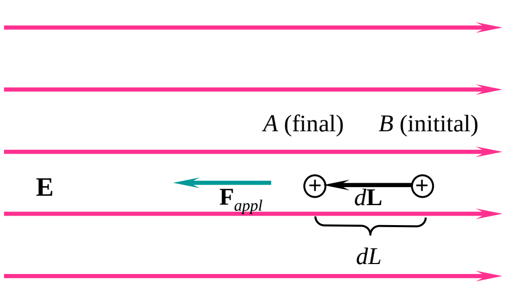
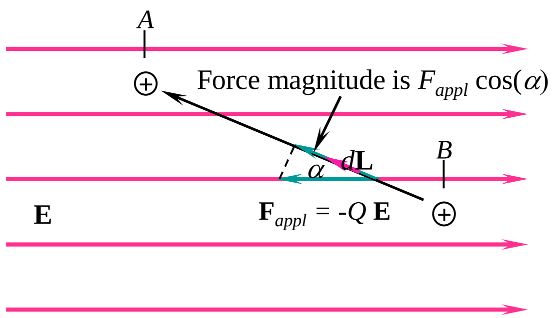
Lab. of Applied ElectroMagnetics

Chap. 4 . Energy / Potential

4.1 전계 내에서 점전하를 이동시키는데 필요한 일

- $\mathbf{F}_{appl} = -Q\,\mathbf{E}$





- $dW = F_{appl}\,dL = QE\,dL = -Q\mathbf{E}\cdot d\mathbf{L}$ [J]
- $dW = F_{appl}\cos(\alpha)\,dL = -Q\mathbf{E}\cdot d\mathbf{L}$

$$W = -Q\int_{\text{init}}^{\text{final}} \mathbf{E}\cdot d\mathbf{L}$$

W<0 : 음의 일. Potential Energy 가 작아짐. 전계가 일함

W>0 : 양의 일. Potential Energy 가 커짐. 외부에서 가해주는 일

$$\boxed{V_{ab} = -\int_b^a \mathbf{E}\cdot d\mathbf{L}} \equiv \frac{W}{Q}$$

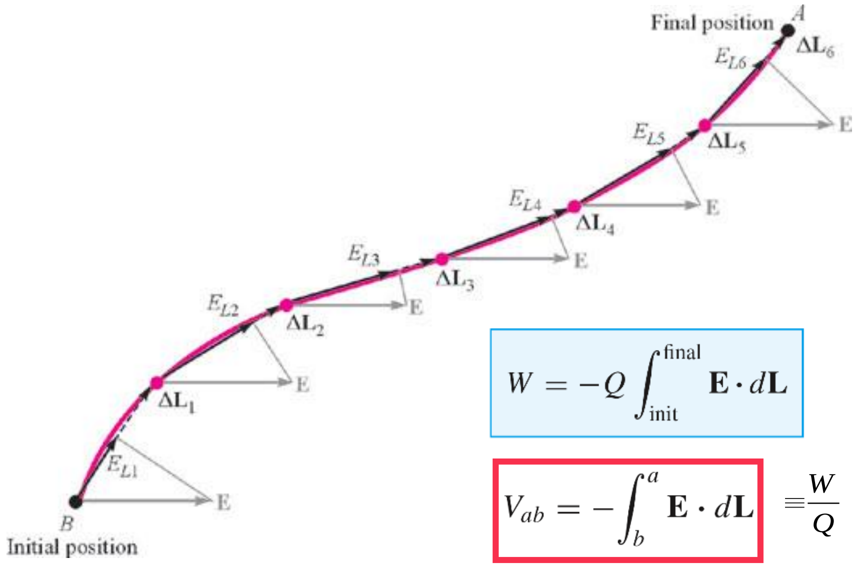
: Electric Potential (전위, 전압)

* Path Independent $\oint \to 0$, A ⇄ B

* Conservation Field. 보존장.

Lab. of Applied ElectroMagnetics

4.2. 선적분 (Line Integral) : $\int_B^A \mathbf{E}\cdot d\mathbf{L}$



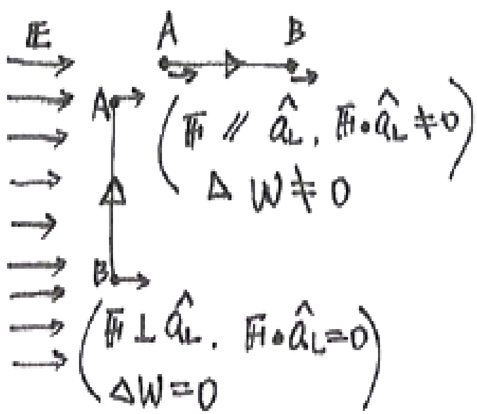
$$W = -Q\int_{\text{init}}^{\text{final}} \mathbf{E}\cdot d\mathbf{L}$$

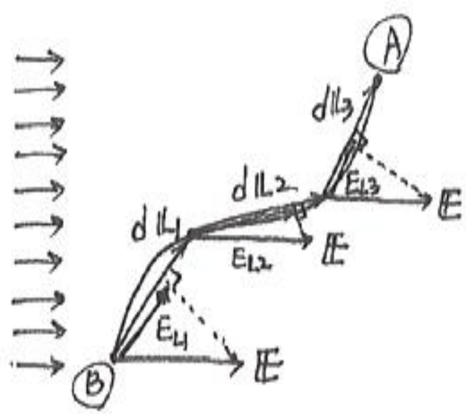
$$\boxed{V_{ab} = -\int_b^a \mathbf{E}\cdot d\mathbf{L}} \equiv \frac{W}{Q}$$

$$\int_B^A \mathbf{E}\cdot d\mathbf{L} \qquad \begin{aligned}\mathbf{E} &= E_x\mathbf{a}_x + E_y\mathbf{a}_y + E_z\mathbf{a}_z\\ d\mathbf{L} &= dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z\end{aligned}$$

$$\int_B^A \mathbf{E}\cdot d\mathbf{L} = \int_B^A (E_x\mathbf{a}_x + E_y\mathbf{a}_y + E_z\mathbf{a}_z)\cdot(dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z)$$

$$= \int_{x_B}^{x_A} E_x\,dx + \int_{y_B}^{y_A} E_y\,dy + \int_{z_B}^{z_A} E_z\,dz$$





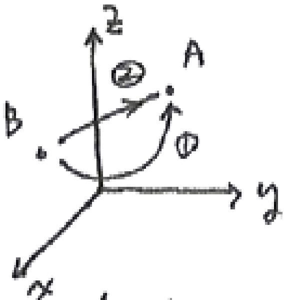
$$W = -Q\int_B^A \mathbf{E}\cdot d\mathbf{L}$$

