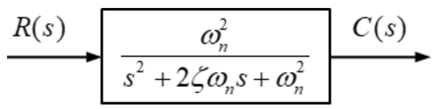
Midterm Exam: Control Systems Eng. (I)
2019/04/16

Student Number: [ ] Name:

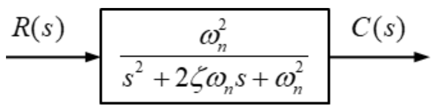
1. (20 points) The system shown in the following figure has a unit step input.



(1) (15 pts) Find the output response as a function of time in the form of $1-A(t)\cos\left(B(t)\right)$. Assume the system is underdamped ($0<\zeta<1$).

(2) (5 pts) Evaluate the settling time (T_s) with the natural logarithm using the result of (1).

2. (20 points) The system shown in the following figure has a unit step input. The peak time (T_p) is found by differentiating $c(t)$. In this problem, we want to use the following relationship to find $\dot{c}(t)$:



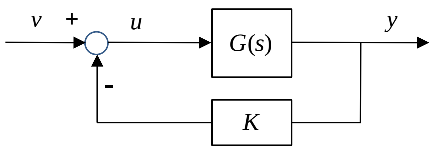
$$L\left(\dot{c}(t)\right)=sC(s)=\frac{\omega_n^2}{(s^2+2\zeta\omega_n s+\omega_n^2)}$$

(1) (15 pts) Find $\dot{c}(t)$

(2) (5 pts) Evaluate the peak time (T_p) using the result of (1).

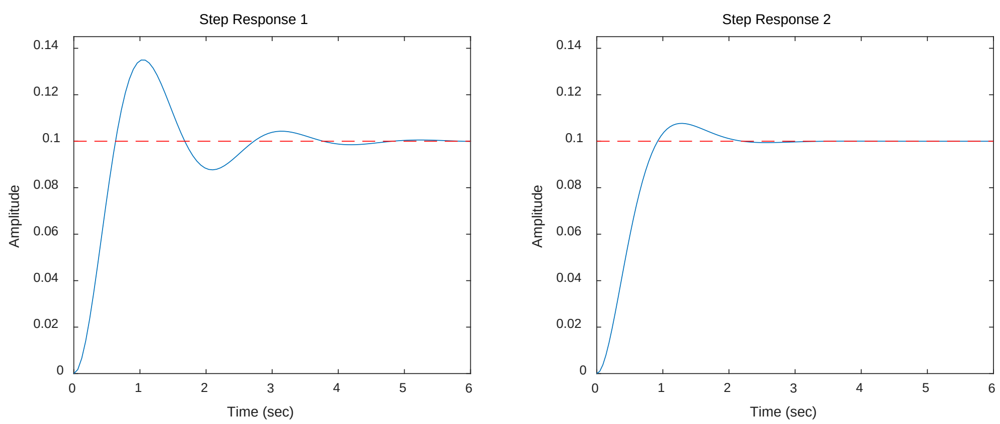
3. (20 points) Consider the closed-loop system below, where the transfer function is either

$$G_1(s)=\frac{1}{s^2+2s+5} \quad \text{or} \quad G_2(s)=\frac{1}{s^2+4s+5}$$



and where $u(t)=-Ky(t)+v(t)$ for some constant K.

Now, consider the two step responses below that were produced with a particular choice of K. **Find this K and determine which step response belongs to which closed-loop system** (as defined by G_1 or G_2). *Note, you must motivate your answer carefully – just giving an answer without justification will give no points, even if the answer happens to be correct.*

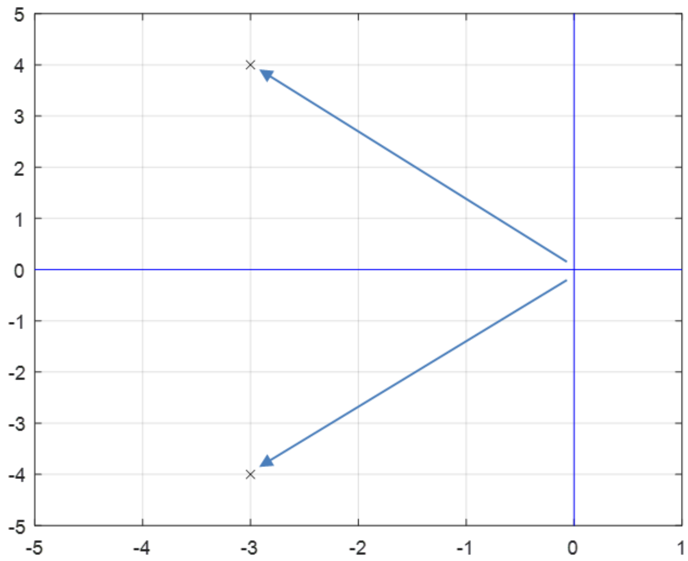


4. (20 points) Given the system represented in state space by the following equations.

$$\dot{\mathrm{x}}(t)=\begin{pmatrix} 0 & 2 \\ -3 & -5 \end{pmatrix}\mathrm{x}(t)+\begin{pmatrix} 0 \\ 1 \end{pmatrix}e^{-t}, \quad \mathrm{y}(t)=\begin{pmatrix} 1 & 3 \end{pmatrix}\mathrm{x}(t), \quad \mathrm{x}(0)=\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Solve for $y(t)$ using state-space and Laplace transform techniques.

5. (10 points) For the given pole plot, find the following values. Use $\pi = 3$.



$$T_p = \frac{\pi}{\omega_d}$$

$$T_s = \frac{4}{\sigma_d}$$

$$\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100\%$$

(1) (2 pts) ζ (2) (2 pts) ω_n (3) (3 pts) T_p (4) (3 pts) T_s.

6. (10 points)

Given a linear time-invariant system with $\dot{\mathrm{x}}(t)=\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\mathrm{x}(t)$, $y=\begin{pmatrix} 1 & 1 \end{pmatrix}\mathrm{x}(t)$, $\mathrm{x}(0)=\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Evaluate the state-transition matrix $(\Phi(t)=e^{At})$ for the system.