生命表

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率的测量

- **死亡率**

例如：粗死亡率

$$CDR = \frac{D(t)}{N(t)} = \frac{\text{某时期的死亡人数}}{\text{该时期的存活人年数}}$$

- **如何计算该时期的存活人年数？一个例子**

$$3 \times 1 + 1 \times \frac{1}{365} + 1 \times \frac{180}{365} = 3.50\text{人年}$$

- **一般的方法：**

$$N(t) = \frac{P(t) + P(t+1)}{2}$$

粗死亡率计算示例

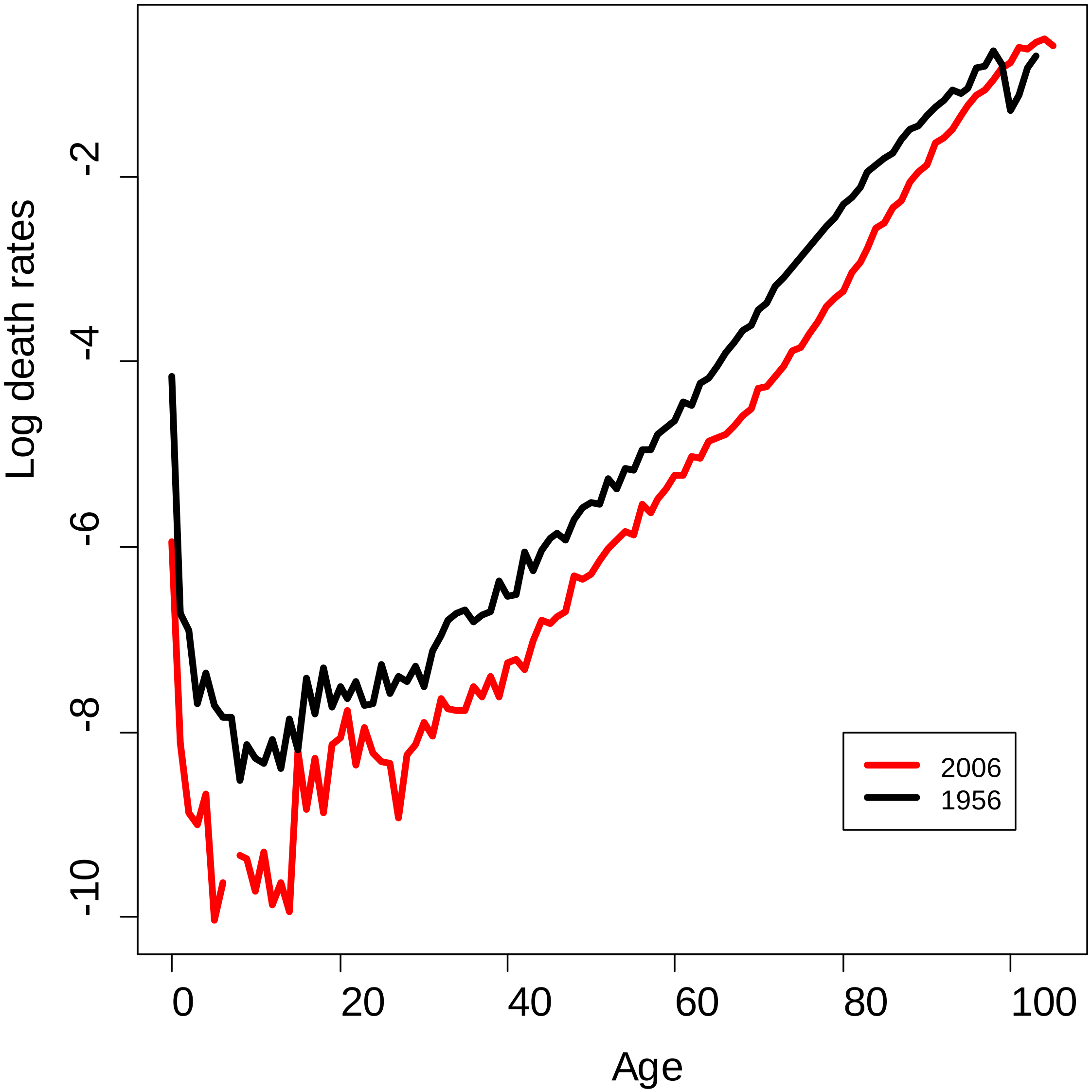
- **1956年瑞典女性的粗死亡率**

$$\frac{D(t=1956)}{N(t=1956)} = \frac{D(t=1956)}{\frac{P(t=1956)+P(t=1957)}{2}} = \frac{33{,}522}{\frac{3{,}651{,}034+3{,}673{,}960}{2}}$$
$$= 0.00915; or\ 9.153\ per\ 1000$$

