Introduction to Wave Optics and Deep Optics

EE367/CS448I: Computational Imaging

stanford.edu/class/ee367

Lecture 14

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Overview & Motivation

Wave optics in a nutshell: coherence, Hyugens-Fresnel principle, wave propagation, wave field vs observable intensity, diffraction-limited resolution, …

Goals:

- Intuitive introduction of fundamentals of wave optics without all the math (not enough time, take an optics class for that)
- Overview of modern approaches combining wave optics with artificial intelligence techniques for various applications

Coherence in a Nutshell

- Incoherent light: emission of light over “large” (i.e., compared to wavelength) area & broad range of wavelengths

- Partially coherent light (one of the following):
  - point or plane wave = spatially coherent
  - monochromatic = temporally coherent

- Coherent light: spatially & temporally coherent

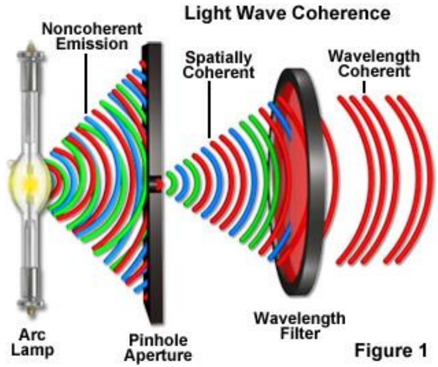
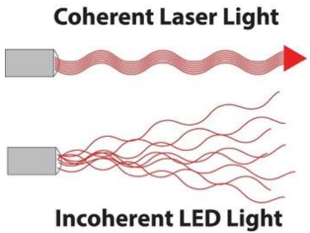


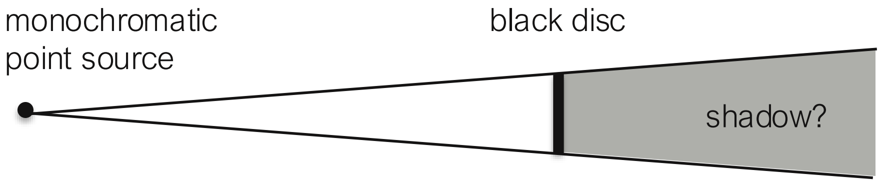
Image courtesy: Zeiss



https://www.informationpalace.com/light-amplification-by-stimulated-emission-of-radiation/

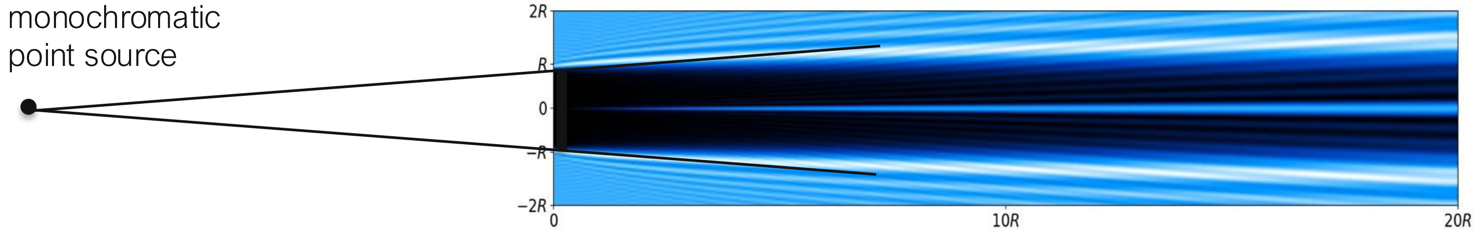
Poisson's Spot

- Common sense says there's a shadow behind an occluder, like a disc
- Wave theory predicts bright spot



Poisson's Spot

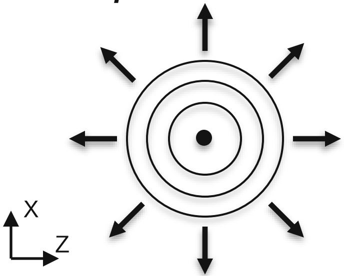
- Common sense says there's a shadow behind an occluder, like a disc
- Wave theory predicts bright spot
- Fresnel predicted, Poisson doubted, and Arago demonstrated it in 1818



Point Sources and Plane Waves

- Point source at $\boldsymbol{x}_0$ and amplitude u_0
- $r = \|\boldsymbol{x} - \boldsymbol{x}_0\|_2, k = \frac{2\pi}{\lambda}$

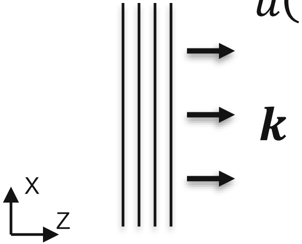
$$u(\boldsymbol{x}) = u_0 \frac{e^{ikr}}{r}$$



point source

- Propagates with velocity c into direction $\boldsymbol{k} = (k_x, k_y, k_z)$
- "what comes out of a laser" or collimated point source

$$u(\boldsymbol{x}) = u_0 e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

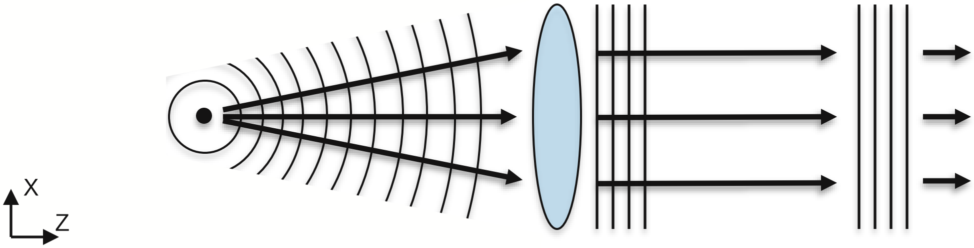


plane wave

Point Sources and Plane Waves

- Point source at $\boldsymbol{x}_0$ and amplitude u_0
- $r = \|\boldsymbol{x} - \boldsymbol{x}_0\|_2, k = \frac{2\pi}{\lambda}$

- Propagates with velocity *c* into direction $\boldsymbol{k} = (k_x, k_y, k_z)$
- "what comes out of a laser" or **collimated point source**



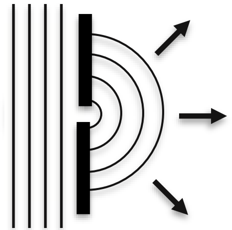
point source

plane wave

Hyugens-Fresnel Principle

Every point on a wavefront is itself the source of spherical wavelets, and the secondary wavelets emanating from different points mutually interfere. The sum of these spherical wavelets forms the wavefront.

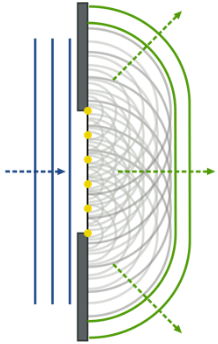
Examples:



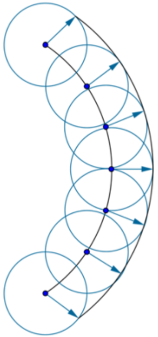
plane wave through slit → spherical wave

plane wave through multiple slits → interfering sph. waves

sph. waves make plane wave



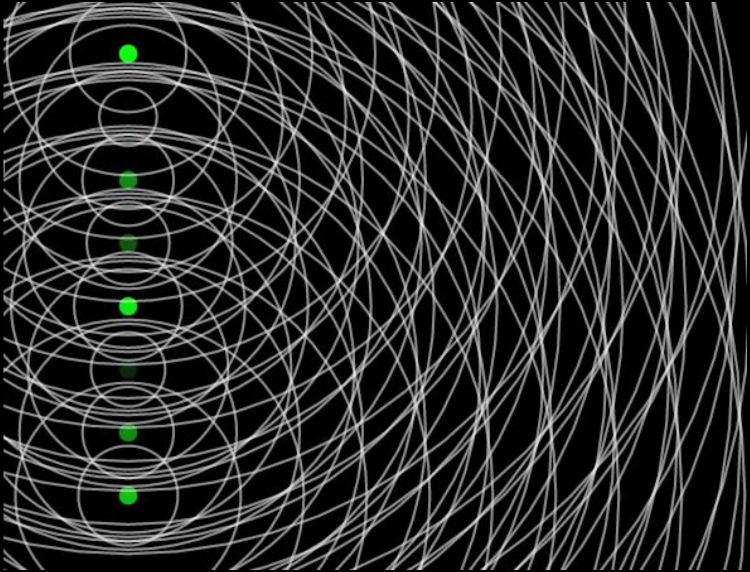
plane wave through big slit



curved wave

Hyugens-Fresnel Principle

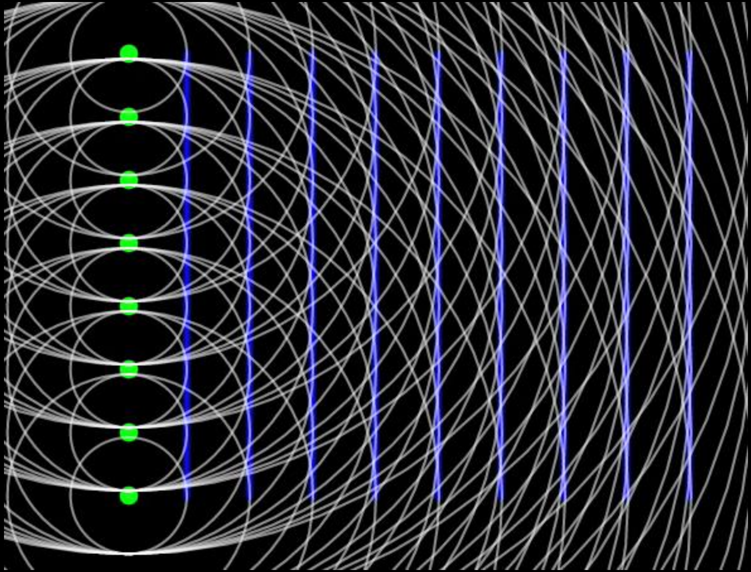
Direction: random



images/videos from [Zhang & Levoy, 2009]

Hyugens-Fresnel Principle

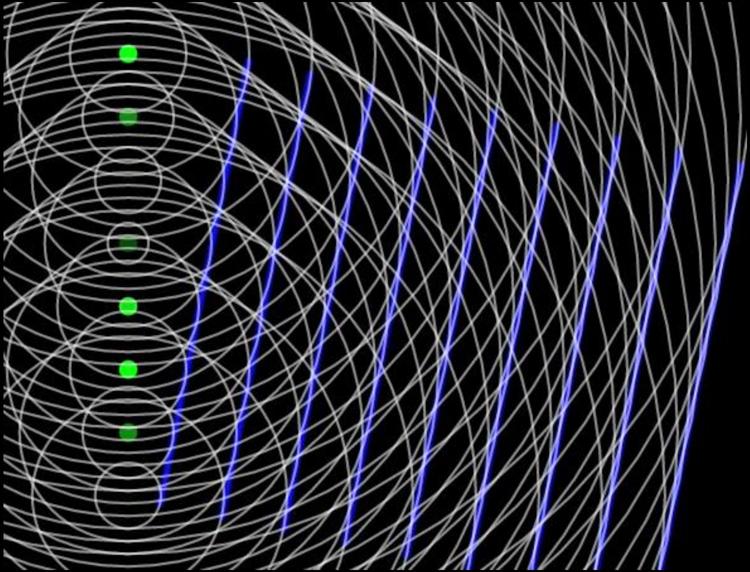
Direction: axial



images/videos from [Zhang & Levoy, 2009]

Hyugens-Fresnel Principle

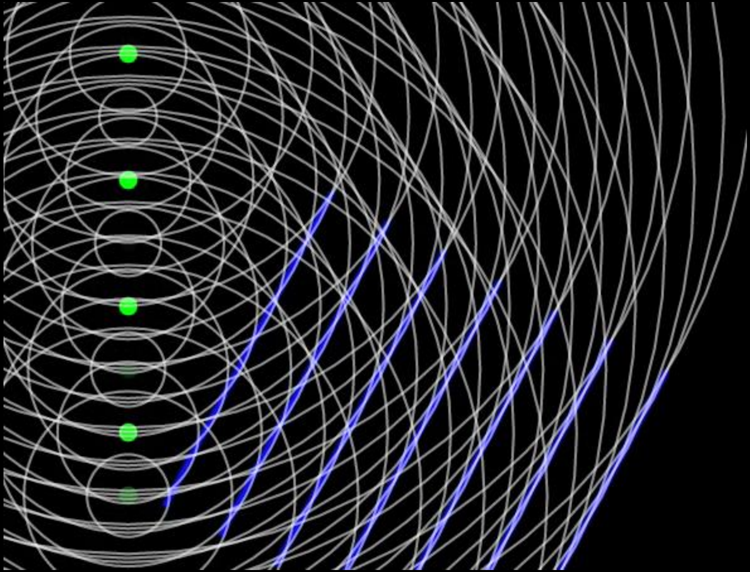
Direction: oblique



images/videos from [Zhang & Levoy, 2009]

