2.6 풀이) 1단계

0	
1	
2	
3	
4	
5	

2단계

0	69
1	8741557278932026
2	0560225434018997
3	60255706566
4	61
5	12

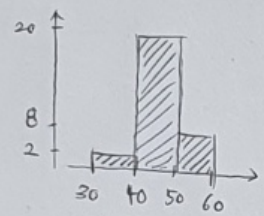
3단계

0	69
1	0 1 2 2 2 3 4 5 5 6 7 7 7 8 8 9
2	0 0 0 1 2 2 4 4 5 5 6 6 7 7 8 9 9
3	0 0 2 5 5 5 6 6 6 6 7
4	1 6
5	1 2

2.7 1) 히스토그램 작성, 분포 설명

남자

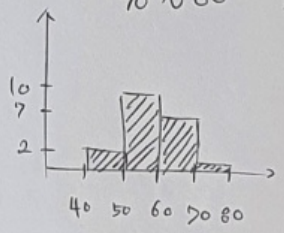
30 ~ 40	2
40 ~ 50	20
50 ~ 60	8



오른쪽으로 치우쳐짐.

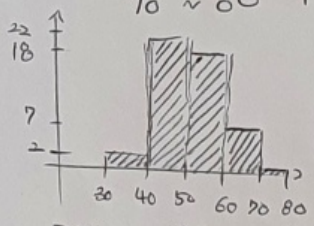
여자

40 ~ 50	2
50 ~ 60	10
60 ~ 70	7
70 ~ 80	1



전체

30 ~ 40	2
40 ~ 50	22
50 ~ 60	18
60 ~ 70	7
70 ~ 80	1



좌우대칭으로 완전히 치우쳐 있음.

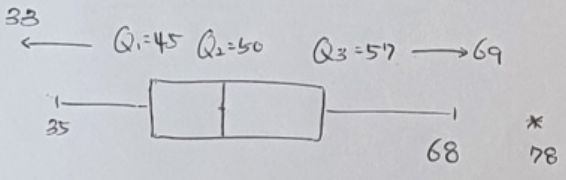
2) 정렬은 Python을 통해 하였음.

$Q_1 = (0.25) \times 50 = [12.5] + 1 = 13$ ⟶ 45

$Q_2 = \text{mean}(25, 26)$ ⟶ 50

$Q_3 = (0.75) \times 50 = [37.5] + 1 = 38$ ⟶ 57

3) $IQR = Q_3 - Q_1 = 12$



2.10 상관계수 계산

$S_{xy} = \sum_{i=1}^{20} x_i y_i - n\bar{x}\bar{y}$, $S_{xx} = \sum_{i=1}^{20} x_i^2 - n\bar{x}^2$, $S_{yy} = \sum_{i=1}^{20} y_i^2 - n\bar{y}^2$

$S_{xy} = 9203 - 20 \cdot {156}/{20} \cdot {1178}/{20} = 14.6$

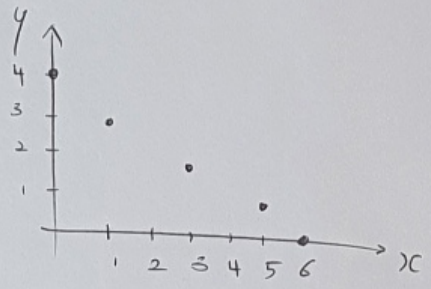
$S_{xx} = 1262 - 20 \times \left(\frac{156}{20}\right)^2 = 45.2$

$S_{yy} = 69390 - 20 \times \left(\frac{1178}{20}\right)^2 = 5.8$

$$r = \frac{S_{xy}}{\sqrt{S_{xx}}\sqrt{S_{yy}}} = \frac{14.6}{\sqrt{45.2}\sqrt{5.8}} = 0.9017$$

∴ 0.9017

2.12 1) 산점도를 그려라.



