기초통계학 1장 연습문제

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3. solve) $A \subset S,\ B \subset S$

$(A \cup B) \cap (A^c \cup B) \cap (A \cup B^c) = ((A \cap A^c) \cup B) \cap (A \cup B^c)$ by 분배법칙

$= (\emptyset \cup B) \cap (A \cup B^c) = B \cap (A \cup B^c) = (A \cap B) \cup (B \cap B^c)$ by 분배법칙

$= (A \cap B) \cup \emptyset = A \cap B$

$\therefore A \cap B$

5 불량품 : 4 정품 : 96

적어도 한개가 불량품 = 1 - 불량품이 하나도 없는 경우

$$P(X=0) = \frac{\binom{4}{0}\binom{96}{10}}{\binom{100}{10}} = \frac{{}_{96}C_{10}}{{}_{100}C_{10}} = \frac{\frac{96!}{10!86!}}{\frac{100!}{10!90!}} = \frac{90 \times 89 \times 88 \times 87}{100 \times 99 \times 98 \times 97}$$

$= 1 - 0.652 = 0.348$

$\therefore 0.348$

6. $R=10,\ W=20,\ B=30$

1) $P(X=2, Y=3) = P(R)^2 \times P(W)^3 \times P(B)^3 = \left(\frac{1}{6}\right)^2 \left(\frac{1}{3}\right)^3 \left(\frac{1}{2}\right)^3 = \frac{1}{36 \times 27 \times 8} = 0.00013$

2) $P(X=2, Y=3) = \dfrac{\binom{10}{2}\binom{20}{3}\binom{30}{3}}{\binom{60}{8}}$

$\therefore 0.00013,\ \dfrac{\binom{10}{2}\binom{20}{3}\binom{30}{3}}{\binom{60}{8}}$

9. $R=3,\ B=9$

두번째 붉은 구슬 = 다섯번째 꺼낸구슬 ⟶ 4번중 한번만 비복원추출로 붉은 구슬을 꺼낸확률

$$P(X=1) = \frac{\binom{9}{3}\binom{3}{1}}{\binom{12}{4}} \times 4 \text{ (순서가 상관이 있음)} = \frac{\frac{9!}{3!6!} \times \frac{3!}{1!2!}}{\frac{12!}{4!8!}} \times 4 = \frac{9 \times 8 \times 7 \times 3}{12 \times 11 \times 10 \times 9} \times 4$$

$\times \frac{2}{8}$ (다섯번째에 붉은 구슬) $= \frac{7}{55}$

$\therefore 0.127$

10. 불량(W) = 3. 정상(R) = 47

1) 꼭 한개가 불량일때, $P(X=1) = \dfrac{\binom{3}{1}\binom{47}{9}}{\binom{50}{10}}$

$\therefore \dfrac{\binom{3}{1}\binom{47}{9}}{\binom{50}{10}}$

2) 많아야 한개가 불량일 때, $P(X=0) + P(X=1) = \dfrac{\binom{3}{0}\binom{47}{10}}{\binom{50}{10}} + \dfrac{\binom{3}{1}\binom{47}{9}}{\binom{50}{10}}$

$\dfrac{\binom{3}{0}\binom{47}{10} + \binom{3}{1}\binom{47}{9}}{\binom{50}{10}}$

기초통계학 2장 연습문제

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3. $f(x) = \dfrac{(|x|+1)^2}{9}$

$E(X) = \sum x f(x) = -1 \cdot \frac{4}{9} + 0 + 1 \cdot \frac{4}{9} = 0$

$E(X^2) = \sum x^2 f(x) = 1 \cdot \frac{4}{9} + 0 + 1 \cdot \frac{4}{9} = \frac{8}{9}$

$E(3X^2 - 2X + 4) = 3 \cdot E(X^2) - 2 \cdot E(X) + 4 = \frac{8}{9} \cdot 3 + 4 = \frac{8}{3} + 4 = \frac{20}{3}$

