Overview

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

Computation Tree Logic

Equivalences and Abstraction

Linear vs branching time

Linear vs branching time

transition system
$\mathcal{T} = (S, Act, \longrightarrow, S_0, AP, L)$

abstraction from actions

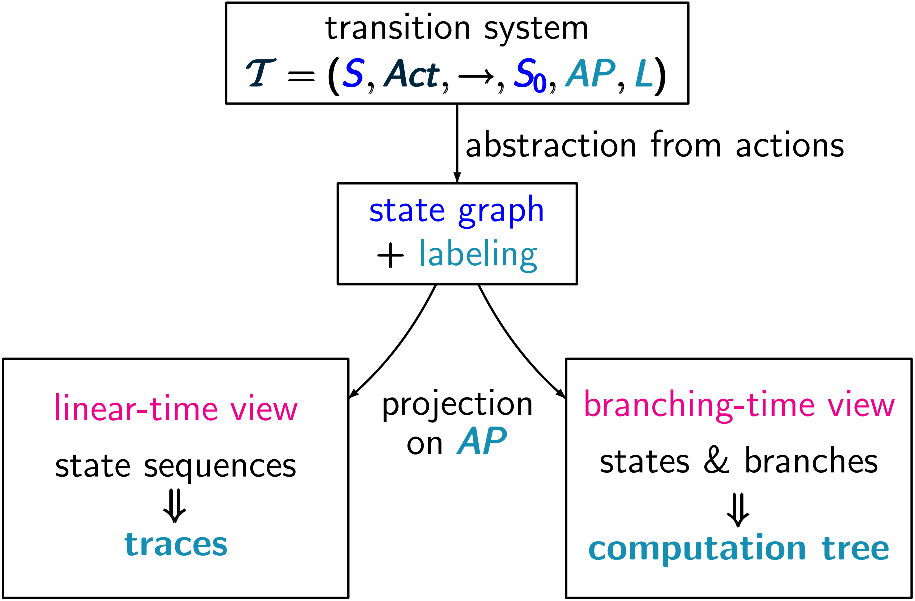
state graph
+ labeling

linear-time view
state sequences

branching-time view
states & branches

Linear vs branching time

CTLSS4.1-1



Computation tree

Computation tree

The computation tree of a transition system $\mathcal{T} = (S, Act, \rightarrow, s_0, AP, L)$ arises by:

- unfolding into a tree
- abstraction from the actions
- projection of the states s to their labels $L(s) \subseteq AP$

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- unfolding $\mathcal{T}_{s_0} = (S, Act, \rightarrow, s_0, AP, L)$ into a tree
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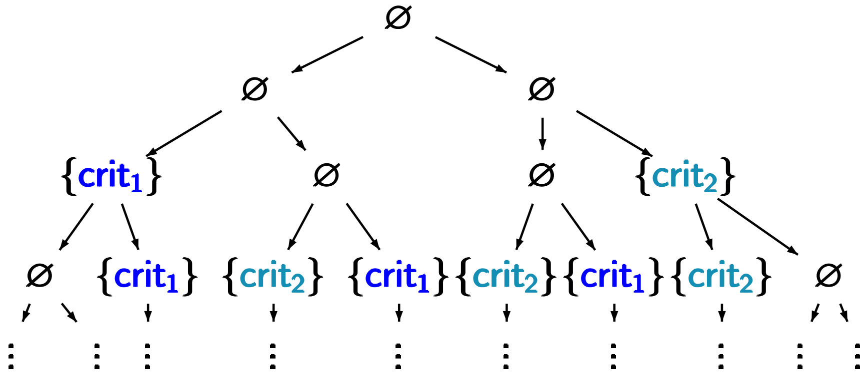
Example: computation tree

mutual exclusion with semaphore and $AP = \{crit_1, crit_2\}$:



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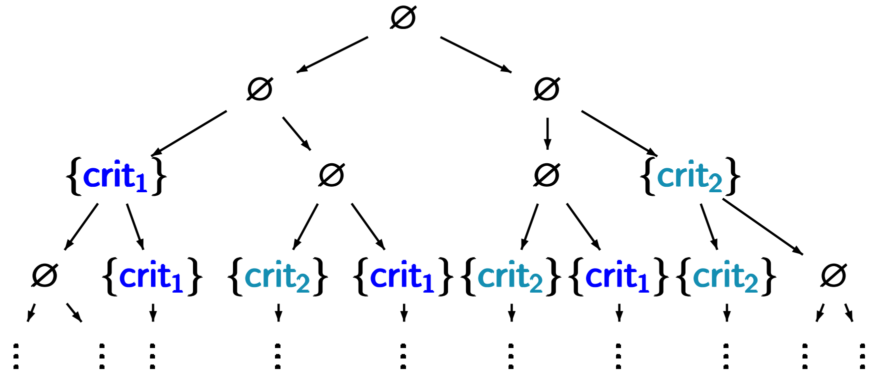

path $\langle nc_1, nc_2\rangle$ $\langle wait_1, nc_2\rangle$ $\langle crit_1, nc_2\rangle$ $\langle crit_1, wait_2\rangle$ ...

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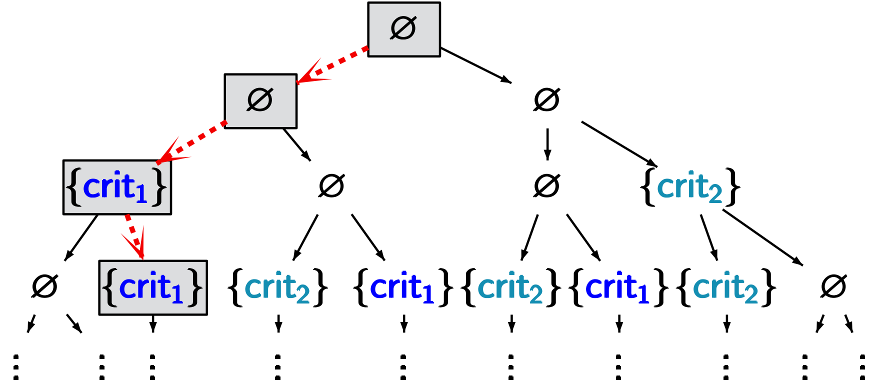

path	$\langle nc_1, nc_2\rangle$	$\langle wait_1, nc_2\rangle$	$\langle crit_1, nc_2\rangle$	$\langle crit_1, wait_2\rangle$	...
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	
trace	$\varnothing$	$\varnothing$	$\{crit_1\}$	$\{crit_1\}$	...

Linear vs. branching time

	linear time	branching time
behavior	path based	state based

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fairness	can be encoded	requires special treatment

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Computation Tree Logic

- syntax and semantics of CTL $\longleftarrow$
- expressiveness of CTL and LTL
- CTL model checking
- fairness, counterexamples/witnesses
- CTL$^+$ and CTL*

Equivalences and Abstraction

Computation Tree Logic (CTL)

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CTL (state) formulas:

$$\Phi \quad ::= \quad \mathit{true} \mid a \mid \Phi_1 \wedge \Phi_2 \mid \neg\Phi \mid \exists\varphi \mid \forall\varphi$$

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Syntax of CTL

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unconditional process fairness $\forall\Box\forall\Diamond \mathit{crit}_1 \wedge \forall\Box\forall\Diamond \mathit{crit}_2$

Example: 15-puzzle

CTLSS4.1-5

6	8	2	12
4	1	13	5
	9	10	14
7	11	15	3

$\overset{?}{\rightsquigarrow}$

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5	6	7	8
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$\uparrow$

states:	game configurations
transitions:	legal moves

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- representation as parallel system:

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CTL specification: seeking for a **witness** for

$$\exists\Diamond \bigwedge_{1 \leq i \leq 15} \text{“}\mathit{piece}\ i \text{ on } \mathit{field}[i]\text{”}$$

Semantics of CTL

define a satisfaction relation $\models$ for CTL formulas over AP and a given TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

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- interpretation of state formulas over the states
- interpretation of path formulas over the paths (infinite path fragments)

Satisfaction relation for path formulas

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Let $\pi = s_0\, s_1\, s_2\, \ldots$ be an infinite path fragment.