Hyugens-Fresnel Principle

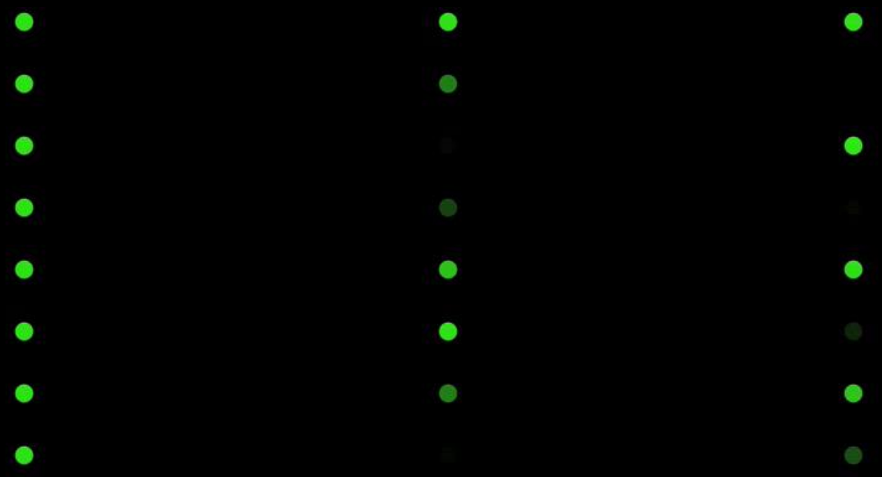
Direction: more oblique



images/videos from [Zhang & Levoy, 2009]

Spatial Frequency

axial | oblique | more oblique



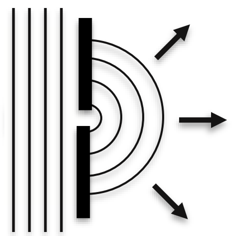
zero spatial frequency | low spatial frequency | higher spatial frequency

images/videos from [Zhang & Levoy, 2009]

Interaction of Wave Field and Thin Object

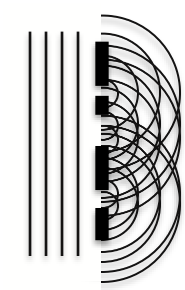
- Field before/after object $u_{in/out}(x)$ are defined by amplitude $a(x)$ and phase $\phi(x)$ of mask (or object) as

$$u_{out}(x) = u_{mask}(x)\, u_{in}(x) = a(x) e^{i\phi(x)}\, u_{in}(x)$$



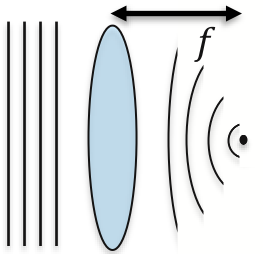
slit

$a(x) = \boldsymbol{\delta}(x - x_0)$
$\phi(x) = 1$



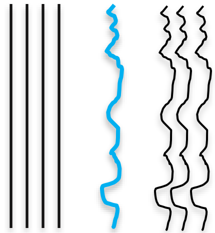
amplitude mask

$a(x) = rand$
$\phi(x) = 1$



lens

$a(x) = 1$
$\phi(x) = -{}^{k}/_{2f}\, x^2$

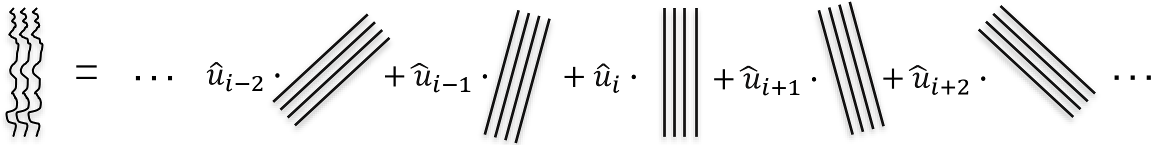


phase mask

$a(x) = 1,$
$\phi(x) = rand$

Plane Wave Decomposition = Fourier Transform

- Every wavefield can be represented as a weighted sum of plane waves

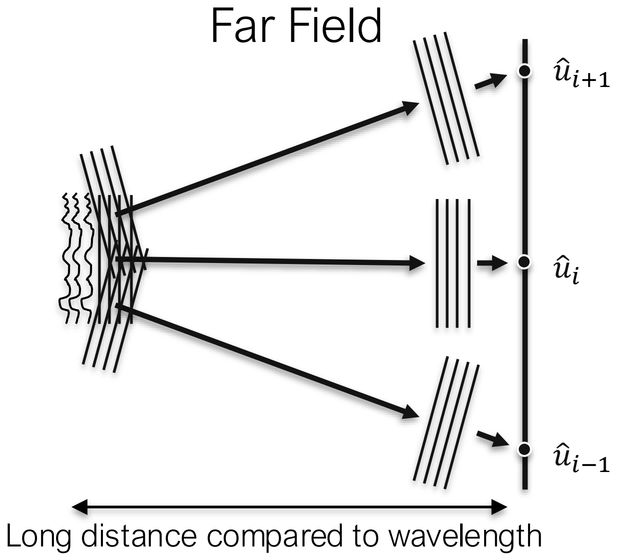


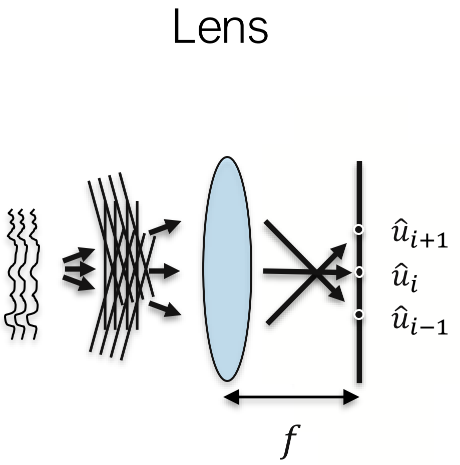
- Decomposition is the Fourier transform: $u(x) = \int \hat{u}(k_x)\, e^{2\pi i x k_x}\, dk_x$

1D plane wave with slope

Optical Fourier Transform via Wave Propagation

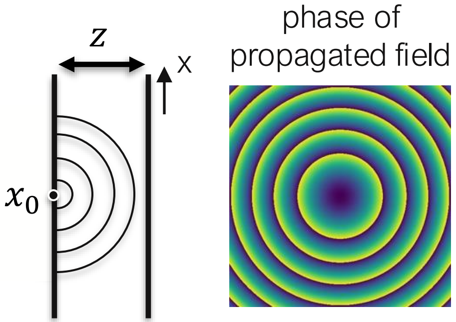
Propagating plane waves in free space is intuitive → long distances in free space or lenses perform optical Fourier transform





Wave Propagation in Free Space

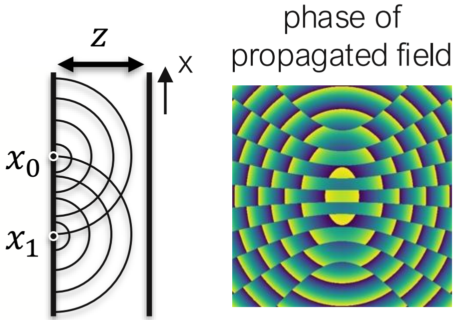
single point source



$$\mathrm{d} = \sqrt{(\boldsymbol{x} - \boldsymbol{x}_0)^2 + z^2}$$

$$u(\boldsymbol{x}) = u_0 \frac{e^{i\frac{2\pi}{\lambda}d}}{d}$$

two point sources



$$d_{0/1} = \sqrt{\left(\boldsymbol{x} - \boldsymbol{x}_{0/1}\right)^2 + z^2}$$

$$u(\boldsymbol{x}) = u_0 \frac{e^{i\frac{2\pi}{\lambda}d_0}}{d_0} + u_1 \frac{e^{i\frac{2\pi}{\lambda}d_1}}{d_1}$$

Wave Propagation in Free Space

In general, free-space propagation by distance *z* is modeled by convolution with a complex-valued propagation kernel or, similarly, a multiplication with a transfer function $\mathcal{H}$ in the Fourier domain.

$$u_{prop}(x,y) = \Im^{-1}\left\{\Im\{u(x',y')\} \cdot \mathcal{H}\left(k_x, k_y, z\right)\right\}$$

$$\mathcal{H}\left(k_x, k_y, z\right) = \begin{cases} e^{-i\frac{2\pi}{\lambda}\sqrt{1-(\lambda k_x)^2-\left(\lambda k_y\right)^2}\,z} & if\ \sqrt{k_x^2+k_y^2} < \frac{1}{\lambda} \\ 0 & otherwise \end{cases}$$

Different propagation operators have different transfer functions, this one is called Angular Spectrum Method (ASM).

[Goodman 2012]

Complex Field vs. Observable Intensity

We cannot directly observe the field, only its squared amplitude or intensity

observable intensity

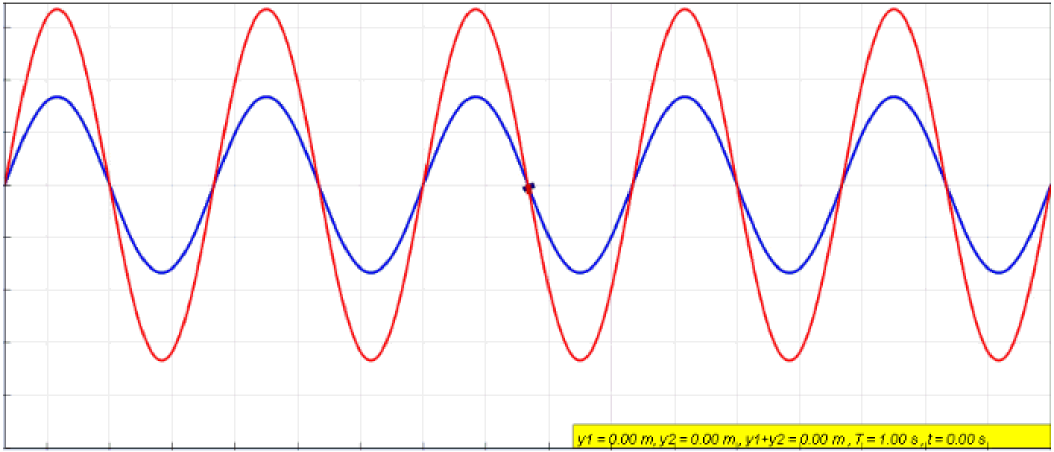
amplitude squared,
no phase information!

$$\mathrm{I}(x,y) = |u(x,y)|^2 = \left|a(x,y)e^{i\phi(x,y)}\right|^2 = a^2(x,y)$$

complex-valued field

Constructive & Destructive Interference

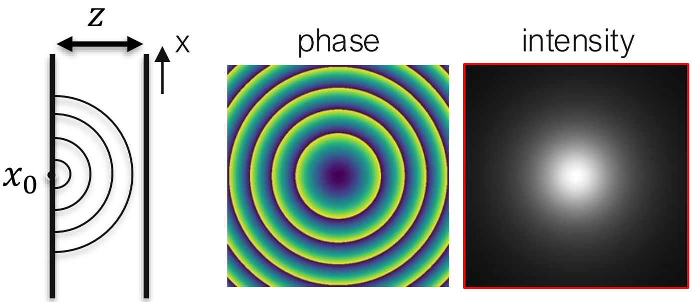
https://en.wikipedia.org/wiki/Wave_interference



$$\left|a_1 e^{i\phi_1} + a_2 e^{i\phi_2}\right|^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos(\phi_1 - \phi_2)$$