$\therefore E(X) = 0, \; E(X^2) = \frac{8}{9}, \; E(3X^2 - 2X + 4) = \frac{20}{3}$

5. $f(0) + f(1) + f(2) + f(3) = 1$

$\therefore$ $f(x)$는 0 ~ 3의 자연수를 확률변수로 갖는 이산형 확률변수

$E(X) = \sum x f(x) = 0 \cdot \frac{3}{10} + 1 \cdot \frac{3}{10} + 2 \cdot \frac{1}{10} + 3 \cdot \frac{3}{10} = \frac{14}{10} = \frac{7}{5}$

$E(X^2) = \sum x^2 f(x) = 0 \cdot \frac{3}{10} + 1 \cdot \frac{3}{10} + 4 \cdot \frac{1}{10} + 9 \cdot \frac{3}{10} = \frac{34}{10} = \frac{17}{5}$

$Var(X) = E(X^2) - E(X)^2 = \frac{17}{5} - \frac{49}{25} = \frac{85-49}{25} = \frac{36}{25}$

$\sigma = \sqrt{Var(X)} = \frac{6}{5}$

$\therefore E(X) = \frac{7}{5}, \; Var(X) = \frac{36}{25}, \; \sigma = \frac{6}{5}$

6. $E(X) = \mu, \; Var(X) = \sigma^2$

$E\left(\frac{X-\mu}{\sigma}\right) = \frac{1}{\sigma} E(X) - \frac{\mu}{\sigma} = \frac{\mu}{\sigma} - \frac{\mu}{\sigma} = 0$

$E\left(\left(\frac{X-\mu}{\sigma}\right)^2\right) = \frac{1}{\sigma^2} E(X^2 - 2\mu X + \mu^2) = \frac{1}{\sigma^2} \{E(X^2) - 2\mu E(X) + \mu^2\}$

$E(X^2) = Var(X) + E(X)^2 = \sigma^2 + \mu^2$

$= \frac{1}{\sigma^2} \{\sigma^2 + \mu^2 - 2\mu^2 + \mu^2\} = \frac{\sigma^2}{\sigma^2} = 1$

8. $M(t) = \frac{1}{3} + \frac{2}{3}e^t$

$E(X) = M'(0) = \frac{2}{3}e^t\Big|_{t=0} = \frac{2}{3}$

$E(X^2) = M''(0) = \frac{2}{3}e^t\Big|_{t=0} = \frac{2}{3}$

$Var(X) = E(X^2) - E(X)^2 = \frac{2}{3} - \frac{4}{9} = \frac{2}{9}$

$E(X^k) = \frac{2}{3}$, $k = 1, \dots$ 이므로, $M(t) = M(0) + \sum_{1}^{\infty} \frac{2}{3}\frac{t^k}{k!} = \frac{1}{3} + \frac{2}{3}\sum_{k=0}^{\infty}\frac{t^k}{k!}$

$= \frac{1}{3}e^{0t} + \frac{2}{3}e^{1t}$

$\therefore f(x) = \begin{cases} \frac{1}{3} & x=0 \\ \frac{2}{3} & x=1 \end{cases}$, $E(X) = \frac{2}{3}$, $Var(X) = \frac{2}{9}$

9. $M_Y(t) = \left(\frac{1}{4} + \frac{3}{4}e^t\right)^{12}$

$E(Y) = M'(0) = 12\cdot\frac{3}{4}e^t\left(\frac{1}{4}+\frac{3}{4}e^t\right)^{11}\Big|_{t=0} = 12\cdot\frac{3}{4} = 9$

$E(Y^2) = M''(0) = 9e^t\left(\frac{1}{4}+\frac{3}{4}e^t\right)^{11} + 9e^t\cdot\frac{3}{4}e^t\left(\frac{1}{4}+\frac{3}{4}e^t\right)^{10}\cdot 11 = 9 + \frac{297}{4}$

