Phase Retrieval and Computer-generated Holography

EE367/CS448I: Computational Imaging

stanford.edu/class/ee367

Lecture 15

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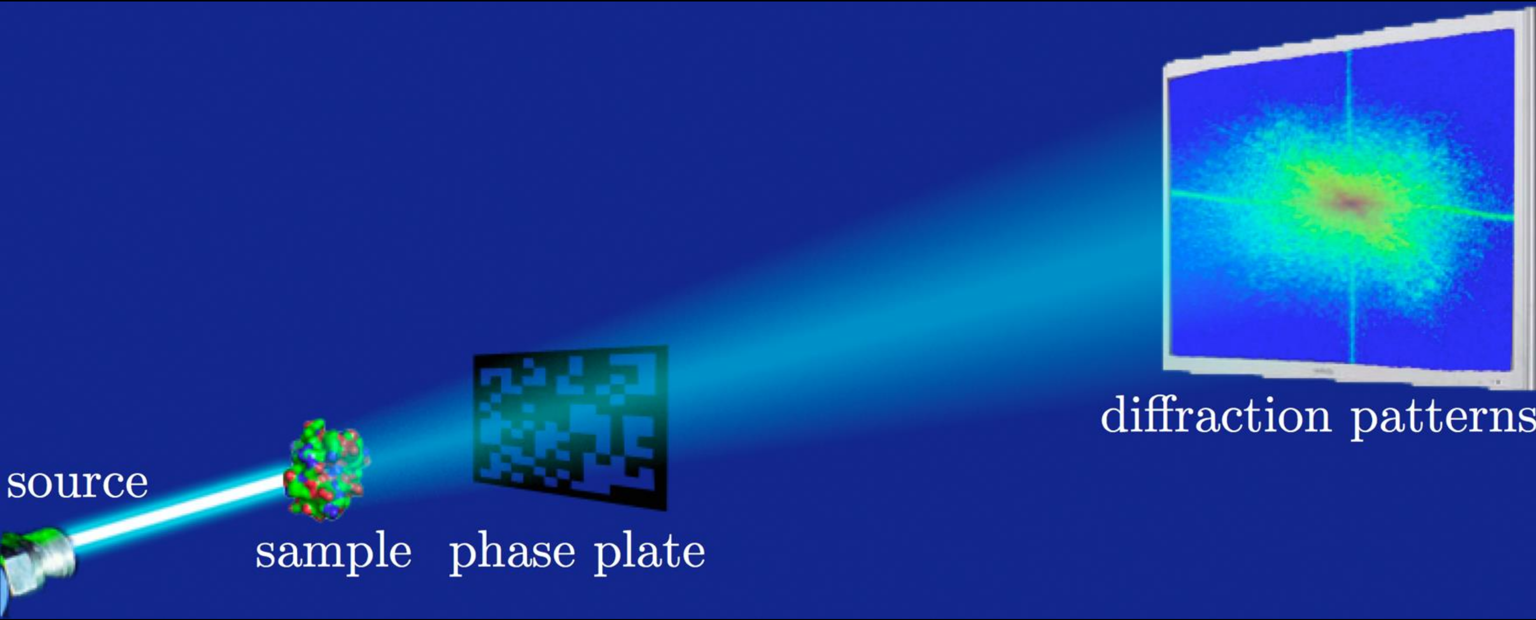
Brief Recap of (some) Wave Optics

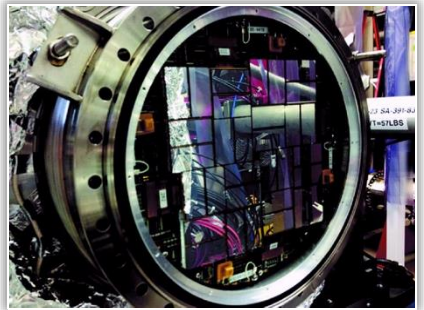
- Free-space wave propagation is often modeled by the Fourier transform of the field, i.e., for large distances or with lenses

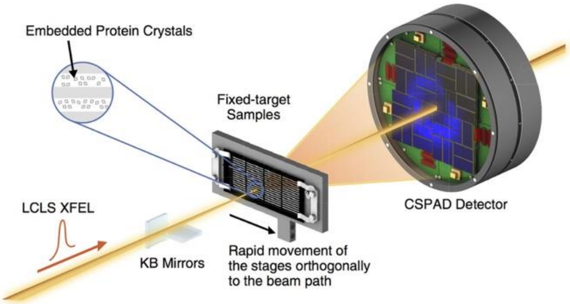
- Cannot measure complex field, only intensity (i.e., amplitude squared)

Phase Retrieval



Phase Retrieval @ SLAC x-ray crystallography or single-particle imaging (SPI)





Applications of Phase Retrieval

- Crystallography
- Transmission electron microscopy
- Astronomy
- Coherent diffractive imaging
- Fourier ptychography
- Lensless imaging
- Computer-generated holography
- …

Phase Retrieval

- Objective function: $$\text{minimize}_x \, J(x) = \frac{1}{2} \| |Fx| - b \|_2^2$$
- *F* is discrete Fourier transform matrix, *b* are amplitude-only measurements, *x* is complex-valued unknown vector

- Subgradient of objective: $$F^H \left((|Fx| - b) \circ \frac{Fx}{|Fx|} \right) \in \nabla J(x)$$
- Absolute value not differentiable, but can work with subgradients

Phase Retrieval

- (Sub)gradient descent:

$$
\begin{aligned}
x^{(k+1)} &= x^{(k)} - \alpha \nabla J(x) \\
&= x^{(k)} - \alpha F^H F x^{(k)} + \alpha F^H \left(b \circ \frac{F x^{(k)}}{\left| F x^{(k)} \right|} \right) \\
&\overset{\alpha=1}{\approx} F^H \left(b \circ \frac{F x^{(k)}}{\left| F x^{(k)} \right|} \right)
\end{aligned}
$$

- Interesting: $b \circ \dfrac{a e^{i\phi}}{|a e^{i\phi}|} = b e^{i\phi}$

Phase Retrieval

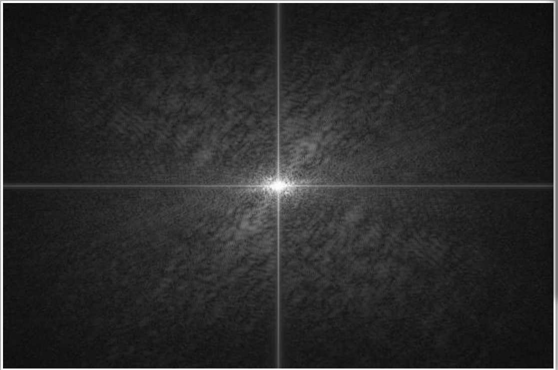
- Generalized Gerchberg-Saxton or Error Reduction (ER) algorithm [GS 1972, Fienup 1982]:

$$x^{(k+1)} = \Pi_{\mathcal{C}}\left(F^H\left(b \circ \frac{Fx^{(k)}}{\left|Fx^{(k)}\right|}\right)\right)$$

- Projection on feasible set $\Pi_{\mathcal{C}}$ enforces additional constraints, such as nonnegativity of x or limited support, via projected (sub)gradient descent
- Approach is 40 years old, but should be a great starting point

