Demand Estimation

MIXTAPE SESSION

Jeff Gortmaker and Ariel Pakes



$$\hat{\theta} = \operatorname*{argmin}_{\theta} g(\theta) W g(\theta)' \quad \text{where} \quad g(\theta) = \frac{1}{N} \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{J}_t} \overbrace{(\delta_{jt} - x'_{jt}\beta)}^{\xi_{jt}(\theta)} \cdot z_{jt}$$
 subject to
$$s_{jt} = \sum_{i \in \mathcal{I}_t} w_{it} \cdot \frac{\exp[\delta_{jt} + \mu_{ijt}(\theta)]}{1 + \sum_{k \in \mathcal{J}_t} \exp[\delta_{kt} + \mu_{ikt}(\theta)]}$$

• On day 2, we added preference heterogeneity μ_{ijt} to improve substitution patterns.

$$\begin{split} \hat{\theta} &= \operatorname*{argmin}_{\theta} g(\theta) W g(\theta)' \quad \text{where} \quad g(\theta) = \frac{1}{N} \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{J}_t} \underbrace{(\delta_{jt} - x'_{jt}\beta)}_{(\delta_{jt} - x'_{jt}\beta)} \cdot z_{jt} \\ \text{subject to} \quad s_{jt} &= \sum_{i \in \mathcal{I}_t} w_{it} \cdot \frac{\exp[\delta_{jt} + x'_{jt}(\Sigma \nu_{it} + \Pi y_{it})]}{1 + \sum_{k \in \mathcal{J}_t} \exp[\delta_{kt} + x'_{kt}(\Sigma \nu_{it} + \Pi y_{it})]} \end{split}$$

- On day 2, we added preference heterogeneity μ_{ijt} to improve substitution patterns.
 - ightarrow Most common form is $\mu_{ijt}=x'_{jt}(\Sigma\nu_{it}+\Pi y_{it})$ for $\nu_{it}\sim N(0,I)$ and y_{it} from census data.
 - \rightarrow Implements random coefficients $\beta_{it} \sim N(\beta + \Pi y_{it}, \Sigma \Sigma')$ on characteristics x_{jt} in utility.

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- Let's go over your second coding exercise.

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 - → Same cereals in each market, so no choice set variation along mushy dimension.
 - → Results in unrealistically limited substitution between similar cereals.
- Also can't estimate a parameter in Π on log income alone.
 - → Market fixed effects are collinear with market-level income means.
 - ightarrow Unrealistic that overall cereal preference doesn't vary with income.

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- Typical example is consumer survey data.
 - \rightarrow Internal surveys conducted by firms.
 - → Ad-hoc surveys conducted by academics.
 - ightarrow Marketing research datasets (e.g. NielsenlQ's Consumer Panel).
 - → Regulatory agencies like the UK's antitrust authority (Reynolds and Walters, 2008).

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- Let's incorporate answers to these questions into estimation.
 - ightarrow We'll set up a general framework and come back to these when we have notation to do so.

Roadmap

Micro BLP Estimation

Choosing Micro Moments

Using More Information

Coding Exercise 3

$$\hat{\theta} = \operatorname*{argmin}_{\theta} g(\theta) W g(\theta)' \quad \text{where} \quad g(\theta) = \frac{1}{N} \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{J}_t} (\delta_{jt}(\theta) - x'_{jt}\beta) \cdot z_{jt}$$

• We can extend our BLP estimator by matching statistics from consumer surveys.

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 - ightarrow Ratios (e.g. mean income given mushy), correlations (e.g. between income and price), etc.
- The resulting "micro BLP" estimator is used a lot in industrial organization.

Micro BLP Popularity

• First popularized by Petrin (2002) and BLP (2004).

