Quiz 1: Control Systems Eng.
2019/03/28

Student Number: [ ] Name: **Solution**

1. (20 points = 2×10 pts)

(1)

$$L\{y'' - y\} = s^2Y(s) - sy(0) - y'(0) - Y(s) = 0$$

$$(s^2 - 1)Y(s) + s - 1 = 0, \quad Y(s) = \frac{-1}{s+1}$$

$$L^{-1}\{Y(s)\} = y(t) = -e^{-t}$$

(2)

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A=[0 1 3 0; 0 0 1 0;0 0 0 1;-7 -9 -2 -3];
B=[0; 5; 8; 2];    % B=[0 5 8 2]';
C=[1 3 4 6];
D=0;
[num, den]=ss2tf(A, B, C, D);
G=tf(num, den)
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      59 s^3 - 164 s^2 - 1621 s - 260
G =  ---------------------------------
       s^4 + 3 s^3 + 2 s^2 + 30 s + 7
```

2. (20 points)

Kirchhoff's current law for the output node yields:

$$C\frac{d}{dt}(v_i - v_o) + \frac{1}{R_1}(v_i - v_o) = \frac{1}{R_2}v_o$$

The Laplace transform of this equation (with zero initial conditions) is

$$Cs\{V_i(s) - V_o(s)\} + \frac{1}{R_1}\{V_i(s) - V_o(s)\} = \frac{1}{R_2}V_o(s)$$

$$\{Cs + \frac{1}{R_1}\}V_i(s) = \{Cs + \frac{1}{R_1} + \frac{1}{R_2}\}V_o(s)$$

The transfer function is:

$$\frac{V_o(s)}{V_i(s)} = \frac{Cs + \frac{1}{R_1}}{Cs + \frac{1}{R_1} + \frac{1}{R_2}} = \frac{s + \frac{1}{R_1C}}{s + \frac{1}{R_1C} + \frac{1}{R_2C}}$$

3. (20 points $= 2\times 10$ pts)

(1) $T(s)=\dfrac{Y(s)}{R(s)}=\dfrac{s^2+7s+2}{s^3+9s^2+26s+24}$

(2) $\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{pmatrix}=\begin{pmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -24 & -26 & -9\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}+\begin{pmatrix}0\\ 0\\ 1\end{pmatrix}r \qquad y=\begin{pmatrix}2 & 7 & 1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}$

4. (20 points) $\quad T(s)=\dfrac{Y(s)}{U(s)}=C(sI-A)^{-1}B+D$

$$(sI-A)^{-1}=\left(s\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}-\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\right)^{-1}=\begin{bmatrix}s & -1\\ 0 & s\end{bmatrix}^{-1}=\frac{1}{s^2}\begin{bmatrix}s & 1\\ 0 & s\end{bmatrix}$$

$$T(s)=\frac{Y(s)}{U(s)}=\begin{bmatrix}1 & 0\end{bmatrix}\frac{1}{s^2}\begin{bmatrix}s & 1\\ 0 & s\end{bmatrix}\begin{bmatrix}0\\ \dfrac{1}{m}\end{bmatrix}+0=\frac{1}{s^2}\begin{bmatrix}s & 1\end{bmatrix}\begin{bmatrix}0\\ \dfrac{1}{m}\end{bmatrix}=\frac{1}{ms^2}$$

5. (20 points)

(1) Linear approximation of a function f at a number a is $f(x)=f(a)+(x-a)f'(a)$.

$$f'(x)=\frac{d}{dx}(1+2x)^{\frac{1}{2}}=\frac{1}{2}(1+2x)^{-\frac{1}{2}}(2)=\frac{1}{\sqrt{1+2x}}$$

The linearization of $f(x)$ at 4 is

$$L(x)=f(4)+f'(4)(x-4)=\left(\sqrt{1+8}\right)+\frac{1}{\sqrt{1+8}}(x-4)$$

$$=3+\frac{1}{3}(x-4)$$

(2) We can use the result of (1) to approximate $f(4.3)$.

$$f(4.3)\approx L(4.3)=3+\frac{1}{3}(4.3-4)=3+\frac{1}{3}(0.3)=3.1$$

(c) Real value at $x=4.3$: $f(4.3)=3.098$

$$\begin{aligned}\text{absolute difference} &= \left|\text{real value} - \text{approximated value}\right|\\ &= \left|3.098-3.1\right|\\ &= 0.002\end{aligned}$$