Phase Retrieval - Example


Ground Truth Image


Simulated measurements

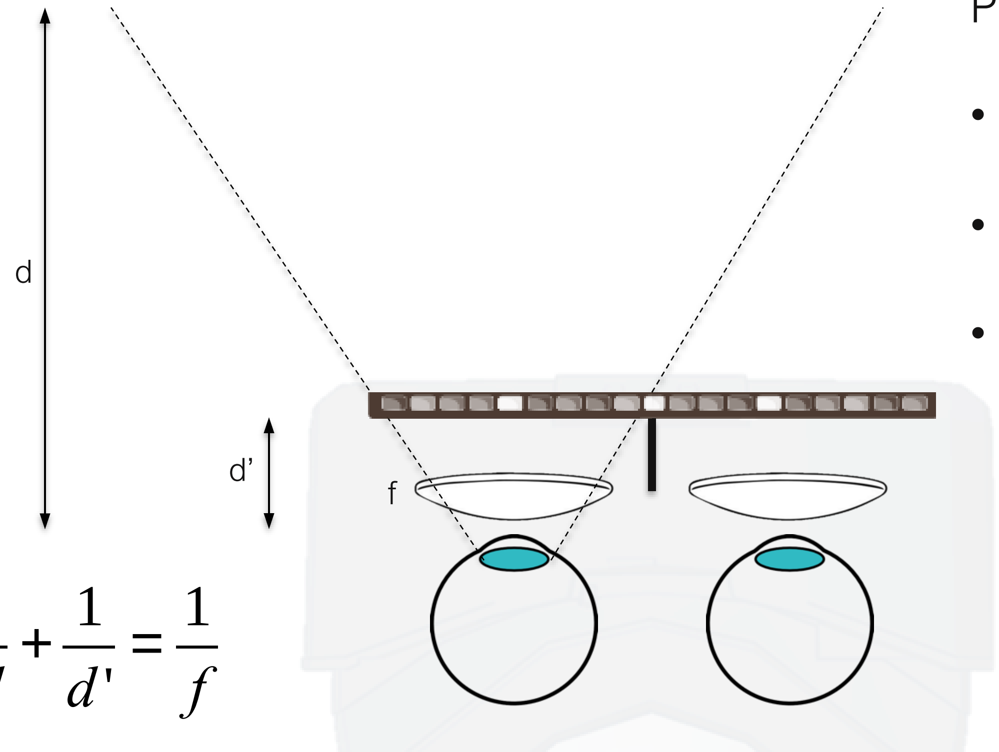

Reconstructed Image

Holographic Displays

https://www.youtube.com/watch?v=sQS6_DbYIAw

Sir Charles Wheatstone, 1838

Virtual Image



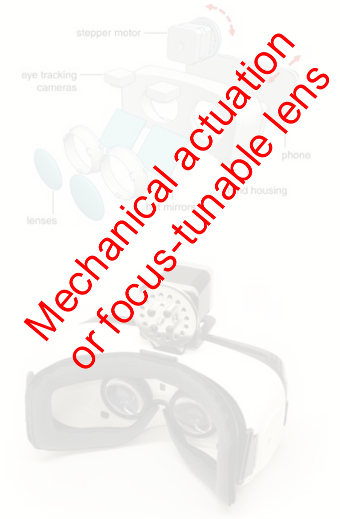
$$\frac{1}{d} + \frac{1}{d'} = \frac{1}{f}$$

Problems:

- fixed focal plane
- no focus cues ☹
- vergence-accommodation conflict (nausea)

Computational Near-eye Displays with Focus Cues

Gaze-contingent Varifocal Displays



Mechanical actuation or focus-tunable lens

Shiwa et al. 1996; Liu et al. 2008;
Konrad et al. 2016; Padmanaban et al. 2016, 2017; …

Multiplane Displays

High-speed SLMs or high system complexity

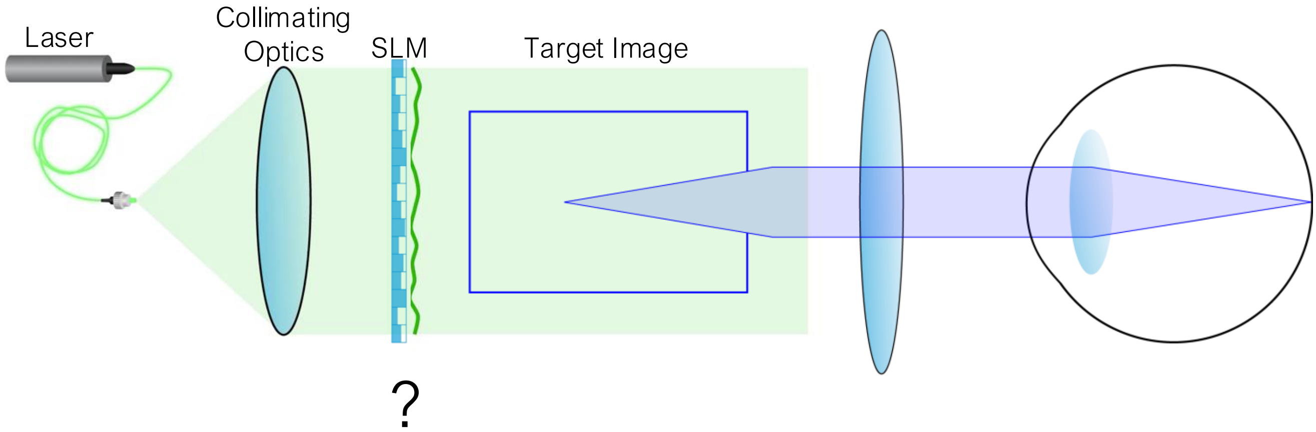
Rolland et al. 2000; Akeley et al. 2004
Liu et al. 2008; Love et al. 2009; …

Near-eye Light Field Displays

Resolution is limited by diffraction

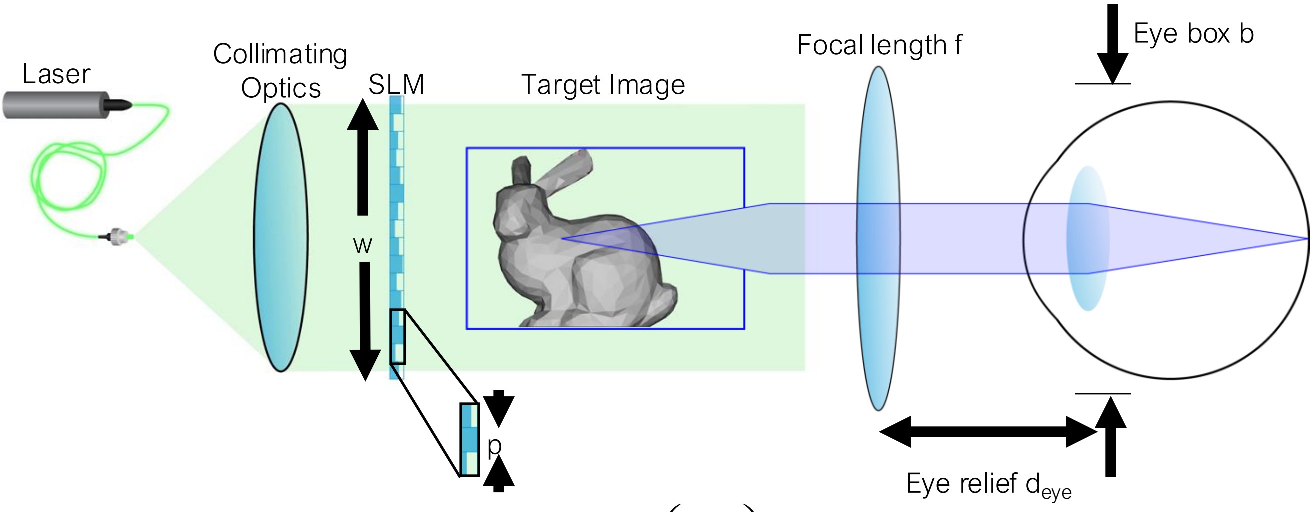
Lanman and Luebke 2013;
Hua and Javidi 2014; Huang et al. 2015

Holographic Near-eye Displays



[Maimone et al. 2017; Padmanaban et al. 2019; Peng et al. 2020; Shi et al. 2021…]

Holographic Near-eye Displays

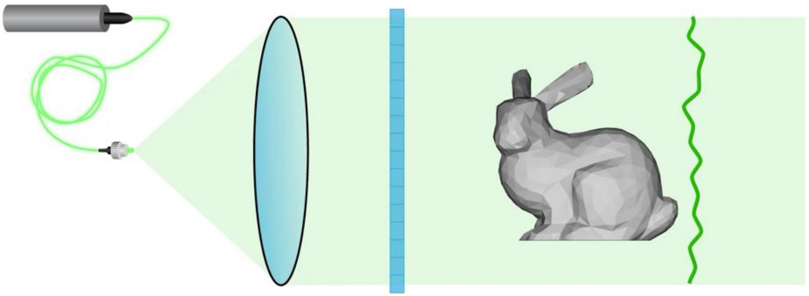


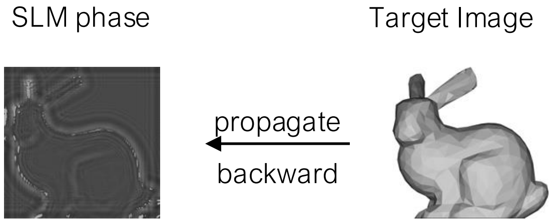
Field of view $FOV \approx 2\tan^{-1}\left(\frac{w}{2f}\right)$

