Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

- syntax and semantics of LTL
- automata-based LTL model checking
- complexity of LTL model checking ⟵

Computation-Tree Logic

Equivalences and Abstraction

Complexity of LTL model checking

main steps of automata-based LTL model checking:

construction of an NBA $\mathcal{A}$ for $\neg\varphi$

persistence checking in the product $\mathcal{T} \otimes \mathcal{A}$

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The **LTL** model checking problem is **PSPACE**-complete

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we show

- just for fun: **coNP**-hardness
- **PSPACE**-completeness

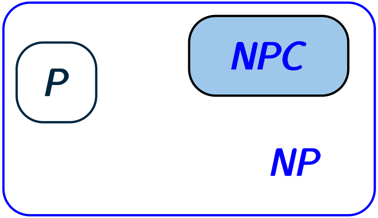
Recall: complexity classes

Complexity classes *P*, *NP*

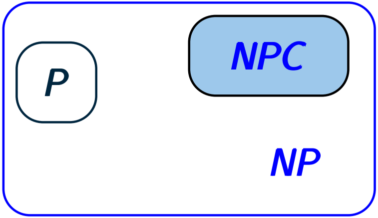
$$P = \text{class of decision problem solvable in deterministic polynomial time}$$

$$NP = \text{class of decision problem solvable in nondeterministic polynomial time}$$

Complexity classes P, NP



NPC = class of NP-complete problems

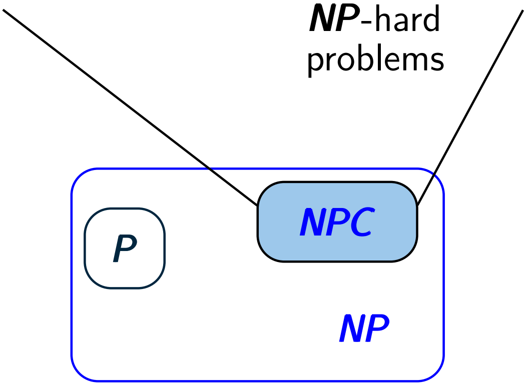


NPC = class of NP-complete problems

(1) $L \in NP$

(2) L is NP-hard, i.e., $K \leq_{poly} L$ for all $K \in NP$

Complexity classes P, NP



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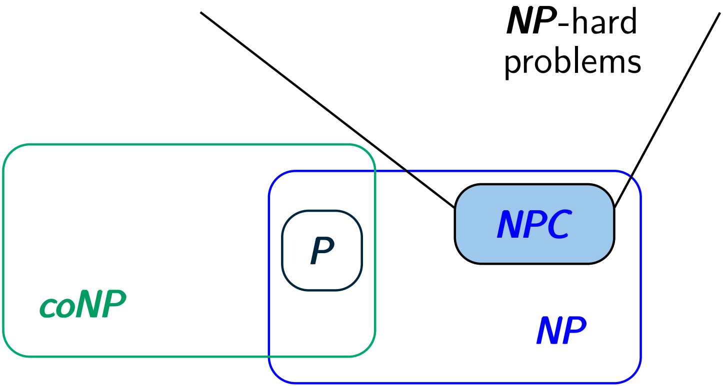
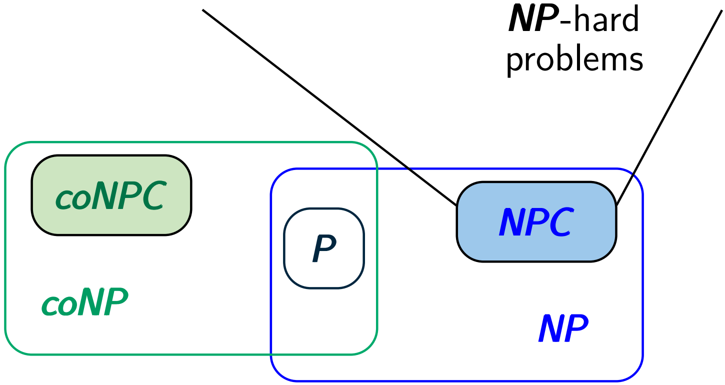


