IMAGE UPSAMPLING WITH RANDOMIZED INTERPOLATION

V. Parubochyi[1], R. Shuwar[1], D. Afanassyev[2]

[1] *Semiconductor Physics Department, Ivan Franko National University of Lviv,*
50 Drahomanov St., Lviv, Ukraine, 79005
eddragonwolf@ukr.net, shuwar@electronics.lnu.edu.ua
[2] *Lviv Polytechnic National University,*
12 Bandera street, Lviv, Ukraine, 79013
denisna64@gmail.com

A new method for upsampling of bitmap images based on randomized interpolation is described. A problem of aliasing, visibility of digital artifacts and accuracy of image analysis is discussed from the viewpoint of the nature of quantization error. A possibility of obtaining a noise-like residual error and attenuation of regular artifacts by the method proposed is demonstrated.

Key words: Sampling theorem, digital artifacts, randomized interpolation, stochastic upsampling method, image analysis.

Introduction

The need for upsampling the images often arises considering the enhancement of visual quality. The goal is to reduce the digital artifacts like pixilation, false colors, etc. to the level behind the threshold of visual acuity, using the excess pixel density of high resolution displays. This idea can be understood as 2D extension of combination of oversampling and digital filtering widely used in reproduction of digital audio [1]. Fig.1 shows the functional diagram of a typical digital audio playback system with oversampling. One should note that such oversampling systems use the integer factor for sampling frequency increase. The zeros are inserted between the original data samples, and then a high order low-pass digital filter is applied to recover the smooth signal. This signal is fed to the D/A converter operating at increased sampling frequency, allowing one to use a simple low order LPF at its output.

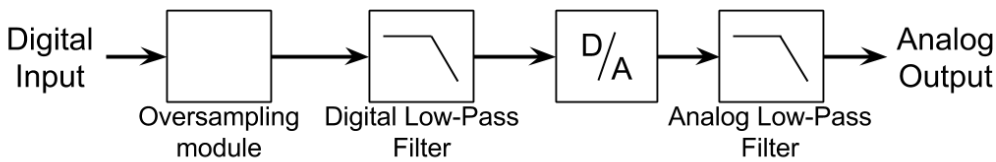


Figure 1: The functional diagram of a typical digital audio playback system with oversampling.

Similarly a high resolution display (for instance, featuring 4K or higher resolution) can be understood as such oversampled D/A converter, while the human vision with naturally limited acuity acts like the low order output LPF [2,3]. The information content of the input image is not increased above the Nyquist limit defined by the original pixel density. Moreover, the enhancement of perceptual visual quality by such method may be accompanied by undesirable side effects resulting in additional errors of reproduction of fine details. These errors may originate from finite accuracy of computation, data truncation and distortion of spatial frequencies spectra.

Special requirements to the accuracy of image reproduction are applied if the upsampling is used as a preprocessing step before image analysis. Such analysis may include the recognition of objects or features in the image as well as correct determination of pixels statistics. Often the extraction of subpixel information may facilitate the accurate segmentation. The upsampling is needed to bring the pixels density of the image to the range for which the algorithm of subsequent analysis is optimized. For instance, the existing OCR software often is optimized for at least 300dpi resolution of the input images for the reliable text recognition. Small characters with reduced count of pixels may be still correctly if one provides their upsampling with removal of digital artifacts.

The residual error of image reproduction can be successfully removed by digital filtering of the upsampled image if this error is of random, noise-like nature. In contrary, the digital representation of analog data generates a quantization error containing coherent spectral components, which interfere with fine detail information. Therefore, the transformation of coherent error spectrum into a randomized one is desirable for enhancement of fine detail recognition in upsampled images.