Eye box $b \gg d_{eye}\frac{l}{p}$

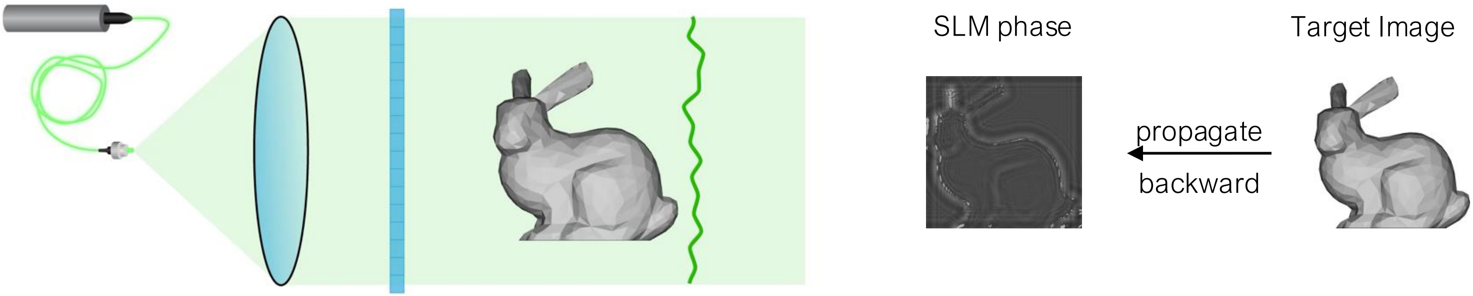
[Padmanaban et al. 2019]

Computer-generated Holography: Direct Methods



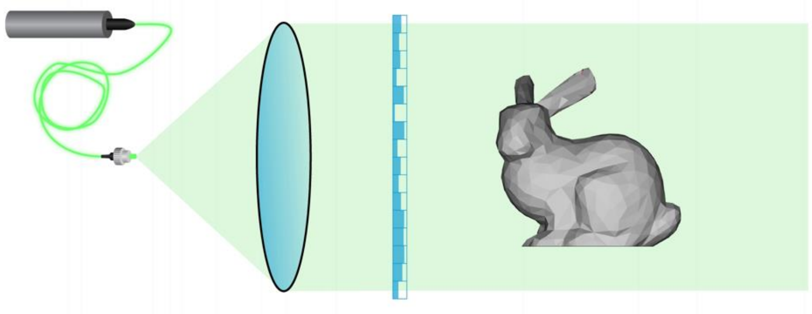


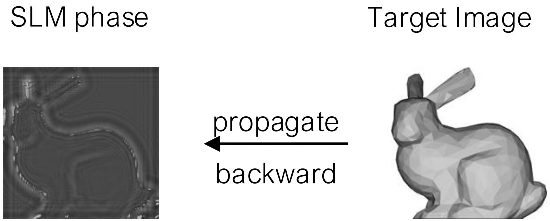
Computer-generated Holography: Direct Methods



1. Only target intensity is provided, need to "make up" some target phase (e.g., 0)
2. Free-space propagation from target to SLM plane
3. Propagated field at SLM plane is complex, but SLM can only address phase → need phase encoding

Computer-generated Holography: Direct Methods





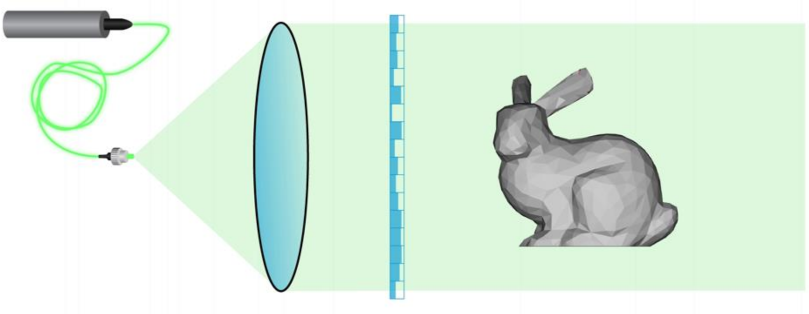
Free-space propagation:

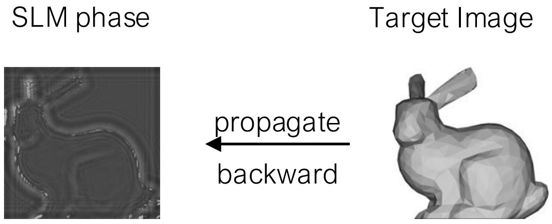
$$u_{slm}(x,y) = \mathcal{F}^{-1}\left\{\mathcal{F}\left\{a(x,y)\,e^{i\phi(x,y)}\right\}\mathcal{H}\left(f_x, f_y, z\right)\right\}$$

$$\mathcal{H}\left(f_x, f_y\right) = \begin{cases} e^{-i\frac{2\pi}{\lambda}\sqrt{1-(\lambda f_x)^2-(\lambda f_y)^2}\,z} & \text{if } \sqrt{f_x^2 + f_y^2} < \frac{1}{\lambda} \\ 0 & \text{otherwise} \end{cases}$$

[Goodman, Fourier Optics]

Computer-generated Holography: Direct Methods





Double phase-amplitude coding:

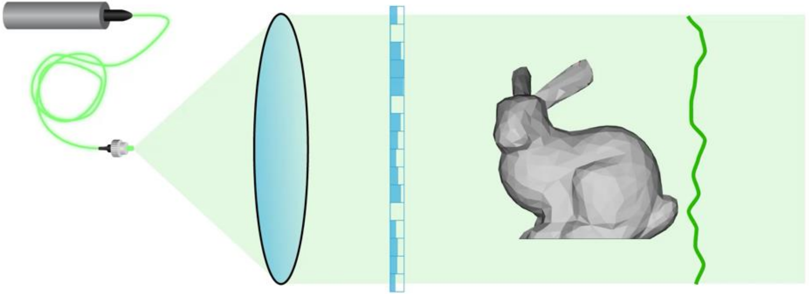
$$u_{slm}(x,y) = a(x,y)\, e^{i\phi(x,y)} = 0.5\left(e^{i\phi_1(x,y)} + e^{i\phi_2(x,y)}\right)$$

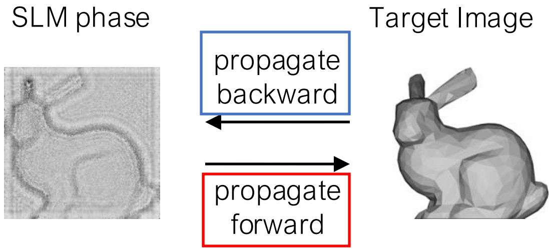
$$\phi_1(x,y) = \phi(x,y) - \cos^{-1}(a(x,y))$$

$$\phi_2(x,y) = \phi(x,y) + \cos^{-1}(a(x,y))$$

[Hsueh and Sawchuk 1978; Maimone et al. 2017]

Computer-generated Holography: Iterative Methods





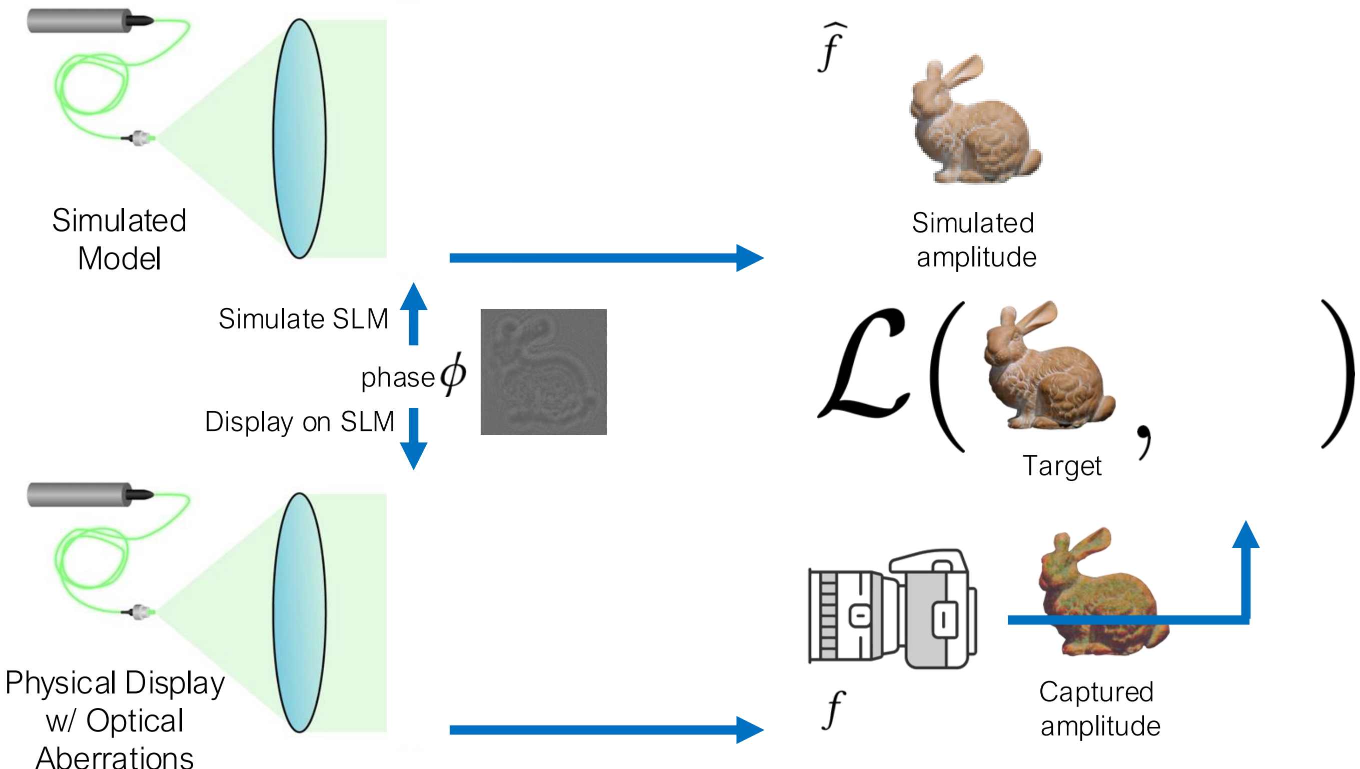
$$\underset{\phi}{\text{minimize}}\ \mathcal{L}\left(|\widehat{f}\,(\phi)\,|, a_{\text{target}}\right)$$