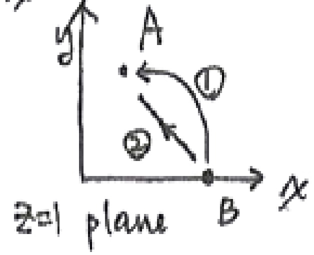
$$= -Q\sum \vec{E}_i\cdot d\vec{L}_i$$

$$= -Q(E_{L1}\Delta L_1 + E_{L2}\Delta L_2 + E_{L3}\Delta L_3)$$

$$W = -Q\int_{\text{init}}^{\text{final}} \mathbf{E}\cdot d\mathbf{L}$$

(Ex 4.1/4.2) 선적분과 경로





$\therefore$ *Path ① = Path ②*

$\mathbf{E} = y\mathbf{a}_x + x\mathbf{a}_y + 2\mathbf{a}_z$ Q = +2 [C]

$B(1, 0, 1) \rightarrow A(0.8, 0.6, 1)$

- Path I : $x^2 + y^2 = 1\,,\ z = 1$ 을 따라 이동. ①
- Path II : $y = -3(x-1)\,,\ z = 1$ 을 따라 이동. ②

sol) ① : $d\vec{L} = dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z$

$$W = -Q\int_B^A \mathbf{E}\cdot d\mathbf{L} = -2\int_B^A (y\hat{a}_x + x\hat{a}_y + 2\hat{a}_z)\cdot(dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z)$$

$$= -2\int_1^{0.8} y\,dx - 2\int_0^{0.6} x\,dy - 4\int_1^1 dz\ ,\quad (\because x^2 + y^2 = 1)$$

$$= -2\int_1^{0.8}\sqrt{1-x^2}\,dx - 2\int_0^{0.6}\sqrt{1-y^2}\,dy - 0$$

$$= -\left[x\sqrt{1-x^2} + \sin^{-1}x\right]_1^{0.8} - \left[y\sqrt{1-y^2} + \sin^{-1}y\right]_0^{0.6} \underline{= -0.96\ [J]}$$

② : $W = -2\int_1^{0.8} y\,dx - 2\int_0^{0.6} x\,dy - 4\int_1^1 dz\ ,\quad (\because y = -3(x-1))$

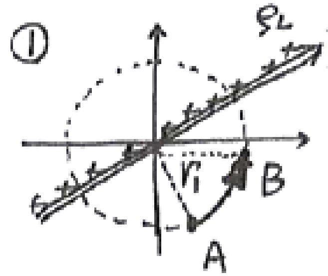
$$= 6\int_1^{0.8}(x-1)\,dx - 2\int_0^{0.6}\left(1-\frac{y}{3}\right)dy\ \underline{= -0.96\,[J]}$$

◎ Sum : $W = -Q\int_B^A \mathbf{E}\cdot d\mathbf{L}$ $V_{ab} = -\int_b^a \mathbf{E}\cdot d\mathbf{L}$ $\vec{E} = -\nabla V$

* 전위 계산과 **Coordinate**

$d\mathbf{L} = dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z$	(rectangular)
$d\mathbf{L} = d\rho\,\mathbf{a}_\rho + \rho\,d\phi\mathbf{a}_\phi + dz\,\mathbf{a}_z$	(cylindrical)
$d\mathbf{L} = dr\,\mathbf{a}_r + r\,d\theta\,\mathbf{a}_\theta + r\sin\theta\,d\phi\,\mathbf{a}_\phi$	(spherical)

* 무한 선전하와 에너지



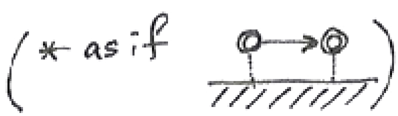
$d\mathbf{L} = \rho_1 \, d\phi \, \mathbf{a}_\phi$

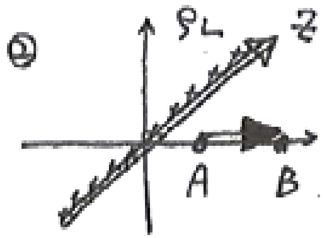
$$\mathbf{E} = E_\rho \mathbf{a}_\rho = \frac{\rho_L}{2\pi\epsilon_0 \rho}\mathbf{a}_\rho$$

$(\Theta \;\; \hat{a}_r \cdot \hat{a}_\phi = 0)$

$$W = -Q\int_{\text{init}}^{\text{final}} \frac{\rho_L}{2\pi\epsilon_0\rho_1}\mathbf{a}_\rho \cdot \rho_1 \, d\phi \, \mathbf{a}_\phi$$

$$= -Q\int_0^{2\pi} \frac{\rho_L}{2\pi\epsilon_0} d\phi \, \mathbf{a}_\rho \cdot \mathbf{a}_\phi = 0$$





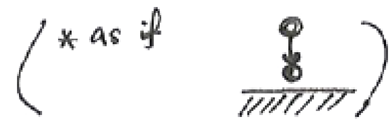
$d\mathbf{L} = d\rho \, \mathbf{a}_\rho$

$$\mathbf{E} = E_\rho \mathbf{a}_\rho = \frac{\rho_L}{2\pi\epsilon_0 \rho}\mathbf{a}_\rho$$

$(\Theta \;\; \hat{a}_r \cdot \hat{a}_r = 1)$

$$W = -Q\int_{\text{init}}^{\text{final}} \frac{\rho_L}{2\pi\epsilon_0\rho}\mathbf{a}_\rho \cdot d\rho \, \mathbf{a}_\rho = -Q\int_a^b \frac{\rho_L}{2\pi\epsilon_0}\frac{d\rho}{\rho}$$

$$= -\frac{Q\rho_L}{2\pi\epsilon_0}\ln\frac{b}{a}$$



* W<0, 즉 A→B 이동시 전계가 일한다.

Potential E 줄어든다.(전압감소)

전하는 전계로부터 힘을 받으며 이동

4.3 전위차, 전위, 전압

- Energy : $W = -Q \int_{\text{init}}^{\text{final}} \mathbf{E} \cdot d\mathbf{L}$
- Potential (Difference) 전위(차), 전압 : $\text{Potential Difference} = \frac{W}{Q} = -\int_{\text{init}}^{\text{final}} \mathbf{E} \cdot d\mathbf{L} \ \ \text{Volts}$

$$\boxed{V_{AB} = V_A - V_B = -\int_B^A \mathbf{E} \cdot d\mathbf{L}}$$

※ 도선전압/공간전압

- 단위전하가 전계 중에서 가지는 Potential Energy. V=W/Q
- 단위 : $V \Rightarrow \frac{W}{Q}[\frac{J}{C} \equiv Volt, V].Volta$
- 전위는 상대적인 양. 기준점에 대한 상대적 차이량

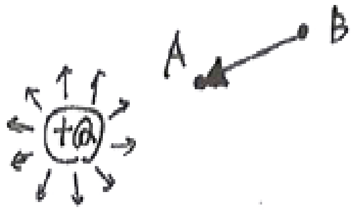
$V_{AB} = V_A - V_B$ 절대전위 : V_B 를 무한원점(또는 대지에 대한전위)로 약속한 환산량

