1. SIMPLE SEQUENTIAL TEST

SINGLE-SAMPLING-PLAN$(p_0, p_1, \alpha, \beta)$
  $n_{\min} \Leftarrow 1,\ n_{\max} \Leftarrow -1$
  $n \Leftarrow n_{\min}$
  while $n_{\max} < 0 \vee n_{\min} < n_{\max}$ **do**
    $x_0 \Leftarrow \tilde{F}^{-1}(\alpha; n, p_0)$
    $x_1 \Leftarrow \tilde{F}^{-1}(1-\beta; n, p_1)$
    if $x_0 \geq x_1 \wedge x_0 \geq 0$ **then**
      $n_{\max} \Leftarrow n$
    else
      $n_{\min} \Leftarrow n + 1$
    if $n_{\max} < 0$ **then**
      $n \Leftarrow 2 \cdot n$
    else
      $n \Leftarrow \lfloor (n_{\min} + n_{\max})/2 \rfloor$
  $n \Leftarrow n_{\max} - 1$
  repeat
    $n \Leftarrow n + 1$
    $c_0 \Leftarrow \lfloor \tilde{F}^{-1}(\alpha; n, p_0) \rfloor$
    $c_1 \Leftarrow \lceil \tilde{F}^{-1}(1-\beta; n, p_1) \rceil$
  until $c_0 \geq c_1$
  return $\langle n, \lfloor (c_0 + c_1)/2 \rfloor \rangle$

Algorithm 2.1: Procedure for finding an optimal single sampling plan using binary search. $\tilde{F}^{-1}(y; n, p)$ can be computed by adding the terms of (2.3) until the sum equals or exceeds y.

SIMPLE-SEQUENTIAL-TEST$(p_0, p_1, \alpha, \beta)$
  $\langle n, c \rangle \Leftarrow$ SINGLE-SAMPLING-PLAN$(p_0, p_1, \alpha, \beta)$
  $m \Leftarrow 0,\ d_m \Leftarrow 0$
  while $d_m \leq c \wedge d_m + n - m > c$ **do**
    $m \Leftarrow m + 1$
    $d_m \Leftarrow d_{m-1} + x_m$
  if $d_m > c$ **then**
    return H_0
  else
    return H_1

Algorithm 2.2: Sequential acceptance sampling procedure based on a single sampling plan.

2. SEQUENTIAL PROBABILITY RATIO TEST

```
SPRT(p_0, p_1, α, β)
  if p_0 = 1 ∨ p_1 = 0 then
    return SIMPLE-SEQUENTIAL-TEST(p_0, p_1, α, β)
  else
    m ⇐ 0,  f_m ⇐ 0
    while log β/(1−α) < f_m < log (1−β)/α do
      m ⇐ m + 1
      f_m ⇐ f_{m−1} + x_m log p_1/p_0 + (1 − x_m) log (1−p_1)/(1−p_0)
    if f_m ≤ log β/(1−α) then
      return H_0
    else
      return H_1
```

SPRT$(p_0, p_1, \alpha, \beta)$
  if $p_0 = 1 \vee p_1 = 0$ **then**
    return SIMPLE-SEQUENTIAL-TEST$(p_0, p_1, \alpha, \beta)$
  else
    $m \Leftarrow 0,\ f_m \Leftarrow 0$
    while $\log \frac{\beta}{1-\alpha} < f_m < \log \frac{1-\beta}{\alpha}$ **do**
      $m \Leftarrow m + 1$
      $f_m \Leftarrow f_{m-1} + x_m \log \frac{p_1}{p_0} + (1 - x_m) \log \frac{1-p_1}{1-p_0}$
    if $f_m \leq \log \frac{\beta}{1-\alpha}$ **then**
      return H_0
    else
      return H_1

Algorithm 2.3: Procedure implementing the sequential probability ratio test.

3. BAYESIAN HYPOTHESIS TESTING

Input : PBLTL property $P_{\geqslant\theta}(\phi)$, acceptance threshold $T \geqslant 1$, prior density g for (unknown) probability p that the system satisfies ϕ

Output: "$H_0 : p \geqslant \theta$ accepted", or "$H_1 : p < \theta$ accepted"

$n := 0$; {number of traces drawn so far}
$x := 0$; {number of traces satisfying ϕ so far}
loop
  $\sigma :=$ draw a sample trace of the system (iid); {Sect. 2}
  $n := n + 1$;
  if $\sigma \models \phi$ **then** {Sect. 3}
    $x := x + 1$
  end
  $\mathcal{B} :=$ BayesFactor(n, x); {Sect. 5}
  if $(\mathcal{B} > T)$ **then return** "H_0 accepted";
  if $(\mathcal{B} < \frac{1}{T})$ **then return** "H_1 accepted"
end loop;

Algorithm 1: Statistical Model Checking by Bayesian Hypothesis Testing

Bayes Factor :

$$\mathcal{B}_n = \frac{\pi_1}{\pi_0} \cdot \left(\frac{1}{F_{(x+\alpha, n-x+\beta)}(\theta)} - 1 \right).$$

$$\forall u \in (0,1) \quad F_{(\alpha,\beta)}(u) \mathrel{\widehat{=}} \int_0^u g(t, \alpha, \beta)\ dt = \frac{1}{B(\alpha, \beta)} \int_0^u t^{\alpha-1}(1-t)^{\beta-1}\ dt \quad (5)$$

$$B(\alpha, \beta) \mathrel{\widehat{=}} \int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt\ .$$

4. BAYESIAN INTERVAL ESTIMATES

Input : BLTL Property ϕ, half-interval size $\delta \in (0, \frac{1}{2})$, interval coverage coefficient $c \in (\frac{1}{2}, 1)$, Prior Beta distribution with parameters α, β for the (unknown) probability p that the system satisfies ϕ

Output: An interval (t_0, t_1) of width 2δ with posterior probability at least c, estimate $\hat{p}$ for the true probability p

$n := 0$; {number of traces drawn so far}
$x := 0$; {number of traces satisfying ϕ so far}
repeat
  $\sigma :=$ draw a sample trace of the system (iid);
  $n := n + 1$;
  if $\sigma \models \phi$ **then** $x := x + 1$;
  $\hat{p} := (x + \alpha)/(n + \alpha + \beta)$; {compute posterior mean}
  $(t_0, t_1) := (\hat{p} - \delta, \hat{p} + \delta)$; {compute interval estimate}
  if $t_1 > 1$ **then** $(t_0, t_1) := (1 - 2 \cdot \delta, 1)$
  else if $t_0 < 0$ **then** $(t_0, t_1) := (0, 2 \cdot \delta)$;
  {compute posterior probability of $p \in (t_0, t_1)$, by (9)}
  $\gamma :=$ PosteriorProb(t_0, t_1)
until $(\gamma \geqslant c)$;
return $(t_0, t_1), \hat{p}$

Algorithm 2: Statistical Model Checking by Bayesian Interval Estimates

Posterior Probability :

$$\int_{t_0}^{t_1} f(u|x_1, \ldots, x_n)du = F_{(x+\alpha, n-x+\beta)}(t_1) - F_{(x+\alpha, n-x+\beta)}(t_0)$$

5.