1、

解:

$$(1)\,f(x)=\frac{1-\cos x+\sin x}{1+\cos x+\sin x}$$

$$=\frac{\sin^2\frac{1}{2}x+\cos^2\frac{1}{2}x-(\cos^2\frac{1}{2}x-\sin^2\frac{1}{2}x)+\frac{1}{2}\sin\frac{1}{2}x\cos\frac{1}{2}x}{\sin^2\frac{1}{2}x+\cos^2\frac{1}{2}x+(\cos^2\frac{1}{2}x-\sin^2\frac{1}{2}x)+\frac{1}{2}\sin\frac{1}{2}x\cos\frac{1}{2}x}=\tan\frac{1}{2}x$$

$\because 1+\cos x+\sin x\neq 0$

$\therefore \sin(x+\frac{\pi}{4})\neq -\frac{\sqrt{2}}{2}$

$\therefore x\neq -\frac{\pi}{2}+2k\pi, x\neq \pi+2k\pi,$

$\therefore f(x)$定义域不关于$x$轴对称

$\therefore f(x)$为非奇非偶函数

$(2)\,f(x)=\tan\frac{1}{2}x$

$\therefore T=2\pi$

2、

$(1)S_{\Delta BCD}=\dfrac{\sqrt{3}}{4}\cdot 36^2=324\sqrt{3},h=\sqrt{36^2-(\dfrac{36}{\sqrt{3}})^2}=\dfrac{36\sqrt{6}}{3}=12\sqrt{6}$

$\therefore V=\dfrac{1}{3}S_{\Delta BCD}h=3888\sqrt{2}$

(2)过$C$点作底面$ABD$的垂线交底面于$H$，

以$H$为坐标原点,$HB$为$x$轴,$HC$为$z$轴，

过$H$作$y$轴分别垂直于$x,z$轴建立坐标系

记内切球球心为$O$,则$O(0,0,3\sqrt{6})$

设$M(x,y,z)$

$x^2+y^2+(z-3\sqrt{6})^2=54$

$C(0,0,12\sqrt{6}),B(12\sqrt{3},0,0)$

$|MB|+\dfrac{1}{3}|MC|=\sqrt{(x-12\sqrt{3})^2+y^2+z^2}+\dfrac{1}{3}\sqrt{x^2+y^2+(z-12\sqrt{6})^2}$

$\because \dfrac{1}{3}\sqrt{x^2+y^2+(z-12\sqrt{6})^2}=\dfrac{1}{3}\sqrt{x^2+y^2+(z-3\sqrt{6}-9\sqrt{6})^2}$

$=\dfrac{1}{3}\sqrt{x^2+y^2+(z-3\sqrt{6})^2-18\sqrt{6}(z-3\sqrt{6})+(9\sqrt{6})^2}$

$=\dfrac{1}{3}\sqrt{x^2+y^2+(z-3\sqrt{6})^2-18\sqrt{6}(z-3\sqrt{6})+8(x^2+y^2+(z-3\sqrt{6})^2)+54}$

$=\dfrac{1}{3}\sqrt{9(x^2+y^2+(z-3\sqrt{6})^2)-18\sqrt{6}(z-3\sqrt{6})+54}=\sqrt{x^2+y^2+(z-3\sqrt{6})^2-2\sqrt{6}(z-3\sqrt{6})+6}$

$=\sqrt{x^2+y^2+(z-4\sqrt{6})^2}$

$\therefore|MB|+\dfrac{1}{3}|MC|=\sqrt{(x-12\sqrt{3})^2+y^2+z^2}+\sqrt{x^2+y^2+(z-4\sqrt{6})^2}\geq\sqrt{(12\sqrt{3})^2+(4\sqrt{6})^2}=4\sqrt{33}$

3、

(1)证明: $a_{n+1}=a_n{}^2-2a_n+2=(a_n-1)^2+1$

$\because a_1=\dfrac{3}{2}>1 \therefore$ 迭代可得 $a_n>1, n\in N^*$

同理 $a_n<2, n\in N^*$ 即 $1<a_n<2, n\in N^*$

$\therefore a_{n+1}-a_n=(a_n-1)(a_n-2)<0$

$\therefore 1<a_{n+1}<a_n<2$

(2)