➢ Q → E → D → V

Lab. of Applied ElectroMagnetics

4.4 점전하에 의한 전위

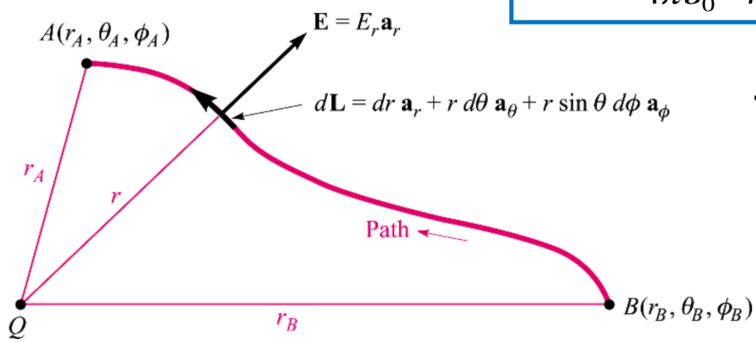
(1) 점전하에의한 Electric Potential :



$$* \begin{cases} \mathbf{E} = E_r \mathbf{a}_r = \dfrac{Q}{4\pi\epsilon_0 r^2}\mathbf{a}_r \\ d\mathbf{L} = dr\,\mathbf{a}_r + r\,d\theta\,\mathbf{a}_\theta + r\sin\theta\,d\phi\,\mathbf{a}_\phi \end{cases}$$

$$V_{AB} = -\int_B^A \mathbf{E}\cdot d\mathbf{L} = -\int_{r_B}^{r_A} \frac{Q}{4\pi\epsilon_0 r^2}\,dr = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

$$\therefore \boxed{V_{AB} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)}$$



- $V_{AB}>0$, 즉 $W_{AB}>0$, E증가, 전압증가(B→A), 외부에서 일을 해주어야 함.

$$* \begin{cases} \left|\vec{E}\right| = \dfrac{Q}{4\pi\varepsilon_0}\cdot\dfrac{1}{r^2} \propto \dfrac{1}{r^2} \\ V = \dfrac{Q}{4\pi\varepsilon_0}\cdot\dfrac{1}{r} \propto \dfrac{1}{r} \end{cases}$$

Lab. of Applied ElectroMagnetics

➢ 무한원점 기준 점전하 전위 : <u>(절대 전위)</u>

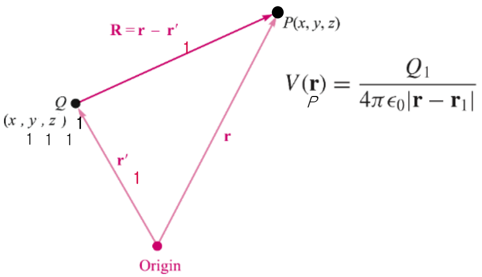
$$V_{AB} = -\int_B^A \vec{E}\cdot d\vec{L} = V_A - V_B = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right) \qquad \text{무한원점 : } r_B \to \infty,\ V_B \to 0$$

$$V_{r\infty} = V_r - V_\infty = -\int_\infty^r \mathbf{E}\cdot d\mathbf{L} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r} - \frac{1}{\infty}\right) = \frac{Q}{4\pi\epsilon_0 r}$$

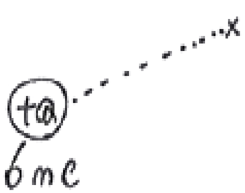
$$\therefore \quad \boxed{V_{AB} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r}[V]}$$

➢ 특정 점을 기준으로 한 전위 : <u>(상대 전위)</u>

$$V = \frac{Q}{4\pi\epsilon_0 r} + C_1$$



(Ex.) P(0.2, -0.4, 0.4)



(a) 절대전위, 무한원점에서 V=0 일 경우 : $V = \dfrac{Q}{4\pi\varepsilon_0 r} = \dfrac{6\times10^{-9}}{4\pi\times8.854\times10^{-12}\times0.2}[V]$

(b) 상대전위, 점(1, 0, 0)에서 V=0 일 경우 :

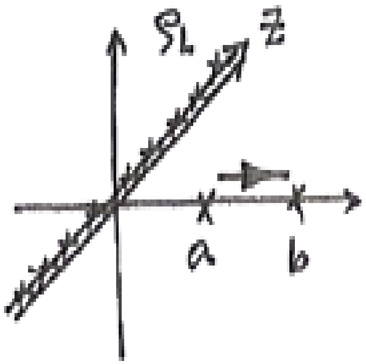
$$V = \frac{Q}{4\pi\varepsilon_0 r} + C_1 \ , \quad \text{r = 1일 때 V=0 이므로} \quad , \quad 0 = \frac{6\times10^{-9}}{4\pi\varepsilon_0 r} + C_1$$

$$\therefore \quad V = \frac{6\times10^{-9}}{4\pi\varepsilon_0 r} - \frac{6\times10^{-9}}{4\pi\varepsilon_0} = \frac{6\times10^{-9}}{4\pi\varepsilon_0}\left(\frac{1}{r} - 1\right)$$

$$\therefore \quad V = \frac{6\times10^{-9}}{4\pi\varepsilon_0}\left(\frac{1}{0.2} - 1\right) = \frac{6\times10^{-9}}{\pi\varepsilon_0}[V]$$

[6nC 에서 20cm 거리 → 약 216 Volt 절대전위]

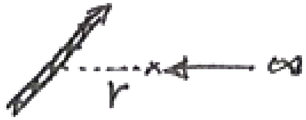
(2) 무한 선전하에 의한 Electric Potential :



$$W = -\frac{Q\rho_L}{2\pi\varepsilon_0}\ln\frac{b}{a}$$

$$\boxed{V_{ab} = -\frac{\rho_L}{2\pi\varepsilon_0}\ln\frac{b}{a}}$$

* 무한원점에서 부터의 전계에 대한 전위 :



$$V_{ab} = \frac{\rho_L}{2\pi\varepsilon_0}\ln r$$

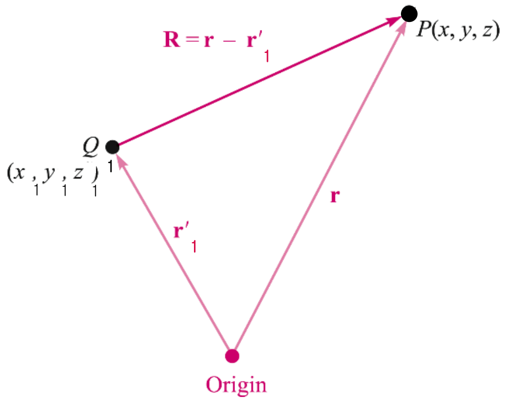
* 방향 및 부호

* 기준점에 대한 전위 :

r = 0 일때 V_0 전위 기준 → $V = \frac{\rho_L}{2\pi\varepsilon_0}\ln r + C$

(3) 체적 전하에 의한 Electric Potential :

