Supplemental Material for "Theoretical Guarantees for Sparse Graph Signal Recovery"

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This document contains additional simulation results for the paper in [1]. First, we evaluate the proposed bounds on the mutual coherence for a distance-based graph. Then, we evaluate the sparse recovery performance for the well-known orthogonal matching pursuit (OMP) and Lasso methods over both the Erdős-Réyni and the distance-based graphs.

S.I. Mutual Coherence

We consider a distance-based graph, defined by the edge weights: [2]

$$W_{k,m} = \begin{cases} \exp\left(-\frac{[dist(k,m)]^2}{2\theta^2}\right) & \text{if } dist(k,m) \leq \gamma, \\ 0 & \text{otherwise,} \end{cases} \tag{S-1}$$

where $dist(i,j)$ is the $2D$ Euclidean distance, θ is the exponential decay rate, and γ is the graph connectivity threshold. In each trial, a graph is constructed by (S-1) from N points drawn from the $2D$ space: $[1,N]\times[1,N]$.

Figure S-1 shows the averaged mutual coherence and its upper and lower bounds as a function of the normalized minimal nodal degree, d_{min}/N, for a graph of size $N=70$ and $\theta=0.2$. The inset reveals a nearly linear relationship between d_{min}/N and γ. Similar to Fig. 1 (top), the mutual coherence decreases as the minimal degree increases, with the proposed bounds forming a narrow range around it. However, unlike Fig. 1 (top), the lower bound here is tighter for all values of d_{min}/N.

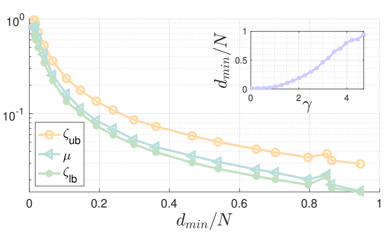
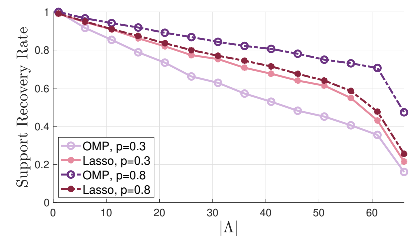


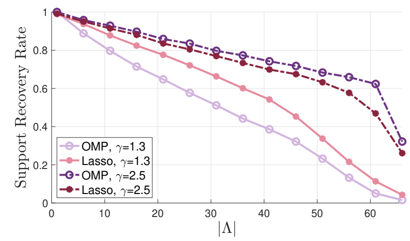
Figure S-1: Distance-based Graph: he averaged mutual coherence, upper, and lower bounds versus the normalized minimal nodal degree, d_{min}/N. In the inset, d_{min}/N is presented versus the edge-edge-connectivity threshold γ.

S.II. Sparse Recovery

We evaluated sparse recovery performance using the support recovery rate, which is defined as the ratio of successful recoveries to the number of trials. In each trial, a random $|\Lambda|\times 1$ support set, Λ, defines the positions of the nonzero elements of the sparse signal $\mathbf{x}_\Lambda$, which are drawn from a standard normal distribution. The measurements are computed as $\mathbf{y}=\mathbf{L}\mathbf{x}$.



(a) Erdős-Rényi graph with $p=0.3$ and $p=0.8$.



(b) Distance-based graph with $\gamma=1.3$ and $\gamma=2.5$.

Figure S-2: Empirical recovery rates of Lasso and OMP versus the support cardinality.

In Fig. S-2, the support recovery rate is shown versus the support cardinality $|\Lambda|$ for (a) the Erdős-Rényi graph with edge-presence probabilities $p=0.3$ and $p=0.8$, and (b) the distance-based graph with $\gamma = 1.3$ and $\gamma = 2.5$, and $\theta = 0.2$. In both cases, recovery rates decrease as support cardinality increases. Additionally, OMP performs significantly better with higher values of p or γ. In contrast, Lasso is moderately affected by changes in p in the Erdős-Rényi graph but is significantly influenced by changes in γ in the distance-based graph. Thus, from Fig. 1 (top) and Fig. S-1, it can be verified that recovery performance improves with an increase in the graph's minimal degree.

References

[1] G. Morgenstern and T. Routtenberg, "Theoretical guarantees for sparse graph signal recovery," *accepted, IEEE Signal Process. Lett.*, 2024.

[2] D. I. Shuman, S. K. Narang, P. Frossard, A. Ortega, and P. Vandergheynst, "The emerging field of signal processing on graphs: Extending high-dimensional data analysis to networks and other irregular domains," *IEEE Signal Process. Mag.*, vol. 30, no. 3, pp. 83–98, 2013.