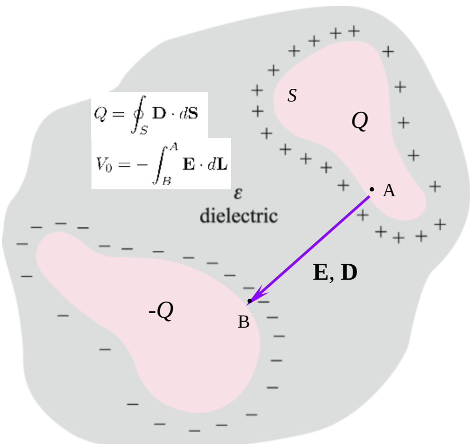
Chap 6. 정전용량

6.1 정전용량의 정의

* Charge & Potential . $\rho \rightarrow \mathbf{E}, \quad \mathbf{D} \rightarrow W, V \quad Q \Leftrightarrow V$

✓ R (Resistance) $R = \dfrac{V}{I}, \quad (V \propto I)$ C (Capacitance) $C = \dfrac{Q}{V}, \quad (Q \propto V)$ L (Inductance) $L = \dfrac{\Phi}{I}, \quad (\Phi \propto I)$



- 정전용량 (Capacitance) C :

$\boxed{C = \dfrac{Q}{V}}$

- 단위전압에 의하여 발생되는 전하량 , 전하에 의하여 생성되는 비율
- 전압에 대한 전하량의 비례상수
- 1 volt 의 전위차를 발생시키기 위해 필요한 전하량
- 도체의 기하학적 구조와 재질 함수

단위 : $C = \dfrac{Q}{V} \quad [\dfrac{Coulomb}{volt} \equiv Faraday, F]$

- 1F : 1 volt 를 위해 1C의 전하가 필요한 구조물
- 1mF , 1μF , 1 nF , 1pF , 1fF

• $\boxed{C = \dfrac{Q}{V} = \dfrac{\oint_S \varepsilon \mathbf{E} \cdot d\mathbf{S}}{-\int \mathbf{E} \cdot d\mathbf{L}}}$ (cf. $R = \dfrac{V}{I} = \dfrac{-\int \mathbf{E} \cdot d\mathbf{L}}{\int_S \sigma \mathbf{E} \cdot d\mathbf{S}}$)

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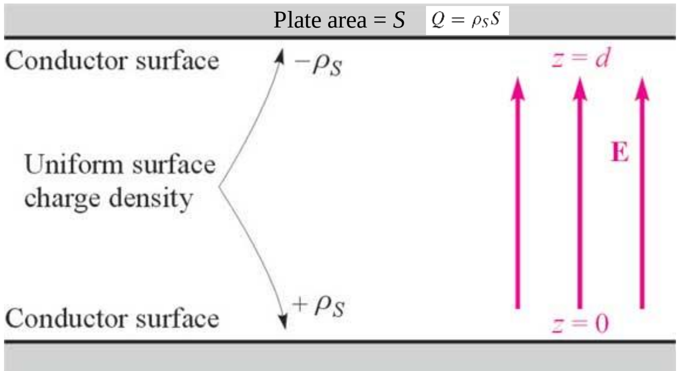
6.2 평행판 커패시터



$$\mathbf{D}\cdot\mathbf{n}_u\big|_{z=d} = \mathbf{D}\cdot(-\mathbf{a}_z) = -\rho_s \;\Rightarrow\; \underline{\mathbf{D} = \rho_s\,\mathbf{a}_z}$$

$$\mathbf{E} = \frac{\rho_S}{\epsilon}\mathbf{a}_z$$

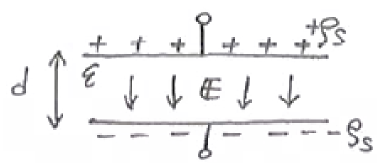
$$V_0 = -\int_{\text{upper}}^{\text{lower}} \mathbf{E}\cdot d\mathbf{L} = -\int_d^0 \frac{\rho_S}{\epsilon}dz = \frac{\rho_S}{\epsilon}d$$

$$Q = \rho_S S \qquad \boxed{C = \frac{Q}{V_0} = \frac{\epsilon S}{d}}$$

$$\mathbf{D}\cdot\mathbf{n}_\ell\big|_{z=0} = \mathbf{D}\cdot\mathbf{a}_z = \rho_s \;\Rightarrow\; \underline{\mathbf{D} = \rho_s\,\mathbf{a}_z}$$

Same result!

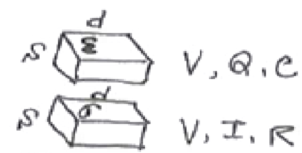
- 평행판 커패시터



$$\mathbf{E} = -\frac{\rho_S}{\varepsilon}\hat{a}_z,\quad \mathbf{D} = \rho_S\hat{a}_z,\quad D_N = \rho_S,\quad Q = \rho_S\cdot S$$

$$V = -\int \mathbf{E}\cdot d\mathbf{L} = -\int_0^d \left(-\frac{\rho_S}{\varepsilon}\right)dz = \frac{\rho_S}{\varepsilon}d$$

$$\therefore V = \frac{\rho_S}{\varepsilon}d = \frac{\rho_S\cdot S}{\varepsilon S}d = \frac{Q}{\varepsilon S}d \qquad \boxed{\therefore C = \varepsilon\frac{S}{d}}$$



(cf. $G = \sigma\dfrac{S}{d}$, $R = \dfrac{d}{\sigma S}$)

- 정전계의 에너지 (Ex.) : $W_E = \int_{vol}\frac{1}{2}\mathbf{D}\cdot\mathbf{E}dv$

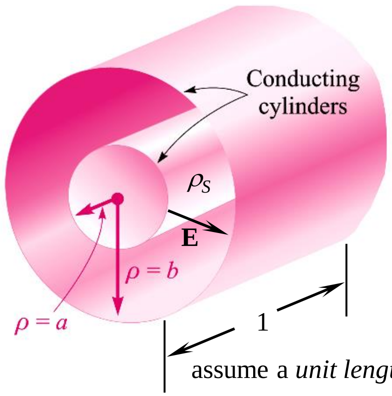
$$W_E = \tfrac{1}{2}\int_{\text{vol}}\epsilon E^2\,dv = \tfrac{1}{2}\int_0^S\int_0^d \tfrac{\epsilon\rho_S^2}{\epsilon^2}dz\,dS = \tfrac{1}{2}\frac{\rho_S^2}{\epsilon}Sd = \tfrac{1}{2}\underbrace{\frac{\epsilon S}{d}}_{C}\underbrace{\frac{\rho_S^2 d^2}{\epsilon^2}}_{V_0^2} = \frac{1}{2}CV^2$$

$$\boxed{W_E = \tfrac{1}{2}C\,V_0^2 = \tfrac{1}{2}Q\,V_0 = \tfrac{1}{2}\frac{Q^2}{C}} \quad \text{(Global)}$$

$$\boxed{= \frac{1}{2}\int \varepsilon\mathbf{E}^2 dv} \quad \text{(Local) Distribution .}$$

6.3 정전용량의 예

(1) 동축 케이블의 C (by ρ_s):



assume a *unit length* in z

$$Q = 2\pi a(1)\rho_s$$

$$\mathbf{E}(\rho) = \frac{a\rho_s}{\epsilon\rho}\mathbf{a}_\rho \text{ V/m} \qquad (a < \rho < b) \qquad \mathbf{E} = 0 \text{ elsewhere}$$

$$V_0 = -\int_b^a \mathbf{E}\cdot d\mathbf{L} = -\int_b^a \frac{a\rho_s}{\epsilon\rho}\mathbf{a}_\rho \cdot \mathbf{a}_\rho\, d\rho = \frac{a\rho_s}{\epsilon}\ln\left(\frac{b}{a}\right)$$

$$\boxed{C = \frac{Q}{V_0} = \frac{2\pi\epsilon}{\ln(b/a)} \text{ F/m}}$$

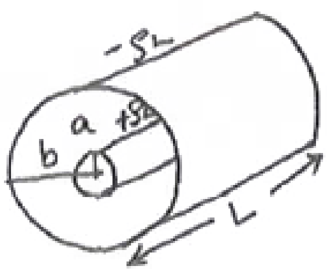
✓ : 단위길이당 C $[F/m]$

✓ $C \propto \dfrac{1}{\ln\frac{b}{a}}$, $a \cong b : c \uparrow$

cf.

✓ 동축 케이블의 C (by ρ_L) :

$$\mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\hat{a}_r$$



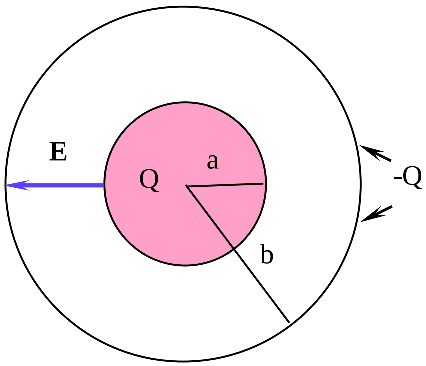
$$V = -\int \mathbf{E}\cdot d\mathbf{L} = -\int_b^a \frac{\rho_L}{2\pi\varepsilon_0}\ln\frac{b}{a}$$

$Q = \rho_L \cdot L$ 이므로 $\rho_L = Q/L$ $\qquad \therefore V = \dfrac{Q/L}{2\pi\varepsilon_0}\ln\dfrac{b}{a}$

$$\boxed{\therefore C = \frac{Q}{V} = \frac{2\pi\varepsilon_0 L}{\ln(b/a)}}$$

: cf. 단위길이당 C [F/m]

(2) 동심 도체구의 C (shall) :



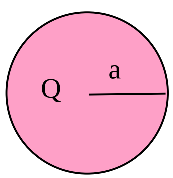
Consider two concentric spherical conductors, having radii *a* and *b*. Equal and opposite charges, *Q*, are on the inner and outer conductors.

$$\mathbf{E} = E_r \mathbf{a}_r = \frac{Q}{4\pi\epsilon r^2}\mathbf{a}_r$$

$$V_0 = -\int_b^a \mathbf{E}\cdot d\mathbf{L} = -\int_b^a \frac{Q}{4\pi\epsilon r^2}\mathbf{a}_r \cdot \mathbf{a}_r\, dr = \underline{\frac{Q}{4\pi\epsilon}\left(\frac{1}{a}-\frac{1}{b}\right)}$$

$$\boxed{C = \frac{Q}{V_0} = \frac{4\pi\epsilon}{(1/a)-(1/b)}} \qquad b \to \infty \quad \boxed{C \to 4\pi\epsilon a}$$

(3) 단일 도체구의 C :



(i) (2)에서 $b \to \infty$: $\boxed{C = 4\pi\varepsilon a \quad [F]}$

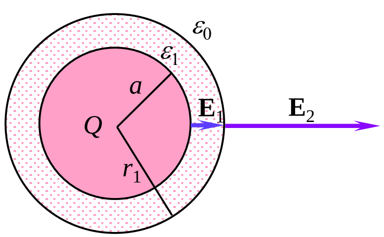
(ii) $V = \dfrac{Q}{4\pi\varepsilon a}$, $C = \dfrac{Q}{V} = 4\pi\varepsilon a \quad [F]$

* $C \propto a$

(ex) a = 1cm 인 구슬 : c=0.556[pF]

a = 64000km 인 지구 : c=0.35[mF]

(4) 유전체로 둘러싸인 구의 C :



$$D_r = \frac{a}{4\pi r^2}$$

$$E_r = \frac{Q}{4\pi\epsilon_1 r^2} \qquad (a < r < r_1)$$

$$= \frac{Q}{4\pi\epsilon_0 r^2} \qquad (r_1 < r)$$

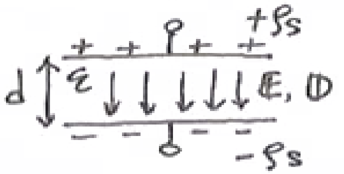
$$V_a - V_\infty = -\int_{r_1}^{a} \frac{Q\,dr}{4\pi\epsilon_1 r^2} - \int_{\infty}^{r_1} \frac{Q\,dr}{4\pi\epsilon_0 r^2}$$

$$= \frac{Q}{4\pi}\left[\frac{1}{\epsilon_1}\left(\frac{1}{a}-\frac{1}{r_1}\right) + \frac{1}{\epsilon_0 r_1}\right] = V_0$$

$$\boxed{C = \frac{4\pi}{\frac{1}{\epsilon_1}\left(\frac{1}{a}-\frac{1}{r_1}\right)+\frac{1}{\epsilon_0 r_1}}}$$

(5) 평판 콘덴서의 C :

① 단일유전체

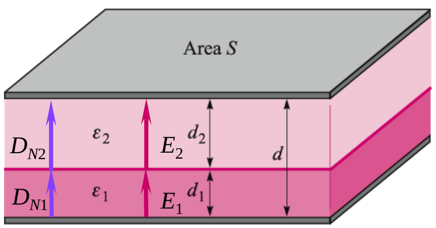


- $E = \dfrac{\rho_S}{\varepsilon}$, $V = -\int \mathbf{E}\cdot d\mathbf{L} = \dfrac{\rho_S}{\varepsilon} d = \dfrac{Q}{\varepsilon S} d$ $\left(\Theta\, Q = \rho_S \cdot S, \quad \rho_S = \dfrac{Q}{S} \right)$

$$\boxed{\therefore C = \frac{Q}{V} = \varepsilon \frac{S}{d}}$$

(or $D = \rho_S = \dfrac{Q}{S}$, $V = E \cdot d = \dfrac{\rho_S}{\varepsilon} d = \dfrac{Q}{\varepsilon S} d$ $\quad \therefore C = \varepsilon \dfrac{S}{d}$)
: same result.