➢ 다중전하에 의한 전압 :



$$V(\mathbf{r}) = \frac{Q_1}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}_1|}$$

$$V(\mathbf{r}) = \sum_{m=1}^{n} \frac{Q_m}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}_m|}$$

$$V(\mathbf{r}) = \int_{\text{vol}} \frac{\rho_v(\mathbf{r}')\,dv'}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}'|}$$

➢ 체적전하에 의한 전압 :

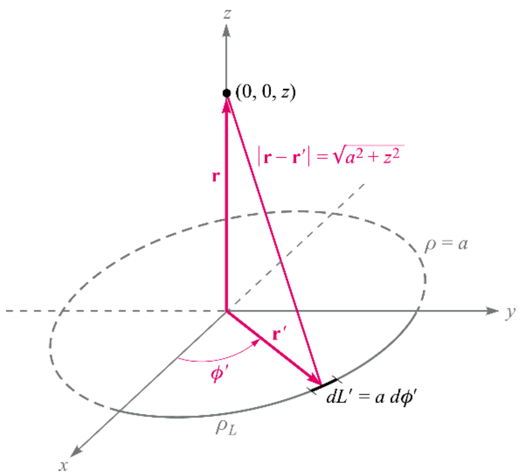
Line Charge: $V(\mathbf{r}) = \int \frac{\rho_L(\mathbf{r}')\,dL'}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}'|}$

Surface Charge: $V(\mathbf{r}) = \int_S \frac{\rho_S(\mathbf{r}')\,dS'}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}'|}$

Volume Charge: $V(\mathbf{r}) = \int_{\text{vol}} \frac{\rho_v(\mathbf{r}')\,dv'}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}'|}$

.cf. (more difficult): $\mathbf{E}(\mathbf{r}) = \int_{\text{vol}} \frac{\rho_v(\mathbf{r}')\,dv'}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}'|^2} \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}$

(Ex) 원형 선 전하에 의한 Electric Potential :



$$dL' = ad\phi'$$

$$\mathbf{r} = z\mathbf{a}_z$$

$$\mathbf{r}' = a\mathbf{a}_\rho$$

$$|\mathbf{r} - \mathbf{r}'| = \sqrt{a^2 + z^2}$$

$$V(\mathbf{r}) = \int \frac{\rho_L(\mathbf{r}')\,dL'}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r}'|}$$

$$V = \int_0^{2\pi} \frac{\rho_L a\,d\phi'}{4\pi\epsilon_0\sqrt{a^2 + z^2}} = \frac{\rho_L a}{2\epsilon_0\sqrt{a^2 + z^2}} \qquad \left(= \frac{\rho_L a}{4\pi\varepsilon_0\sqrt{a^2 + z^2}} \int_0^{2\pi} d\phi' \right)$$

$$\boxed{V = \frac{\rho_L a}{2\varepsilon_0\sqrt{a^2 + z^2}}}$$

z = 0. 원점에서 $\boxed{V = \dfrac{\rho_L a}{2\varepsilon_0}}$ $.cf.[\vec{E}] = 0$

z = ∞. 무한점에서 $V = 0$

4.5 보존장

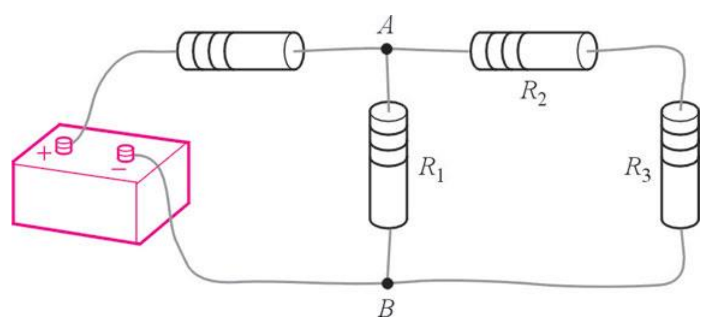
- $V_{ab} = -\int_b^a \mathbf{E} \cdot d\mathbf{L}$

- Q를 A→B →A 로 이동시킬 때의 일 :

$$V_{AB} + V_{BA} = -\int_B^A \vec{E} \cdot d\vec{L} - \int_A^B \vec{E} \cdot d\vec{L}$$

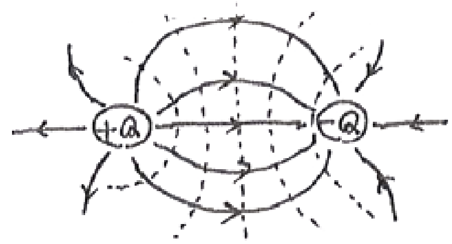
$$\begin{cases} 左 = & V_{AB} + V_{BA} = 0 \\ 右 = & -\int_B^A \vec{E} \cdot d\vec{L} = -\int_A^B \vec{E} \cdot d\vec{L} \end{cases} \Rightarrow \boxed{\oint \vec{E} \cdot d\vec{L} = 0}$$

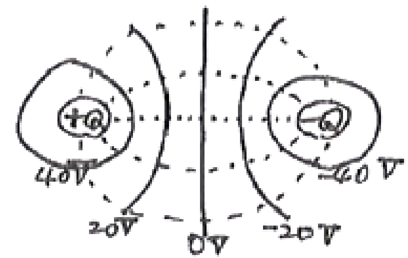
: 보존장. Conservative Field, {Kirchhoff 법칙.} $\left(\sum_i V_i = 0 \right)$



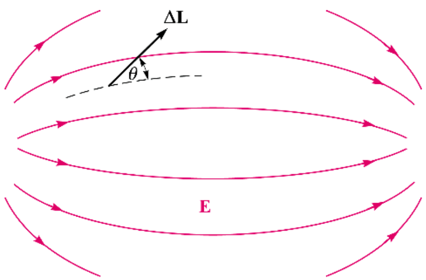
4.6 전위 경도(Potential Gradient)

- Flux Line & Equi-Potential Line





$$V_{ab} = -\int_b^a \mathbf{E} \cdot d\mathbf{L}$$



$$\Delta V \doteq -\mathbf{E} \cdot \Delta\mathbf{L}$$

$$\Delta V \doteq -E \Delta L \cos\theta$$

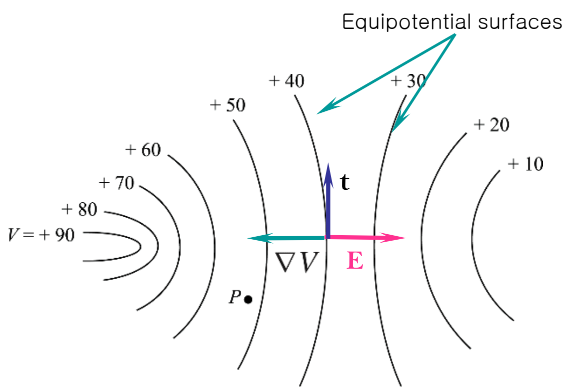
$$\frac{dV}{dL} = -E\cos\theta$$

$$\left.\frac{dV}{dL}\right|_{\max} = E$$

$$\mathbf{E} = -\left.\frac{dV}{dL}\right|_{\max} \mathbf{a}_N$$

$\mathbf{a}_N$: unit vector in the direction of *increasing* potential

E points in the direction of maximum rate of *decrease* in potential
→ in the direction of the *negative gradient* of *V*.



