ECE 364: Programming Methods for Machine Learning, Spring 2025

Midterm 2 – Sample

- **You will have 75 minutes (1.25 hours) to solve all the problems. Most have multiple parts.** Don't spend too much time on questions you don't understand and focus on answering as much as you can!
- ***BUDGET YOUR TIME WISELY***. I highly recommend working on the questions you know first and the questions you need to think about second.
- *No* resources are allowed for use during the exam except a cheatsheet and scratch paper on the back of the exam. **Do not tear out the cheatsheet or the scratch paper!** It messes with the auto-scanner.
- You should write your answers *completely* in the space given for the question. We will not grade parts of any answer written outside of the designated space.
- Please *use a dark-colored pen* unless you are *absolutely* sure your pencil writing is forceful enough to be legible when scanned. We reserve the right to deduct points if we have difficulty reading the uploaded document.
- **Don't cheat.** C'mon, be cool, be honest.
- **Good luck!**

Name: ______________________________

NetID: ______________________________

Date: ______________________________

1. **True/False** (30 points)

For each question, circle whether the statement is true or false.

(a) **TRUE** False In PCA, the eigenvectors of the covariance matrix correspond to the principal component directions, and the associated eigenvalues represent the amount of variance explained along those directions.

(b) True **FALSE** The K-Means algorithm is guaranteed to find the globally optimal clustering configuration for any dataset.

(c) **TRUE** False Mode collapse is a common challenge in the GAN training.

(d) True **FALSE** The filters (or kernels) in a convolutional layer share weights to reduce the number of parameters.

(e) **TRUE** False A goal of activation function is to introduce nonlinearity

(f) **TRUE** False The forget gate in LSTM decides what information to remove from the cell state.

(g) True **FALSE** LSTM networks suffer from vanishing gradients more severely than standard RNNs.

(h) **TRUE** False Principal components are always orthogonal to each other.

(i) **TRUE** False The number of filters in a CNN layer determines the depth of the output feature map.

(j) True **FALSE** PCA is a supervised learning technique used for dimension reduction.

(k) **TRUE** False Learned positional embeddings cannot generalize to sequence lengths longer than those seen in training.

(l) True **FALSE** BERT is a left-to-right (unidirectional) language model.

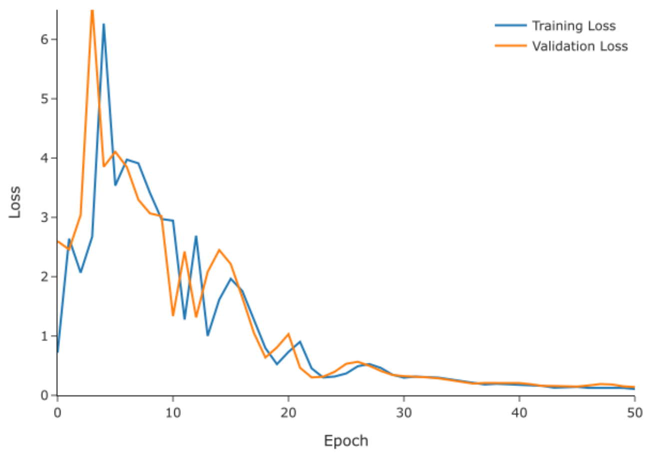
(m) True **FALSE** SSD relies on max-pooling layers to generate its multi-scale feature maps for detection.

(n) True **FALSE** In SSD, default boxes (anchors) are generated only at the final convolutional feature map.

(o) True **FALSE** The sigmoid and tanh activation functions can be used interchangably since they both have a “S”-curve shape.

2. **Debugging a Training Job** (10 points)

You have been assigned to build a classification model to detect whether emails are spam or not. After dedicating several days to the project, you have designed a promising model. But, during training, you observe the following behavior in the training and validation losses:



(a) Which of the following is a possible issue with the training? Circle your answer.

Underfitting Overfitting Learning Rate No issue

Solution:
Learning Rate

(b) Justify your answer. If you chose any option other than "No issue", suggest a possible remedy to mitigate the issue. Use no more than four sentences in total for your answer.

Solution:
The loss fluctuates significantly during the initial epochs, which is not expected. The issue may be caused by a higher learning rate than required. A possible remedy would be to reduce the learning rate. If stochastic gradient descent is used for training, increasing the batch size may also help.

3. **Neural Networks for Simple Functions** (10 points)

In this problem, you will be hand designing a simple neural network to model a specific function. Assume $x \in \mathbb{R}$ and provide appropriate weights $w_0, w_1 \in \mathbb{R}^2$. In other words, the neural network has one input neuron, 2 hidden neurons, and one output neuron.

