1、试用裂项法证明以下公式：

$(1)1^2+2^2+\cdots+n^2=\dfrac{n(n+1)(2n+1)}{6}$

$(2)1^3+2^3+\cdots+n^3=(\dfrac{(n+1)n}{2})^2$

2、对于已知正项数列$\{a_n\},\{b_n\}$，有$\begin{cases}a_{n+1}=a_n+\dfrac{1}{b_n}\\ b_{n+1}=b_n+\dfrac{1}{a_n}\end{cases},n\in N^*$，求证：$a_{50}+b_{50}>20$

3、已知数列$\{a_n\}$满足$a_n=\dfrac{n}{t+1},(n,t\in N^*,t\geq 3,n\leq t)$，证明:

$(1)a_n<e^{a_n-1}$

$(2)\dfrac{1}{a_1}+\dfrac{1}{a_2}+\cdots+\dfrac{1}{a_n}>(t+1)ln(n+1)$

$(3)(a_1)^t+(a_2)^t+\cdots+(a_n)^t<1$

4、数列$\{a_n\}$满足$a_1=1,a_{n+1}=\left(1+\dfrac{1}{n^2+n}\right)a_n,n\in N^*$

证明:$\dfrac{2n}{n+1}\leq a_n<\dfrac{en}{n+1}$

5、已知数列$\{a_n\}$的首项$a_1=\dfrac{1}{2},\dfrac{1}{a_{n+1}}=\dfrac{1}{2}\left(a_n+\dfrac{1}{a_n}\right),n\in N^*$,记$b_n=\dfrac{(a_n-a_{n+1})^2}{a_na_{n+1}}$,

其前n项和为T_n,证明：$T_n<\dfrac{3}{10}$

6、已知数列$\{a_n\},\{b_n\}$满足$a_1=1,a_2=4,a_{n+2}-a_n=4a_{n+1},a_nb_n=a_{n+1},n\in N^*$

(1)求b_1,b_2,b_3

(2)求证：$|b_{n+1}-b_n|\leq\dfrac{1}{4}\cdot\dfrac{1}{17^{n-1}}$

(3)求证：$|b_{2n}-b_n|\leq\dfrac{1}{64}\cdot\dfrac{1}{17^{n-2}}$

7、已知数列$\{a_n\}$满足$a_n>0,a_1=2,$且$(n+1){a_{n+1}}^2=\mathrm{n}{a_n}^2+a_n,n\in N^*$

(1)证明:$a_n>1$

(2)证明:$\dfrac{{a_2}^2}{4}+\dfrac{{a_3}^2}{9}+\cdots+\dfrac{{a_n}^2}{n^2}<\dfrac{7}{4}$

8、已知数列$\{a_n\}$满足$a_1 = 1$ 且$a_{n+1} = \frac{1}{2}a_n + \left(\frac{1}{2}\right)^{n+1} a_n{}^2$,求证：

(1)$0 < a_{n+1} < a_n \le 1, n \in N^*$

(2)当$n \in N^*$时,$\frac{3 \cdot 2^{n-1}}{2 \cdot 4^{n-1} + 1} \le a_n \le \frac{3 \cdot 2^{n-1}}{4^{n-1} + 2}$

9、设数列$\{a_n\}$满足$|a_n - \frac{a_{n+1}}{2}| \le 1, n \in N^*$

(1)证明: $|a_n| \ge 2^{n-1}(|a_1|-2)$

(2)*若*$|a_n| \le \left(\frac{3}{2}\right)^n, n \in N^*$, *证明*: $|a_n| \le 2, n \in N^*$

10、已知数列$\{a_n\}$中，满足$a_1 = \frac{1}{2}, a_{n+1} = \sqrt{\frac{a_n + 1}{2}}$，记$\{a_n\}$的前$n$项和为$S_n$

(1)证明: $a_{n+1} > a_n$

(2)求$\{a_n\}$的通项

(3)证明: $S_n > n - \frac{27 + \pi^2}{54}$