2) ∴ 추세선이 음의 기울기를 가지므로 (-) 부호 일치.

3) $\bar{x} = 3$, $\bar{y} = 2$, $n = 5$

$\sum_{i=1}^{5} x_i^2 = 71$, $\sum_{i=1}^{5} y_i^2 = 30$, $\sum_{i=1}^{5} x_i y_i = 14$

$S_{xy} = 71 - 45 = 26$, $S_{yy} = 30 - 20 = 10$, $S_{xy} = 14 - 30 = -16$

$$r = \frac{S_{xy}}{\sqrt{S_{xx}}\sqrt{S_{yy}}} = \frac{-16}{\sqrt{26}\sqrt{10}} = -0.99227$$

∴ −0.99227

4)

V	3	5	15	9	13
W	5	4	1	3	2

$\bar{V} = 9$, $\bar{W} = 3$, $n = 5$, $\sum_{i=1}^{5} V_i^2 = 509$, $\sum_{i=1}^{5} W_i^2 = 55$, $\sum_{i=1}^{5} V_i W_i = 103$

$S_{vw} = 103 - 5 \cdot 3.9 = -32$

$S_{vv} = 509 - 5 \cdot 81 = 104$

$S_{ww} = \sum_{i=1}^{5} w_i^2 - n\bar{w}^2 = 55 - 5 \cdot 9 = 10$

$$r = \frac{S_{vw}}{\sqrt{S_w}\sqrt{S_{ww}}} = \frac{-32}{\sqrt{104}\sqrt{10}} = -0.99227\cdots$$

$\therefore -0.99227$, same with $r(X, Y)$

3.4 1) B와 C 둘다 일어난다.

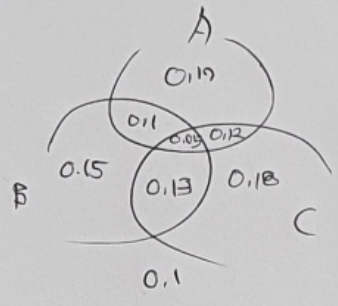
$P(BC) = P(ABC) + P(\bar{A}BC) = 0.05 + 0.13 = 0.18$ ∴ 0.18

2) B와 C가 일어난다.

$P(B \cup C) = P(B) + P(C) - P(BC) = 0.43 + 0.48 - 0.18 = 0.73$ ∴ 0.73

3) B는 일어나고 C는 일어나지 않는다.

$P(B\bar{C}) = 0.26$ ∴ 0.26



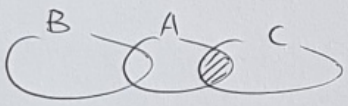
4) A, B, C 중 1개만 일어난다.

$P(\bar{A}\bar{B}\bar{C}) = 0.17 + 0.15 + 0.18 = 0.5$ ∴ 0.5

3.6 1) $P(A) = 0.6$, $P(C) = 0.25$

$P(AC) = P(C) - P(\bar{A}C) = 0.15$

∵ $P(AC) = P(A)P(C)$ ∴ A, C는 독립

2) 

$P(C) = 0.25$, $P(A\bar{B}) = P(A) - P(AB) = P(A) - P(B)P(A|B)$

$= 0.6 - 0.2 = 0.4$

$P(A\bar{B}C) = P(AC) = 0.15$

∵ $P(A\bar{B}C) \neq P(A\bar{B})P(C)$ ∴ 독립이 아니다.