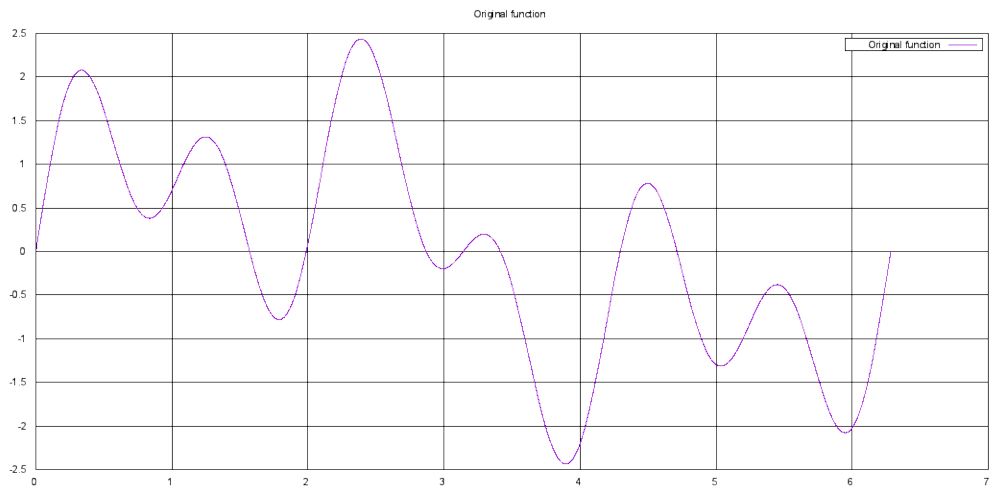
The authors propose a new method of image upsampling dealing with such transformation of the error spectrum. The method is based on introduction of a random dither into 2D sampling scheme with subsequent linear interpolation based on known Delaunay triangulation and application of a smooth low-pass filter.

The applications of image scaling methods with advanced requirements for accuracy of fine detail reproduction include various tasks of computer vision, medical and geophysical imaging, object detection, content based image retrieval, etc. Usual first steps of the methods of image analysis used in these applications are segmentation and edge detection. The need for upsampling arises if the pixels density of the original image is not enough for accurate partitioning of the image and a subpixel structure may be reconstructed resulting in increase of procedure accuracy. Although an image segment may contain necessary number of pixels for its correct representation as defined by sampling theorem, it may be necessary to apply a kind of brick wall low-pass filter to avoid aliasing. Such filters possess a drawback of oscillating impulse response, that itself generates the visual artifacts like false edges and ringing, which may complicate the subsequent analysis procedures. The known regular upsampling schemes like one described above for digital audio playback use the digital realizations of low-pass reconstruction filters with group delay correction, which somewhat improves impulse response. The 2D extensions of such filters needed for upsampling also utilize methods for smooth interpolation between original pixels, among them the Lanczos algorithm is thought to be a best compromise one [4]. Indeed, such methods are still not satisfactory and generate residual aliases. Sometimes the false contours arising from high order interpolation methods may be considered the useful side effect, for instance, to enhance edges for visual observation. Indeed, it will be always undesirable for informational analysis. For instance, it is found that the simple bilinear interpolation can be considered more appropriate for such purposes than more elaborate high order methods like bicubic or Lanczos ones.

