Chap 7. 정상자계

- $\left[\begin{array}{l}\text{전기의 역사 (마찰전기, Electra, 라이덴 병)} \\ \text{자기의 역사 (나침반, Coulomb, Oersted)}\end{array}\right]$ + Oersted, Faraday, Maxwell

- Maxwell Eq : $\left[\begin{array}{llll}\nabla \cdot \vec{D} = \rho & \text{: Gauss' s Law} & \nabla \cdot \vec{B} = 0 & \\ \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} & \text{: Faraday' s Law} & \nabla \times \vec{H} = \vec{J} + \dfrac{\partial \vec{D}}{\partial t} & \text{: Ampere's Law}\end{array}\right]$

- Material : $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$ $\quad \vec{P} = \varepsilon_0 \chi_e \vec{E}$ $\quad \vec{D} = \varepsilon \vec{E}$

  $\vec{B} = \mu_0(\vec{H} + \vec{M})$ $\quad \vec{M} = \chi \vec{H}$ $\quad \vec{B} = \mu \vec{H}$

- Q? / I? : 정지한 전하 $\longrightarrow$ $\vec{E}$ $\quad$ * 전하가 움직이면 → 전류

  일정한 전류 $\longrightarrow$ $\vec{H}$ $\quad$ * 움직임이란 ? : Relative Motion

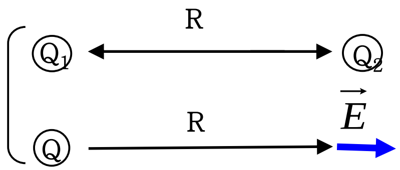
- 1905, Einstein – Relativistic Theory → " On the Electrodynamics of Moving Bodies "

- Classical Electromagnetics $\left[\begin{array}{l} \xrightarrow{\text{High Speed}} \text{Relativistic Theory} \\ \xrightarrow{\text{Small Size}} \text{Quantum Theory}\end{array}\right.$

Lab. of Applied ElectroMagnetics

7.1 Biot– Savart's Law

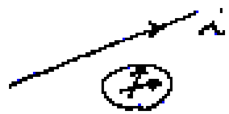
• 1784, Coulomb :



$F = k\dfrac{Q_1 \cdot Q_2}{R^2}$: Coulomb Force ($F = G\cdot\dfrac{m_1 m_2}{R^2}$: Newton Force)

$\vec{E} = k\cdot\dfrac{Q}{R^2}\hat{a}_r$: Electric Field Intensity

• 1819, Oersted : Hans Christian Oersted



(전기)

(자기)

• 1820, Ampere : Andre – Marie Ampere : 편미분 방정식으로 정리

Charge Q [C, Coulomb] ⟶ Electric Field Intensity $\vec{E}$ [C/m] , [N/C]

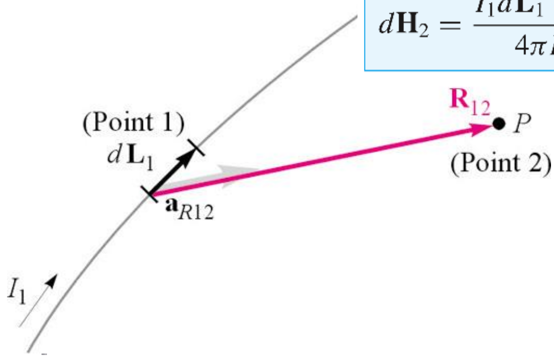
Current I [A, Ampere] ⟶ Magnetic Field Intensity $\vec{H}$ [A/m] , [Oe]

• Biot – Savart 실험식 : 무한히 작은 전류소에 의하여 발생하는 자기장

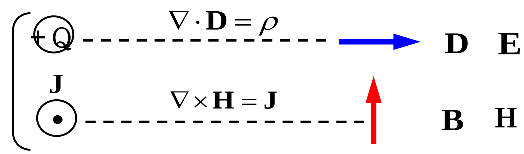
$$d\mathbf{E}_2 = \frac{dQ_1\mathbf{a}_{R12}}{4\pi\epsilon_0 R_{12}^2}$$

Free space

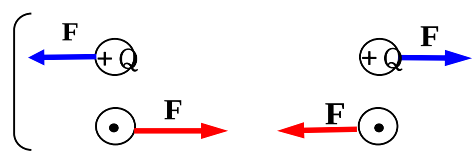
$$\boxed{d\mathbf{H}_2 = \frac{I_1 d\mathbf{L}_1 \times \mathbf{a}_{R12}}{4\pi R_{12}^2}}$$

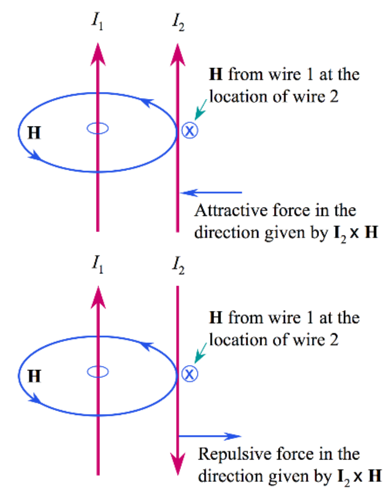


✓ Direction :



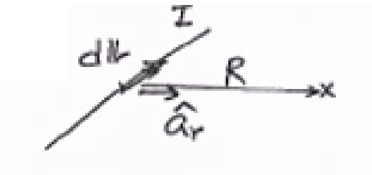
✓ Force :





• *Bio – Savart 식에 의한 자계의 세기*

(1) 선전류의 경우 :

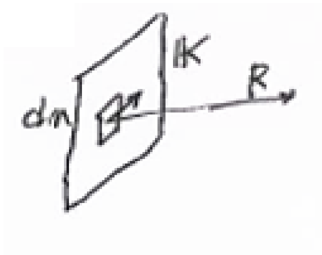


• I : 선전류 , $d\mathbf{L}$: 방향벡터

$$\boxed{\mathbf{H} = \oint \frac{I d\mathbf{L} \times \mathbf{a}_R}{4\pi R^2}}$$

cf : $\mathbf{E} = \int_v \frac{\rho \cdot \hat{a}_r}{4\pi R^2} dv$

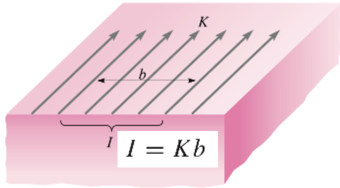
(2) 면전류의 경우 :



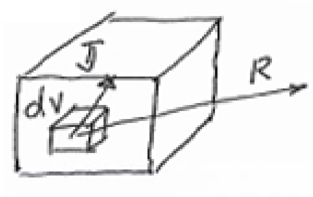
• K : 표면전류밀도 (Surface current density) [A/m]

$Id\mathbf{L} \longrightarrow \mathbf{K}dS$ $I = \int \mathbf{K}dN$

$$\boxed{\mathbf{H} = \int_s \frac{\mathbf{K} \times \mathbf{a}_R dS}{4\pi R^2}}$$



(3) 체적전류의 경우:

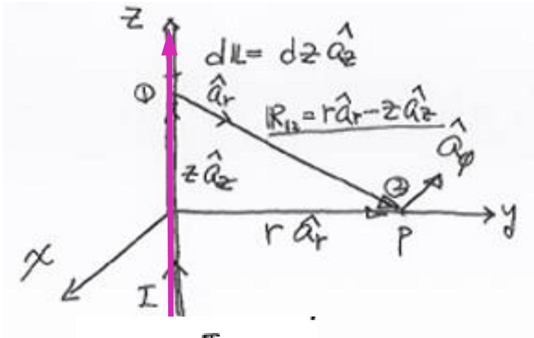


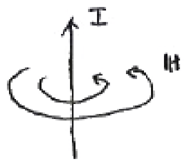
• J : 전류밀도 (Current density) [A/m^2]

$Id\mathbf{L} \longrightarrow \mathbf{J}dv$ $I = \int \mathbf{J}dv$

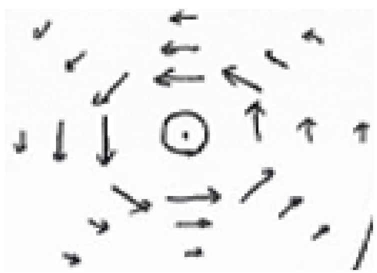
$$\boxed{\mathbf{H} = \int_{\text{vol}} \frac{\mathbf{J} \times \mathbf{a}_R dv}{4\pi R^2}}$$

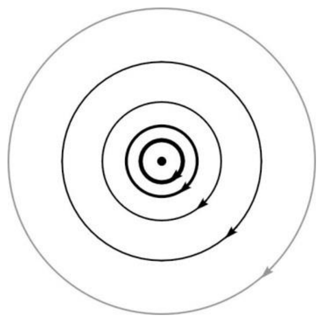
(Ex) 무한 직선전류에 의한 자계의 세기를 Biot – Savart 법칙으로 구하시오.





$$\therefore \mathbf{H}_2 = \frac{I}{2\pi r}\hat{a}_\phi$$





$$\mathbf{R} = \mathbf{r} - \mathbf{r}' = \rho\mathbf{a}_\rho - z'\mathbf{a}_z \qquad \mathbf{a}_R = \frac{\rho\mathbf{a}_\rho - z'\mathbf{a}_z}{\sqrt{\rho^2 + z'^2}}$$

Biot – Savart : $d\mathbf{H} = \dfrac{I\,d\mathbf{L}\times\mathbf{a}_R}{4\pi R^2} = \dfrac{I\,dz'\mathbf{a}_z\times(\rho\mathbf{a}_\rho - z'\mathbf{a}_z)}{4\pi(\rho^2+z'^2)^{3/2}}$

$$\mathbf{H}_2 = \int_{-\infty}^{\infty}\frac{I\,dz'\mathbf{a}_z\times(\rho\mathbf{a}_\rho - z'\mathbf{a}_z)}{4\pi(\rho^2+z'^2)^{3/2}} \qquad \because \begin{bmatrix} \hat{a}_z\times\hat{a}_z = 0 \\ \hat{a}_r\times\hat{a}_z = \hat{a}_\phi \end{bmatrix}$$

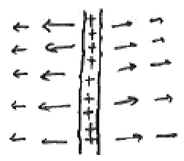
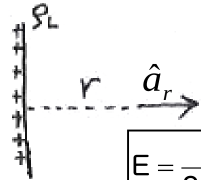
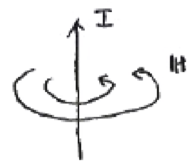
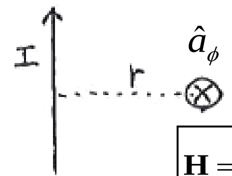
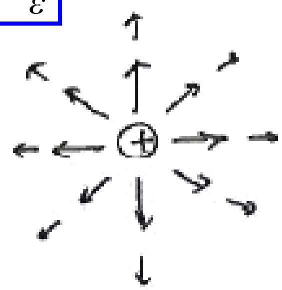
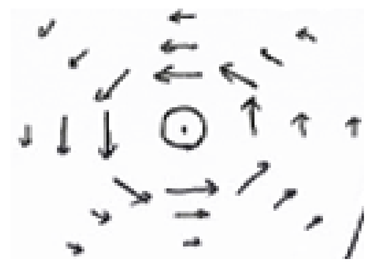
$$= \frac{I}{4\pi}\int_{-\infty}^{\infty}\frac{\rho\,dz'\mathbf{a}_\phi}{(\rho^2+z'^2)^{3/2}}$$

$$= \frac{I\rho\mathbf{a}_\phi}{4\pi}\left.\frac{z'}{\rho^2\sqrt{\rho^2+z'^2}}\right|_{-\infty}^{\infty}$$