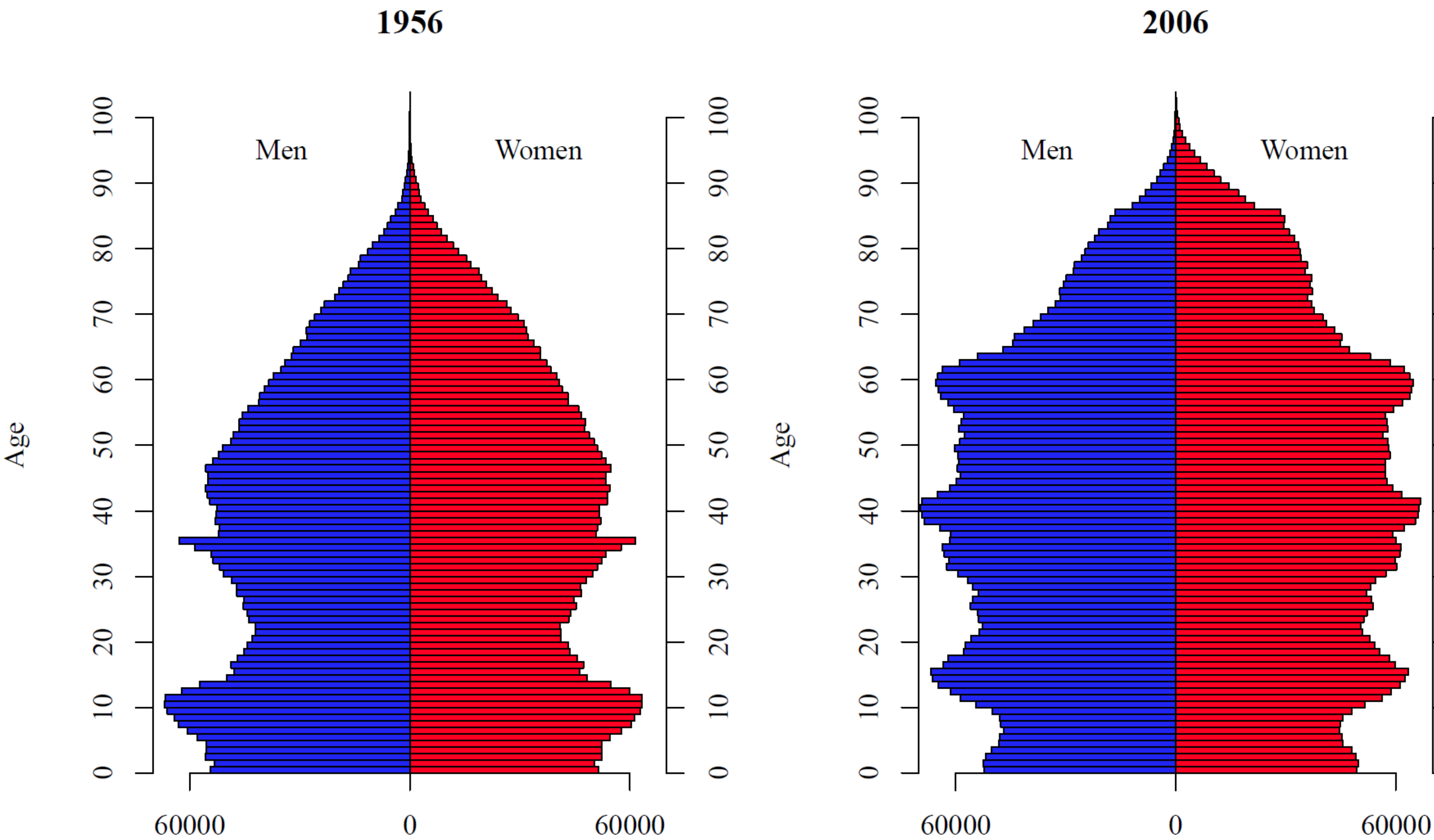
- **2006年瑞典女性的粗死亡率**

$$\frac{D(t=2006)}{N(t=2006)} = \frac{D(t=2006)}{\frac{P(t=2006)+P(t=2007)}{2}} = \frac{47{,}000}{\frac{4{,}561{,}160+4{,}589{,}732}{2}}$$
$$= 0.01027; or\ 10.27\ per\ 1000$$

年龄别死亡率，瑞典，女性



HMD:2012



1956
2006
Men
Women
Age
0
10
20
30
40
50
60
70
80
90
100
60000
0
60000

粗死亡率与年龄别死亡率的关系

- $CDR = \frac{D}{N} = \frac{\sum_{x=0}^{\infty} {}_nD_x}{N} = \frac{\sum_{x=0}^{\infty} \frac{{}_nD_x}{{}_nN_x} * {}_nN_x}{N}$

$$= \sum_{x=0}^{\infty} \frac{{}_nD_x}{{}_nN_x} * \frac{{}_nN_x}{N} = \sum_{x=0}^{\infty} {}_nM_x * {}_nC_x$$

生命表

- 英国数学家约翰 格兰特编制第一张生命表《关于死亡表的自然的和政治的观察》
- 死亡率分析的核心工具
- 生命表可以用于估计死亡力（u_x）、存活人数（l_x）、死亡分布（f_x）、平均预期寿命（e_x）

