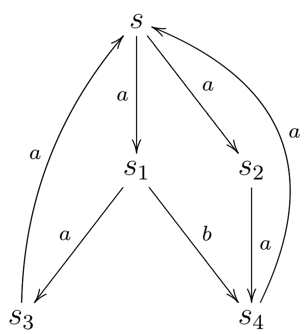
Exercise 1*

Consider the following labelled transition system.



1. Decide whether the state s satisfies the following formulae of Hennessy-Milner logic:

   - $s \stackrel{?}{\models} \langle a \rangle tt$
   - $s \stackrel{?}{\models} \langle b \rangle tt$
   - $s \stackrel{?}{\models} [a] ff$
   - $s \stackrel{?}{\models} [b] ff$
   - $s \stackrel{?}{\models} [a]\langle b \rangle tt$
   - $s \stackrel{?}{\models} \langle a \rangle\langle b \rangle tt$
   - $s \stackrel{?}{\models} [a]\langle a \rangle[a][b] ff$
   - $s \stackrel{?}{\models} \langle a \rangle\big(\langle a \rangle tt \wedge \langle b \rangle tt\big)$
   - $s \stackrel{?}{\models} [a]\big(\langle a \rangle tt \vee \langle b \rangle tt\big)$
   - $s \stackrel{?}{\models} \langle a \rangle\big([b][a] ff \wedge \langle b \rangle tt\big)$
   - $s \stackrel{?}{\models} \langle a \rangle\big([a](\langle a \rangle tt \wedge [b] ff) \wedge \langle b \rangle ff\big)$

2. Compute the following sets according to the denotational semantics for Hennessy-Milner logic.

   - $[\![[a][b] ff]\!] = ?$
   - $[\![\langle a \rangle\big(\langle a \rangle tt \wedge \langle b \rangle tt\big)]\!] = ?$
   - $[\![[a][a][b] ff]\!] = ?$
   - $[\![[a]\big(\langle a \rangle tt \vee \langle b \rangle tt\big)]\!] = ?$

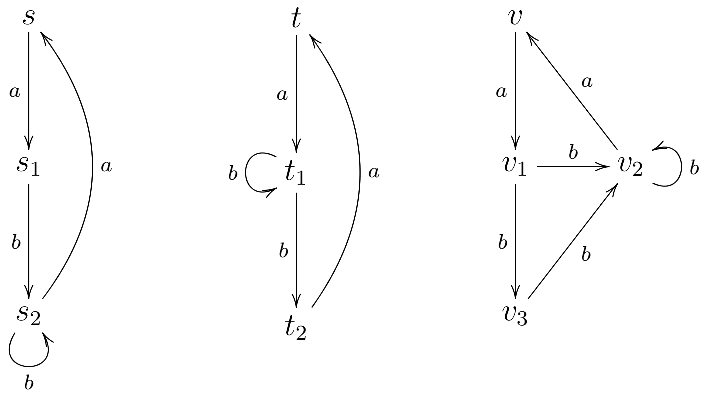
Exercise 2

Find (one) labelled transition system with an initial state s such that it satisfies (at the same time) the following properties:

- $s \models \langle a \rangle (\langle b \rangle \langle c \rangle tt \wedge \langle c \rangle tt)$
- $s \models \langle a \rangle \langle b \rangle ([a] ff \wedge [b] ff \wedge [c] ff)$
- $s \models [a] \langle b \rangle ([c] ff \wedge \langle a \rangle tt)$

Exercise 3*

Consider the following labelled transition system.



It is true that $s \not\sim t$, $s \not\sim v$ and $t \not\sim v$. Find a distinguishing formula of Hennessy-Milner logic for the pairs

- s and t
- s and v
- t and v.

Exercise 4*

For each of the following CCS expressions decide whether they are strongly bisimilar and if no, find a distinguishing formula in Hennessy-Milner logic.

- $b.a.Nil + b.Nil$ and $b.(a.Nil + b.Nil)$
- $a.(b.c.Nil + b.d.Nil)$ and $a.b.c.Nil + a.b.d.Nil$
- $a.Nil \mid b.Nil$ and $a.b.Nil + b.a.Nil$

- $(a.Nil \,|\, b.Nil) + c.a.Nil$ and $a.Nil \,|\, (b.Nil + c.Nil)$

Home exercise: verify your claims in CWB (use the `strongeq` and `checkprop` commands) and check whether you found the shortest distinguishing formula (use the `dfstrong` command).

Exercise 5 (optional)

Prove that for every Hennessy-Milner formula F and every state $p \in Proc$:

$$p \models F \quad \text{if and only if} \quad p \in [\![F]\!].$$

Hint: use structural induction on the structure of the formula F.