Exam #2: Control Systems Eng.(I)
2020/06/02

Student Number: [ ] Name: **Solution**

1. Characteristic equation: $1+G(s)H(s)=0$

$$1+\left(K_p+\frac{K_i}{s}\right)\left(\frac{1}{(s+2)(s+10)}\right)=0$$

$$1+\frac{K_p s+K_i}{s(s+2)(s+10)}=0,\quad s(s+2)(s+10)+K_p s+K_i=0$$

$$s^3+12s^2+(K_p+20)s+K_i=0$$

$\rightarrow$ Applying Routh-Hurwitz criterion $\rightarrow$

$$\begin{array}{c|cc} s^3 & 1 & (K_p+20) \\ s^2 & 12 & K_i \\ s^1 & \dfrac{12(K_p+20)-K_i}{12} & 0 \\ s^0 & K_i & \end{array}$$

$$\frac{12(K_p+20)-K_i}{12}>0 \text{ and } K_i>0 \quad\rightarrow\quad K_i>0 \text{ and } K_p>\frac{K_i}{12}-20$$

2.

(1) (10pts) $\dfrac{C(s)}{R(s)}=\dfrac{G(s)}{1+G(s)}$

$$E(s)=R(s)-C(s)=R(s)-\frac{G(s)}{1+G(s)}R(s)=\frac{1}{1+G(s)}R(s)$$

$$E(s)=\frac{1}{1+\dfrac{K}{s^2+3s+2}}\left(\frac{1}{s}\right)=\left(\frac{1}{s}\right)\cdot\frac{s^2+3s+2}{s^2+3s+2+K}$$

$$e_{ss}=\lim_{t\to\infty}e(t)=1-0.75=0.25$$

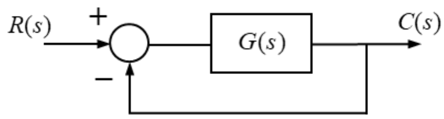
```
clc, clear all
numg = [6]; deng = [1 3 2];
sys1=tf(numg, deng)
sys=feedback(sys1,1)
step(sys)
set(gca,'ytick',0:0.05:1)
title('Unit Step Response')
```

(2) (10pts) $e_{ss}=\lim_{s\to 0}s\cdot E(s)=\lim_{s\to 0}s\cdot\left(\dfrac{1}{s}\right)\dfrac{s^2+3s+2}{s^2+3s+2+K}=\dfrac{2}{2+K}$

$$\frac{2}{2+K}=\frac{1}{4}$$

$\rightarrow K=6$

3.



(i) (5 points) Find K:

$$G(s)=\frac{\dfrac{16}{s(s+4)}}{1+(Ks)\dfrac{16}{s(s+4)}}=\frac{\dfrac{16}{s(s+4)}}{\dfrac{s^2+4s+16Ks}{s(s+4)}}=\frac{16}{s^2+4s+16Ks}$$

$$\frac{C(s)}{R(s)}=\frac{G(s)}{1+G(s)}=\frac{\dfrac{16}{s^2+4s+16Ks}}{1+\dfrac{16}{s^2+4s+16Ks}}=\frac{16}{s^2+(16K+4)s+16}=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$\omega_n^2=16\rightarrow\omega_n=4 \text{ and } \zeta=0.8$$

$$2\zeta\omega_n=16K+4\rightarrow 2(0.8)(4)=16K+4$$

$$K=0.15$$

$$\frac{C(s)}{R(s)}=\frac{\dfrac{16}{s(s+4)}}{1+(1+Ks)\dfrac{16}{s(s+4)}}=\frac{\dfrac{16}{s(s+4)}}{\dfrac{s^2+4s+16(Ks+1)}{s(s+4)}}=\frac{16}{s^2+4s+16(Ks+1)}=\frac{16}{s^2+(16K+4)s+16}=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

(ii) (15 points) Find steady state error:

$$E(s)=R(s)-C(s)=R(s)-E(s)G(s),\quad E(s)=\frac{R(s)}{1+G(s)}$$

$$e_{ss}=\lim_{s\to 0}s\cdot E(s)=\lim_{s\to 0}\frac{s\cdot\left(\dfrac{1}{s^2}\right)}{1+\dfrac{16}{s^2+4s+16Ks}}$$

$$=\lim_{s\to 0}\frac{1}{s+\dfrac{16s}{s^2+(16K+4)s}}=\frac{4+16K}{16}=\frac{1+4(0.15)}{4}=\frac{1.6}{4}=0.4$$

$$\rightarrow e_{ss}=0.4$$

4.

