

Demand Estimation

MIXTAPE SESSION

Jeff Gortmaker and Ariel Pakes



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subject to $s_{jt} = \sum_{i \in \mathcal{I}_t} w_{it} \cdot \frac{\exp[\delta_{jt} + \mu_{ijt}(\theta)]}{1 + \sum_{k \in \mathcal{J}_t} \exp[\delta_{kt} + \mu_{ikt}(\theta)]}$

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- Let's go over your second coding exercise.

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 - Recall the linear regression intuition from the last class.
- Can't credibly estimate a standard deviation in Σ on the mushy dummy.
 - Same cereals in each market, so no choice set variation along mushy dimension.
 - Results in unrealistically limited substitution between similar cereals.
- Also can't estimate a parameter in Π on income alone.
 - Market fixed effects are collinear with market-level income means.
 - Unrealistic that overall cereal preference doesn't vary with income.

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 - Marketing research datasets (e.g. NielsenIQ’s Consumer Panel).
 - Regulatory agencies like the UK’s antitrust authority (Reynolds and Walters, 2008).

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- Imagine surveying people at the supermarket who purchased cereal.
 - “What was your annual income last year?”
 - “What would you have purchased if your mushy cereal wasn’t available?”

Roadmap

Micro BLP Estimation

Choosing Micro Moments

Using More Information

Coding Exercise 3

Micro BLP Estimator

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 - Ratios (e.g. mean income given mushy), correlations (e.g. between income and price), etc.

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- Don't have to be averages \bar{v} . Can match any smooth function $f(\bar{v})$.
 - Ratios (e.g. mean income given mushy), correlations (e.g. between income and price), etc.
- The resulting “micro BLP” estimator is used a lot in industrial organization.

Micro BLP Popularity

- First popularized by
Petrin (2002) and BLP (2004).

Paper	Industry	Country	Years
Petrin (2002)	Automobiles	United States	1981–1993
Berry, Levinsohn, and Pakes (2004)	Automobiles	United States	1993
Thomadsen (2005)	Fast Food	United States	1999
Goeree (2008)	Personal Computers	United States	1996–1998
Ciliberto and Kuminoff (2010)	Cigarettes	United States	1993–2002
Nakamura and Zerom (2010)	Coffee	United States	2000–2004
Beresteanu and Li (2011)	Automobiles	United States	1999–2006
Li (2012)	Automobiles	United States	1999–2006
Copeland (2014)	Automobiles	United States	1999–2008
Starc (2014)	Health Insurance	United States	2004–2008
Ching, Hayashi, and Wang (2015)	Nursing Homes	United States	1999
Li, Xiao, and Liu (2015)	Automobiles	China	2004–2009
Nurski and Verboven (2016)	Automobiles	Belgium	2010–2011
Barwick, Cao, and Li (2017)	Automobiles	China	2009–2011
Murry (2017)	Automobiles	United States	2007–2011
Wollmann (2018)	Commercial Vehicles	United States	1986–2012
Li (2018)	Automobiles	China	2008–2012
Li, Gordon, and Netzer (2018)	Digital Cameras	United States	2007–2010
Backus, Conlon, and Sinkinson (2021)	Cereal	United States	2007–2016
Grieco, Murry, and Yurukoglu (2021)	Automobiles	United States	1980–2018
Neilson (2021)	Primary Schools	Chile	2005–2016
Armitage and Pinter (2022)	Automobiles	United States	2009–2017
Döpper, MacKay, Miller, and Stiebale (2022)	Retail	United States	2006–2019
Durrmeyer (2022)	Automobiles	France	2003–2008
Weber (2022)	Trucks	United States	2010–2018
Bodéré (2023)	Preschools	United States	2010–2018
Montag (2023)	Laundry Machines	United States	2005–2015
Conlon and Rao (2023)	Distilled Spirits	United States	2007–2013

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- We'll use the standardized framework for PyBLP from [Conlon and Gortmaker \(2023\)](#).