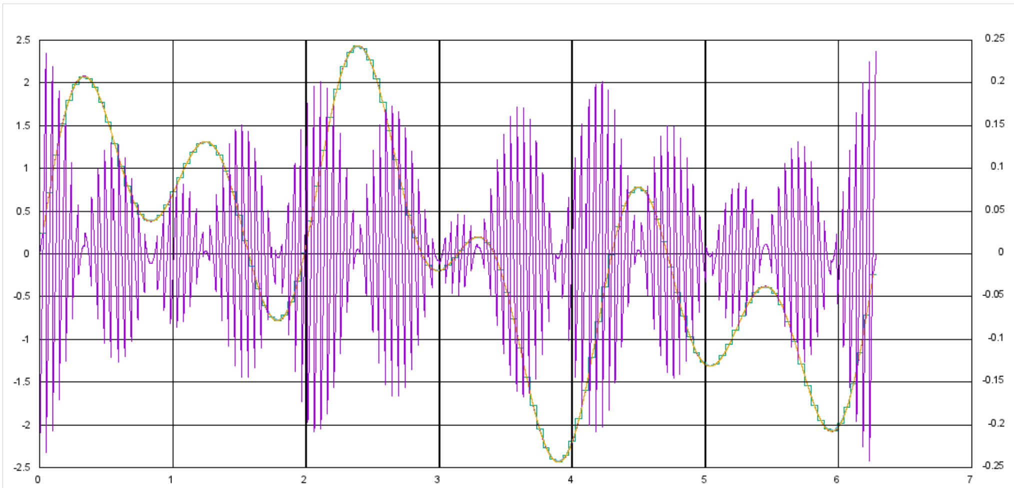
There exists a possibility to transform the regular residual error spectra of uniform sampling into random noise by using the extension of sampling theorem. It is known that the continuous function can be reproduced by a nonuniformly spaced set of samples if their mean frequency is at least twice the upper limit of the spectrum of the input function (like the Nyquist limit). This condition results in suppression or considerable attenuation of coherent aliases and appearance of a broadband noise trace instead of them [5]. Applied to images, this residual noise will look exactly like a grain observable in analog photography. This grain can be understood exactly as a manifestation of reconstruction error of randomized sampling, while the plurality of silver halide crystals in a photo emulsion serve as a stochastic image sensor. The available digital image sensors are regular lattices of pixels. As the pixels are of finite size, the act as finite window low-pass filters, which average the light over a defined square area. The transfer function of such filter is the same as that of a boxcar integrator [3,6]. Indeed it is not immune from aliasing as the frequencies above $f_n/2$ (f_n is the Nyquist frequency) are not enough attenuated. The regular color filter array like the Bayer one mostly used in digital cameras increases this aliasing further, as the size of a pixel is much smaller than the period of sublattice of each primary color. The lens of the camera acts as a low-pass filter before the digital sensor, but often its transfer function does not attenuate the spatial frequencies above of the $f_n/2$ for individual color layers. Previously the digital cameras had the antialiasing filter installed in front of the image sensor. This filter splits each incoming beam into four ones by means of two birefringent plates, thus reducing the spatial frequency by the factor of two. It corresponds to the size of Bayer array cell, which consists of four pixels (two green, one red and one blue). This filter reduces the appearance of such visual artifacts as moire or color pixilation, but at the expense of reduced resolution. The modern image sensors feature much smaller pixels, so it is a common practice to avoid the use of such low-pass filters. Indeed, the aliases still appear in the images containing well – defined fine structure, the only way to suppress them is the subsequent digital processing.

The pixilation artifacts due to finite size of pixels in the image resulting from such acquisition procedure are visible if they are larger than the circle of confusion of the eye. The same aliasing takes place with zero order interpolation of one – dimension signals digitized by an integrating A/D converter. Fig. 2 illustrates the appearance and amplitude of such aliases using a sum of harmonic oscillations as the input signal.

One can clearly see a massive coherent aliasing component (violet line), which falls into the spectral range of the input signal. A desirable result hare would be a transformation of this regular error into some kind of random noise. This result can be achieved by introducing an irregular distribution of samples during the data acquisition process. Indeed, the spatial distribution of pixels in commonly available image sensors is regular therefore such aliasing components may be present in an acquired image. Nevertheless it is worth attention to apply a randomization to the grid of interpolation nodes in the procedure of upsampling[6,7]. This solution can reduce the residual regular artifacts in the resulting interpolated image.



a)



b)

Figure 2: a): input signal y = sin(x) + sin(3 * x) + sin(6 * x); b): error of zero order interpolation (right y axis) after integrating A/D conversion.

Stochastic upsampling method

Randomization is implemented by jittering of interpolation nodes within a circular window of fixed radius, centered at initial pixel position. The input pixel is a square of unity size, and we use a circular window with a radius less than half of this size $r \leq 0,5$. The jitter value for each pixel is determined by the random number generator with normal distribution. This random number generator is more effective then analogue normal distributed generator, because it minimizes displacement in the vicinity of border of randomization window and maximizes pixel displacement probability in the area of initial position of the pixel. This approach allows one to destroy regular pixel grid, but provides minimal image distortion.

The computation of resulting pixels comprises two stages. The first one includes building of the general image model and its conversion. The second one is the computing of the new image based on general model generated at the first stage. We suggest a triangulation grid based of the jittered nodes as a general model. Particularly we employ here the Delaunay triangulation. However, the question about kind of the best triangulation method for accurate reproduction of image structure is open. Based on this model we implement next processing steps, the expansion and generation of the final image. Generally this approach allows one to perform asymmetric scaling, but here we consider only symmetric scaling without change of original aspect ratio. The new image is generated on the base of expanded triangular grid. Every triangle is represented in 3D space, where z-coordinates of each triangle vertex have equal brightness belonging to the jittered pixel contained in this vertex of the triangle. Using this representation, we compute the pixels of the new image which falling into this triangle. The color components of these interpolated pixels are computed by 2D linear interpolation as soon as they belong to a fragment of a plane defined by three neighbour interpolation nodes. This procedure is performed independently on all channels of the color image using same triangular grid.