$Var(Y) = E(Y^2) - E(Y)^2 = 9 + \frac{297}{4} - 81 = \frac{297}{4} - 72 = \frac{297-288}{4} = \frac{9}{4}$

$\therefore E(Y) = 9,\ Var(Y) = \frac{9}{4}$

12. $f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$

$M(t) = \sum_{x=0}^{\infty} e^{tx}\cdot\frac{\lambda^x e^{-\lambda}}{x!} = e^{-\lambda}\sum_{x=0}^{\infty}\frac{(\lambda e^t)^x}{x!} = e^{-\lambda}\cdot e^{\lambda e^t} = e^{\lambda(e^t-1)}$

$E(X) = M'(t=0) = \lambda e^t\cdot e^{\lambda(e^t-1)} = \lambda$

$\therefore E(X) = \sum_{x=0}^{\infty} x\cdot\frac{\lambda^x e^{-\lambda}}{x!} = M'(0) = \lambda$

13. $f(x) = \binom{n}{x}p^x(1-p)^{n-x}$ if $x=0$, $\binom{n}{0}p^0(1-p)^{n-0} = (1-p)^n$

$x=1$, $\binom{n}{1}p^1(1-p)^{n-1} = \binom{n}{1}\left(\frac{p}{1-p}\right)\cdot(1-p)^n$

$x=2$, $\binom{n}{2}p^2(1-p)^{n-2} = \binom{n}{2}\left(\frac{p}{1-p}\right)^2(1-p)^n$

$\therefore f(x) = \begin{cases} (1-p)^n, & x=0 \\ \left(\frac{n-x+1}{x}\right)\left(\frac{p}{1-p}\right)f(x-1) & (1\le x\le n) \end{cases}$

$= \left(\frac{n-1}{2}\right)\left(\frac{p}{1-p}\right)\underbrace{\left(\frac{n}{1}\right)\left(\frac{p}{1-p}\right)(1-p)^n}_{f(1)}$

$\vdots$

$x=k = \frac{n-k+1}{k}\cdot\frac{p}{1-p}\cdot f(k-1)$

14. $\sum_{x=0}^{n} e^{-\lambda}\frac{\lambda^{x}}{x!} = \frac{1}{x!}\int_{\lambda}^{\infty} e^{-t}t^{n}dt = \frac{1}{x!}\left[t^{n}(-e^{-t})\right]_{\lambda}^{a} + \frac{1}{x!}\int_{\lambda}^{\infty} n\cdot e^{-t}\cdot t^{n-1}dt$

$$= \frac{1}{x!}\lambda^{n}e^{-\lambda} + \frac{n}{x!}\lambda^{n-1}e^{-\lambda} + \frac{n\cdot n-1}{x!}\int_{\lambda}^{\infty} e^{-t}\cdot t^{n-2}dt.$$

$$= \frac{1}{x!}\lambda^{n}e^{-\lambda} + \frac{n}{x!}\lambda^{a-1}e^{-\lambda} + \frac{n(n-1)}{x!}\lambda^{n-2}e^{-\lambda} + \cdots$$

$$= \frac{1}{x!}\sum_{i=0}^{\infty}\lambda^{i}e^{-\lambda}$$

기초통계학 3장 연습문제

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3. $f(x) = \frac{1}{2}, \quad -1 \le x \le 1$

$$E(x) = \int_{-1}^{1} x \cdot \frac{1}{2} dx = \frac{1}{4}x^2 \Big]_{-1}^{1} = 0$$

$$Var(x) = \int_{-1}^{1} x^2 \frac{1}{2} dx - 0 = \frac{1}{6}x^3 \Big]_{-1}^{1} = \frac{1}{3}$$

$\therefore E(x) = 0, \ Var(x) = \frac{1}{3}$

7. $X \sim N(650, 625)$

1) $P(600 \le X < 660) = P\left(\frac{600-650}{\sqrt{625}} \le \frac{X-650}{\sqrt{625}} < \frac{600-625}{\sqrt{625}}\right)$

