Tutorial 9 – Solutions

Exercise 1*:

Consider the following four alternative definitions of TCCS agent M:

- $M_1 \stackrel{\text{def}}{=} \epsilon(3).(\epsilon(2).a.M_1 + b.M_1)$
- $M_2 \stackrel{\text{def}}{=} \epsilon(5).a.M_2 + \epsilon(3).b.M_2$
- $M_3 \stackrel{\text{def}}{=} \epsilon(3).(\epsilon(2).a.M_3 + \tau.M_3)$
- $M_4 \stackrel{\text{def}}{=} \epsilon(5).a.M_4 + \epsilon(3).\tau.M_4$

For which of the above four definitions do we have $M_i \xrightarrow{\epsilon(4)}$.

1. For which of the four definitions do we have $M_i \xrightarrow{4}$. In the affirmative cases use the SOS rules for TCCS to prove the delay-transition as well as identify the target process P_i such that $M_i \xrightarrow{\epsilon(4)} P_i$.

   - M_1 has a transition $M_1 \xrightarrow{4} \epsilon(1).a.M_1 + b.M_1$ which we prove below:

$$\dfrac{\dfrac{\dfrac{\overline{\epsilon(2).a.M_1 \xrightarrow{1} \epsilon(1).a.M1} \qquad \overline{b.M_1 \xrightarrow{1} b.M1}}{\epsilon(2).a.M_1 + b.M_1 \xrightarrow{1} \epsilon(1).a.M_1 + b.M_1}}{\epsilon(3)(\epsilon(2).a.M_1 + b.M_1) \xrightarrow{4} \epsilon(1).a.M_1 + b.M_1}}{M_i \xrightarrow{4} \epsilon(1).a.M_1 + b.M_1}$$

   - M_2 has a transition $M_2 \xrightarrow{4} \epsilon(1).a.M_2 + b.M_2$ which we prove below:

$$\dfrac{\dfrac{\overline{\epsilon(5).a.M_2 \xrightarrow{4} \epsilon(1).a.M_2} \qquad \dfrac{\overline{b.M_2 \xrightarrow{1} b.M_2}}{\epsilon(3).b.M_2 \xrightarrow{4} b.M_2}}{\epsilon(5).a.M_2 + \epsilon(3).b.M_2 \xrightarrow{4} \epsilon(1).a.M_2 + b.M_2}}{M_2 \xrightarrow{4} \epsilon(1).a.M_2 + b.M_2}$$

   - M_3 cannot delay four time units, since a tau transition i enabled after just three time units.
   - M_4 has the same problem as M_3.

2. Discuss the general relationship between process terms $\epsilon(d).(P + Q)$ and $\epsilon(d).P + \epsilon(d).Q$.

   The above processes are equivalent as, unlike action prefixes, delay prefixes are distributive. This is because delays to not resolve non-determinism.

Exercise 2*:

Consider the agent M and the three variants of agent N:

- $M \stackrel{\text{def}}{=} \epsilon(3).(\epsilon(2).\bar{a}.M + \bar{b}.M)$
- $N_1 \stackrel{\text{def}}{=} \epsilon(5).b.N_1 + \epsilon(3).a.N_1$
- $N_2 \stackrel{\text{def}}{=} \epsilon(3).(\epsilon(2).a.N_2 + \tau.N_2)$
- $N_3 \stackrel{\text{def}}{=} \epsilon(5).\tau.N_3 + \epsilon(3).b.N_3$

Indicate the values of i for which *a)* $M \mid N_i \xrightarrow{3}$, *b)* $M \mid N_i \xrightarrow{5}$, and *c)* $M \mid N_i \xrightarrow{8}$. In the affirmative cases give proper proofs using the SOS rules for TCCS.

- $M \mid N_1$ can delay for both three and five time units but cannot delay for eight time units, since after five time units both b and b are enabled for synchronization and must be engaged. The proofs for the two former are as follows:
  a)

$$\dfrac{\dfrac{\dfrac{}{\epsilon(3).(\epsilon(2).\bar{a}.M + \bar{b}.M) \xrightarrow{3} \epsilon(2).\bar{a}.M + \bar{b}.M}}{M \xrightarrow{3} \epsilon(2).\bar{a}.M + \bar{b}.M} \qquad \dfrac{\dfrac{\dfrac{}{\epsilon(5).b.N_1 \xrightarrow{3} \epsilon(2).b.N_1} \quad \dfrac{}{\epsilon(3).a.N_1 \xrightarrow{3} a.N_1}}{\epsilon(5).b.N + \epsilon(3).a.N \xrightarrow{3} \epsilon(2).b.N_1 + a.N_1}}{N_1 \xrightarrow{3} \epsilon(2).b.N_1 + a.N_1}}{M \mid N_1 \xrightarrow{3} \epsilon(2).\bar{a}.M + \bar{b}.M \mid \epsilon(2).b.N_1 + a.N_1}$$

