# 

- You will have 75 minutes (1.25 hours) to solve all the problems. Most have multiple parts. Don't spend too much time on questions you don't understand and focus on answering as much as you can!
- BUDGET YOUR TIME WISELY. I highly recommend working on the questions you know first and the questions you need to think about second.
- No resources are allowed for use during the exam except a cheatsheet and scratch paper on the back of the exam. **Do not tear out the cheatsheet or the scratch paper!** It messes with the auto-scanner.
- You should write your answers *completely* in the space given for the question. We will not grade parts of any answer written outside of the designated space.
- Please use a dark-colored pen unless you are absolutely sure your pencil writing is forceful enough to be legible when scanned. We reserve the right to deduct points if we have difficulty reading the uploaded document.
- Don't cheat. C'mon, be cool, be honest.
- Good luck!

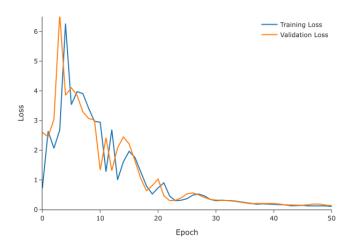
Name: .			
NetID:			
Date:			

1. <b>True/False</b> For each que	estion, ci	(30 points) rcle whether the statement is true or false.
(a) True	False	In PCA, the eigenvectors of the covariance matrix correspond to the principal component directions, and the associated eigenvalues represent the amount of variance explained along those directions.
(b) True	False	The K-Means algorithm is guaranteed to find the globally optimal clustering configuration for any dataset.
(c) True	False	Mode collapse is a common challenge in the GAN training.
(d) True	False	The filters (or kernels) in a convolutional layer share weights to reduce the number of parameters.
(e) True	False	A goal of activation function is to introduce nonlinearity
(f) True	False	The forget gate in LSTM decides what information to remove from the cell state.
(g) True	False	LSTM networks suffer from vanishing gradients more severely than standard RNNs.
(h) True	False	Principal components are always orthogonal to each other.
(i) True	False	The number of filters in a CNN layer determines the depth of the output feature map.
(j) True	False	PCA is a supervised learning technique used for dimension reduction.
(k) True	False	Learned positional embeddings cannot generalize to sequence lengths longer than those seen in training.
(l) True	False	BERT is a left-to-right (unidirectional) language model.
(m) True	False	SSD relies on max-pooling layers to generate its multi-scale feature maps for detection.
(n) True	False	In SSD, default boxes (anchors) are generated only at the final convolutional feature map.
(o) True	False	The sigmoid and tanh activation functions can be used interchangably since they both have a "S"-curve shape.

## 2. Debugging a Training Job

(10 points)

You have been assigned to build a classification model to detect whether emails are spam or not. After dedicating several days to the project, you have designed a promising model. But, during training, you observe the following behavior in the training and validation losses:



(a) Which of the following is a possible issue with the training? Circle your answer.

Underfitting

Overfitting

Learning Rate

No issue

(b) Justify your answer. If you chose any option other than "No issue", suggest a possible remedy to mitigate the issue. Use no more than four sentences in total for your answer.

# 3. Neural Networks for Simple Functions

(10 points)

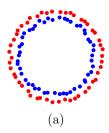
In this problem, you will be hand designing a simple neural network to model a specific function. Assume  $x \in \mathbb{R}$  and provide appropriate weights  $w_0, w_1 \in \mathbb{R}^2$ . In other words, the neural network has one input neuron, 2 hidden neurons, and one output neuron.

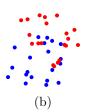
Find  $w_0, w_1 \in \mathbb{R}^2$  such that  $f(x) = w_1^T \sigma(w_0 x) = x, \forall x \in \mathbb{R}$  where  $\sigma = \text{ReLU}$  (note:  $w_0 x$  here is a vector-scalar product:  $\mathbb{R}^2 \times \mathbb{R} \to \mathbb{R}^2$ , e.g.  $[0, 1]^T x = [0, x]^T$ ). Show why your answer is correct.

# 4. K-means and GMM

(10 points)

(a) For each figure below, indicate whether it is possible for K-Means, GMM, neither, or both algorithms to produce the clustering assignments shown (as indicated by the two colors). In 1–2 sentences, briefly explain your reasoning.





- (b) Which algorithm (K-Means or GMM) typically has more parameters to learn for the same number of clusters, and why?
- (c) Which of the following expressions is used to compute cluster responsibilities in the E-step of the EM algorithm for GMM?

