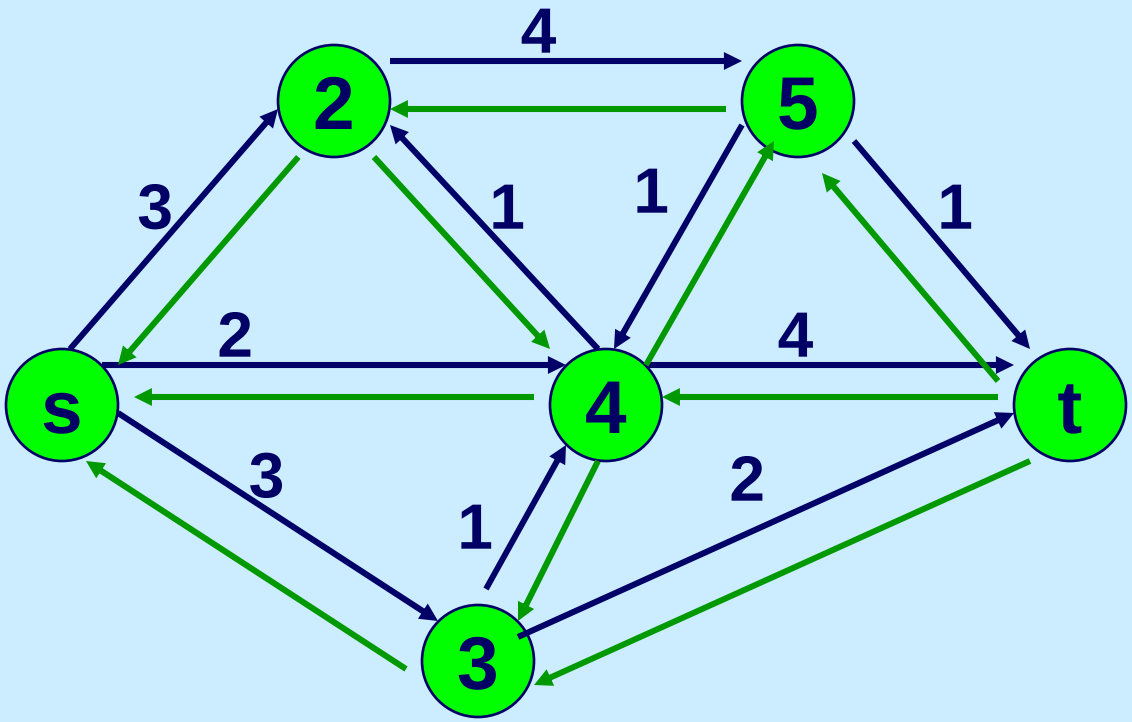
15.082 and 6.855J

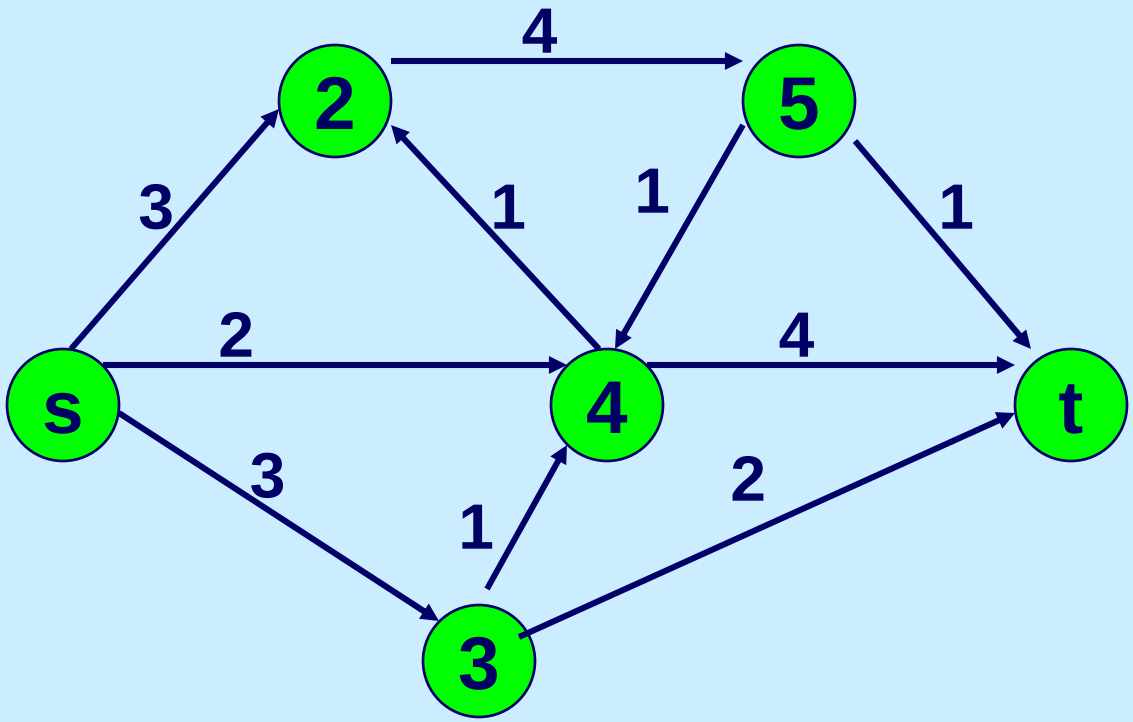
The Goldberg-Tarjan Preflow Push Algorithm for the Maximum Flow Problem

Preflow Push



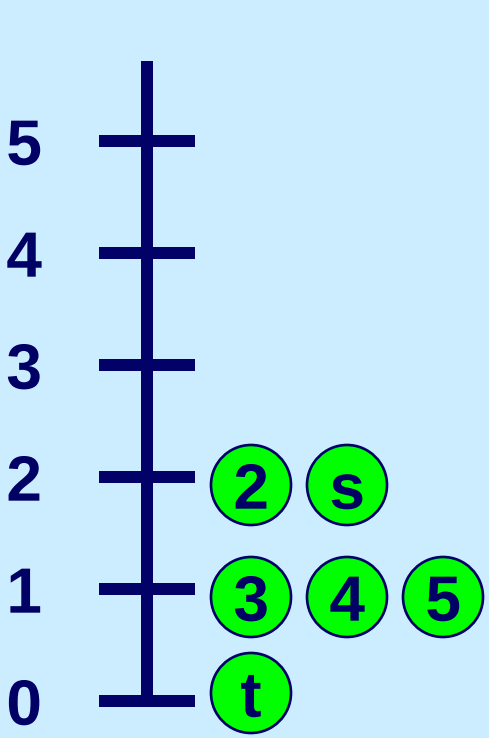
This is the original network, plus reversals of the arcs.

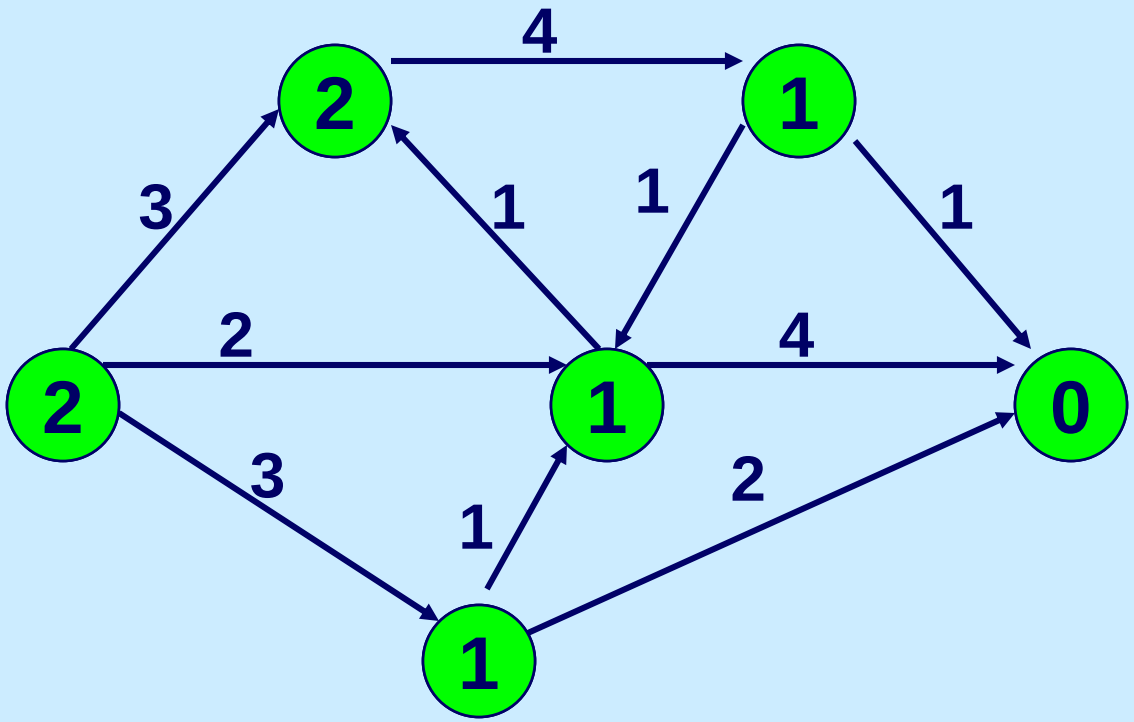
Preflow Push



This is the original network, and the original residual network.