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\pi \models \bigcirc \Phi & \text{iff} & s_1 \models \Phi \\
\pi \models \Phi_1 \, \mathsf{U} \, \Phi_2 & \text{iff} & \text{there exists } j \geq 0 \text{ such that} \\
 & & \qquad s_j \models \Phi_2 \\
 & & \qquad s_k \models \Phi_1 \text{ for } 0 \leq k < j
\end{array}
$$

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$\pi \models \Phi_1 \, \mathsf{U} \, \Phi_2$	iff	there exists $j \geq 0$ such that $s_j \models \Phi_2$ and $s_k \models \Phi_1$ for $0 \leq k < j$

semantics of derived operators:

$\pi \models \Diamond \Phi$ iff there exists $j \geq 0$ with $s_j \models \Phi$

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$\pi \models \Box\Phi$ iff for all $j \geq 0$ we have: $s_j \models \Phi$

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Satisfaction relation for state formulas

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$s \models \exists\varphi$	iff	there is a path $\pi \in \mathit{Paths}(s)$ s.t. $\pi \models \varphi$
$s \models \forall\varphi$	iff	for each path $\pi \in \mathit{Paths}(s)$: $\pi \models \varphi$

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satisfaction set for state formula Φ:

$$\mathit{Sat}(\Phi) \stackrel{\text{def}}{=} \{s \in S : s \models \Phi\}$$

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$$s \models \exists \bigcirc \Phi \quad \text{iff} \quad \text{there exists } \pi = s\, s_1\, s_2 \ldots \in Paths(s) \text{ s.t. } \pi \models \bigcirc \Phi$$

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$s \models \exists\bigcirc\Phi$ iff there exists $\pi = s\, s_1\, s_2\, \ldots \in \mathit{Paths}(s)$
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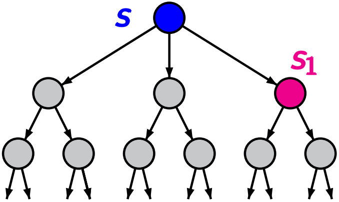
$\exists\bigcirc\Phi$



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$\pi \models \bigcirc \Phi$

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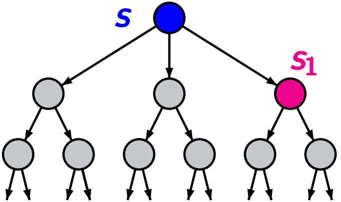

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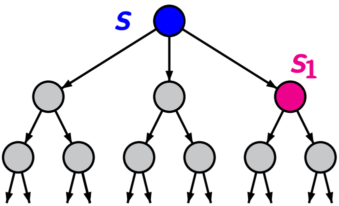

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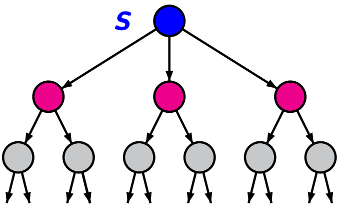
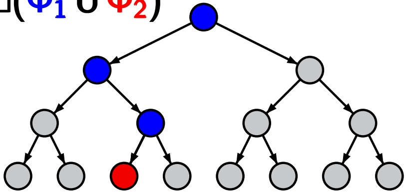