3.10 $X = 0, 1, 2$

$P(X=0) = \frac{14}{20} \times \frac{25}{30} = \frac{7}{12}$

$P(X=1) = \frac{6}{20} \times \frac{25}{30} + \frac{14}{20} \times \frac{5}{30} = \frac{11}{30}$

$P(X=2) = \frac{6}{20} \times \frac{5}{30} = \frac{1}{20}$

∴

X	P(X=x)
0	7/12
1	11/30
2	1/20

3.15 1) 확률분포

x	f(x)
4,000,000	1/5000
1,000,000	3/5000
100,000	95/5000
5000	425/5000
0	4476/5000

2) 기댓값 $E(X) = \sum x f(x)$

$= 4,000,000 \times \frac{1}{5000} + 1,000,000 \times \frac{3}{5000}$

$+ 100,000 \times \frac{95}{5000} + 5,000 \times \frac{425}{5000}$

$= 800 + 600 + 1900 + 425 = 3,725$

∴ 3,725

3) ∵ 상금이 5000원 이하일 확률

$P(X \leq 5000) = \frac{425}{5000} + \frac{4476}{5000} = \frac{4,901}{5000}$

3.18 1) 표에서 F(x)열 완성

$F(1) = 0.07$

$F(2) = f(1) + f(2) = 0.19$

$F(3) = F(2) + f(3) = 0.44$

$F(4) = F(3) + f(4) = 0.72$

$F(5) = F(4) + f(5) = 0.9$

$F(6) = F(5) + f(6) = 1$

X	F(x)
1	0.07
2	0.19
3	0.44
4	0.72
5	0.9
6	1

2) $F(x) \rightarrow f(x)$

$f(i) = F(i) - F(i-1) \quad (i \geq 2), \quad F(1) = f(1)$

$f(6) = F(6) - F(5) = 0.1$

$f(5) = F(5) - F(4) = 0.18$

$f(4) = F(4) - F(3) = 0.28$

$f(3) = F(3) - F(2) = 0.25$

$f(2) = F(2) - F(1) = 0.12$

4.3 믿는 확률 $P = 0.2$

$n = 20$

1) 20명 모두가 믿지 않는다 $P[X=0] = \binom{20}{0}(0.2)^0(0.8)^{20} = 0.1074$

2) 7명 이상 믿는다. $P[X \geq 7] = 1 - P[X \leq 6] = 1 - \binom{20}{6}(0.2)^6(0.8)^{14}$

누적 이항 분포표 : $1 - 0.913$ $\therefore 0.087$

3) 평균 : $np = 10$

표준편차 : $\sqrt{npq} = \sqrt{8}$ $\therefore 10, 2\sqrt{2}$

4.5 $n = 14$, $P = 0.4$

1) $P[4 \leq X \leq 9] = P[X \leq 9] - P[X \leq 4]$ ∵ 누적 이항 분포표.

$= 0.982 - 0.279 = 0.703$ $\therefore 0.703$

2) $P[4 < X \leq 9] = P[X \leq 9] - P[X \leq 3]$

$= 0.982 - 0.124 = 0.858$ $\therefore 0.858$.

3) $P[4 < X < 9] = P[X \leq 8] - P[X \leq 3]$

$= 0.942 - 0.124 = 0.818$ $\therefore 0.818$

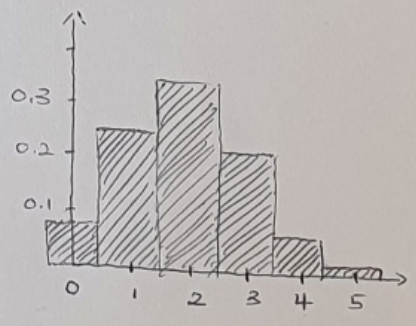
4) $E(X) = np = 14 \cdot 0.4 = 5.6$ $\therefore 5.6$

5) $Sd(X) = \sqrt{npq} = \sqrt{5.6}$ $\therefore 2.3664$

4.6 1) $n=5$, $P=0.4$ 확률분포를 나열하라

X	f(x)	
0	${}_5C_0(0.4)^0(0.6)^5$	0.07776
1	${}_5C_1(0.4)^1(0.6)^4$	0.2592
2	${}_5C_2(0.4)^2(0.6)^3$	0.3456
3	${}_5C_3(0.4)^3(0.6)^2$	0.2304
4	${}_5C_4(0.4)^4(0.6)^1$	0.0768
5	${}_5C_5(0.4)^5(0.6)^0$	0.01024

2) 확률 히스토그램을 그려라.