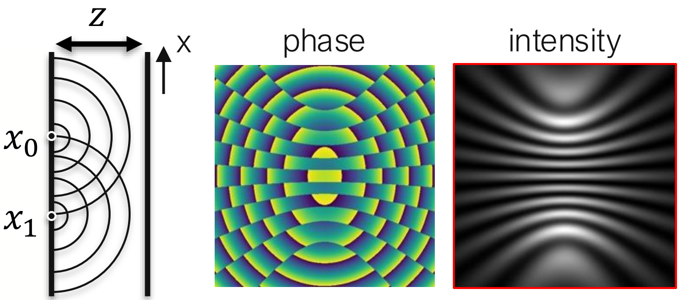
Constructive & Destructive Interference

single point source



no interference

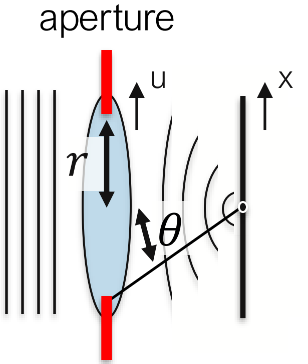
two point sources



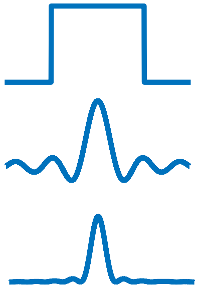
interference

$$i(\boldsymbol{x}) = |u(\boldsymbol{x})|^2$$

Diffraction-limited Resolution

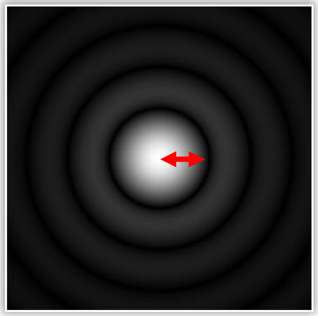


1D:
- Aperture is $rect(ru)$
- Wave at sensor is $sinc(krx)$
- Intensity is $|sinc(krx)|^2, k = \frac{2\pi}{\lambda}$



2D:
- Aperture is $circ(r)$
- Wave at sensor is
$$jinc(kr\sin\theta) = \frac{J_1(kr\sin\theta)}{kr\sin\theta}$$
- Intensity is $|jinc(kr\sin\theta)|^2$ $\longrightarrow$

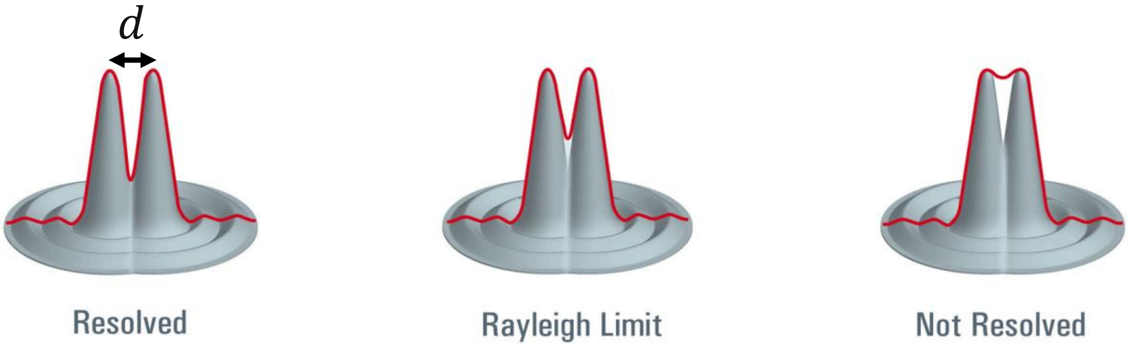
Airy disk



First minimum at radius
$$1.22\frac{\lambda}{2n\sin\theta}$$

$n\sin\theta$ related to f-number or NA of lens

Diffraction-limited Resolution

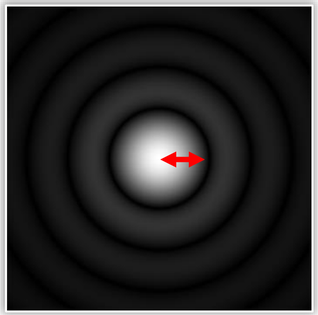


Can resolve 2 points if distance d is at least:

- Rayleigh Limit $1.22\,\dfrac{\lambda}{2n\sin\theta}$
- Abbe Limit $\dfrac{\lambda}{2n\sin\theta}$

n is refractive index of medium

Airy disk



First minimum at radius

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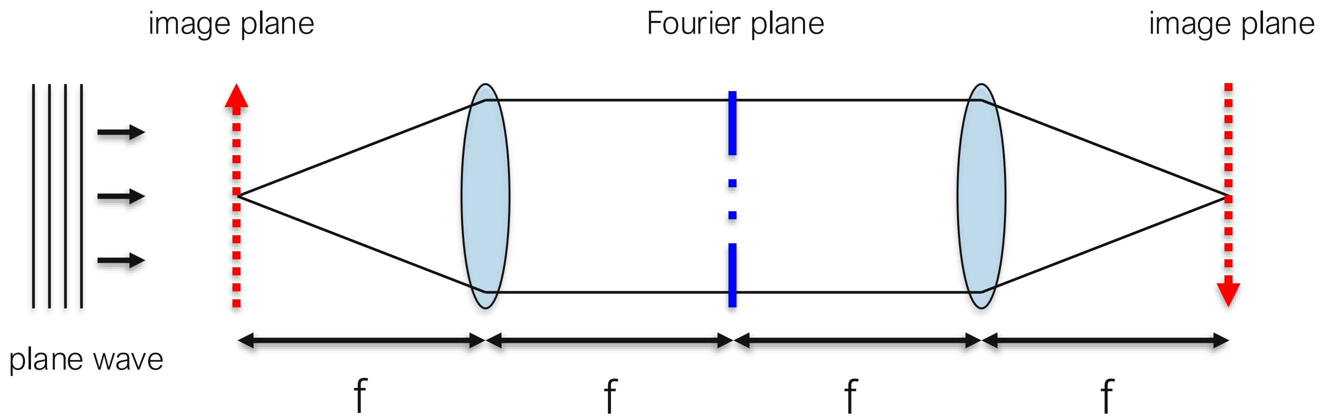
$n\sin\theta$ related to f-number or NA of lens

https://www.leica-microsystems.com/science-lab/microscope-resolution-concepts-factors-and-calculation/

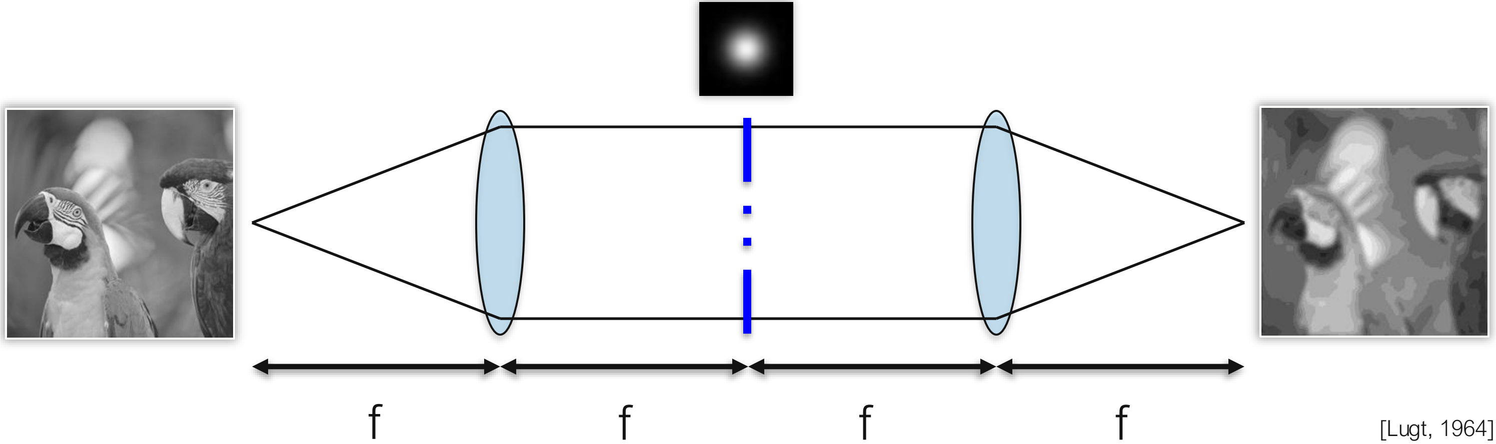
4f system

- 4f system consists of 2 lenses spaced at 2x their focal length f
- Image plane is copied at 4f distance
- Fourier plane between lenses



Optical Correlator

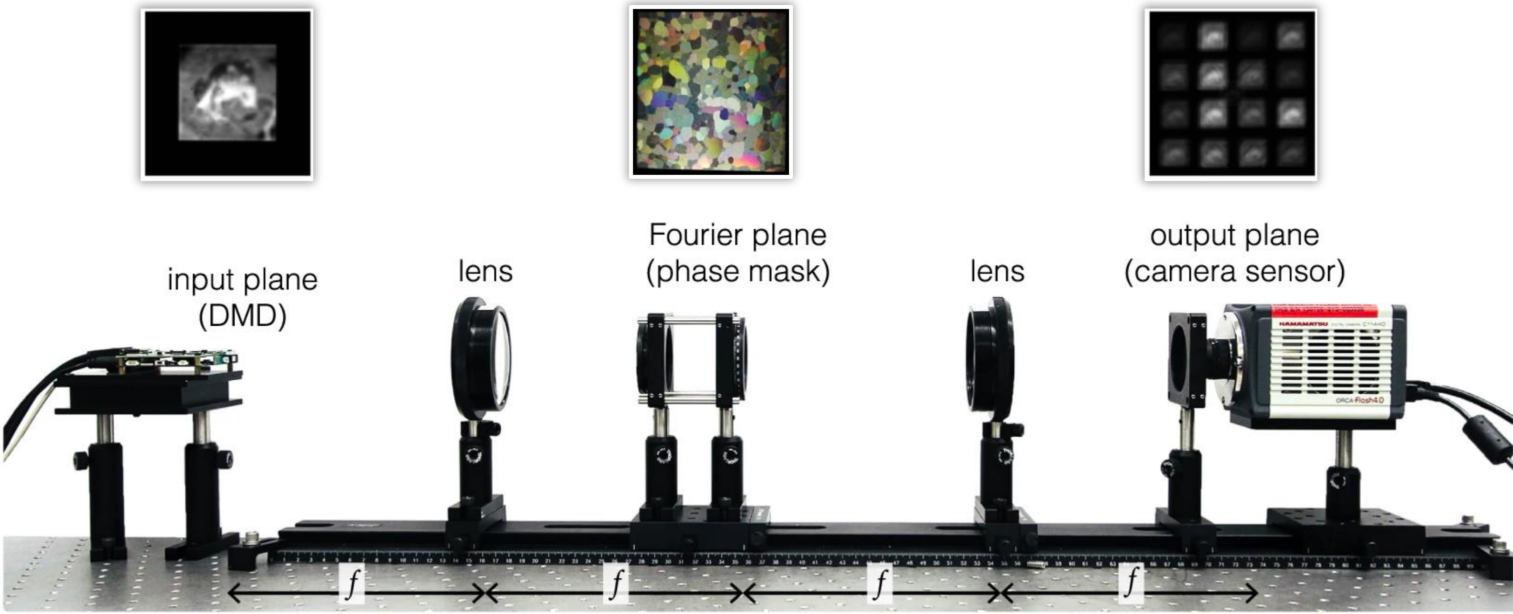
- Amplitude or phase mask placed at Fourier plane performs optical filtering / correlation
- Can implement low-, high-, or bandpass filter optically!