$a_{k+1}=(a_k-1)^2+1>1$

当 $k\ge 2$ 时, $\dfrac{6}{2^k+3}<1<a_{k+1}$

当 $k=1$ 时, $a_2=\dfrac{5}{4}>\dfrac{6}{2^{2-1}+3}=\dfrac{6}{5}$

$a_1=\dfrac{3}{2}=\dfrac{6}{2^{1-1}+3}$

$\therefore a_n\ge\dfrac{6}{2^{n-1}+3}, n\in N^*$

对右边不等式进行归纳证明如下:

$(i)$ 当 $n=1$ 时, $a_1=\dfrac{3}{2}=\dfrac{2^{1-1}+2}{2^{1-1}+1}$ 即命题对 $n=1$ 成立

$(ii)$ 假设当 $n=k$ 时命题成立即 $a_k\le\dfrac{2^{k-1}+2}{2^{k-1}+1}=1+\dfrac{1}{2^{k-1}+1}$

$$a_{k+1}=(a_k-1)^2+1\le\frac{1}{(2^{k-1}+1)^2}+1=\frac{1}{2^{2k-2}+2^k+1}+1<\frac{1}{2^k+1}+1=\frac{2^k+2}{2^k+1}$$

即命题对 $n=k+1$ 也成立

由 $(i)(ii)$ 可知命题对 $n\in N^*$ 成立

$\therefore \dfrac{6}{2^{n-1}+3}\le a_n\le\dfrac{2^{n-1}+2}{2^{n-1}+1}$

(3)证明:

$\because a_n>1 \therefore S_n>n \because a_n\le\dfrac{2^{n-1}+2}{2^{n-1}+1}<1+\dfrac{1}{2^{n-1}}$

$\therefore S_n<n+\displaystyle\sum_{i=1}^{n}\frac{1}{2^{i-1}}<n+2 \therefore n<S_n<n+2$

(4) $a_{n+1}-1=(a_n-1)^2$

$\because a_n>1$

$\therefore \ln(a_{n+1}-1)=2\ln(a_n-1)$

$\therefore \ln(a_n-1)=2^{n-1}\ln(a_1-1)=\ln(\dfrac{1}{2})^{2^{n-1}}$

$\therefore a_n-1=(\dfrac{1}{2})^{2^{n-1}}$ 即 $a_n=1+(\dfrac{1}{2})^{2^{n-1}}$

4、

$(1)\because e=\frac{\sqrt{6}}{3}$

$\therefore \frac{c}{a}=\frac{\sqrt{6}}{3}$即$\frac{c^2}{a^2}=\frac{2}{3}$

$\therefore c^2=2b^2, a^2=3b^2$

令$l: y=kx+m$

$x^2+3(kx+m)^2=3b^2$

$\therefore (3k^2+1)x^2+6kmx+3m^2-3b^2=0$

$\therefore x_A+x_B=-\frac{6km}{3k^2+1}$

$\therefore y_A+y_B=-\frac{6km}{3k^2+1}\cdot k+2m=\frac{2m}{3k^2+1}$

$\because (x_A,y_A)+(x_B,y_B)=(x_A+x_B,y_A+y_B)=(-\frac{6km}{3k^2+1},\frac{2m}{3k^2+1})$与$(3,-1)$共线

$\therefore k=1$

$(2) l: y=x-c$即$m=-c$

设$M(x_0,y_0)$,则

$x_0=x_A\sin\alpha+x_B\cos\alpha$

$y_0=y_A\sin\alpha+y_B\cos\alpha$

由(1)得$\begin{cases} x_A+x_B=\frac{3}{2}c \\ x_Ax_B=\frac{3}{4}b^2 \end{cases}$

而$y_Ay_B=(x_A-c)(x_B-c)=\frac{3}{4}b^2-\frac{3}{2}c^2+c^2=-\frac{1}{4}b^2$

$\therefore x_Ax_B+3y_Ay_B=0$

$\because \begin{cases} \frac{{x_A}^2}{3b^2}+\frac{{y_A}^2}{b^2}=1 \\ \frac{{x_B}^2}{3b^2}+\frac{{y_B}^2}{b^2}=1 \end{cases}$