A. 
$$\arg\min_{k} |x_i - \mu_k|^2$$

A. 
$$\arg\min_{k} |x_i - \mu_k|^2$$
B.  $\frac{\pi_k, \mathcal{N}(x_i \mid \mu_k, \Sigma_k)}{\sum_{j=1}^K \pi_j, \mathcal{N}(x_i \mid \mu_j, \Sigma_j)}$ 
C.  $\sum_{k=1}^K |x_i - \mu_k|^2$ 

C. 
$$\sum_{k=1}^{K} |x_i - \mu_k|^2$$

D. 
$$\sum_{i=1}^{n} |x_i - \bar{x}|^2$$

#### 5. Attention is All You Need!

(15 points)

The Transformer architecture integrates attention mechanisms with multi-layer perceptrons (MLPs). The attention mechanism takes three inputs – queries (Q), keys (K), and values (V). Each has a shape of  $B \times N \times d_{\text{model}}$ . Here, B, N, and  $d_{\text{model}}$  represent the batch size, sequence length, and model dimension (or hidden size), respectively. Given h as the number of attention heads, the following process is used to compute attention

**Step 1.** For each head i, we apply learned projection matrices as

$$Q_i = QW_i^Q, K_i = KW_i^K, V_i = VW_i^V$$

where  $W_i^Q, W_i^K, W_i^V \in \mathbb{R}^{d_{\text{model}} \times d_h}$  and  $d_h = d_{\text{model}}/h$ .

**Step 2.** For each head, attention is computed as follows

$$A_i = \text{Attention}(Q_i, K_i, V_i) = \text{softmax}\left(\frac{Q_i K_i^T}{\sqrt{d_h}}\right) V_i$$

Step 3. Concatenate the attention across heads and apply one more linear transform

MultiHeadAttention
$$(Q, K, V) = \operatorname{concat}(A_1, A_2 \dots A_h)W^O$$

where  $W_O \in \mathbb{R}^{d_{\text{model}} \times d_{\text{model}}}$ .

Complete the code below to implement multi-head attention. Use the provided definitions as a guide. For this question, you can ignore additional details such as attention masks, dropouts, and optimized implementations.

```
1 # Even though you should not need to import anything else, feel
2 # free to do so.
3 import torch
4 import torch.nn as nn
5 from torch.nn.functional import softmax
 class MultiHeadAttention(nn.Module):
      def __init__(self, d_model: int, h: int) -> None:
          assert d_model % h == 0, "d_model must be divisible by h"
12
          self.d_model = d_model
          self.h = h
          self.d_head = self.d_model // self.h
14
          # Combine h weight matrices of d_head into one
16
          self.wq = nn.Linear(self.d_model, self.d_model, bias=False)
          self.wk = nn.Linear(self.d_model, self.d_model, bias=False)
18
          self.wv = nn.Linear(self.d_model, self.d_model, bias=False)
19
          self.wo = nn.Linear(self.d_model, self.d_model, bias=False)
20
          # Space to add more class variables as required
22
23
24
25
26
27
```

```
29
       def forward(self, Q: torch.Tensor, K: torch.Tensor, V: torch.Tensor) ->
30
       torch.Tensor:
            0.00
31
32
            Inputs
33
            Q: B \times N \times d_{model}
34
            K: B x N x d_model
35
            V: B x N x d_model
36
            0.000
            # Complete this
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66
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69
70
71
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78
79
80
81
82
83
            # Function end
84
```

## 6. Output and Parameter Count

(15 points)

Consider the following network:

```
model = nn.Sequential(
    nn.Conv2d(1, 4, 3, padding=1),
    nn.MaxPool2d(2,2),
    nn.Conv2d(4, 8, 3, padding=1),
    nn.MaxPool2d(2,2),
    nn.Flatten(),
    nn.Linear(8*7*7, 10)
)
```

(a) Suppose the input is (1, 1, 28, 28). What are the output shapes after each layer?

(b) Determine the total learnable parameters in the model (no biases).

# 7. Layer-Output Computation

Consider the two-channel input

$$X_1 = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, \quad X_2 = \begin{pmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{pmatrix}$$

(10 points)

and depthwise kernels

$$K_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad K_2 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

(a) Compute the two output feature-maps of Conv2d(in\_channels=2,out\_channels=2,groups=2,kernel\_size=2) (bias=0).

This page is for additional scratch work!