Initialize Distances

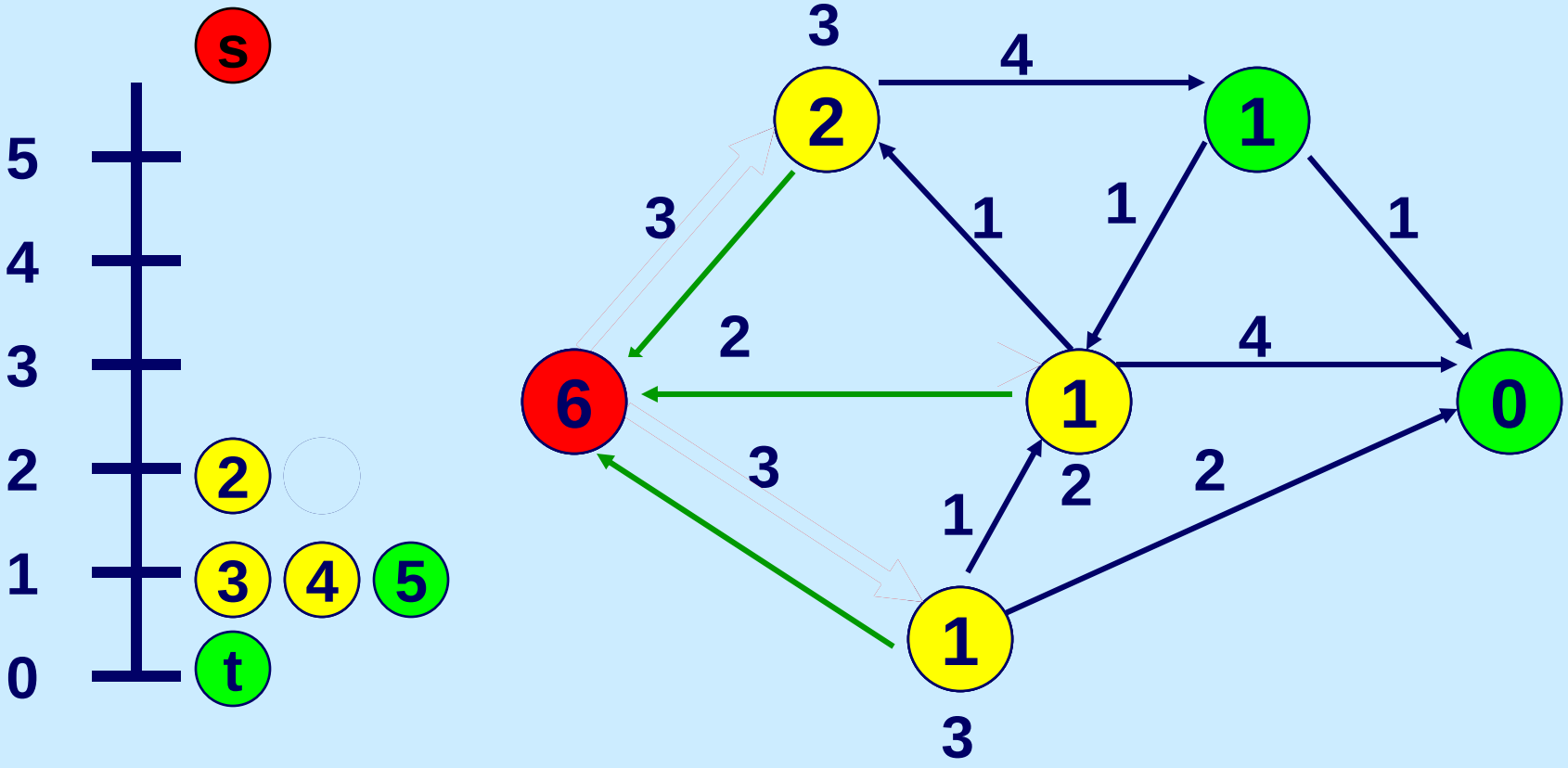




The node label henceforth will be the distance label.

d(j) is at most the distance of j to t in G(x)

Saturate Arcs out of node s

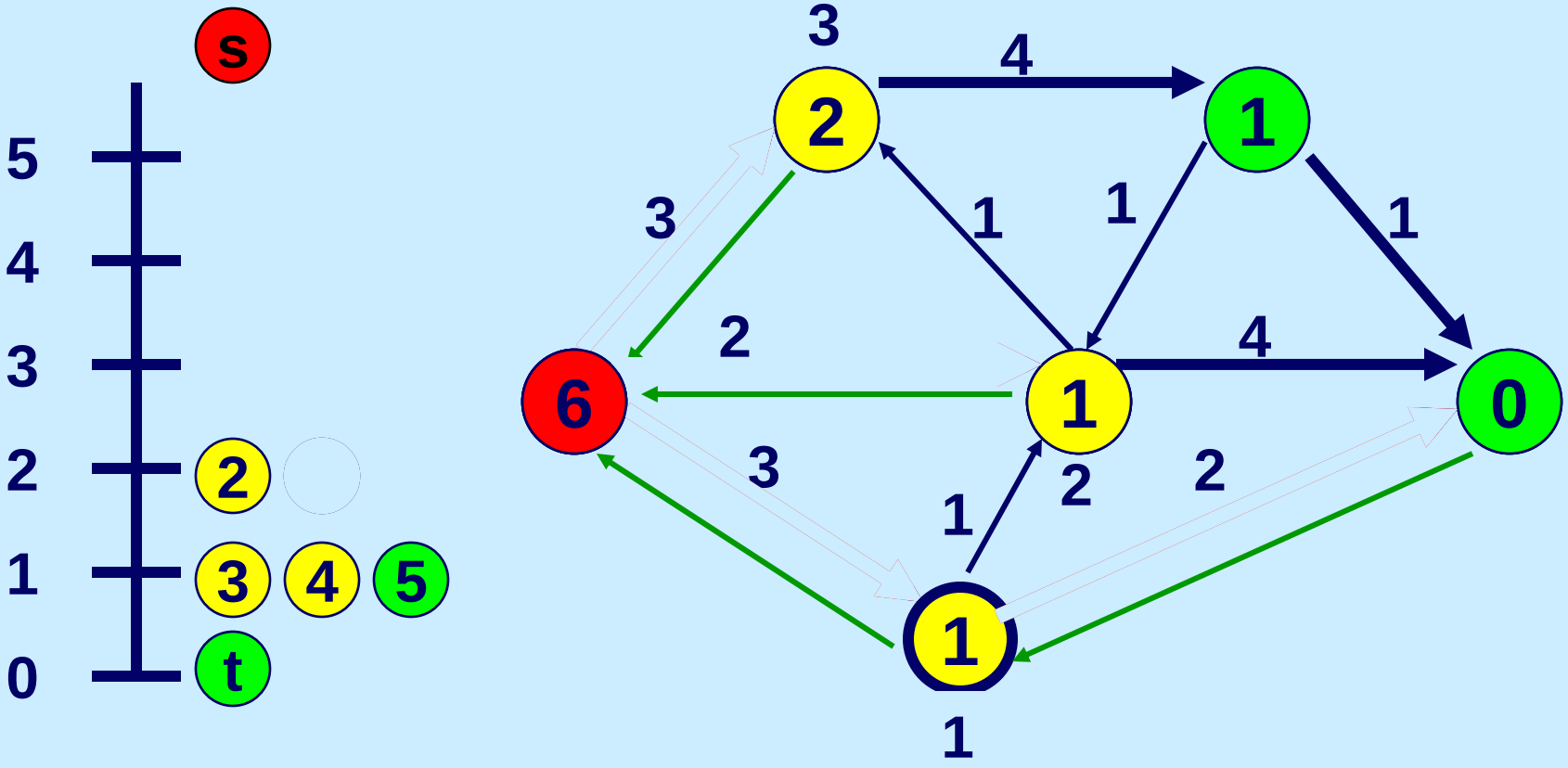


Saturate arcs out of node s.

Move excess to the adjacent arcs

Relabel node s after all incident arcs have been saturated.