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- There are two new components.
 1. Micro statistics $f(\bar{v}) = [f_1(\bar{v}), \dots, f_M(\bar{v})]'$.
 2. Their model analogues $f(v(\theta)) = [f_1(v(\theta)), \dots, f_M(v(\theta))]'$.

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- Statistically, we need $f(\bar{v}) \rightarrow f(v(\theta))$ as the micro dataset expands.
 - This gives what we'll call $m = 1, \dots, M$ different "micro moments."
 - Quite different from our "aggregate moments" $\mathbb{E}[\xi_{jt} \cdot z_{jt}] = 0$.

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- Different weights w_{dijt} , values v_{pijt} , and functions $f_m(\cdot)$ support most summary stats.

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 - In practice, you specify a second function to compute a matrix of values for each market t .
- Lastly, you need to define your **micro moment** m .
 - The identity function $f_m(\bar{v}_p) = \bar{v}_p$ just matches the mean surveyed income.
 - You also need to specify the actual value of the micro statistic \bar{v}_1 .

Model Analogues

$$f_m(\bar{v}_p) \rightarrow f_m(v_p(\theta))$$

- For each guess of θ , PyBLP will compute the model analogue $f_m(v_p(\theta))$.

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 - Expected micro value $v_{pi_n j_n t_n}$ divided by the probability of being selected.
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 1. In market $t_n \in \mathcal{T}$ with equal probability.

Model Analogues

$$f_m\left(\frac{1}{N_d} \sum_{n \in \mathcal{N}_d} v_{pi_n j_n t_n}\right) \rightarrow f_m\left(\frac{\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}_t} w_{it} \cdots v_{pijt}}{\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}_t} w_{it} \cdots}\right)$$

- For each guess of θ , PyBLP will compute the model analogue $f_m(v_p(\theta))$.
- The model analogue $v_p(\theta)$ of a micro part \bar{v}_p is a conditional expectation.
 - Expected micro value $v_{pi_n j_n t_n}$ divided by the probability of being selected.
- Recall the data generating process for surveyed consumers $n \in \mathcal{N}_d$.
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 4. Selected to be in the survey with known probability $w_{di_n j_n t_n}$.

Roadmap

Micro BLP Estimation

Choosing Micro Moments

Using More Information

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- My advice for adding micro moments is similar.
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 - Start by choosing a single micro moment that “targets” the parameter.
- What if you could estimate a parameter with either aggregate or micro variation?
 - Could just choose the variation that seems more “credible.” Often the micro moment.
 - Can use both. Micro moments can reduce large SEs from limited aggregate variation.

Targeting Micro Moments

- Simplest case: 1 characteristic x_{jt} (e.g. price), 1 demographic y_{it} (e.g. income).

$$u_{ijt} = \beta_1 + \pi_1 y_{it} + (\beta_x + \pi_x y_{it}) x_{jt} + \xi_{jt} + \varepsilon_{ijt}$$

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- To target π_x , you can match “ $\mathbb{E}[y_{it} \cdot x_{jt} \mid j \neq 0]$ ” or similar, e.g. “ $\mathbb{C}(y_{it}, x_{jt} \mid j \neq 0)$.”
 - Relationship between income and price targets how income shifts price sensitivity.
 - Other common examples include “ $\mathbb{E}[y_{it} \mid x_{jt} < \bar{x}]$ ” and “ $\mathbb{E}[x_{jt} \mid y_{it} < \bar{y}]$.”

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 - Other common examples include “ $\mathbb{E}[y_{it} \mid x_{jt} < \bar{x}]$ ” and “ $\mathbb{E}[x_{jt} \mid y_{it} < \bar{y}]$.”
- Micro data is not directly informative about “linear parameters” β_1 or β_x .
 - Mean utility $\delta_{jt} = \beta_1 + \beta_x x_{jt} + \xi_{jt}$ is already pinned down by market shares s_{jt} .

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 - Survey asks consumers which $k_n \neq j_n$ they'd choose if their first choice weren't available.
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 - Survey asks consumers which $k_n \neq j_n$ they'd choose if their first choice weren't available.
 - Micro weights and values now just have an extra index: w_{dijkt} and v_{pijkt} .
- Direct measures of substitution are very informative about Σ .
 - Recall the red bus/blue bus example that motivated adding preference heterogeneity.
 - Each second choice is like observing a new market with the first choice removed.