$$Y(s)=\left(\frac{\lambda}{s+\lambda}\right)\left(\frac{1}{s-1}\right)\left(-3Y(s)+\frac{1}{s}\big(R(s)-Y(s)\big)+W(s)\right)$$

$$s(s+\lambda)(s-1)Y(s)=\lambda(-3sY(s)+R(s)-Y(s)+sW(s))$$

$$\underbrace{(s^3+(\lambda-1)s^2+2\lambda s+\lambda)}_{\text{characteristic polynomial}}Y(s)=\lambda(R(s)+sW(s))$$

$\rightarrow$ Applying Routh-Hurwitz criterion $\rightarrow$

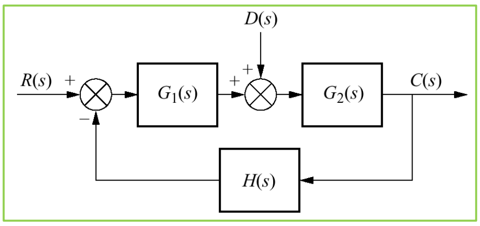
$$\begin{array}{c|cc} s^3 & 1 & 2\lambda \\ s^2 & (\lambda-1) & \lambda \\ s^1 & \dfrac{2\lambda(\lambda-1)-\lambda}{\lambda-1} & 0 \\ s^0 & \lambda & \end{array}$$

$$(\lambda-1)>0,\quad \lambda>0,\quad \frac{2\lambda(\lambda-1)-\lambda}{\lambda-1}>0$$

$$\frac{2\lambda(\lambda-1)-\lambda}{\lambda-1}>0\rightarrow\frac{\lambda(2\lambda-3)}{\lambda-1}>0\rightarrow\lambda>\frac{3}{2}$$

5

(1) Find steady-state error $E(s) = R - C$, and $e(\infty) = \lim_{s \to 0} sE(s)$



input of G_1: $R - CH$

input of G_2 : $G_1E + D = G_1(R - CH) + D$

output: $C = G_2\left[G_1(R - CH) + D\right] \cdots (a)$

steady-state error $E = R - C,\ C = R - E \ \cdots (b)$

$(b) \to (a)$;

$$R - \underline{E} = G_1G_2R - G_1G_2(CH) + G_2D = G_1G_2R - G_1G_2(R - E)H + G_2D = G_1G_2R - G_1G_2RH + \underline{G_1G_2EH} + G_2D$$

arrange for E,

$$E(1 + G_1G_2H) = R - G_1G_2R + G_1G_2RH - G_2D$$

$$\therefore E = \frac{1 - G_1G_2 + G_1G_2H}{1 + G_1G_2H}R - \frac{G_2}{1 + G_1G_2H}D = \left(1 - \frac{G_1G_2}{1 + G_1G_2H}\right)R - \left(\frac{G_2}{1 + G_1G_2H}\right)D$$

$$e(\infty) = \lim_{s \to 0} sE(s) = 1 - \lim_{s \to 0}\left(\frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H(s)}\right) - \lim_{s \to 0}\left(\frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)}\right)$$

$$= 1 - \lim_{s \to 0}\left(\frac{\dfrac{K_1K_2}{s+2}}{1 + \dfrac{K_1K_2(s+1)}{s+2}}\right) - \lim_{s \to 0}\left(\frac{\dfrac{K_2}{s+2}}{1 + \dfrac{K_1K_2(s+1)}{s+2}}\right) = 1 - \frac{\dfrac{K_1K_2}{2}}{1 + \dfrac{K_1K_2}{2}} - \frac{\dfrac{K_2}{2}}{1 + \dfrac{K_1K_2}{2}} = \frac{2 - K_2}{2 + K_1K_2}$$

(2) $$S_{e:K_1} = \frac{K_1}{e}\frac{\delta e}{\delta K_1} = \frac{K_1}{\dfrac{2 - K_2}{2 + K_1K_2}}\left(\frac{-(2 - K_2)K_2}{(2 + K_1K_2)^2}\right) = \frac{-K_1K_2}{2 + K_1K_2} = \frac{-(10)(0.1)}{2 + (10)(0.1)} = -\frac{1}{3} = -0.33$$

(3) $$S_{e:K_2} = \frac{K_2}{e}\frac{\delta e}{\delta K_2} = \frac{K_2}{\dfrac{2 - K_2}{2 + K_1K_2}}\left(\frac{-(2 + K_1K_2) - (2 - K_2)K_1}{(2 + K_1K_2)^2}\right) = \frac{2(K_1 + 1)K_2}{(K_2 - 2)(K_1K_2 + 2)} = \frac{2(11)(0.1)}{-1.9(3)} = -\frac{2.2}{5.7} = -0.39$$