$Post(s) \subseteq Sat(\Phi)$

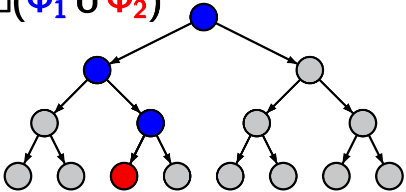
Semantics of until

$\exists(\Phi_1 \, \mathsf{U} \, \Phi_2)$

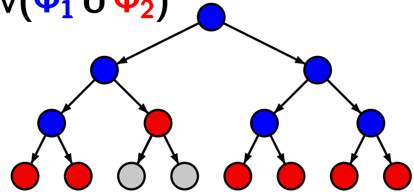


Semantics of until

$\exists(\Phi_1 \, \mathsf{U} \, \Phi_2)$

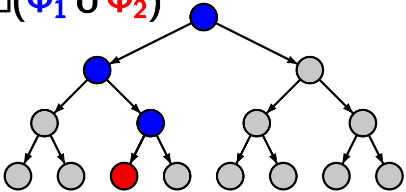


$\forall(\Phi_1 \, \mathsf{U} \, \Phi_2)$

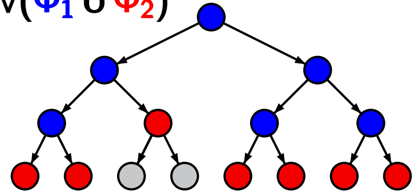


Semantics of until and eventually

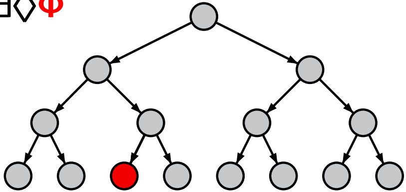
$\exists(\Phi_1 \,\mathsf{U}\, \Phi_2)$



$\forall(\Phi_1 \,\mathsf{U}\, \Phi_2)$

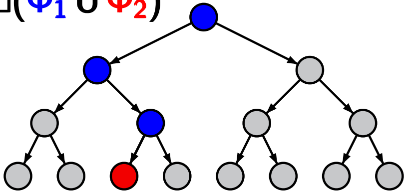


$\exists\Diamond\Phi$

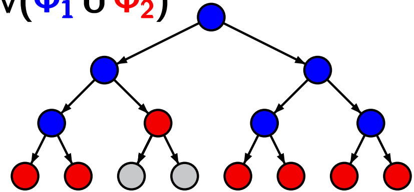


Semantics of until and eventually

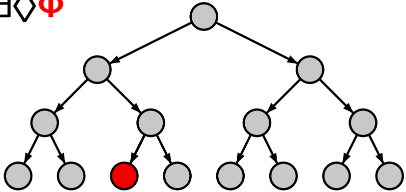
$\exists(\Phi_1 \, \mathsf{U} \, \Phi_2)$



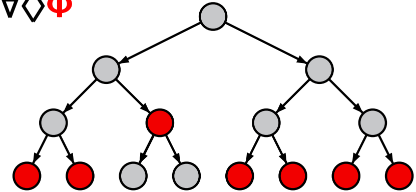
$\forall(\Phi_1 \, \mathsf{U} \, \Phi_2)$