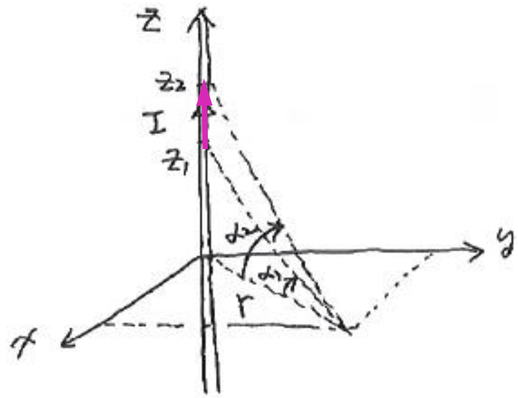
$$\boxed{\mathbf{H}_2 = \frac{I}{2\pi\rho}\mathbf{a}_\phi}$$

$\int \frac{1}{(x^2+a^2)^{\frac{3}{2}}}dx = \frac{x}{a^2\sqrt{x^2+a^2}}$	$\int \frac{1}{\sqrt{(x^2+a^2)}}dx = \ln(x+\sqrt{x^2+a^2})$	$\int \frac{1}{(x^2+a^2)}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$
$\int \frac{x}{(x^2+a^2)^{\frac{3}{2}}}dx = \frac{-1}{\sqrt{x^2+a^2}}$	$\int \frac{x}{\sqrt{(x^2+a^2)}}dx = \sqrt{x^2+a^2}$	$\int \frac{x}{(x^2+a^2)}dx = \frac{1}{2}\ln(x^2+a^2)$

✓ <u>선전하에 의한 전계 / 선전류에 의한 자계 :</u>

전계	자계
 	 
$E = \dfrac{\rho_L}{2\pi\varepsilon_0 r}\hat{a}_r$	$\mathbf{H} = \dfrac{I}{2\pi r}\hat{a}_\phi$
$\boxed{\nabla \bullet E = \dfrac{\rho}{\varepsilon}}$	$\boxed{\nabla \times H = J}$
	
(i) r 이 같은 지역에서 $\lvert E\rvert$ 는 같다. 즉, E 가 같은 점을 이으면 동심원 (ii) $\lvert E\rvert$ 의 크기는 $\frac{1}{r}$ 에 비례 (iii) $\lvert E\rvert$ 의 방향은 $\hat{a}_r$	(i) r 이 같은 지역에서 $\lvert\mathbf{H}\rvert$ 는 같다. 즉, H 가 같은 점을 이으면 동심원 (ii) $\lvert\mathbf{H}\rvert$ 의 크기는 $\frac{1}{r}$ 에 비례 (iii) $\lvert\mathbf{H}\rvert$ 의 방향은 $\hat{a}_\phi$

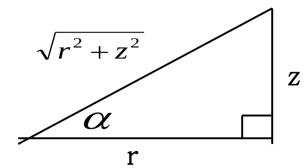
(Ex) 유한 직선전류에 의한 자계의 세기.



$$\mathbf{H}_2 = \frac{I \cdot r \cdot \hat{a}_\phi}{4\pi}\int_{z_1}^{z_2} \frac{dz}{(r^2+z^2)^{\frac{3}{2}}} = \frac{I \cdot r \cdot \hat{a}_\phi}{4\pi}\left[\frac{z}{r^2\sqrt{r^2+z^2}}\right]_{z_1}^{z_2}$$

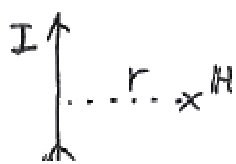
$$= \frac{I\hat{a}_\phi}{4\pi r}\cdot[\sin\alpha]_{\alpha_1}^{\alpha_2}$$

$$\boxed{= \frac{I}{4\pi r}(\sin\alpha_2 - \sin\alpha_1)\hat{a}_\phi}$$



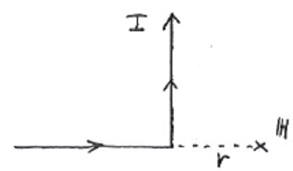
$$\sin\alpha = \frac{z}{\sqrt{r^2+z^2}}$$

* (ex)



$$\alpha_1 = -90^\circ, \quad \alpha_2 = 90^\circ$$

$$H = \frac{I}{2\pi r}\hat{a}_\varphi$$

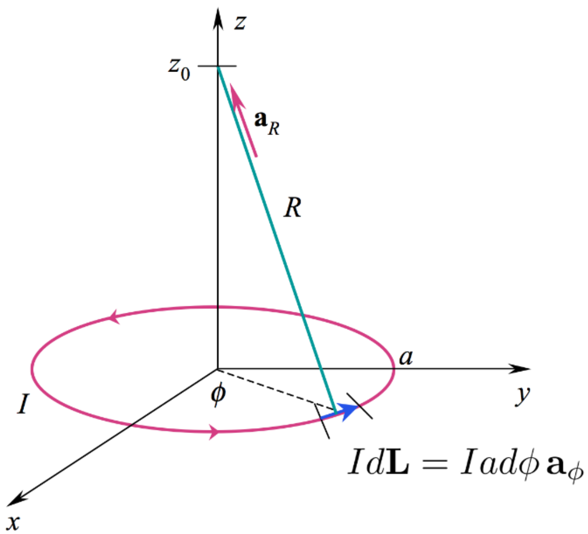


$$\alpha_1 = 0 \quad , \quad \alpha_2 = 90^\circ$$

$$H = \frac{I}{4\pi r}\hat{a}_\varphi$$

(Ex 7.1)

(Ex) 원형 선전류에 의한 자계의 세기.



$$R = \sqrt{a^2 + z_0^2} \qquad \mathbf{a}_R = \frac{z_0\,\mathbf{a}_z - a\,\mathbf{a}_\rho}{\sqrt{a^2 + z_0^2}}$$

$$Id\mathbf{L} = Iad\phi\,\mathbf{a}_\phi$$

$$\mathbf{H} = \int \frac{Id\mathbf{L} \times \mathbf{a}_R}{4\pi R^2} = \int_0^{2\pi} \frac{Iad\phi\,\mathbf{a}_\phi \times (z_0\,\mathbf{a}_z - a\,\mathbf{a}_\rho)}{4\pi(a^2 + z_0^2)^{3/2}} = \int_0^{2\pi} \frac{Iad\phi\,(z_0\cancel{\mathbf{a}_\rho} + a\,\mathbf{a}_z)}{4\pi(a^2 + z_0^2)^{3/2}}$$

$$\mathbf{a}_\rho = \cos\phi\,\mathbf{a}_x + \sin\phi\,\mathbf{a}_y$$

$$\boxed{\mathbf{H} = \frac{I(\pi a^2)\mathbf{a}_z}{2\pi(a^2 + z_0^2)^{3/2}}}$$

$$\mathbf{m} = I(\pi a^2)\,\mathbf{a}_z$$

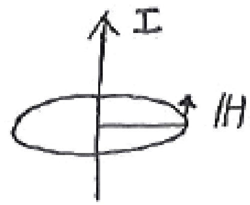
If Z=0 : H=I/2a

7.2. Ampere 의 주회법칙

- (전계) : Gauss 의 법칙 : $\oint \mathbf{D}\cdot d\mathbf{s} = Q$

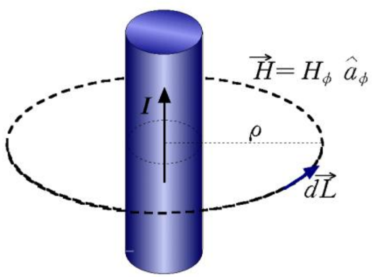
어느 한 폐곡면을 따라 더한 D의 합은 ($\mathbf{D}\cdot d\mathbf{s}$)이 폐곡면으로 둘러싸인 내부의 전하량의 값과 같다.

- (자계) : Ampere’s Circuital Law : $\oint \mathbf{H}\cdot d\mathbf{L} = I$



어느 한 폐곡면을 따라 더한 H의 합은 ($\mathbf{H}\cdot d\mathbf{L}$)이 폐곡선 면을 관통하는 전류의 값과 같다.

< Ex > 무한 직선전류에 의한 자계의 세기를 Ampere의 법칙으로 구하시오.

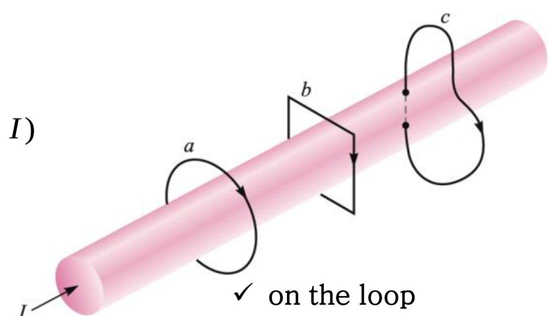


$$\boxed{\oint \mathbf{H}\cdot d\mathbf{L} = I}$$

$$\oint \mathbf{H}\cdot d\mathbf{L} = \int_0^{2\pi} H_\phi \rho\, d\phi = H_\phi \rho \int_0^{2\pi} d\phi = H_\phi 2\pi\rho \quad (= I)$$

$$H_\phi = \frac{I}{2\pi\rho}$$

$$\boxed{\therefore \mathbf{H} = \frac{I}{2\pi r}\hat{a}_\phi}$$

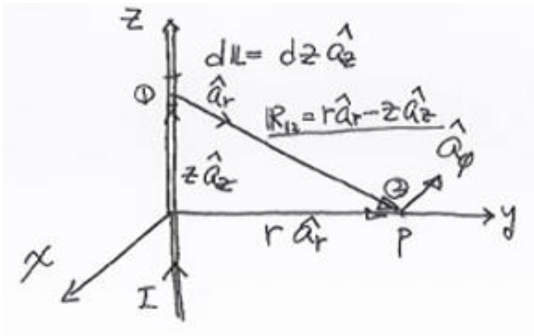


✓ on the loop

$$\begin{bmatrix} \mathbf{H} = H_\phi \hat{a}_\phi \\ |\mathbf{H}| = Const = H_\phi \end{bmatrix}$$

Lab. of Applied ElectroMagnetics

(Ex) 무한 직선전류에 의한 자계의 세기를 Biot – Savart 법칙으로 구하시오.



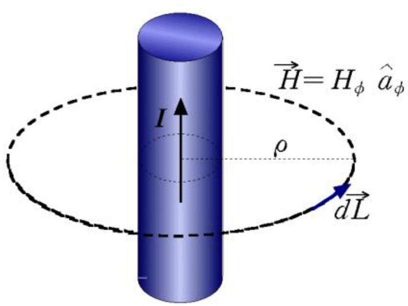
$$\hat{a}_r = \frac{\mathbf{R}_{12}}{|\mathbf{R}_{12}|} = \frac{r\hat{a}_r - z\hat{a}_z}{\sqrt{r^2+z^2}}$$

$$\text{Biot – Savart} \ : \ d\mathbf{H} = \frac{I d\mathbf{L}\times\mathbf{a}_R}{4\pi R^2} = \frac{I\,dz'\mathbf{a}_z\times(\rho\mathbf{a}_\rho - z'\mathbf{a}_z)}{4\pi(\rho^2+z'^2)^{3/2}}$$

$$\mathbf{H}_2 = \int_{-\infty}^{\infty}\frac{I\,dz'\mathbf{a}_z\times(\rho\mathbf{a}_\rho - z'\mathbf{a}_z)}{4\pi(\rho^2+z'^2)^{3/2}} = \frac{I}{4\pi}\int_{-\infty}^{\infty}\frac{\rho\,dz'\mathbf{a}_\phi}{(\rho^2+z'^2)^{3/2}} = \frac{I\rho\mathbf{a}_\phi}{4\pi}\left.\frac{z'}{\rho^2\sqrt{\rho^2+z'^2}}\right|_{-\infty}^{\infty}$$

$$\boxed{\therefore \mathbf{H} = \frac{I}{2\pi r}\hat{a}_\phi}$$

(Ex) 무한 직선전류에 의한 자계의 세기를 Ampere의 법칙으로 구하시오.