Select, then relabel/push

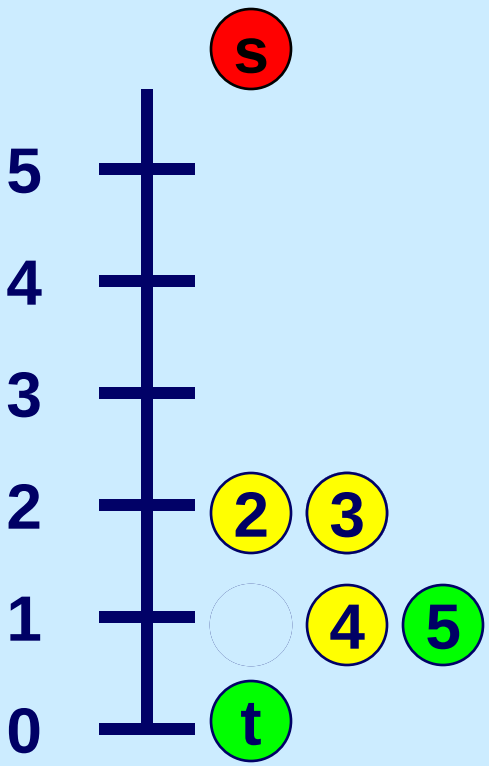


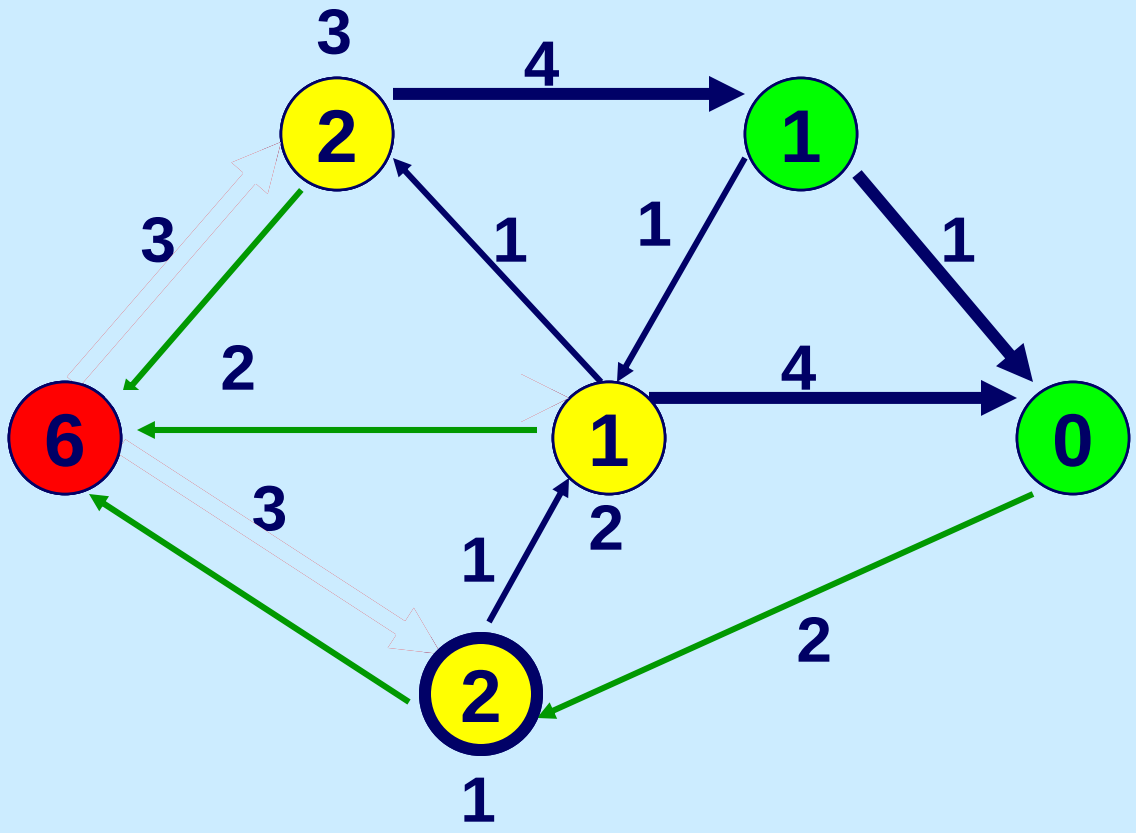
Select an active node, that is, one with excess

Push/Relabel

Update excess after a push

Select, then relabel/push

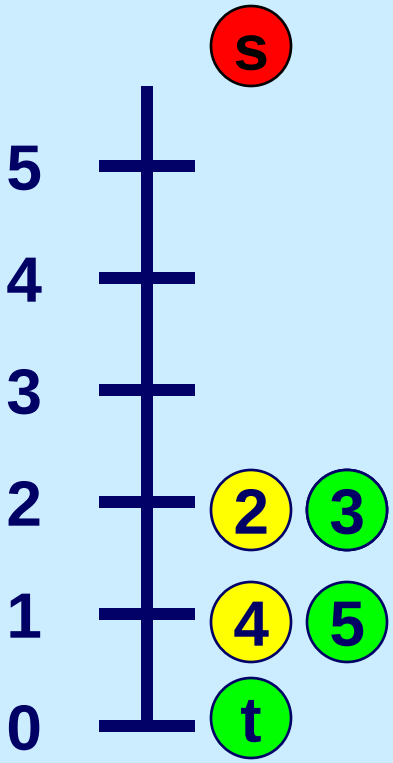


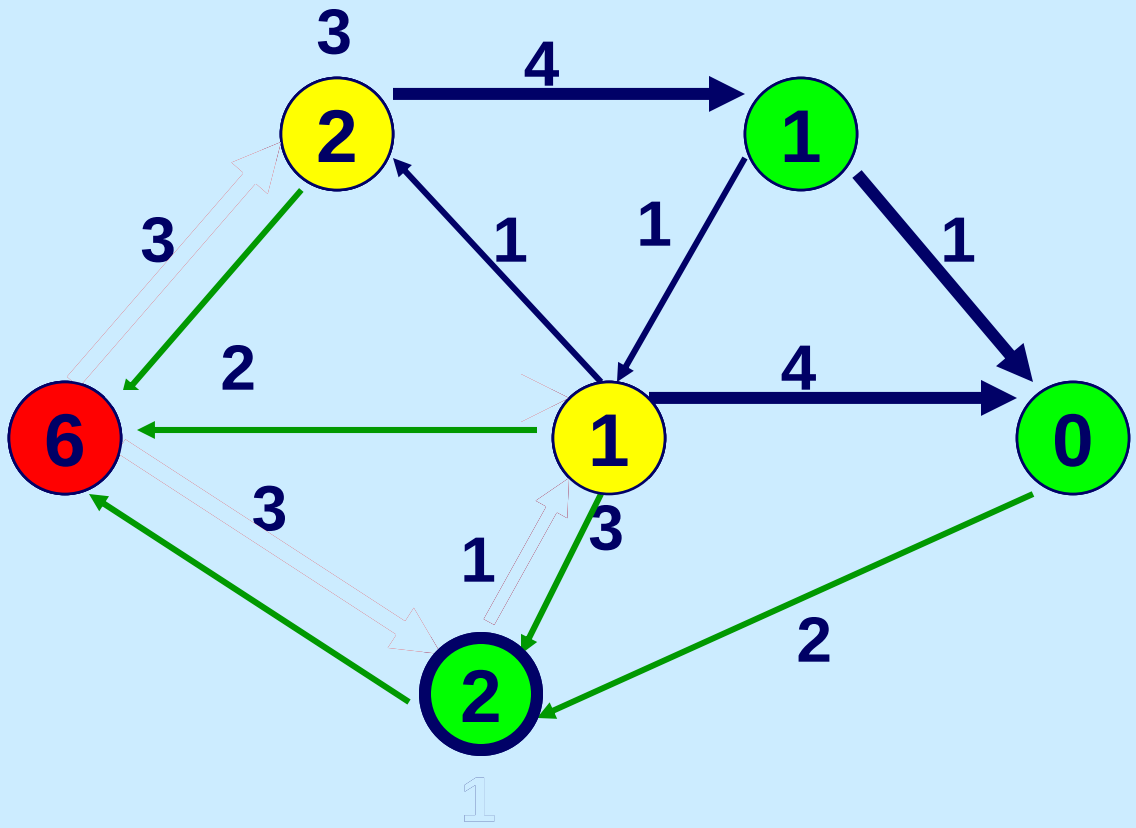


Select an active node, that is, one with excess

No arc incident to the selected node is admissible. So relabel.