$\widehat{f}$ Free-space propagation model

f Unknown physical propagation, $\widehat{f} \neq f$

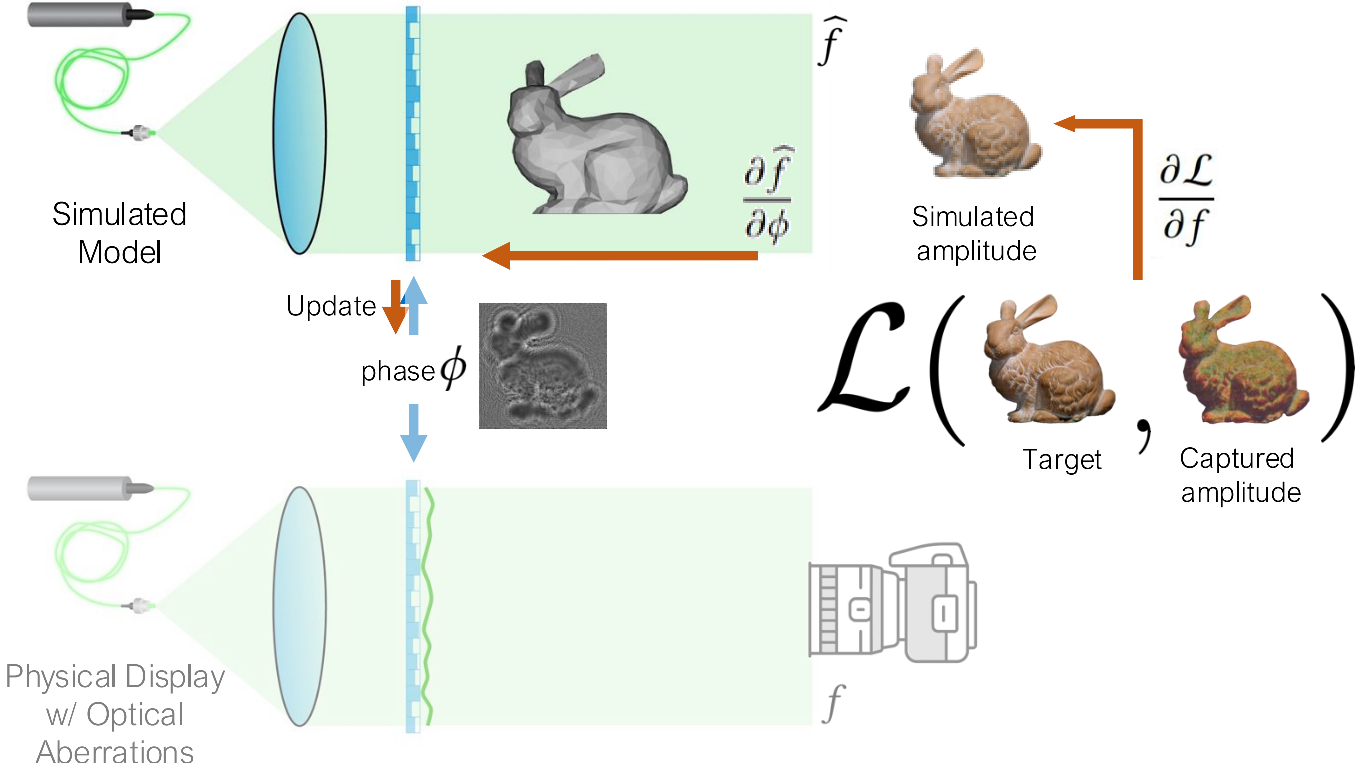
Iterations: $$\phi^{(k)} \leftarrow \phi^{(k-1)} - \alpha \boxed{\left(\frac{\partial \mathcal{L}}{\partial \phi}\right)^T} \mathcal{L}\left(\boxed{\left|\widehat{f}\left(\phi^{(k-1)}\right)\right|}, a_{\text{target}}\right)$$

Camera-in-the-loop (CITL) Optimization



[Peng et al., SIGGRAPH & SIGGRAPH Asia 2020]

Camera-in-the-loop (CITL) Optimization



[Peng et al., SIGGRAPH & SIGGRAPH Asia 2020]

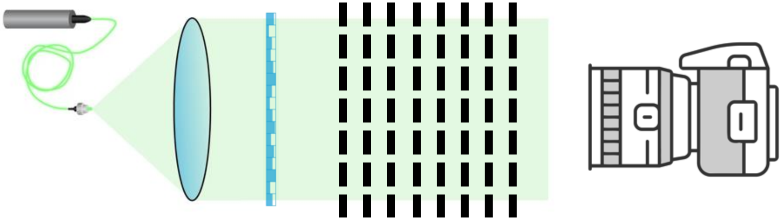
Camera-in-the-loop (CITL) Hologram Optimization

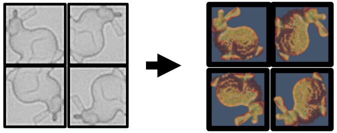
SGD with the ASM Model | Proposed CITL CGH

[Peng et al., SIGGRAPH & SIGGRAPH Asia 2020]

Neural Holography

Physical Optics

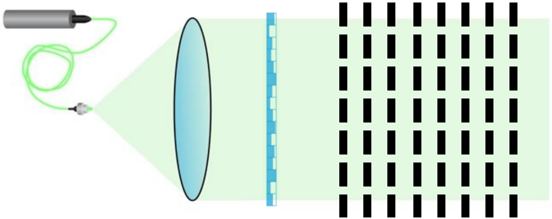


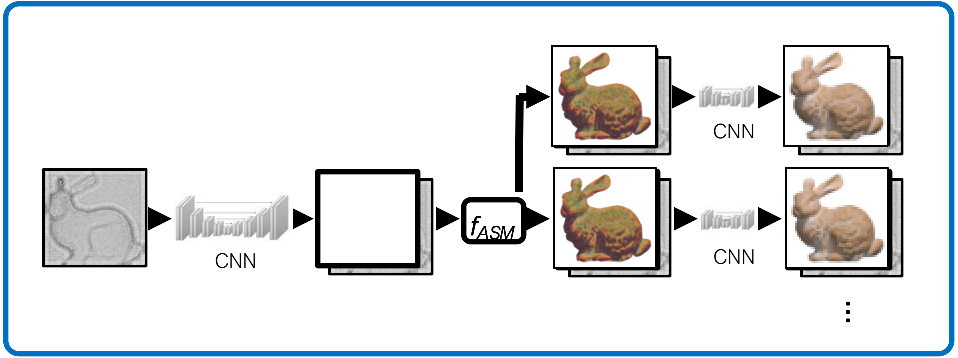


[Peng et al., SIGGRAPH Asia 2020; Choi et al., SIGGRAPH ASIA 2021; SIGGRAPH 2022; ...]