3) $E(X) = \sum xf(x) = 0.2592 + 2 \cdot 0.3456 + 3 \cdot 0.2304 + 4 \cdot 0.0768 + 5 \cdot 0.01024 = 2$

$Var(X) = \sum x^2 f(x) - \mu^2 = 0.2592 + 4 \cdot 0.3456 + 9 \cdot 0.2304 + 16 \cdot 0.0768$

$+ 25 \cdot 0.01024 - 4 = 5.2 - 4 = 1.2$

4) $E(X) = np = 2$ $Var(X) = npq = 1.2$

∴ same as (3)'s result

5.3 $\mu = 150$, $\sigma = 5$

1) $P[X < b] = 0.975$

$P\left[Z < \frac{b-150}{5}\right] = 0.975$

$\frac{b-150}{5} = 1.96$

$\therefore b = 159.8$

2) $P[X < b] = 0.025$

$P\left[Z < \frac{b-150}{5}\right] = 0.025$

$\frac{b-150}{5} = -2.81$

$\therefore b = 135.95$

5.4 $\mu = 582$, $\sigma = 75$

1) 표준점수 : $(696 - 582)/75$ $\therefore 1.52$

2) $\frac{x - 582}{75} = -0.8$ $\therefore 522$

$x = 522$

3) $380 \le X \le 560$

$\frac{380 - 582}{75} \le Z \le \frac{560 - 582}{75}$ $\therefore -2.693 \le Z \le -0.293$

-2.693 -0.293

4) $-1.2 \le Z \le 1.2$

$-1.2 \le \frac{X - 582}{75} \le 1.2$

$582 - 1.2 \times 75 \le X \le 582 + 1.2 \times 75$ $\therefore 492 \le X \le 672$.

$492 \le X \le 672$

5.6 $P[Z \le a] = 0.6$ $a = 0.26$ (0.6026)

$X \le a \cdot \sigma + \mu = 0.26 \times 8 + 70 = 72.8$ $\therefore 72.8$

5.8 이항분포 $E(X) = np = 120$ $\sigma = \sqrt{Var(X)} = \sqrt{84}$

1) $P[X \ge 105]$

$P\left[Z \ge \frac{105 - 120}{\sqrt{84}}\right] = P[Z \ge -1.636] = 1 - P[Z < -1.636] = 1 - 0.0505 = 0.9495$

2) $P[X < 105] = 0.0505$ 이므로 확률이 굉장히 작으므로.

많이 나올 가능성이 없어보임.

6.2 모집단 = {2, 4, 6, 8}, $n=2$

1) 가능한 표본 = {(2,2), (2,4), (2,6), (2,8), (4,2), (4,4), (4,6), (4,8)
(6,2), (6,4), (6,6), (6,8), (8,2), (8,4), (8,6), (8,8)}

$\bar{x}$ = 2, 3, 4, 5, 6, 7, 8

2) 표본평균

$\bar{x}$	$f(\bar{x})$
2	1/16
3	1/8
4	3/16
5	1/4
6	3/16
7	1/8
8	1/16

3) 모집단 분포, 모평균, 모표준편차.

x	$f(x)$
2	1/4
4	1/4
6	1/4
8	1/4

$\mu = \sum x f(x) = \frac{1}{2} + 1 + \frac{3}{2} + 2 = 5$

$\sigma^2 = \sum x^2 f(x) - \mu^2$

$= (1+4+9+16) - 25 = 5$

$\sigma = \sqrt{5}$ $\therefore \mu = 5, \sigma = \sqrt{5}$

4) 2)에 대한 표본분포의 평균, 분산

$\mu = \sum \bar{x} f(\bar{x}) = \frac{1}{8} + \frac{3}{8} + \frac{6}{8} + \frac{10}{8} + \frac{9}{8} + \frac{7}{8} + \frac{4}{8} = \frac{40}{8} = 5$