Select, then relabel/push

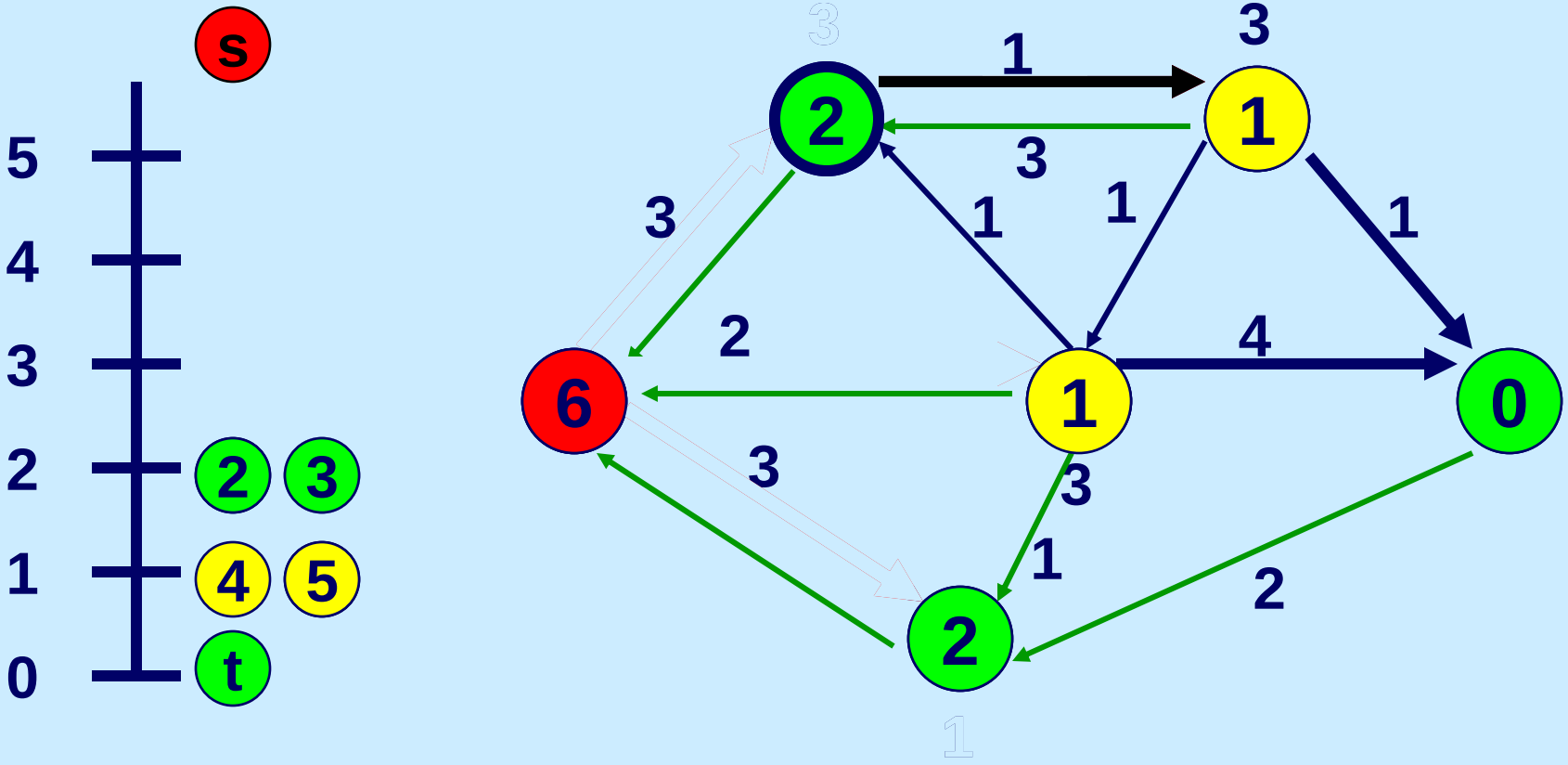




Select an active node, that is, one with excess

Push/Relabel

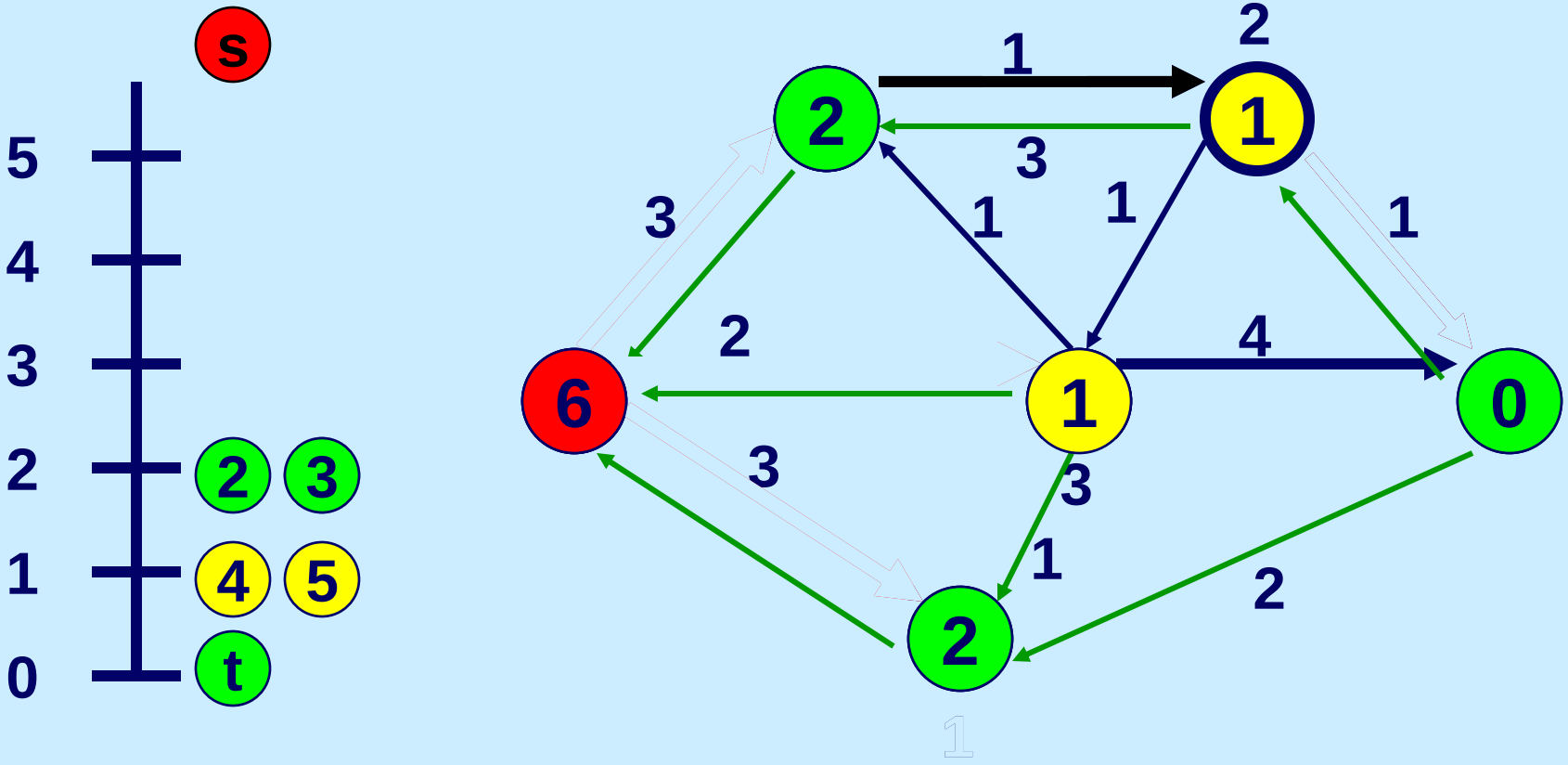
Select, then relabel/push



Select an active node.