Neural Holography

Physical Optics

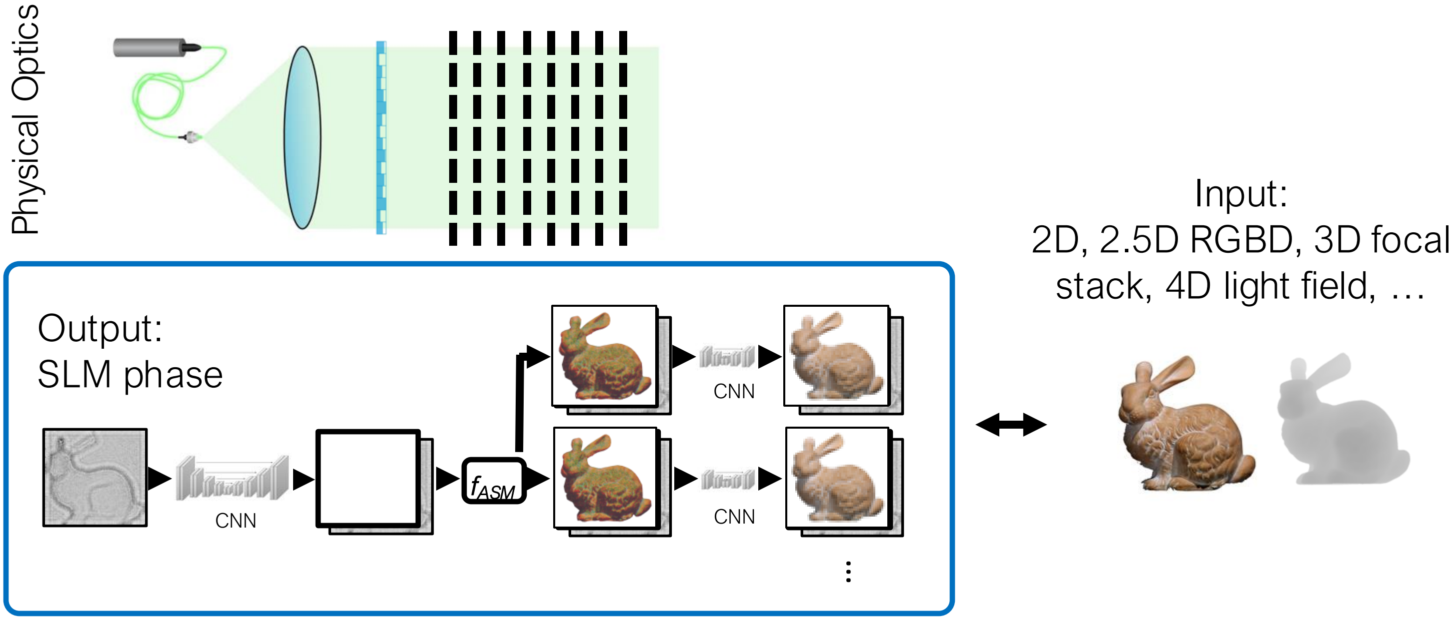




Camera-calibrated Wave Propagation Model

[Peng et al., SIGGRAPH Asia 2020; Choi et al., SIGGRAPH ASIA 2021; SIGGRAPH 2022; ...]

Neural Holography



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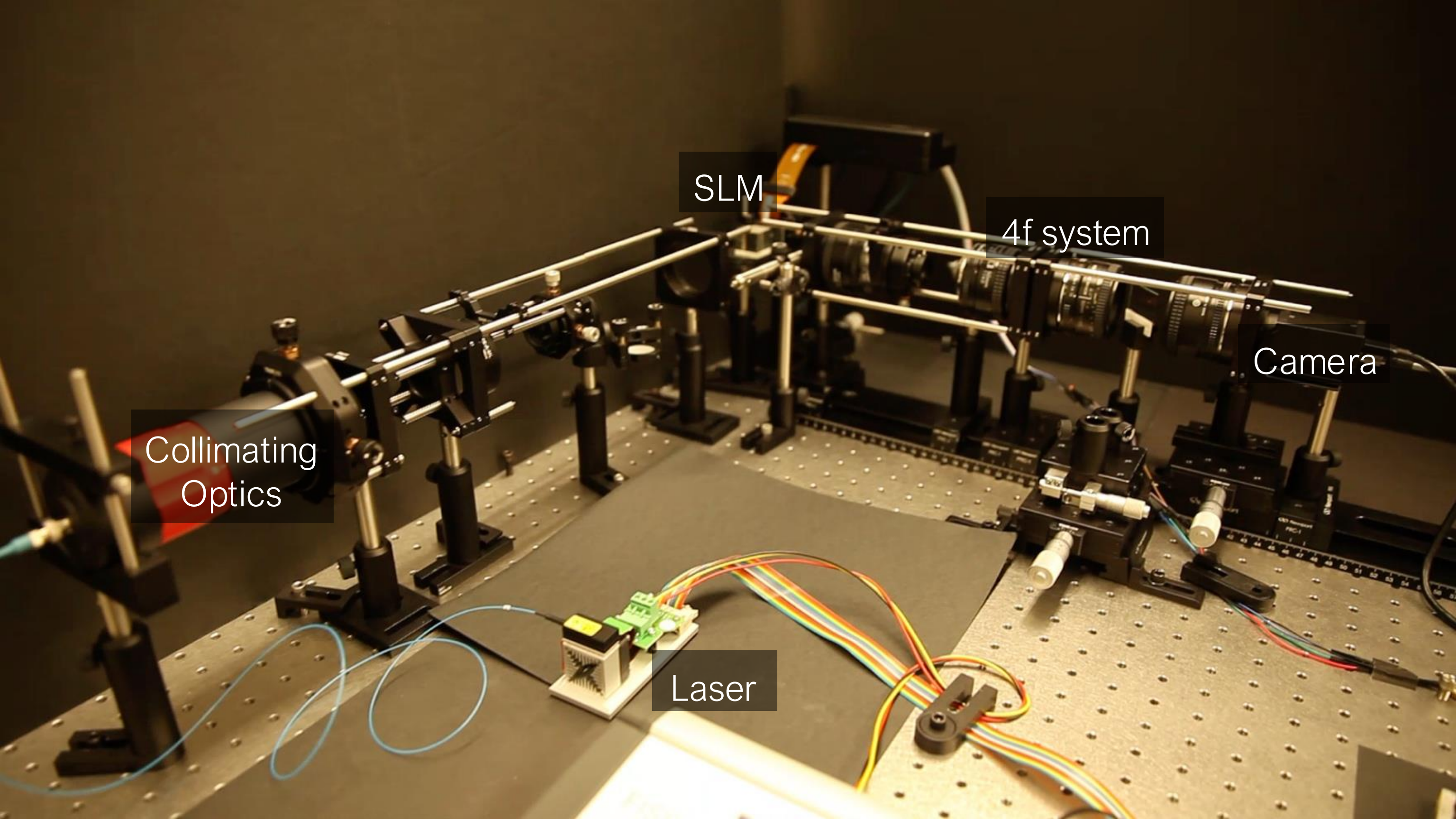
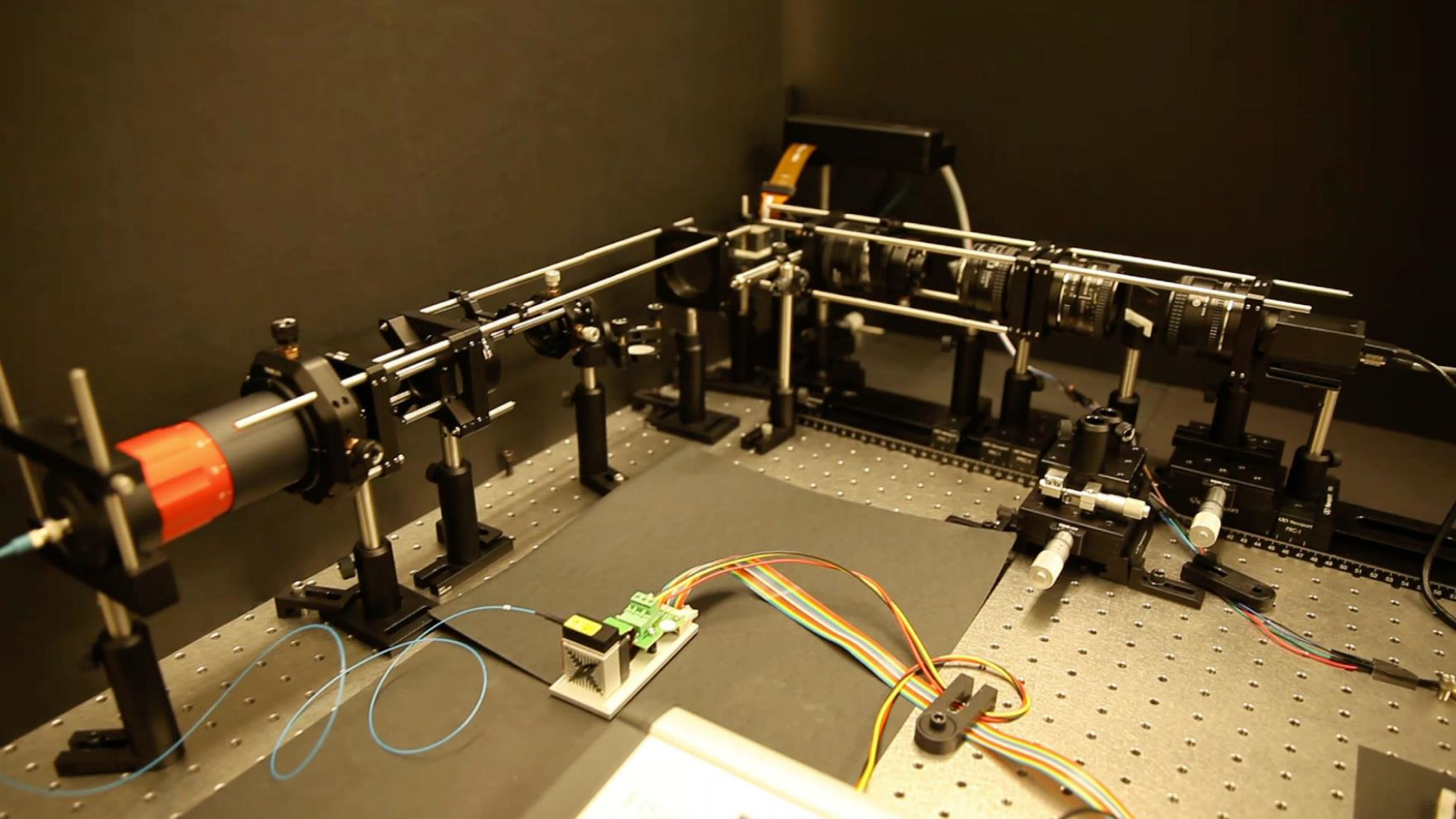
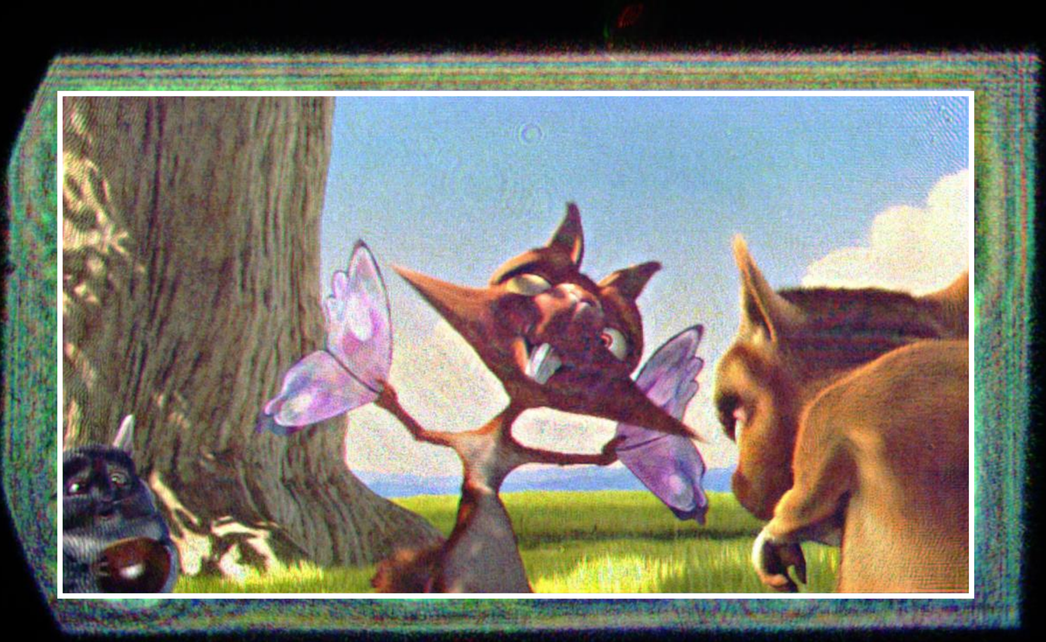