② 두 유전체의 직렬구조



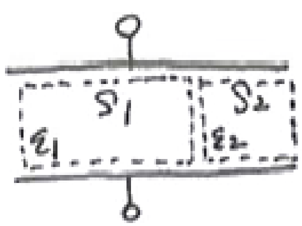
$$D_{N1} = D_{N2}, \quad \varepsilon_1 E_1 = \varepsilon_2 E_2 \qquad \therefore E_2 = \frac{\varepsilon_1}{\varepsilon_2} E_1$$

$$V_0 = E_1 d_1 + E_2 d_2 = E_1 d_1 + \frac{\varepsilon_1}{\varepsilon_2} E_1 d_2 = (d_1 + \frac{\varepsilon_1}{\varepsilon_2} d_2) E_1 \qquad E_1 = \frac{V_0}{d_1 + d_2(\epsilon_1/\epsilon_2)}$$

$Q = \rho_S S$ 이므로 $\rho_{S1} = D_1 = \epsilon_1 E_1 = \dfrac{V_0}{\dfrac{d_1}{\epsilon_1} + \dfrac{d_2}{\epsilon_2}} = D_2)$

$$\boxed{C = \frac{Q}{V_0} = \frac{\rho_S S}{V_0} = \frac{1}{\dfrac{d_1}{\epsilon_1 S} + \dfrac{d_2}{\epsilon_2 S}} = \frac{1}{\dfrac{1}{C_1} + \dfrac{1}{C_2}}} \qquad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

③ 두 유전체의 병렬구조 :



$$V_0 = E_1 d = E_2 d \qquad E_1 = E_2 = \frac{V_0}{d} \qquad \begin{aligned} D_1 &= \varepsilon_1 E_1 = \rho_{S1} \\ D_2 &= \varepsilon_2 E_2 = \rho_{S2} \end{aligned}$$

$$Q = \rho_{S1} \cdot S_1 + \rho_{S2} \cdot S_2 = \varepsilon_1 E_1 S_1 + \varepsilon_2 E_2 S_2 = \varepsilon_1 \frac{V_0}{d} S_1 + \varepsilon_2 \frac{V_0}{d} S_2 = (\varepsilon_1 \frac{S_1}{d} + \varepsilon_2 \frac{S_2}{d}) V_0$$

$$\boxed{\therefore C = \frac{Q}{V_0} = \varepsilon_1 \frac{S_1}{d} + \varepsilon_2 \frac{S_2}{d} = C_1 + C_2}$$

* V는 동일하므로 $Q = Q_1 + Q_2$

< 직렬연결 >

$R = R_1 + R_2$

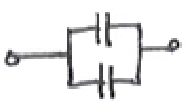
$L = L_1 + L_2$

$C = C_1 // C_2$

< 병렬연결 >

$R = R_1 // R_2$

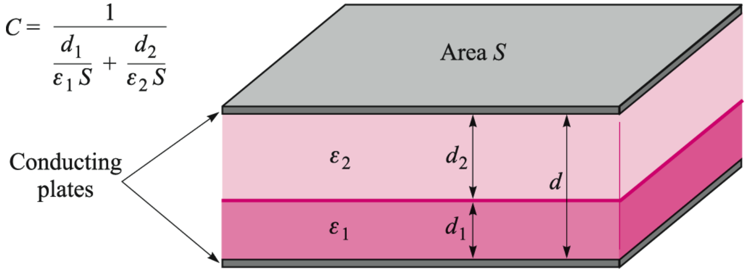
$L = L_1 // L_2$



$C = C_1 + C_2$

- Condensor의 병렬연결 :

$$C = \frac{1}{\frac{d_1}{\varepsilon_1 S} + \frac{d_2}{\varepsilon_2 S}}$$

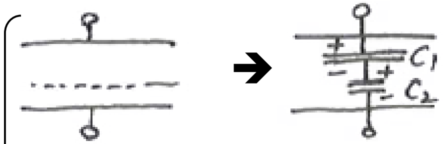


<직렬연결>

$R = R_1 + R_2$

$L = L_1 + L_2$

$C = C_1 // C_2$

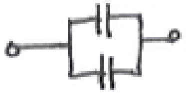


$C = C_1//C_2$ $\quad$ $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

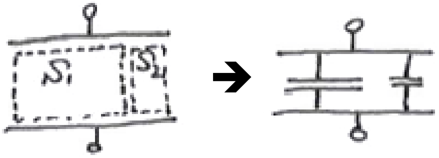
< 병렬연결 >

$R = R_1 // R_2$

$L = L_1 // L_2$



$C = C_1 + C_2$



$C = C_1 + C_2$

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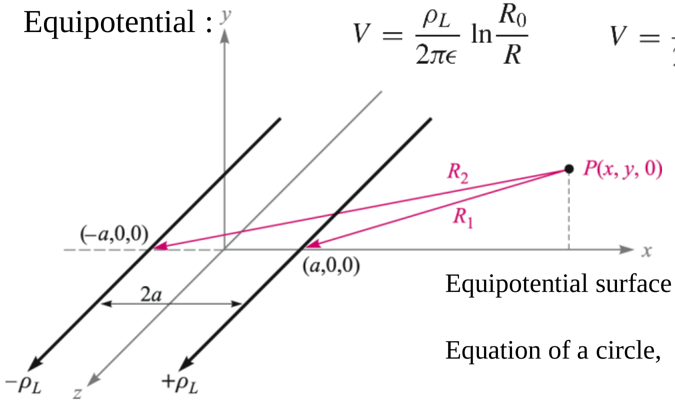
6.4 평행도선의 정전용량

- Equipotential :



$$V = \frac{\rho_L}{2\pi\epsilon}\ln\frac{R_0}{R} \qquad V = \frac{\rho_L}{2\pi\epsilon}\left(\ln\frac{R_{10}}{R_1} - \ln\frac{R_{20}}{R_2}\right) = \frac{\rho_L}{2\pi\epsilon}\ln\frac{R_{10}R_2}{R_{20}R_1}$$

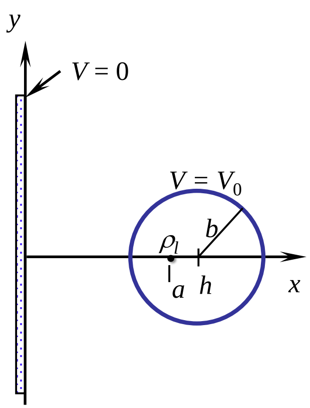
Expressing R_1 and R_2 in terms of x and y,

$$V = \frac{\rho_L}{2\pi\epsilon}\ln\sqrt{\frac{(x+a)^2+y^2}{(x-a)^2+y^2}} = \frac{\rho_L}{4\pi\epsilon}\ln\frac{(x+a)^2+y^2}{(x-a)^2+y^2} \qquad K_1 = \frac{(x+a)^2+y^2}{(x-a)^2+y^2}$$

Equipotential surface on which $V = V_1$ $\quad K_1 = e^{4\pi\epsilon V_1/\rho_L} \qquad x^2 - 2ax\frac{K_1+1}{K_1-1} + y^2 + a^2 = 0$

Equation of a circle, displaced along the x axis by distance h, and having radius b

$$\left(x - a\frac{K_1+1}{K_1-1}\right)^2 + y^2 = \left(\frac{2a\sqrt{K_1}}{K_1-1}\right)^2 \qquad b = \frac{2a\sqrt{K_1}}{K_1-1} \qquad h = a\frac{K_1+1}{K_1-1}$$



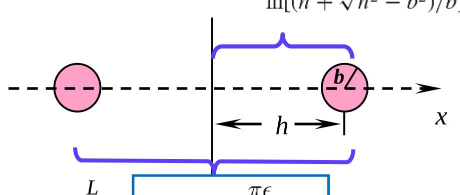
$$bK_1 - 2h\sqrt{K_1} + b = 0 \qquad \sqrt{K_1} = \frac{h \pm \sqrt{h^2-b^2}}{b} \qquad a = \sqrt{h^2-b^2}$$

equivalent line charge, ρ_l: $\rho_L = \frac{4\pi\epsilon V_0}{\ln K_1} \qquad \sqrt{K_1} = e^{2\pi\epsilon V_0/\rho_L}$

Given h, b, and V_0, find a, ρ_l, and K_1. $\quad C = \frac{\rho_L L}{V_0} = \frac{4\pi\epsilon L}{\ln K_1} = \frac{2\pi\epsilon L}{\ln\sqrt{K_1}}$

$$\boxed{C = \frac{2\pi\epsilon L}{\ln[(h+\sqrt{h^2-b^2})/b]} = \frac{2\pi\epsilon L}{\cosh^{-1}(h/b)}}$$

- Capacitance : $C = \frac{2\pi\epsilon L}{\ln[(h+\sqrt{h^2-b^2})/b]} = \frac{2\pi\epsilon L}{\cosh^{-1}(h/b)}$



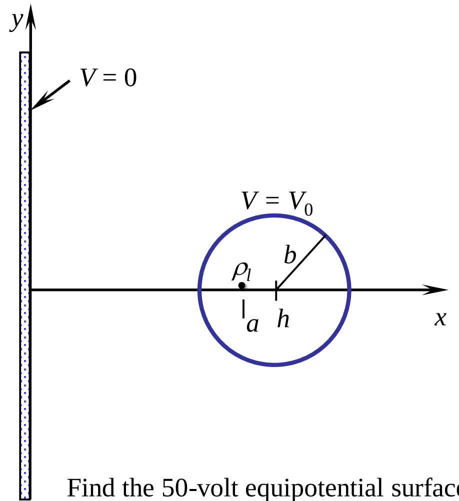
$$\boxed{C = \frac{\pi\epsilon}{\cosh^{-1}(h/b)}}$$

if $b << h$, then

$$\ln\left[\left(h+\sqrt{h^2-b^2}\right)/b\right] \doteq \ln[(h+h)/b] \doteq \ln(2h/b)$$

$$\boxed{C \doteq \frac{\pi\epsilon L}{\ln(2h/b)}}$$

(Ex) b = 5 mm, h = 13 mm, and V_0 = 100 V. Find ρ_l and C.



$$a = \sqrt{h^2 - b^2} = \sqrt{13^2 - 5^2} = 12 \text{ mm}$$

$$\sqrt{K_1} = \frac{h + \sqrt{h^2 - b^2}}{b} = \frac{13 + 12}{5} = 5$$

$$\rho_L = \frac{4\pi\epsilon V_0}{\ln K_1} = \frac{4\pi \times 8.854 \times 10^{-12} \times 100}{\ln 25} = 3.46 \text{ nC/m}$$

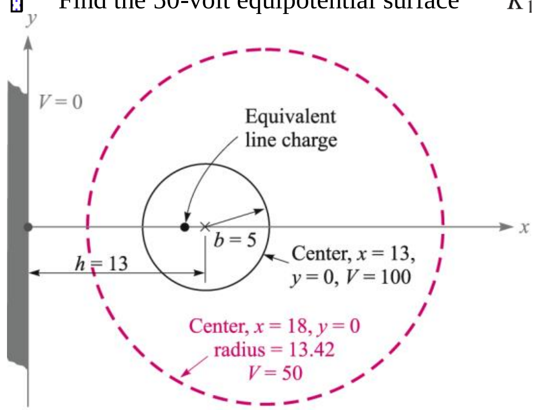
$$C = \frac{2\pi\epsilon}{\cosh^{-1}(h/b)} = \frac{2\pi \times 8.854 \times 10^{-12}}{\cosh^{-1}(13/5)} = 34.6 \text{ pF/m}$$

Find the 50-volt equipotential surface

$$K_1 = e^{4\pi\epsilon V_1/\rho_L} = e^{4\pi \times 8.854 \times 10^{-12} \times 50/3.46 \times 10^{-9}} = 5.00$$

$$b = \frac{2a\sqrt{K_1}}{K_1 - 1} = \frac{2 \times 12\sqrt{5}}{5 - 1} = 13.42 \text{ mm}$$

$$h = a\frac{K_1 + 1}{K_1 - 1} = 12\frac{5 + 1}{5 - 1} = 18 \text{ mm}$$



$h = 13$, $b = 5$, $\therefore K_1 = 25$; $\therefore \rho_L = 3.46 \times 10^{-9}$ C/m, $\therefore a = 12$

If $V_1 = 50$, $K_1 = 5$, $h = 18$, $b = 13.42$, ρ_L unchanged

$$C = \frac{2\pi\varepsilon_0 L}{\ln 5} = 34.6 \text{ pF/m}$$

Scaling :

The same capacitance and charge densities would result as long as proportions are maintained in the dimensions; i.e., it doesn't matter whether the dimensions are given in micrometers, meters, or kilometers!

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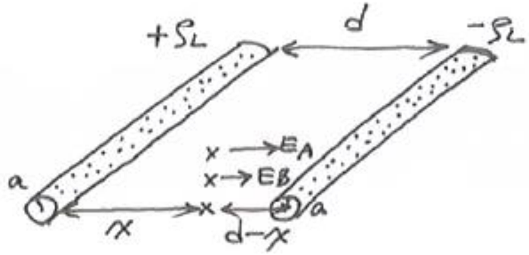
- 평행도선의 정전용량



- $$\mathbf{E}_A = \frac{\rho_L}{2\pi\varepsilon_0 x}\hat{a}_x \qquad \mathbf{E}_B = \frac{\rho_L}{2\pi\varepsilon_0 (d-x)}\hat{a}_x$$

$$\mathbf{E} = \mathbf{E}_A + \mathbf{E}_B = \frac{\rho_L}{2\pi\varepsilon_0}\left(\frac{1}{x} + \frac{1}{d-x}\right)\hat{a}_x \quad [V/m]$$

- $$V_{AB} = -\int_{d-a}^{a} E dx = -\frac{\rho_L}{2\pi\varepsilon_0}\int_{d-a}^{a}\left(\frac{1}{x} + \frac{1}{d-x}\right)dx$$

$$= \frac{\rho_L}{2\pi\varepsilon_0}\left(\ln\frac{d-a}{a} - \ln\frac{a}{d-a}\right)$$

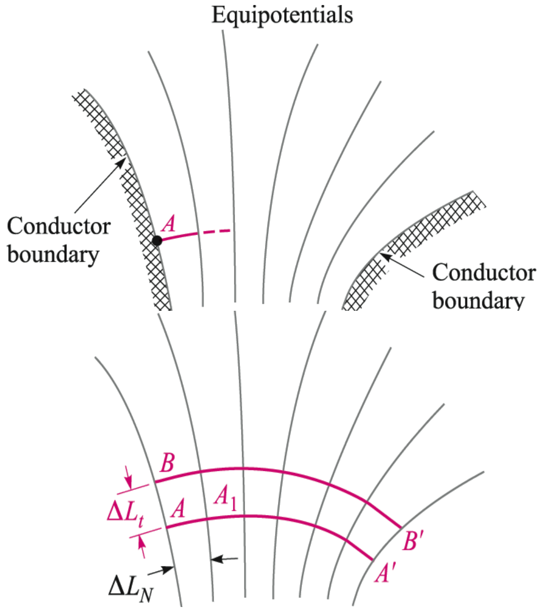
$$= \frac{\rho_L}{\pi\varepsilon_0}\ln\frac{d-a}{a} \quad [V]$$

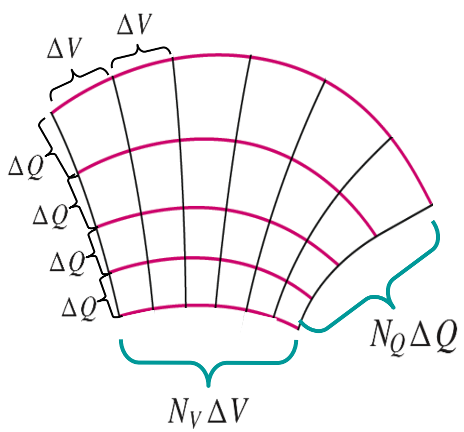
$$\boxed{\therefore C = \frac{\rho_L}{V_{AB}} = \frac{\pi\varepsilon_0}{\ln\dfrac{d-a}{a}} \quad [F/m]}$$

- If) d >> a : $\ln\frac{d-a}{a} \cong \ln\frac{d}{a}$

$$\boxed{\therefore C = \frac{\pi\varepsilon_0}{\ln(d/a)}}$$

6. 5 2D에서 C추정을 위한 전계 그림 사용





Two Lines of **D** are shown (sketched in red), and spacings between adjacent field lines and between adjacent equipotential surfaces are noted. All field lines *must* intersect equipotentials at 90^0.