$\exists\lozenge\Phi$

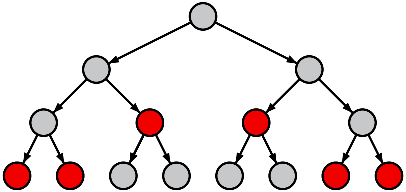


$\forall\lozenge\Phi$

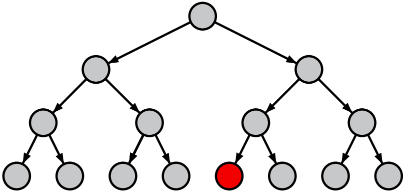


Semantics of eventually and always

$\forall\Diamond\Phi$

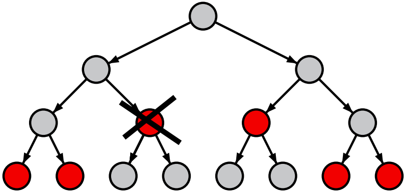


$\exists\Diamond\Phi$

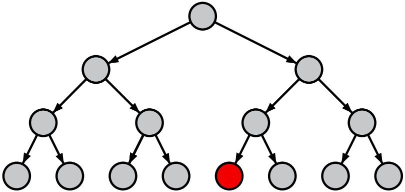


Semantics of eventually and always

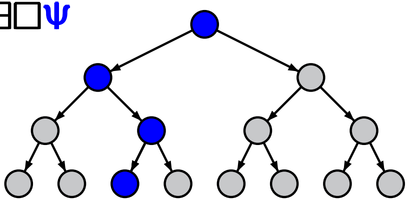
$\neg \forall \Diamond \Phi$



$\exists \Diamond \Phi$

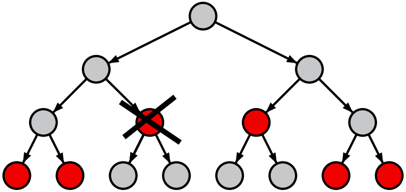


$\exists \Box \Psi$

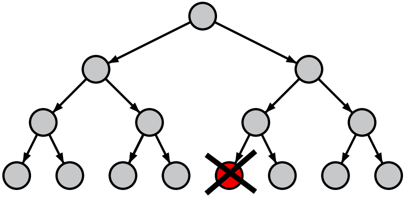


Semantics of eventually and always

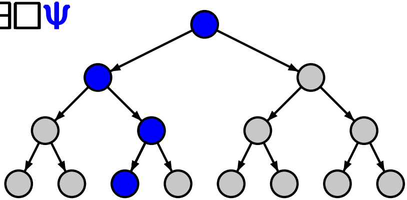
$\neg\forall\Diamond\Phi$



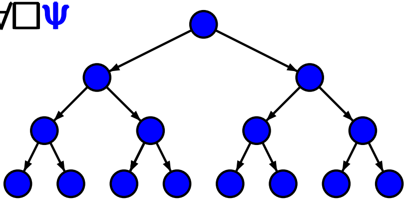
$\neg\exists\Diamond\Phi$



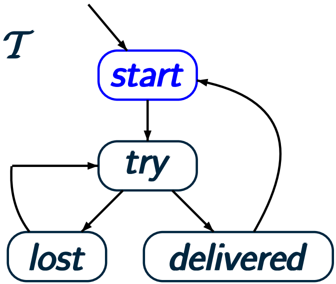
$\exists\Box\Psi$



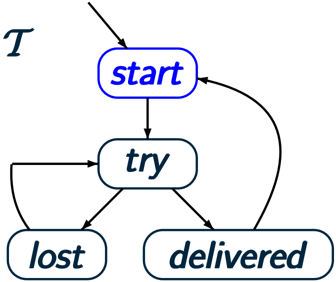
$\forall\Box\Psi$



Example for CTL semantics



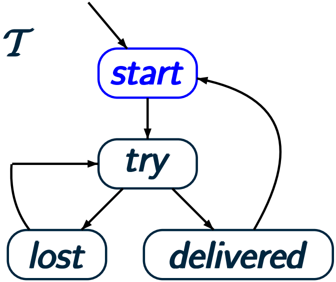
Example for CTL semantics



CTL formula

$\Phi = \forall\Box\forall\Diamond \textit{start}$

Example for CTL semantics

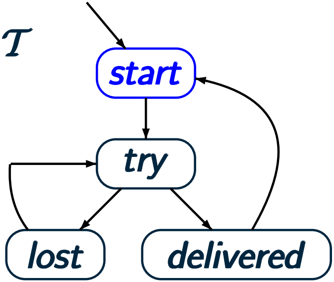


CTL formula

$\Phi = \forall\Box \boxed{\forall\Diamond start}$

$Sat(\forall\Diamond start) = \textbf{?}$

Example for CTL semantics

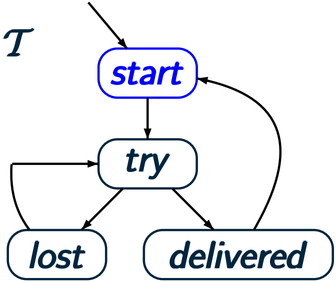


CTL formula

$\Phi = \forall\Box \boxed{\forall\Diamond start}$

$Sat(\forall\Diamond start) = \{start, delivered\}$

Example for CTL semantics

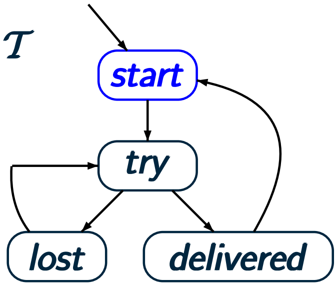


CTL formula

$$\Phi = \forall\Box\forall\Diamond start \quad \widehat{=} \quad \forall\Box(start \vee delivered)$$