人口学的一些表示法

死亡数:　$_{n}D_{x}$

年龄区间的长度（年）

开始年龄（年）

$_{10}D_{5}$ = 在年龄5岁 和 **15 = 5+10** 岁间的死亡数

人口学的一些表示法

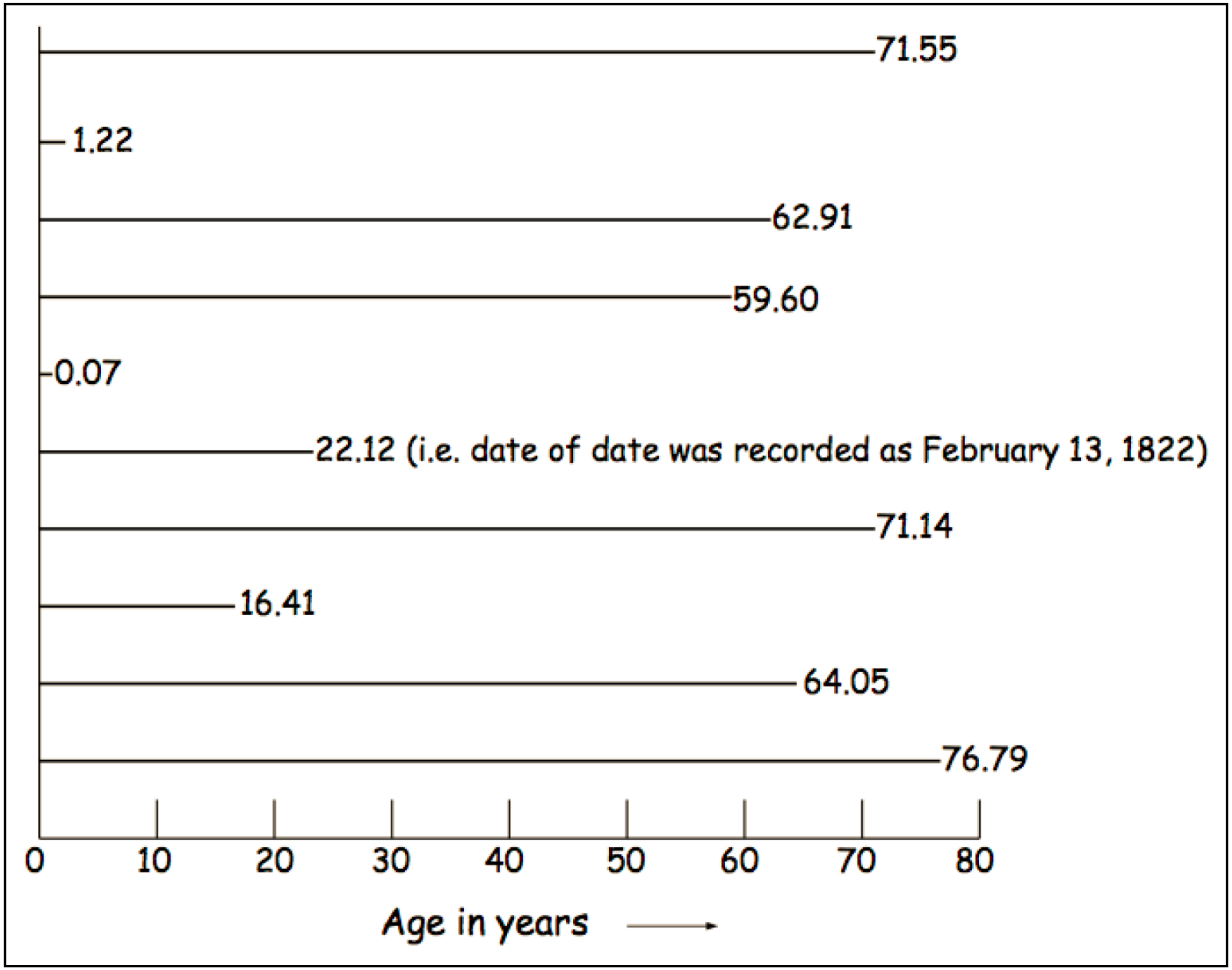
存活到年龄x的人数 $= l_x$

$$l_x - l_{x+n} = {}_n D_x$$

ω = 最高年龄

$l_\omega = 0$

假想出生队列的死亡年龄和生命线

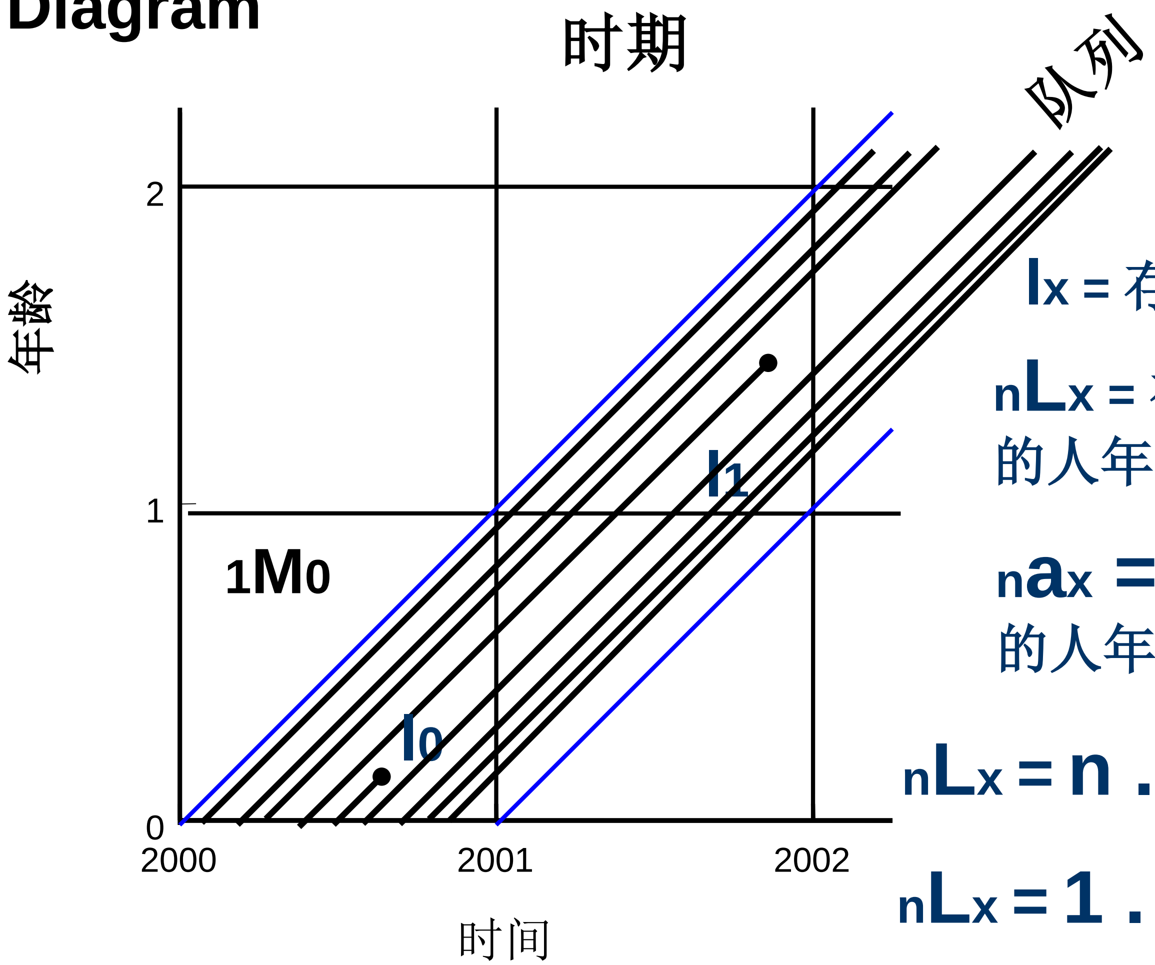


一个假想出生队列的生命表

Exact age x	Number left alive at age x l_x	Number dying between ages x and x+n ${}_nd_x$	Probability of dying between ages x and x+n ${}_nq_x$	Probability of surviving between ages x and x+n ${}_np_x$	Person-years lived between ages x and x+n ${}_nL_x$	Person-years lived above age x $T_x = \sum_{a=x}^{\infty} {}_nL_a$	Expectation of life at age x $e_x^o = \frac{T_x}{l_x}$	Death rate in the cohort between ages x and x+n ${}_nm_x$	Average person-years lived in the interval by those dying in the interval ${}_na_x$
0	10	1	0.1	0.9	9.07	445.86	44.586	0.110	0.07
1	9	1	0.111	0.889	32.22	436.79	48.532	0.031	0.22
5	8	0	0	1	40	404.57	50.571	0.000	-
10	8	1	0.125	0.875	76.41	364.57	45.571	0.013	6.41
20	7	1	0.143	0.857	62.12	288.16	41.166	0.016	2.12
30	6	0	0	1	60	226.04	37.673	0.000	-
40	6	0	0	1	60	166.04	27.673	0.000	-
50	6	1	0.167	0.833	59.6	106.04	17.673	0.017	9.6
60	5	2	0.4	0.6	36.96	46.44	9.288	0.054	3.48
70	3	3	1	0	9.48	9.48	3.160	0.316	3.16
80	0	0	-	-					