$= P\left(\frac{-50}{25} \le Z < \frac{-25}{25}\right) = P(-2 \le Z < -1) = \phi(-1) - \phi(-2)$

$= 0.1587 - 0.0228 = 0.1359 \qquad \therefore 0.1359$

2) $P(|X-650| \le C) = 0.9544$

$P(-C \le X - 650 \le C) = P\left(-\frac{C}{\sqrt{625}} \le \frac{X-650}{\sqrt{625}} \le \frac{C}{\sqrt{625}}\right)$

$= P\left(-\frac{C}{25} \le Z \le \frac{C}{25}\right) \triangleq k \qquad 1-k = 0.9544.$

$k = 0.0456$

$\frac{k}{2} = 0.0228$

$\therefore \frac{C}{25} = 2 \qquad \therefore C = 50$

$\therefore C = 50$

8. $X \sim N(\mu, \sigma^2)$ if, $Y = |X - \mu|$

$f_Y(y) = F_Y'(y) = (P(Y \le y))' = (P(|X-\mu| \le y))' = P(-y \le X - \mu \le y)'$

$= P\left(\frac{-y}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{y}{\sigma}\right)'$ ∵ make standard normal distribution

$= \left(\phi\left(\frac{y}{\sigma}\right) - \phi\left(-\frac{y}{\sigma}\right)\right)' = \left(2\phi\left(\frac{y}{\sigma}\right) - 1\right)' = \frac{2}{\sigma}\phi\left(\frac{y}{\sigma}\right) \qquad \therefore \frac{2}{\sigma}\phi\left(\frac{y}{\sigma}\right)$

10.

$$F(x) = \begin{cases} 0 & x<0 \\ \frac{x^2}{4} & 0 \le x < 1 \\ \frac{x+1}{4} & 1 \le x < 2 \\ 1 & 2 \le x \end{cases} \qquad f(x) = \begin{cases} \frac{x}{2} & 0 \le x < 1 \\ \frac{1}{4} & 1 \le x < 2 \end{cases}$$

$$f(2) = f(1) = \frac{1}{4}$$

1) $E(x) = \int_0^1 x \frac{x}{2} dx + \frac{1}{4} + \int_1^2 x \frac{1}{4} dx + \frac{1}{2} = \frac{3}{4} + \frac{1}{6} + \frac{3}{8} = \frac{18+4+9}{24} = \frac{31}{24}$

$E(X^2) = \int_0^1 x^2 \frac{x}{2} dx + \frac{1}{4} + \int_1^2 x^2 \cdot \frac{1}{4} dx + 1 = \frac{1}{8} + \frac{1}{4} + 1 + \frac{7}{12} = \frac{14+24+6+3}{24} = \frac{47}{24}$

$Var(X) = E(X^2) - E(X)^2 = \frac{47}{24} - \left(\frac{31}{24}\right)^2 = \frac{1128-961}{576} = \frac{167}{576}$

$\therefore E(X) = \frac{31}{24}, \; Var(X) = \frac{167}{576}$

2) $P(\frac{1}{4} < x < 1) = \int_{\frac{1}{4}}^1 \frac{x}{2} dx = \frac{x^2}{4} \Big]_{\frac{1}{4}}^{1} = \frac{1}{4} - \frac{1}{64} = \frac{15}{64}$

$P(X=1) = f(1) = \frac{1}{4}$

$P(X = \frac{1}{2}) = 0$

$P(\frac{1}{2} \le X < 2) = \int_{\frac{1}{2}}^1 \frac{x}{2} + \frac{1}{4} + \int_1^2 \frac{1}{4} dx = \frac{x^2}{4} \Big]_{\frac{1}{2}}^{1} + \frac{1}{4} + \frac{x}{4} \Big]_1^2 = \frac{3+4+4}{16} = \frac{11}{16}$

$\therefore \frac{15}{64}, \frac{1}{4}, 0, \frac{11}{16}$

기초통계학 4장 과제

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1. $f(x,y) = \frac{x+y}{32}$, $x=1,2$. $y=1,2,3,4$