The volume between the two red field lines forms a "tube" of flux, of amount $\Delta\psi$.
(This is the same as the charge on the conductor that is bounded by the tube.)

1. 도체는 등전위
2. **E & D** 는 등전위면과 수직
3. **E & D** 는 도체와 수직
4. **Q → ψ**

$$E = \frac{1}{\epsilon}\frac{\Delta\Psi}{\Delta L_t}$$

$$E = \frac{\Delta V}{\Delta L_N}$$

$$\frac{1}{\epsilon}\frac{\Delta\Psi}{\Delta L_t} = \frac{\Delta V}{\Delta L_N}$$

$$\boxed{\frac{\Delta L_t}{\Delta L_N} = \text{constant} = \frac{1}{\epsilon}\frac{\Delta\Psi}{\Delta V}}$$

In other words, the sketch is done such that the ratio $\Delta L_t \,/\, \Delta L_N$ is fixed.
The easiest way to do this is to make $\Delta L_t = \Delta L_N$
So draw the sketch such that *each grid segment is approximately square*

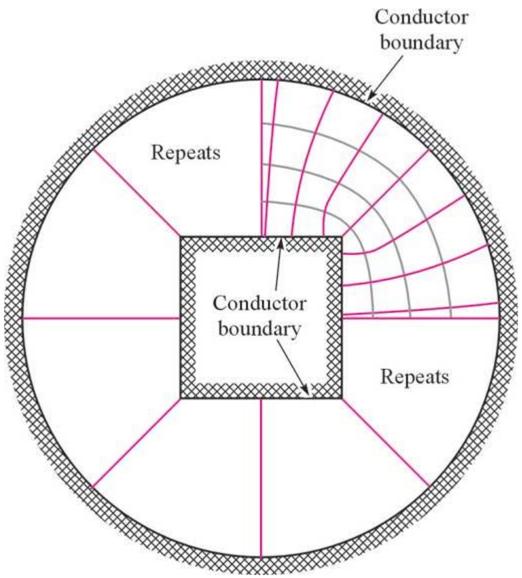
$$Q = N_Q \Delta Q = N_Q \Delta\Psi$$

$$V_0 = N_V \Delta V$$

$$C = \frac{N_Q \Delta Q}{N_V \Delta V}$$

$$\boxed{C = \frac{N_Q}{N_V}\epsilon\frac{\Delta L_t}{\Delta L_N} = \epsilon\frac{N_Q}{N_V}}$$

(Ex) Capacitance per unit length.

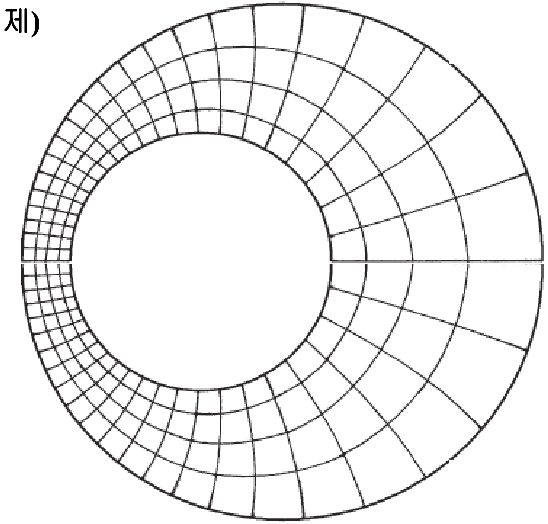


In this case, division of the range parallel to the conductors into an integral number of squares was not achieved. Instead, over one-eighth of the distance around the perimeter, we have 3.25 divisions.

Between conductors there are exactly four squares.

$$C = \epsilon \frac{N_Q}{N_V} = \epsilon_0 \frac{8 \times 3.25}{4} = \underline{57.6\,\mathrm{pF/m}}$$

(응용예제)



6. 6 프와송 및 라플라스 방정식

◆ 전기장 지배 방정식 :

Gauss's law : $\nabla \cdot \mathbf{D} = \rho_v$ (Maxwell 1'st eq.)

Materal : $\mathbf{D} = \epsilon \mathbf{E}$ Potential : $\mathbf{E} = -\nabla V$

$$\nabla \cdot \mathbf{D} = \nabla \cdot (\epsilon \mathbf{E}) = -\nabla \cdot (\epsilon \nabla V) = \rho_v$$

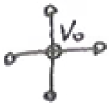
$$\boxed{\nabla \cdot \nabla V = -\frac{\rho_v}{\epsilon}}$$

$$\nabla V = \frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z$$

$$\nabla \cdot \nabla V = \frac{\partial}{\partial x}\left(\frac{\partial V}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial V}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial V}{\partial z}\right) = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$\nabla \cdot \nabla$ is abbreviated ∇^2 (Laplacian operator)

(전기장 지배방정식) $$\boxed{\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho_v}{\epsilon}}$$ Poisson's equation



$\boxed{\nabla^2 V = 0}$ Laplace's equation: (if $\rho = 0$: $\nabla^2 V = 0$)

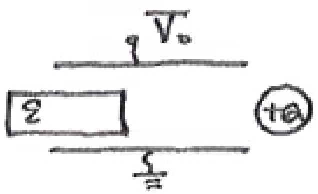
◆ **Laplace Operator . Laplacian :** $\nabla^2 V = \nabla \cdot \nabla V$

직각좌표계 : $\nabla^2 V = \dfrac{\partial^2 V}{\partial x^2} + \dfrac{\partial^2 V}{\partial y^2} + \dfrac{\partial^2 V}{\partial z^2} = 0$ (rectangular)

원통좌표계 : $\nabla^2 V = \dfrac{1}{\rho}\dfrac{\partial}{\partial \rho}\left(\rho \dfrac{\partial V}{\partial \rho}\right) + \dfrac{1}{\rho^2}\left(\dfrac{\partial^2 V}{\partial \phi^2}\right) + \dfrac{\partial^2 V}{\partial z^2}$ (cylindrical)

구좌표계 : $\nabla^2 V = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2 \dfrac{\partial V}{\partial r}\right) + \dfrac{1}{r^2 \sin\theta}\dfrac{\partial}{\partial \theta}\left(\sin\theta \dfrac{\partial V}{\partial \theta}\right) + \dfrac{1}{r^2 \sin^2\theta}\dfrac{\partial^2 V}{\partial \phi^2}$ (spherical)

◆ $$\left[\begin{array}{cc:c} \rho \rightarrow \mathbf{E}, & \mathbf{D} \rightarrow V & \rho \rightarrow V \\ (\nabla \cdot \mathbf{D} = \rho) & (\mathbf{E} = -\nabla V) & \nabla^2 V = -\dfrac{\rho}{\varepsilon} \end{array}\right]$$



6. 7 라플라스 방정식의 예

➢ <u>Potential Bar :</u>

직각좌표계 :

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho}{\varepsilon} \qquad 1-D, \quad \rho = 0 \quad :$$

$$\frac{\partial^2 V}{\partial x^2} = 0, \quad \frac{dV^2}{dx^2} = 0, \quad \frac{dV}{dx} = A, \qquad V = Ax + B$$

B.C. : $\begin{cases} V(x_1) = V_1 : \\ V(x_2) = V_2 : \end{cases}$ $\quad A = \dfrac{V_1 - V_2}{x_1 - x_2}, \quad B = \dfrac{V_2 x_1 - V_1 x_2}{x_1 - x_2}$

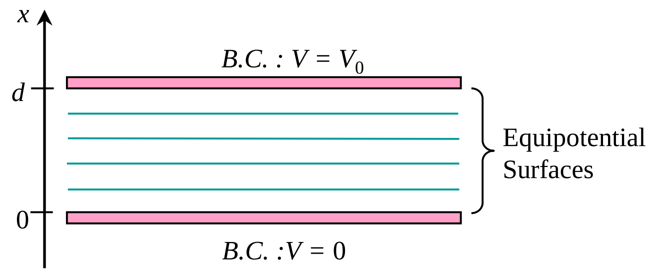
$$\therefore V = \frac{1}{x_1 - x_2}(V_1(x - x_2) - V_2(x - x_1))$$

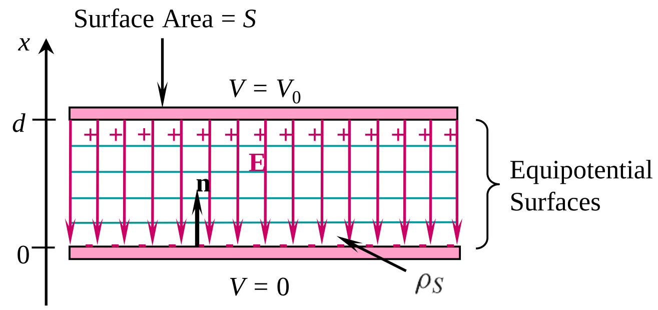
* V(0) = 0 , V(0) = V_0 일 경우 $A = \dfrac{V_0}{d}, \quad B = 0$

$$\boxed{\therefore V = \frac{V_0 x}{d}}$$

Lab. of Applied ElectroMagnetics

(Ex 6.2) Parallel Plate Capacitor :





Boundary conditions:

1. $V = 0$ at $x = 0$
2. $V = V_0$ at $x = d$

- Potential :

Laplace's equation: $\frac{d^2V}{dx^2} = 0$ $\quad \frac{dV}{dx} = A \quad V = Ax + B$

Boundary conditions: 1. $V = 0$ at $x = 0$ $\quad 0 = A(0) + B \Rightarrow B = 0$

2. $V = V_0$ at $x = d$ $\quad V_0 = Ad \Rightarrow A = \frac{V_0}{d}$

$$\boxed{V = \frac{V_0 x}{d}}$$

- Capacitance :

$$V = V_0 \frac{x}{d}$$

$$\mathbf{E} = -\nabla V = -\frac{V_0}{d}\mathbf{a}_x$$

$$\mathbf{D} = -\epsilon \frac{V_0}{d}\mathbf{a}_x$$

At the lower plate surface ($z = 0$):

$$\mathbf{D}_S = \mathbf{D}\big|_{x=0} = -\epsilon \frac{V_0}{d}\mathbf{a}_x \qquad \mathbf{n} = \mathbf{a}_x$$

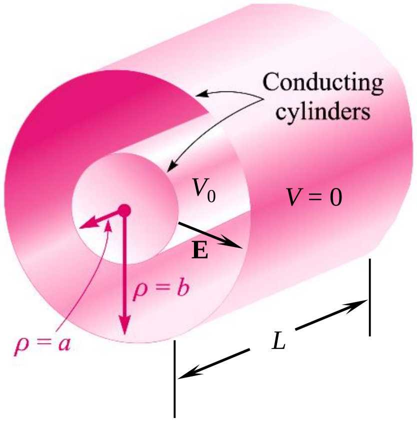
$$\mathbf{D}\cdot\mathbf{n}\big|_s = \rho_s \qquad D_N = -\epsilon \frac{V_0}{d} = \rho_S$$

$$Q = \int_S \frac{-\epsilon V_0}{d} dS = -\epsilon \frac{V_0 S}{d}$$

$$\boxed{C = \frac{|Q|}{V_0} = \frac{\epsilon S}{d}}$$

Lab. of Applied ElectroMagnetics

(Ex 6.3) Coaxial Transmission Line :



Boundary conditions:
1. $V = 0$ at $\rho = b$
2. $V = V_0$ at $\rho = a$

- Potential : $\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \frac{\partial V}{\partial r}) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}(\sin\theta \frac{\partial V}{\partial \theta}) + \frac{1}{r^2 \sin^2\phi}\frac{\partial^2 V}{\partial \phi^2}$

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) = 0 \qquad \rho \frac{dV}{d\rho} = A \qquad V = A \ln \rho + B$$

Boundary conditions:
1. $V = 0$ at $\rho = b$
2. $V = V_0$ at $\rho = a$

$$0 = A\ln(a) + B \quad \Rightarrow \quad B = -A\ln(a)$$

$$V_0 = A\ln(a) - A\ln(b) = A\ln(a/b) \quad \Rightarrow \quad A = -\frac{V_0}{\ln(b/a)}$$

$$\boxed{V(\rho) = -\frac{V_0}{\ln(b/a)}[\ln(\rho) - \ln(b)] = V_0 \frac{\ln(b/\rho)}{\ln(b/a)}}$$

- Capacitance :

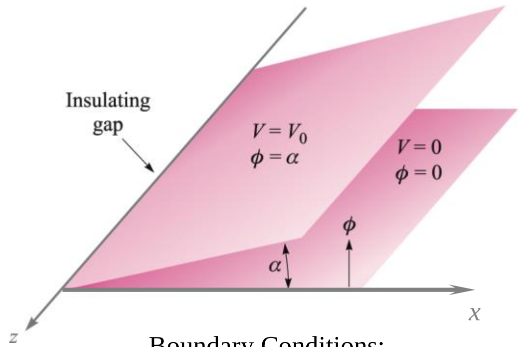
$$\mathbf{E} = -\nabla V = -\frac{dV}{d\rho}\mathbf{a}_\rho = \frac{V_0}{\rho}\frac{1}{\ln(b/a)}\mathbf{a}_\rho$$

$$\rho_s = \mathbf{D}\cdot\mathbf{a}_\rho\Big|_{\rho=a} = \frac{\epsilon V_0}{a}\frac{1}{\ln(b/a)} \ \mathrm{C/m^2}$$

$$Q = \int_S \rho_s \, da = 2\pi a L \, \rho_s = \frac{2\pi\epsilon L V_0}{\ln(b/a)} \ \mathrm{C}$$

$$\boxed{C = \frac{Q}{V_0} = \frac{2\pi\epsilon L}{\ln(b/a)} \ \mathrm{F}}$$

(Ex 6.4) Angled Plate :



Boundary Conditions:
1. $V = 0$ at $\phi = 0$
2. $V = V_0$ at $\phi = \alpha$

- Potential :

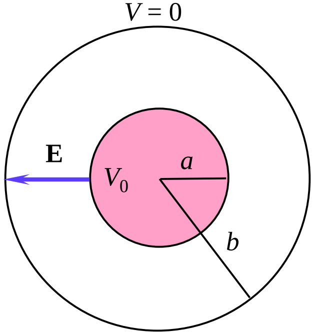
$$\nabla^2 V = \frac{1}{\rho^2}\frac{d^2V}{d\phi^2} = 0 \quad (\rho > 0) \qquad \frac{dV}{d\phi} = A \qquad V(\phi) = A\phi + B$$

Boundary Conditions:
1. $V = 0$ at $\phi = 0$ $\qquad 0 = A(0) + B \Rightarrow B = 0$
2. $V = V_0$ at $\phi = \alpha$ $\qquad V_0 = A\alpha \Rightarrow A = \frac{V_0}{\alpha}$

$$\boxed{V(\phi) = V_0\frac{\phi}{\alpha}}$$

$$\mathbf{E} = -\nabla V = -\frac{1}{\rho}\frac{dV}{d\phi}\mathbf{a}_\phi = -\frac{V_0}{\alpha\rho}\mathbf{a}_\phi$$