队列存活人年数: ${}_nL_x$

Lexis Diagram



l_x = 存活到x的人数

${}_nL_x$ = 在x和x+n之间存活的人年数

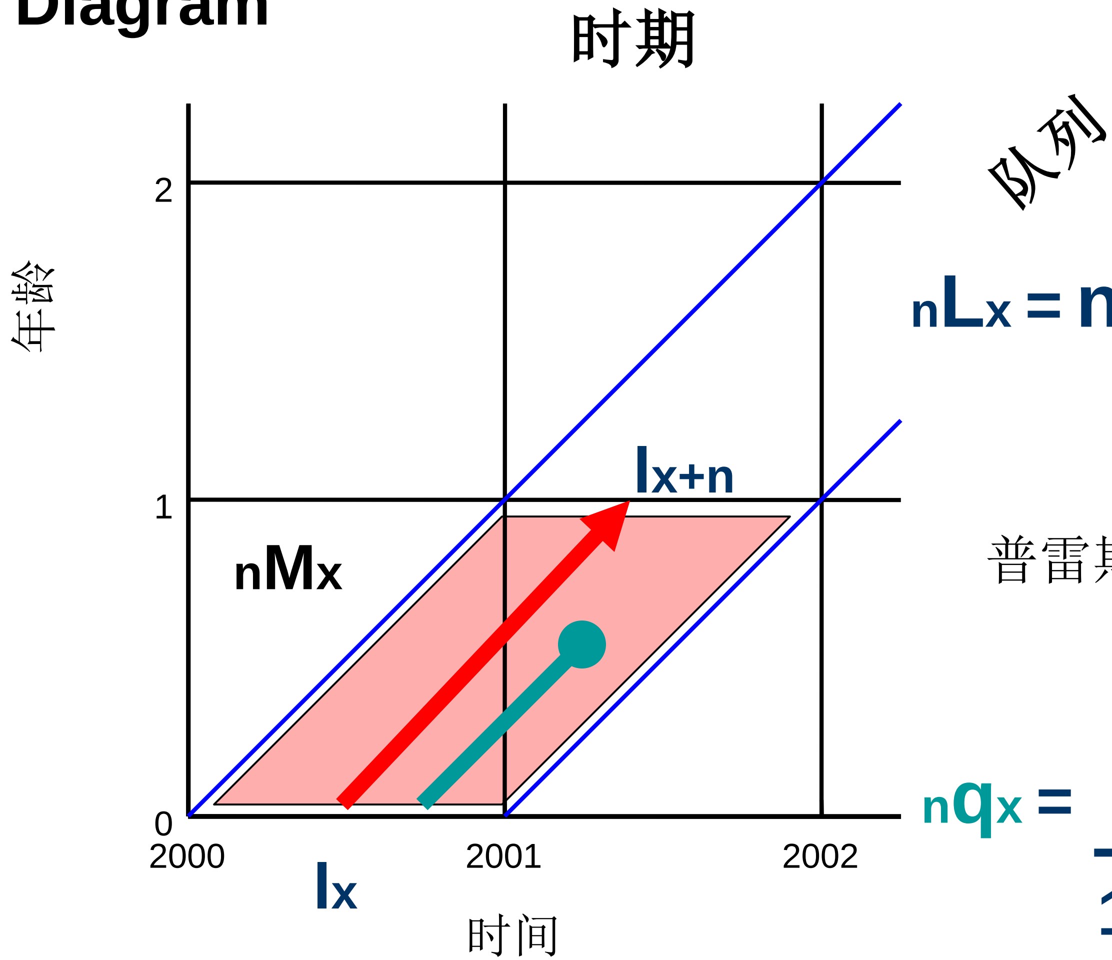
${}_na_x$ = 死者平均存活的人年数.

$${}_nL_x = n \,.\, l_{x+n} + {}_na_x \,.\, {}_nd_x$$

$${}_nL_x = 1 \,.\, 9 \quad + \; 0.07 \,.\, 1$$

队列死亡概率: $_nq_x$

Lexis Diagram



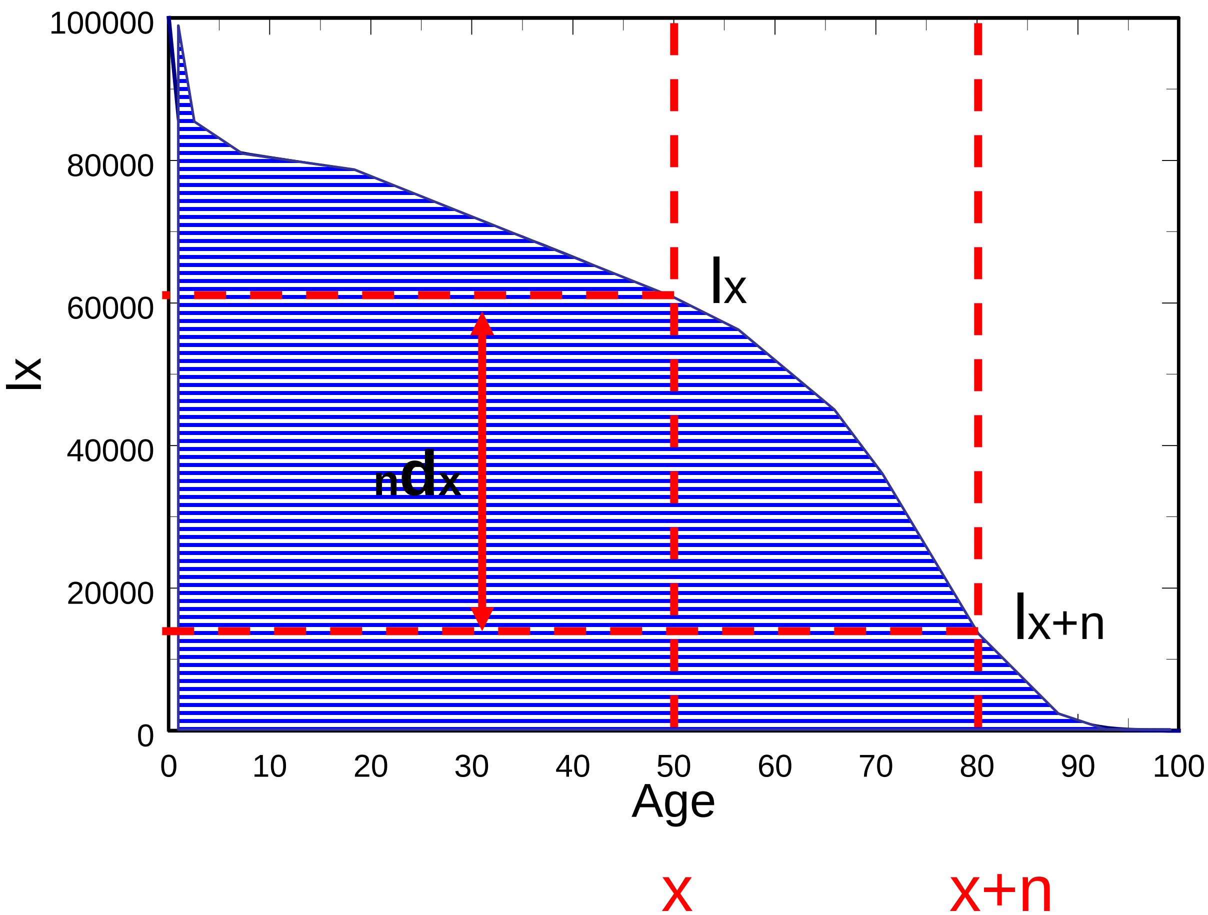
$$_nL_x = n\ .\ l_{x+n} + {_na_x}\ .\ {_nd_x}$$

普雷斯顿等 第37 页

$$_nq_x = \frac{n\ .\ {_nm_x}}{1 + (n - {_na_x})\ {_nm_x}}$$

$_nM_x \rightarrow {_nm_x} \rightarrow {_nq_x} = 1 - {_np_x}$

队列生命表的概率



x岁的存活人数**:** l_x

死亡数: $_nd_x = l_x - l_{x+n}$

存活概率:

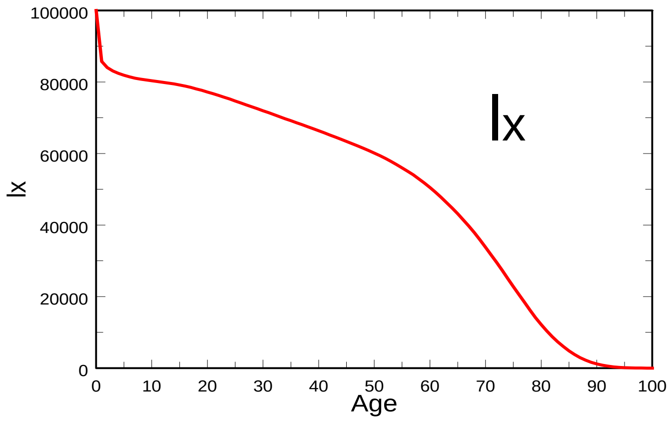
$_np_x = l_{x+n} / l_x$

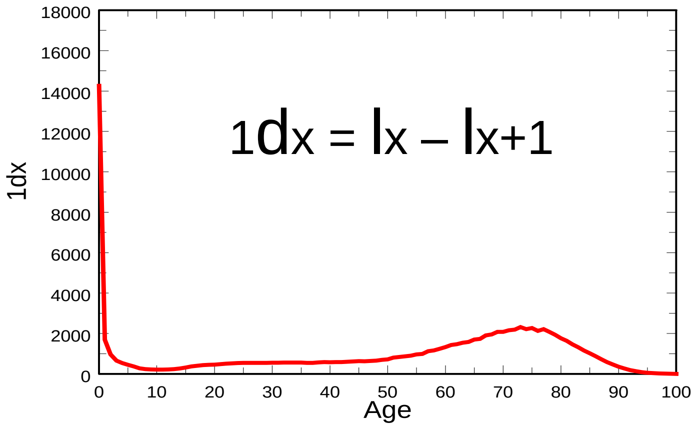
死亡概率:

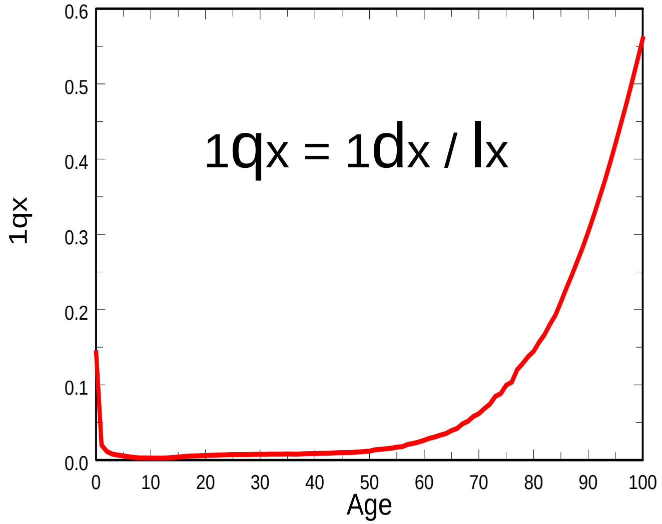
$_nq_x = 1 - l_{x+n} / l_x$

$= {_nd_x} / l_x$

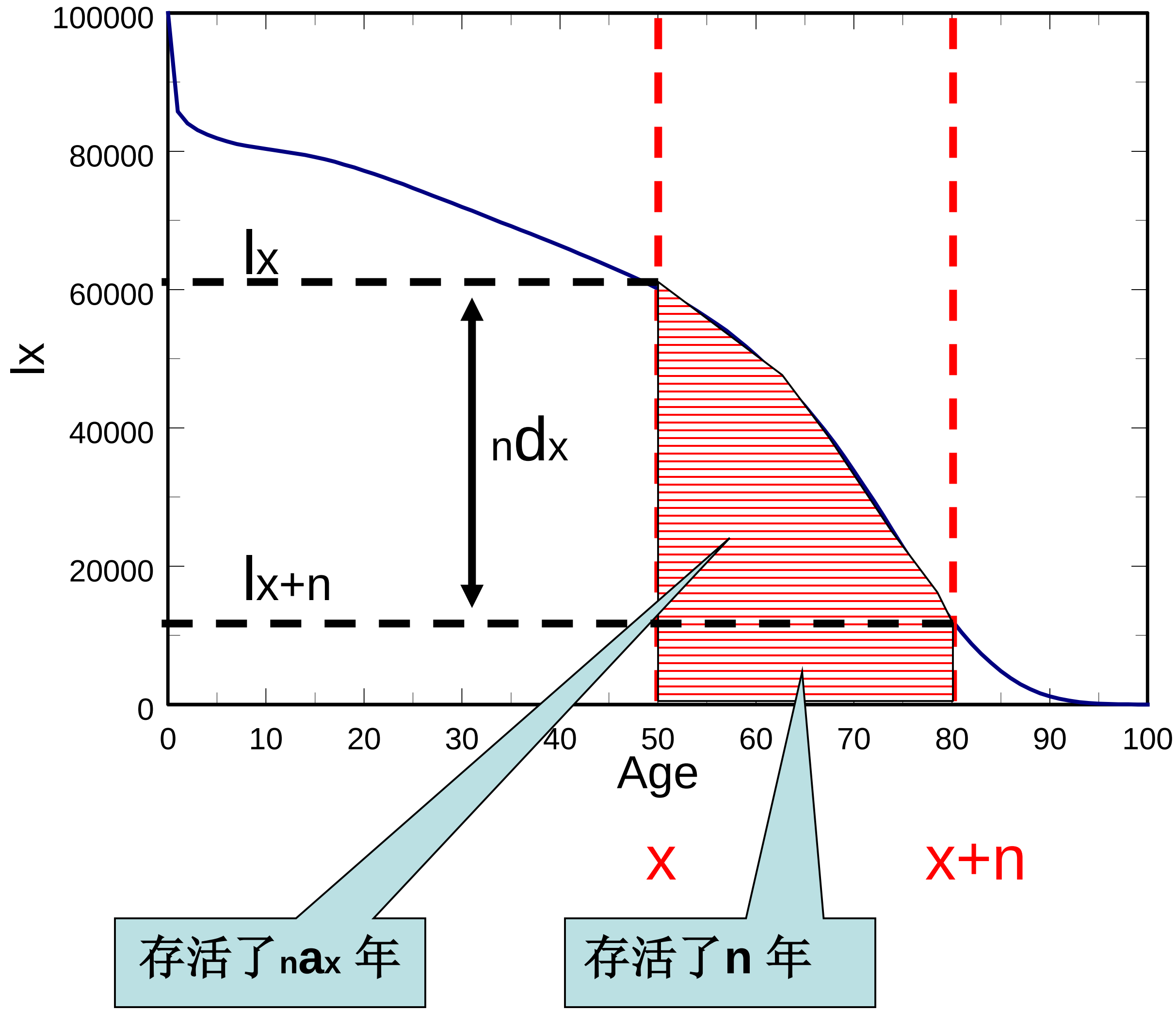
生命表函数


lx
100000
80000
60000
40000
20000
0
0 10 20 30 40 50 60 70 80 90 100
Age
lx


1dx = lx – lx+1
18000
16000
14000
12000
10000
8000
6000
4000
2000
0
0 10 20 30 40 50 60 70 80 90 100
Age
1dx