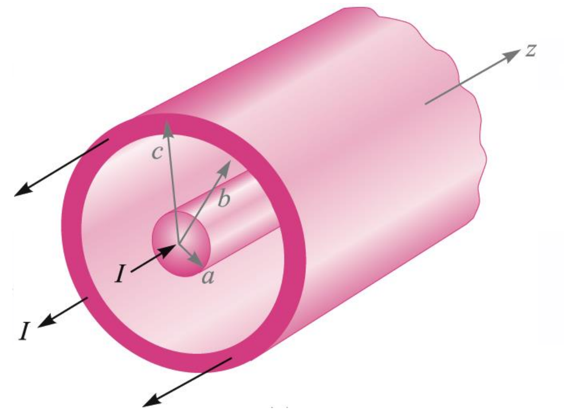
$$\boxed{\oint \mathbf{H}\cdot d\mathbf{L} = I}$$

$$\oint \mathbf{H}\cdot d\mathbf{L} = \int_0^{2\pi} H_\phi \rho\, d\phi = H_\phi\rho\int_0^{2\pi} d\phi = H_\phi 2\pi\rho \quad (= I)$$

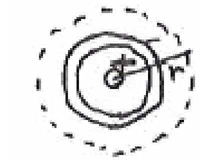
$$H_\phi = \frac{I}{2\pi\rho}$$

$$\boxed{\therefore \mathbf{H} = \frac{I}{2\pi r}\hat{a}_\phi}$$

< Ex > 동축케이블의 자계와 Magnetic Shielding. (자기차폐)

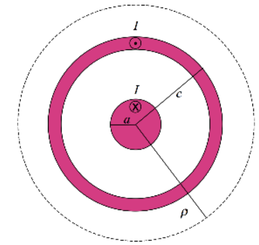


(i) r > c :

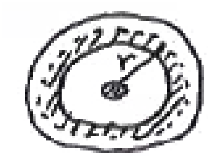


$$\oint \mathbf{H}\cdot d\mathbf{L} = 0$$

$$\boxed{\therefore H_\phi = 0}$$



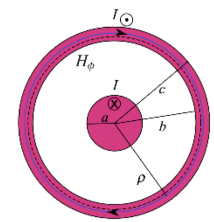
(ii) b < r < c : 면내부전류 $= I - I \times \dfrac{\pi r^2 - \pi b^2}{\pi c^2 - \pi b^2}$ (외통총면적 : $\pi c^2 - \pi b^2$, r까지 면적 : $\pi r^2 - \pi b^2$)



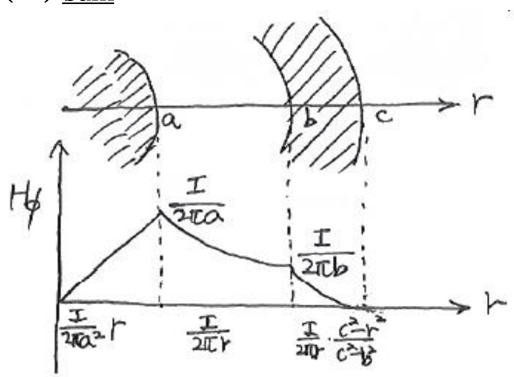
$$\therefore \oint \mathbf{H}\cdot d\mathbf{L} = I\cdot\left(1-\frac{r^2-b^2}{c^2-b^2}\right)$$

$$2\pi r H_\phi = I\cdot\frac{c^2-r^2}{c^2-b^2}$$

$$\boxed{\therefore H_\phi = \frac{I}{2\pi r}\cdot\frac{c^2-r^2}{c^2-b^2}}$$



(*) sum

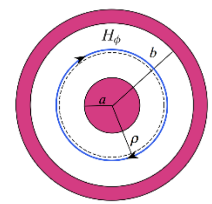


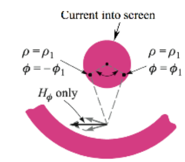
- (i) 경계면에서 i는 연속
- (ii) 케이블 외부(r>c)의 자계는 zero: → 자기차폐 (Shielding)

(iii) a < r < b : 면내부전류 = I

$$\oint \mathbf{H}\cdot d\mathbf{L} = I$$

$$\boxed{\therefore H_\phi = \frac{I}{2\pi r}}$$

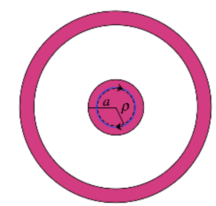




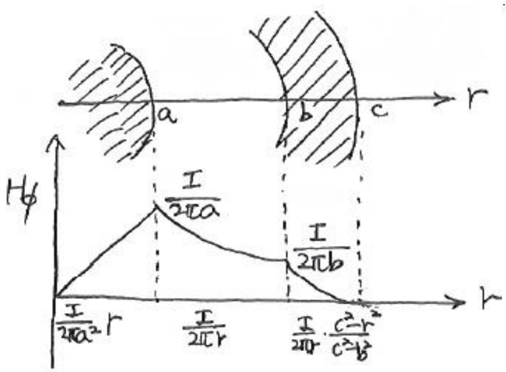
(iv) r < a : 면내부전류 $= \dfrac{2\pi r^2}{2\pi a^2}\times I = \dfrac{r^2}{a^2} I$

$$\oint \mathbf{H}\cdot d\mathbf{L} = \frac{r^2}{a^2} I$$

$$\boxed{\therefore H_\phi = \frac{I\cdot r}{2\pi a^2}}$$



(*) sum

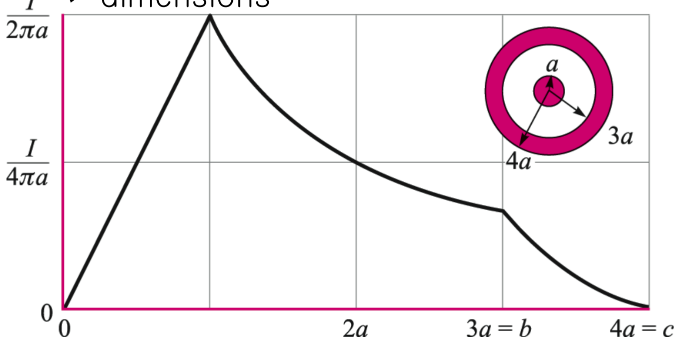


(i) 경계면에서 i는 연속

(ii) 케이블 외부(r>c)의 자계는 zero:

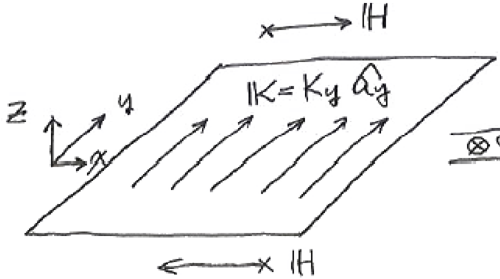
→ 자기차폐 (Shielding)

✓ dimensions

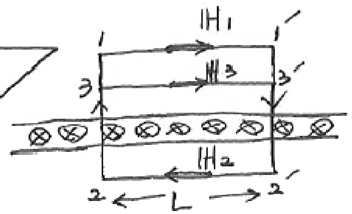


Lab. of Applied ElectroMagnetics

< Ex > 평면전류에 의한 자계의 세기



$\mathbf{H} = \frac{1}{2}\mathbf{K} \times \mathbf{a}_N$



$$\oint \mathbf{H} \cdot d\mathbf{L} = I$$

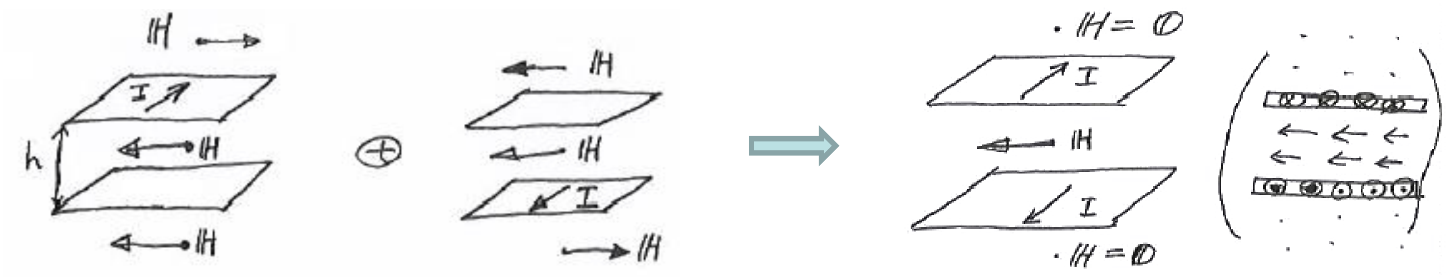
$$\begin{cases} \text{1-2 Loop}: \ H_1 \cdot L + H_2 \cdot (-L) = K_y \cdot L & \rightarrow \ H_1 - H_2 = K_y \\ \text{2-3 Loop}: \ H_3 \cdot L + H_2 \cdot (-L) = K_y L & \rightarrow \ H_3 - H_2 = K_y \end{cases}$$

$\therefore H_1 = H_3$ ($\because$ Loop내부 전류가 동일)

$$\therefore H_x = \begin{cases} \frac{1}{2}K_y & (z > 0) \\ -\frac{1}{2}K_y & (z < 0) \end{cases} \quad (\,!\ |\mathbf{H}| \cancel{\propto} f(r)\,)$$

$\mathbf{H} = \frac{1}{2}\mathbf{K} \times \mathbf{a}_N$

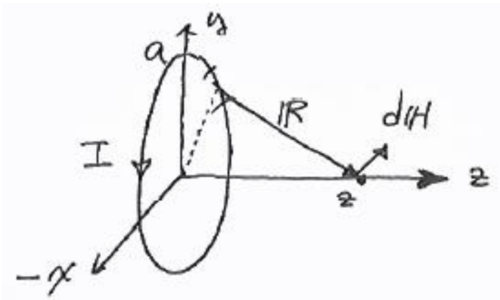
< Ex > 양 평면전류에 의한 자계의 세기



$\mathbf{H} = \mathbf{K} \times \mathbf{a}_N$

Lab. of Applied ElectroMagnetics

< Ex > 원형코일에 의한 자계의 세기



$\oint \mathbf{H}\cdot d\mathbf{L} = I$, Ampere 의 법칙 ?