$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

$$dV = -\mathbf{E} \cdot d\mathbf{L} = -E_x\,dx - E_y\,dy - E_z\,dz$$

$$E_x = -\frac{\partial V}{\partial x}$$

$$E_y = -\frac{\partial V}{\partial y}$$

$$E_z = -\frac{\partial V}{\partial z}$$

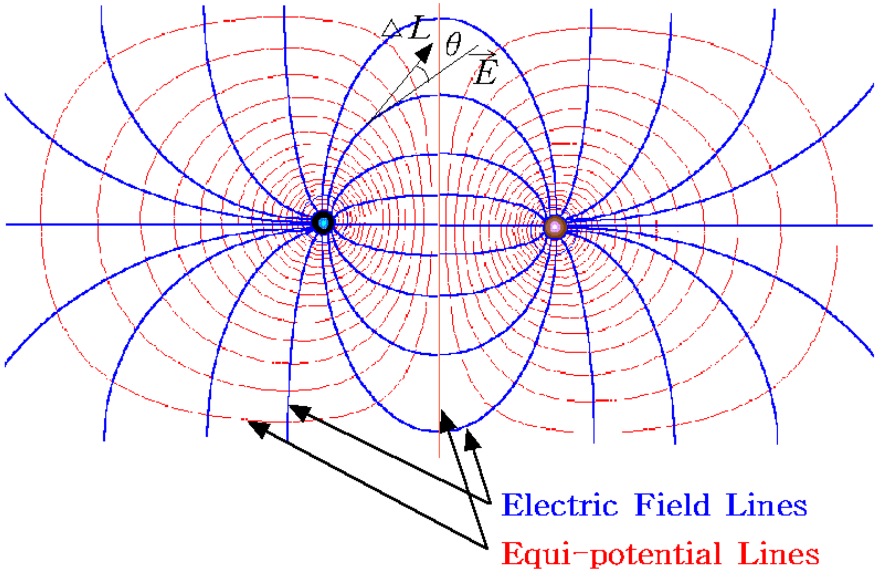
$$\boxed{\mathbf{E} = -\left(\frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z\right)}$$

$$\nabla = \frac{\partial}{\partial x}\mathbf{a}_x + \frac{\partial}{\partial y}\mathbf{a}_y + \frac{\partial}{\partial z}\mathbf{a}_z$$

$$\boxed{\mathbf{E} = -\nabla V}$$

✓ E is equal to the *negative gradient* of *V*

Lab. of Applied ElectroMagnetics



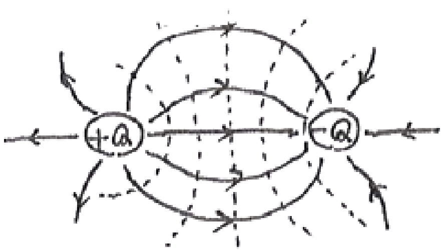
$$\boxed{V_{ab} = -\int_b^a \mathbf{E} \cdot d\mathbf{L}}$$

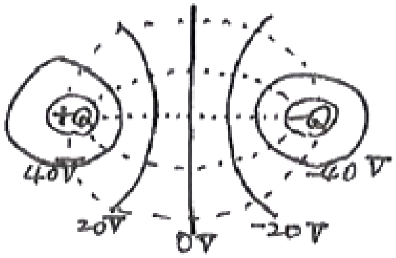
$$\vec{E} = -\text{grad } V = -\left(\frac{\partial V}{\partial x} \hat{a}_x + \frac{\partial V}{\partial y} \hat{a}_y + \frac{\partial V}{\partial z} \hat{a}_z \right)$$

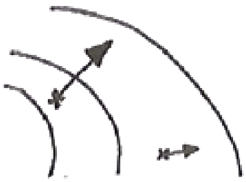
$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$

$$E = -\left.\frac{dV}{dL}\right|_{\max} \hat{a}_N = -\text{grad } V = -\nabla V$$

$$\boxed{\mathbf{E} = -\nabla V}$$







Lab. of Applied ElectroMagnetics

$$\mathbf{E} = -\nabla V$$

$$\begin{cases} \nabla V = \dfrac{\partial V}{\partial x}\mathbf{a}_x + \dfrac{\partial V}{\partial y}\mathbf{a}_y + \dfrac{\partial V}{\partial z}\mathbf{a}_z & \text{(rectangular)} \\ \nabla V = \dfrac{\partial V}{\partial \rho}\mathbf{a}_\rho + \dfrac{1}{\rho}\dfrac{\partial V}{\partial \phi}\mathbf{a}_\phi + \dfrac{\partial V}{\partial z}\mathbf{a}_z & \text{(cylindrical)} \\ \nabla V = \dfrac{\partial V}{\partial r}\mathbf{a}_r + \dfrac{1}{r}\dfrac{\partial V}{\partial \theta}\mathbf{a}_\theta + \dfrac{1}{r\sin\theta}\dfrac{\partial V}{\partial \phi}\mathbf{a}_\phi & \text{(spherical)} \end{cases}$$

➢ **Sum :**

$$\left[\; V_{known} \underset{V=-\int \vec{E}\cdot d\vec{L}}{\overset{\vec{E}=-\nabla V}{\rightleftarrows}} \vec{E} \underset{\vec{E}=\vec{D}/\varepsilon_0}{\overset{\vec{D}=\varepsilon_0\vec{E}}{\rightleftarrows}} \vec{D} \underset{\nabla\cdot\vec{D}=\rho}{\overset{\nabla\cdot\vec{D}=\rho}{\rightleftarrows}} \rho \;\right]$$

Potential/Energy Field/Flux Charge

(Ex.) $V \rightarrow \vec{E} \rightarrow \vec{D} \rightarrow \rho \;\; at\ P$

$V = 2x^2y - 5z$, 점 P(−4, 3, 6) 에서 $\vec{E} = ?$, $\rho = ?$

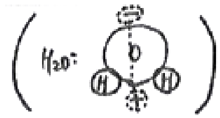
sol .) $\vec{E} = -\nabla V = -\dfrac{\partial V}{\partial x}\hat{a}x - \dfrac{\partial V}{\partial y}\hat{a}y - \dfrac{\partial V}{\partial z}\hat{a}z = -4xy\hat{a}x - 2x^2\hat{a}y + 5\hat{a}z$