$$Sat(\forall\Diamond start) = \{start, delivered\}$$

Example for CTL semantics



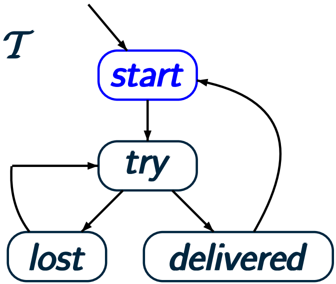
CTL formula

$$\Phi = \forall\Box\forall\Diamond start \quad \widehat{=} \quad \forall\Box(start \vee delivered)$$

$$Sat(\forall\Diamond start) = \{start, delivered\}$$

$$Sat(\Phi) = \varnothing$$

Example for CTL semantics



$\mathcal{T} \not\models \forall\Box\forall\Diamond start$

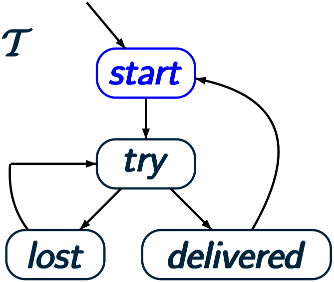
CTL formula

$\Phi = \forall\Box\forall\Diamond start \quad \widehat{=} \quad \forall\Box(start \vee delivered)$

$$Sat(\forall\Diamond start) = \{start, delivered\}$$

$$Sat(\Phi) = \varnothing$$

Example for CTL semantics



$\mathcal{T} \not\models \forall\Box\forall\Diamond start$

"infinitely often *start*"

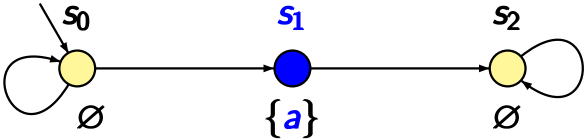
CTL formula

$\Phi = \forall\Box\forall\Diamond start \quad \widehat{=} \quad \forall\Box(start \vee delivered)$

$$Sat(\forall\Diamond start) = \{start, delivered\}$$

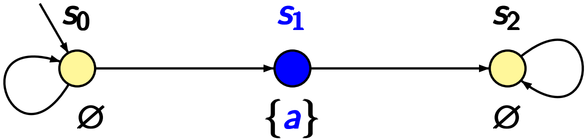
$$Sat(\Phi) = \varnothing$$

Example: CTL semantics



does $\mathcal{T} \models \exists\bigcirc\forall\Box\neg a$ hold ?

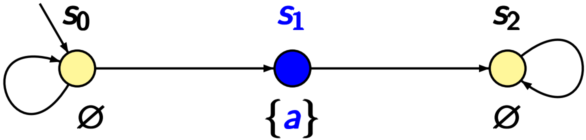
Example: CTL semantics



does $\mathcal{T} \models \exists\bigcirc\forall\Box\neg a$ hold **?**

answer: no

Example: CTL semantics

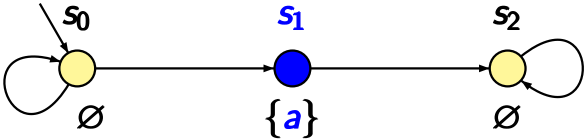


does $\mathcal{T} \models \exists\bigcirc\forall\Box\neg a$ hold ?

answer: no

$$\boxed{Sat(\forall\Box\neg a) \;=\; \{s_2\}}$$

Example: CTL semantics

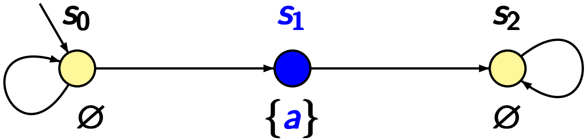


does $\mathcal{T} \models \exists\bigcirc\forall\Box\neg a$ hold ?

answer: no

$$\begin{array}{rcl} Sat(\forall\Box\neg a) & = & \{s_2\} \\ Sat(\exists\bigcirc\forall\Box\neg a) & = & \{s_2, s_1\} \end{array}$$

Example: CTL semantics

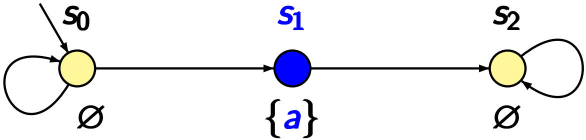


does $\mathcal{T} \models \exists\bigcirc\forall\Box\neg a$ hold ?

answer: no

does $\mathcal{T} \models \forall\Box\exists\bigcirc\neg a$ hold ?