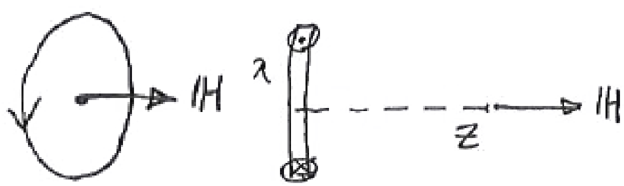
$$d\mathbf{H} = \frac{Id\mathbf{L}\times \hat{a}_r}{4\pi R^2} \quad \begin{cases} R=\sqrt{a^2+z^2} & \hat{a}_\phi \times \hat{a}_r\big|_z = \dfrac{a}{R} \\ Id\mathbf{L} = Iad\phi\cdot\hat{a}_\phi \end{cases}$$

$$\therefore H_z = \oint \frac{Iad\phi\cdot\hat{a}_\phi\times\hat{a}_r}{4\pi(a^2+z^2)} = \frac{I}{4\pi}\int_0^{2\pi}\frac{a\cdot\dfrac{a}{R}}{a^2+z^2}d\phi = \frac{I\cdot a^2}{4\pi(a^2+z^2)^{3/2}}\cdot\int_0^{2\pi}d\phi$$

$$= \frac{I\cdot a^2}{2(a^2+z^2)^{3/2}}$$

$$\boxed{\therefore H_z = \frac{I\cdot a^2}{2(a^2+z^2)^{3/2}}}$$

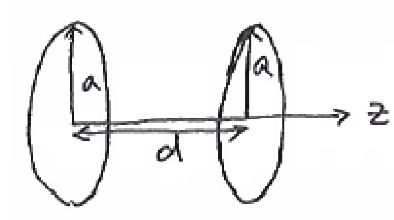
❖ At center , z = 0 :



$$\boxed{H = \frac{I}{2a}\hat{a}_z}$$

Pusan National University
Lab. of Applied ElectroMagnetics

< Ex > Helm-Holtz Coil 에 의한 자계의 세기



$$H_z = \frac{I \cdot a^2}{2}\left[\frac{1}{(a^2+z^2)^{3/2}} + \frac{1}{(a^2+(z-d)^{3/2}}\right]$$

$$\left.\frac{dH_z}{dz}\right|_{z=\frac{d}{2}} = 0$$

즉, 두 코일 정 중앙에서는 자계 변화 없음

✓ a = d 일 때, 즉 코일간 거리가 반지름과 같은 경우 ➔ Helm-Holtz Coil

이 경우, $\left.\frac{dH_z}{dz}\right|_{z=\frac{a}{2}} = 0$ $\left.\frac{d^2H_z}{dz^2}\right|_{z=\frac{a}{z}} = 0$

$$\boxed{H_z = \frac{Ia^2}{2}\left[\frac{1}{(a^2+z^2)^{3/2}} + \frac{1}{(a^2+(z-a)^2)^{3/2}}\right]}$$

✓ $z = \frac{a}{2}$ 에서의 자계의 세기 : $H_z(z=\frac{a}{2}) = \frac{Ia^2}{2}\left[\frac{1}{(\frac{5}{4}a^2)^{3/2}} + \frac{1}{(\frac{5}{4}a^2)^{3/2}}\right] = \frac{Ia^2}{2}\cdot\frac{2}{(\frac{5}{4})^{3/2}\cdot a^2} = (\frac{5}{4})^{3/2}\cdot\frac{I}{a} \cong 0.7155\cdot\frac{I}{a}$

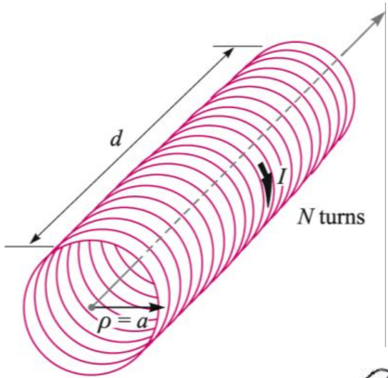
✓ N Turn 감겨 있을 경우 : $\boxed{H_z|_{\frac{a}{2}} = 0.7\cdot\frac{NI}{a}}$

✓ a= 10cm, 지름 20cm, 거리 10cm 일 때 1A 10Turn 이면 0.88G

$$B_z|_{\frac{a}{2}} = \frac{0.7\mu_0}{a}NI = \frac{0.7\times 4\pi\times 10^{-7}\times NI}{a[cm]\times 10^{-2}}$$

$$\boxed{B_z|_{\frac{a}{2}} = 0.88\times\frac{NI}{a[cm]}[G]}$$

< Ex > Solenoid 의 자계의 세기 (Coil)

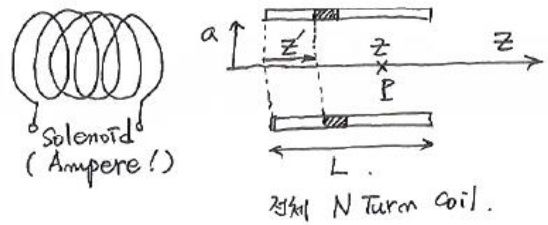


- Density of turns = N/d. $dI = \dfrac{N}{d} I dz$
- In single loop : $\mathbf{H} = \dfrac{I(\pi a^2)\mathbf{a}_z}{2\pi(a^2+z_0^2)^{3/2}}$

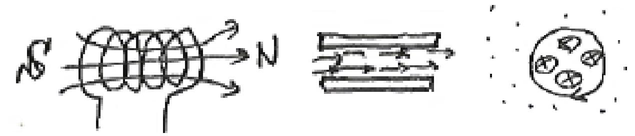
$$d\mathbf{H} = \frac{(N/d)Idz(\pi a^2)\mathbf{a}_z}{2\pi(a^2+z^2)^{3/2}}$$

$$\mathbf{H} = \int d\mathbf{H} = \int_{-d/2}^{d/2} \frac{(N/d)Idz(\pi a^2)\mathbf{a}_z}{2\pi(a^2+z^2)^{3/2}} = \frac{NIa^2}{2d}\mathbf{a}_z \int_{-d/2}^{d/2} \frac{dz}{(a^2+z^2)^{3/2}} = \frac{NIa^2}{2d}\mathbf{a}_z \frac{d}{a^2\sqrt{a^2+(d/2)^2}} = \frac{NI\,\mathbf{a}_z}{2\sqrt{a^2+(d/2)^2}}$$

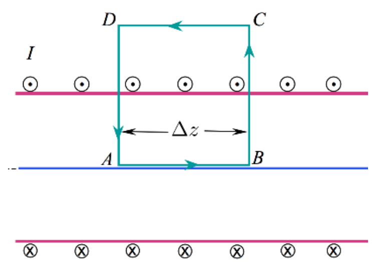
$$\mathbf{H} = \frac{NI\,\mathbf{a}_z}{2\sqrt{a^2+(d/2)^2}} \qquad \boxed{\mathbf{H} \doteq \frac{NI}{d}\mathbf{a}_z} \quad : \text{if } d >> a$$



$$\boxed{H_z = \frac{NI}{L}}$$

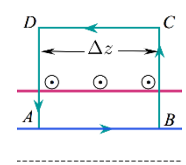


✓ Uniformity :



$$\oint \mathbf{H}\cdot d\mathbf{L} = \int_A^B H_z\,dz + \int_B^C \cancel{H_\rho}\,d\rho + \int_C^D H_{z,out}\,dz + \int_D^A \cancel{H_\rho}\,d\rho = I_{encl} = \frac{NI}{d}\Delta z$$

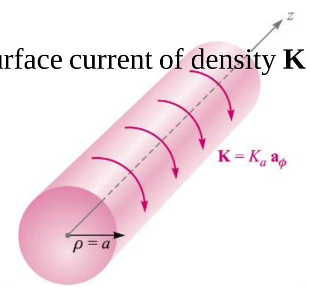
$$\oint \mathbf{H}\cdot d\mathbf{L} = \underbrace{\int_A^B H_z\,dz}_{(NI/d)\Delta z} + \int_C^D \cancel{H_{z,out}}\,dz = I_{encl} = \frac{NI}{d}\Delta z$$



H_z = NI/d

Magnetic field = Constant throughout the coil cross-section.

✓ Continuous surface current of density $\mathbf{K} = K_a\,\mathbf{a}_\phi$ A/m.



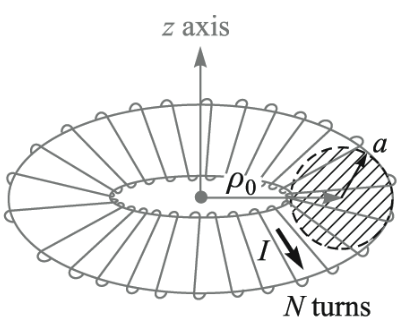
$$\mathbf{K} = K_a\,\mathbf{a}_\phi = \frac{NI}{d}\mathbf{a}_\phi \quad \text{A/m}$$

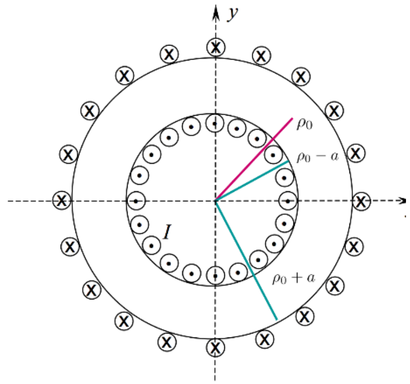
$$\mathbf{H}(\rho = z = 0) = \frac{K_a d\,\mathbf{a}_z}{2\sqrt{a^2+(d/2)^2}} \doteq K_a\,\mathbf{a}_z \ (d >> a) \quad \text{A/m}$$

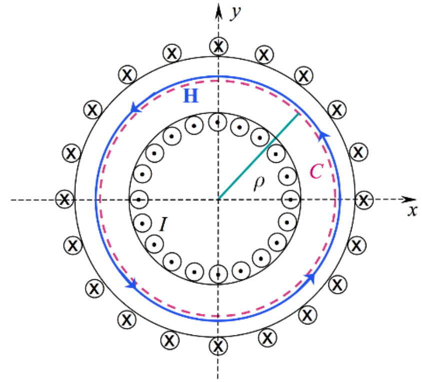
On-axis field magnitude near the center = the surface current density.

Lab. of Applied ElectroMagnetics

< Ex > Toroidal Coil 의 자계의 세기





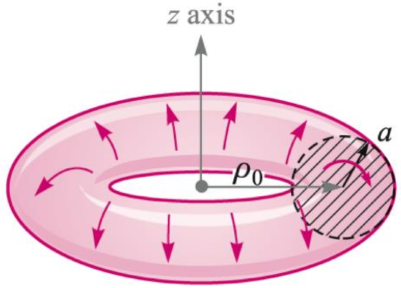


$$\mathbf{H} = H_\phi \, \mathbf{a}_\phi$$

$$\oint_C \mathbf{H} \cdot d\mathbf{L} = 2\pi\rho \, H_\phi = I_{encl} = NI$$

$$(\rho_0 - a < \rho < \rho_0 + a)$$

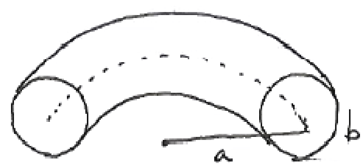
$$\boxed{H_\phi = \frac{NI}{2\pi\rho}}$$

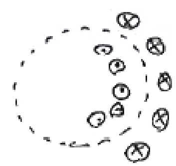


$$\oint_C \mathbf{H} \cdot d\mathbf{L} = 2\pi\rho H_\phi = I_{encl} = 2\pi(\rho_0 - a) K_a$$

$$H_\phi = \frac{\rho_0 - a}{\rho} K_a$$

- Toroidal Coil :





(i) r < a : $\oint \mathbf{H} \cdot d\mathbf{L} = 0, \quad \mathbf{H} = \mathbf{0}$

(ii) r > b : $\oint \mathbf{H} \cdot d\mathbf{L} = 0, \quad \mathbf{H} = \mathbf{0}$

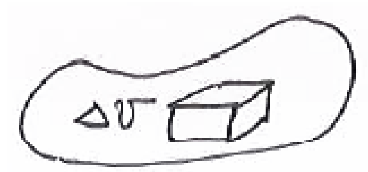
(iii) a < r < b : 면 내부전류는 NI 이므로

$$\oint \mathbf{H} \cdot d\mathbf{L} = NI \quad 2\pi r H_\phi = NI, \quad H_\phi = \frac{NI}{2\pi r}$$

$$\boxed{\therefore \mathbf{H} = \frac{NI}{2\pi r} \hat{a}_\phi}$$

7.3 & 7.4 벡터장의 회전, Stokes 정리

• (전계) Divergence :



div D $= \nabla \cdot \mathbf{D} = \lim_{\Delta v \to 0} \dfrac{\oint_s \mathbf{D} \cdot d\mathbf{S}}{\Delta v}$ (meaning)

$= \rho$ (Gauss)

❖ $\boxed{\oint_s \mathbf{D} \cdot d\mathbf{S} = Q}$ (적분형)

↓

$\boxed{\nabla \cdot \mathbf{D} = \rho}$ (미분형) : Gauss's Law

$$\begin{cases} Q = \int_v \rho dv & \text{(def.)} \\ \oint_s \mathbf{D} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{D} dv & \text{(Green Theorem)} \end{cases}$$

