Midterm Exam: Control Systems Eng.(I)
2020/05/12

Student Number: [ ] Name: **Solution**

1.

$$Z_1(s)=R_1+\frac{1}{C_1 s}=\frac{s+1}{s},\quad Z_2(s)=\frac{R_2\dfrac{1}{C_2 s}}{R_2+\dfrac{1}{C_2 s}}=\frac{2}{4s+1}$$

$$\frac{V_o(s)}{V_i(s)}=\frac{Z_1(s)+Z_2(s)}{Z_1(s)}=\frac{4s^2+7s+1}{4s^2+5s+1}$$

2.

(1) $$T(s)=\frac{Y(s)}{X(s)}=\frac{\dfrac{1}{9s}}{1+\dfrac{1}{9s}\cdot\dfrac{1}{4s}}=\frac{\dfrac{1}{9s}}{\dfrac{36s^2+1}{36s^2}}=\frac{4s}{36s^2+1}$$

(2) use: $\cos at \leftrightarrow \dfrac{s}{s^2+a^2}$ and $\delta(t)\leftrightarrow 1$

$$\frac{Y(s)}{X(s)}=\frac{4s}{36s^2+1}=\frac{1}{9}\frac{s}{\left(s^2+\left(\dfrac{1}{6}\right)^2\right)}\leftrightarrow y(t)=\frac{1}{9}\cos\frac{1}{6}t \quad \text{or} \quad y(t)=\frac{1}{9}\cos\left(\frac{1}{6}t\right)\cdot u(t)$$

3.

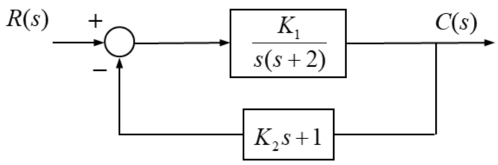
$$f(x)\approx f(x_0)+m_a\delta x=5\cos(\frac{\pi}{2})+\left.\frac{df(x)}{dx}\right|_{x=\frac{\pi}{2}}(\delta x)=0-5\sin(\frac{\pi}{2})(\delta x)=-5\delta x$$

Or $$f(x)\approx f(x_0)+\left.\frac{df(x)}{dx}\right|_{x=x_0}\frac{(x-x_0)}{1!}+\cdots=5\cos(\frac{\pi}{2})-\sin(\frac{\pi}{2})(x-\frac{\pi}{2})=-5(x-\frac{\pi}{2})$$

4. $$T=\frac{\dfrac{K_1}{s(s+2)}}{1+\dfrac{K_1}{s(s+2)}(K_2 s+1)}=\frac{K_1}{s^2+(2+K_1K_2)s+K_1}=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

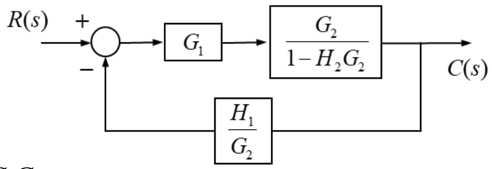
$$\omega_n^2=K_1 \ \rightarrow\ K_1=16$$

$$2\zeta\omega_n=2+K_1K_2 \ \rightarrow\ K_2=\frac{3.6}{16}=4.44$$



5.

$$T(s)=\frac{\dfrac{G_1G_2}{1-G_2H_2}}{1+\left(\dfrac{G_1G_2}{1-H_2G_2}\right)\left(\dfrac{H_1}{G_2}\right)}=\frac{\dfrac{G_1G_2}{1-G_2H_2}}{\dfrac{G_2(1-H_2G_2)+G_1G_2H_1}{G_2(1-H_2G_2)}}$$



$$=\frac{G_1G_2}{1-G_2H_2}\cdot\frac{G_2(1-H_2G_2)}{G_2(1-H_2G_2)+G_1G_2H_1}=\frac{G_1G_2}{(1-H_2G_2)+G_1H_1}=\frac{G_1G_2}{1+G_1H_1-H_2G_2}$$

6. (1) – [C] (2) – [A] (3) – [D] (4) – [B]

7.

$$(sI-A)=\begin{bmatrix} s & -1 \\ 0 & (s+2)\end{bmatrix},\quad (sI-A)^{-1}=\frac{1}{s(s+2)}\begin{bmatrix} s+2 & 1 \\ 0 & s\end{bmatrix}$$

$$T(s)=C(sI-A)^{-1}B+D=\frac{1}{s(s+2)}\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix} s+2 & 1 \\ 0 & s\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}+0=\frac{1}{s(s+2)}\begin{bmatrix}s+2 & 1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}=\frac{1}{s(s+2)}$$

8. (1) – [C] (2) – [A] (3) – [B] (4) – [D]

9.

$$\dot{x}_1=-x_1+x_2$$

$$\dot{x}_2=-x_1+2u$$

$$y=0.5(x_1+\dot{x}_1)=0.5(x_1-x_1+x_2)=0.5x_2$$

$$\therefore\quad \dot{\mathrm{x}}=\begin{bmatrix}-1 & 1\\ -1 & 0\end{bmatrix}\mathrm{x}+\begin{bmatrix}0\\2\end{bmatrix}u,\ \ y=\begin{bmatrix}0 & 0.5\end{bmatrix}\mathrm{x}\quad \text{for}\quad \mathrm{x}=\begin{bmatrix}x_1\\x_2\end{bmatrix}$$

10.

$$L^{-1}\left[(sI-A)^{-1}\right]=\Phi(t)=e^{At},\qquad (sI-A)^{-1}=\Phi(s)=\frac{\begin{bmatrix}s+6 & 1\\ -5 & s\end{bmatrix}}{s^2+6s+5}$$

$$(sI-A)=\begin{bmatrix}s & -1\\ 5 & s+6\end{bmatrix},\qquad A=\begin{bmatrix}s & 0\\ 0 & s\end{bmatrix}-\begin{bmatrix}s & -1\\ 5 & s+6\end{bmatrix}=\begin{bmatrix}0 & 1\\ -5 & -6\end{bmatrix}\quad\Rightarrow\quad A=\begin{bmatrix}0 & 1\\ -5 & -6\end{bmatrix}$$