Push/Relabel

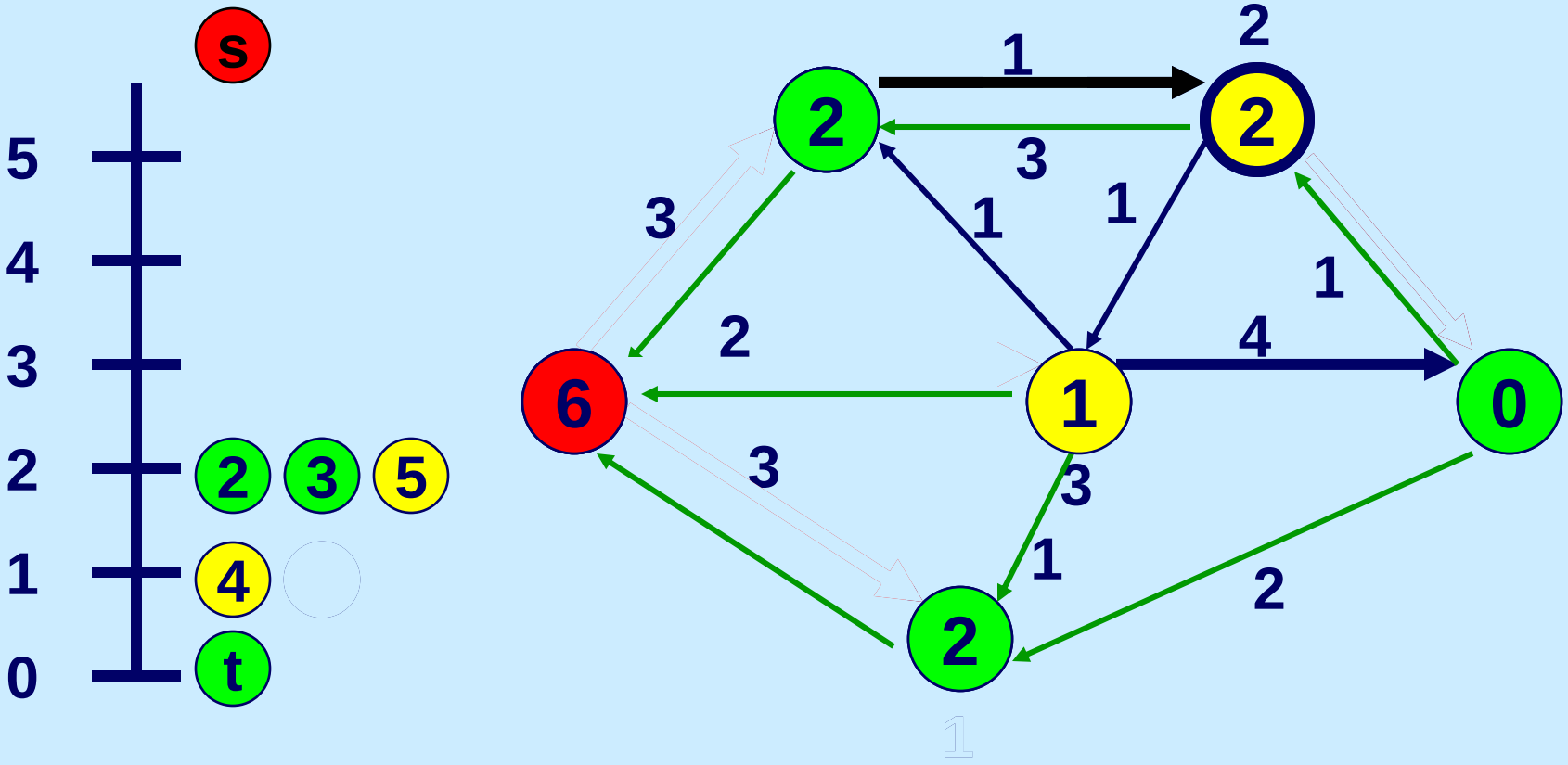
Select, then relabel/push



Select an active node.

Push/Relabel

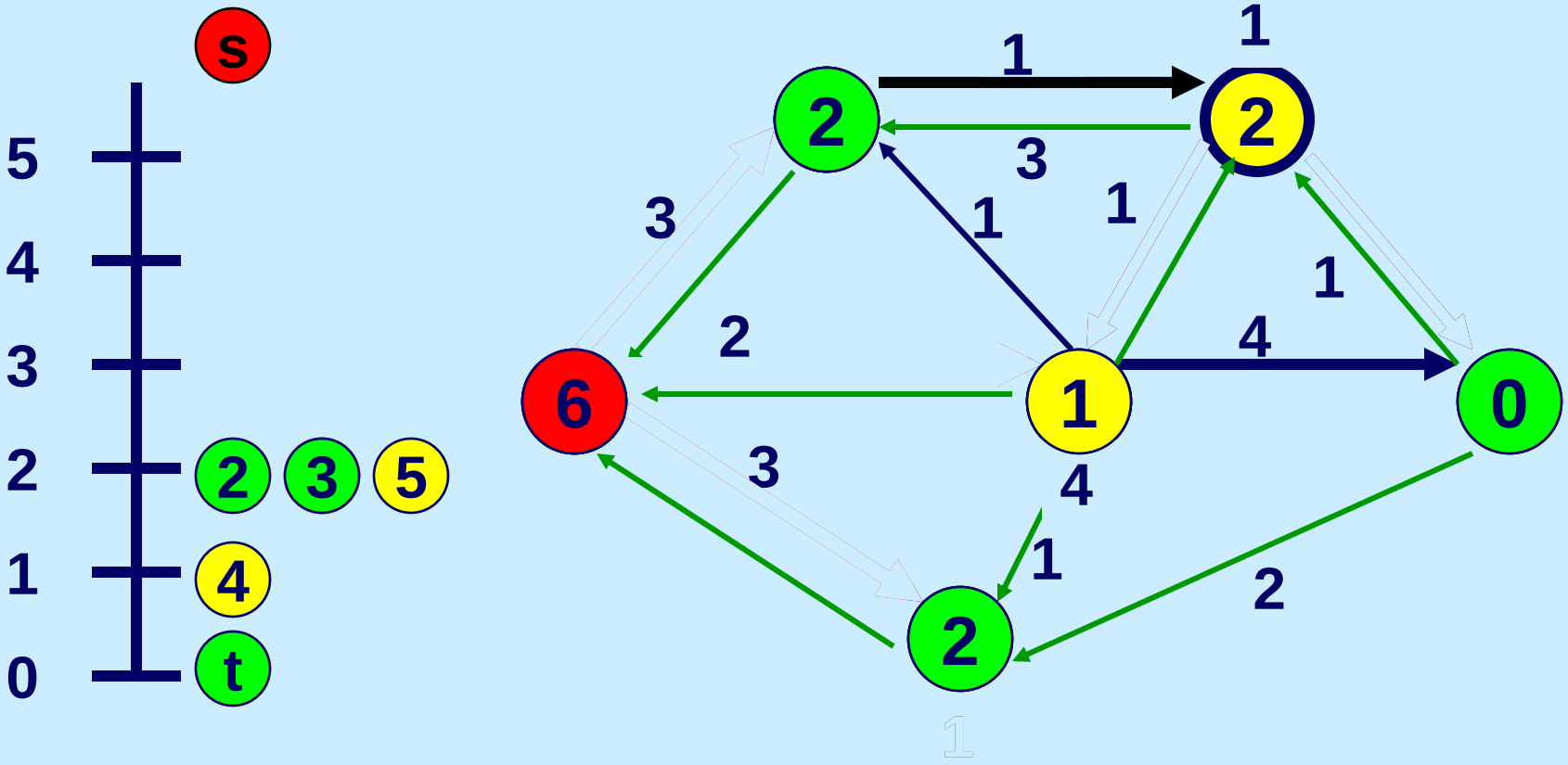
Select, then relabel/push



Select an active node.

There is no incident admissible arc. So Relabel.