(Ex 6.5) Concentric Sphere :



Boundary Conditions:
1. $V = 0$ at $r = b$
2. $V = V_0$ at $r = a$

- Potential :

$$\nabla^2 V = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dV}{dr}\right) = 0 \qquad \frac{d}{dr}\left(r^2\frac{dV}{dr}\right) = 0 \qquad \frac{dV}{dr} = \frac{A}{r^2} \qquad V(r) = -\frac{A}{r} + B$$

Boundary Conditions:
1. $V = 0$ at $r = b$ $\qquad 0 = -\frac{A}{b} + B \Rightarrow B = \frac{A}{b}$
2. $V = V_0$ at $r = a$ $\qquad V_0 = -\frac{A}{a} + \frac{A}{b} \Rightarrow A = \frac{V_0}{(1/b) - (1/a)}$

$$\boxed{V(r) = V_0\frac{(1/r) - (1/b)}{(1/a) - (1/b)}}$$

- Capacitance :

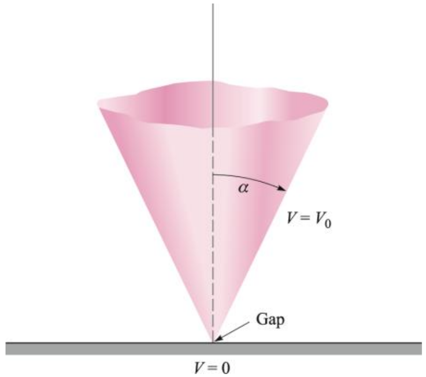
$$\mathbf{E} = -\nabla V = \frac{dV}{dr}\mathbf{a}_r = \frac{V_0}{r^2[(1/a) - (1/b)]}\mathbf{a}_r \quad \text{V/m}$$

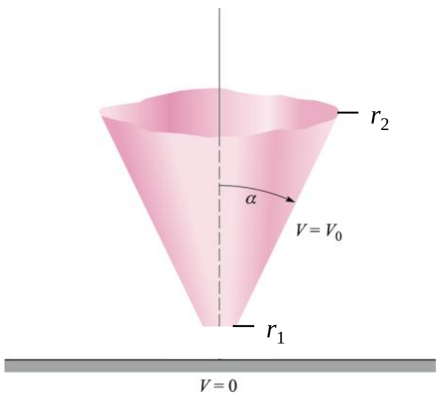
$$\rho_s = \mathbf{D}\cdot\mathbf{a}_r\Big|_{r=a} = \frac{\epsilon V_0}{a^2[(1/a) - (1/b)]} \quad \text{C/m}^2$$

$$Q = \int_S \rho_s\, da = 4\pi a^2\rho_s = \frac{4\pi\epsilon V_0}{[(1/a) - (1/b)]} \quad \text{C}$$

$$\boxed{C = \frac{Q}{V_0} = \frac{4\pi\epsilon}{[(1/a) - (1/b)]} \quad \text{F}}$$

(Ex 6.6) θ – Dependent Potential Field :





Boundary Conditions:

1. $V = 0$ at $\theta = \pi/2$

2. $V = V_0$ at $\theta = \alpha$

- Potential :

$$\frac{1}{r^2 \sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dV}{d\theta}\right) = 0$$

✓ r and θ cannot be zero : $\sin\theta\frac{dV}{d\theta} = A$

$$V = \int \frac{A\,d\theta}{\sin\theta} + B \;=\; A\ln\left(\tan\frac{\theta}{2}\right) + B$$

Boundary Conditions:

1. $V = 0$ at $\theta = \pi/2$

2. $V = V_0$ at $\theta = \alpha$

$$0 = A\ln\tan\left(\frac{\pi}{4}\right) + B = B \quad\Rightarrow\quad B = 0$$

$$V_0 = A\ln\tan\left(\frac{\alpha}{2}\right) \quad\Rightarrow\quad A = \frac{V_0}{\ln\tan(\alpha/2)}$$

$$\boxed{V(\theta) = V_0\frac{\ln\tan(\theta/2)}{\ln\tan(\alpha/2)}}$$

- Capacitance :

$$\mathbf{E} = -\nabla V = \frac{-1}{r}\frac{\partial V}{\partial\theta}\mathbf{a}_\theta = -\frac{V_0}{r\sin\theta\ln\left(\tan\frac{\alpha}{2}\right)}\mathbf{a}_\theta$$

$$\rho_s = \mathbf{D}\cdot\mathbf{a}_\theta\Big|_{\theta=\alpha} = \frac{-\epsilon V_0}{r\sin\alpha\ln\left[\tan(\alpha/2)\right]}$$

$$Q = \int_0^{2\pi}\int_{r_1}^{r_2}\frac{-\epsilon V_0}{r\sin\alpha\ln\left[\tan(\alpha/2)\right]}\,r\sin\alpha\,dr\,d\phi = \frac{-2\pi\epsilon V_0(r_2 - r_1)}{\ln\left[\tan(\alpha/2)\right]}$$

$$\boxed{C = \frac{Q}{V_0} = \frac{-2\pi\epsilon(r_2 - r_1)}{\ln\left[\tan(\alpha/2)\right]}}$$

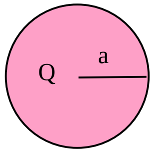
This is an approximate result because we have neglected the fringing fields that will occur at the cone edges. Fringing fields will be more important for smaller α .

✓ Note that the capacitance is in fact positive (as it should be). Do you see why?

Lab. of Applied ElectroMagnetics

◆ SUM

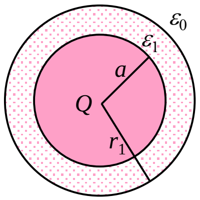
(1) 단일 도체구 :



$$V = \frac{Q}{4\pi\varepsilon a}$$

$$\boxed{C = 4\pi\varepsilon a \quad [F]}$$

(2) 유전체 구 :

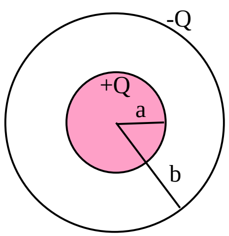


$$D_r = \frac{a}{4\pi r^2}$$

$$V_0 = \frac{Q}{4\pi}\left[\frac{1}{\epsilon_1}\left(\frac{1}{a} - \frac{1}{r_1}\right) + \frac{1}{\epsilon_0 r_1}\right]$$

$$\boxed{C = \frac{4\pi}{\frac{1}{\epsilon_1}\left(\frac{1}{a} - \frac{1}{r_1}\right) + \frac{1}{\epsilon_0 r_1}}}$$

(3) 동심 도체구의 C (shall) :

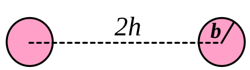


$$\mathbf{E} = E_r \mathbf{a}_r = \frac{Q}{4\pi\epsilon r^2}\mathbf{a}_r$$

$$V_0 = \frac{Q}{4\pi\epsilon}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$\boxed{C = \frac{Q}{V_0} = \frac{4\pi\epsilon}{(1/a) - (1/b)}}$$

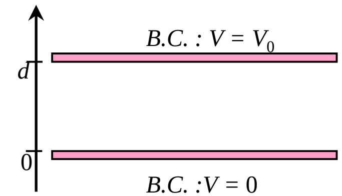
(4) 평행도선의 정전용량



$$V = \frac{\rho_L}{2\pi\epsilon}\ln\frac{R_0}{R}$$

$$\boxed{C \doteq \frac{\pi\epsilon L}{\ln(2h/b)}}$$

(5) Parallel Plate Capacitor :

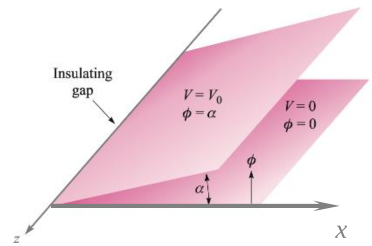


$$\mathbf{E} = \frac{\rho_S}{\epsilon}\mathbf{a}_z$$

$$V_0 = \frac{\rho_S}{\epsilon}d$$

$$\boxed{C = \frac{Q}{V_0} = \frac{\epsilon S}{d}}$$

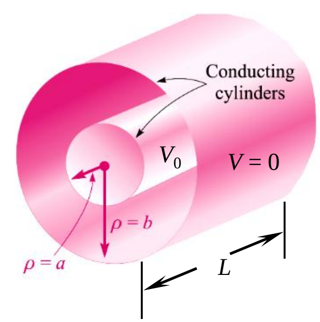
(6) Angled Plate :



$$\mathbf{E} = -\frac{V_0}{\alpha\rho}\mathbf{a}_\phi$$

$$\boxed{V(\phi) = V_0\frac{\phi}{\alpha}}$$

(7) 동축 케이블 :

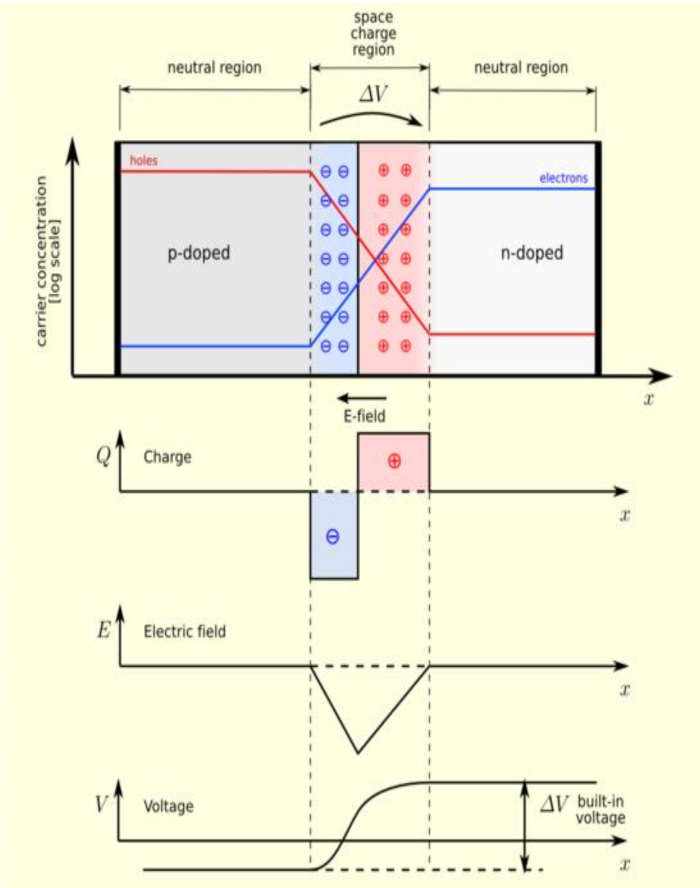


$$\mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\hat{a}_r$$

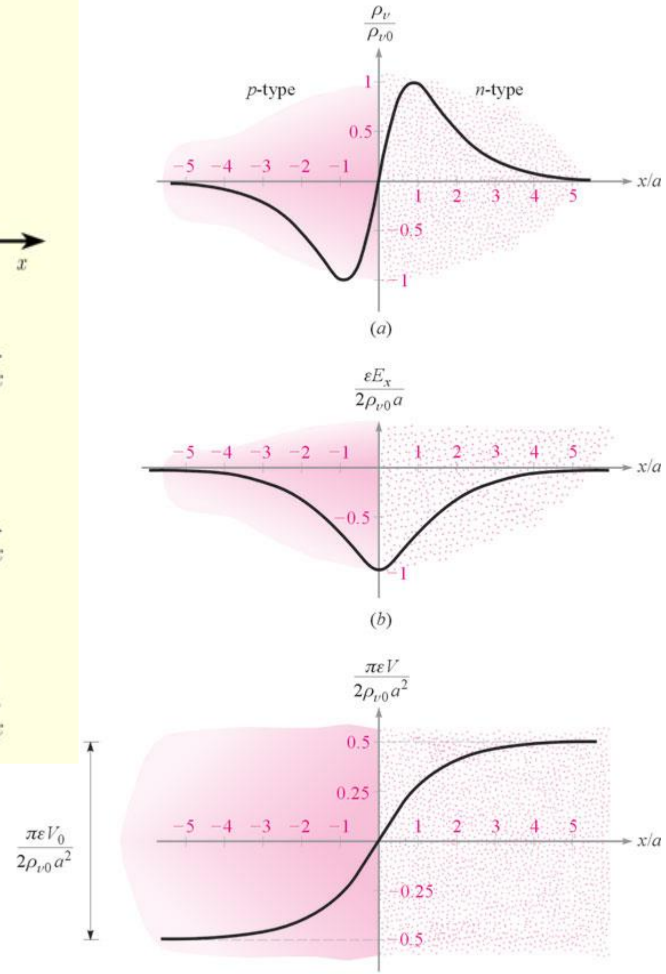
$$\therefore V = \frac{\rho_L}{2\pi\varepsilon_0}\ln\frac{b}{a}$$

$$\boxed{\therefore C = \frac{Q}{V} = \frac{2\pi\varepsilon_0 L}{\ln(b/a)}}$$

6. 8 P-N 접합의 정전용량



$$C_{j0} = \sqrt{\frac{\epsilon_{si} q}{2}\frac{N_A N_D}{N_A + N_D}\frac{1}{V_0}} = \frac{C_{j0}}{\sqrt{1 - \frac{V_R}{V_0}}},$$



(a)

(b)

(c)

$$p_{n,f} = \frac{p_{p,f}}{\exp \frac{V_0 - V_F}{V_T}}.$$

$$\Delta n_p \approx \frac{N_D}{\exp \frac{V_0}{V_T}}(\exp \frac{V_F}{V_T} - 1).$$

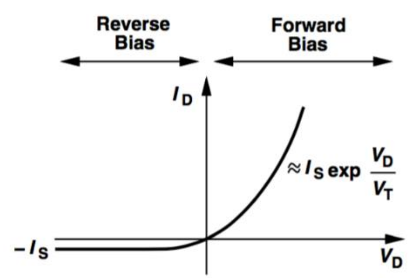
$$\begin{cases} \frac{d\varepsilon(x)}{dx} = \frac{q}{\epsilon} Nd \quad (0 < x < x_{no}) \\ \frac{d\varepsilon(x)}{dx} = -\frac{q}{\epsilon} Na \quad (-x_{po} < x < 0) \end{cases}$$

$$\boxed{\frac{d^2 V(x)}{dx^2} = -\frac{\rho(x)}{\epsilon} = -\frac{d\varepsilon(x)}{dx}}$$

V(x) : x 대한 전위	[V]
ρ(x) : x 대한 전하밀도	[C/㎤]
ϵ : 유전율	[F/cm]
ε(x) : x 대한 전계강도	[V/cm]

$$V_0 = \frac{kT}{q}\ln\frac{N_A N_D}{n_i^2}.$$

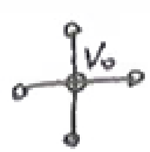
$$I_{tot} = I_S(\exp\frac{V_F}{V_T} - 1),$$