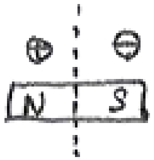
$$\vec{E}\Big|_P = \underline{48\hat{a}x - 32x^2\hat{a}y + 5\hat{a}z \;\; [V/m]}$$

$$\vec{D} = \varepsilon_0\vec{E} = -35.4xy\hat{a}x - 17.71x^2\hat{a}y + 44.3\hat{a}z \;\; [pC/m^2]$$

$$\rho_v = \nabla\cdot\vec{D} = -35.4y\,[pC/m^3] \qquad \rho_v\big|_P = -106.2\,[pC/m^3] = \underline{-0.106\,[nC/m^3]}$$

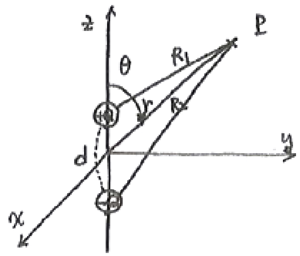
4.7 전기 쌍극자 (Electric Dipole)

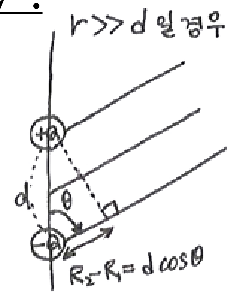




- What is dipole? Where are they?
- Monopole / Dipole
  - Electric Monopole / Electric Dipole
  - Magnetic Monopole / Magnetic Dipole

- **전기 쌍극자에 의한 전위 V :**





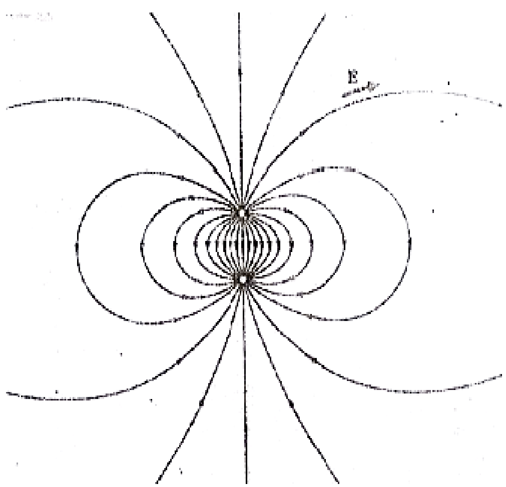
$$V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{Q}{4\pi\varepsilon_0}\cdot\frac{R_2 - R_1}{R_1 R_2}$$

$$\therefore \boxed{V = \frac{Q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2}} \quad \left(\begin{aligned} &\because R_2 - R_1 = d\cos\theta \\ &R_1 \cdot R_2 = r^2 \end{aligned}\right)$$

Point Charge 에 의한 $V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r}$, $V \propto \frac{1}{r}$

Dipole Moment 에 의한 $V = \ // \ \cdot\frac{Q}{r^2}$, $V \propto \frac{1}{r^2}$

- **전기쌍극자에 의한 전계 E :**



$$\mathbf{E} = -\nabla V = -\left(\frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{a}_\phi\right)$$

$$= -\left[\frac{\partial}{\partial r}\left(\frac{Qd\cos\theta}{4\pi\varepsilon_0 r^2}\right)\hat{a}_r + \frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{Qd\cos\theta}{4\pi\varepsilon_0 r^2}\right)\hat{a}_\theta + 0\right]$$

$$= -\left(-\frac{Qd\cos\theta}{2\pi\varepsilon_0 r^3}\hat{a}_r - \frac{Qd\sin\theta}{4\pi\varepsilon_0 r^2}\hat{a}_\theta\right)$$

$$\boxed{\therefore \mathbf{E} = \frac{Qd}{4\pi\varepsilon_0 r^3}(2\cos\theta\hat{a}_r + \sin\theta\hat{a}_\theta)}$$

Point Charge 에 의한 $\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}\hat{a}_r$ $\quad |\mathbf{E}| \propto \frac{1}{r^2}$

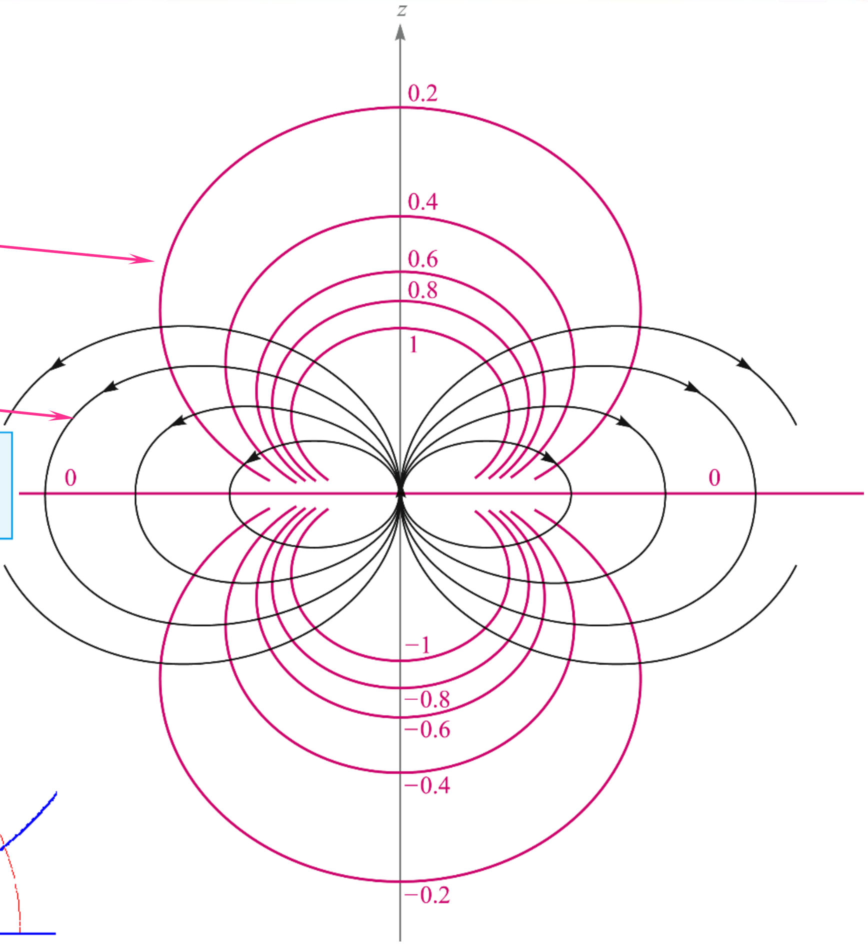
Dipole Moment에 의한 $\sim$ $\quad |\mathbf{E}| \propto \frac{1}{r^3}$

