Checking whether (or not) a number is divisible by 4). You are given a binary number and need to output if this number is divisible by 4.

   b The game of TicTacToe. You are given a completed tic-tac-toe board and you need to determine who won. (this won't have a clean regular expression. Just define some encoding and describe how you would build the expression, you don't need to write the whole expression out.) Hint: think about how many games of TicTacToe there are.

3. **Regular expressions I:** For each of the following languages over the alphabet $\{0, 1\}$, give a regular expression that describes that language, and briefly argue why your expression is correct.

   (a) $L_A = \{w || w| \leq 5\}$

   (b) $L_B = \{w | w \text{ is any string not in } 0^* + 1^*\}$

   (c) $L_C = \left\{w | w \text{ is any string not in } (01^+)^*\right\}$

   (d) $L_D = \{w | w \text{ every odd position is a } 1\}$

4. **Regular expressions II:** For each of the regular expressions, give a brief (1-2 sentence) English description of the language that regular expression represents.

   (a) $\left((0^*10^*10^*)^*\right)^2 \qquad \Sigma = \{0, 1\}$

   (b) $\emptyset(0+1)^*1 \qquad \Sigma = \{0, 1\}$

   (c) $(\varepsilon + 1)(01)^*(\varepsilon + 0) \qquad \Sigma = \{0, 1\}$

   (d) $1^*|1 + (0+1)0(0+1)^*|0 \qquad \Sigma = \{0, 1, |\}$