1) $f_X(x) = \sum_{i=1}^{4} \frac{x+y}{32} = \frac{4x+(1+2+3+4)}{32} = \langle \frac{2x+5}{16} \rangle$

2) $f_Y(y) = \sum_{1}^{2} \frac{x+y}{32} = \frac{2y+(1+2)}{32} = \langle \frac{2y+3}{32} \rangle$

3) $P(X>Y) = f(2,1) = \langle \frac{3}{32} \rangle$

4) $P(Y=2X) = f(1,2)+f(2,4) = \frac{3}{32} + \frac{6}{32} = \langle \frac{9}{32} \rangle$

5) $P(X+Y=3) = f(1,2)+f(2,1) = \frac{3}{32} + \frac{3}{32} = \frac{6}{32} = \langle \frac{3}{16} \rangle$

6) $P(X \le 3-Y) = P(X+Y \le 3) = f(1,1)+f(1,2)+f(2,1) = \frac{2+3+3}{32} = \langle \frac{1}{4} \rangle$

7) $\because f(x,y) \ne f(x)f(y)$ 〈 ∴ X와 Y는 독립이 아니다. 〉

8) $\mu_x = \Sigma x f(x) = 1 \cdot \frac{7}{16} + 2 \cdot \frac{9}{16} = \langle \frac{25}{16} \rangle$

$\mu_y = \Sigma y f(y) = 1 \cdot \frac{5}{32} + 2 \cdot \frac{7}{32} + 3 \cdot \frac{9}{32} + 4 \cdot \frac{11}{32} = \frac{5+14+27+44}{32} = \frac{90}{32} \langle = \frac{45}{16} \rangle$

9) $\sigma_x^2 = \Sigma x^2 f(x) - \mu_x^2 = 1 \cdot \frac{7}{16} + 4 \cdot \frac{9}{16} - (\frac{25}{16})^2 = \frac{43}{16} - \frac{25^2}{16^2} = \frac{688-625}{256} = \langle \frac{63}{256} \rangle$

$\sigma_y^2 = \Sigma y^2 f(y) - \mu_y^2 = 1 \cdot \frac{5}{32} + 4 \cdot \frac{7}{32} + 9 \cdot \frac{9}{32} + 16 \cdot \frac{11}{32} - (\frac{45}{16})^2$

$= \frac{5+28+81+176}{32} - \frac{45^2}{256} = \frac{2320-2025}{256} = \langle \frac{295}{256} \rangle$

$cov(X,Y) = \Sigma\Sigma xy f(x,y) - \mu_x \mu_y = 1 \cdot 1 \cdot \frac{2}{32} + 1 \cdot 2 \cdot \frac{3}{32} + 1 \cdot 3 \cdot \frac{4}{32} + 1 \cdot 4 \cdot \frac{5}{32}$
$+ 2 \cdot 1 \cdot \frac{3}{32} + 2 \cdot 2 \cdot \frac{4}{32} + 2 \cdot 3 \cdot \frac{5}{32} + 2 \cdot 4 \cdot \frac{6}{32}$
$- (\frac{25}{16} \cdot \frac{45}{16})$

$= \frac{2+6+12+20+6+16+30+48}{32} - \frac{1125}{256} = \frac{1120-1125}{256} = -\frac{5}{256}$

$\rho = \frac{cov(x,y)}{\sqrt{\sigma_x^2}\sqrt{\sigma_y^2}} = \frac{-\frac{5}{256}}{\sqrt{\frac{63}{256}}\sqrt{\frac{295}{256}}} = \frac{-5}{\sqrt{63}\sqrt{295}} = \langle -0.0367 \rangle$

10. $g(x|y) = \frac{f(x,y)}{f_Y(y)} = \frac{\frac{x+y}{32}}{\frac{2y+3}{32}} = \langle \frac{x+y}{2y+3} \rangle$

