ECE 374 B ✧ Fall 2023
Homework 4

- Groups of up to three people can submit joint solutions. Each problem should be submitted by exactly one person, and the beginning of the homework should clearly state the Gradescope names and email addresses of each group member. In addition, whoever submits the homework must tell Gradescope who their other group members are.
- **Submit your solutions electronically on the course Gradescope site as PDF files.** please use the LaTeX solution template on the course web site. If you must submit scanned handwritten solutions, please use a black pen on blank white paper and a high-quality scanner app (or an actual scanner, not just a phone camera).

☞ Some important course policies ☜

- **You may use any source at your disposal**—paper, electronic, or human—but you ***must*** cite ***every*** source that you use, and you ***must*** write everything yourself in your own words. See the academic integrity policies on the course web site for more details.
- **Avoid the Three Deadly Sins!** Any homework or exam solution that breaks any of the following rules will be given an ***automatic zero***, unless the solution is otherwise perfect. Yes, we really mean it. We're not trying to be scary or petty (Honest!), but we do want to break a few common bad habits that seriously impede mastery of the course material.
  - Always give complete solutions, not just examples.
  - Always declare all your variables, in English. In particular, always describe the specific problem your algorithm is supposed to solve.
  - Never use weak induction.

See the course web site for more information.

If you have any questions about these policies,
please don't hesitate to ask in class, in office hours, or on Piazza.

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1. Solve the following recurrence relations. For parts (a) and (b), give an exact solution. For parts (c) and (d), give an asymptotic one. In both cases, justify your solution.

   (a) $W(n) = W(n-1) + 2\log n + 1; W(0) = 0$

   (b) $X(n) = 5X(n-1) + 3; X(1) = 3$

   (c) $Y(n) = Y(n/2) + 2Y(n/3) + 3Y(n/4) + n^2$

   (d) $Z(n) = Z(n/15) + Z(n/10) + 2Z(n/6) + \sqrt{n}$

2. Suppose you are given a stack of n pancakes of different sizes. You want to sort the pancakes so that smaller pancakes are on top of larger pancakes. The only operation you can perform is *flip* - insert a spatula under the top k pancakes, for some integer k between 1 and n, and flip them all over.

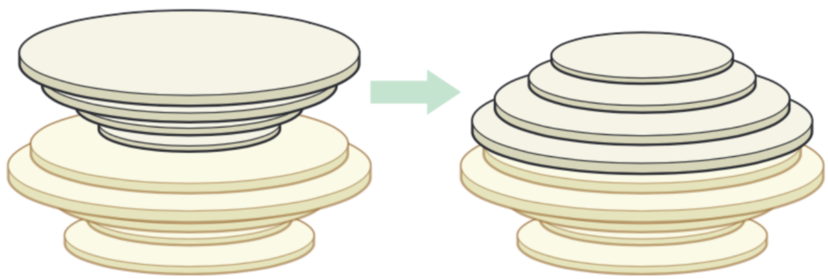

Figure 1.19. Flipping the top four pancakes.

   (a) Describe an algorithm to sort an arbitrary stack of n pancakes using $O(n)$ flips. *Exactly* how many flips does your algorithm perform in the worst case? [*Hint: This problem has nothing to do with the Tower of Hanoi.*]

   (b) For every positive integer n, describe a stack of n pancakes that requires $\Omega(n)$ flips to sort.

   (c) Now suppose one side of each pancake is burned. Describe an algorithm to sort an arbitrary stack of n pancakes, so that the burned side of every pancake is facing down, using $O(n)$ flips. *Exactly* how many flips does your algorithm perform in the worst case?

3. Suppose we are given an array $A[1..n]$ of n integers, which could be positive, negative, or zero, sorted in increasing order so that $A[1] \le A[2] \le \cdots \le A[n]$. Suppose we wanted to count the number of times some integer value x occurs in A. Describe an algorithm (as fast as possible) which returns the number of elements containing value x.

4. Given an arbitrary array $A[1..n]$, describe an algorithm to determine in $O(n)$ time whether A contains more than $n/4$ copies of any value. **Do not use hashing, or radix sort, or any other method that depends on the precise input values.**