[Lugt, 1964]

Optical CNN

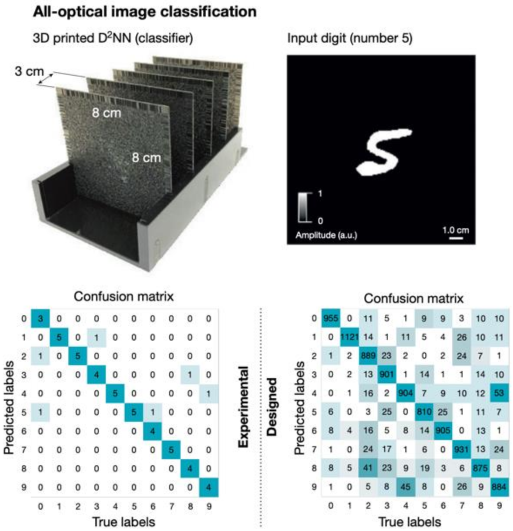
- Copy input image & convolve with different kernels = CNN



[Chang et al., 2018]

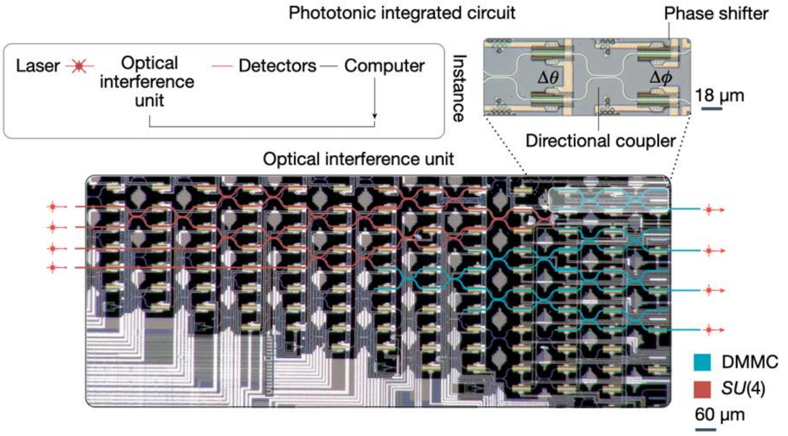
Optical Computing

Deep Diffractive Neural Networks



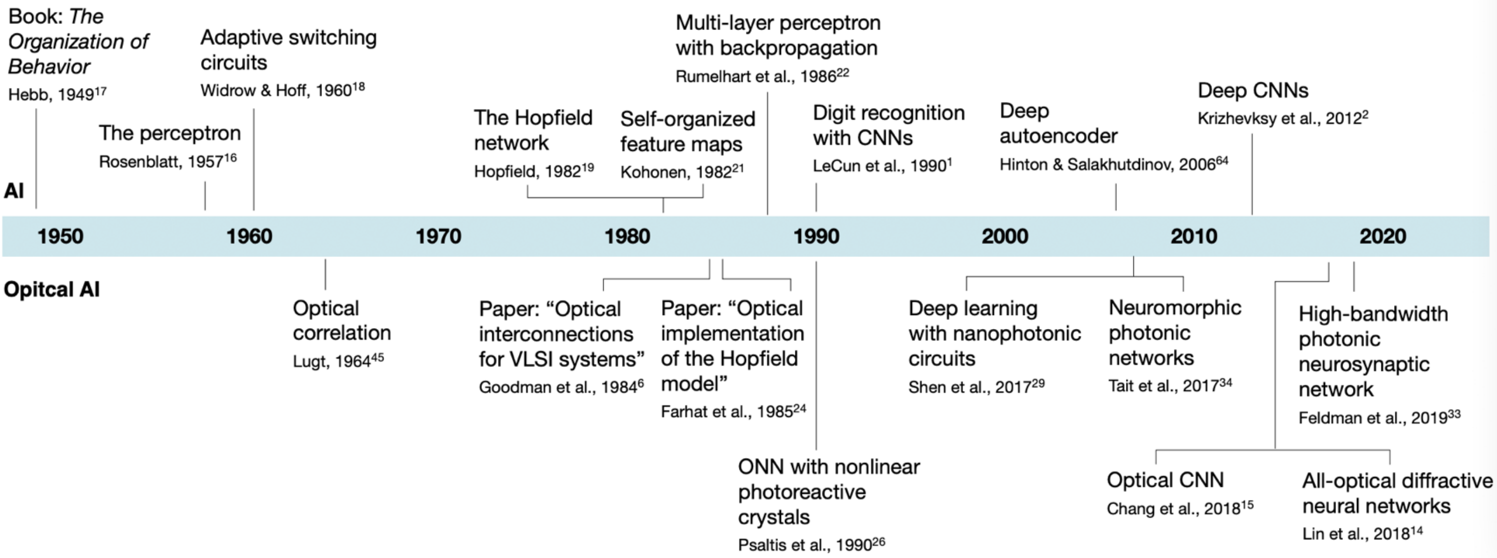
[Lin et al., 2018]

Photonic Integrated Circuit



[Shen et al., 2017]

AI and Related Optical Implementations

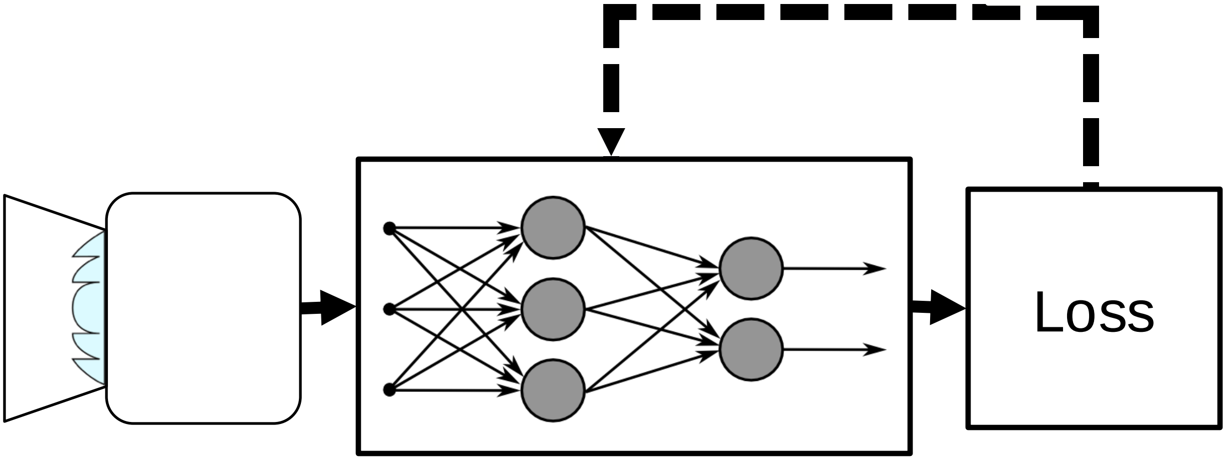


[Wetzstein et al., 2020]

Deep Optics

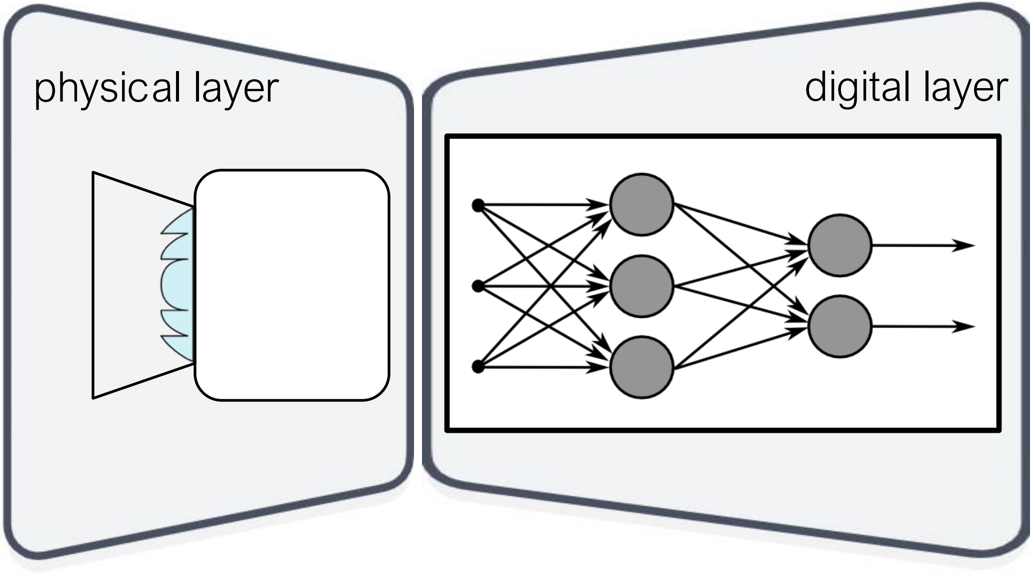
End-to-end Optimization of Optics and Image Processing

Deep Optics



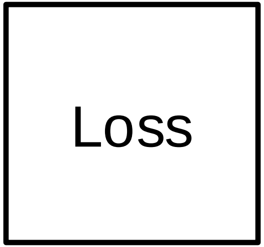
Jointly optimize optics and image processing end-to-end!

Deep Optics

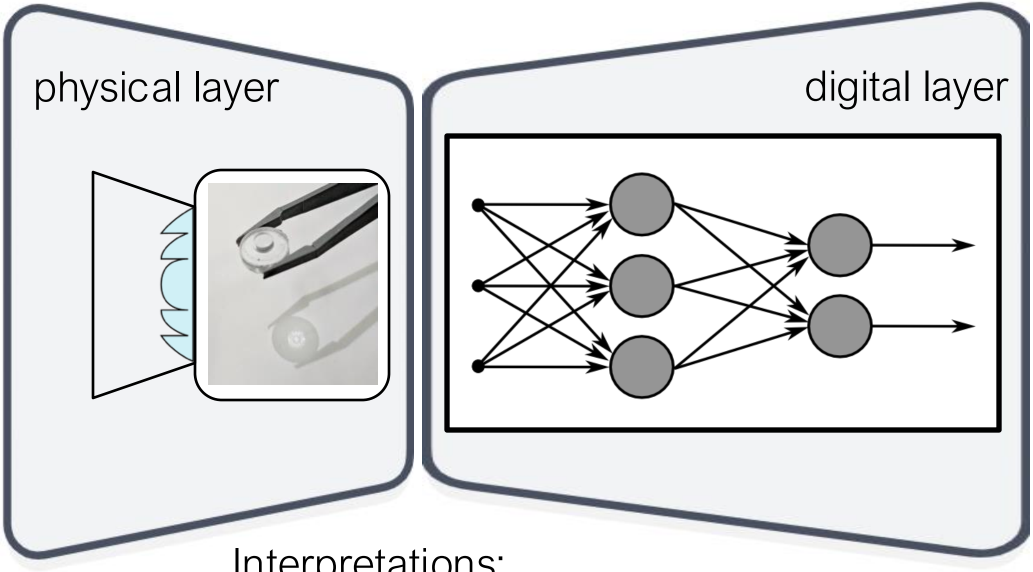


Training:

end-to-end in simulation



Deep Optics



Inference:

fabricate lens or other physical components, run network

Interpretations:

- Optical encoder, electronic decoder system
- Hybrid optical-electronic neural network

Case Study:

Image Classification in Low Light

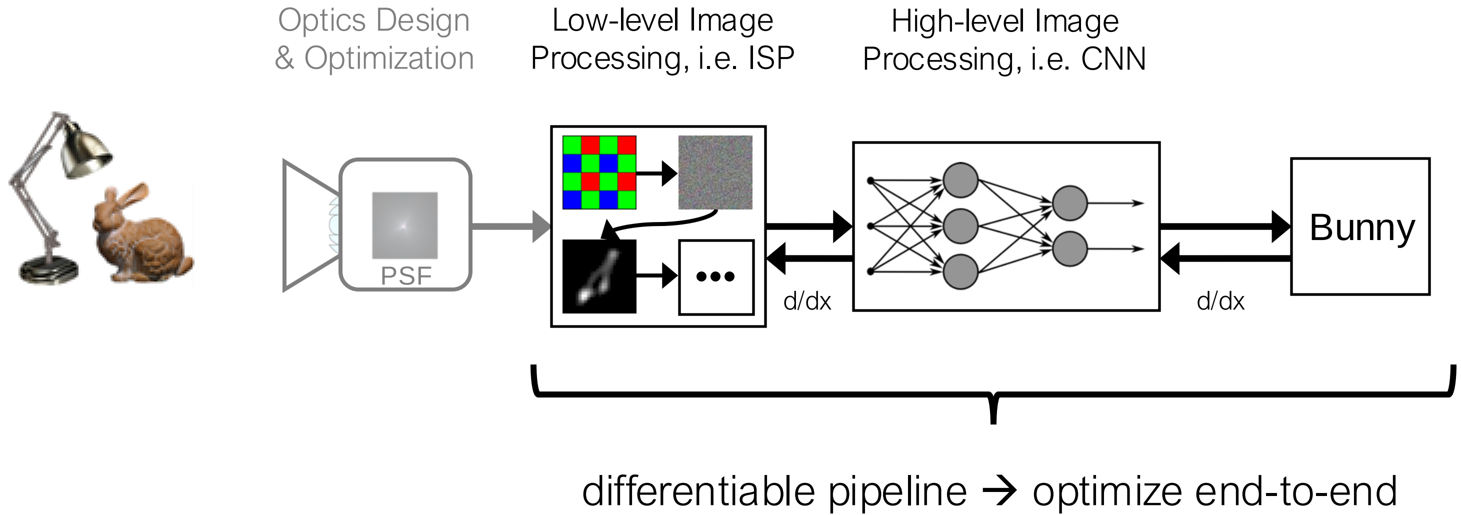
S. Diamond, V. Sitzmann, S. Boyd, G. Wetzstein, F. Heide "**Dirty Pixels: Optimizing Image Classification Architectures for Raw Sensor Data**", arXiv preprint arXiv:1701.06487, 2017

S. Diamond, V. Sitzmann, F. Heide, G. Wetzstein "**Unrolled Optimization with Deep Priors**", arXiv preprint: arXiv:1705:08041, 2018