$$coNP = \{ \overline{L} : L \in NP \}$$

$\uparrow$ complement of L

Complexity classes P, NP, $coNP$



$coNPC$ = class of $coNP$-complete problems

(1) $L \in coNP$

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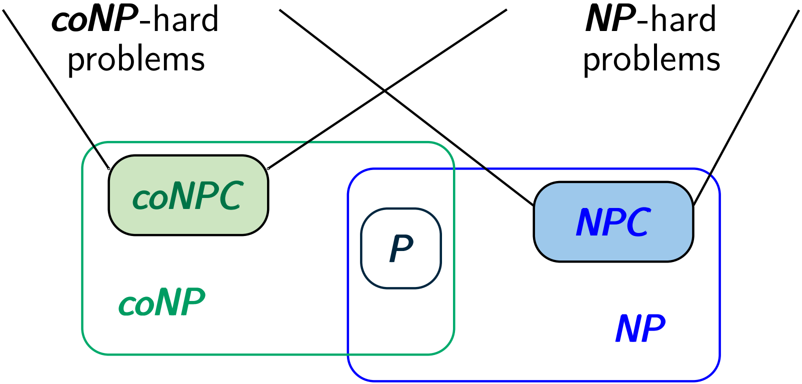
Complexity classes *P*, *NP*, *coNP*



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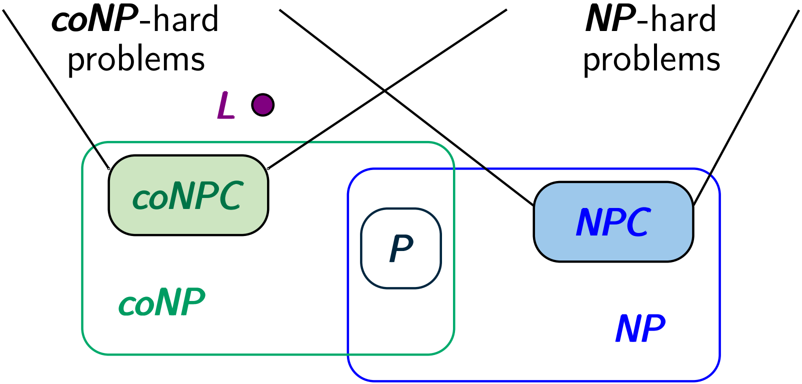

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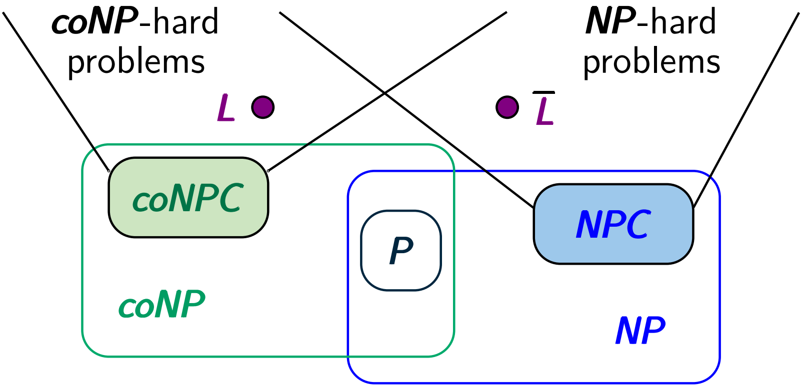