Example: CTL semantics



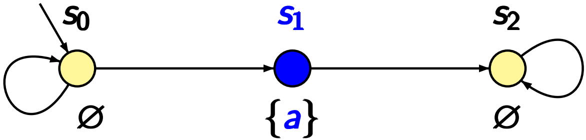
does $\mathcal{T} \models \exists\bigcirc\forall\Box\neg a$ hold ?

answer: no

does $\mathcal{T} \models \forall\Box\exists\bigcirc\neg a$ hold ?

answer: yes

Example: CTL semantics



does $\mathcal{T} \models \exists\bigcirc\forall\Box\neg a$ hold ?

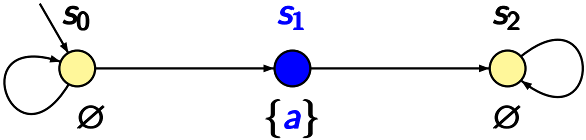
answer: no

does $\mathcal{T} \models \forall\Box\exists\bigcirc\neg a$ hold ?

answer: yes

$$\boxed{\mathit{Sat}(\exists\bigcirc\neg a) = \{s_0, s_1, s_2\}}$$

Example: CTL semantics



does $\mathcal{T} \models \exists\bigcirc\forall\Box\neg a$ hold ?

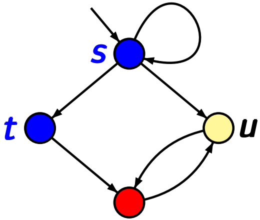
answer: no

does $\mathcal{T} \models \forall\Box\exists\bigcirc\neg a$ hold ?

answer: yes

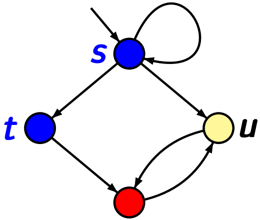
$$\boxed{\begin{array}{rcl} Sat(\exists\bigcirc\neg a) & = & \{s_0, s_1, s_2\} \\ Sat(\forall\Box\exists\bigcirc\neg a) & = & \{s_0, s_1, s_2\} \end{array}}$$

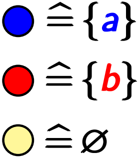
Example: CTL semantics



$\mathcal{T} \models \exists\Box\exists(a \cup b)$?

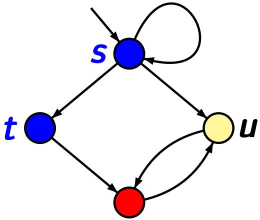
Example: CTL semantics





$\mathcal{T} \models \exists\Box\exists(a \,\mathsf{U}\, b)$ $\quad\checkmark$ as $s \models \exists(a \,\mathsf{U}\, b)$

Example: CTL semantics



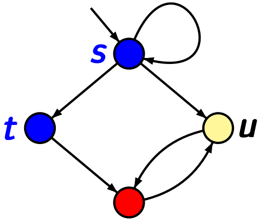
$\bullet \mathrel{\widehat{=}} \{a\}$

$\bullet \mathrel{\widehat{=}} \{b\}$

$\circ \mathrel{\widehat{=}} \varnothing$

$\mathcal{T} \models \exists\Box\exists(a \,\mathsf{U}\, b)$ $\quad\surd\quad$ as $s\,s\,s\ldots \models \Box\exists(a \,\mathsf{U}\, b)$

Example: CTL semantics



$\bullet \mathrel{\widehat{=}} \{a\}$

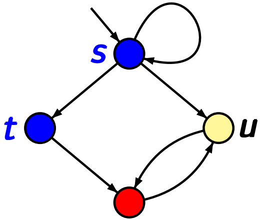
$\bullet \mathrel{\widehat{=}} \{b\}$

$\circ \mathrel{\widehat{=}} \varnothing$

$\mathcal{T} \models \exists\Box\exists(a \,\mathsf{U}\, b)$ $\quad\checkmark\quad$ as $s\,s\,s\,\ldots \models \Box\exists(a \,\mathsf{U}\, b)$