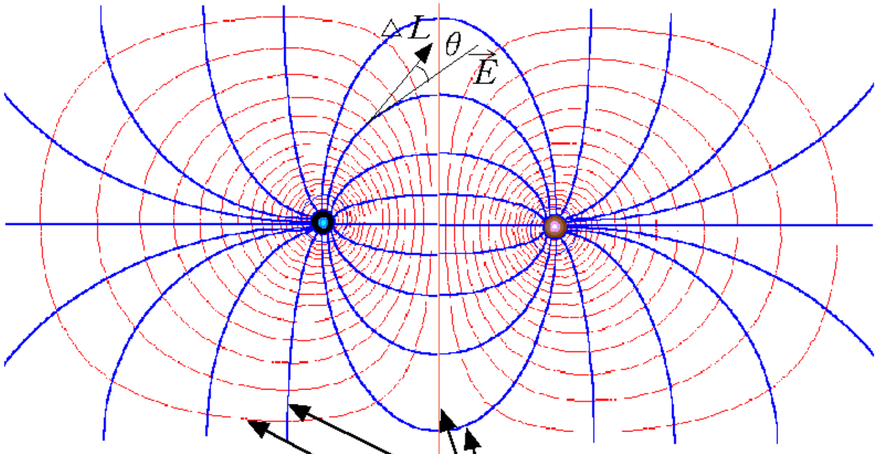
$$V = \frac{Qd\cos\theta}{4\pi\epsilon_0 r^2}$$

Equipotential surface

Electric field streamline

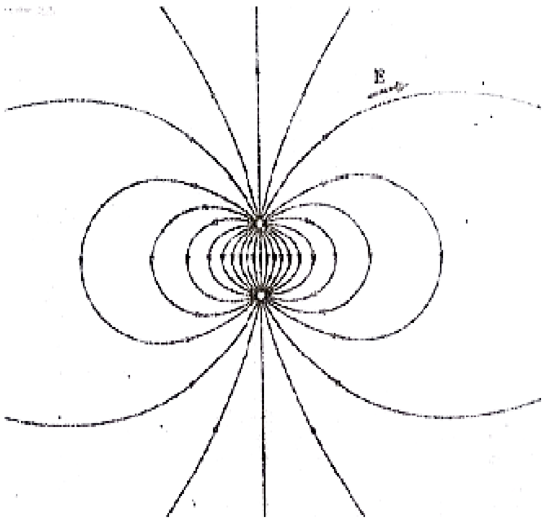
$$\mathbf{E} = \frac{Qd}{4\pi\epsilon_0 r^3}(2\cos\theta\,\mathbf{a}_r + \sin\theta\,\mathbf{a}_\theta)$$





Lab. of Applied ElectroMagnetics

- **전기쌍극자 모멘트 (Electric Dipole moment) P :**



$$\boxed{\mathbf{p} = Q\mathbf{d}}$$; 쌍극자 모멘트

$\left(\begin{array}{l} |\vec{P}| \propto Q, d \\ 방향 \,//\, \vec{d} \end{array}\right)$ $\begin{array}{l} d \to \vec{d}\,; 크기는 거리, 방향은 -Q \to +Q \\ \vec{a}r : radial, \mathbf{d}\cdot\mathbf{a}_r = d\cos\theta \end{array}$

$$\boxed{V = \frac{\mathbf{p}\cdot\mathbf{a}_r}{4\pi\epsilon_0 r^2}} \qquad V = \frac{1}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}'|^2}\mathbf{p}\cdot\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}$$

Point Charge 에 의한 $V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r}$, $V \propto \frac{1}{r}$

Dipole Moment 에 의한 $V =$ // $\cdot\frac{Q}{r^2}$, $V \propto \frac{1}{r^2}$

$$\boxed{\therefore \mathbf{E} = \frac{Qd}{4\pi\varepsilon_0 r^3}(2\cos\theta\hat{a}_r + \sin\theta\hat{a}_\theta)}$$

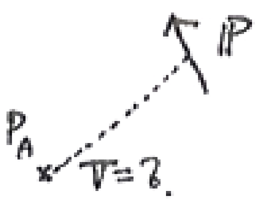
Point Charge 에 의한 $\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}\hat{a}_r$ $|\mathbf{E}| \propto \frac{1}{r^2}$

Dipole Moment에 의한 ~ $|\mathbf{E}| \propto \frac{1}{r^3}$

- **Sum** : Electric Dipole Moment

$$\left\{\begin{array}{ll} \text{Moment :} & \mathbf{P} = Q\mathbf{d} \\ \text{Potential :} & V = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{Qd\cos\theta}{r^2} = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{\mathbf{P}\cdot\hat{a}_r}{r^2} \\ \text{Field :} & \mathbf{E} = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{Qd}{r^3}(2\cos\theta\hat{a}_r + \sin\theta\hat{a}_\theta) \end{array}\right\}$$

(Ex) D(1,2,-1) 점에 $\mathbf{P} = -4\hat{a}_x + 5\hat{a}_y + 3\hat{a}_z$ $[nC\cdot m]$가 있을 경우 $P_A(0,0,0)$ 에서의 전위는?

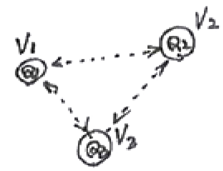


$$\hat{a}_r = \|(0,0,0)-(1,2,-1)\| = \frac{1}{\sqrt{6}}(-1,-2,1)$$

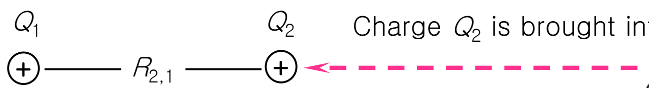
$$\left[\begin{array}{l} \mathbf{r}-\mathbf{r}' = (0,0,0)-(1,2,-1) = (-1,-2,1) \\ |\mathbf{r}-\mathbf{r}'| = \sqrt{1+4+1} = \sqrt{6} \end{array}\right.$$

$$\therefore V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\mathbf{P}\cdot\hat{a}_r}{r^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-4,5,3)\cdot(-1,-2,1)}{\sqrt{6}}\times 10^{-9} = \boxed{-1.835[V]}$$

4.8 정전계의 에너지



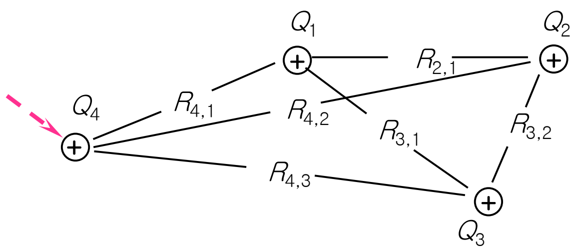
· 2Q



Charge Q_2 is brought into position from infinity.