A classification task

Low-Light
Mobile
Imaging
Scenario

BM3D → Inception-v4 Classification



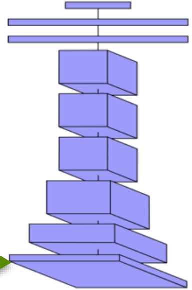
Learning Image Processing

Unrolling Image Optimization

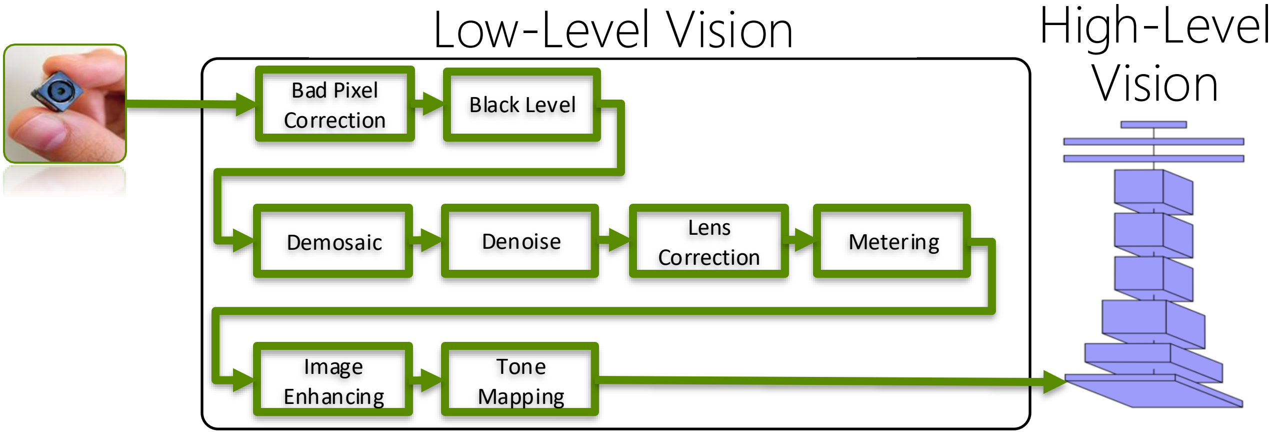
Low-Level Vision

- Physical image formation
- Prior and hyperparameters free
- Differentiable

High-Level Vision



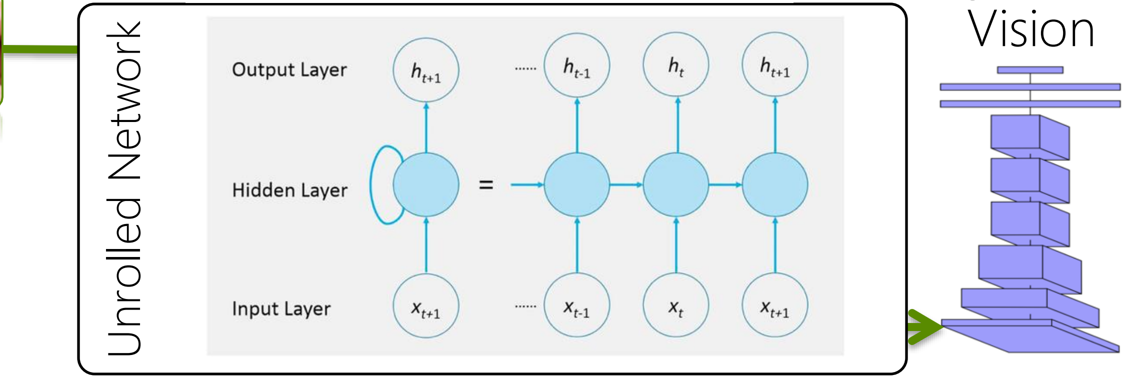
Unrolling Image Optimization



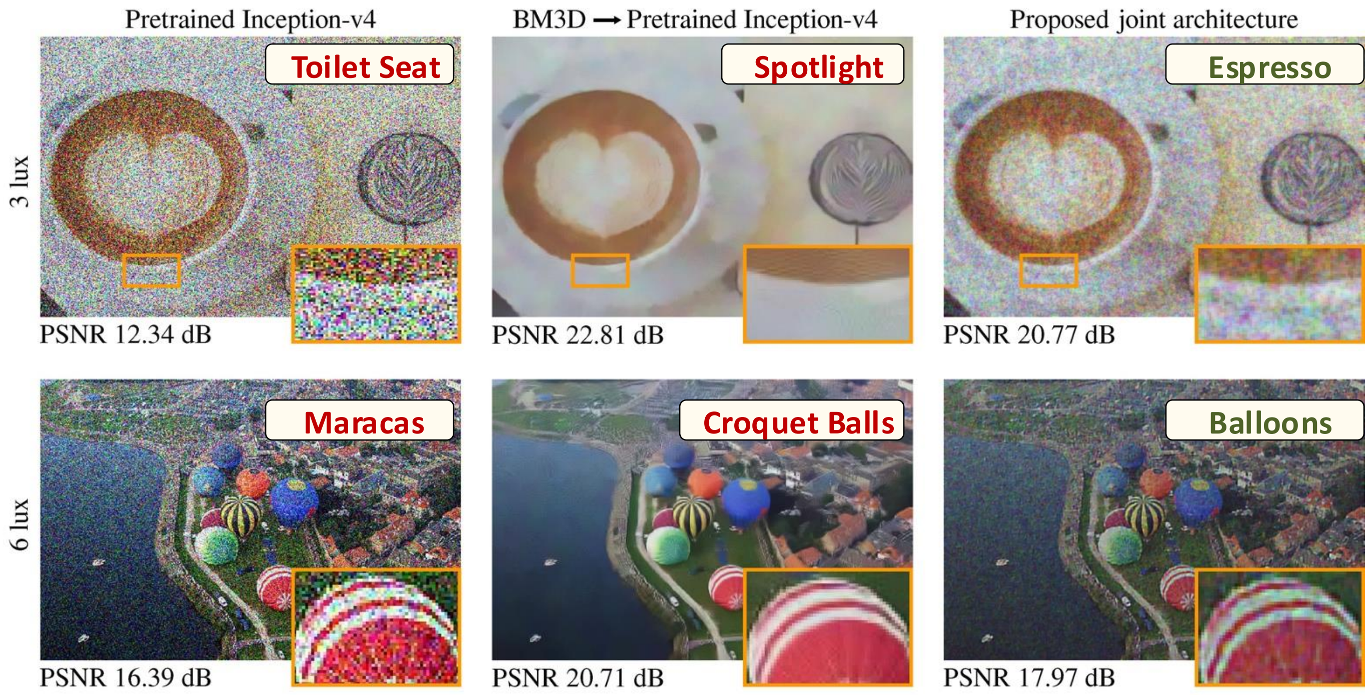
Unrolling Image Optimization

Low-Level Vision

High-Level Vision



Low-Light Classification

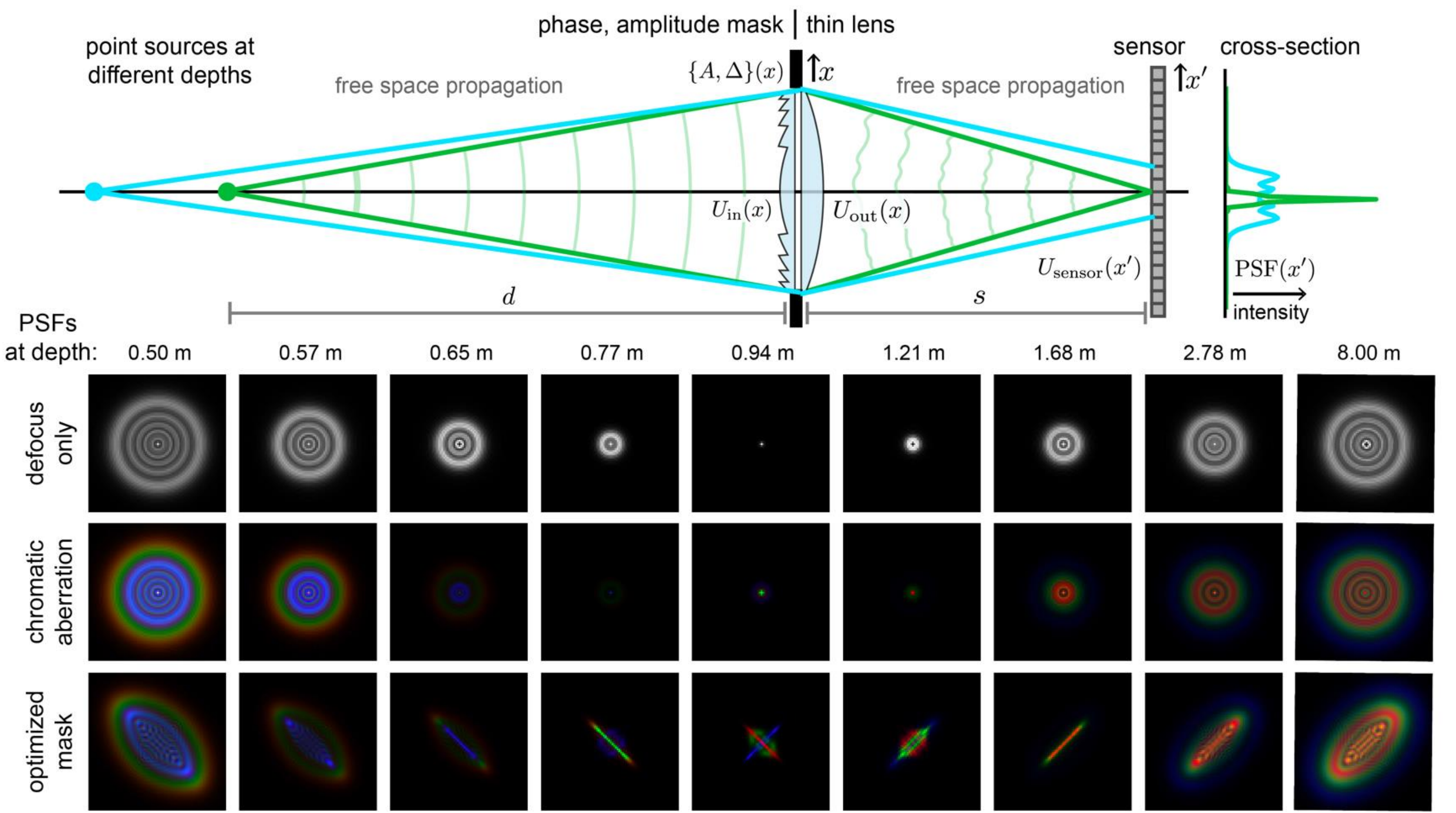


Case Study:

Monocular Depth Estimation

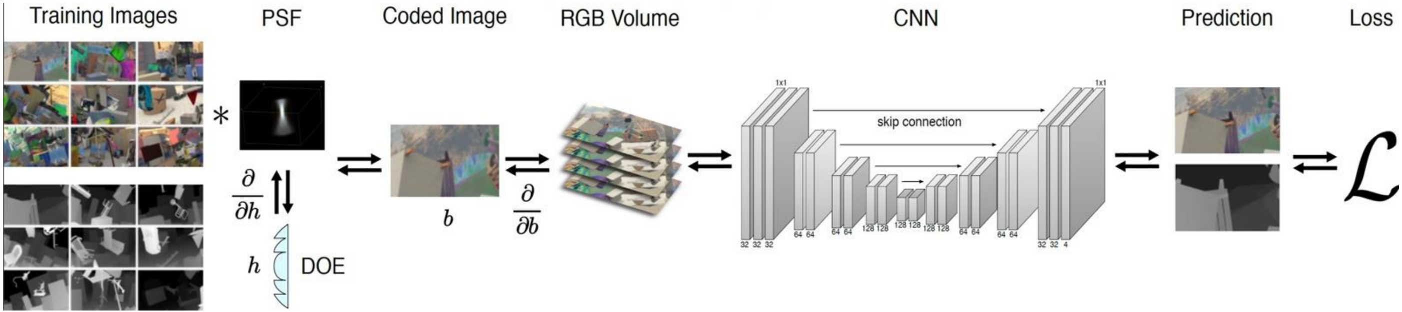
J. Chang, G. Wetzstein "**Deep Optics for Monocular Depth Estimation and 3D Object Detection**", ICCV 2019

H. Ikoma, C. Nguyen, Y. Peng, G. Wetzstein "**Depth from Defocus with Learned Optics for Imaging and Occlusion-aware Depth Estimation**", ICCP 2021