  b)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\epsilon(2).\bar{a}.M \xrightarrow{2} \bar{a}.M} \quad \dfrac{}{\bar{b}.M \xrightarrow{2} \bar{b}.M}}{\epsilon(2).\bar{a}.M + \bar{b}.M \xrightarrow{2} \bar{a}.M + \bar{b}.M}}{\epsilon(3).(\epsilon(2).\bar{a}.M + \bar{b}.M) \xrightarrow{5} \bar{a}.M + \bar{b}.M}}{M \xrightarrow{5} \bar{a}.M + \bar{b}.M} \qquad \dfrac{\dfrac{\dfrac{}{\epsilon(5).b.N_1 \xrightarrow{5} b.N_1} \quad \dfrac{\dfrac{}{a.N_1 \xrightarrow{2} a.N_1}}{\epsilon(3).a.N_1 \xrightarrow{5} a.N_1}}{\epsilon(5).b.N + \epsilon(3).a.N \xrightarrow{5} b.N_1 + a.N_1}}{N_1 \xrightarrow{5} b.N_1 + a.N_1}}{M \mid N_1 \xrightarrow{5} \bar{a}.M + \bar{b}.M \mid b.N_1 + a.N_1}$$

- $M \mid N_2$ can only delay for no more than three time units as N_2 itself has an enabled tau transition after three time units.
  a)

$$\dfrac{\dfrac{\dfrac{}{\epsilon(3).(\epsilon(2).\bar{a}.M + \bar{b}.M) \xrightarrow{3} \epsilon(2).\bar{a}.M + \bar{b}.M}}{M \xrightarrow{3} \epsilon(2).\bar{a}.M + \bar{b}.M} \qquad \dfrac{\dfrac{}{\epsilon(3).(\epsilon(2).b.N + \tau.N) \xrightarrow{3} \epsilon(2).b.N_2 + \tau.N_2}}{N_2 \xrightarrow{3} \epsilon(2).b.N_2 + \tau.N_2}}{M \mid N_2 \xrightarrow{3} \epsilon(2).\bar{a}.M + \bar{b}.M \mid \epsilon(2).b.N_2 + \tau.N_2}$$

- $M \mid N_3$ can delay up to three time units after which M and N_3 are enabled to synchronized on the b action and must do so.
  a)

$$\dfrac{\dfrac{\dfrac{}{\epsilon(3).(\epsilon(2).\bar{a}.M + \bar{b}.M) \xrightarrow{3} \epsilon(2).\bar{a}.M + \bar{b}.M}}{M \xrightarrow{3} \epsilon(2).\bar{a}.M + \bar{b}.M} \qquad \dfrac{\dfrac{\dfrac{}{\epsilon(5).\tau.N_3 \xrightarrow{3} \epsilon(2).\tau.N_3} \quad \dfrac{}{\epsilon(3).b.N_3 \xrightarrow{3} b.N_3}}{\epsilon(5).\tau.N + \epsilon(3).b.N \xrightarrow{3} \epsilon(2).\tau.N_3 + b.N_3}}{N_3 \xrightarrow{3} \epsilon(2).\tau.N_3 + b.N_3}}{M \mid N_3 \xrightarrow{3} \epsilon(2).\bar{a}.M + \bar{b}.M \mid \epsilon(2).\tau.N_3 + b.N_3}$$

Exercise 4:

- **TCCS process:**

$$
\begin{array}{rcl}
T & \stackrel{\text{def}}{=} & set_5.T_5 + set_{10}.T_{10} + set_{30}.T_{30} \\
T_5 & \stackrel{\text{def}}{=} & \epsilon(5).\overline{to}.T + T \\
T_{10} & \stackrel{\text{def}}{=} & \epsilon(10).\overline{to}.T + T \\
T_{30} & \stackrel{\text{def}}{=} & \epsilon(30).\overline{to}.T + T
\end{array}
$$

- **Timed Automata:** A solution model can be seen in Figure 1.

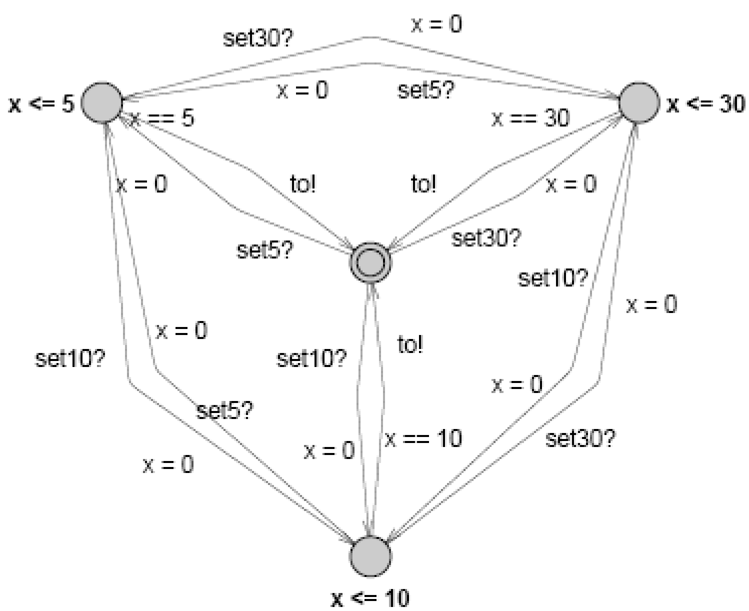

Figure 1

Exercise 5:

– Timed automaton