Second Choice Moments

- Extend the simple example from before with unobserved preference heterogeneity:

$$u_{ijt} = \beta_1 + \sigma_1 \nu_{1it} + \pi_1 y_{it} + (\beta_x + \sigma_x \nu_{2it} + \pi_x y_{it}) x_{jt} + \xi_{jt} + \varepsilon_{ijt}$$

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- To target σ_x , want a measure of how much people substitute within x_{jt} .
 - In your exercise, you'll match the share " $\mathbb{P}(\text{mushy}_{jt} \text{ and } \text{mushy}_{kt} \mid j \neq 0)$."
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- **Diversion ratios** straightforward to interpret and collect.

Outside Substitution and Market Size

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- On day 1, we discussed how choosing a market size is neither easy nor innocuous.
 - Assuming a small market size M_t means assuming a small outside share s_{0t} .
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 - Typically, this results in relatively high inside qualities ξ_{jt} .
 - Implies little substitution to the outside good in counterfactuals.
- Directly matching an outside diversion ratio will help discipline outside substitution.
 - If $\hat{\sigma}_1$ is large, many people will dislike all inside goods and usually choose $j = 0$.
 - This reduces the *effective* market size, helping to compensate for a too-large M_t .
 - See [Zhang \(2023\)](#) for more on how σ_1 can help and other solutions.

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 - **Compatibility**: Aggregate and micro data may come from different sampling schemes.
 - **Clarity**: Matching a single statistic makes it clear where identification comes from.
- But adding more info can greatly increase the precision of our estimates.
 - Ideally we'd observe a complete micro dataset $\{t_n, j_n, k_n, y_{i_n t_n}\}_{n \in \mathcal{N}_d}$.

Maximum Likelihood

- If we only had micro data, we may want to just work with its log likelihood:

$$\log \mathcal{L}(\theta, \delta) = \sum_{n \in \mathcal{N}_d} \mathbb{P}(t_n, j_n, k_n, y_{i_n t_n} \mid n \in \mathcal{N}_d; \theta, \delta)$$

- This classic approach proceeds into two steps:
 1. Find the $\hat{\theta} = (\hat{\Sigma}, \hat{\Pi})$ and mean utilities $\hat{\delta}$ that maximize $\log \mathcal{L}(\theta, \delta)$.
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- This classic approach proceeds into two steps:
 1. Find the $\hat{\theta} = (\hat{\Sigma}, \hat{\Pi})$ and mean utilities $\hat{\delta}$ that maximize $\log \mathcal{L}(\theta, \delta)$.
 2. Run an IV regression of $\hat{\delta}_{jt}$ on x_{jt} to recover linear parameters $\hat{\beta}$ like in day 1.
- For a modern take on this MLE approach, see [Grieco, Murry, Pinkse and Sagl \(2023\)](#).
 - Combine 1, 2, and the likelihood for aggregate market shares into a single objective.
 - Their Julia package [Grumps.jl](#) efficiently handles the high-dimensional $\delta = \{\delta_{jt}\}_{j,t}$.

Optimal Micro Moments

- In micro BLP, **optimal micro moments** match the first-order conditions in MLE:

$$f^*(\bar{v}) = \frac{1}{N_d} \sum_{n \in \mathcal{N}_d} \frac{\partial \mathbb{P}(t_n, j_n, k_n, y_{i_n t_n} \mid n \in \mathcal{N}_d; \theta)}{\partial \theta}$$

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- These use all the information in a micro dataset (**Conlon and Gortmaker, 2023**).
 - Intuition for statistical efficiency here is just that MLE is efficient.
- Can be a bit tricky to compute, but only a few lines of code with PyBLP.
 - Like optimal IVs, can update along with the weighting matrix for a second GMM step.

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- We'll post solutions to all the exercises.

Good luck with estimating your own demand systems!

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