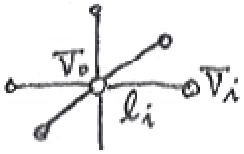


◆ Numerical Simulation : $\boxed{\therefore \nabla^2 V = -\dfrac{\rho}{\varepsilon}}$

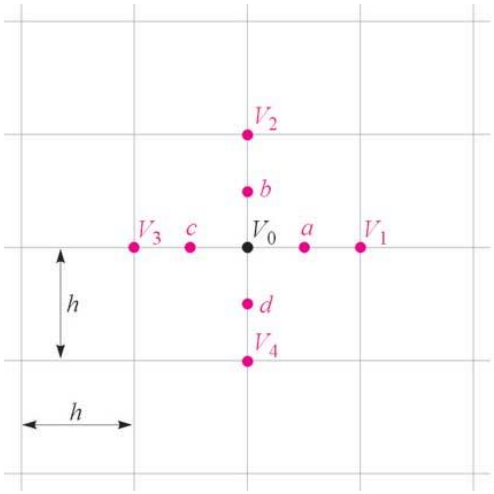
- 실험적 사상법 (Mapping Method)
- 반복 계산법 (Iteration Mehod) – Laplacian : Mean Value Theorem
- 수치해석적 방법

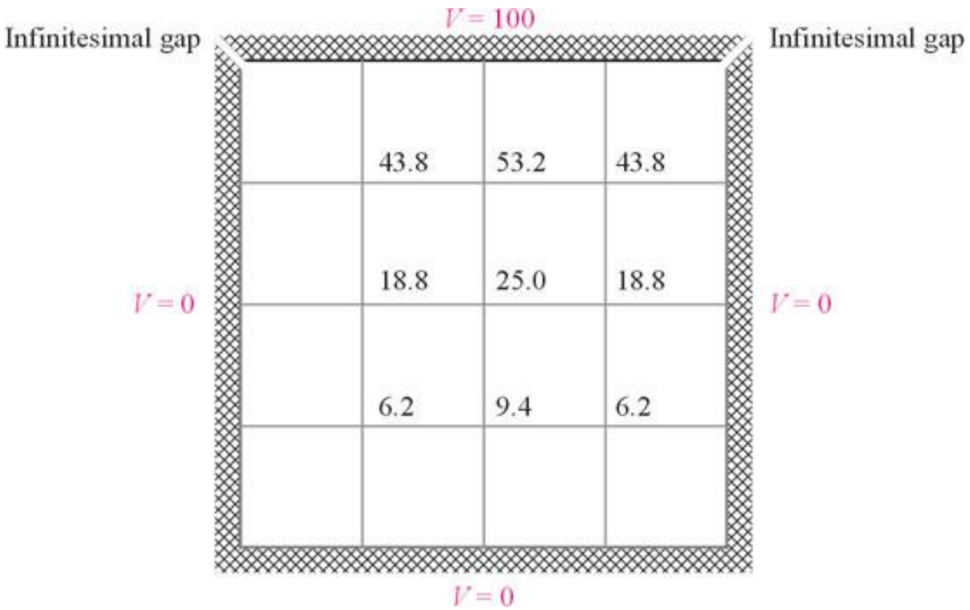
* Mean Value Theorem : $\nabla^2 V = 0$

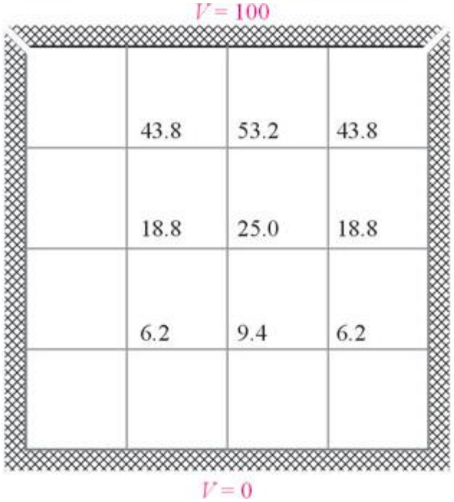




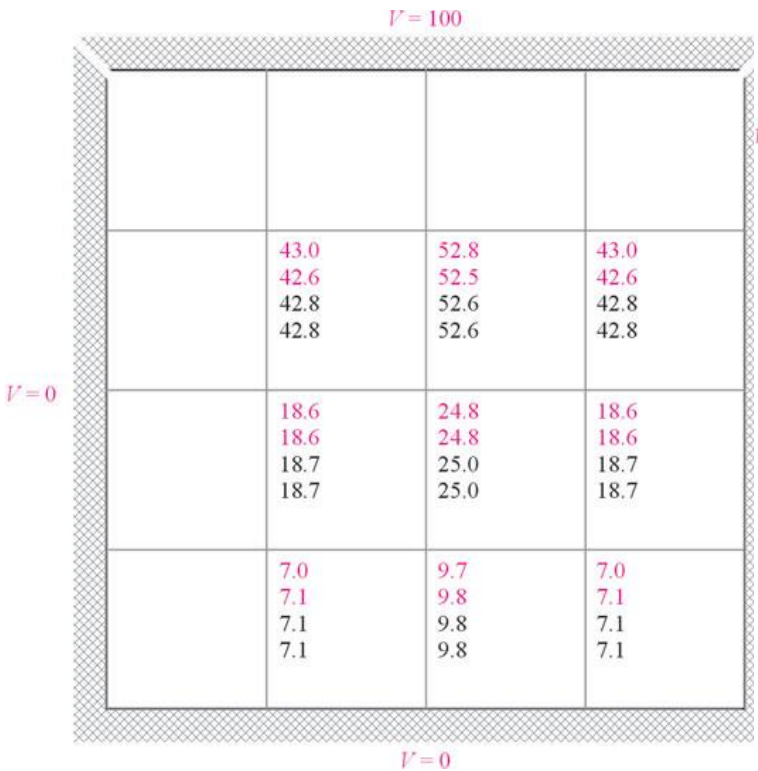
$$V_0 = \frac{\sum_i l_i V_i}{\sum_i l_i}$$



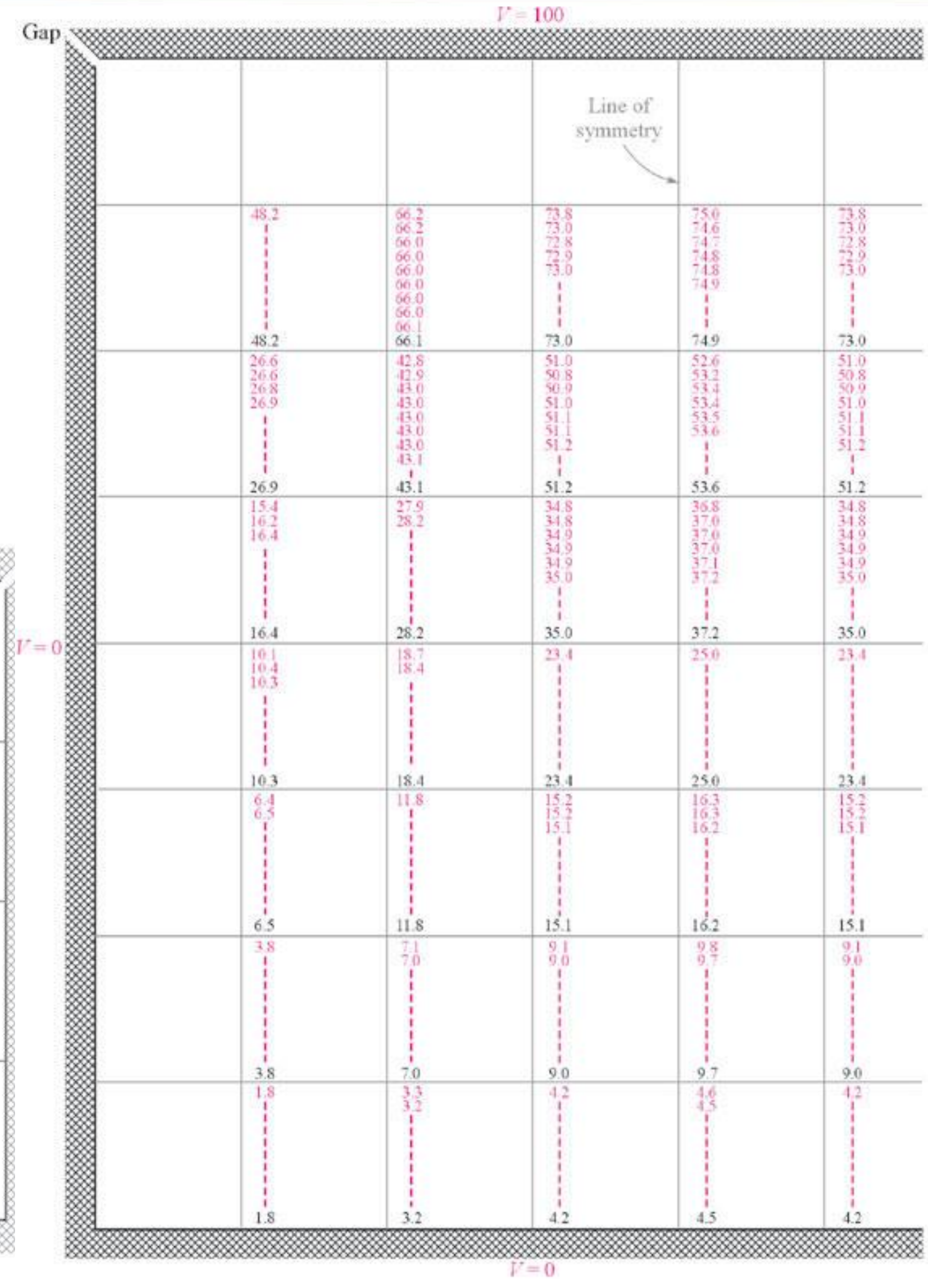




V = 100
43.8
53.2
43.8
18.8
25.0
18.8
6.2
9.4
6.2
V = 0

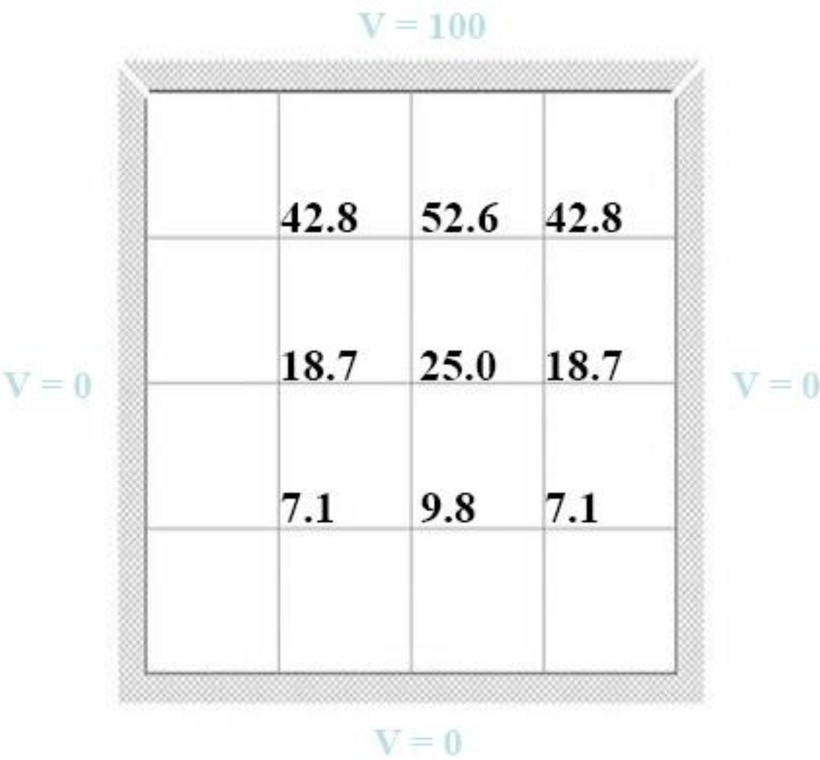


V = 100
V = 0
43.0 42.6 42.8 42.8
52.8 52.5 52.6 52.6
43.0 42.6 42.8 42.8
18.6 18.6 18.7 18.7
24.8 24.8 25.0 25.0
18.6 18.6 18.7 18.7
7.0 7.1 7.1 7.1
9.7 9.8 9.8 9.8
7.0 7.1 7.1 7.1
V = 0

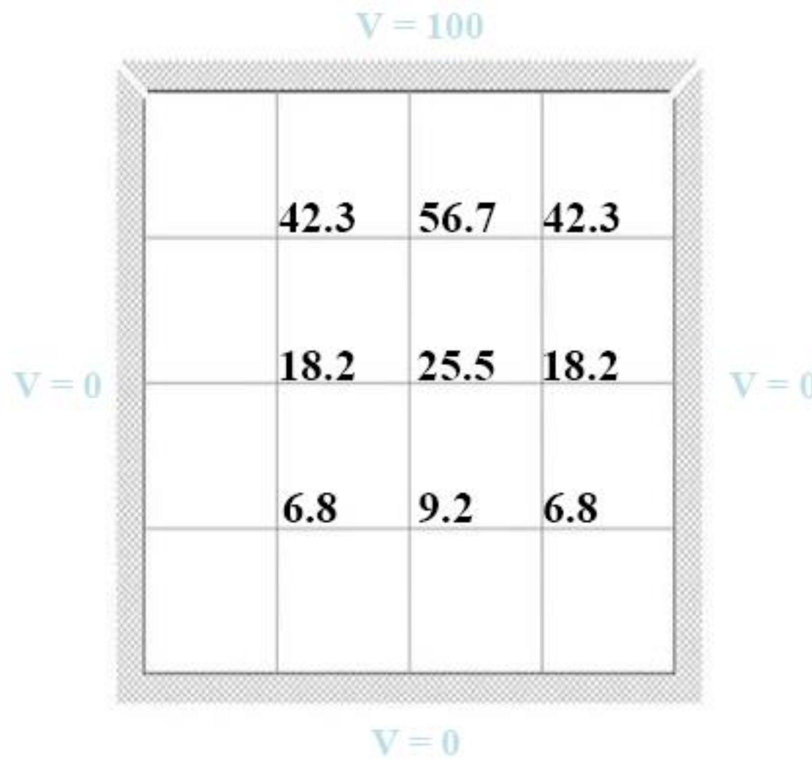


V = 100
Gap
Line of symmetry
V = 0
48.2 66.1 73.0 74.9 73.0
26.9 43.1 51.2 53.6 51.2
16.4 28.2 35.0 37.2 35.0
10.3 18.4 23.4 25.0 23.4
6.5 11.8 15.1 16.2 15.1
3.8 7.0 9.0 9.7 9.0
1.8 3.2 4.2 4.5 4.2
V = 0