SLM
4f system
Camera
Collimating Optics
Laser



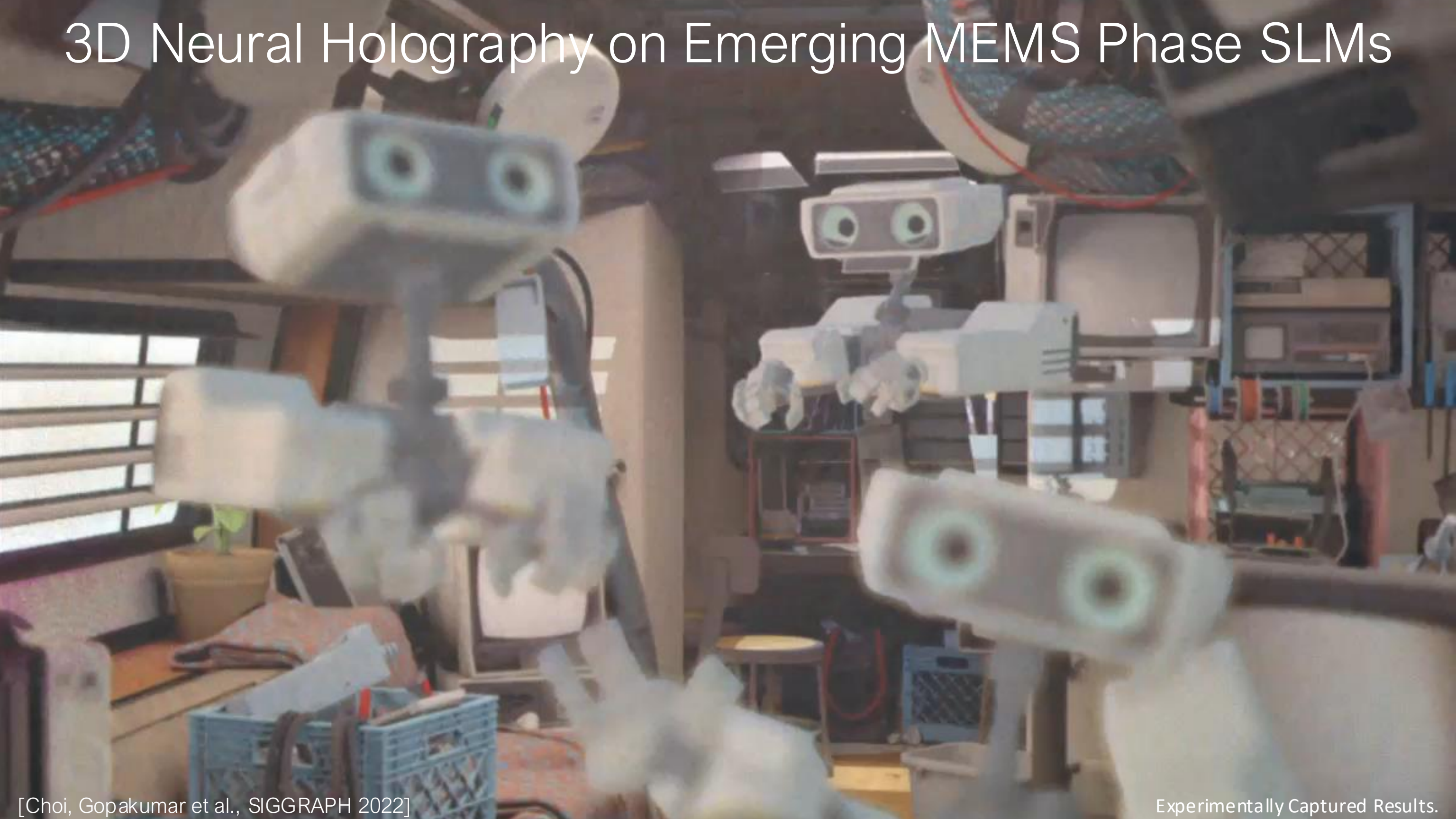
Gerchberg–Saxton

Neural Holography (CITL) 2020 Results



[Peng et al., SIGGRAPH & SIGGRAPH Asia 2020]

3D Neural Holography on Emerging MEMS Phase SLMs



[Choi, Gopakumar et al., SIGGRAPH 2022]

Experimentally Captured Results.