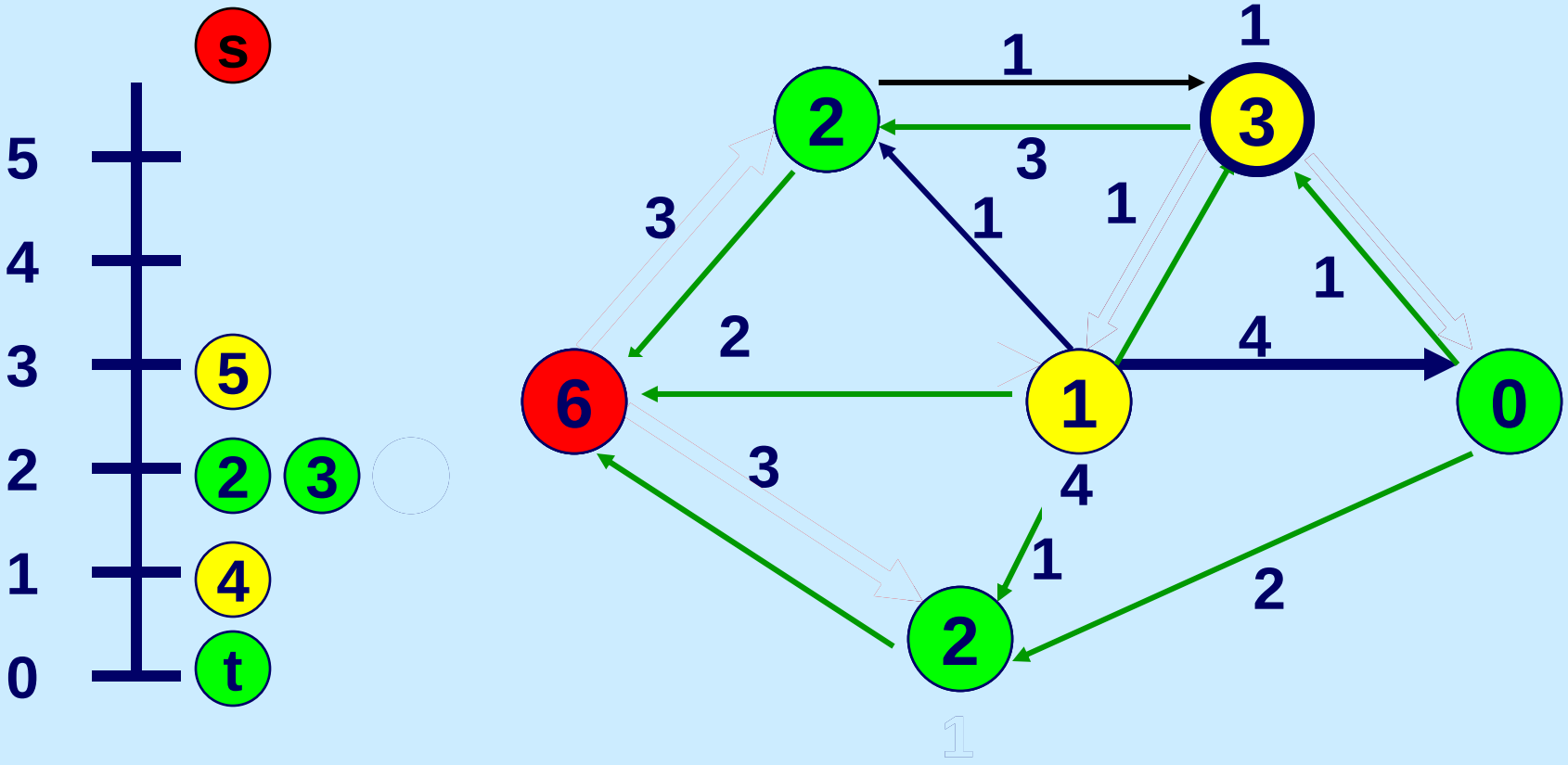
Select, then relabel/push



Select an active node.

Push/Relabel

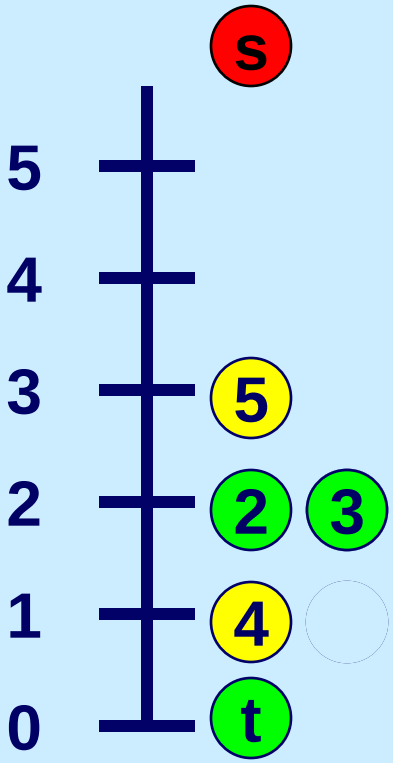
Select, then relabel/push

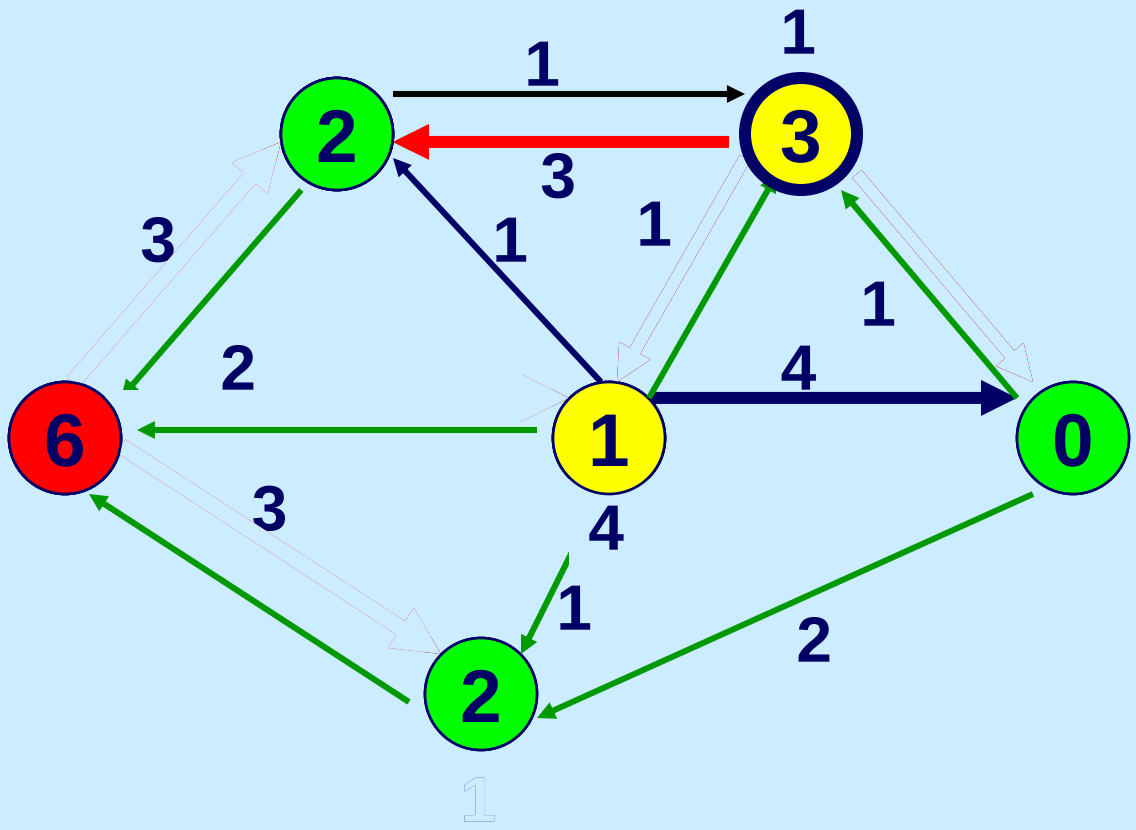


Select an active node.

There is no incident admissible arc. So relabel.

Select, then relabel/push

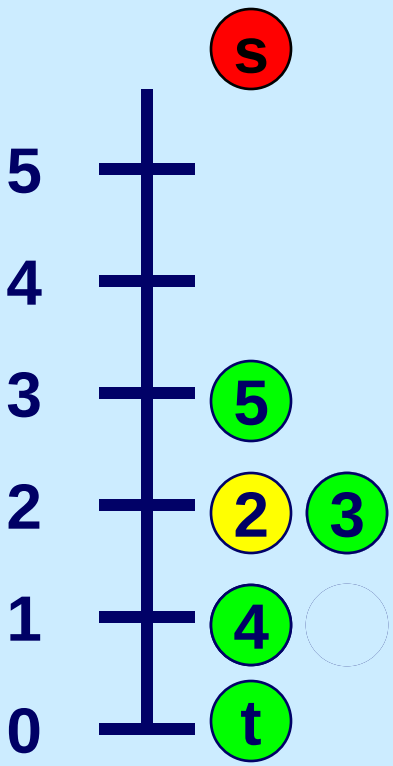


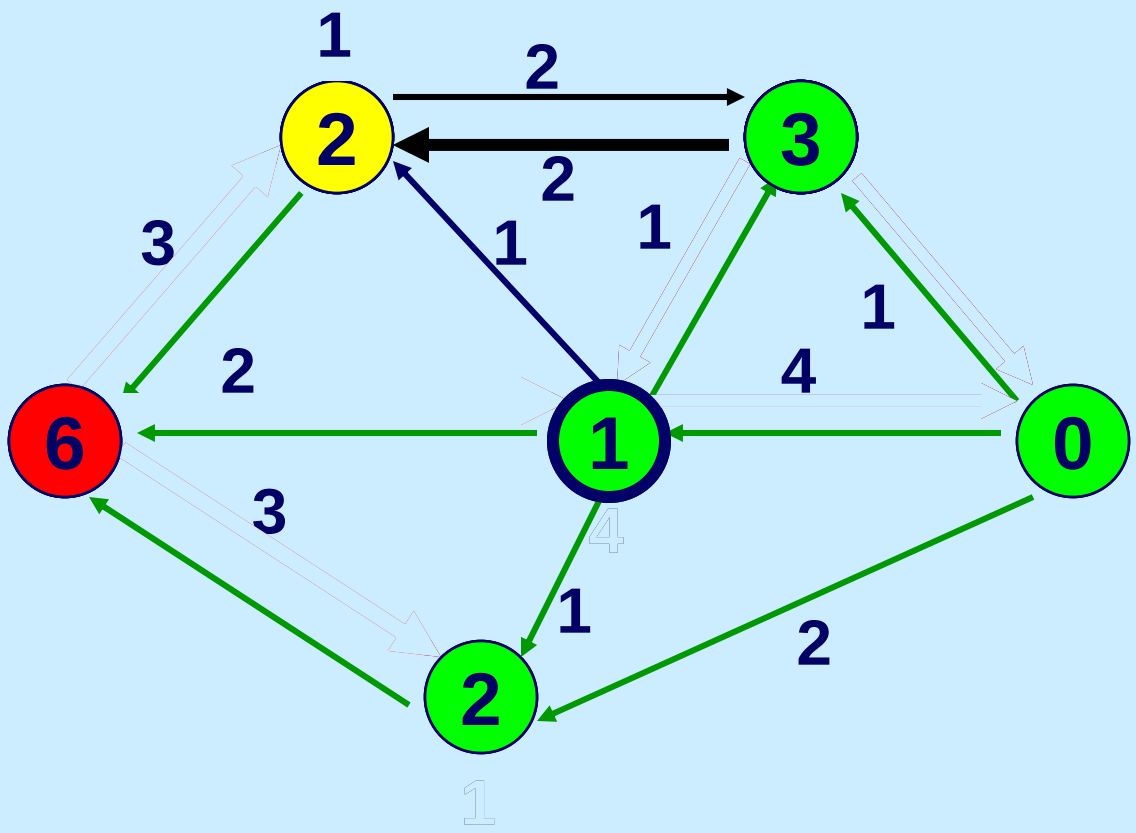


Select an active node.