Find $w_0, w_1 \in \mathbb{R}^2$ such that $f(x) = w_1^T \sigma(w_0 x) = x, \forall x \in \mathbb{R}$ where $\sigma = \text{ReLU}$ (note: $w_0 x$ here is a vector-scalar product: $\mathbb{R}^2 \times \mathbb{R} \to \mathbb{R}^2$, e.g. $[0,1]^T x = [0,x]^T$). Show why your answer is correct.

Solution:

In order to achieve this function, we can set $w_0 = [1,-1]^T, w_1 = [1,-1]^T$. We can see that this works because applying our answer to our function, we get

$$f(x) = w_1^T \sigma(w_0 x) = [1,-1]^{T^T} \sigma([1,-1]^T x)$$

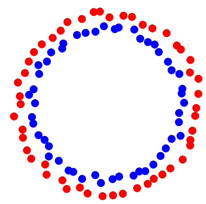
$$= [1,-1]\sigma([x,-x]^T) = \begin{cases} [1,-1][x,0]^T = x & ,x > 0 \\ [1,-1][0,0]^T = 0 & ,x = 0 \\ [1,-1][0,-x]^T = x & ,x < 0 \end{cases}$$

$$\Rightarrow f(x) = x, \forall x \in \mathbb{R}$$

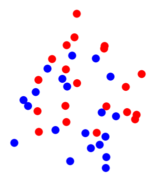
(ReLU: $\sigma(x) = max(0,x)$).

4. **K-means and GMM** (10 points)

(a) For each figure below, indicate whether it is possible for K-Means, GMM, neither, or both algorithms to produce the clustering assignments shown (as indicated by the two colors). In 1–2 sentences, briefly explain your reasoning.



(a)



(b)

Solution:

(a) Neither

(b) GMM only

(b) Which algorithm (K-Means or GMM) typically has more parameters to learn for the same number of clusters, and why?

Solution:
GMM, because for each cluster it estimates a mean, a covariance matrix (or variance in 1D), and a mixing coefficient. K-Means only estimates cluster centers.

(c) Which of the following expressions is used to compute cluster responsibilities in the E-step of the EM algorithm for GMM?

A. $\arg\min_k |x_i - \mu_k|^2$

B. $\dfrac{\pi_k, \mathcal{N}(x_i \mid \mu_k, \Sigma_k)}{\sum_{j=1}^{K} \pi_j, \mathcal{N}(x_i \mid \mu_j, \Sigma_j)}$

C. $\sum_{k=1}^{K} |x_i - \mu_k|^2$

D. $\sum_{i=1}^{n} |x_i - \bar{x}|^2$

Solution:

(b). The E-step computes the probability (responsibility) that each point belongs to each component using Bayes' rule.

5. **Attention is All You Need!** (15 points)

The Transformer architecture integrates attention mechanisms with multi-layer perceptrons (MLPs). The attention mechanism takes three inputs – queries (Q), keys (K), and values (V). Each has a shape of $B \times N \times d_{\text{model}}$. Here, B, N, and d_{model} represent the batch size, sequence length, and model dimension (or hidden size), respectively. Given h as the number of attention heads, the following process is used to compute attention

Step 1. For each head i, we apply learned projection matrices as

$$Q_i = QW_i^Q, K_i = KW_i^K, V_i = VW_i^V$$

where $W_i^Q, W_i^K, W_i^V \in \mathbb{R}^{d_{\text{model}} \times d_h}$ and $d_h = d_{\text{model}}/h$.

Step 2. For each head, attention is computed as follows

$$A_i = \text{Attention}(Q_i, K_i, V_i) = \text{softmax}\left(\frac{Q_i K_i^T}{\sqrt{d_h}}\right) V_i$$

Step 3. Concatenate the attention across heads and apply one more linear transform

$$\text{MultiHeadAttention}(Q, K, V) = \text{concat}(A_1, A_2 \dots A_h) W^O$$

where $W_O \in \mathbb{R}^{d_{\text{model}} \times d_{\text{model}}}$.

Complete the code below to implement multi-head attention. Use the provided definitions as a guide. For this question, you can ignore additional details such as attention masks, dropouts, and optimized implementations.

```python
# Even though you should not need to import anything else, feel
# free to do so.
import torch
import torch.nn as nn
from torch.nn.functional import softmax



class MultiHeadAttention(nn.Module):
    def __init__(self, d_model: int, h: int) -> None:
        assert d_model % h == 0, "d_model must be divisible by h"
        self.d_model = d_model
        self.h = h
        self.d_head = self.d_model // self.h

        # Combine h weight matrices of d_head into one
        self.wq = nn.Linear(self.d_model, self.d_model, bias=False)
        self.wk = nn.Linear(self.d_model, self.d_model, bias=False)
        self.wv = nn.Linear(self.d_model, self.d_model, bias=False)
        self.wo = nn.Linear(self.d_model, self.d_model, bias=False)

        # Space to add more class variables as required