$\mathcal{T} \models \exists((\exists\bigcirc a) \,\mathsf{U}\, b)$ $\quad$ **?**

Example: CTL semantics



$\bullet \mathrel{\widehat{=}} \{a\}$ (blue)

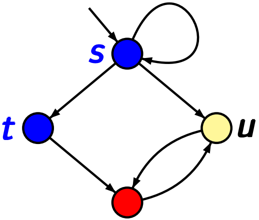
$\bullet \mathrel{\widehat{=}} \{b\}$ (red)

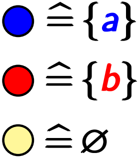
$\bullet \mathrel{\widehat{=}} \varnothing$ (yellow)

$\mathcal{T} \models \exists\Box\exists(a \,\mathsf{U}\, b)$ $\quad \surd \quad$ as $s\,s\,s\,\ldots \models \Box\exists(a \,\mathsf{U}\, b)$

$\mathcal{T} \not\models \exists((\exists\bigcirc a) \,\mathsf{U}\, b)$ $\quad$ as $t \not\models \exists\bigcirc a$, $u \not\models \exists\bigcirc a$

Example: CTL semantics



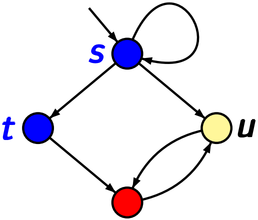


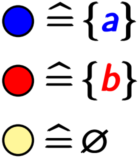
$\mathcal{T} \models \exists\Box\exists(a \,\mathsf{U}\, b)$ $\quad\surd\quad$ as $s\,s\,s\,\ldots \models \Box\exists(a \,\mathsf{U}\, b)$

$\mathcal{T} \not\models \exists((\exists\bigcirc a) \,\mathsf{U}\, b)$ $\quad$ as $t \not\models \exists\bigcirc a$, $u \not\models \exists\bigcirc a$

$\mathcal{T} \models \exists(a \,\mathsf{U}\, \forall(\neg a \,\mathsf{U}\, b))$ $\quad$?

Example: CTL semantics

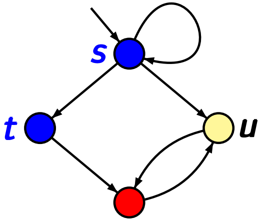


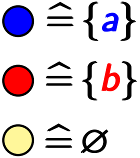


$\mathcal{T} \models \exists\Box\exists(a \,\mathsf{U}\, b)$ $\quad\surd\quad$ as $s\,s\,s\,\ldots \models \Box\exists(a \,\mathsf{U}\, b)$

$\mathcal{T} \not\models \exists((\exists\bigcirc a) \,\mathsf{U}\, b)$ $\quad$ as $t \not\models \exists\bigcirc a$, $u \not\models \exists\bigcirc a$

$\mathcal{T} \models \exists(a \,\mathsf{U}\, \forall(\neg a \,\mathsf{U}\, b))$ $\quad\surd$

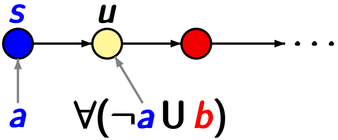




$\mathcal{T} \models \exists\Box\exists(a \,\mathsf{U}\, b)$ $\quad\checkmark\quad$ as $s\,s\,s\ldots \models \Box\exists(a \,\mathsf{U}\, b)$

$\mathcal{T} \not\models \exists((\exists\bigcirc a) \,\mathsf{U}\, b)$ $\quad$ as $t \not\models \exists\bigcirc a$, $u \not\models \exists\bigcirc a$

$\mathcal{T} \models \exists(a \,\mathsf{U}\, \forall(\neg a \,\mathsf{U}\, b))$ $\quad\checkmark$



$\models a \,\mathsf{U}\, \forall(\neg a \,\mathsf{U}\, b)$