$\sigma^2 = \sum \bar{x}^2 f(\bar{x}) - \mu^2$

$= \frac{4}{16} + \frac{18}{16} + \frac{48}{16} + \frac{100}{16} + \frac{108}{16} + \frac{98}{16} + \frac{64}{16} = \frac{440}{16} - 25 = 27.5 - 25$

$= 2.5 = \frac{5}{2}$

$\sigma = \frac{\sqrt{5}}{\sqrt{2}}$ $\therefore \mu_{모} = \mu_{표}$

$\sigma_{모}/\sqrt{n} = \sigma_{표}$ ($n=2$)

6.3 모집단 = {2.4.6}, $n=2$

1) 표본평균 $\bar{x}$, s^2 계산.

{(2,2), (2.4), (2,6), (4,2), (4,4), (4,6), (6.2), (6.4), (6.6)}

$\bar{x}$ = 2, 3, 4, 5, 6

s^2 =

2) $\bar{X}$ 표본평균 표본분포.

$\bar{x}$	$f(\bar{x})$
2	1/9
3	2/9
4	1/3
5	2/9
6	1/9

3) s^2 표본분포.

6.7 1) $N=462$, $\sigma=3.5$, $n=9$

모집단 : 표본분포 $P(\bar{X} \le 455) = P\left(z \le \frac{455-462}{3.5/3}\right)$

$= P\left(z \le \frac{-21}{3.5}\right) = 0$

$\therefore 0$

2) $P(X \le 455) = P\left(z \le \frac{455-462}{3.5}\right) = P(z \le -2) = 1 - 0.9772$

$= 0.0228$

$0.0228 \times 100 = 2.28$

$\therefore 2.28\%$

6.11 $\mu=2$ $\sigma=0.6$ $n=50$

$P(90 \le \bar{X} \le 112.5) \rightarrow P(1.5 \le \bar{X} \le 1.875)$

$P\left(\frac{1.5-2}{0.6/\sqrt{50}} \le z \le \frac{1.875-2}{0.6/\sqrt{50}}\right) = P\left(-\frac{5}{6}\sqrt{50} \le z \le -1.4731\right)$

-5.892

$\simeq P(z \le -1.4731)$

$\therefore 0.0708$

6.5 1) $\frac{\sigma}{4} : \frac{\sigma}{\sqrt{n_1}} = \frac{\sigma}{4}$ $\therefore n_1 = 16$

2) $\frac{\sigma}{8} = \frac{\sigma}{\sqrt{n_2}}$ $\therefore n_2 = 64$

3) $0.15\sigma = \frac{\sigma}{\sqrt{n_3}}$

$n_3 = \left[\frac{100}{15}\right]^2$ $\therefore n_3 = 45$

7. 3 $n=55$ $\bar{x}=126.9$, $s=10.5$

1) 점추정 : 모수 = 126.9

추정량 : $\bar{X}$

$S.E(\bar{x})=\frac{\sigma}{\sqrt{55}}$, $S.E(\bar{x})(\text{추정량})=\frac{10.5}{\sqrt{55}}$

95% 오차한계 $1-\alpha=0.954$, $\alpha=0.046$

오차한계 $= z_{\alpha/2}\frac{10.5}{\sqrt{55}} = z_{0.023}\frac{10.5}{\sqrt{55}} = 1.995\frac{10.5}{\sqrt{55}}$

$\therefore \bar{x}=126.9$, $d=1.995\frac{10.5}{\sqrt{55}}$

2) 90% 신뢰구간 $1-\alpha=0.9$ $\alpha=0.1$, $\frac{\alpha}{2}=0.05$

$\left(\bar{x}-z_{0.05}\frac{10.5}{\sqrt{55}}, \bar{x}+z_{0.05}\frac{10.5}{\sqrt{55}}\right)$.