```

```python

    def forward(self, Q: torch.Tensor, K: torch.Tensor, V: torch.Tensor) ->
    torch.Tensor:
        """
        Inputs
        ------
        Q: B x N x d_model
        K: B x N x d_model
        V: B x N x d_model
        """
        # Complete this













































        # Function end
```

Solution:

```python
# Even though you should not need to import anything else, feel
# free to do so.
import torch
import torch.nn as nn
from torch.nn.functional import softmax

class MultiHeadAttention(nn.Module):
    def __init__(self, d_model: int, h: int) -> None:
        assert d_model % h == 0, "d_model must be divisible by h"
        self.d_model = d_model
        self.h = h
        self.d_head = self.d_model // self.h

        # Combine h weight matrices of d_head into one
        self.wq = nn.Linear(self.d_model, self.d_model, bias=False)
        self.wk = nn.Linear(self.d_model, self.d_model, bias=False)
        self.wv = nn.Linear(self.d_model, self.d_model, bias=False)
        self.wo = nn.Linear(self.d_model, self.d_model, bias=False)

        # Space to add more class variables as required
        self.scaling = self.d_head**-0.5

    def forward(self, Q: torch.Tensor, K: torch.Tensor, V: torch.Tensor) -> torch.Tensor:
        """
        Inputs
        ------
        Q: B x N x d_model
        K: B x N x d_model
        V: B x N x d_model
        """
        # Complete this
        B, N, d_model = Q.size()
        query_states = self.wq(Q) * self.scaling # B x N x d_model
        key_states = self.wk(K) # B x N x d_model
        val_states = self.wv(V) # B x N x d_model

        query_states = query_states.view(B, N, self.h, self.d_head).transpose(1, 2).contiguous() # B x h x N x d_head
        key_states = key_states.view(B, N, self.h, self.d_head).transpose(1, 2).contiguous() # B x h x N x d_head
        val_states = val_states.view(B, N, self.h, self.d_head).transpose(1, 2).contiguous() # B x h x N x d_head

        query_states = query_states.view(-1, N, self.d_head) # B*h x N x d_head
        key_states = key_states.view(-1, N, self.d_head) # B*h x N x d_head
        val_states = val_states.view(-1, N, self.d_head) # B*h x N x d_head

        attn_weights = query_states @ key_states.transpose(1, 2) # B*h x N x N
        attn_weights = softmax(attn_weights, dim=-1) # B*h x N X N

        attn_outputs = attn_weights @ val_states # B*h x N x d_head
        attn_outputs = attn_outputs.view(B, self.h, N, self.d_head) # B x h x N x d_head
```

```
        attn_outputs = attn_outputs.transpose(1, 2) # B x N x h x d_head
        attn_outputs = attn_outputs.reshape(B, N, d_model) # B x N x d_model
        attn_outputs = self.wo(attn_outputs) # B x N x d_model

        return attn_outputs
        # Function end
```

6. **Output and Parameter Count** (15 points)

Consider the following network:

```
model = nn.Sequential(
    nn.Conv2d(1, 4, 3, padding=1),
    nn.MaxPool2d(2,2),
    nn.Conv2d(4, 8, 3, padding=1),
    nn.MaxPool2d(2,2),
    nn.Flatten(),
    nn.Linear(8*7*7, 10)
)
```

(a) Suppose the input is $(1, 1, 28, 28)$. What are the output shapes after each layer?

Solution:

After Conv1: $(1, 4, 28, 28)$
Pool1: $(1, 4, 14, 14)$
Conv2: $(1, 8, 14, 14)$
Pool2: $(1, 8, 7, 7)$
Flatten: $(1, 392)$
Linear: $(1, 10)$

(b) Determine the total learnable parameters in the model (including biases).

Solution:

Conv1: $4 \times 1 \times 3 \times 3 + 4 = 36 + 4 = 40$
Conv2: $8 \times 4 \times 3 \times 3 + 8 = 288 + 8 = 296$
Linear: $392 \times 10 + 10 = 3920 + 10 = 3930$
Total: $40 + 296 + 3930 = 4266$.

7. **Layer-Output Computation** (10 points)

Consider the two-channel input

$$X_1 = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, \quad X_2 = \begin{pmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{pmatrix}$$

and depthwise kernels

$$K_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad K_2 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

(a) Compute the two output feature-maps of `Conv2d(in_channels=2,out_channels=2,groups=2,kernel_size=2)` (bias=0).

Solution:

Channel 1: $\begin{pmatrix} 6 & 8 \\ 12 & 14 \end{pmatrix}$, Channel 2: $\begin{pmatrix} 28 & 24 \\ 16 & 12 \end{pmatrix}$

This page is for additional scratch work!