Push/Relabel

Select, then relabel/push

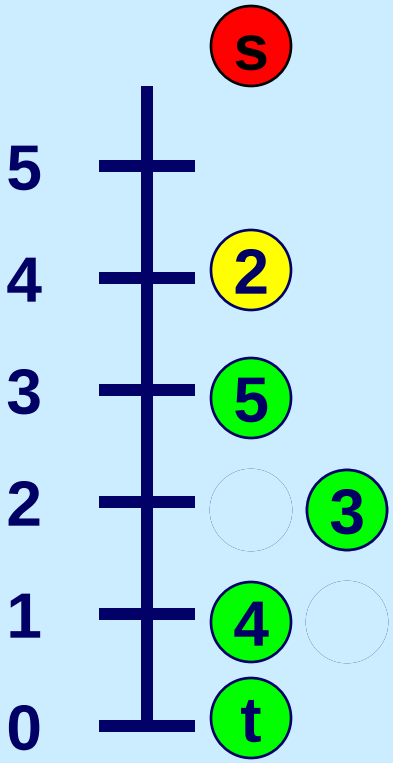


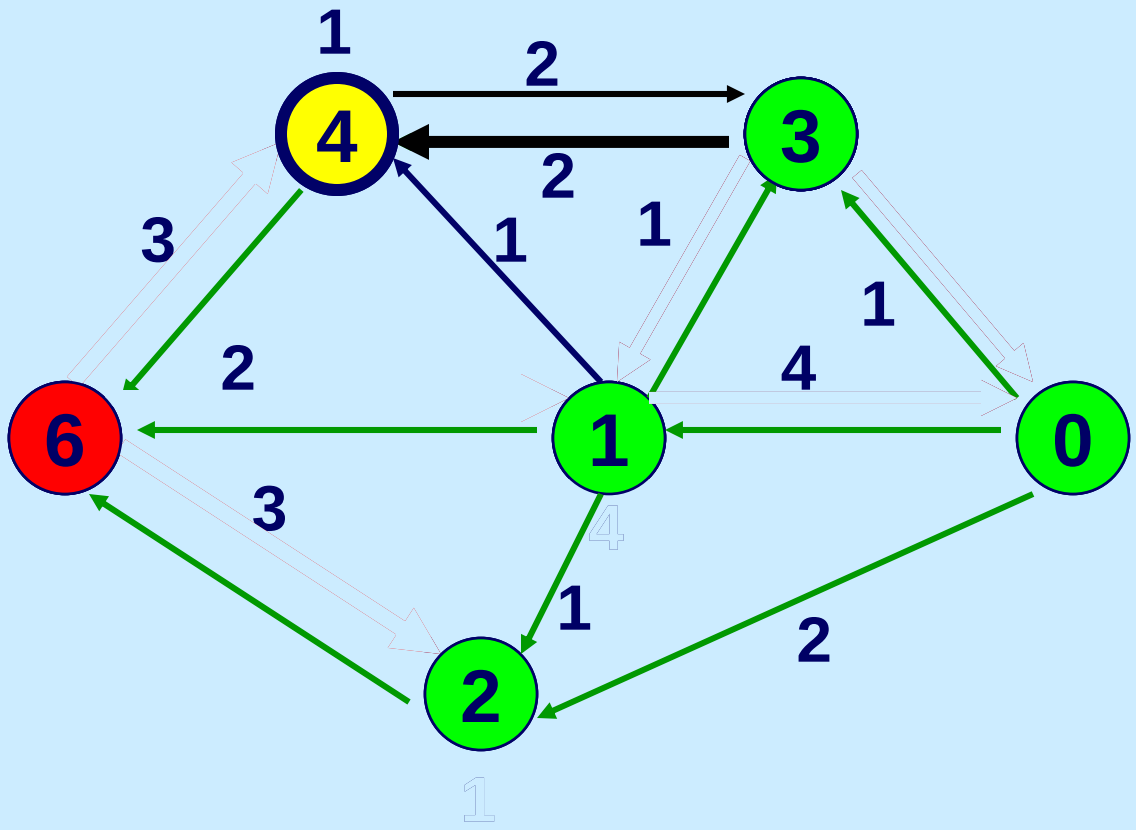


Select an active node.

Push/Relabel

Select, then relabel/push

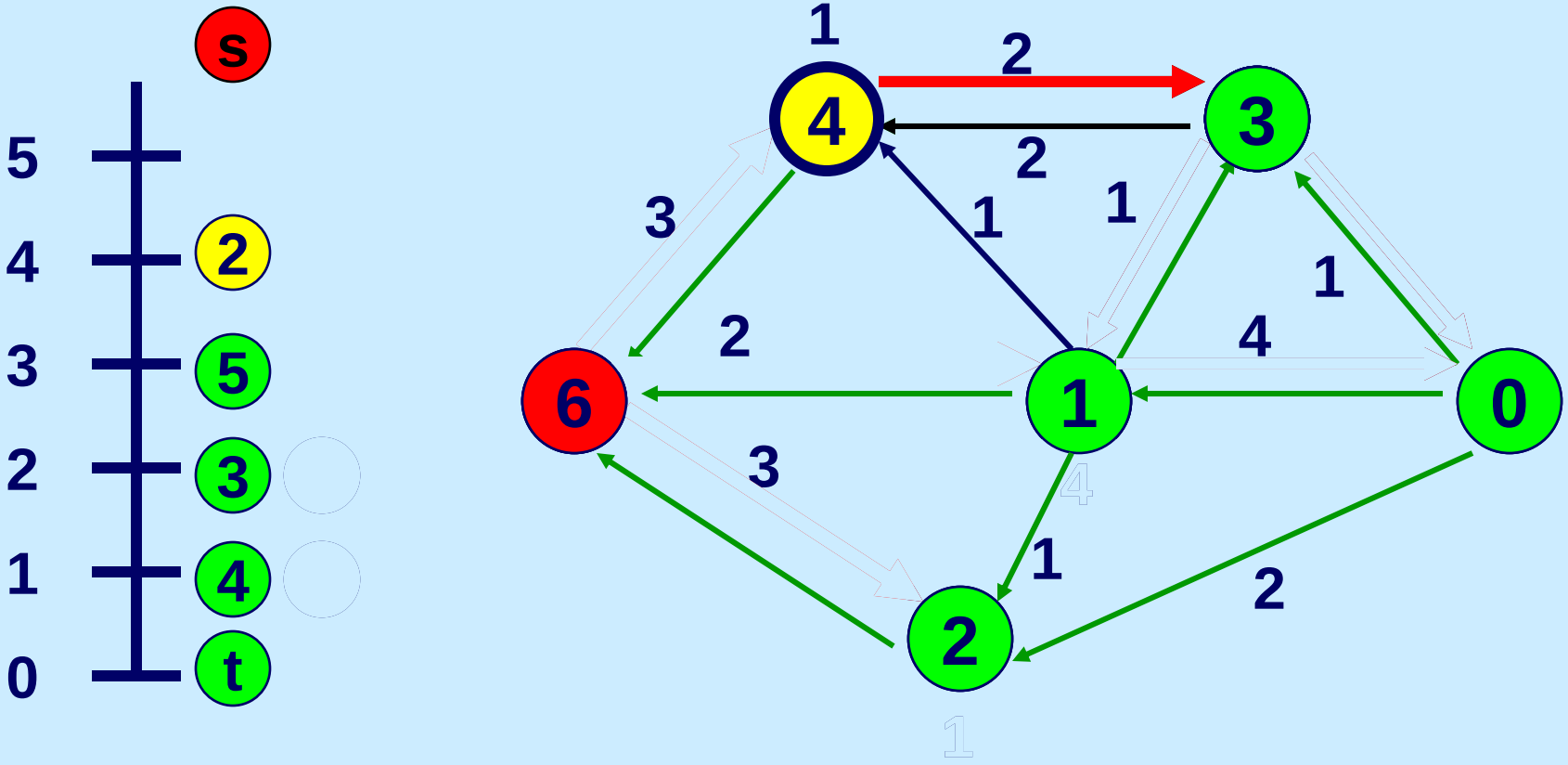




Select an active node.