11. $h(y|x) = \frac{f(x,y)}{f_X(x)} = \frac{\frac{x+y}{32}}{\frac{2x+5}{16}} = \langle \frac{x+y}{2(2x+5)} \rangle$

12. $E(Y|X=1) = \sum_{1}^{4} y \cdot h(y|x=1) = \sum_{1}^{4} y \cdot \frac{y+1}{14} = 1 \cdot \frac{2}{14} + 2 \cdot \frac{3}{14} + 3 \cdot \frac{4}{14} + 4 \cdot \frac{5}{14}$

$$= \frac{2+6+12+20}{14} = \frac{40}{14} = \frac{20}{7} \quad \therefore \frac{20}{7}$$

13. $E(Y|X=2) = \sum_{1}^{4} y \cdot h(y|x=2) = \sum_{1}^{4} y \cdot \frac{y+2}{18} = 1 \cdot \frac{3}{18} + 2 \cdot \frac{4}{18} + 3 \cdot \frac{5}{18} + 4 \cdot \frac{6}{18}$

$$= \frac{3+8+15+24}{18} = \frac{50}{18} = \frac{25}{9} \quad \therefore \frac{25}{9}$$

14. $P(1 \le y \le 3 | X=1) = \sum_{1}^{3} \frac{y+1}{14} = \frac{2}{14} + \frac{3}{14} + \frac{4}{14} = \frac{9}{14} \quad \therefore \frac{9}{14}$

15. $Var(Y|X=1) = E(Y^2|X=1) - E(Y|X=1)^2 = \sum_{1}^{4} y^2 \cdot \frac{y+1}{14} - \left(\frac{20}{7}\right)^2 = 1 \cdot \frac{2}{14} + 4 \cdot \frac{3}{14} + 9 \cdot \frac{4}{14}$

$$+ 16 \cdot \frac{5}{14} - \frac{400}{49}$$

$$= \frac{2+12+36+80}{14} - \frac{400}{49} = \frac{130}{14} - \frac{400}{49} = \frac{65 \cdot 7}{49} - \frac{400}{49} = \frac{55}{49}$$

$$\therefore \frac{55}{49}$$

5. $f(x,y) = e^{-x-y} \quad (0 \le x < \infty, 0 \le y < \infty)$

1) $f_X(x) = \int_0^\infty e^{-x-y} dy = -e^{-x}[e^{-y}]_0^\infty = +e^{-x}$

$f_Y(y) = \int_0^\infty e^{-x-y} dx = -e^{-y}[e^{-x}]_0^\infty = +e^{-y}$

$\because f(x,y) = f_X(x) f_Y(y) = e^{-x-y}$, X와 Y는 독립이다.

2) $P(X<Y) = \int_0^\infty \int_x^\infty e^{-x-y} dy dx = \int_0^\infty -e^{-x} \cdot [e^{-y}]_x^\infty dx = \int_0^\infty e^{-2x} dx = \left[-\frac{1}{2} e^{-2x}\right]_0^\infty$

$$\left(= \frac{1}{2}\right)$$

2) $P(X>1, Y>1) = \int_1^\infty \int_1^\infty e^{-x-y} dy dx = \int_1^\infty -e^{-x}[e^{-y}]_1^\infty dx = \int_1^\infty e^{-x-1} dx = e^{-1}[-e^{-x}]_1^\infty$

$$(= e^{-2})$$

4) $P(X=Y)$ ∵ 연속분포에서의 한점의 확률은 0이다. $\therefore P(X=Y) = 0$

5) $P(X<2) = \int_0^2 f_X(x) dx = \int_0^2 +e^{-x} dx = [-e^{-x}]_0^2 = 1 - e^{-2} \quad \therefore 1 - e^{-2}$