1qx = 1dx / lx
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0 10 20 30 40 50 60 70 80 90 100
Age
1qx

$_nL_x$ 和 $_nm_x$



死亡数: ndx = lx - lx+n

在x和x+n间的存活人年数: nLx = n.lx+n + nax.ndx

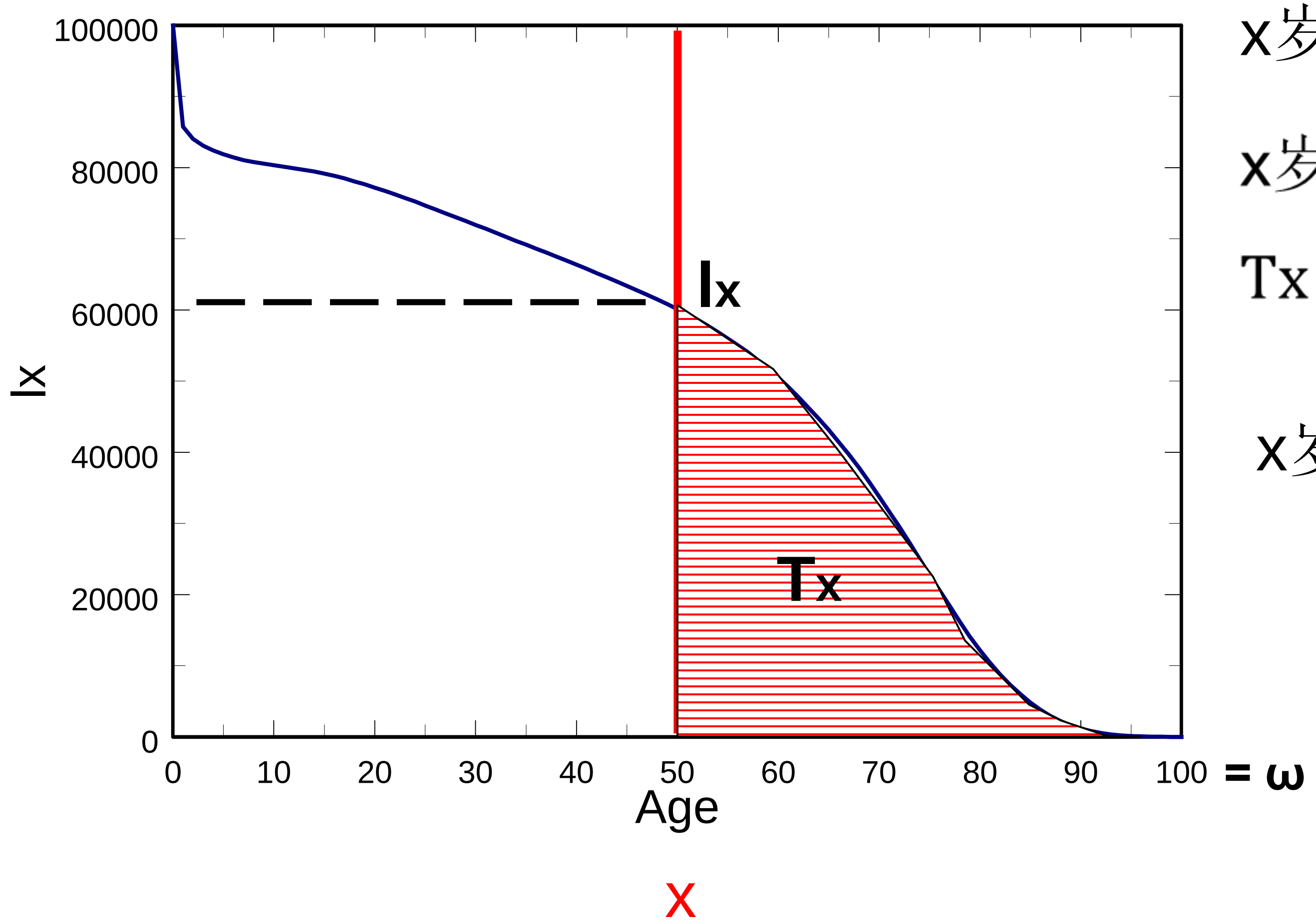
队列死亡率:

nmx = ndx / nLx

时期死亡率:

nMx = nDx / nNx

T_x 和 e_x



x岁的存活人数: lx

x岁以上的存活人年数:

$$Tx = \sum_{x}^{\omega} nLx$$

x岁的预期寿命:

$$e_x = T_x / l_x$$

时期生命表，奥地利，男性，1992

Age x	${}_nN_x$	${}_nD_x$	${}_nm_x$	${}_na_x$	${}_nq_x$	${}_np_x$	l_x	${}_nd_x$	${}_nL_x$	T_x	e_x^o
0	47,925	419	0.008743	0.068	0.008672	0.991328	100,000	867	99,192	7,288,901	72.889
1	189,127	70	0.000370	1.626	0.001479	0.998521	99,133	147	396,183	7,189,709	72.526
5	234,793	36	0.000153	2.500	0.000766	0.999234	98,986	76	494,741	6,793,526	68.631
10	238,790	46	0.000193	3.143	0.000963	0.999037	98,910	95	494,375	6,298,785	63.682
15	254,996	249	0.000976	2.724	0.004872	0.995128	98,815	481	492,980	5,804,410	58.740
20	326,831	420	0.001285	2.520	0.006405	0.993595	98,334	630	490,106	5,311,431	54.014
25	355,086	403	0.001135	2.481	0.005659	0.994341	97,704	553	487,127	4,821,324	49.346
30	324,222	441	0.001360	2.601	0.006779	0.993221	97,151	659	484,175	4,334,198	44.613
35	269,963	508	0.001882	2.701	0.009368	0.990632	96,492	904	480,384	3,850,023	39.900
40	261,971	769	0.002935	2.663	0.014577	0.985423	95,588	1,393	474,686	3,369,639	35.252
45	238,011	1,154	0.004849	2.698	0.023975	0.976025	94,195	2,258	465,777	2,894,953	30.734
50	261,612	1,866	0.007133	2.676	0.035082	0.964918	91,937	3,225	452,188	2,429,176	26.422
55	181,385	2,043	0.011263	2.645	0.054861	0.945139	88,711	4,867	432,096	1,976,988	22.286
60	187,962	3,496	0.018600	2.624	0.089062	0.910938	83,845	7,467	401,480	1,544,893	18.426
65	153,832	4,366	0.028382	2.619	0.132925	0.867075	76,377	10,152	357,713	1,143,412	14.971
70	105,169	4,337	0.041238	2.593	0.187573	0.812427	66,225	12,422	301,224	785,699	11.864
75	73,694	5,279	0.071634	2.518	0.304102	0.695898	53,803	16,362	228,404	484,475	9.005
80	57,512	6,460	0.112324	2.423	0.435548	0.564452	37,441	16,307	145,182	256,070	6.839
85+	32,248	6,146	0.190585	5.247	1.000000	0.000000	21,134	21,134	110,889	110,889	5.247