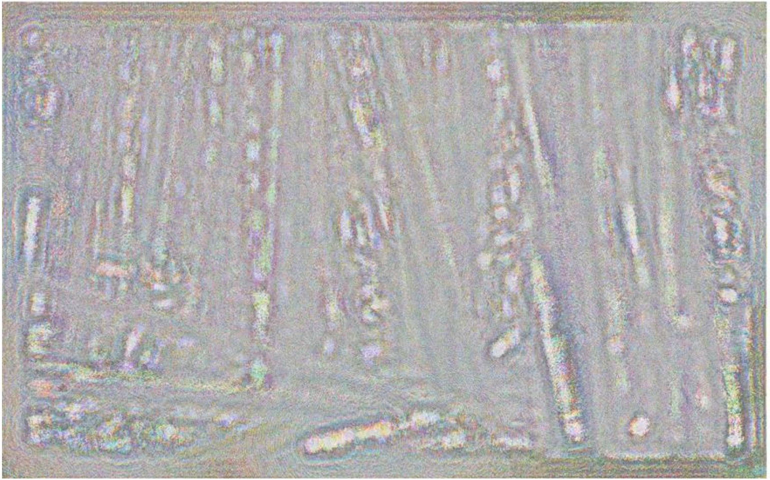
3D Neural Holography on Emerging MEMS Phase SLMs

[Choi, Gopakumar et al., SIGGRAPH 2022]

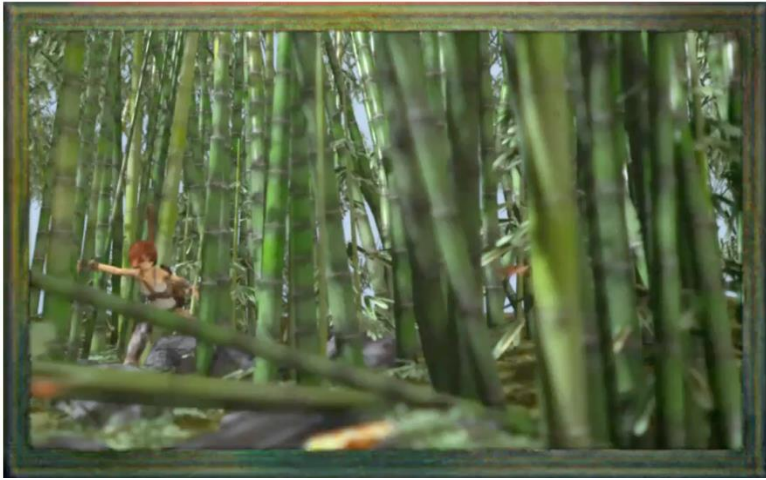
Experimentally Captured Results.

3D Neural Holography on Emerging MEMS Phase SLMs

Displayed patterns on phase SLM



Holograms captured with our prototype

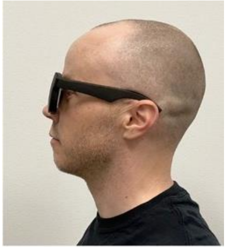


[Choi, Gopakumar et al., SIGGRAPH 2022]

Experimentally Captured Results.

Additional Benefits of Holographic Near-eye Displays

Thin VR Display Form Factors



Maimone et al., SIGGRAPH 2020

Kim et al., SIGGRAPH 2022

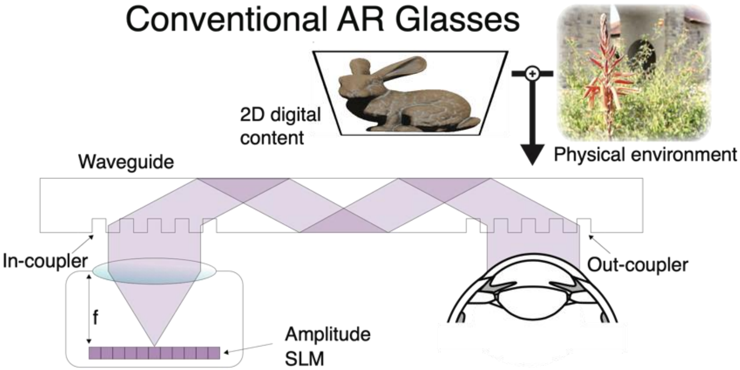
Other:

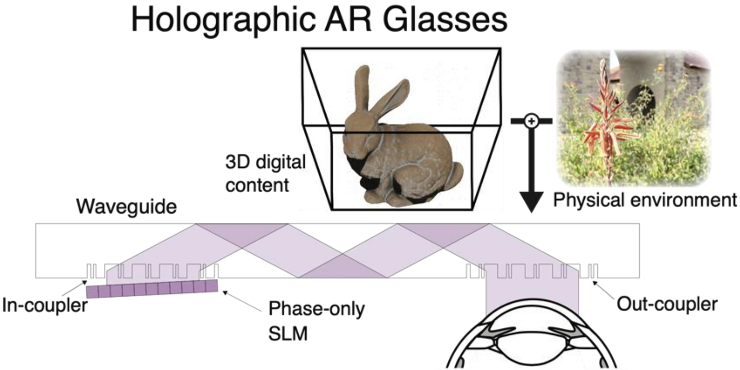
- Light-efficient AR Displays
- Prescription correction (including astigmatism and higher-orders)
- Correcting optical aberrations

…

Gopakumar, Lee, …, Wetzstein, “Full-colour 3D holographic augmented-reality displays with metasurface waveguides”, Nature 2024
Photo by Andrew Brodhead