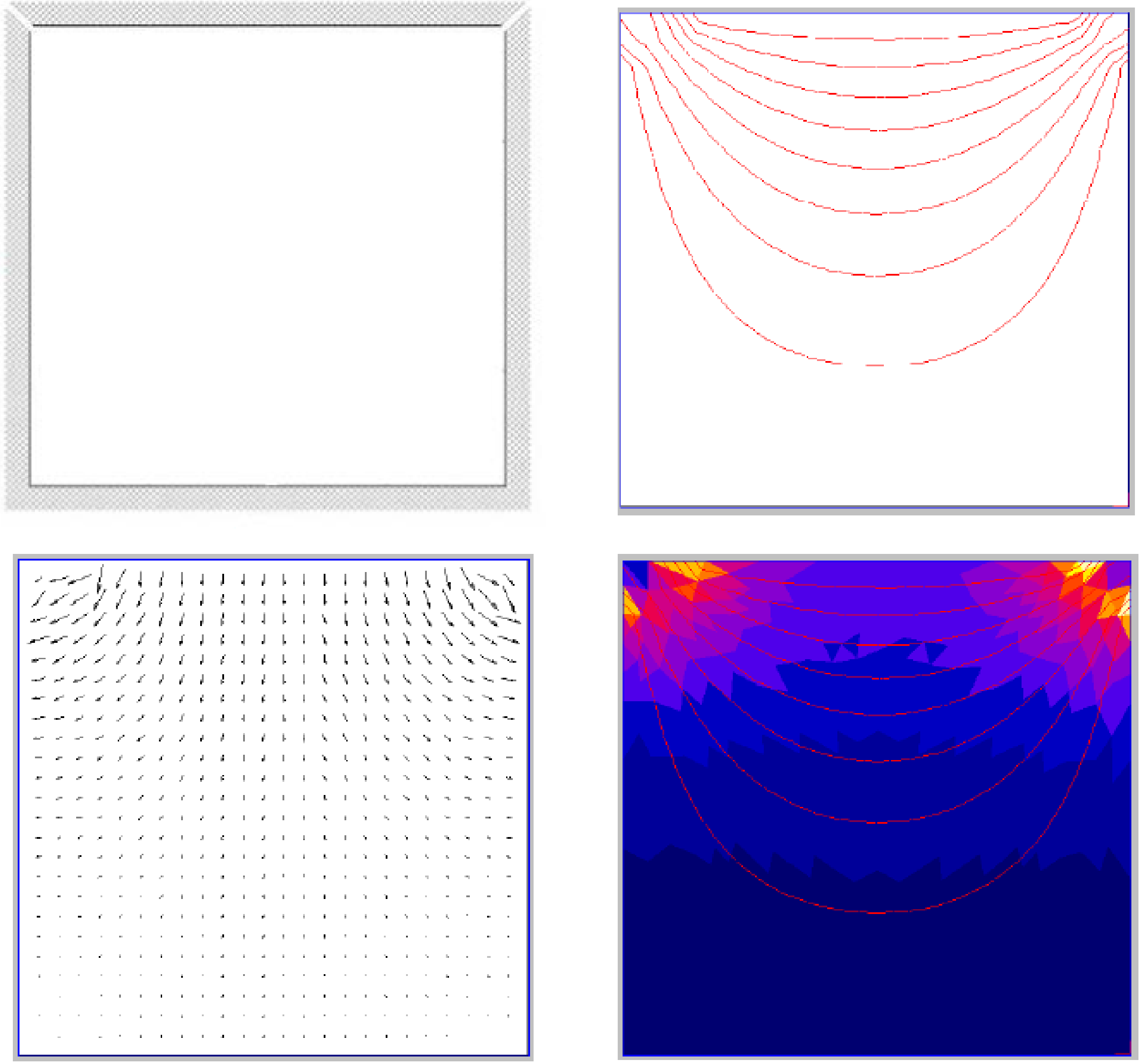
실험적 사상법 (축차법) 과 **FEM** 의 비교



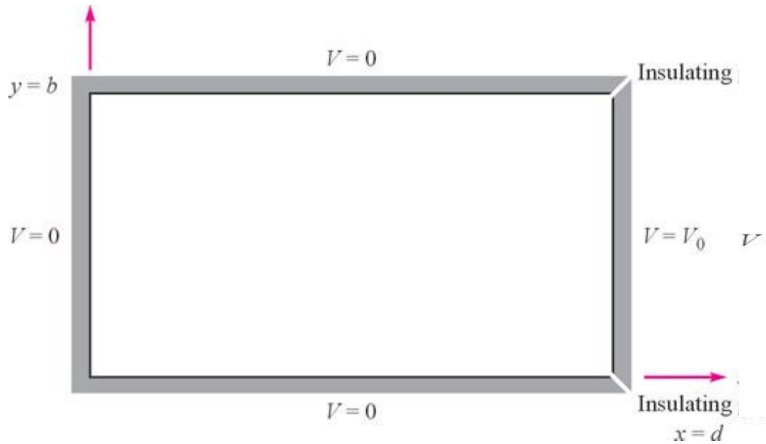
실험적 사상법(축차법), 4번 계산 후

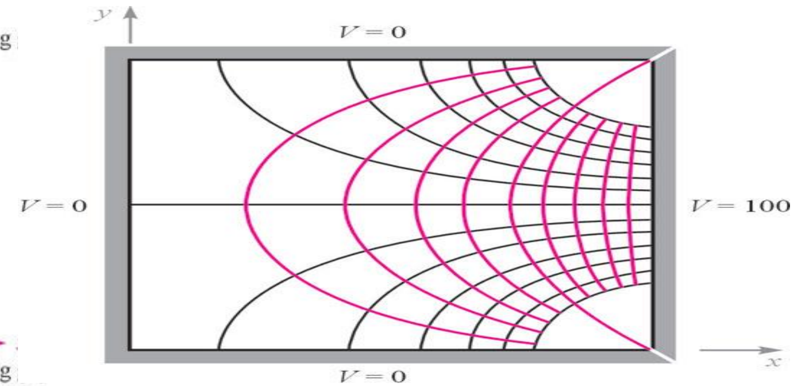


FEM



< Ex >



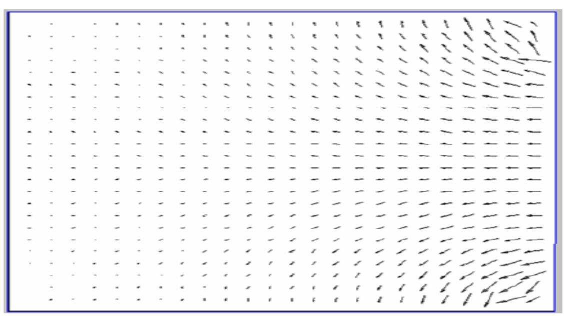


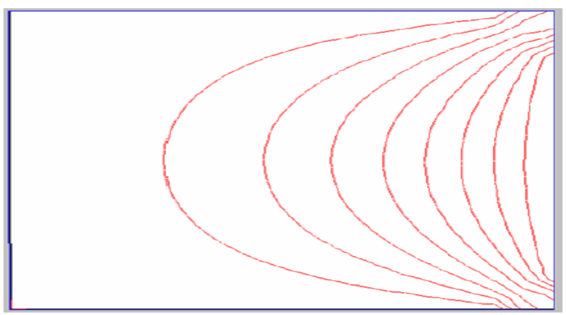
◆ How to solve poisson (or Laplace) Equation? : $V = \frac{4V_0}{\pi}\sum_{m=1,}^{\infty}\frac{1}{m}\cdot\frac{\sinh\frac{m\pi x}{b}}{\sinh\frac{m\pi d}{b}}\cdot\sin\frac{m\pi y}{b}$

$\rho \rightarrow v$ / $V \rightarrow \mathbf{E} \rightarrow \mathbf{F}$

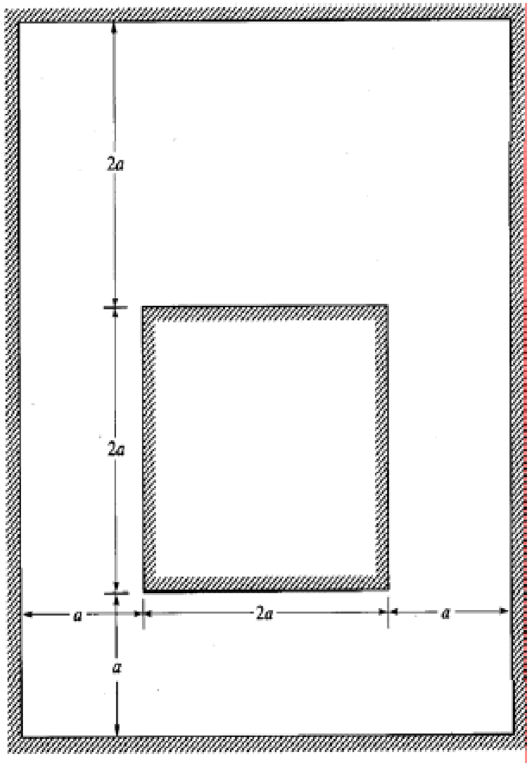
① 수식을 이용 → Bessel 함수, Legendre함수, Fourier급수, 변수분리법

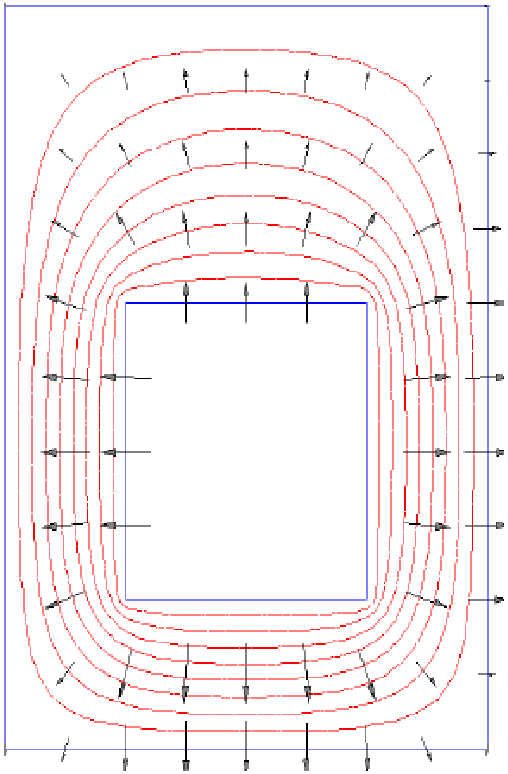
② computer 이용 → FDM , FEM , BEM , MMM

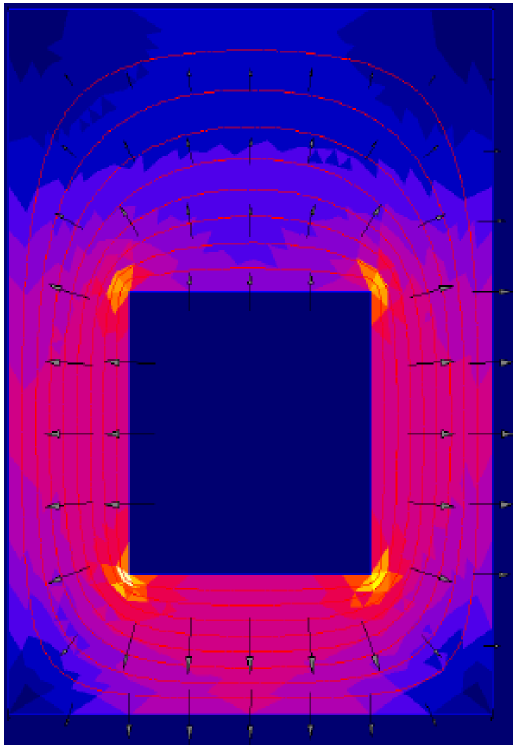




연습문제 6.13 사각도체 전위

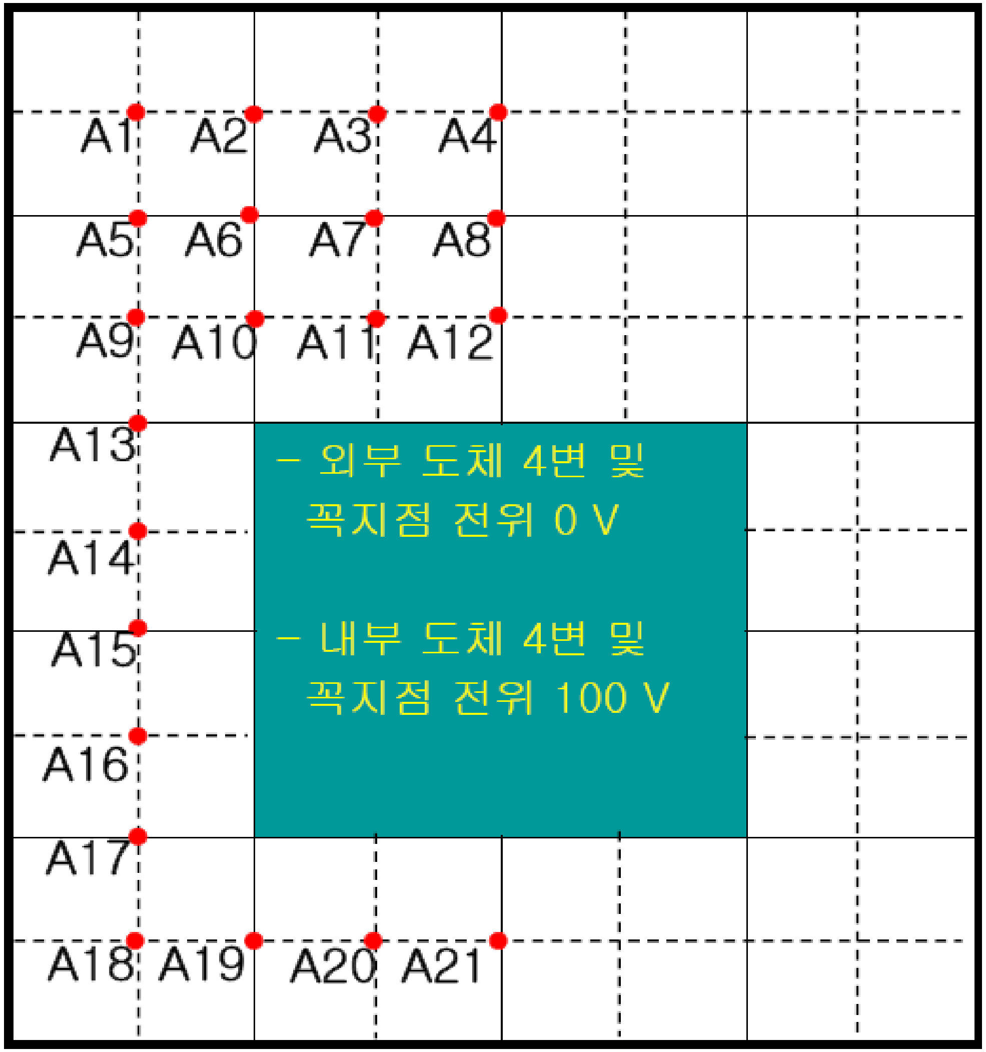






실험적 사상법

A1	8.0770
A2	14.7535
A3	18.6088
A4	19.8217
A5	17.5545
A6	32.3280
A7	39.8602
A8	42.0692
A9	29.8131
A10	57.1440
A11	66.4347
A12	68.7346
A13	44.5539
A14	48.4026
A15	49.0563
A16	47.8226
A17	42.2342
A18	21.1143
A19	42.2231
A20	47.7780
A21	48.8890

