Exercise 1

a) What are all the possible values that can be stored in the variable x after the execution of the following parallel program when assignment is assumed to be an atomic instruction?

$$\texttt{x:=10; }\big(\texttt{(x:=x*2; x:=x-11; x:=x+2)} \parallel \texttt{x:=x-5}\big)$$

b) Does the following parallel program satisfy the given pre and post condition?

$$\texttt{[x=0] (x++} \parallel \texttt{x++) [x=2]}$$

Exercise 2

Let R be a binary relation on a set A. Let us define the binary relation

$$E \stackrel{\text{def}}{=} \{(x, x) \mid x \in A\}.$$

It is trivially true that $R \cup E$ is a reflexive relation.

- Argue that $R \cup E$ is a reflexive closure of R.

Exercise 3

Let R be a binary relation on a set A. Let us define the binary relation

$$R^{-1} \stackrel{\text{def}}{=} \{(y, x) \mid (x, y) \in R\}.$$

- Argue that $R \cup R^{-1}$ is a symmetric relation.
- Argue that $R \cup R^{-1}$ is a symmetric closure of R.

Exercise 4

Consider a binary relation $succ \subseteq \mathbb{N} \times \mathbb{N}$ defined by

$$succ = \{(n, n+1) \mid n \in \mathbb{N}\} .$$

Answer the following questions:

- What relation is the transitive closure of $succ$?
- What relation is the reflexive and transitive closure of $succ$?
- What relation is the symmetric, reflexive and transitive closure of $succ$?

Exercise 5*

Let us consider the following labelled transition system.

$$\begin{tikzcd} s \arrow[r, "a"] & s_1 \arrow[d, "a"] \\ s_3 \arrow[u, "a"] & s_2 \arrow[l, "a"'] \end{tikzcd}$$

- Define the labelled transition system as a triple $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$.
- What is the reflexive closure of the binary relation $\xrightarrow{a}$? (A drawing is fine.)
- What is the symmetric closure of the binary relation $\xrightarrow{a}$? (A drawing is fine.)
- What is the transitive closure of the binary relation $\xrightarrow{a}$? (A drawing is fine.)

Exercise 6

Let us consider the following CCS definition of a coffee machine.

$$\mathrm{CM} \stackrel{\mathrm{def}}{=} coin.\overline{coffee}.\mathrm{CM}$$

- Give a CCS process which describes a coffee machine that may behave like CM but may also steal the money it receives and fail at any time.

Exercise 7

Assume a given labelled transition system $T = (Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$ such that the sets $Proc$ and Act are finite.

- Does it imply that $\xrightarrow{a}$ is also a finite set? Why?
- Draw an example of an LTS with four states and two actions.
- How can your example be described by a sequential fragment of CCS (with Nil, action prefixing, nondeterminism and recursive definitions of names)?
- Show that in general any finite LTS T can be described by using only a sequential fragment of CCS.