Homework 2

- **Submit your solutions electronically on the course Gradescope site as PDF files.** If you plan to typeset your solutions, please use the LaTeX solution template on the course web site. If you must submit scanned handwritten solutions, please use a black pen on blank white paper and a high-quality scanner app (or an actual scanner, not just a phone camera). We will mark difficult to read solutions as incorrect and move on.
- **Every homework problem must be done *individually*.** Each problem needs to be submitted to Gradescope before 6AM of the due data which can be found on the course website: https://ecealgo.com/homeworks.html.
- For nearly every problem, **we have covered all the requisite knowledge required to complete a homework assignment prior to the "assigned" date.** This means that there is no reason not to begin a homework assignment as soon as it is assigned. Starting a problem the night before it is due a recipe for failure.

Policies to keep in mind

- **You may use any source at your disposal**—paper, electronic, or human—but you ***must*** cite ***every*** source that you use, and you ***must*** write everything yourself in your own words. See the academic integrity policies on the course web site for more details.
- **Being able to clearly and concisely explain your solution is a part of the grade you will receive.** Before submitting a solution ask yourself, if you were reading the solution without having seen it before, would you be able to understand it within two minutes? If not, you need to edit. Images and flow-charts are very useful for concisely explain difficult concepts.

See the course web site (https://ecealgo.com) for more information.

If you have any questions about these policies,
please don't hesitate to ask in class, in office hours, or on Piazza.

1. Given an arbitrary regular language L on some alphabet Σ, prove that it is closed under the following operations. In other words, prove the following languages are regular.

   (a) $L^R = \{w^R \mid w \in L\}$

   (b) $\texttt{subseq}(L) := \{x \in \Sigma^* | x \text{ is a subsequence of some } y \in L\}$.

   [*Hint*: given an NFA (or DFA) for L, construct an NFA for $func(L)$. Give a formal description of your construction. Provide an explanation of how your NFA works, including the meaning of each state. A formal proof of correctness of your NFA is not required.]

2. Let

$$\Sigma_{3R} = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \ldots, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

   Σ_{3R} contains all size 3 columns of 0s and 1s. A string of symbols in Σ_{3R} gives three rows of 0s and 1s. Consider each row to be a binary number and let

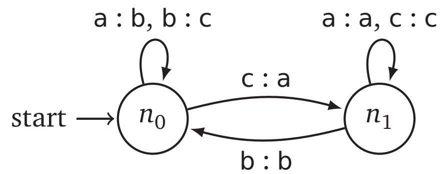
$$B = \{w \in \Sigma_{3R}^* \mid \text{the bottom row of } w \text{ is the sum of the top 2 rows}\}.$$

   For example,

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \in B, \quad \text{but} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \notin B.$$

   Show that B is regular. (Hint: Working with B^R is easier. Use the result of part (a).)

3. A *finite-state transducer* (FST) is a type of deterministic finite automaton whose output is a string instead of just *accept* or *reject*. The following is the state diagram of finite state transducer FST_0.



Each transition of an FST is labeled at least an input symbol and an output symbol, separated by a colon (:). There can also be multiple input-output pairs for each transitions, separated by a comma (,). For instance, the transition from n_0 to itself can either take a or b as an input, and outputs b or c respectively.

When an FST computes on an input string $s := \overline{s_0 s_1 \ldots s_{n-1}}$ of length n, it takes the input symbols $s_0, s_1, \ldots, s_{n-1}$ one by one, starting from the starting state, and produces corresponding output symbols. For instance, the input string abccba produces the output string bcacbb, while cbaabc produces abbbca.

(a) Assume that FST_1 has an input alphabet Σ_1 and an output alphabet Γ_1, give a formal definition of this model and its computation. (Hint: An FST is a 5-tuple with no accepting states. Its transition function is of the form $\delta : Q \times \Sigma \to Q \times \Gamma$.)

(b) Give a formal description of FST_0.

(c) Give a state diagram of an FST with the following behavior. Its input and output alphabets are $\{\mathsf{T}, \mathsf{F}\}$. Its output string is inverted on the positions with indices divisible by 3 and is identical on all the other positions. For instance, on an input TFTTFTFT it should output FFTFFTTT.

4. **Another language transformation:** Given an arbitrary regular language L on some alphabet Σ, prove that regular languages are closed under the following operation:

$$\mathsf{cycle}(L) := \{xy \mid x, y \in \Sigma^*, yx \in L\} \tag{1}$$