The complexity estimation of this stochastic upsampling procedure is determined by complexities of its stages. Let W and H is width and height of the input image, then complexity of the randomization is equal to $O(W * H)$, because each pixel of the image is jittered only once and the general count of pixels is equal to $W * H$. The complexity estimation of the triangulation is equal to $O((W * H) * log(W * H))$. This estimation is based on quick increase approach for constructing a Delaunay triangulation without edge flipping. The complexity estimation of scaling can be performed similarly to that for the randomization such as scaling is also performed once for each pixel. That means the complexity estimation of scaling is equal to $O(W * H)$. Complexity estimation of the generation of the final image is most complex, because it depends not only on the number of pixels, but also on the number of triangles of the grid, which can be calculated as $t(k, h) = h + 2(k - h) - 2$, where $k = W * H$ – the number of pixels, and h – number of the nodes in the triangular grid which formed a convex hull, which bounded triangulation. h can be computed as $h = 2 * (W - 1) + 2 * (H - 1)$, then $t(W * H, h)$ is equal following:

$$t(W * H, h) = h + 2 * (W * H - h) - 2 =$$
$$2 * (W - 1) + 2 * (H - 1) + 2 * (W * H - 2 * (W - 1) - 2 * (H - 1)) - 2 =$$
$$2 * W * H - 2 * (W - 1) - 2 * (H - 1) - 2 = 2 * W * H - 2 * W - 2 * H + 2 =$$
$$2(W(H - 1) - (H - 1)) = 2((W - 1) * (H - 1)).$$

Complexity of final image generation using number of the triangles and scaling coefficient can be estimated as

$$O(t(W * H, h) * log(e^2)) = O(2((W - 1) * (H - 1)) * log(e^2)).$$

Thus, main complexity of the method is complexity of triangulation and that of final image generation, but always it is less or equal then logarithmic complexity. Moreover, complexity of the generation is always less than that of the triangulation if the scaling factor is small. Therefore the total complexity of the stochastic upsampling method can be estimated as

$$O((W * H) * log(W * H)).$$

Results and discussion

The proposed method of the stochastic upsampling method was tested using an input image containing a curve defined by a described analytic function:

$$f(x) = \frac{h}{(1+(10*(\frac{x}{w}))^2)},$$

where h and w – height and width of the image.

The main advantage of this image is the presence of all slopes of boundary tilt from almost vertical to almost horizontal. Moreover, this image can be generated with arbitrary resolution which gives a possibility to compare results computed with various scaling factors with the ideal image. Another advantage of arbitrary resolution is the possibility to estimate the degree of deviation of the test image after processing from the ideal image.

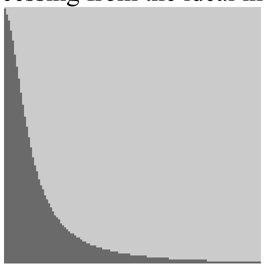
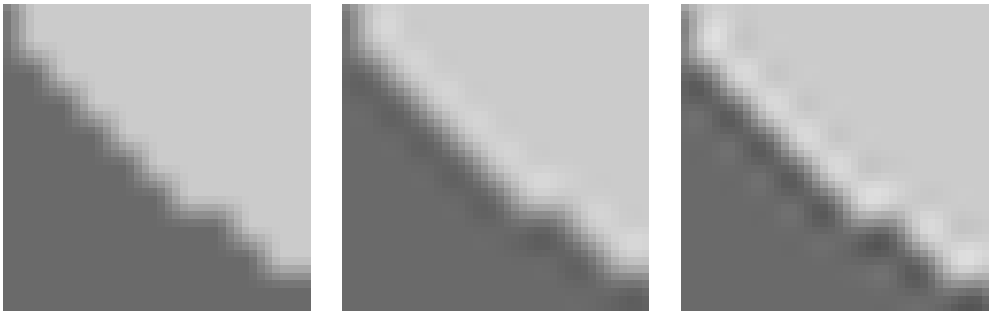


Figure 3: Test image.