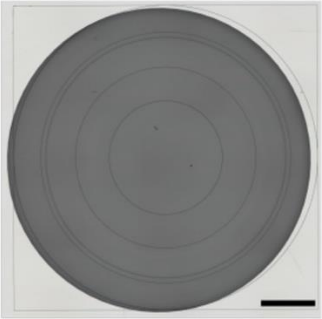
phase, amplitude mask | thin lens
point sources at different depths
free space propagation
$\{A, \Delta\}(x)$
x
free space propagation
sensor
x'
cross-section
$U_{\text{in}}(x)$
$U_{\text{out}}(x)$
$U_{\text{sensor}}(x')$
$\text{PSF}(x')$
intensity
d
s
PSFs at depth:
0.50 m
0.57 m
0.65 m
0.77 m
0.94 m
1.21 m
1.68 m
2.78 m
8.00 m
defocus only
chromatic aberration
optimized mask

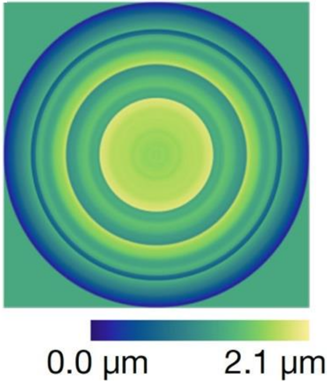
Pipeline



Ikoma et al., ICCP 2021

Prototype

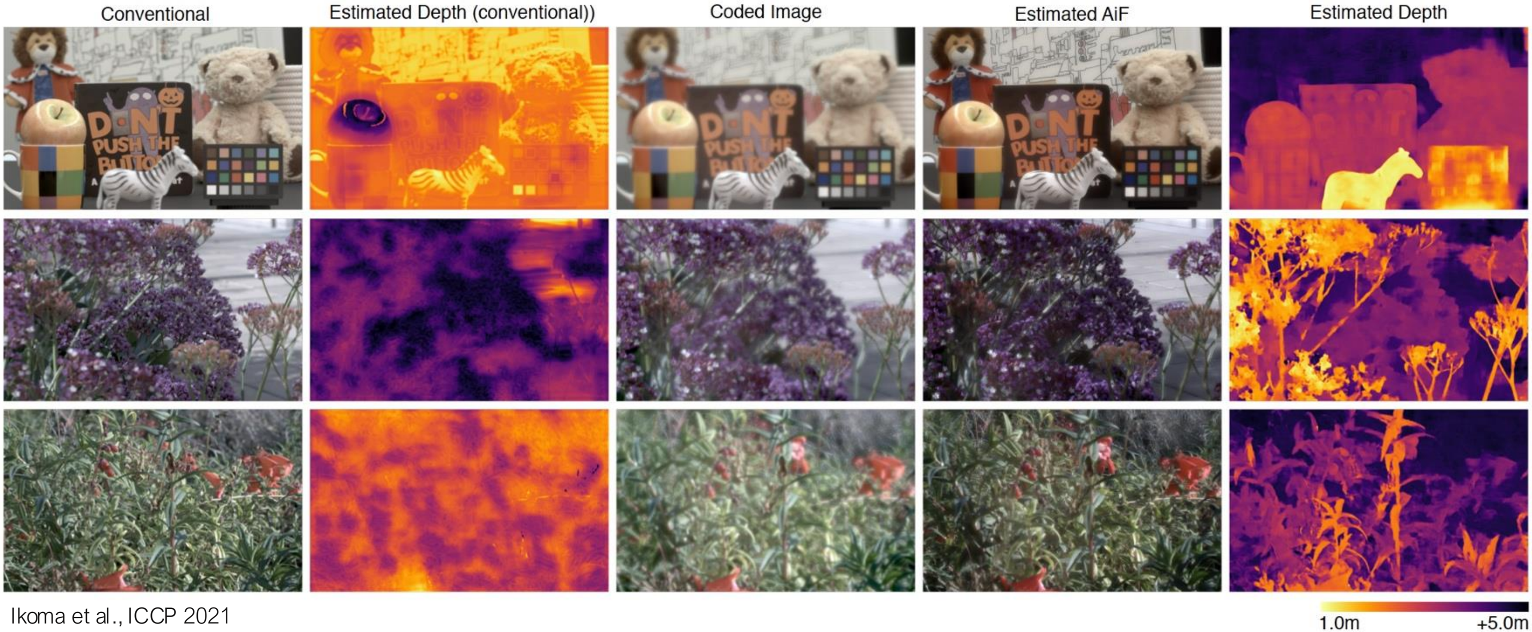




Ikoma et al., ICCP 2021

Results

- Estimate RGB image and depth map from a single optically coded image



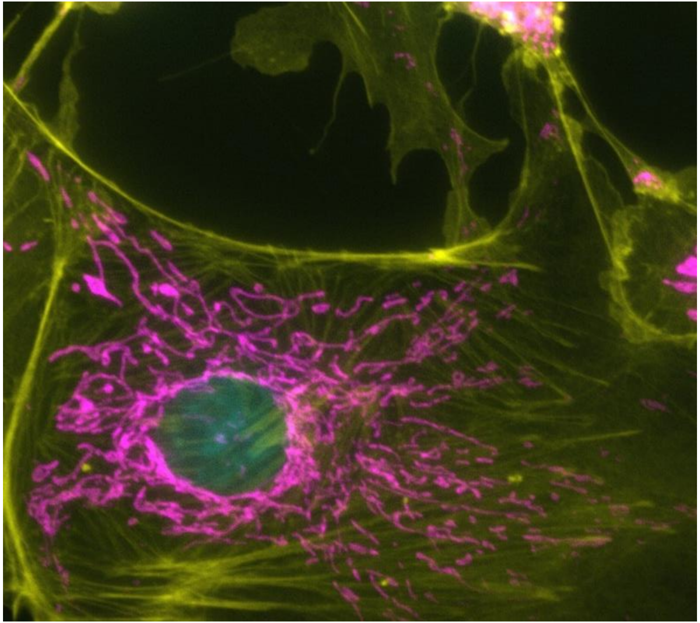
Ikoma et al., ICCP 2021

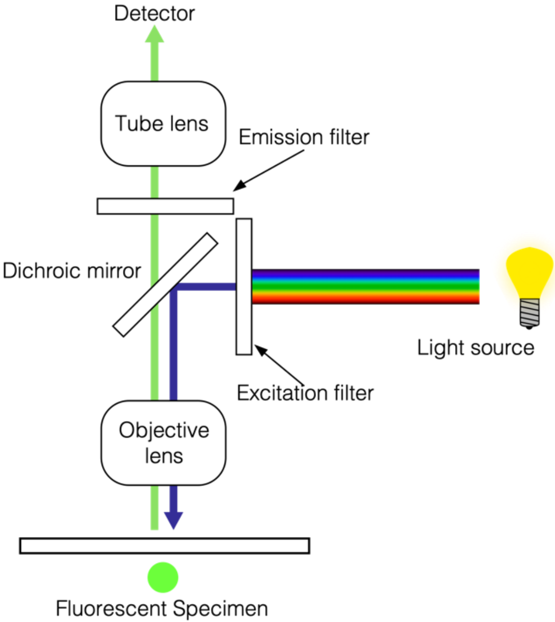
Case Study:

3D Localization Microscopy

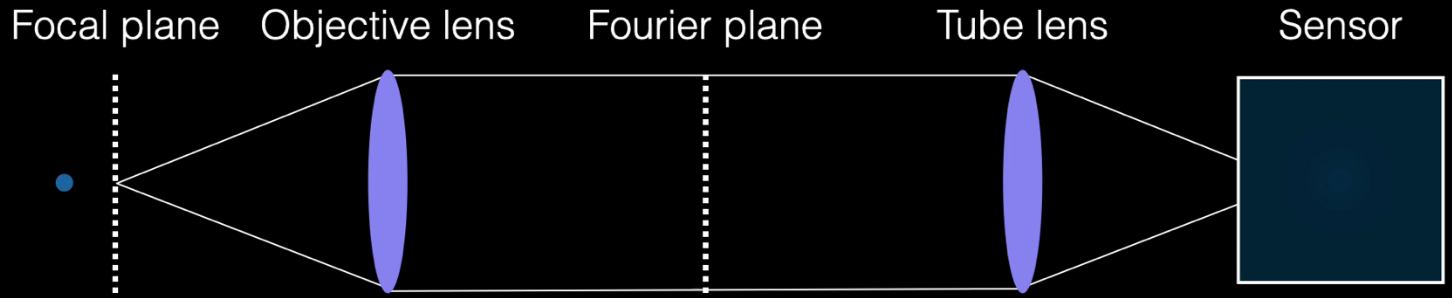
H. Ikoma, T. Kudo, Y. Peng, M. Broxton, **“Deep learning multi-shot 3D localization microscopy using hybrid optical-electronic computing”**, in submission

Fluorescence Microscopy

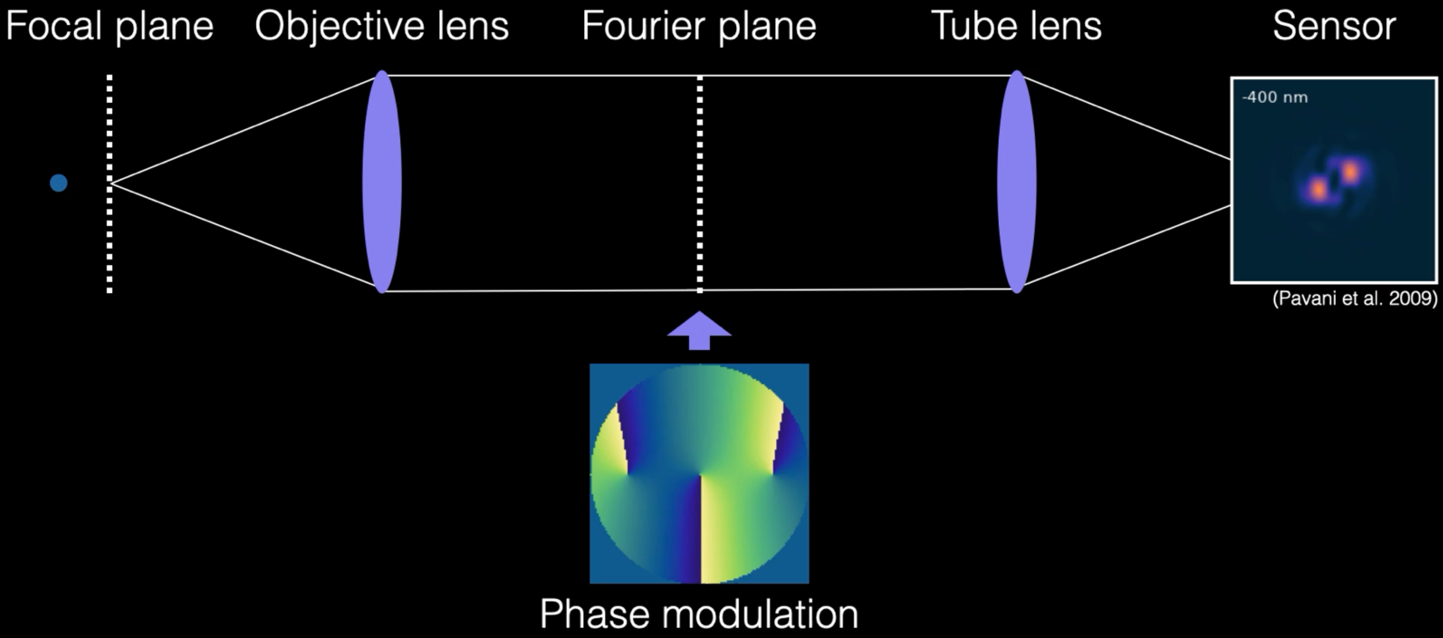




PSF of a widefield microscope

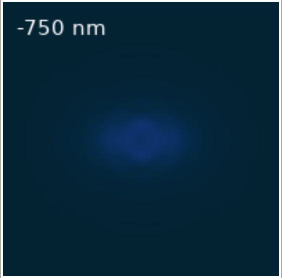


PSF engineering - Double Helix PSF

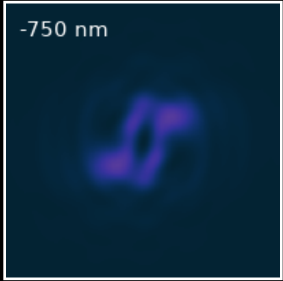


Other PSFs for Single Molecule Localization

Astigmatism



Double helix

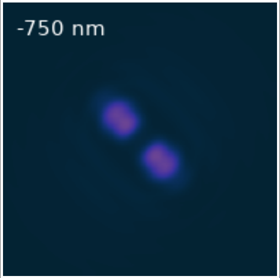


(Pavani et al. 2009)

Biplane

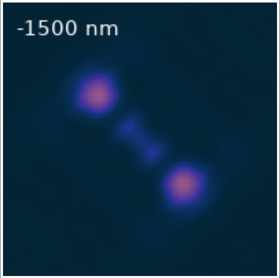
(Juette et al. 2008)