• (자계) Curl , Rotation :

curl H = $\nabla \times \mathbf{H} = \lim_{\Delta S_n \to 0} \dfrac{\oint_l \mathbf{H} \cdot d\mathbf{L}}{\Delta S_n}$ (meaning)

$= \mathbf{J}$ (Ampere)

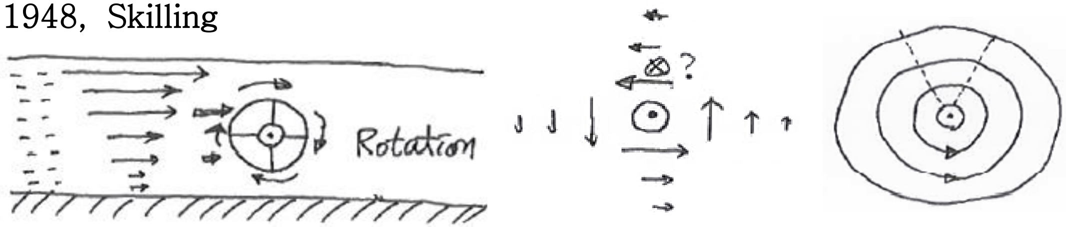
$$\begin{cases} I = \int_S \mathbf{J} \cdot d\mathbf{S} & \text{(def.)} \\ \oint_l \mathbf{H} \cdot d\mathbf{L} = \int_S \nabla \times \mathbf{H} \cdot d\mathbf{S} & \text{(Stokes Theorem)} \end{cases}$$

❖ $\boxed{\oint_l \mathbf{H} \cdot d\mathbf{L} = I}$ (적분형)

↓

$\boxed{\nabla \times \mathbf{H} = \mathbf{J}}$ (미분형) : Ampere's Circutal Law

- <u>Curl 의 측정</u> : 1948, Skilling



(a) (b)

- <u>Curl 의 계산:</u>

$$Curl\ \ H = \nabla \times H = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \underline{\underline{\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\hat{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\hat{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\hat{a}_z}}$$

x → y → z

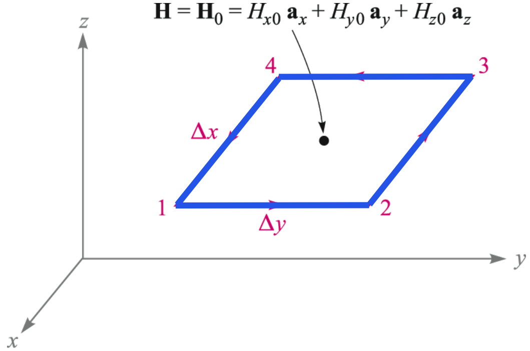
(직각좌표계): $= \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\hat{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\hat{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\hat{a}_z$

(원통좌표계) $= \left(\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z}\right)\hat{a}_\rho + \left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho}\right)\hat{a}_\phi + \left[\frac{1}{\rho}\frac{\partial(\rho H_\phi)}{\partial \rho} - \frac{1}{\rho}\frac{\partial H_\rho}{\partial \phi}\right]\hat{a}_z$

(구 좌표계) $= \frac{1}{r\sin\theta}\left[\frac{\partial(H_\phi \sin\theta)}{\partial \theta} - \frac{\partial H_\theta}{\partial \phi}\right]\hat{a}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial H_r}{\partial \phi} - \frac{\partial(rH_\phi)}{\partial r}\right]\hat{a}_\theta + \frac{1}{r}\left[\frac{\partial(rH_\theta)}{\partial r} - \frac{\partial H_r}{\partial \theta}\right]\hat{a}_\phi$

Lab. of Applied ElectroMagnetics

➢ 벡터의 미분 : Curl (회전)



$$(\mathbf{H}\cdot\Delta\mathbf{L})_{1-2} = H_{y,1-2}\Delta y \qquad H_{y,1-2} \doteq H_{y0} + \frac{\partial H_y}{\partial x}\left(\frac{1}{2}\Delta x\right) \qquad (\mathbf{H}\cdot\Delta\mathbf{L})_{1-2} \doteq \left(H_{y0} + \frac{1}{2}\frac{\partial H_y}{\partial x}\Delta x\right)\Delta y$$

$$(\mathbf{H}\cdot\Delta\mathbf{L})_{1-2} \doteq \left(H_{y0} + \frac{1}{2}\frac{\partial H_y}{\partial x}\Delta x\right)\Delta y$$

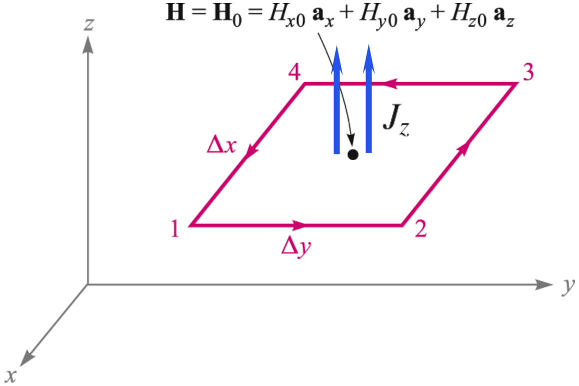
$$(\mathbf{H}\cdot\Delta\mathbf{L})_{3-4} \doteq \left(H_{y0} - \frac{1}{2}\frac{\partial H_y}{\partial x}\Delta x\right)(-\Delta y)$$

$$(\mathbf{H}\cdot\Delta\mathbf{L})_{2-3} \doteq \left(H_{x0} + \frac{1}{2}\frac{\partial H_x}{\partial y}\Delta y\right)(-\Delta x)$$

$$(\mathbf{H}\cdot\Delta\mathbf{L})_{4-1} \doteq \left(H_{x0} - \frac{1}{2}\frac{\partial H_x}{\partial y}\Delta y\right)(\Delta x)$$

$$\oint \mathbf{H}\cdot d\mathbf{L} \doteq (\mathbf{H}\cdot\Delta\mathbf{L})_{1-2} + (\mathbf{H}\cdot\Delta\mathbf{L})_{2-3} + (\mathbf{H}\cdot\Delta\mathbf{L})_{3-4} + (\mathbf{H}\cdot\Delta\mathbf{L})_{4-1}$$

$$\oint \mathbf{H}\cdot d\mathbf{L} \doteq \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\Delta x\Delta y$$



$$\oint \mathbf{H}\cdot d\mathbf{L} \doteq \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\Delta x\Delta y \doteq J_z\Delta x\Delta y \qquad \frac{\oint \mathbf{H}\cdot d\mathbf{L}}{\Delta x\Delta y} \doteq \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \doteq J_z$$

$$\lim_{\Delta x,\Delta y\to 0}\frac{\oint \mathbf{H}\cdot d\mathbf{L}}{\Delta x\Delta y} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = J_z$$

$$\lim_{\Delta y,\Delta z\to 0}\frac{\oint \mathbf{H}\cdot d\mathbf{L}}{\Delta y\Delta z} = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = J_x$$

$$\lim_{\Delta z,\Delta x\to 0}\frac{\oint \mathbf{H}\cdot d\mathbf{L}}{\Delta z\Delta x} = \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = J_y$$

$$\lim_{\Delta x,\Delta y\to 0}\frac{\oint \mathbf{H}\cdot d\mathbf{L}}{\Delta x\Delta y} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = J_z$$

$$(\text{curl }\mathbf{H})_N = \lim_{\Delta S_N\to 0}\frac{\oint \mathbf{H}\cdot d\mathbf{L}}{\Delta S_N}$$

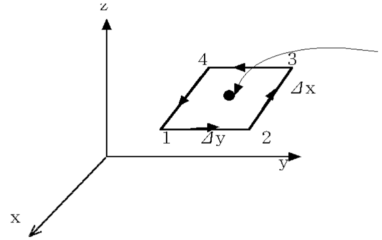
$$\text{curl }\mathbf{H} = \nabla\times\mathbf{H}$$

$$\text{curl }\mathbf{H} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

$$\text{curl }\mathbf{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\mathbf{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\mathbf{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\mathbf{a}_z$$

➢ 벡터의 미분 : Curl (회전)

- 정의 : $(Curl\,\vec{H})_n = \lim_{\triangle S_n \to 0} \dfrac{\oint \vec{H}\cdot d\vec{l}}{\triangle S_n}$



$$\vec{H} = \vec{H_0} = H_{x0}\,\hat{a}_x + H_{y0}\,\hat{a}_y + H_{z0}\,\hat{a}_z$$

$$Curl\,\vec{H} = \nabla \times \vec{H}$$

- 계산 : $Curl\,\vec{H} = \left(\dfrac{\partial H_z}{\partial y} - \dfrac{\partial H_y}{\partial z}\right)\hat{a}_x + \left(\dfrac{\partial H_x}{\partial z} - \dfrac{\partial H_z}{\partial x}\right)\hat{a}_y + \left(\dfrac{\partial H_y}{\partial x} - \dfrac{\partial H_x}{\partial y}\right)\hat{a}_z = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$

$$(\text{직각좌표계}) = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\hat{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\hat{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\hat{a}_z$$

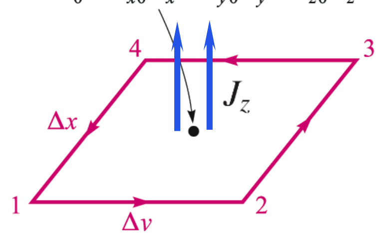
$$(\text{원통좌표계}) = \left(\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z}\right)\hat{a}_\rho + \left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho}\right)\hat{a}_\phi + \left[\frac{1}{\rho}\frac{\partial(\rho H_\phi)}{\partial \rho} - \frac{1}{\rho}\frac{\partial H_\rho}{\partial \phi}\right]\hat{a}_z$$

$$(\text{구 좌표계}) = \frac{1}{r\sin\theta}\left[\frac{\partial(H_\phi \sin\theta)}{\partial \theta} - \frac{\partial H_\theta}{\partial \phi}\right]\hat{a}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial H_r}{\partial \phi} - \frac{\partial(rH_\phi)}{\partial r}\right]\hat{a}_\theta + \frac{1}{r}\left[\frac{\partial(rH_\theta)}{\partial r} - \frac{\partial H_r}{\partial \theta}\right]\hat{a}_\phi$$

- Stoke's Thm : $\oint \vec{H}\cdot d\vec{L} \equiv \int_S (\nabla \times \vec{H})\cdot d\vec{S}$

➢ Ampere's Law :

$\mathbf{H} = \mathbf{H}_0 = H_{x0}\,\mathbf{a}_x + H_{y0}\,\mathbf{a}_y + H_{z0}\,\mathbf{a}_z$



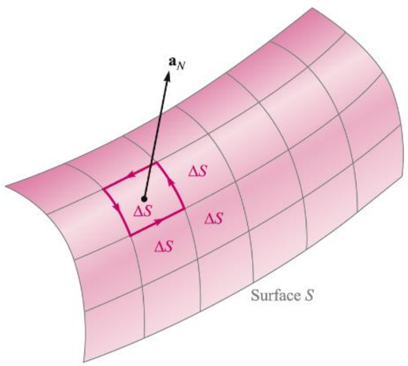
$$\lim_{\triangle x \triangle y \to 0} \frac{\oint \vec{H}\cdot d\vec{l}}{\triangle x \triangle y} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = J_z$$

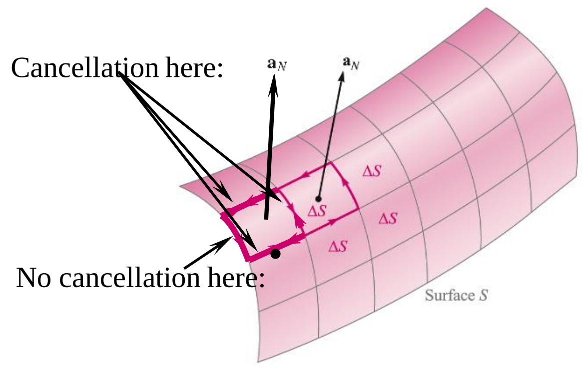
$$\lim_{\triangle y \triangle z \to 0} \frac{\oint \vec{H}\cdot d\vec{l}}{\triangle y \triangle z} = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = J_x$$