Push/Relabel

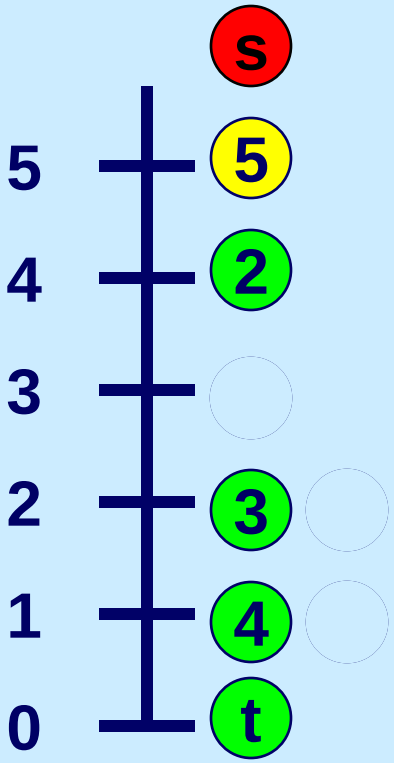
Select, then relabel/push

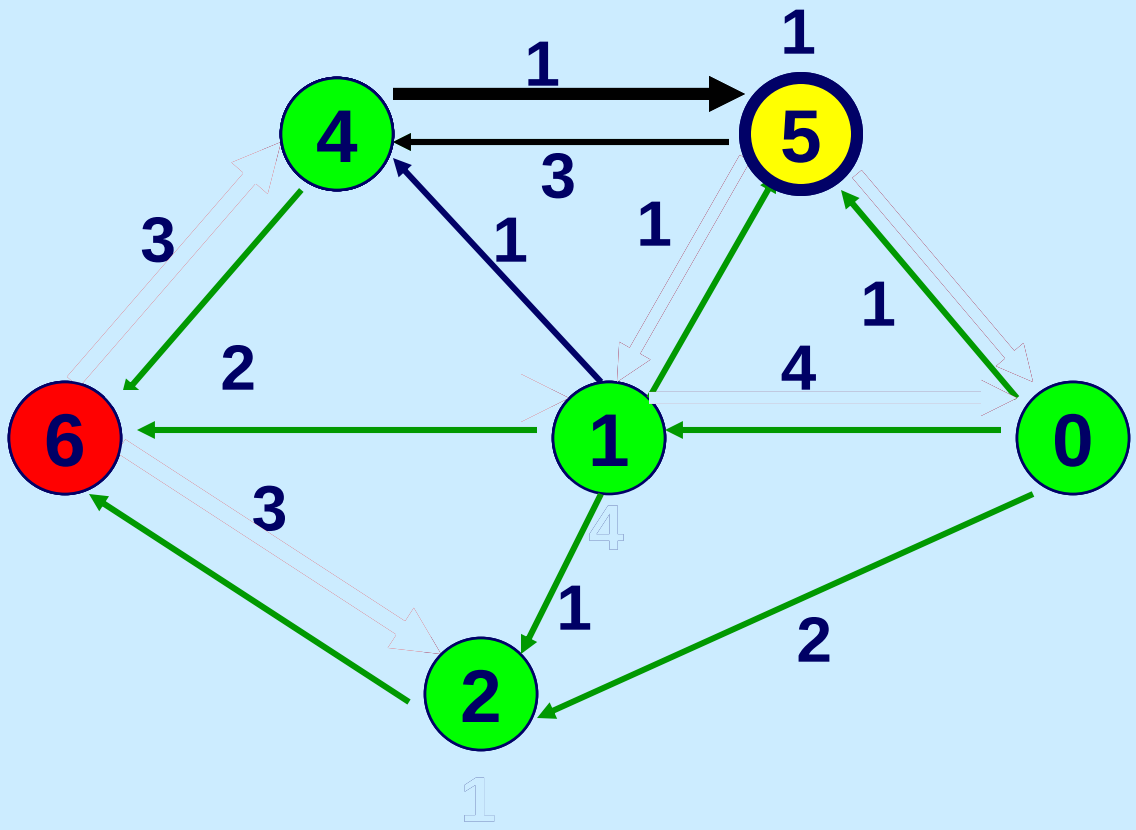


Select an active node.

Push/Relabel

Select, then relabel/push

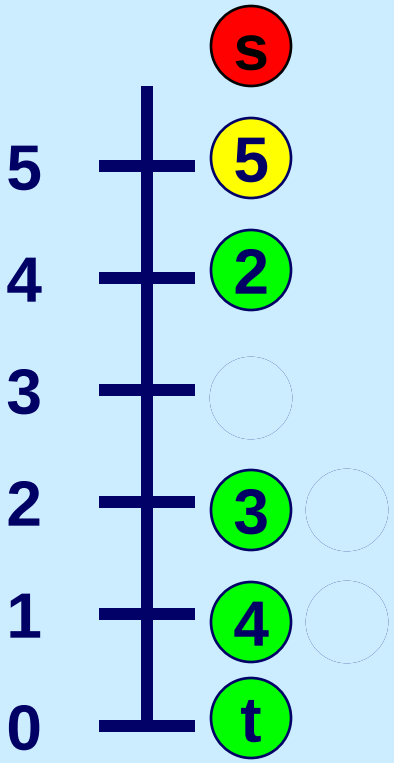


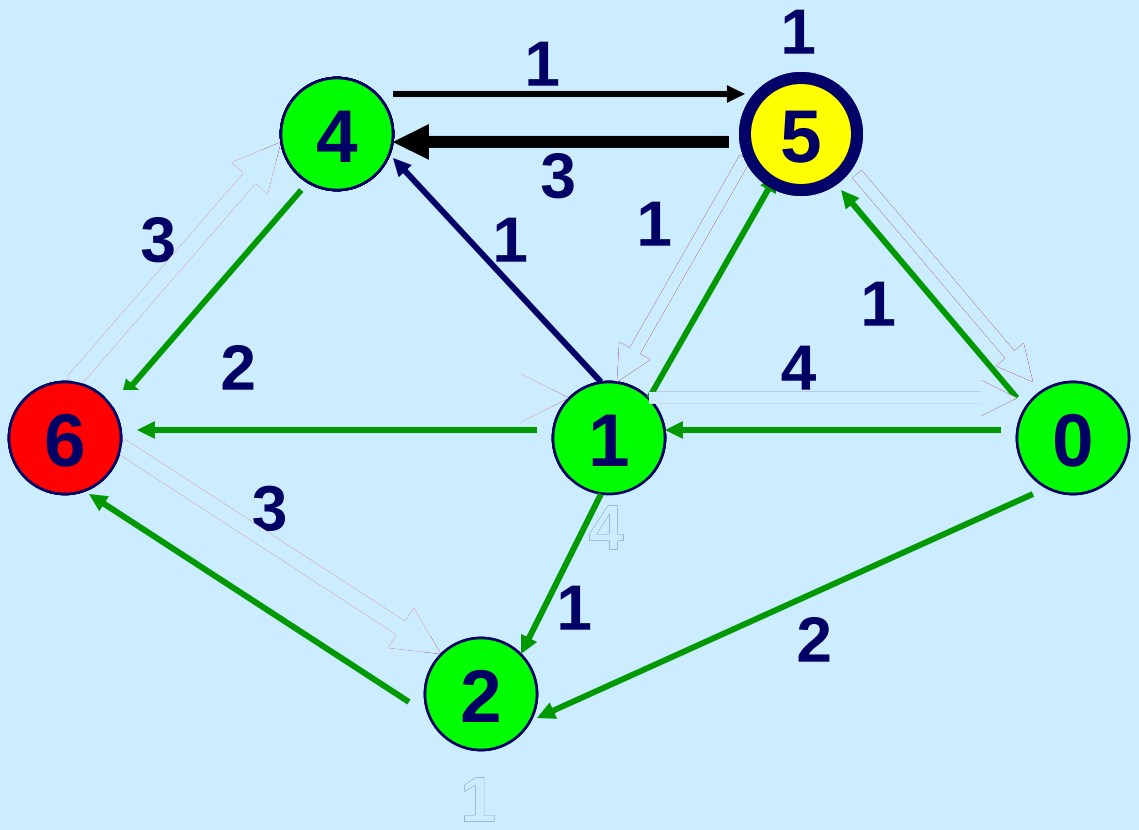


Select an active node.

Push/Relabel

Select, then relabel/push





One can keep pushing flow between nodes 2 and 5 until eventually all flow returns to node s.

There are no paths from nodes 2 and 5 to t, and there are ways to speed up the last iterations.