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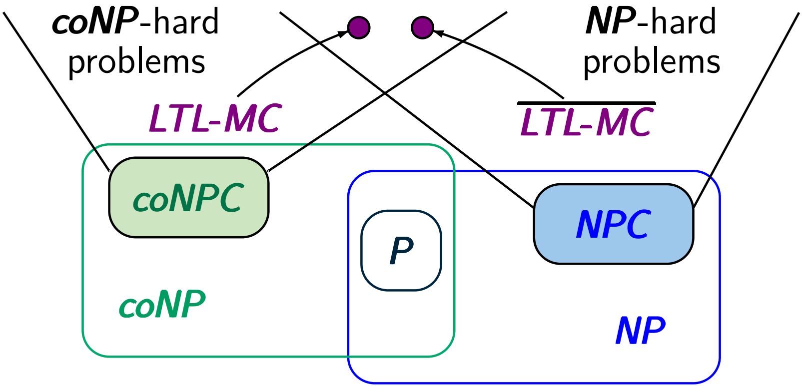

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***coNP*-hardness**

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proof by a polynomial reduction

$$\mathit{HP} \;\leq_{\mathit{poly}}\; \overline{\mathit{LTL\text{-}MC}}$$

Hamilton path problem

complement of the **LTL** model checking problem

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given: finite directed graph G

question: does G has a Hamilton path **?**, i.e., a path that visits each node exactly once

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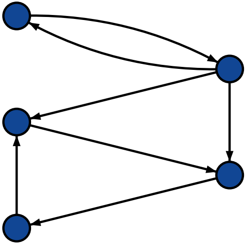

Hamilton path problem *HP*

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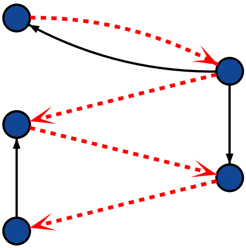

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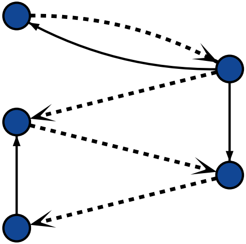

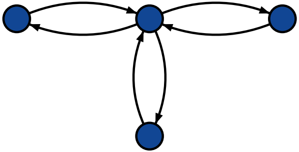

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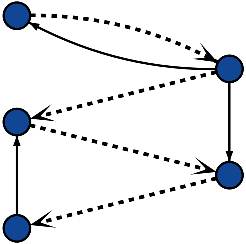

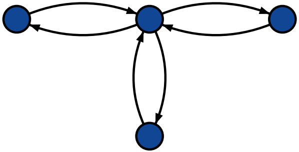

has no Hamilton path

HP is known to be ***NP***-complete

Polynomial reduction

$$HP \quad \leq_{poly} \quad \overline{\textit{LTL-MC}}$$

Hamilton path problem

complement of the **LTL** model checking problem

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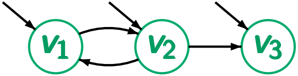
node-set V of G $\quad \widehat{=} \quad$ states of $\mathcal{T}$



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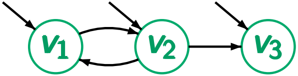

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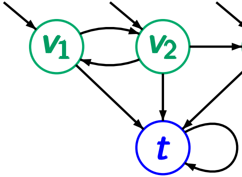
additional trap state t



Polynomial reduction

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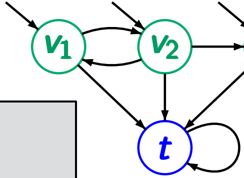

$\varphi = ?$

Polynomial reduction

HP $\leq_{poly}$ $\overline{\textit{LTL-MC}}$

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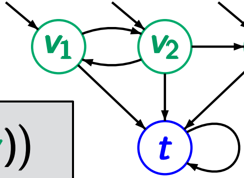

$$\varphi = \bigwedge_{v \in V} (\Diamond v \wedge \Box(v \rightarrow \bigcirc \Box \neg v))$$

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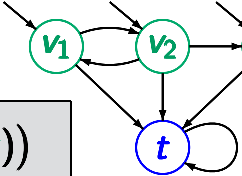

$$\varphi = \neg \bigwedge_{v \in V} \left(\Diamond v \wedge \Box (v \rightarrow \bigcirc \Box \neg v) \right)$$

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We just saw:

The **LTL** model checking problem is ***coNP***-hard

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We now prove:

The **LTL** model checking problem is ***PSPACE***-complete

For the interested student !!!