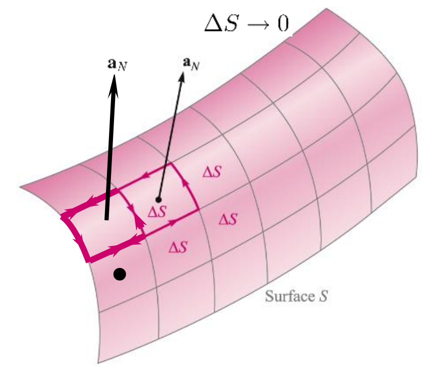
$$\lim_{\triangle z \triangle x \to 0} \frac{\oint \vec{H}\cdot d\vec{l}}{\triangle z \triangle x} = \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = J_y$$

$$\boxed{\nabla \times \mathbf{H} = \mathbf{J}}$$

➤ Stokes' Theorem :







$$\frac{\oint \mathbf{H}\cdot d\mathbf{L}_{\Delta S}}{\Delta S} \doteq (\nabla\times\mathbf{H})\cdot\mathbf{a}_N$$

$$\oint \mathbf{H}\cdot d\mathbf{L}_{\Delta S} \doteq (\nabla\times\mathbf{H})\cdot\mathbf{a}_N\Delta S = (\nabla\times\mathbf{H})\cdot\Delta\mathbf{S}$$

$$\sum_{\substack{\text{all surface}\\ \text{elements}}} \oint \mathbf{H}\cdot d\mathbf{L}_{\Delta S} \doteq \sum_{\substack{\text{all surface}\\ \text{elements}}} \nabla\times\mathbf{H}\cdot\mathbf{a}_N\Delta S$$

$$\underbrace{\sum_{\substack{\text{all surface}\\ \text{elements}}} \oint \mathbf{H}\cdot d\mathbf{L}_{\Delta S}} \doteq \underbrace{\sum_{\substack{\text{all surface}\\ \text{elements}}} \nabla\times\mathbf{H}\cdot\mathbf{a}_N\Delta S}$$

In the limit, this side becomes the path integral of **H** over the outer perimeter because all interior paths cancel

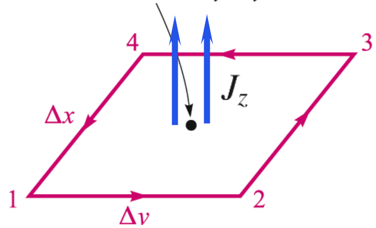
In the limit, this side becomes the integral of the curl of **H** over surface S

$$\boxed{\oint \mathbf{H}\cdot d\mathbf{L} \equiv \int_S (\nabla\times\mathbf{H})\cdot d\mathbf{S}}$$

Lab. of Applied ElectroMagnetics

- **Ampere's Law :**

$$\mathbf{H} = \mathbf{H}_0 = H_{x0}\,\mathbf{a}_x + H_{y0}\,\mathbf{a}_y + H_{z0}\,\mathbf{a}_z$$

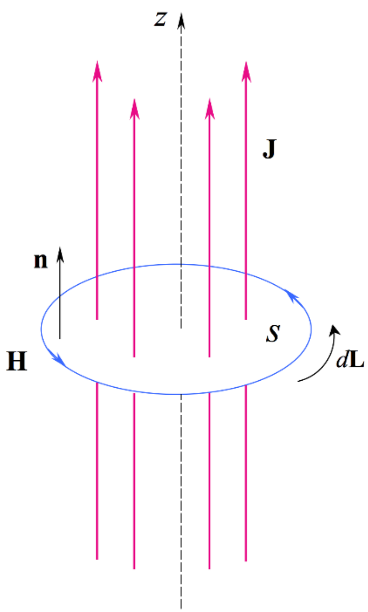


$$\lim_{\triangle x \triangle y \to 0} \frac{\oint \vec{H} \cdot d\vec{l}}{\triangle x \triangle y} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = J_z$$

$$\lim_{\triangle y \triangle z \to 0} \frac{\oint \vec{H} \cdot d\vec{l}}{\triangle y \triangle z} = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = J_x$$

$$\lim_{\triangle z \triangle x \to 0} \frac{\oint \vec{H} \cdot d\vec{l}}{\triangle z \triangle x} = \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = J_y$$

$$\boxed{\nabla \times \mathbf{H} = \mathbf{J}}$$



$$\boxed{\nabla \times \mathbf{H} = \mathbf{J}}$$

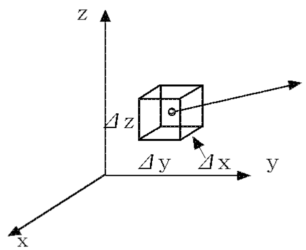
$$\int_S (\nabla \times \mathbf{H}) \cdot d\mathbf{S} = \int_S \mathbf{J} \cdot d\mathbf{S} = \oint \mathbf{H} \cdot d\mathbf{L}$$

$$\boxed{\oint \mathbf{H} \cdot d\mathbf{L} = I}$$

Lab. of Applied ElectroMagnetics

➢ 벡터의 미분 : Divergence (발산)

- 정의 : $\text{div A} = \lim_{\Delta V \to 0} \dfrac{\oint_s \vec{A}\cdot\vec{ds}}{\Delta v}$



체적내 중심점 : $\vec{D_0} = D_{x0}\,\hat{a}_x + D_{y0}\,\hat{a}_y + D_{z0}\,\hat{a}_z$

$$\oint_s \mathrm{D}\cdot\mathrm{ds} = \int_{front} + \int_{back} + \int_{left} + \int_{right} + \int_{top} + \int_{bottom}$$

$$\int_{front} + \int_{back} = \frac{\partial D_x}{\partial x}\Delta x\Delta y\Delta z$$

$$\int_{right} + \int_{left} = \frac{\partial D_y}{\partial y}\Delta x\Delta y\Delta z$$

$$\int_{top} + \int_{bottom} = \frac{\partial D_z}{\partial z}\Delta x\Delta y\Delta z$$

$$\int_{front} \doteq D_{front}\cdot\Delta s_{front} = D_{x\,front}\Delta y\Delta z$$

$$D_{x\,front} \doteq D_{x0} + \frac{\Delta x}{2}\times D_x \text{ 변화율 (with x)}$$

$$\doteq D_{x0} + \frac{\Delta x}{2}\frac{\partial D_x}{\partial x}$$

$$= \int_{front}\left(D_{x0} + \frac{\Delta x}{2}\frac{\partial D_x}{\partial x}\right)\Delta y\Delta z$$

- Gauss : $\text{Charge q in } \Delta v = \left(\dfrac{\partial D_x}{\partial x} + \dfrac{\partial D_y}{\partial y} + \dfrac{\partial D_z}{\partial z}\right)\times\ \Delta\text{vol}$

$$= \nabla\cdot\vec{D}\ \ \Delta\text{volume}$$

$$\therefore\ Q = \int \rho dv = \int_{vol} \nabla\cdot D\,dv$$

$$\boxed{\nabla\cdot\vec{D} = \rho_v}$$

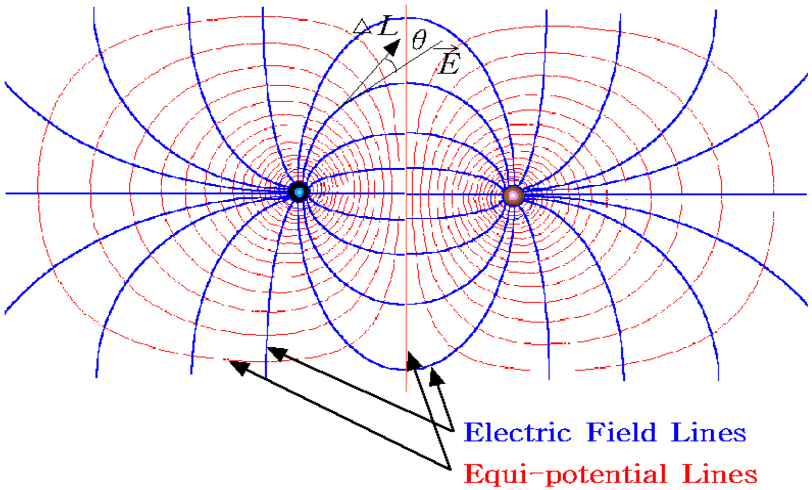
- 계산 : $\nabla\cdot\vec{D} = \dfrac{\partial D_x}{\partial x} + \dfrac{\partial D_y}{\partial y} + \dfrac{\partial D_z}{\partial z}$

➢ 벡터의 미분 : Gradient (경도)

- Gradient :

$$\Delta V = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

- Field/Potential :



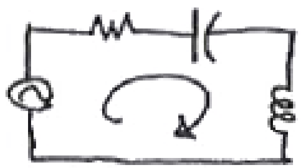
$$V = -\int \vec{E} \cdot \overrightarrow{dL}$$

$$\vec{E} = -\text{grad } V = -\left(\frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z \right)$$

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$

$$E = -\left.\frac{dV}{dL}\right|_{\max} \hat{a}_N = -\text{grad } V = -\nabla V$$

- (KVL)



$$\oint \mathbf{E}\cdot d\mathbf{L} = 0$$

$$\nabla \times \mathbf{E} = \mathbf{0}$$

$$\oint \mathbf{E}\cdot d\mathbf{L} = \int_S \nabla \times \mathbf{E}\cdot d\mathbf{S} \quad \text{(Stokes)}$$

$$V = \int \mathbf{E}\cdot d\mathbf{L} = 0$$

$$\boxed{\therefore \sum_i V_i = 0}$$

- (KCL) :

$$\nabla \times \mathbf{H} = \mathbf{J}$$

$$\nabla \cdot (\nabla \times \mathbf{H}) \equiv 0$$

✓ Curl의 Divergence는 항상 Zero

put $\nabla \cdot (\nabla \times \mathbf{Z}) == T$

$$\int_v \nabla \cdot (\nabla \times \mathrm{Z}) dv = \int_v T dv$$

$$\oint_s (\nabla \times \mathbf{Z}) \cdot d\mathbf{S} = \int_v T dv$$

(Divergence)

$v \rightarrow 0$: $T \rightarrow 0$

$$\therefore \nabla \cdot J = 0$$

$$\int (\nabla \cdot J) dv = \int J \cdot dS = I$$

$$\boxed{\therefore \sum_i I_i = 0}$$

7.5. 자속, 자속밀도

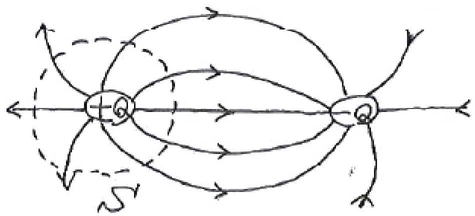
$\oint_S \mathbf{B}\cdot d\mathbf{S} = 0 \qquad \int_v \nabla\cdot\mathbf{B}dv = 0 \qquad \boxed{\therefore \nabla\cdot\mathbf{B} = 0}$

◆ 전속과 자속 (Electric Flux, Magnetic Flux)

• 전속 Ψ [Coulomb, C]

$$\Psi = \int_s \mathbf{D}\cdot d\mathbf{S} \quad : \text{(def.)}$$

$= Q$: (Gauss)



$$\int_v \nabla\cdot\mathbf{D}dv = \int_v \rho dv$$

$$\boxed{\therefore \nabla\cdot\mathbf{D} = \rho}$$

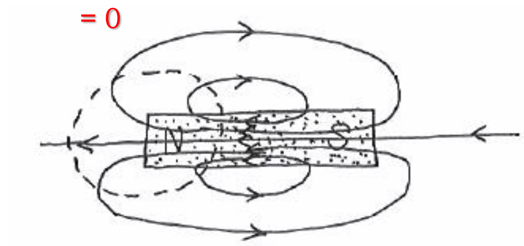
✓ D : 전속밀도 $[{}^{C}/_{m^2}]$

(Electric Flux Density)

• 자속 Φ [Weber, Wb]

$$\Phi = \int_s \mathbf{B}\cdot d\mathbf{S} \quad : \text{(def.)}$$

$= 0$



$$\oint_S \mathbf{B}\cdot d\mathbf{S} = \int_v \nabla\cdot\mathbf{B}\, dv = 0$$

$\boxed{\nabla\cdot\mathbf{B} = 0}$ (There is NO magnetic monopole.)