时期生命表编制

- 观测数据

$$ {}_nN_x = x\text{至}x+n\text{岁年中人口数} $$

$$ {}_nD_x = x\text{至}x+n\text{岁当年死亡人口数} $$

时期生命表编制步骤

1.计算${}_n m_x \cong {}_n M_x = \frac{{}_n D_x}{{}_n N_x}$,

2.${}_n a_x$,5岁以下用寇尔-德曼公式计算,5岁以上采用Keyfitz和Fliger的数据

3.$_n q_x = \frac{n \cdot {}_n m_x}{1 + (n - {}_n a_x) \cdot {}_n m_x}$

4.$_n p_x = 1 - {}_n q_x$

5.$l_0 = 100,000, l_{x+n} = l_x \cdot {}_n p_x$

6.$_n d_x = l_x - l_{x+n}$

7.$_n L_x = n \cdot l_{x+n} + {}_n a_x \cdot {}_n d_x$(开放年龄组${}_\infty L_x = \frac{l_x}{{}_\infty m_x}$)

8.$T_x = \sum_{a=x}^{\infty} {}_n L_a$

9.$e_x^0 = \frac{T_x}{l_x}$

5岁以下年龄组和开放年龄组

- 寇尔-德曼（1984）关于 a的经验公式
- 开放年龄组[85,+]

$$ {}_{\infty}d_x = l_x, {}_{\infty}q_x = 1, {}_{\infty}p_x = 0 $$

5岁以下年龄组和开放年龄组

	男性	女性
${}_1a_0$ 的值		
当 ${}_1m_0 \geq 0.107$	0.330	0.350
当 ${}_1m_0 \prec 0.107$	0.045+2.684* ${}_1m_0$	0.053+2.8* ${}_1m_0$
${}_4a_1$ 的值		
当 ${}_1m_0 \geq 0.107$	1.352	1.361
当 ${}_1m_0 \prec 0.107$	1.652-2.816* ${}_1m_0$	1.522-1.518* ${}_1m_0$

$$ {}_\infty m_x = \frac{{}_\infty d_x}{{}_\infty L_x} \rightarrow {}_\infty L_x = \frac{{}_\infty d_x}{{}_\infty m_x} \rightarrow {}_\infty L_x = \frac{{}_\infty l_x}{{}_\infty m_x} $$

计算总人口的a_0^T

- $a_0^T = \frac{a_0^F D_0^F + a_0^M D_0^M}{D_0^F + D_0^M}$

- 这里的a_0^T就是总人口的a值，a_0^{F}和a_0^M分别是女性和男性的a值。D_0^F和D_0^M分别是生命表中的女性和男性0岁人口的死亡数。

${}_{n}a_{x}$函数

Austria, male, 1992.

年龄	n	nax
0	1	0.068
1	4	1.626
5	5	2.500
10	5	3.143
15	5	2.724
20	5	2.520
25	5	2.481
30	5	2.601
35	5	2.701
40	5	2.663
45	5	2.698
50	5	2.676
55	5	2.645
60	5	2.624
65	5	2.619
70	5	2.593
75	5	2.518
80	5	2.423
85		5.247

年龄	n	nax
0	1	0.07
1	1	0.5
2	1	0.5
3	1	0.5
4	1	0.5
5	1	0.5
6	1	0.5
7	1	0.5
8	1	0.5
9	1	0.5
10	1	0.5
11	1	0.5
12	1	0.5
13	1	0.5
14	1	0.5
15	1	0.5
16	1	0.5
17	1	0.5
18	1	0.5
19	1	0.5
20	1	0.5
21	1	0.5
22	1	0.5
23	1	0.5
…		
109	1	0.5
110		1.41

m_{110+}=0.71929

开放年龄组

队列死亡率:

$$ {}_n m_x = {}_n d_x / {}_n L_x $$

开放年龄组死亡率:

$$ {}_\infty m_x = {}_\infty d_x / {}_\infty L_x $$

$$ l_x = {}_\infty d_x \quad \Rightarrow \quad {}_\infty L_x = l_x / {}_\infty m_x $$

$$ {}_\infty e_x = {}_\infty L_x / {}_\infty l_x = 1 / {}_\infty m_x = {}_\infty a_x $$

人类死亡率数据库(HMD)

Human Mortality Databas

www.mortality.org

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The Human Mortality Database

John R. Wilmoth, *Director*	University of California, Berkeley
Vladimir Shkolnikov, *Co-Director*	Max Planck Institute for Demographic Research

The Human Mortality Database (HMD) was created to provide detailed mortality and population data to researchers, students, journalists, policy analysts, and others interested in the history of human longevity. The project began as an outgrowth of earlier projects in the Department of Demography at the University of California, Berkeley, USA, and at the Max Planck Institute for Demographic Research in Rostock, Germany (see history). It is the work of two teams of researchers in the USA and Germany (see research teams), with the help of financial backers and scientific collaborators from around the world (see acknowledgements).

We seek to provide open, international access to these data. At present the database contains detailed population and mortality data for the following 37 countries or areas:

Australia	Finland	Lithuania	Spain
Austria	France	Luxembourg	Sweden
Belarus	Germany	Netherlands	Switzerland
Belgium	Hungary	New Zealand	Taiwan
Bulgaria	Iceland	Norway	U.K.
Canada	Ireland	Poland	U.S.A.
Chile	Israel	Portugal	Ukraine
Czech Republic	Italy	Russia	
Denmark	Japan	Slovakia	
Estonia	Latvia	Slovenia	

For more information, please begin by reading an overview of the database. If you have comments or questions, or trouble gaining access to the data, please write to us (hmd@mortality.org).