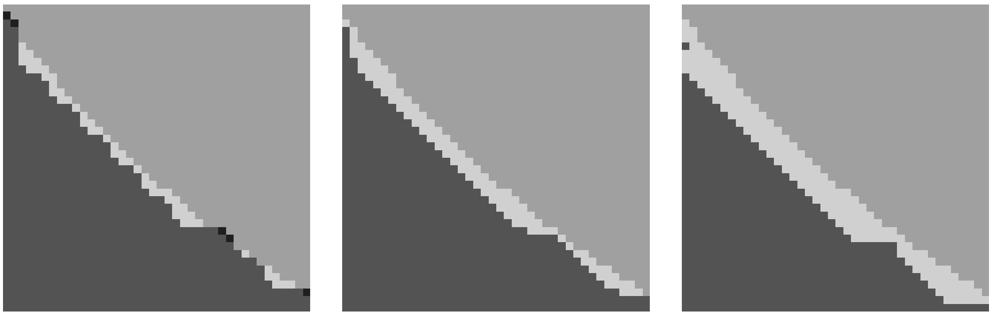
a) b) c)

Figure 4: Fragment of the test image upsampled by factor of four using: a) bilinear interpolation; b) bicubic interpolation; c) Lanczos method.

The results of application of standard methods for upsampling of this test image are given in Fig. 4. These fragments of the test image have pixels increased by factor of four to estimate level of distortion of the output image. As the various shades of colors scale complicate this estimation, all images were binarized. The fragments of difference images for binarized output images and ideal one are shown in the Fig. 5. It is evident that the result of bilinear interpolation is closest to ideal image, but the boundary is uneven and highly pixelated. The results of the bicubic interpolation and the Lanczos method show clearly visible ringing or false contours at the boundary. Although this ringing will disappear after binarization, it leads to some boundary shift (Figures 5.b and 5.c), which will introduce additional error during subsequent image analysis. The number of pixels having differences in color values against the ones from the original image is largest for the Lanczos method, while that for the bilinear interpolation is the smallest (see Table 1).

The result of stochastic upsampling is represented in the Fig. 7. Obviously it is much closer to the ideal image. The count of different pixels is also given in Table 1.

Indeed the stochastic upsampling method generates an irregular edge between objects within the image. These edges spoil visual appearance and can be a problem for ally appealing. Moreover this irregular structure can be problem for edge detection algorithms based on the brightness differences resulting in creation of false contours making the objects recognition impossible.



a) b) c)

Figure 5: Fragment of the difference image of the ideal image and the binarized one upsampled by factor of four using: a) bilinear interpolation; b) bicubic interpolation; c) Lanczos method.

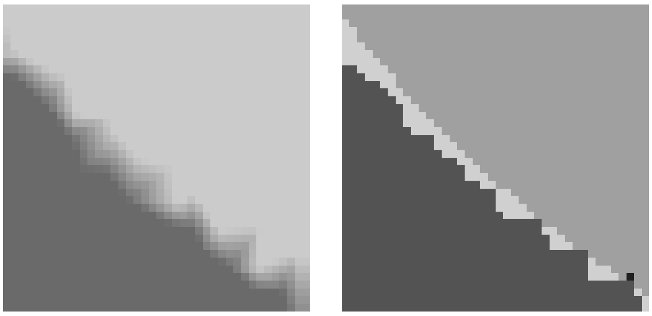
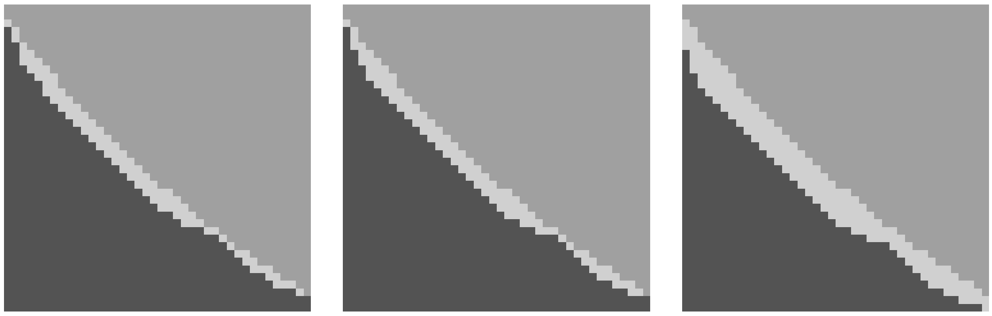


Figure 6: Fragment of the test image upsampled by factor of four using the proposed stochastic method and its difference image.