Paper	Industry	Country	Years
Petrin (2002)	Automobiles	United States	1981-1993
Berry, Levinsohn, and Pakes (2004)	Automobiles	United States	1993
Thomadsen (2005)	Fast Food	United States	1999
Goeree (2008)	Personal Computers	United States	1996 - 1998
Ciliberto and Kuminoff (2010)	Cigarettes	United States	1993-2002
Nakamura and Zerom (2010)	Coffee	United States	2000-2004
Beresteanu and Li (2011)	Automobiles	United States	1999-2006
Li (2012)	Automobiles	United States	1999-2006
Copeland (2014)	Automobiles	United States	1999-2008
Starc (2014)	Health Insurance	United States	2004-2008
Ching, Hayashi, and Wang (2015)	Nursing Homes	United States	1999
Li, Xiao, and Liu (2015)	Automobiles	China	2004-2009
Nurski and Verboven (2016)	Automobiles	Belgium	2010-2011
Barwick, Cao, and Li (2017)	Automobiles	China	2009-2011
Murry (2017)	Automobiles	United States	2007 - 2011
Wollmann (2018)	Commercial Vehicles	United States	1986 - 2012
Li (2018)	Automobiles	China	2008-2012
Li, Gordon, and Netzer (2018)	Digital Cameras	United States	2007-2010
Backus, Conlon, and Sinkinson (2021)	Cereal	United States	2007-2016
Grieco, Murry, and Yurukoglu (2021)	Automobiles	United States	1980 - 2018
Neilson (2021)	Primary Schools	Chile	2005-2016
Armitage and Pinter (2022)	Automobiles	United States	2009-2017
Döpper, MacKay, Miller, and Stiebale (2022)	Retail	United States	2006-2019
Durrmeyer (2022)	Automobiles	France	2003-2008
Weber (2022)	Trucks	United States	2010-2018
Bodéré (2023)	Preschools	United States	2010-2018
Montag (2023)	Laundry Machines	United States	2005 - 2015
Conlon and Rao (2023)	Distilled Spirits	United States	2007-2013

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- We'll use the standardized framework for PyBLP from Conlon and Gortmaker (2023).

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- There are two new components.
 - 1. Micro statistics $f(\overline{v}) = [f_1(\overline{v}), \dots, f_M(\overline{v})]'$.
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 - 2. Their model analogues $f(v(\theta)) = [f_1(v(\theta)), \dots, f_M(v(\theta))]'$.
- Statistically, we need $f(\overline{v}) \to f(v(\theta))$ as the micro dataset expands.
 - \rightarrow This gives what we'll call $m=1,\ldots,M$ different "micro moments."
 - ightarrow Quite different from our "aggregate moments" $\mathbb{E}[\xi_{jt}\cdot z_{jt}]=0$.

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• Different weights w_{dijt} , values v_{pijt} , and functions $f_m(\cdot)$ support most summary stats.

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- Lastly, you need to define your micro moment m.
 - \rightarrow The identity function $f_m(\overline{v}_p) = \overline{v}_p$ just matches the mean surveyed income.
 - \rightarrow You also need to specify the actual value of the micro statistic \overline{v}_1 .

$$f_m(\overline{v}_p) \to f_m(v_p(\theta))$$

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- The model analogue $v_p(\theta)$ of a micro part \overline{v}_p is a conditional expectation.
 - ightarrow Expected micro value $v_{pi_nj_nt_n}$ divided by the probability of being selected.
- Recall the data generating process for surveyed consumers $n \in \mathcal{N}_d$.

$$f_m\left(\frac{1}{N_d}\sum_{n\in\mathcal{N}_d}v_{pi_nj_nt_n}\right)\to f_m\left(\frac{\sum_{t\in\mathcal{T}}\cdots v_{pijt}}{\sum_{t\in\mathcal{T}}\cdots}\right)$$

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- The model analogue $v_p(\theta)$ of a micro part \overline{v}_p is a conditional expectation.
 - ightarrow Expected micro value $v_{pi_nj_nt_n}$ divided by the probability of being selected.
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 - 4. Selected to be in the survey with known probability $w_{di_nj_nt_n}$.

Roadmap

Micro BLP Estimation

Choosing Micro Moments

Using More Information

Coding Exercise 3

Adding Micro Moments

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 - ightarrow Start by choosing a single micro moment that "targets" the parameter.
- What if you could estimate a parameter with either aggregate or micro variation?
 - ightarrow Could just choose the variation that seems more "credible." Often the micro moment.
 - $\,\rightarrow\,$ Can use both. Micro moments can reduce large SEs from limited aggregate variation.

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 - ightarrow Relationship between income and price targets how income shifts price sensitivity.
 - o Other common examples include " $\mathbb{E}[y_{it} \mid x_{jt} < \overline{x}]$ " and " $\mathbb{E}[x_{jt} \mid y_{it} < \overline{y}]$."

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- Micro data is not directly informative about "linear parameters" β_1 or β_x .
 - \rightarrow Mean utility $\delta_{jt} = \beta_1 + \beta_x x_{jt} + \xi_{jt}$ is already pinned down by market shares s_{jt} .