6) $P(X/Y \le a) = P(X \le aY) = \int_0^\infty \int_0^{ay} e^{-x-y} dx dy = \int_0^\infty -e^{-y}[e^{-x}]_0^{ay} dy$

$$= \int_0^\infty -e^{-y}(e^{-ay} - 1) dy = \int_0^\infty e^{-y} - e^{-(a+1)y} dy$$

$$= [-e^{-y}]_0^\infty + \left[\frac{1}{a+1} e^{-(a+1)y}\right]_0^\infty = 1 - \frac{1}{a+1}$$

7) $Z = \frac{X}{Y}$ $\quad f_Z(z) = F_Z'(z) = (P_Z(Z \le z))'$

$$P_Z(Z \le z) = P\left(\frac{X}{Y} \le z\right) = P(X \le zY) = 1 - \frac{1}{z+1} \quad \because \text{6) pdf}$$

$$f_Z(z) = F_Z'(z) = \left(1 - \frac{1}{z+1}\right)' = \frac{1}{(z+1)^2} \qquad \therefore \frac{1}{(z+1)^2}$$

7. $f(x,y) = \frac{e^{-y}}{y}$ $\quad (0<x<y,\ 0<y<\infty)$

$$E(X^3 \mid Y=y) = \int_0^y x^3 f_{X|Y}(x)\,dx \quad \cdots \text{①}$$

$$f_{X|Y}(x) = \frac{f(x,y)}{f_Y(y)} = \frac{\frac{e^{-y}}{y}}{\int_0^y \frac{e^{-y}}{y}\,dx} = \frac{1}{y} \quad \cdots \text{②}$$

② 결과를 ①에 대입, $E(X^3 \mid Y=y) = \int_0^y x^3 \cdot \frac{1}{y}\,dx = \frac{y^3}{4}$ $\qquad \therefore E(X^3 \mid Y=y) = \frac{y^3}{4}$

$$E(XY) = \int_0^\infty \int_0^y xy \cdot \frac{e^{-y}}{y}\,dx\,dy = \int_0^\infty e^{-y}\left[\frac{1}{2}x^2\right]_0^y dy = \frac{1}{2}\int_0^\infty y^2 e^{-y}\,dy$$

$$= \frac{1}{2}\left(\left[y^2 - e^{-y}\right]_0^\infty + \int_0^\infty 2y e^{-y}\,dy\right)$$

$$= \frac{1}{2}\left(\left[-2ye^{-y}\right]_0^\infty + \int_0^\infty 2e^{-y}\,dy\right) = \frac{1}{2}\cdot 2 = 1$$

$$E(Y) = \int_0^\infty y \cdot e^{-y}\,dy = \left[-ye^{-y}\right]_0^\infty + \int_0^\infty e^{-y}\,dy = 1$$

$$E(X) = E(E(X|Y)) = E\left(\int_0^y x f_{X|Y}(x)\,dx\right) = E\left(\int_0^y x \cdot \frac{1}{y}\,dx\right) = E\left(\frac{1}{2}y\right) = \frac{1}{2}$$

$$Cov(X,Y) = E(XY) - E(X)E(Y) = 1 - \frac{1}{2}\cdot 1 = \frac{1}{2}$$

$$Var(X) = E(X^2) - (E(X))^2 = E(E(X^2|Y)) - E(X)^2 = E\left(\int_0^y x^2 f_{X|Y}(x)\,dx\right) - E(X)^2$$

$$= E\left(\frac{1}{3}Y^2\right) \cdots \text{②}$$

$$Var(Y) = E(Y^2) - E(Y)^2 = \int_0^\infty y^2 f_Y(y)\,dy = \int_0^\infty y^2 e^{-y}\,dy = 2 \quad \because E(XY) \text{ 에서 계산}$$

$$- E(Y)^2 = 2 - 1 = 1$$

$$\therefore Var(X) = E\left(\frac{1}{3}Y^2\right) - E(X)^2 = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$$

$$\therefore Corr(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)}\sqrt{Var(Y)}} = \frac{\frac{1}{2}}{\sqrt{\frac{5}{12}}} = \sqrt{\frac{3}{5}}$$