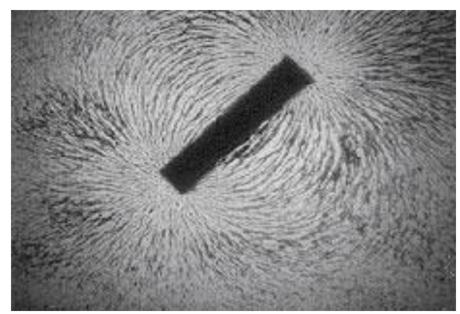
✓ B : 자속밀도

(Magnetic Flux Density)

MKS $[{}^{Wb}/_{m^2} \equiv Tesla]$

CGS [Gauss, G]

→ 1 [T] = 10^4 [G]



- Maxwell's Equations for Static Fields

$$\nabla \cdot \mathbf{D} = \rho_v$$ Gauss' Law for the electric field

$$\nabla \times \mathbf{E} = 0$$ Conservative property of the static electric field

$$\nabla \times \mathbf{H} = \mathbf{J}$$ Ampere's Circuital Law

$$\nabla \cdot \mathbf{B} = 0$$ Gauss' Law for the Magnetic Field

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = Q = \int_{\mathrm{vol}} \rho_v \, dv$$

$$\oint \mathbf{E} \cdot d\mathbf{L} = 0$$

$$\oint \mathbf{H} \cdot d\mathbf{L} = I = \int_S \mathbf{J} \cdot d\mathbf{S}$$

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$$

where, in free space: $\mathbf{D} = \epsilon_0 \mathbf{E}$ $\quad \epsilon_0 = 8.854 \times 10^{-12}\ \mathrm{F/m}$

$\mathbf{B} = \mu_0 \mathbf{H}$ $\quad \mu_0 = 4\pi \times 10^{-7}\ \mathrm{H/m}$

Lab. of Applied ElectroMagnetics

◆ In Matter

: 유전체, 자성체, 도체, 반도체, 초전도체

(1) Def : • Field ? Flux ?

$\frac{\text{Field}}{\text{Flux}}$

	CGS	MKS
진공에서	$\mathbf{B}=\mathbf{H}$	$\mathbf{B}=\mu_0\mathbf{H}$
매질에서	$\mathbf{B}=\mu_r\mathbf{H}$	$\mathbf{B}=\mu_0\mu_r\mathbf{H}$

? μ

Why Field / Flux ? Why H / B ?

μ_0 (or ε_0) : 단위계 조정상수

$\boxed{\mu_r = \frac{B}{H}}$: H에 대한 B의 비율

물질고유상수. 부록 C

Al(1) , Co(60) , Fe(1000)

철분(100), 철심(3000), Supermalloy(10만)

(2) Rel : (전계) $\boxed{\mathbf{D}=\varepsilon\mathbf{E}}$ $\varepsilon=\varepsilon_0\cdot\varepsilon_r$

ε : 유전율 (permitivity)

ε_r : 비유전율 (relative permitivity) ,

물질고유상수. 부록 (유리5 , 도자기 6)

ε_0 : 진공유전율 , $\frac{1}{36\pi}\times10^{-9}$ $\boxed{=8.854\times10^{-12}[F/m]}$

(자계) $\boxed{\mathbf{B}=\mu\mathbf{H}}$ $\mu=\mu_0\cdot\mu_r$

μ : 투자율 (permeability)

μ_r : 비투자율 (relative permeability) ,

물질고유상수. 부록(철 1000)

μ_0 : 진공투자율 , $\boxed{4\pi\times10^{-7}[H/m]}$

(3) Unit : $\varepsilon=\frac{\mathbf{D}}{\mathbf{E}}$ $[\frac{C/m^2}{V/m}=\frac{C}{V}\cdot\frac{1}{m}\equiv\frac{F}{m}]$,

$\boxed{Q=CV}$ $C=\frac{V}{Q}$ $[\frac{C}{V}\equiv Farad]$: Capacitance

$\mu=\frac{\mathbf{B}}{\mathbf{H}}$ $[\frac{Wb/m^2}{A/m}=\frac{Wb}{A}\cdot\frac{1}{m}\equiv\frac{H}{m}]$

$\boxed{\Phi=LI}$ $L=\frac{\Phi}{I}$ $[\frac{Wb}{A}\equiv Henry]$: Inductance

✓ ε or μ = $\frac{\text{flux density}}{\text{field intensity}}$

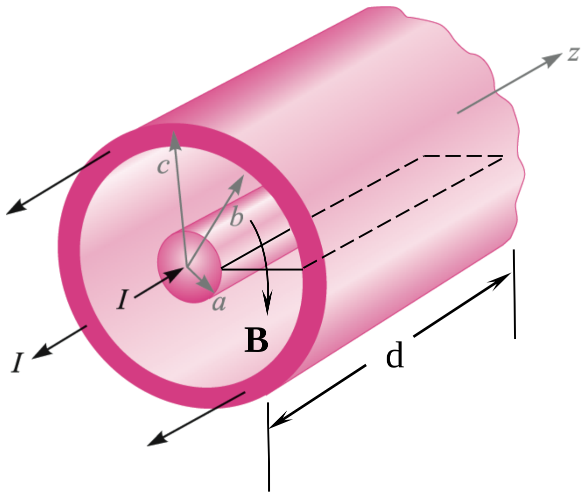
$\varepsilon_0:\mu_0=8.854\times10^{-12}:4\pi\times10^{-7}=1:15$만

(μ0는 15만배 큼)

$\frac{1}{\sqrt{\varepsilon_0\cdot\mu_0}}=2.998\times10^8=C[m/\sec]$

Lab. of Applied ElectroMagnetics

< Ex > 동축선로 에서의 자계의 세기와 인덕턴스



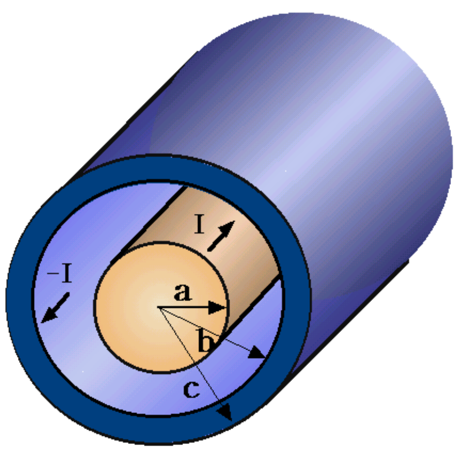
- $\oint \mathbf{H}\cdot d\mathbf{L} = I$

$$H_\phi = \frac{I}{2\pi\rho} \quad (a < \rho < b)$$

$$\mathbf{B} = \mu_0 \mathbf{H} = \frac{\mu_0 I}{2\pi\rho}\mathbf{a}_\phi$$

- $\Phi = \int_S \mathbf{B}\cdot d\mathbf{S} = \int_0^d \int_a^b \frac{\mu_0 I}{2\pi\rho}\mathbf{a}_\phi \cdot d\rho\, dz\, \mathbf{a}_\phi = \frac{\mu_0 I}{2\pi}\int_0^l dz \cdot \int_a^b \frac{1}{r} dr$

$$\boxed{\Phi = \frac{\mu_0 I d}{2\pi}\ln\frac{b}{a}}$$



$$\therefore L = \frac{\Phi}{I} = \boxed{\frac{\mu_0 l}{2\pi}\ln\frac{b}{a}} \ [\,\mathrm{H}\,] \qquad \begin{bmatrix} L \propto l & \\ b \cong a & : \quad L大 \end{bmatrix}$$

Lab. of Applied ElectroMagnetics

7.6. 자기 스칼라 , 벡터 포텐셜

(1) 자기 스칼라 포텐셜 (V_m)

정의 (전계) V_e : Electric Scalar Potential , 전위 , 전압. $\mathbf{E} = -\nabla V_e$ $(\mathbf{E} : 3C, \quad V : 1C)$

(자계) V_m : Magnetic Scalar Potential , 자위. $\mathbf{H} = -\nabla V_m$

Dimension : V_e ⟶ [Volt]

V_m ⟶ [Ampere]

$\nabla \cdot \mathbf{B} = \mu_0 \nabla \cdot \mathbf{H} = \mu_0 \cdot (-\nabla V_m) \equiv 0$ $\boxed{\therefore \nabla^2 V_m = 0}$ Where J = 0, Laplace Equation

한계 $\nabla \times \mathbf{H} = \nabla \times (-\nabla V_m) = \mathbf{J}$ $\nabla \times (\nabla Z) \equiv 0$ ∴ J = 0 인 범위 내에서만 V_m 을 정의됨

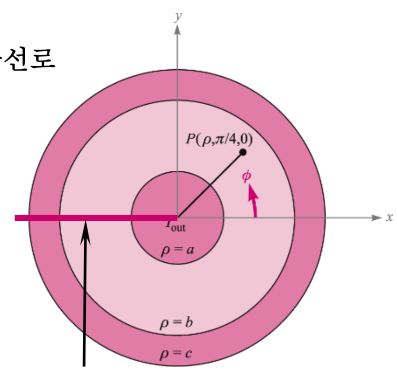
다가함수

(전계) : $\nabla \times \mathbf{E} = \mathbf{0}$, $\boxed{\oint \mathbf{E} \cdot d\mathbf{L} = 0}$, $V_{ab} = \int_a^b \mathbf{E} \cdot d\mathbf{L}$ Ve : 보존계 : a 와 b 의 위치함수. 적분 경로에는 무관하게 unique

(자계) : $\nabla \times \mathbf{H} = \mathbf{0}$, $\boxed{\oint \mathbf{H} \cdot d\mathbf{L} = I,}$ $V_{m,ab} = \int_a^b \mathbf{H} \cdot d\mathbf{L}$ Vm : 비보존계 : a에서 b까지의 적분경로가 도선을 한바퀴 돌 때마다 I만큼 증가

not 1가함수 적분경로에 의지하는 비 보존계.

< Ex > 동축선로



Barrier at $\phi = \pi$

$$\mathbf{H} = \frac{I}{2\pi r}\hat{a}_\phi = -\nabla V_m \hat{a}_\phi = -\frac{1}{r}\cdot\frac{\partial V_m}{\partial \phi}\hat{a}_\phi$$

$$\frac{\partial V_m}{\partial \phi} = -\frac{I}{2\pi}, \quad V_m = \int_0^\phi \left(-\frac{I}{2\pi}\right) d\phi = -\frac{I}{2\pi}\phi$$

At point P ($\phi = \pi/4$) : $V_{mP} = -\frac{I}{8} \quad \left(\phi = \frac{\pi}{4}\right)$

$$\phi = \frac{\pi}{4}, \quad \frac{9}{4}\pi, \quad \frac{17}{4}\pi, \ldots\ldots \longrightarrow \boxed{V_m = \frac{I}{2\pi}\left(2n - \frac{1}{4}\right)\pi}$$

✓ 다가함수

(2) 자기 벡터 포텐셜 (A)

• 유도 : $\nabla \cdot \mathbf{B} = 0$ 이며 $\nabla \cdot (\nabla \times ?) = 0$ 은 항상 만족하므로 $\boxed{\mathbf{B} = \nabla \times \mathbf{A}}$ 로 정의하면

$\nabla \cdot \mathbf{B} = 0$ 은 항상 자동으로 만족된다.