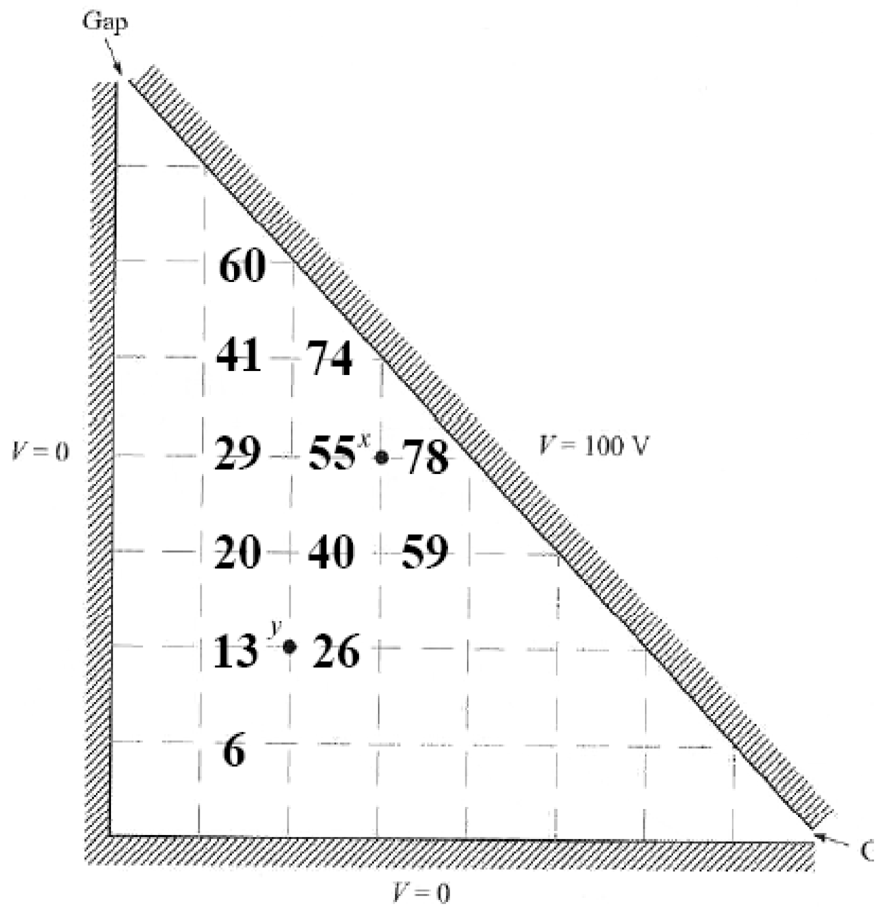


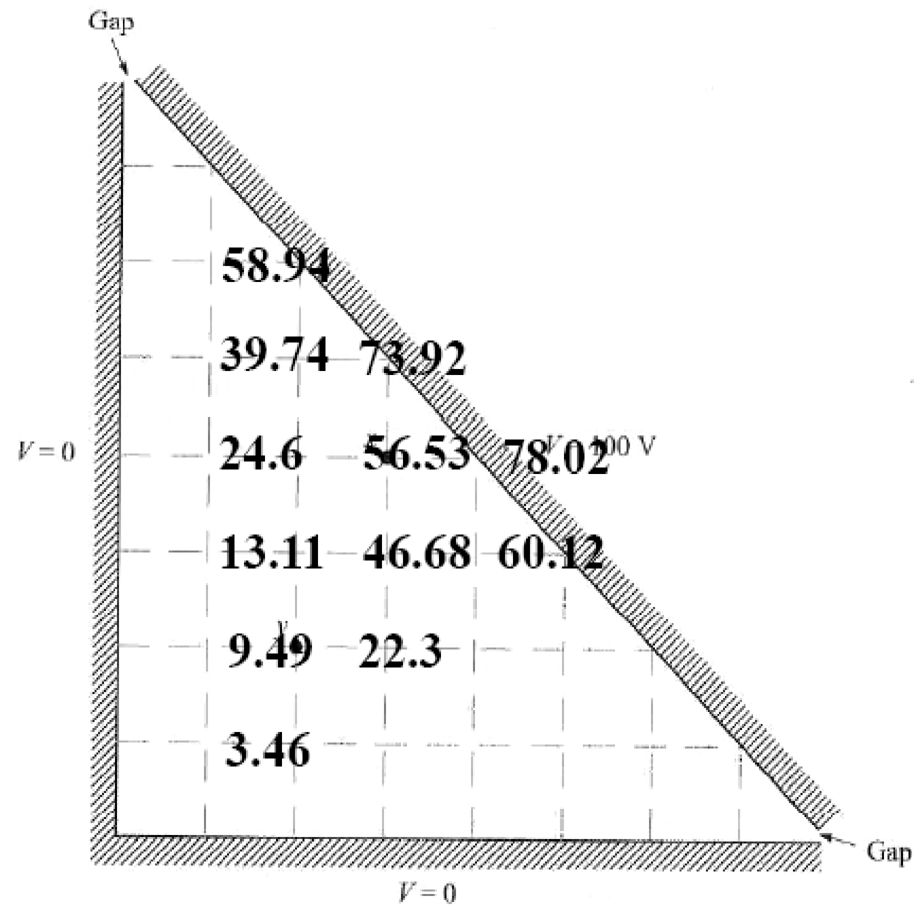
F.E.M

A1	8.21
A2	14.5
A3	18.2
A4	19.88
A5	18.52
A6	31.32
A7	40.01
A8	41.063
A9	28.99
A10	58.05
A11	66.22
A12	68.46
A13	44.58
A14	47.7
A15	48.56
A16	47.44
A17	42.058
A18	21.054
A19	42.111
A20	47.521
A21	48.687

Lab. of Applied ElectroMagnetics

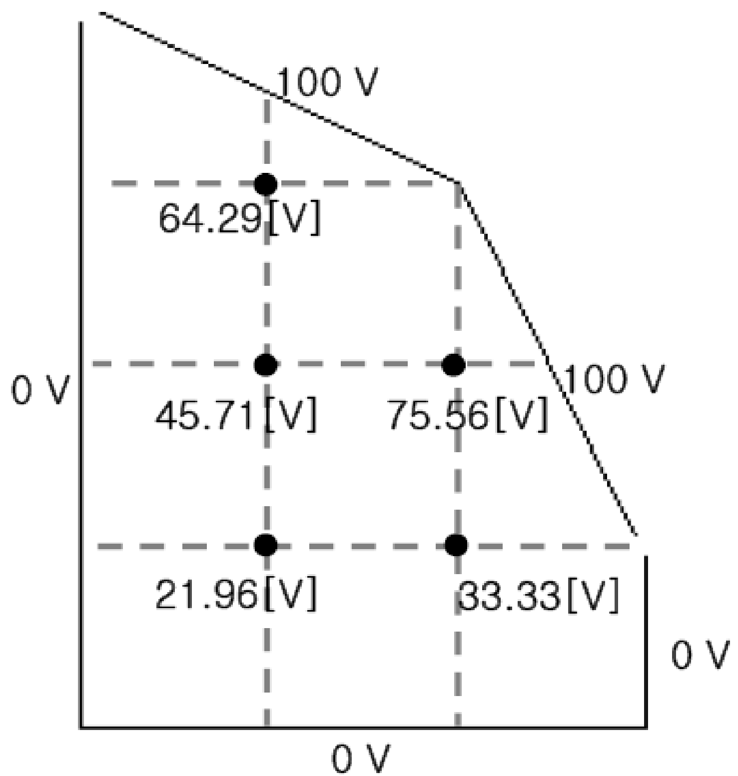
(7th, Edition) 연습문제 7.30, 삼각구조의 전위



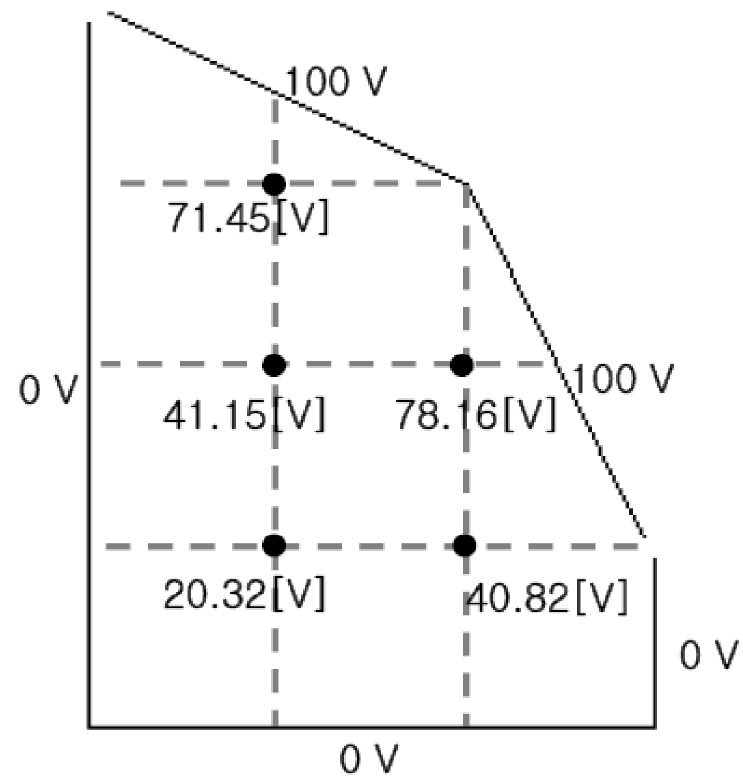


(7th, Edition) 연습문제 7.34, 각형구조의 전위

$$V_0 = \frac{V_1}{\left(1+\frac{h_1}{h_3}\right)\left(1+\frac{h_1 h_3}{h_4 h_2}\right)} + \frac{V_2}{\left(1+\frac{h_2}{h_4}\right)\left(1+\frac{h_2 h_4}{h_1 h_3}\right)} + \frac{V_3}{\left(1+\frac{h_3}{h_1}\right)\left(1+\frac{h_3 h_1}{h_2 h_4}\right)} + \frac{V_4}{\left(1+\frac{h_4}{h_2}\right)\left(1+\frac{h_4 h_2}{h_3 h_1}\right)}$$