Saddle-point

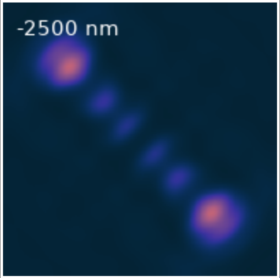


(Shechtman et al. 2014)

Tetrapod (3μm)

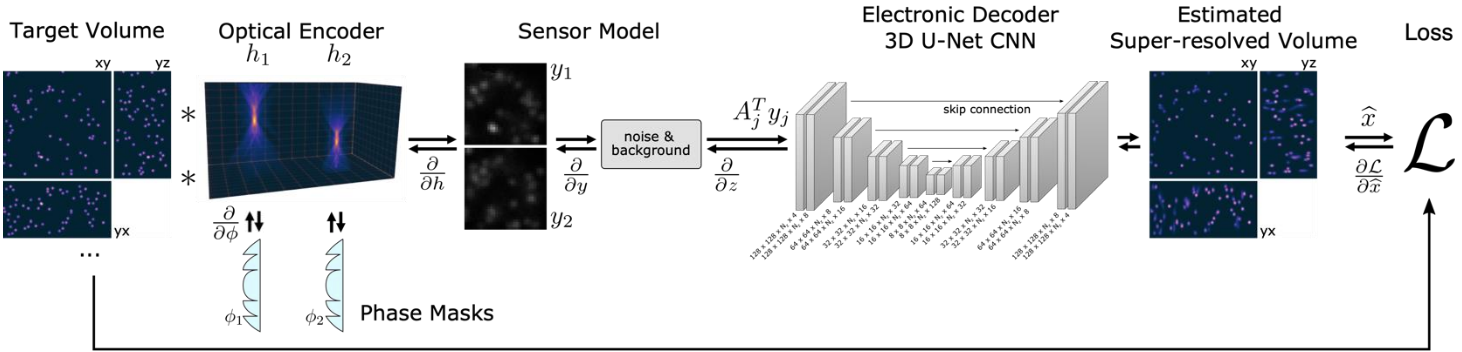


Tetrapod (5μm)

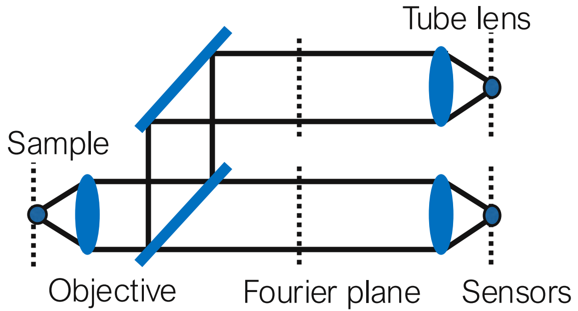


(Shechtman et al. 2015)

Deep Learning–based PSF Engineering



Multi-path imaging system



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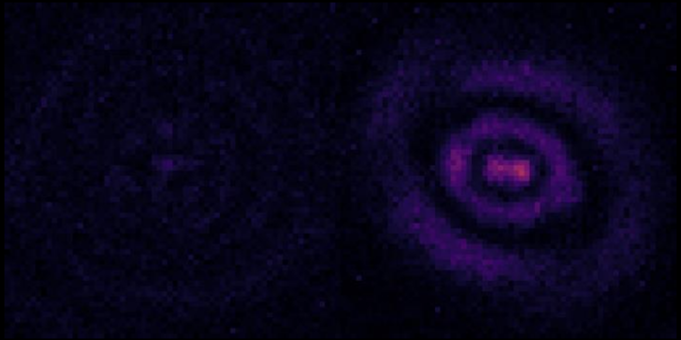
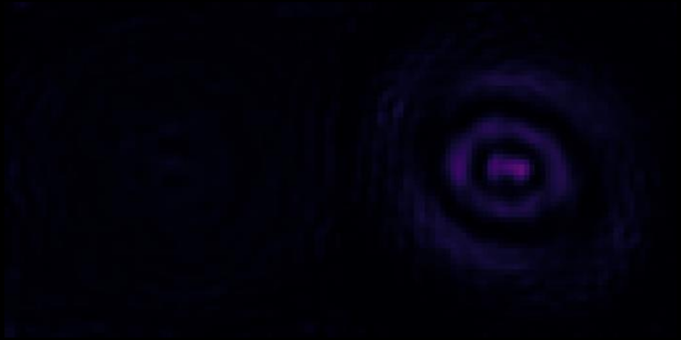
Optimized Two-shot 3D PSFs

Captured



Simulation



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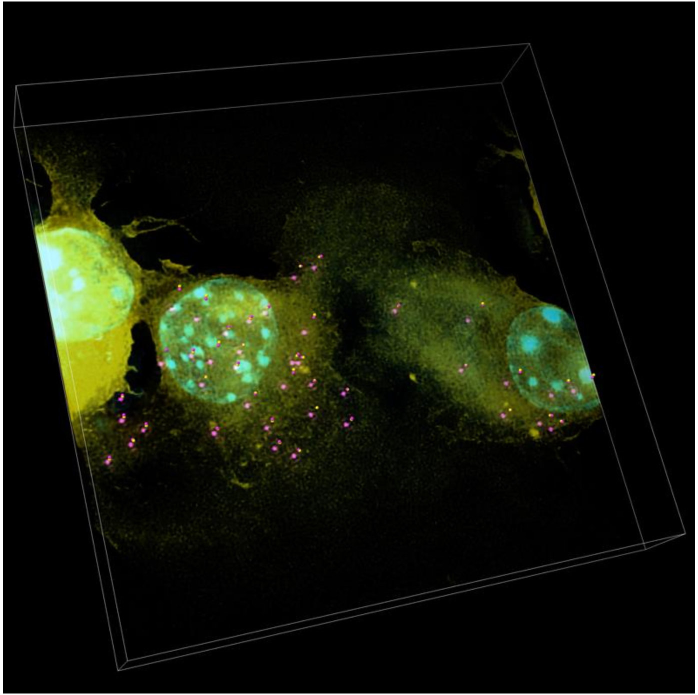
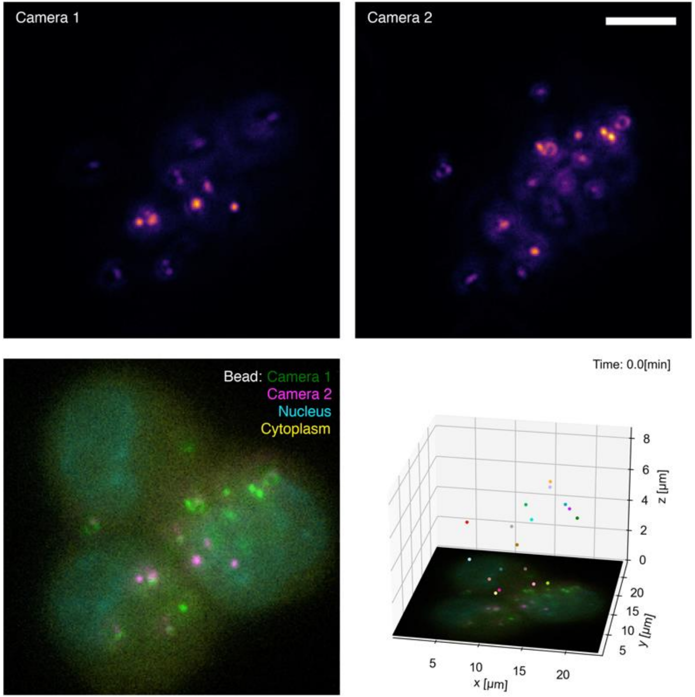
Preliminary Results



Fixed cells with beads



Time lapse of live cells

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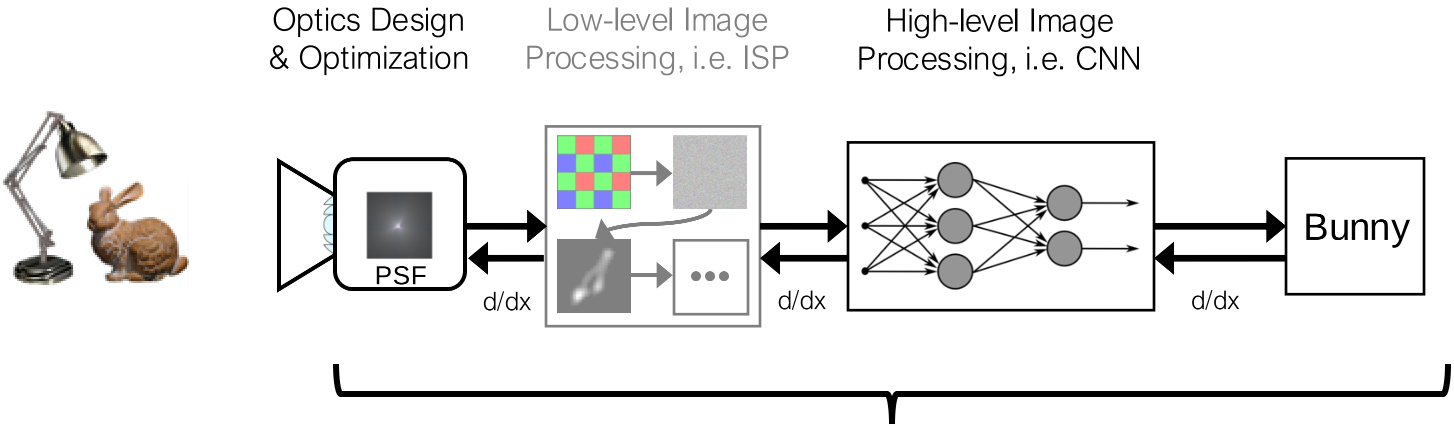
Case Study:

Hybrid Optical-Electronic Computing

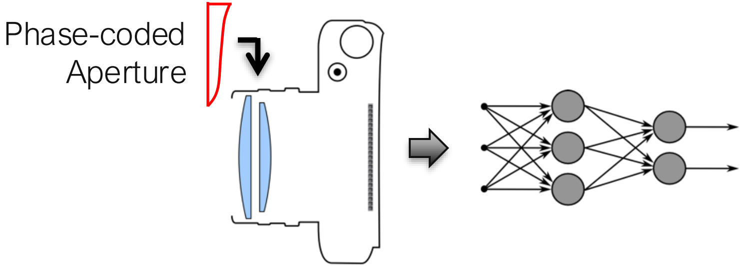
J. Chang, V. Sitzmann, X. Dun, W. Heidrich, G. Wetzstein "**Hybrid optical-electronic convolutional neural networks with optimized diffractive optics for image classification**", Scientific Reports, 2018