将生命表视为静止人口

生命表基数= 总出生数= 总死亡数

封闭人口

所以，这个人口是零增长

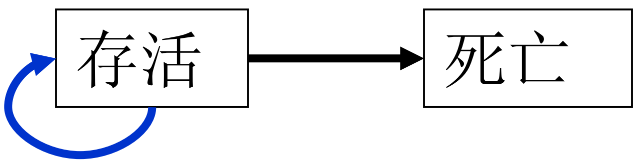
年龄别死亡率是固定的

CDR = 1 / e0　　每个生命时间死亡一人

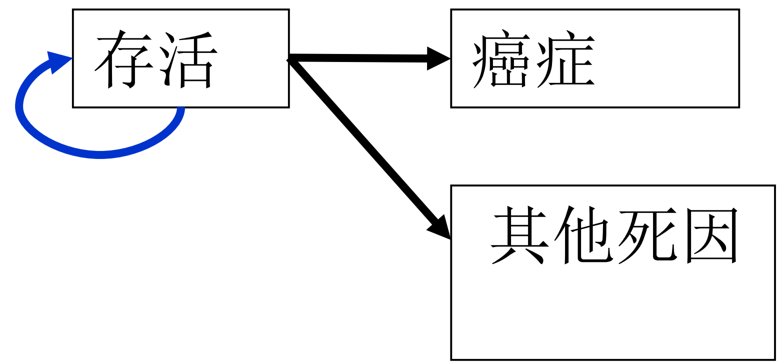
CBR = CDR

增减生命表

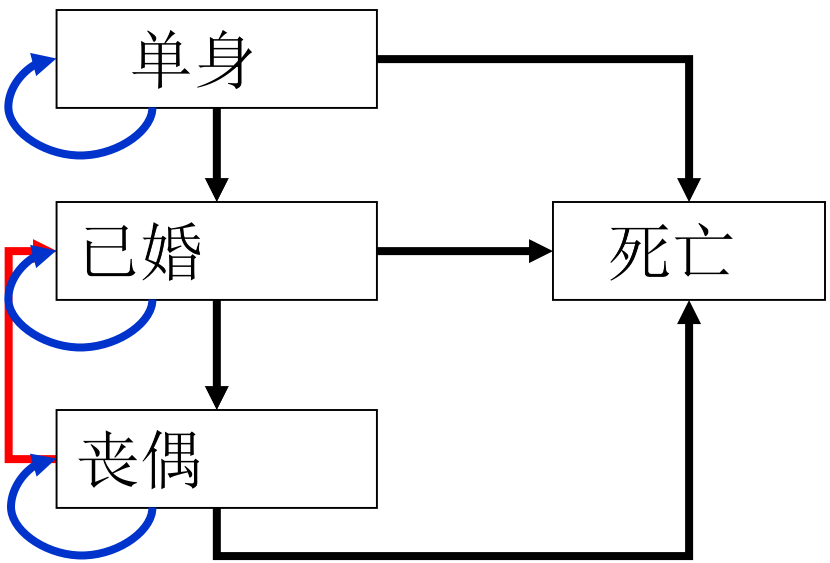
单减

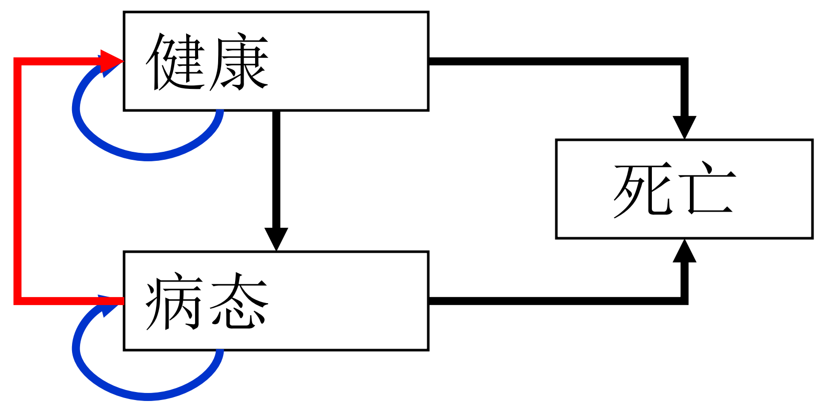


多减



增-减





连续变量的生命表

$$
\begin{aligned}
{}_n d_x &= l(x) - l(x+n) \\
\mu(x) &= \lim_{n \to 0} \frac{{}_n d_x}{{}_n L_x} = \lim_{n \to 0} {}_n m_x \\
\mu(x) &= \lim_{n \to 0} {}_n m_x = \lim_{n \to 0} \left[\frac{l(x) - l(x+n)}{n \cdot l(x)} \right] \\
\mu(x) &= \lim_{n \to 0} \left[\frac{l(x) - l(x+n)}{n \cdot l(x)} \right] = \frac{-d \ln(l(x))}{dx} \\
-\int_y^z \mu(x) dx &= \ln l(z) - \ln(y)
\end{aligned}
$$

连续变量的生命表

$$e^{-\int_y^z \mu(x)dx} = \frac{l(z)}{l(y)}$$

$$l(z) = l(y)e^{-\int_y^z \mu(x)dx}$$

$$l(a) = l(0)e^{-\int_0^a \mu(x)dx}$$

连续变量的生命表

$$ {}_np_x = \frac{l(x+n)}{l(x)} = e^{-\int\limits_x^{x+n}\mu(a)da} $$

$$ {}_nd_x = \int\limits_x^{x+n} l(a)\mu(a)da = \int\limits_x^{x+n} l(x)e^{\int\limits_x^a \mu(y)dy}\mu(a)da = l(x)\int\limits_x^{x+n} e^{\int\limits_x^a \mu(y)dy}\mu(a)da $$

$$ {}_nq_x = \frac{{}_nd_x}{l(x)} = \int\limits_x^{x+n} e^{\int\limits_x^a \mu(y)dy}\mu(a)da $$

连续变量的生命表

$$
{}_nL_x = \int\limits_x^{x+n} l(a)da = \int\limits_x^{x+n} l(x)e^{\int\limits_x^a \mu(y)dy}da = l(x)\int\limits_x^{x+n} e^{\int\limits_x^a \mu(y)dy}da
$$

$$
{}_nm_x = \frac{{}_nd_x}{{}_nL_x} = \frac{\int\limits_x^{x+n} l(a)\mu(a)da}{\int\limits_x^{x+n} l(a)da} = \frac{\int\limits_x^{x+n} e^{-\int\limits_x^a \mu(y)dy}\mu(a)da}{\int\limits_x^{x+n} e^{-\int\limits_x^a \mu(y)dy}da}
$$

$$
{}_na_x = \frac{\int\limits_x^{x+n} l(a)\mu(a)(a-x)da}{\int\limits_x^{x+n} l(a)\mu(a)da} = \frac{\int\limits_x^{x+n} e^{-\int\limits_x^a \mu(y)dy}\mu(a)(a-x)da}{\int\limits_x^{x+n} e^{-\int\limits_x^a \mu(y)dy}\mu(a)da}
$$

连续变量的生命表

$$T_x = \int_x^{\infty} l(a)da = l(x)\int_x^{\infty} e^{-\int_x^a \mu(y)dy}$$

$$e_x^o = \frac{T_x}{l_x} = \frac{\int_x^{\infty} l(a)da}{l(x)} = \int_x^{\infty} e^{-\int_x^a \mu(y)dy} da = \frac{\int_x^{\infty} l(a)\mu(a)(a-x)da}{\int_x^{\infty} l(a)\mu(a)da}$$