$\therefore \frac{{x_0}^2}{3b^2}+\frac{{y_0}^2}{b^2}=\frac{(x_A\sin\alpha+x_B\cos\alpha)^2}{3b^2}+\frac{3(y_A\sin\alpha+y_B\cos\alpha)^2}{3b^2}$

$=\frac{({x_A}^2+3{y_A}^2)\sin^2\alpha+({x_B}^2+3{y_B}^2)\cos^2\alpha+2(x_Ax_B+3y_Ay_B)\sin\alpha\cos\alpha}{3b^2}$

$=\frac{3b^2(\sin^2\alpha+\cos^2\alpha)}{3b^2}=1$

$\therefore M$在椭圆上

5、

(1) $f(x)=\ln(x+1)-x,(x>0)$

$f'(x)=-\dfrac{x}{x+1}<0\therefore f(x)$单调递减而$f(0)=0\therefore -m\le 0$即$m\ge 0$

(2)令$F(x)=\ln(x+1)-x+\dfrac{1}{2}x^2,x>0$

$F'(x)=\dfrac{1}{x+1}-1+x=\dfrac{x^2}{x+1}>0$

$\therefore F(x)$单调递增而$F(0)=0\therefore \ln(x+1)-x>-\dfrac{1}{2}x^2$

而$-\dfrac{1}{2}(\sqrt{2m})^2+m=0\therefore x_1>\sqrt{2m}$

(3)证明: $g(x)=\ln(x+1)-\dfrac{1}{2}x,(x>-1)$

$\therefore g(x-1)+g(x)=\ln(x+1)x-x+\dfrac{1}{2},(x>0)$

令$h(x)=\ln(x+1)x-x+\dfrac{1}{2},x>0$

$h'(x)=\dfrac{2x+1}{(x+1)x}-1=\dfrac{-x^2+x+1}{x^2+x}$

$h'(1)>0,h'(2)<0$,令$h'(x_0)=0$则$1<x_0<2$

$\therefore h(x)$在$(1,x_0)$上递增,$(x_0,2)$上递减

而$h(1)=\ln 2-\dfrac{1}{2},h(2)=\ln 6-\dfrac{3}{2}>h(1)$

$\therefore$当$x\in(1,2)$时,$h(x)_{\min}>h(1)=\ln 2-\dfrac{1}{2}$

$\therefore g(x-1)+g(x)>\ln 2-\dfrac{1}{2},1<x<2$

$g(x)=\ln(x+1)-\dfrac{1}{2}x,g'(x)=\dfrac{1}{x+1}-\dfrac{1}{2},g'(1)=0$

$\therefore x_1<1<x_2,x_3<1<x_4,g(x)$在$(0,1)$单调递增,$(1,+\infty)$单调递减

$(i)$若$x_2>2$,则$x_2-x_3>2-1=1$

(ii)若$x_2\le 2$则$1<x_2\le 2$

$g(x_2-1)+g(x_2)>\ln 2-\dfrac{1}{2}=g(x_3)+g(x_2)$

$\therefore g(x_2-1)>g(x_3)$

$\because x_2-1<1,x_3<1\therefore x_2-1>x_3$即$x_2-x_3>1$

$\because b<a\therefore b<\dfrac{\ln 2-\frac{1}{2}}{2}$而$g(2)=\ln 3-1>\dfrac{\ln 2-\frac{1}{2}}{2}>b=g(x_4)\therefore x_4>2$而$x_1<1$

$\therefore x_4-x_1>1\therefore |x_4-x_3|+|x_2-x_1|=x_4-x_3+x_2-x_1=(x_4-x_2)+(x_3-x_1)>1+1=2$