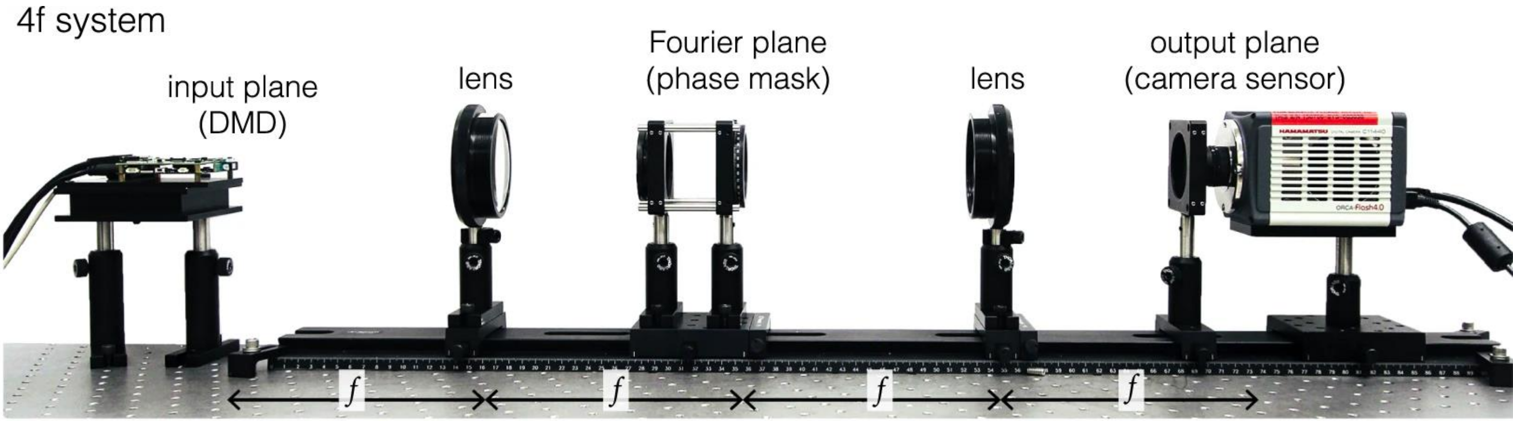


differentiable pipeline → optimize end-to-end

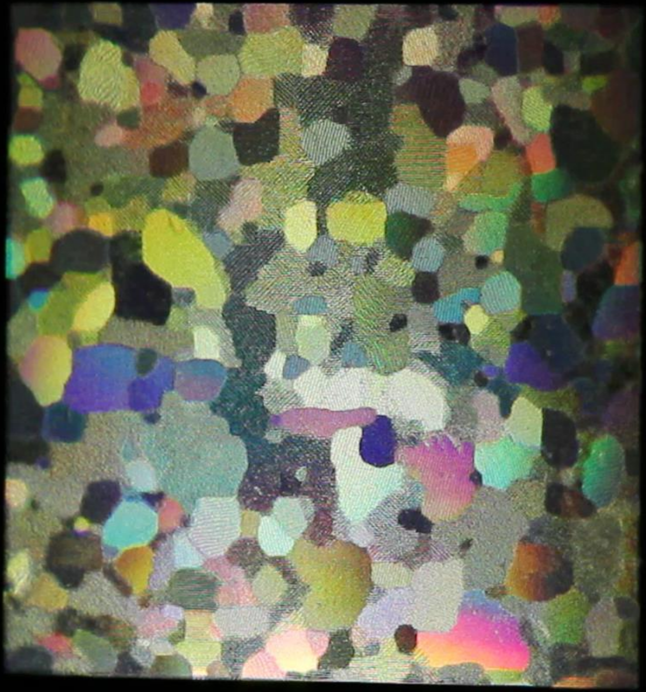
Learning Optics & CNN

Hybrid Optical-Electronic CNNs

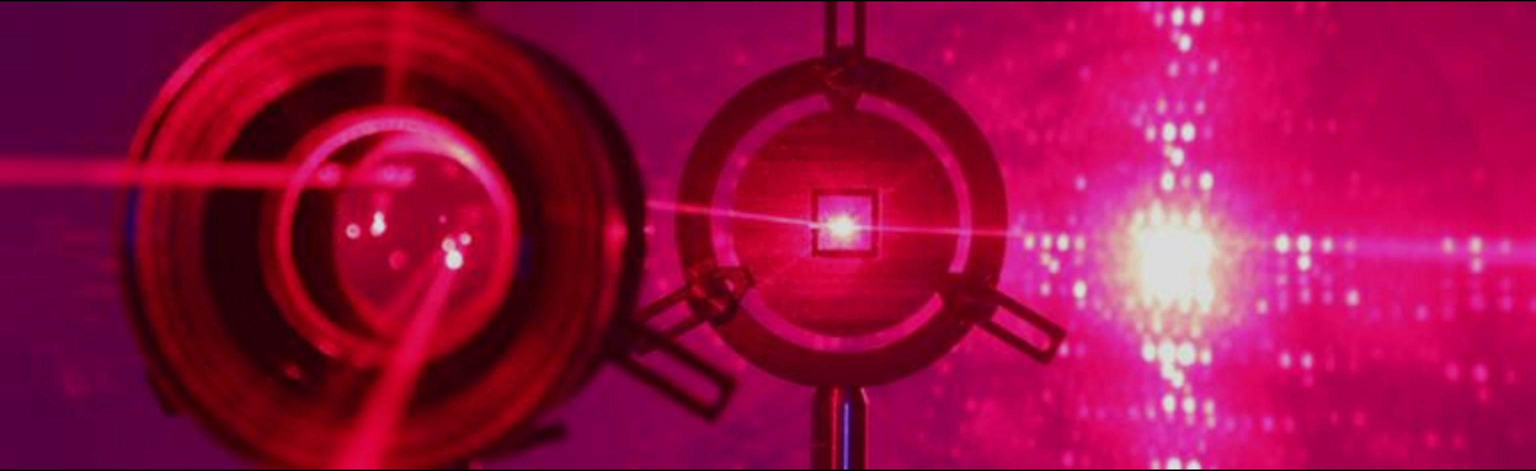




Chang et al., Scientific Reports 2018



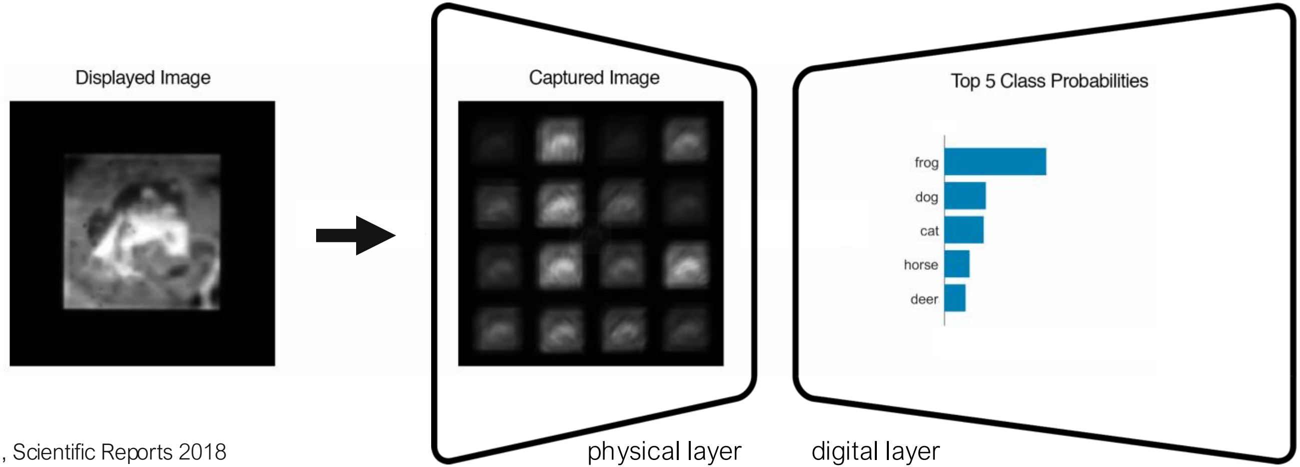
Chang et al., Scientific Reports 2018



Hybrid Optical-Electronic CNNs

current results:

- 2x classification accuracy for same power
- half power for same classification accuracy



physical layer digital layer

Chang et al., Scientific Reports 2018

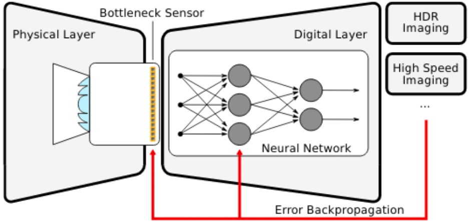
Case Study:

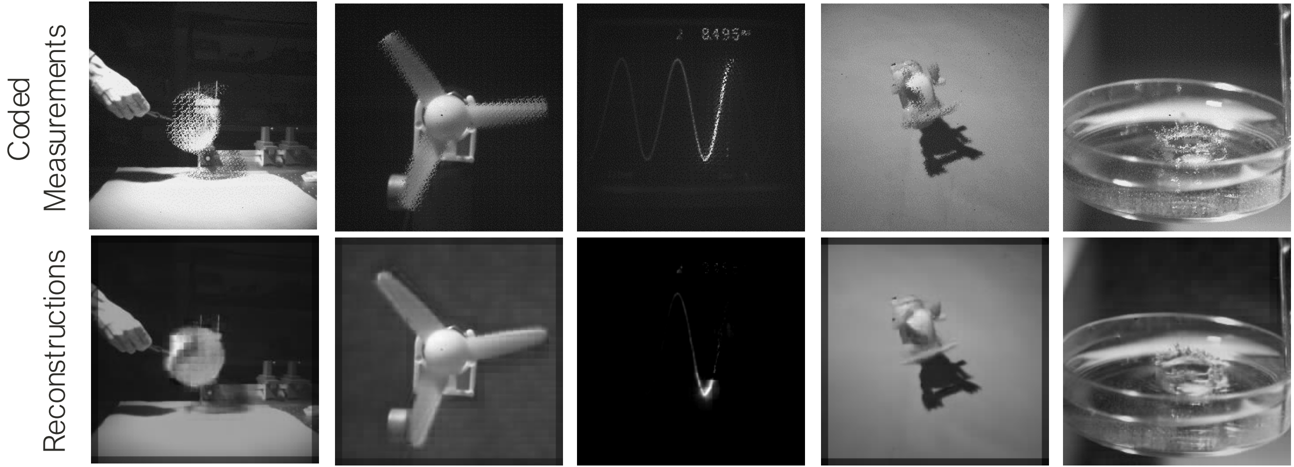
Neural Sensors

J. Martel, L. Muller, S. Carey, P. Dudek, G. Wetzstein "**Neural Sensors: Optimizing Pixel Exposures for HDR Imaging and Video Compressive Sensing with Programmable Sensors**", IEEE TPAMI (Proc. ICCP) 2020

Y. Li, M. Qi, R. Gulve, M. Wei, R. Genov, K. Kutulakos, W. Heidrich "**End-to-End Video Compressive Sensing Using Anderson-Accelerated Unrolled Networks**", ICCP 2020

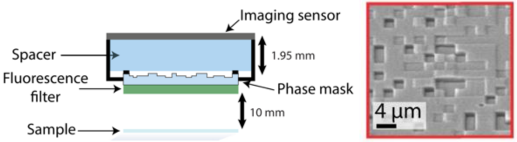
Neural Sensors





Martel et al., ICCP 2020

Other Examples



HDR Imaging Metzler EDOF Imaging Sitzmann Flat / Lensless Cameras

et al. CVPR 2020 Sun et al. et al. SIGGRAPH 2018 Boominathan et al. TPAMI/ICCP 2020

CVPR 2020

Gordon Wetzstein

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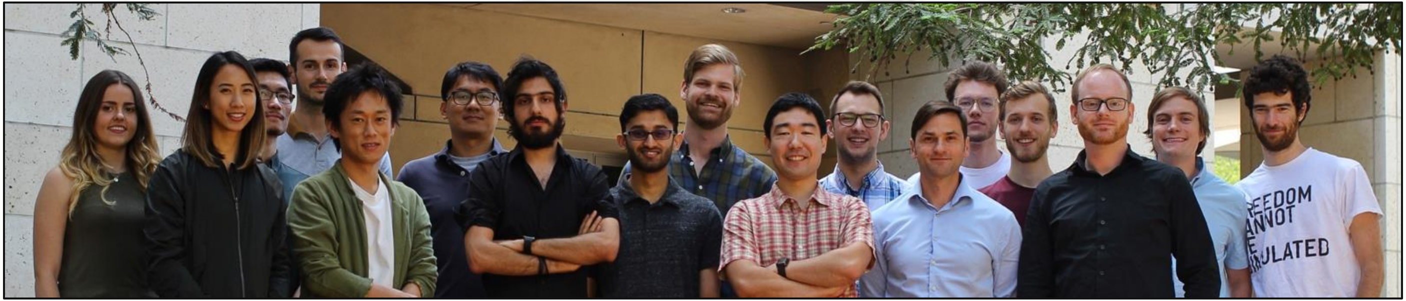
Computational Imaging Lab

Stanford University EE & CS

G. Wetzstein, A. Ozcan, S. Gigan, S. Fan, D. Englund, M. Soljacic, C. Denz, D. Miller, D. Psaltis, “**Inference in artificial intelligence with deep optics and photonics**”, Nature (review article), 2020

computationalimaging.org



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References and Further Reading

Wave Optics:

- J. Goodman, “Introduction to Fourier Optics”, Oxford University Press, 2012
- A. Lugt, “Signal detection by complex spatial filtering”, IEEE Trans. Information Theory, 1964
- J. Chang, V. Sitzmann, X. Dun, W. Heidrich, G. Wetzstein, “Hybrid optical-electronic convolutional neural networks with optimized diffractive optics for image classification”, Scientific Reports 2018
- G. Wetzstein, A. Ozcan, S. Gigan, S. Fan, D. Englund, M. Soljačić, C. Denz, D. Miller, D. Psaltis, “Inference in artificial intelligence with deep optics and photonics”, Nature (review paper), 2020
- Y. Shen, N. Harris, S. Skirlo, M. Prabhu, T. Baehr-Jones, M. Hochberg, X. Sun, S. Zhao, H. Larochelle, D. Englund, M. Soljačić, “Deep learning with coherent nanophotonic circuits”, Nature Photonics, 2017
- X. Lin, Y. Rivenson, N. Yardimci, M. Veli, Y. Luo, M. Jarrahi, A. Ozcan, “All-optical machine learning using diffractive deep neural networks”, Science, 2018
- Z. Zhang, M. Levoy, “Wigner distributions and how they relate to the light field”, ICCP 2009

Deep Optics:

See individual slides