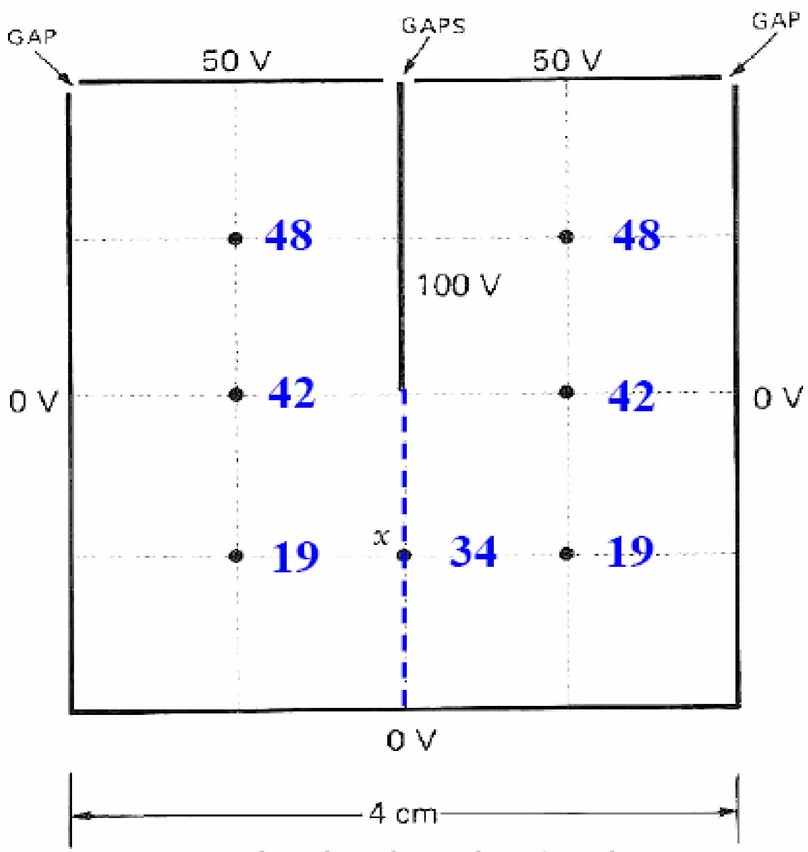
실험적 사상법에 의한 해석



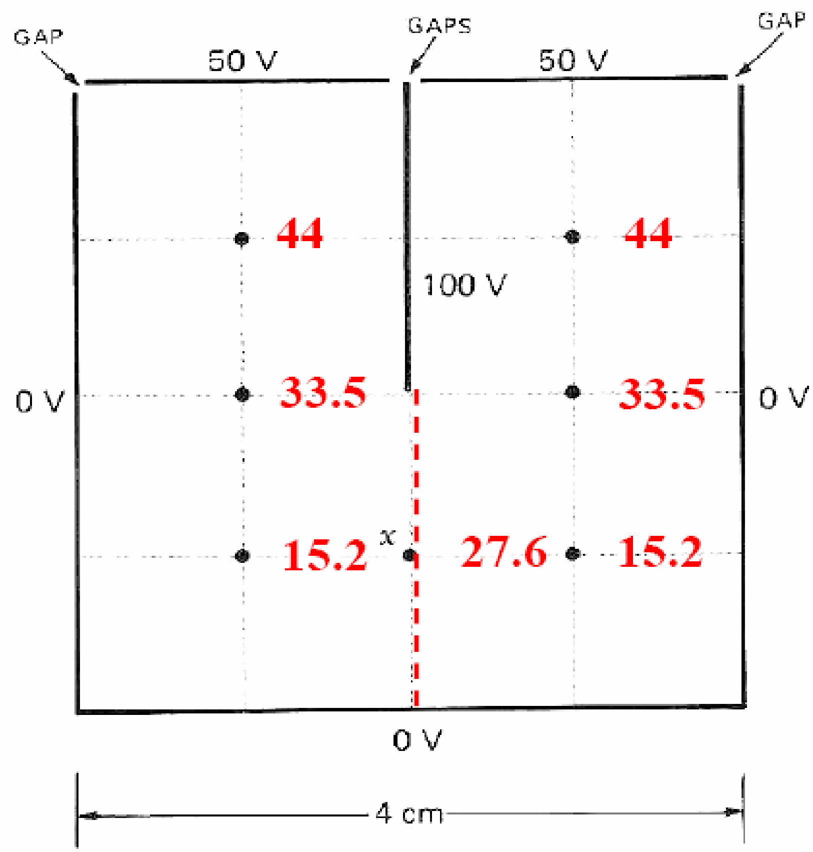
F.E.M. 해석

Lab. of Applied ElectroMagnetics

(7th, Edition) 연습문제 7.35, 침:평판 구조의 전위



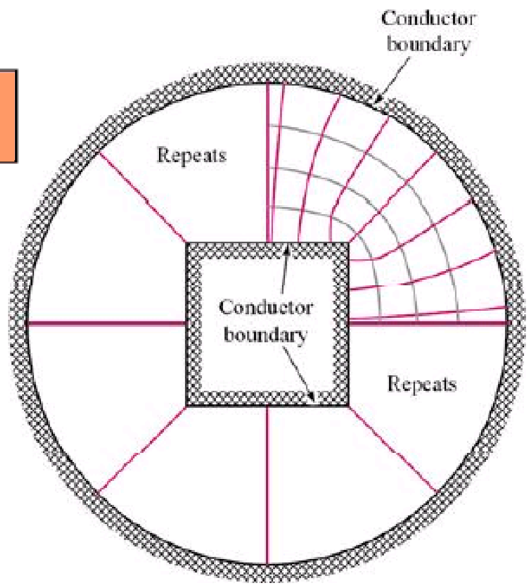
실험적 사상법

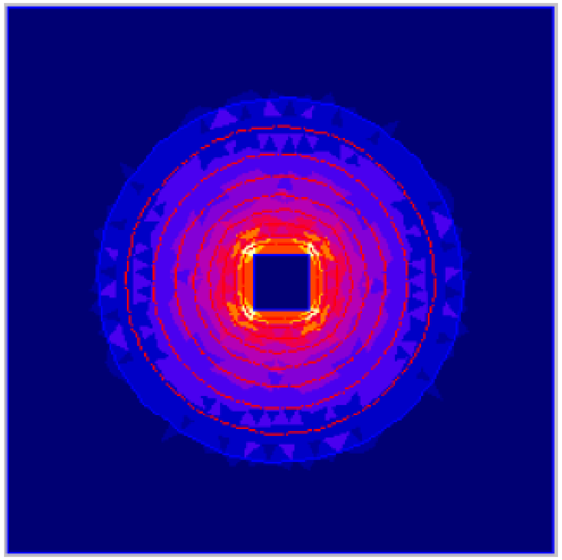


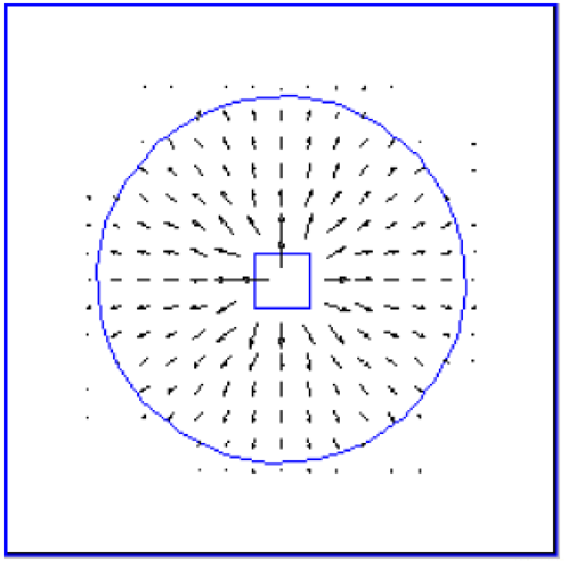
F.E.M

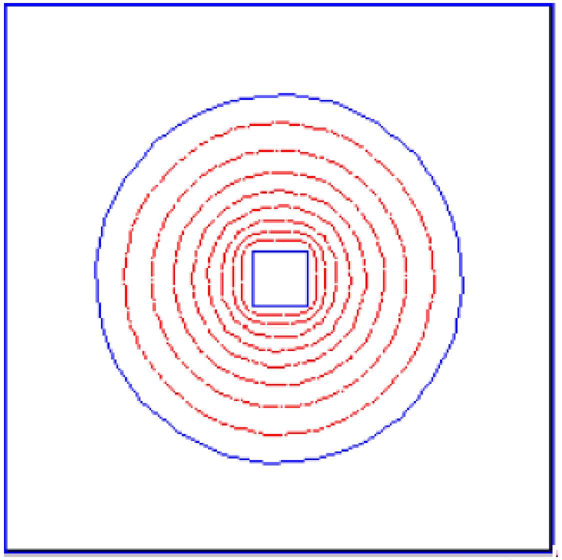
Lab. of Applied ElectroMagnetics

본문 그림 6.8

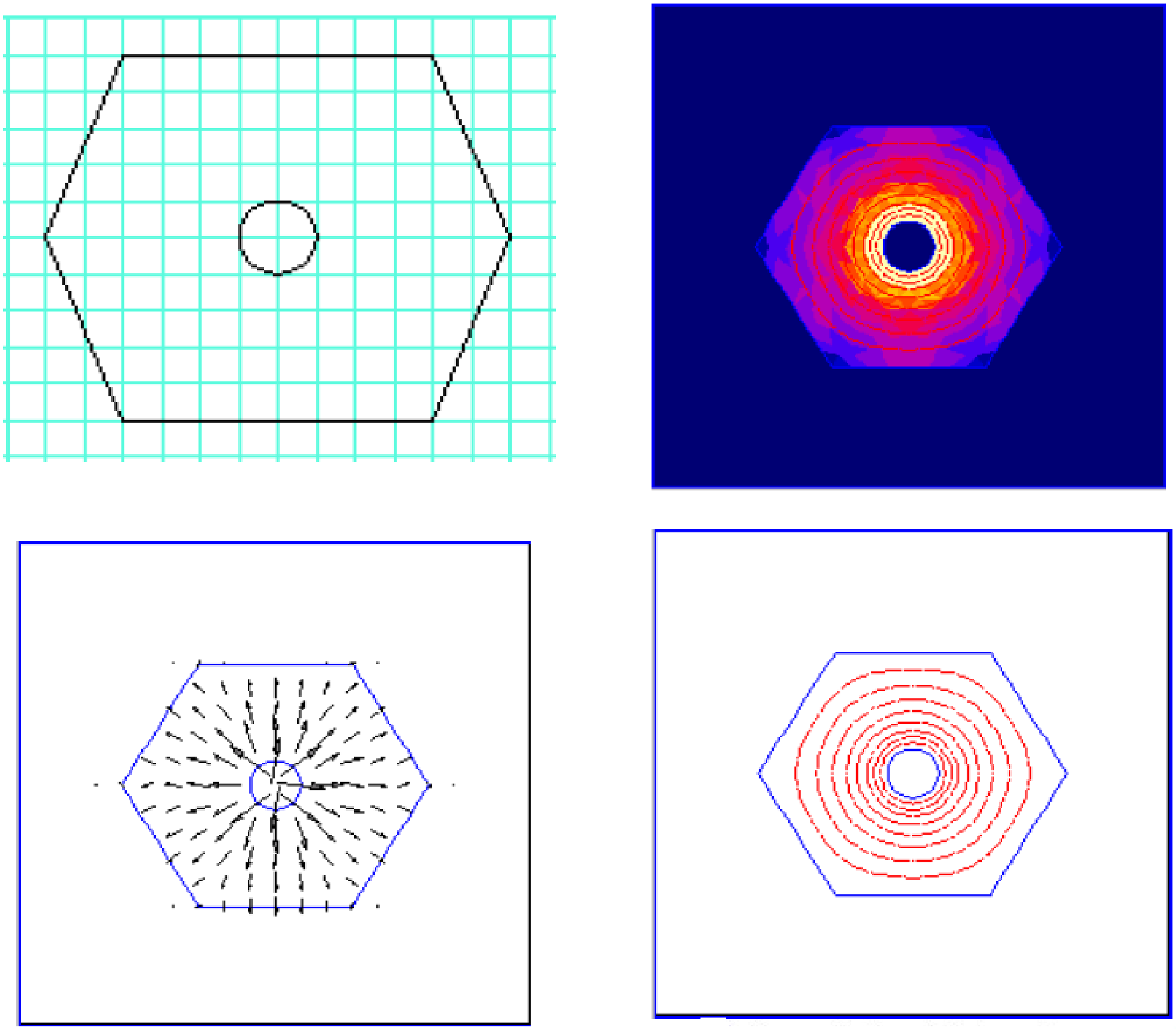




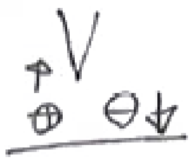




전기집진기



< Ex > 코로나 방전 (Corona   Discharge)



- a 가 ionize
- corona discharge
- 소음 , 빛

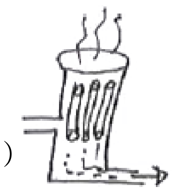
- \+ corona : 평탄균일
- − corona : 불균일, 솔가지모양, Tree구조, 소음발생

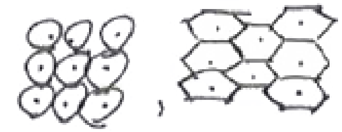
▪ 정전 침전기, 매연흡착용

• 1820년 독일

- 1906년 Berkley 대학
- F. Cottrol 제작중

▪ 포항제철 매연흡착기 (올리버트위스트)



()

(ex)

- 도선 반지름 : $r_1 = 1mm$
- 관의 반지름 : $r_2 = 200mm$
- 공기항복전압 : $3\times10^6[V/m]$ $= 3만[V/m]$

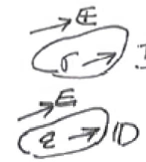
$$: V = E_r \cdot r \cdot \ln\frac{r_2}{r_1}$$: 내 도체표면 $r = r_1 = 1mm$, $E_r = 3\times10^6[V/m]$

$\rightarrow V = 15.9kV$, 만 5000 volt 이상에서 발생함

Lab. of Applied ElectroMagnetics

➢ **R & C :**

- $\begin{cases} \text{도 체 :} & \mathbf{D} = \varepsilon \mathbf{E}_\varepsilon \quad \mathbf{E}_\sigma = -\nabla V_\sigma \\ \text{유전체 :} & \mathbf{J} = \sigma \mathbf{E}_\sigma \quad \mathbf{E}_\varepsilon = -\nabla V_\varepsilon \end{cases}$



- $$\left(\begin{array}{ll} R = \dfrac{V_\sigma}{I} = \dfrac{-\int \mathbf{E} \cdot d\mathbf{L}}{\sigma \oint \mathbf{E}_\sigma \cdot d\mathbf{S}} & \left(\because I = \int \mathbf{J} \cdot d\mathbf{S} = \sigma \int \mathbf{E}_\sigma \cdot d\mathbf{S} \right) \\ C = \dfrac{Q}{V_e} = \dfrac{\varepsilon \oint_S \mathbf{E}_\varepsilon \cdot d\mathbf{S}}{-\int \mathbf{E} \cdot d\mathbf{L}} & \left(\because Q = \int \mathbf{D} \cdot d\mathbf{S} = \varepsilon \int \mathbf{E}_\varepsilon \cdot d\mathbf{S} \right) \end{array} \right) \rightarrow \boxed{R \cdot C = \frac{\varepsilon}{\sigma}} \quad \left(\begin{array}{l} C = \dfrac{\varepsilon}{\sigma} \cdot \dfrac{1}{R} \\ R = \dfrac{\varepsilon}{\sigma} \cdot \dfrac{1}{C} \end{array} \right)$$

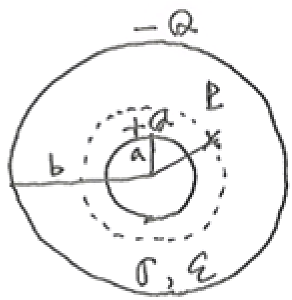
ε, σ : known , R측정 → C 계산

< EX > 동심 도체구

$$\left(\begin{array}{l} C = 4\pi\varepsilon \cdot \dfrac{ab}{b-a} \\ R = \dfrac{1}{4\pi\sigma} \cdot \dfrac{b-a}{ab} \end{array} \right)$$

$$\boxed{RC = \frac{\varepsilon}{\sigma}}$$

$$C = \frac{\varepsilon}{\sigma} \cdot \frac{1}{R}$$



$(a)\ E = \dfrac{Q}{4\pi\varepsilon r^2}\ a_r , \qquad V = \dfrac{Q}{4\pi\varepsilon r}$

$(b)\ V_{ab} = -\int_a^b E \cdot dr = \dfrac{Q}{4\pi\varepsilon} \int_a^b -\dfrac{dr}{r^2} = \dfrac{Q}{4\pi\varepsilon}\left(\dfrac{1}{a} - \dfrac{1}{b}\right) = \dfrac{Q}{4\pi\varepsilon} \cdot \dfrac{b-a}{ab}$

$(c)\ C = \dfrac{Q}{V} = \dfrac{\cancel{Q}}{\dfrac{\cancel{Q}}{4\pi\varepsilon} \cdot \dfrac{b-a}{ab}} = 4\pi\varepsilon \cdot \dfrac{ab}{b-a}$

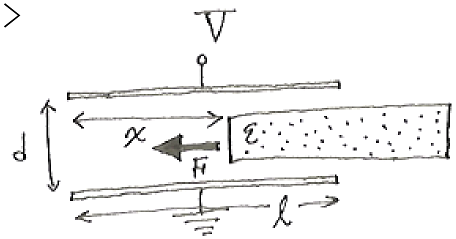
$(d)\ I_{ab} = \int J \cdot ds = \int \sigma E \cdot ds = \dfrac{\sigma Q}{4\pi\varepsilon r^2} \cdot 4\pi r^2 = \dfrac{\sigma Q}{\varepsilon}$

$(e)\ R_{ab} = \dfrac{V}{I} = \dfrac{-\int_a^b E \cdot dL}{\int_s \sigma E \cdot dS} = \dfrac{\dfrac{Q}{4\pi\varepsilon} \cdot \dfrac{b-a}{ab}}{\dfrac{\sigma Q}{\varepsilon}} = \dfrac{1}{4\pi\sigma} \cdot \dfrac{b-a}{ab}$

$(f)\ R \cdot C = \left(\dfrac{1}{\cancel{4\pi}\sigma} \cdot \dfrac{\cancel{b-a}}{ab} \right) \cdot \left(\cancel{4\pi}\varepsilon \cdot \dfrac{ab}{\cancel{b-a}} \right) = \dfrac{\varepsilon}{\sigma}$

➢ Electric Force : $\overline{W} = \frac{1}{2} CV^2$, $F = \frac{\partial \overline{w}}{\partial x}$ $\underline{V = Constant}$

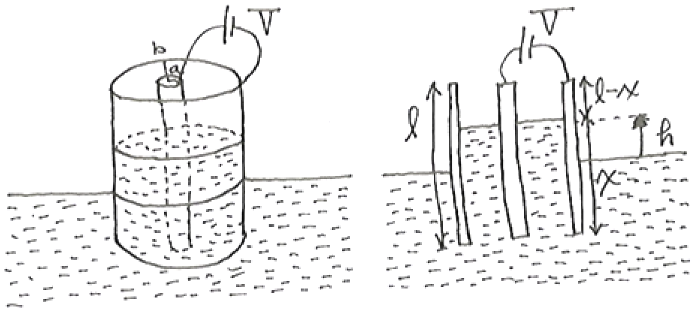
< EX >



(a) C

(b) F

< EX > ρ : 밀도, m : 질량 = 밀도 × 부피 = $\rho \cdot \overline{V}ol$, $F = ma = mG$



(a) C

(b) F

(c) h :