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 - \rightarrow Survey asks consumers which $k_n \neq j_n$ they'd choose if their first choice weren't available.
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 - ightarrow Micro weights and values now just have an extra index: w_{dijkt} and v_{pijkt} .
- Direct measures of substitution are very informative about Σ .
 - \rightarrow Recall the red bus/blue bus example that motivated adding preference heterogeneity.
 - ightarrow Each second choice is like observing a new market with the first choice removed.

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- To target σ_x , want a measure of how much people substitute within x_{jt} .
 - \rightarrow In your exercise, you'll match the share " $\mathbb{P}(\text{mushy}_{it} \text{ and mushy}_{kt} \mid j \neq 0)$."
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- Diversion ratios straightforward to interpret and collect.

Outside Substitution and Market Size

- Estimating a σ_1 is important if you're interested in inside-outside substitution.
 - → How many consumers will stop purchasing soda if all sodas are taxed?
 - ightarrow How much larger will a market grow if innovation reduces production costs?

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- On day 1, we discussed how choosing a market size is neither easy nor innocuous.
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 - \rightarrow Assuming a small market size M_t means assuming a small outside share s_{0t} .
 - \rightarrow Typically, this results in relatively high inside qualities ξ_{jt} .
 - ightarrow Implies little substitution to the outside good in counterfactuals.
- Directly matching an outside diversion ratio will help discipline outside substitution.
 - ightarrow If $\hat{\sigma}_1$ is large, many people will dislike all inside goods and usually choose j=0.
 - ightarrow This reduces the effective market size, helping to compensate for a too-large $M_t.$
 - ightarrow See Zhang (2023) for more on how σ_1 can help and other solutions.

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Information Tradeoffs

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 - → Confidentiality: Data providers may want to protect respondent identities.
 - → Compatibility: Aggregate and micro data may come from different sampling schemes.
 - → Clarity: Matching a single statistic makes it clear where identification comes from.
- But adding more info can greatly increase the precision of our estimates.
 - ightarrow Ideally we'd observe a complete micro dataset $\{t_n,j_n,k_n,y_{i_nt_n}\}_{n\in\mathcal{N}_d}$.

Maximum Likelihood

If we only had micro data, we may want to just work with its log likelihood:

$$\log \mathcal{L}(\theta, \delta) = \sum_{n \in \mathcal{N}_d} \mathbb{P}(t_n, j_n, k_n, y_{i_n t_n} \mid n \in \mathcal{N}_d; \theta, \delta)$$

- This classic approach proceeds into two steps:
 - 1. Find the $\hat{\theta} = (\hat{\Sigma}, \hat{\Pi})$ and mean utilities $\hat{\delta}$ that maximize $\log \mathcal{L}(\theta, \delta)$.
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- This classic approach proceeds into two steps:
 - 1. Find the $\hat{\theta} = (\hat{\Sigma}, \hat{\Pi})$ and mean utilities $\hat{\delta}$ that maximize $\log \mathcal{L}(\theta, \delta)$.
 - 2. Run an IV regression of $\hat{\delta}_{it}$ on x_{it} to recover linear parameters $\hat{\beta}$ like in day 1.
- For a modern take on this MLE approach, see Grieco, Murry, Pinkse and Sagl (2023).
 - \rightarrow Combine 1, 2, and the likelihood for aggregate market shares into a single objective.
 - ightarrow Their Julia package Grumps.jl efficiently handles the high-dimensional $\delta=\{\delta_{jt}\}_{j,t}$.

Optimal Micro Moments

• In micro BLP, optimal micro moments match the first-order conditions in MLE:

$$f^*(\overline{v}) = \frac{1}{N_d} \sum_{n \in \mathcal{N}} \frac{\partial \mathbb{P}(t_n, j_n, k_n, y_{i_n t_n} \mid n \in \mathcal{N}_d; \theta)}{\partial \theta}$$

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- These use all the information in a micro dataset (Conlon and Gortmaker, 2023).
 - \rightarrow Intuition for statistical efficiency here is just that MLE is efficient.
- Can be a bit tricky to compute, but only a few lines of code with PyBLP.
 - $\,\rightarrow\,$ Like optimal IVs, can update along with the weighting matrix for a second GMM step.

Roadmap

Micro BLP Estimation

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 - → Estimating a nesting parameter.
- We'll post solutions to all the exercises.

Good luck with estimating your own demand systems!

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