Demand Estimation

Mixtape Session

Jeff Gortmaker and Ariel Pakes

Who Am I?

- A fifth-year Economics PhD candidate at Harvard University.

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- A fifth-year Economics PhD candidate at Harvard University.
- Making BLP-style estimation more accessible to researchers.
  - → Best practices papers (Conlon and Gortmaker, 2020, 2023).
  - → Open-source Python package (PyBLP).
  - → This course!

This Course

- Three days, 6pm-9pm.
    1. Today: BLP model, pure logit, price endogeneity.
    2. Wednesday: Mixed logit, identification, numerical best practices.
    3. Friday: Micro BLP, consumer survey data, other extensions.

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- Three coding exercises, one after each day.
  - $\rightarrow$ Try these on your own or with your classmates' help. Use Discord rooms!
  - $\rightarrow$ I'll do the first two exercises live at the start of days 2 and 3. We'll post solutions.

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- Typically used for counterfactual analysis of something that hasn't happened.
  - $\rightarrow$ Need a structural model when we can't just estimate a treatment effect.
- Running example: What if we halved an important product's price?
  - $\rightarrow$ Practitioners: Increased sales vs. cannibalization?
  - $\rightarrow$ Regulators: Revenue loss from eliminating a tax?
  - $\rightarrow$ Academics: Welfare consequences?

Roadmap

The BLP Model

Pure Logit Estimation

Price Endogeneity

Coding Exercise 1

Model Overview

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  - $\rightarrow$ Different demographics and preferences.
- Individuals are faced with choices denoted by $j \in \mathcal{J}_t$.
  - $\rightarrow$ Products, hospitals, candidates, etc.
  - $\rightarrow$ Outside option $j = 0$: no purchase, no treatment, no vote, etc.

Utility Maximization

$$\max_{j \in \mathcal{J}_t \cup \{0\}} u_{ijt}$$

- Individuals choose an alternative to maximize (indirect) utility u_{ijt}.
  - $\rightarrow$ We will specify a function for u_{ijt} and use revealed preference to estimate it.

Utility Maximization

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- We will parameterize δ_{jt} and μ_{ijt} and make a convenient assumption about ε_{ijt}.

Aggregate Market Shares

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- We'll match these to observed quantities $q_{jt} = s_{jt}\cdot M_t$ in our data.

Choosing a Market Size

- In our data, we observe quantities $q_{jt} = s_{jt} \cdot M_t$.
  - $\rightarrow$ Need to divide by some market size M_t to get our model's market shares s_{jt}.
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- Sometimes the choice of market size is straightforward.
  - $\rightarrow$ Market size for drugs to treat a condition is how many people have that condition.
- But typically, the choice of market size is neither easy nor innocuous.
  - $\rightarrow$ E.g. how many choices of which cereal to buy are made every day in a specific city?
  - $\rightarrow$ Population $\times$ max cereals per day? Foot traffic estimate $\times$ max cereals per trip?

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- You should try different assumptions and see how they change your results.
  - $\rightarrow$ In general, the bigger the market size, the more substitution to the outside good.
  - $\rightarrow$ We'll learn how to discipline these assumptions with data on day 3.

Identification and Normalizations

$$\max_{j \in \mathcal{J}_t \cup \{0\}} u_{ijt} = \delta_{jt} + \mu_{ijt} + \varepsilon_{ijt}$$

- We will estimate our utility function with revealed preference.
  - $\rightarrow$ Holding μ_{ijt} fixed, a higher quantity $q_{jt} > q_{kt}$ implies a higher mean utility $\delta_{jt} > \delta_{kt}$.

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$$u_{ijt} > u_{ikt} \quad \overset{b>0}{\Longleftrightarrow} \quad a + b \cdot u_{ijt} > a + b \cdot u_{ikt}$$

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- Now that our model can in theory be identified, how do we estimate it?

Roadmap

The BLP Model

Pure Logit Estimation

Price Endogeneity

Coding Exercise 1

Pure Logit Model

$$\max_{j \in \mathcal{J}_t \cup \{0\}} u_{ijt} = \delta_{jt} + \cancel{\mu_{ijt}}^{\,0} + \varepsilon_{ijt}$$

- Start with the simplest case: no heterogenous utility. We'll add μ_{ijt} back on day 2.

Pure Logit Model

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- We can recover mean utilities from observed market shares (Berry, 1994).
  - $\rightarrow$ If we specify a function for δ_{jt}, we'll have a linear regression!

Pure Logit Estimating Equation

$$\log \frac{s_{jt}}{s_{0t}} = \delta_{jt}$$

- Running example: What if we halved an important product's price?
  - $\rightarrow$ In your exercise, products j are breakfast cereals; markets t are city-quarters.
  - $\rightarrow$ If we estimate the model, we can change p_{jt} and estimate how consumers react.

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  - $\rightarrow$ In your exercise, p_{jt} is per serving; x_{jt} includes a constant, a "mushy" dummy, etc.
- Interpret the regression error ξ_{jt} as unobserved product quality not in our data.
  - $\rightarrow$ Unobserved characteristics, advertising, average taste variation, "demand shocks," etc.

Interpreting Parameters

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- Let's say we estimate this equation. How to interpret our parameter estimates?

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  - $\rightarrow$ Instead, report own-price elasticities, or a quantity-weighted average/median.
  - $\rightarrow$ You can derive elasticities by differentiating the multinomial logit expression for s_{jt}.

$$\eta_{jjt} = \frac{\partial \log q_{jt}}{\partial \log p_{jt}} = \frac{\partial q_{jt}}{\partial p_{jt}} \frac{p_{jt}}{q_{jt}} = \frac{\partial s_{jt}}{\partial p_{jt}} \frac{p_{jt}}{s_{jt}} = \alpha \cdot p_{jt} \cdot (1 - s_{jt})$$

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  - $\rightarrow$ Instead, report β/α, the dollar willingness to pay for mushyness.