$\left(126.9-1.645\frac{10.5}{\sqrt{55}}, 126.9+1.645\frac{10.5}{\sqrt{55}}\right)$

$(124.571, 129.23)$ $\therefore (124.57, 129.23)$.

7.5 $\alpha=0.1$, $d=2.5$

$n=\left[\frac{z_{0.05}\sigma}{d}\right]^2$ $\sigma=\frac{d\sqrt{n}}{z_{0.05}}=\frac{2.5\sqrt{108}}{1.645}=15.8$.

$\alpha=0.05$, $d=1.8$ $\rightarrow$ $n=?$

$n=\left[\frac{z_{0.025}\sigma}{d}\right]^2=\left[\frac{1.96\times15.8}{1.8}\right]^2=295.99\rightarrow296$.

$\therefore 296$.

7.12.

$H_0 : \mu = 75$

$H_1 : \mu > 75$

$n = 56$. $\bar{x} = 77.04$, $s = 6.8$.

1) 검정통계량 $n > 30$, $\bar{X} \sim N\left(75, \frac{6.8}{\sqrt{56}}\right)$

$$z = \frac{\bar{x} - 75}{\frac{6.8}{\sqrt{56}}}$$

기각역 $\mu > 75$, 유의수준 0.05

$P[z \geq z_\alpha] = 0.05$.

$z_\alpha = 1.645$.

$\therefore z = \frac{\bar{x} - 75}{\frac{6.8}{\sqrt{56}}}$, $R : z \geq 1.645$.

2) $\frac{77.04 - 0.5}{6.8/\sqrt{56}} = 2.245$.

$\therefore$ H_0 기각되고 $\mu > 75$ 의 주장이 지지된다.

3) $P\text{-val} = P[z \geq 2.245] = 0.01225$.

$\therefore \alpha$ 보다 충분히 작다.

7.14 1) $1 - \alpha = 0.95$ $\alpha = 0.05$ $d = 0.08$.

$|\bar{x} - \mu| < 0.08 \rightarrow n ?$ $\hat{p} = 0.6$.

$$n \geq \left[\frac{z_{0.025} \times 0.6}{d}\right] = \left[\frac{1.96 \times 0.6}{0.08}\right] \text{올림} = 0.24\left[\frac{1.96}{0.08}\right] = 145.$$

$\therefore 145$

2) $\hat{p} = 0.68$

$1 - \alpha = 0.95$ $\alpha = 0.05$

$\left(\hat{p} - z_{0.025}\sqrt{\frac{\hat{p}\hat{q}}{n}}, \hat{p} + z_{0.025}\sqrt{\frac{\hat{p}\hat{q}}{n}}\right)$.

$\left(0.68 - 1.96\sqrt{\frac{0.68 \times 0.32}{68}}, 0.68 + 1.96\sqrt{\frac{0.68 \times 0.32}{68}}\right)$

$\therefore (0.57, 0.79)$

7.16. $\hat{p} = \frac{38}{130}$ $\hat{q} = \frac{92}{130}$ $n = 130$.

$H_0 : p_0 = 0.25$

$H_1 : p_0 > 0.25$

가설검정통계량 $Z = \frac{\hat{p} - p_0}{\sqrt{p_0 q_0 / n}}$

기각역 $[R = Z \geq Z_{\alpha/2}]$

$R : Z \geq Z_{\alpha/2}$

ex) $\alpha = 0.05$

$Z \geq 1.96$.

$\therefore H_0$ 기각되지 않는다.

$$p\text{-value} = \frac{\frac{38}{130} - 0.25}{\sqrt{\frac{0.25 \times 0.75}{130}}} = 1.114$$

$P(Z \geq 1.114) = P(Z \geq 1.114) = 0.134$

$\therefore$ 유의수준 α를 0.13 이상 크게 해야 하므로

증거가 약하다.