CS 350 Homework #4 Tim Duffy

1) M/M/1 simulation:

a. Lambda = 50 and Ts = 0.015 and simulation time = 100

$\rho = \lambda * Ts = 50 * 0.015 = 0.75$
$q = \rho / (1 - \rho) = 0.75 / 0.25 = 3$
$Tq = q / \lambda = 3/50 = 0.06$ seconds = 60 msec
$Tw = Tq - Ts = 60$ msec - 15 msec = 45 msec
$w = \lambda * Tw = 50 * 0.045 = 2.25$

b. Lambda = 65 and Ts = 0.015 and simulation time = 100

$\rho = \lambda * Ts = 65 * 0.015 = 0.975$
$q = \rho / (1 - \rho) = 0.975 / 0.025 = 39$
$Tq = q / \lambda = 39/50 = 0.78$ seconds = 780 msec
$Tw = Tq - Ts = 780$ msec - 15 msec = 765 msec
$w = \lambda * Tw = 50 * 0.765 = 38.25$

c. Lambda = 65 and Ts = 0.020 and simulation time = 100

$\rho = \lambda * Ts = 65 * 0.020 = 1.3$

The system is at 130% utilization, which will create an infinitely long queue
q = infinity
$Tq = q / \lambda$ = infinity
$Tw = Tq - Ts$ = infinity
$w = \lambda * Tw$ = infinity

In parts a,b, and c, my simulation produces results very similar to the calculated values

2) M/M/K simulation:

a. K = 5, Lambda = 30, Ts = 0.03 and simulation time = 100

$\rho = \lambda * Ts = 30 * 0.03 = 0.9$

$q = \rho / (1-\rho) - [(K+1)\, \rho^{(K+1)} / (1-\rho^{(K+1)})]$
$= (0.9 / 0.1) - (6*(0.9^6) / 1 - 0.9^6)$
$= 9 - (3.188646 / 0.468559)$
q = 2.1947823

Pr("Rejection") = Pr(Sk) = $(1-\rho)\rho^K / (1-\rho^{K+1})$
$= 0.1*(0.9^5) / (1-0.9^6) = 0.059049 / 0.468559 =$ **0.12602255**

$\lambda' = \lambda * (1 -$ Pr("Rejection")) $= 30 * 0.87397745 =$ **26.2193235**

$Tq = q / \lambda'$
Tq = 2.1947823 / 26.2193235 = 0.0837085785 seconds **= 83.7085785 msec**

My simulation produces values somewhat closer to 3 than to 2 for q.
The confidence interval calculated for Tq is always very wide, which suggests that there is a very large standard deviation for the response times of individual processes. The sample mean and the average Tq computed above both fall within this interval, but often differ by around 15 to 20 msec.

b. K = 5, Lambda = 50, Ts = 0.03 and simulation time = 100

$\rho = \lambda * Ts = 50 * 0.03 = 1.5$

$q = \rho / (1-\rho) - [(K+1)\, \rho^{(K+1)} / (1-\rho^{(K+1)})]$
$= (1.5 / -0.5) - (6*(1.5^6) / 1 - 1.5^6)$
$= -3 - (68.34375 / -10.390625)$
q = 3.57744361

Pr("Rejection") = Pr(Sk) = $(1-\rho)\rho^K / (1-\rho^{K+1})$
$= -0.5*(1.5^5) / (1-1.5^6) = -3.796875 / -10.390625 =$ **0.365413534**

$\lambda' = \lambda * (1 - \text{Pr("Rejection")}) = 50 * 0.634586466 =$ **31.7293233**

$Tq = q / \lambda'$
Tq = 3.57744361 / 31.7293233 = 0.112748815 seconds **= 112.748815 msec**

My simulation produces results for q that are very near the expected value. My simulation has again calculated a very wide confidence interval for Tq. The sample mean and the expected value of Tq often differ by up to 30 msec.

c/d. The analysis for the system will be the same as above (with Poisson service times). However, the results of the simulation should (and when tested, actually do) be closer to the numbers computed above. When computing Tq and q, we use the mean service rate. So if the service time for each process is constant, the sample mean will match exactly with the mean used in our calculations.

As stated before, when I run my simulation with fixed service times, the results become much closer to the numbers computed above. They are still not exactly the same since event arrivals are still Poisson.

3. ?

4. The CPU is the bottleneck of the system, since it must process a large number of processes each second. An arrival rate of 50 process per second or greater will cause this system to buckle.