• $\nabla \times \mathbf{H} = \frac{1}{\mu_0} \nabla \times \nabla \times \mathbf{A}$ (= J)

• B : 3C $\longrightarrow$ A : 3C 전위를 도입하는 장점은 ?

A 로 정의되는 Biot – Savart's law

$$\left(\begin{aligned} \mathbf{A} &= \int_l \frac{\mu_0}{4\pi R} I d\mathbf{L} && \text{(선전류)} \\ &= \int_S \frac{\mu_0}{4\pi R} \mathbf{K} dS && \text{(면전류)} \\ &= \int_v \frac{\mu_0}{4\pi R} \mathbf{J} dS && \text{(체적전류)} \end{aligned} \right)$$

cf. 전계에서

$$V = \int_v \frac{\rho_v}{4\pi R \varepsilon_0} dv$$

✓ A의 방향 : $d\mathbf{A} = \frac{\mu_0 I}{4\pi R} d\mathbf{L}$ $\quad d\mathbf{A} \ // \ d\mathbf{L}$

A 벡터의 방향은 전류가 흐르는 방향과 같다 !

Lab. of Applied ElectroMagnetics

7.7. 정상자계법칙 유도

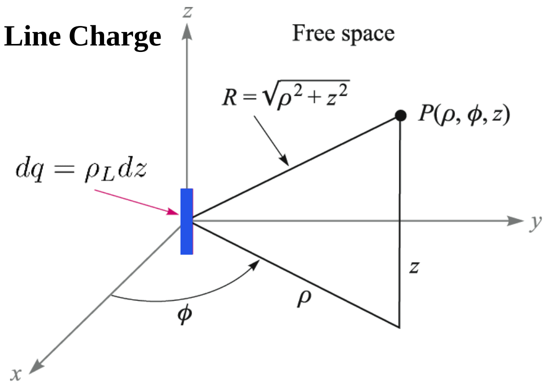
- $\nabla \cdot \mathbf{B} = 0 \qquad \nabla \cdot \nabla \times \mathbf{A} = 0 \qquad \mathbf{B} = \nabla \times \mathbf{A}$

  $\nabla \times \mathbf{H} = \mathbf{J} = \dfrac{1}{\mu_0} \nabla \times \nabla \times \mathbf{A}\,, \quad \nabla \times \nabla \times \mathbf{A} \equiv \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} = -\nabla^2 \mathbf{A} \qquad \nabla \cdot \mathbf{A} = 0$ (∵ 적분 volume 내에 전류 loop 이 모두 있을경우)

  $\boxed{\therefore \nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}}$: Poisson Equation. cf. $\nabla^2 V = -\dfrac{\rho}{\varepsilon}$

- $\boxed{\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}} \qquad \nabla^2 \mathbf{A} = \nabla^2 A_x \mathbf{a}_x + \nabla^2 A_y \mathbf{a}_y + \nabla^2 A_z \mathbf{a}_z \qquad \begin{pmatrix} \nabla^2 A_x = -\mu_0 J_x \\ \nabla^2 A_y = -\mu_0 J_y \\ \nabla^2 A_z = -\mu_0 J_z \end{pmatrix}$

- **Line Charge**

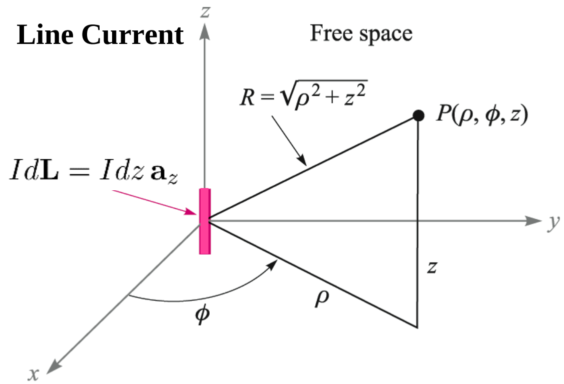


Scalar Electrostatic Potential

$$\boxed{dV = \frac{dq}{4\pi\epsilon_0 R} = \frac{\rho_L dL}{4\pi\epsilon_0 R}}$$

$$V = \int \frac{\rho_L dL}{4\pi\epsilon_0 R}$$

Line Current



$$d\mathbf{A} = \frac{\mu_0 I\, dz\, \mathbf{a}_z}{4\pi\sqrt{\rho^2 + z^2}}$$

$$d\mathbf{H} = \frac{1}{\mu_0} \nabla \times d\mathbf{A} = \frac{1}{\mu_0}\left(-\frac{\partial dA_z}{\partial \rho}\right)\mathbf{a}_\phi$$

$$= \frac{I\, dz}{4\pi} \frac{\rho}{(\rho^2 + z^2)^{3/2}} \mathbf{a}_\phi$$

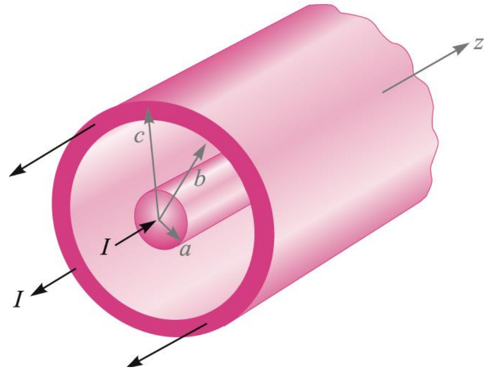
✓ The same as Biot-Savart Result

Vector Magnetic Potential

$$\boxed{d\mathbf{A} = \frac{\mu_0 I d\mathbf{L}}{4\pi R} = \frac{\mu_0 I dz\, \mathbf{a}_z}{4\pi R}}$$

$$\boxed{\mathbf{A} = \int_S \frac{\mu_0 \mathbf{K}\, dS}{4\pi R}} \quad \boxed{\mathbf{A} = \oint \frac{\mu_0 I\, d\mathbf{L}}{4\pi R}} \quad \boxed{\mathbf{A} = \int_{\text{vol}} \frac{\mu_0 \mathbf{J}\, dv}{4\pi R}}$$

< Ex > 동축케이블 에서의 자기 벡터 포텐셜



- $\mathbf{A} = A_z \hat{a}_z$ 도체사이에서 $\nabla^2 A_z = 0$ (∵ 전류 zero 부분 !)

- 원통좌표계에서 $\nabla^2 A_z = \frac{1}{r}\cdot\frac{\partial}{\partial r}(r\frac{\partial A_z}{\partial r}) + \frac{1}{r^2}\cdot\cancel{\frac{\partial^2 A_z}{\partial \phi^2}} + \cancel{\frac{\partial^2 A_z}{\partial z^2}} = 0$ ✓ Az 의 변화 !

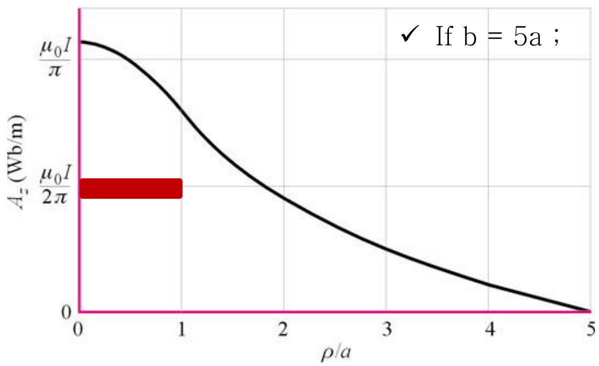
$A_z = f(r)$ 이므로 $\frac{1}{r}\cdot\frac{\partial}{\partial r}(r\frac{\partial A_z}{\partial r}) = 0, \quad r\frac{\partial A_z}{\partial r} = C \longrightarrow A_z = C_1 \ln r + C_2$

B.C. (i) r = b 에서 $A_z = 0 \rightarrow A_z = C_1 \ln\frac{r}{b}$

(ii) $\nabla\times\mathbf{A} = -\frac{\partial A_z}{\partial r}\hat{a}_\phi = -\frac{C_1}{r}\hat{a}_\phi = \mathbf{B} \qquad \mathbf{H} = \frac{\mathbf{B}}{\mu_0} = -\frac{C_1}{\mu_0 r}\hat{a}_\phi$

$$\oint \mathbf{H}\cdot d\mathbf{L} = I = \int_0^{2\pi}(-\frac{C_1}{\mu_0 r})\hat{a}_\phi \cdot r d\phi \hat{a}_\phi = -\frac{2\pi C_1}{\mu_0} \qquad \therefore C_1 = -\frac{\mu_0 I}{2\pi}$$

$$\boxed{\therefore A_z = \frac{\mu_0 I}{2\pi}\ln\frac{b}{r}}, \quad H_\phi = \frac{I}{2\pi r}$$



Lab. of Applied ElectroMagnetics

◆ Sum : field, flux, unit

	전계	자계
Flux	$\Psi \quad [C]$	$\Phi \quad [Wb]$
Flux Density	$\mathbf{D} \quad [C/m^2]$	$\mathbf{B} \quad [Wb/m^2 \equiv T]$
Field Intensity	$\mathbf{E} \quad [V/m^2]$	$\mathbf{H} \quad [A/m]$
Material Constant	$\varepsilon \quad [F/m]$	$\mu \quad [H/m]$

◆ Sum : Equations

미분형	적분형
$\nabla\cdot\mathbf{D}=\rho$ $\nabla\times\mathbf{E}=\mathbf{0}$	$\oint_S \mathbf{D}\cdot d\mathbf{S}=Q$ $\oint_l \mathbf{E}\cdot d\mathbf{L}=0$
$\nabla\cdot\mathbf{B}=\mathbf{0}$ $\nabla\times\mathbf{H}=\mathbf{J}$	$\oint_S \mathbf{B}\cdot d\mathbf{S}=0$ $\oint_l \mathbf{H}\cdot d\mathbf{L}=I$

In dynamic field	**Maxwell**
• $\nabla\cdot\mathbf{D}=\rho$	**(Gauss)**
* $\nabla\times\mathbf{E}=-\dfrac{\partial\mathbf{B}}{\partial t}$	**(Faraday)**
• $\nabla\cdot\mathbf{B}=0$	**(Gauss)**
* $\nabla\times\mathbf{H}=\mathbf{J}+\dfrac{\partial\mathbf{D}}{\partial t}$	**(Ampere)**

$$\left[\begin{array}{ll} \mathbf{D}=\varepsilon\mathbf{E},\quad \mathbf{D}=\varepsilon_0\mathbf{E}+\mathbf{P} & \text{, P : Polarization 분극} \\ \mathbf{B}=\mu\mathbf{H},\quad \mathbf{B}=\mu_0\mathbf{H}+\mathbf{M} & \text{, M : Magnetization 자화} \end{array}\right]$$

$$\left[\begin{array}{ll} \mathbf{E}=-\nabla\mathbf{V} & \text{, V : Electric Scalar Potential} \\ \mathbf{B}=\nabla\times\mathbf{A} & \text{, A : Magnetic Vector Potential} \end{array}\right]$$

➢ Sum :

$$\left[\begin{array}{lll} \text{(전계) :} & Q \xrightarrow[\text{Gauss}]{\nabla\cdot\mathbf{D}=\rho} \begin{bmatrix} E \\ D \end{bmatrix}_{\varepsilon} \longrightarrow & W, V \\ \text{(자계) :} & I \xrightarrow[\text{Ampere}]{\nabla\times\mathbf{H}=\mathbf{J}} \begin{bmatrix} H \\ B \end{bmatrix}_{\mu} \longrightarrow & W, A \end{array}\right]$$

$$\left[\begin{array}{l} \mathbf{F}=Q\mathbf{E} \\ \mathbf{F}=\mathbf{J}\times\mathbf{B} \end{array}\right]$$