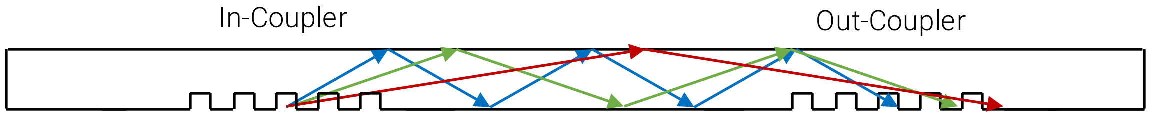
Pairing Holography with Waveguide AR





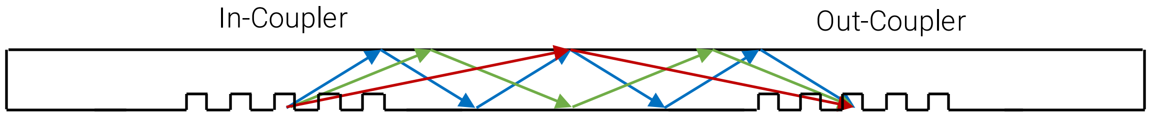
Gopakumar, Lee et al., Nature 2024

Waveguide Geometry for 3D Holograms



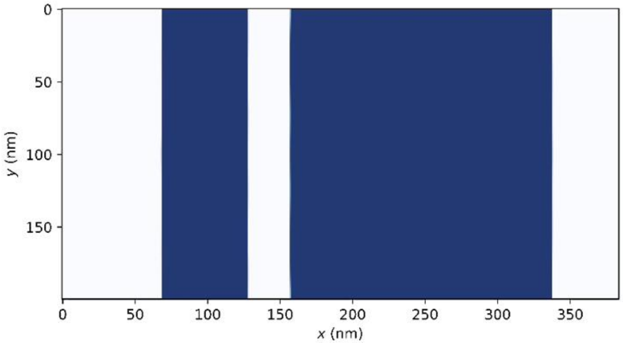
Gopakumar, Lee et al., Nature 2024

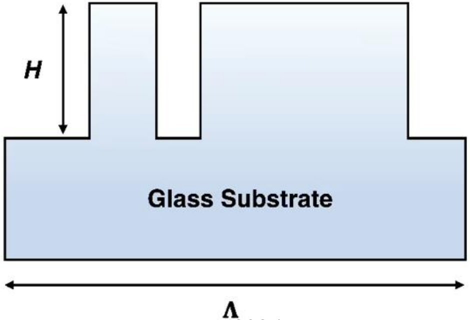
Waveguide Geometry for 3D Holograms

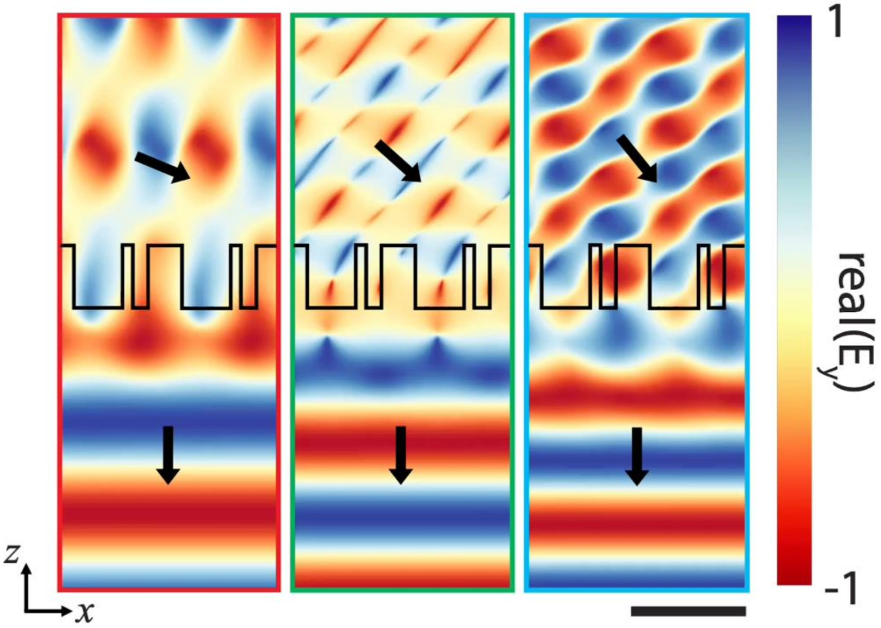


Gopakumar, Lee et al., Nature 2024

Inverse-designed Metasurface Waveguide

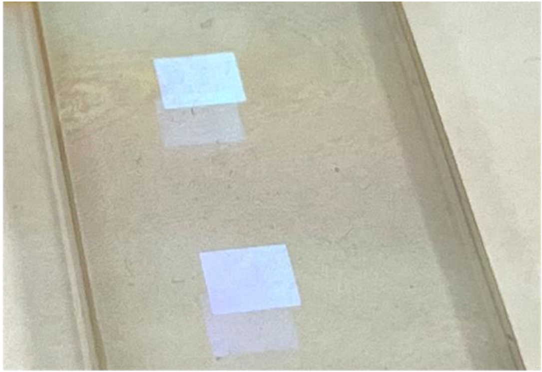


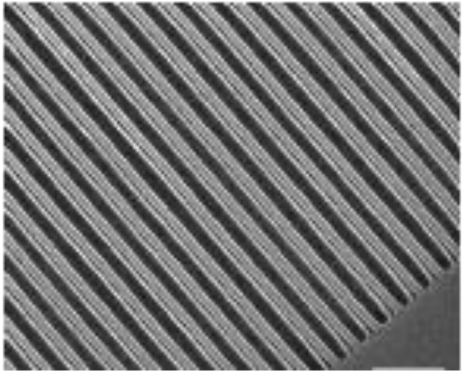


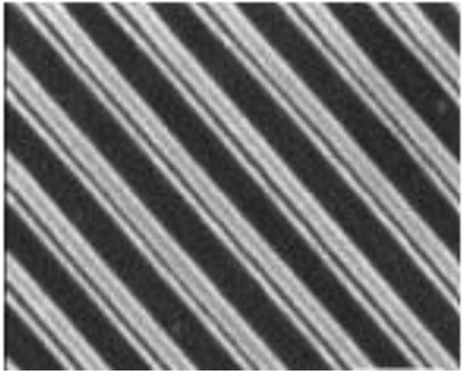


Gopakumar, Lee et al., Nature 2024

Fabricated Metasurface Waveguide

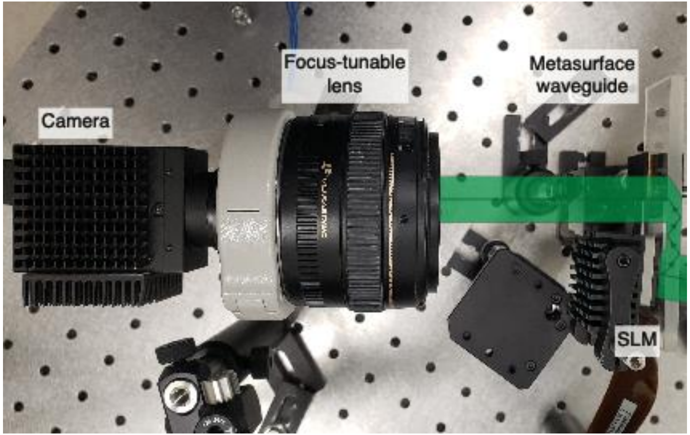






Gopakumar, Lee et al., Nature 2024

Metasurface Waveguide Holography Setup



Gopakumar, Lee et al., Nature 2024

Experimental Results

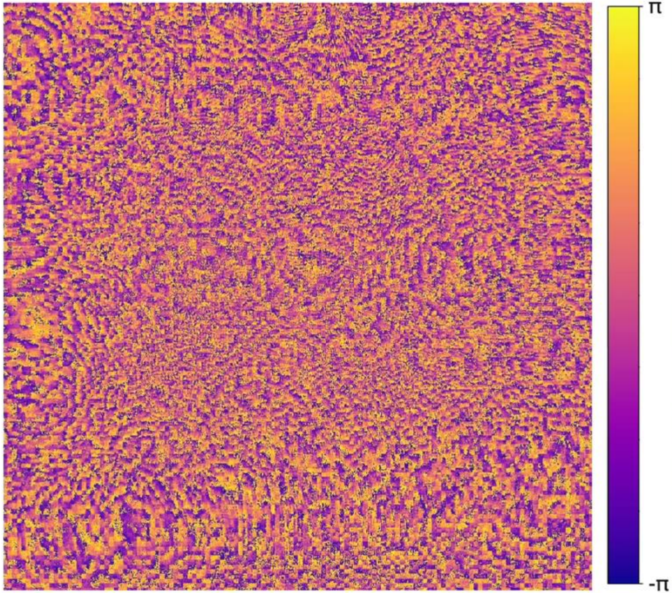
Conventional Wave Propagation Model

Our Learned Physical Waveguide Model

Gopakumar, Lee et al., Nature 2024

Experimental 3D Results

SLM Phase Modulation Pattern



View of 3D Hologram

Gopakumar, Lee et al., Nature 2024

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Computational Imaging Lab
Stanford University EE & CS

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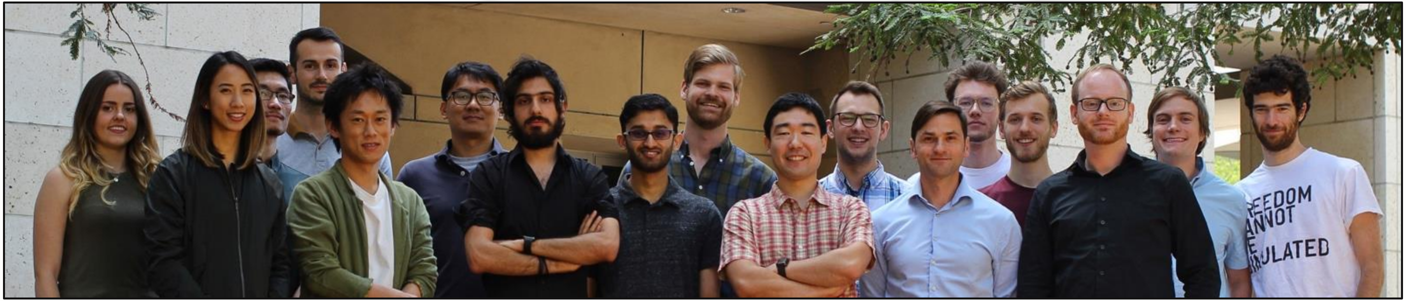

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References and Further Reading

Phase Retrieval

- R. Gerchberg, W. Saxton, "A Practical Algorithm for the Determination of Phase from Image and Diffraction Plane Pictures", Optik 1972
- J. Fienup, "Phase retrieval algorithms: a comparison", Applied Optics 1982
- …

Holographic Near-eye Displays

- A. Maimone, A. Georgiou, J.S. Kollin, "Holographic near-eye displays for virtual and augmented reality", ACM SIGGRAPH 2017
- N. Padmanaban, Y. Peng, G. Wetzstein, Holographic near-eye displays based on overlap-add stereograms", ACM SIGGRAPH Asia 2019
- Y. Peng, S. Choi, N. Padmanaban, G. Wetzstein, "Neural Holography with Camera-in-the-loop Training", ACM SIGGRAPH Asia 2020
- S. Choi, J. Kim, Y. Peng, G. Wetzstein, "Optimizing image quality for holographic near-eye displays with Michelson Holography", OSA Optica 2021
- S. Choi, M. Gopakumar, Y. Peng, J. Kim, G. Wetzstein, "Neural 3D Holography: Learning Accurate Wave Propagation Models for 3D Holographic Virtual and Augmented Reality Displays"
- Y. Peng, S. Choi, J. Kim, G. Wetzstein, "Speckle-free Holography with Partially Coherent Light Sources and Camera-in-the-loop Calibration", Science Advances 2021
- L. Shi, B. Li, C. Kim, P. Kellnhofer, W. Matusik, "Towards real-time photorealistic 3D holography with deep neural networks", Nature 2021

Computational Near-eye Displays with Focus Cues

- See review paper/talk for overview: G. Wetzstein, "Computational Eyeglasses and Near-Eye Displays with Focus Cues", SPIE AR/VR/MR Conference 2020