ECE 374 B ✧ Spring 2022

Homework 9

- Groups of up to three people can submit joint solutions. Each problem should be submitted by exactly one person, and the beginning of the homework should clearly state the Gradescope names and email addresses of each group member. In addition, whoever submits the homework must tell Gradescope who their other group members are.
- **Submit your solutions electronically on the course Gradescope site as PDF files.** If you plan to typeset your solutions, please use the LaTeX solution template on the course web site. If you must submit scanned handwritten solutions, please use a black pen on blank white paper and a high-quality scanner app (or an actual scanner, not just a phone camera).

☞ Some important course policies ☜

- **You may use any source at your disposal**—paper, electronic, or human—but you ***must*** cite ***every*** source that you use, and you ***must*** write everything yourself in your own words. See the academic integrity policies on the course web site for more details.
- **Avoid the Three Deadly Sins!** Any homework or exam solution that breaks any of the following rules will be given an ***automatic zero***, unless the solution is otherwise perfect. Yes, we really mean it. We're not trying to be scary or petty (Honest!), but we do want to break a few common bad habits that seriously impede mastery of the course material.
  - Always give complete solutions, not just examples.
  - Always declare all your variables, in English. In particular, always describe the specific problem your algorithm is supposed to solve.
  - Never use weak induction.

See the course web site for more information.

If you have any questions about these policies,
please don't hesitate to ask in class, in office hours, or on Piazza.

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itemsep=4ex plus 0.5fil A *domino* is a 1×2 rectangle divided into two squares, each of which is labeled with an integer[1]. In a *legal arrangement* of dominos, the dominos are lined up end-to-end so that the numbers on adjacent ends match. An example of a legal arrangement of dominos is given below:

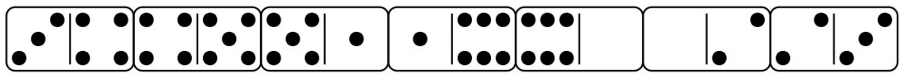

Figure 1. A legal arrangement of dominos in which every integer between 0 and 6 appears twice.

For each of the following problems, either describe and analyze a polynomial-time algorithm or prove that the problem is NP-*complete*:

(a) Given an arbitrary bag D of dominos, is there a legal arrangement of *all* the dominos in D?

(b) Given an arbitrary bag D of dominos and an integer n, is there a legal arrangement of dominos from D in which every integer between 1 and n appears exactly twice?

iitemsep=4ex plus 0.5fiil Let M be a Turing machine, let w be an arbitrary input string, and let s and t be positive integers. We say that M accepts w *in space* s if M accepts w after accessing at most the first s cells on its tape, and M accepts w *in time* t if M accepts w after at most t transitions.

(a) Prove that the following langauges are decidable:

i. $\left\{\langle M, w\rangle \;\middle|\; M \text{ accepts } w \text{ in time } |w|^2\right\}$

ii. $\left\{\langle M, w\rangle \;\middle|\; M \text{ accepts } w \text{ in space } |w|^2\right\}$

(b) Prove that the following languages are undecidable:

i. $\left\{\langle M\rangle \;\middle|\; M \text{ accepts at least one string } w \text{ in time } |w|^2\right\}$

ii. $\left\{\langle M\rangle \;\middle|\; M \text{ accepts at least one string } w \text{ in space } |w|^2\right\}$

iiitemsep=4ex plus 0.5fiiil For each of the following decision problems, either sketch an algorithm or prove that the problem is undecidable. Recall that w^R denote the reversal of string w. For each problem, the input is an encoding $\langle M, w\rangle$ of a Turing machine M and its input string w.

(a) Does M either accept w or reject w^R?

(b) If we run M on input w, does M ever change a symbol on its tape?

(c) If we run M on input w, does M ever leave its start state?

(d) If we run M on input w, does M ever reenter its start state?

[1] These integers are usually represented by pips, exactly like dice. On a standard domino, the number of pips on each side is between 0 and 6, although one can buy sets with up to 9 or even 12 pips on each side; we will allow arbitrary integer labels. A standard set of dominos contains exactly one domino for each possible unordered pair of labels; we do *not* assume that the inputs to our problems have this property.