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Coding Exercise 1

Endogeneity Concerns

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- Typically, we expect price to be strongly correlated with unobserved quality.
  $\rightarrow$ Firms know more than us about demand when setting prices.
  $\rightarrow$ Often, $\mathbb{C}(p_{jt}, \xi_{jt}) > 0$, so $\hat{\alpha} < 0$ is biased towards zero. $\mathbb{C}$ means covariance.

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- Today we'll focus on handling just price endogeneity for simplicity.

Fixed Effects

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- Adding product and market fixed effects to x_{jt} can eliminate a lot of bias.
  - $\rightarrow$ E.g. if p_{jt} is correlated with fixed effects ξ_j and/or ξ_t in $\xi_{jt} = \xi_j + \xi_t + \Delta\xi_{jt}$.
  - $\rightarrow$ But do need multiple observations per product and market to add ξ_j and ξ_t.

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  - $\rightarrow$ But do need multiple observations per product and market to add ξ_j and ξ_t.
  - $\rightarrow$ Aside: Related to dynamic panel approach. Let $\xi_{jt} = \rho\xi_{jt-1} + \Delta\xi_{jt}$, estimate ρ.

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  - $\rightarrow$ E.g. if p_{jt} is correlated with fixed effects ξ_j and/or ξ_t in $\xi_{jt} = \xi_j + \xi_t + \Delta\xi_{jt}$.
  - $\rightarrow$ But do need multiple observations per product and market to add ξ_j and ξ_t.
  - $\rightarrow$ Aside: Related to dynamic panel approach. Let $\xi_{jt} = \rho\xi_{jt-1} + \Delta\xi_{jt}$, estimate ρ.
- Modern grocery scanner datasets have many thousands of products/markets.
  - $\rightarrow$ Dummies take too much memory, so we "absorb" them, i.e. iteratively de-mean.
  - $\rightarrow$ Stata: Reghdfe. R: Fixest. Python: PyFixest. Coding exercise: PyBLP via PyHDFE.

Fixed Effects

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- Modern grocery scanner datasets have many thousands of products/markets.
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  - $\rightarrow$ Stata: Reghdfe. R: Fixest. Python: PyFixest. Coding exercise: PyBLP via PyHDFE.
- Helpful but insufficient: ξ_{jt} typically varies by product *and* market.
  - $\rightarrow$ And if prices don't, then we don't have any variation left to estimate α.

Instrumental Variables

$$\delta_{jt} = \alpha p_{jt} + x'_{jt}\beta + \xi_{jt}$$

- With or without fixed effects, a carefully-chosen IV can be a good solution.
  - $\rightarrow$ Relevance: $\mathbb{C}(p_{jt}, z_{jt}) \neq 0$. Exclusion: $\mathbb{C}(\xi_{jt}, z_{jt}) = 0$.

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- Always run a first-stage regression of p_{jt} on z_{jt} and x_{jt}.
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  - $\rightarrow$ Does the sign of the coefficient on z_{jt} make sense?
  - $\rightarrow$ Is the instrument strong, or should you worry about weak instruments?
- Many places to look. I'll discuss the most common ones.

Typical Instruments for Price

$$\delta_{jt} = \alpha p_{jt} + x'_{jt}\beta + \xi_{jt}$$

- Typically, prices are marginal costs plus a markup term.
  - $\rightarrow$ We want valid instruments that shift costs and/or markups.

Typical Instruments for Price

$$\delta_{jt} = \alpha p_{jt} + x'_{jt}\beta + \xi_{jt}$$

- Typically, prices are marginal costs plus a markup term.
- Cost-shifters: Measures of input prices, tariffs, etc.
  $\rightarrow$ Consumers should only care about them through their effect on prices.

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- Typically, prices are marginal costs plus a markup term.
- Cost-shifters: Measures of input prices, tariffs, etc.
- Hausman: Current price of the same product averaged across *other* locations.
  - $\rightarrow$ Need costs to be correlated across locations, but not unobserved quality.
  - $\rightarrow$ Helpful that retailers tend to do "uniform pricing" (DellaVigna and Gentzkow, 2019).

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- Hausman: Current price of the same product averaged across *other* locations.
- Waldfogel: Average consumer characteristics in *nearby* locations.
  $\rightarrow$ With uniform pricing, your neighbors' demographics will affect your prices.

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- Hausman: Current price of the same product averaged across *other* locations.
- Waldfogel: Average consumer characteristics in *nearby* locations.
- BLP: Average characteristics x_{kt} of *competing* products $k \neq j$.
  - $\rightarrow$ Characteristics of competing products affect markups.
  - $\rightarrow$ We'll come back to these later, since they can also serve a different purpose.

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- Waldfogel: Average consumer characteristics in *nearby* locations.
- BLP: Average characteristics x_{kt} of *competing* products $k \neq j$.
- I recommend starting with just one. A straightforward cost-shifter if you have it.

Roadmap

The BLP Model

Pure Logit Estimation

Price Endogeneity

Coding Exercise 1

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- Try to do the first exercise before day 2's class, when I'll do it live.
    1. Getting set up with Python and PyBLP.
    2. Pure logit estimation.
    3. Running the price cut counterfactual.

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  - $\rightarrow$ Do the substitution patterns you estimate seem reasonable?

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  → Do the substitution patterns you estimate seem reasonable?

- If you have time, try the supplemental exercises.
  → Statistical inference.
  → Modeling the supply side.
  → Checking your code by simulating data.

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