Table 1. Estimation of the different pixels between output images and the ideal one.

Method	Over edge, p	Under edge, p	Sum, p
Bilinear	667	3595	4262
Bicubic	3921	7322	11243
Lanczos	5296	8571	13867
Stochastic upsampling	713	3295	4008

These edges contain stochastic noise and can be successfully smoothed by a dedicated low-pass filter applied to the upsampled image. Here we suggest a high order low-pass filter of Bessel type (Figure 7, 8). This filter provides desirable attenuation of the stochastic noise while having a smooth transient response without ringing. Such filter would be insufficient to suppress the pixilation and ringing produced by regular interpolation methods. Fig. 8 shows that the output image after such filtering has more smooth border and appears much closer to the ideal one. Moreover, applying this same filter to results of standard method is less effective than to results of the stochastic upsampling method (Table 2). Also high order of the filter doesn't destroy the defect structure of images computed with using regular grid of pixels. Destroying of the defect structure can be implemented only using this filter with large window. In this case, the image smoothing is increased but the fine detail resolution is lost.



a) b) c)

Figure 7: Fragment of the difference image of the ideal image and binarized one upsampled by factor of four using: a) bilinear interpolation; b) bicubic interpolation; c) Lanczos method; and smoothed by high order Bessel filter.

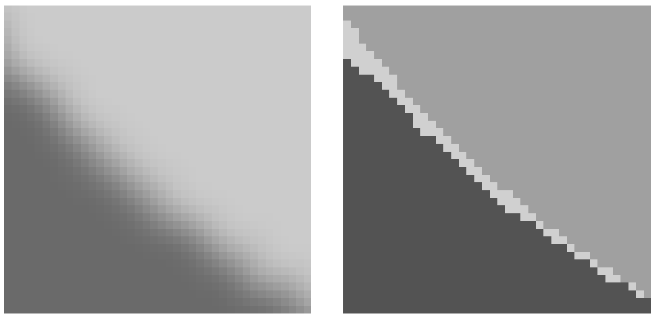


Figure 8: Fragment of the test image upsampled by factor of four using the proposed stochastic upsampling method and smoothed by high order Bessel filter, and corresponding difference image.

Table 2: Estimation of the different pixels between output images smoothed using high Bessel filter and the ideal image.

Method	Over edge, p	Under edge, p	Sum, p
Bilinear	3890	6794	10684
Bicubic	5207	8805	14012
Lanczos	4492	9067	13559
Stochastic upsampling	3688	6682	10370

These results reveal that the choice of the methods for of triangulation and output filtering requires the further studies. But we can already mention the efficiency and advantages of the stochastic upsampling for digital image processing.

References

1. Watkinson, John. An introduction to digital audio, 2nd edition, Taylor & Francis, Jul 24, 2013, 419p.

2. Howard E. Burdick, Digital Imaging: Theory and Applications, McGraw-Hill, Jan 1, 1997, 315 p.
3. Jähne, Bernd. Digital Image Processing, 6th edition, Springer, 2005, 608p, ISBN: 978-3-540-24035-8.
4. Ken Turkowski. Filters for common resampling tasks, 10 april 1990. Graphics gems I, Academic Press Professional, Inc., 1990, 147-165., April 1990.
5. Bilinskis, Ivars. Digital Alias-free Signal Processing, Wiley, 2007, 454 p, ISBN: 978-0-470-02738-7.
6. R L Cook, Stochastic sampling in computer graphics, ACM Transactions on Graphics, Vol. 5, No. 1, January 1986, Pages 51-72.
7. Afanassyev, D. Degtyarenko, I. Stochastic subsampling and pixel clustering, Modern Problems of Radio Engineering Telecommunications and Computer Science (TCSET), 2012 International Conference Proceedings, p.532, IEEE Explore, 2012.