Mathematical Statistics (I) Assignment 3

Spring, 2021

1. Let $X_1, \cdots, X_n$ be a random sample from a population with pdf

$$f(x) = e^{-(x-\theta)} I(\theta < x < \infty),$$

where $-\infty < \theta < \infty$. (The pdf of shifted exponential distribution.) And let $Y_n = \min\{X_1, \cdots, X_n\}$. Prove that $Y_n \xrightarrow{P} \theta$.

2. Let $X_1, \cdots, X_n$ be a random sample from a population with cdf F and let

$$Y_n = \max\{X_1, \cdots, X_n\}.$$

Find the limiting distribution of $Z_n = n(1 - F(Y_n))$.

3. Let $\bar{X_n}$ be the mean of a random sample of size n from a Poisson distribution with parameter $\lambda = 1$ and let $Y_n = \sqrt{n}(\bar{X_n} - 1)$.
(a) Find the mgf $M_n(t)$ of Y_n.
(b) Find the limiting distribution of Y_n by finding the limit of $M_n(t)$.
(c) Find the limiting distribution of $\sqrt{n}(\sqrt{\bar{X_n}} - 1)$.

4. Let $X_1, \cdots, X_n$ be a random sample from a Poisson distribution with mean μ.

a. What is the asymptotic mean and asymptotic variance of $\bar{X} = \sum_{i=1}^{n} X_i/n$?

b. Show that $g(\mu) = \sqrt{\mu}$ is the 'variance stabilizing transform' by showing that the asymptotic variance of $\sqrt{\bar{X}}$ does not depend on μ.

5. Let $\underline{X}_1, \underline{X}_2, \cdots$ be a sequence of k-dimensional random vectors. Show that

$$\underline{X}_n \xrightarrow{D} N_k(\mu, \Sigma) \quad \text{if and only if} \quad a'\underline{X}_n \xrightarrow{D} N_1(a'\mu, a'\Sigma a),\ \forall a \in R^k.$$