$$W_E(2\text{ charges}) = Q_2 V_{2,1} = \frac{Q_2 Q_1}{4\pi\epsilon_0 R_{2,1}}$$

· 4Q



$$W_E(4\text{ charges}) = Q_2V_{2,1} + Q_3V_{3,1} + Q_3V_{3,2} + Q_4V_{4,1} + Q_4V_{4,2} + Q_4V_{4,3}$$

$$V_{4,1} = \frac{Q_1}{4\pi\epsilon_0 R_{4,1}} \qquad V_{4,2} = \frac{Q_2}{4\pi\epsilon_0 R_{4,2}} \qquad V_{4,3} = \frac{Q_3}{4\pi\epsilon_0 R_{4,3}}$$

$$W_E(4\text{ charges}) = Q_1V_{1,2} + Q_1V_{1,3} + Q_2V_{2,3} + Q_1V_{1,4} + Q_2V_{2,4} + Q_3V_{3,4}$$

$$2W_E = Q_1(V_{1,2}+V_{1,3}+V_{1,4}) + Q_2(V_{2,1}+V_{2,3}+V_{2,4}) + Q_3(V_{3,1}+V_{3,2}+V_{3,4}) + Q_4(V_{4,1}+V_{4,2}+V_{4,3})$$

$$V_1 = V_{1,2} + V_{1,3} + V_{1,4}$$
$$V_2 = V_{2,1} + V_{2,3} + V_{2,4}$$
$$V_3 = V_{3,1} + V_{3,2} + V_{3,4}$$
$$V_4 = V_{4,1} + V_{4,2} + V_{4,3}$$

$$W_E(4\text{ charges}) = \frac{1}{2}(Q_1V_1 + Q_2V_2 + Q_3V_3 + Q_4V_4) = \frac{1}{2}\sum_{m=1}^{4} Q_m V_m$$

· nQ

Extending for n charges: $W_E(n\text{ charges}) = \frac{1}{2}\sum_{m=1}^{n} Q_m V_m \qquad V_m = \sum_{p=1}^{n} V_{m,p} \qquad (p \neq m)$

Continuous charge ($dq = \rho_v\, dv$) : $W_E = \frac{1}{2}\int_{\text{vol}} \rho_v V\, dv$

Maxwell's first equation : $W_E = \frac{1}{2}\int_{\text{vol}} \rho_v V dv = \frac{1}{2}\int_{\text{vol}} (\nabla\cdot\mathbf{D})V\, dv \;= \frac{1}{2}\int_{\text{vol}} [\nabla\cdot(V\mathbf{D}) - \mathbf{D}\cdot(\nabla V)]\, dv$

$$\nabla\cdot(V\mathbf{D}) \equiv V(\nabla\cdot\mathbf{D}) + \mathbf{D}\cdot(\nabla V)$$

$$W_E = \frac{1}{2}\oint_S \cancel{(V\mathbf{D})}\cdot d\mathbf{S} - \frac{1}{2}\int_{\text{vol}} \mathbf{D}\cdot(\nabla V)\, dv \;= -\frac{1}{2}\int_{vol} \mathbf{D}\cdot\nabla V dv$$

$$V \doteq k_1\left(\frac{1}{r}\right) \quad D \doteq k_2\left(\frac{1}{r^2}\right) \quad VD \doteq k_1k_2\left(\frac{1}{r^3}\right)$$

$$\mathbf{E} = -\nabla V$$

$$\boxed{W_E = \int_{vol} \frac{1}{2}\mathbf{D}\cdot\mathbf{E} dv}$$

$$\boxed{w_E = \frac{1}{2}\mathbf{D}\cdot\mathbf{E} = \frac{1}{2}\epsilon_0 E^2 \quad \text{J/m}^3}$$

- Sum : $W = \frac{1}{2}\int_{Vol} \rho_v V dv = \frac{1}{2}\int_{Vol} \varepsilon_0 E^2 dv$

  ρ 와 V로부터 정전계의 에너지계산
  E의 분포로부터 // //

- Electric Pressure : $dW_e = \frac{1}{2}\mathbf{D}\cdot\mathbf{E}dv = \frac{1}{2}\varepsilon_0 \mathbf{E}^2 dv$

  Energy : W
  Force : $\frac{dW}{dx}$
  Pressure : $\frac{dW}{dv}$

  $$\boxed{\frac{dW_e}{dv} = \frac{1}{2}\mathbf{D}\cdot\mathbf{E} = \frac{1}{2}\varepsilon_0 E^2}$$

  (cf. magnetic pressure : $\frac{B^2}{2\mu_0}$)

(Ex) 동축 케이블에 축적된 에너지

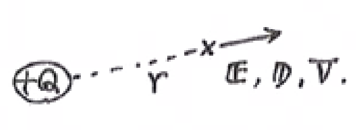
$$\mathbf{E} = \frac{a\rho_s}{\varepsilon_0 r}\hat{a}_r$$

$$W_e = \frac{1}{2}\int_0^L \int_0^{2\pi} \int_a^b \varepsilon_0 \cdot \left(\frac{a\rho_S}{\varepsilon_0 r}\right)^2 \cdot r dr d\phi dz \boxed{= \frac{\pi L a^2 \rho_S{}^2}{\varepsilon_0} \ln\frac{b}{a}}$$

$$V_a = -\int_b^a E_r dr = -\int \frac{a\rho_S}{\varepsilon_0 r} dr = \frac{a\rho_S}{\varepsilon_0}\ln\frac{b}{a}$$

➢ Sum : $Q \rightarrow \mathbf{E}, \mathbf{D} \rightarrow W, V$ (Chap 2 , 3 , 4)

① 점전하 :



< From Coulomb >

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q_2}{r^2} \hat{a}_r$$

$$\boxed{\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} \hat{a}_r}$$

< From Gauss >

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = Q$$

$$4\pi r^2 \cdot D_r = Q$$

$$\mathbf{D} = \frac{Q}{4\pi r^2} \hat{a}_r$$

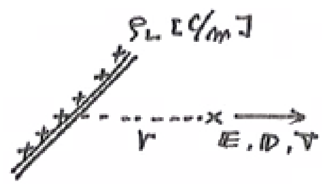
$$\boxed{\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} \hat{a}_r}$$

$$V = -\int \mathbf{E} \cdot d\mathbf{L}$$

$$= -\int \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} dr$$

$$\boxed{= \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r}}$$

② 선전하 :



$$\mathbf{E} = \int_{-\infty}^{\infty} \frac{1}{4\pi\varepsilon_0} \cdot \frac{\rho_L dl}{r^2} \hat{a}_r$$

$$\vdots$$

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = Q$$

$$2\pi r L \cdot D_r = \rho_L \cdot L$$

$$\mathbf{D} = \frac{\rho_L}{2\pi r} \hat{a}_r$$

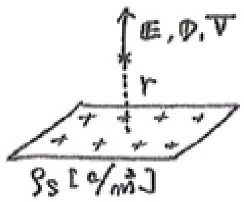
$$\boxed{\mathbf{E} = \frac{1}{2\pi\varepsilon_0} \cdot \frac{\rho_L}{r} \hat{a}_r}$$

$$V = -\int \mathbf{E} \cdot d\mathbf{L}$$

$$= -\int \frac{\rho_L}{2\pi\varepsilon_0} \cdot \frac{dr}{r}$$

$$\boxed{= \frac{\rho_L}{2\pi\varepsilon_0} \cdot \ln r}$$

③ 면전하 :



$$\mathbf{E}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{4\pi\varepsilon_0}\cdot\frac{\rho_S}{r^2}dxdy\,\hat{a}_r$$

$\vdots$

$$\oint_S \mathbf{D}\cdot d\mathbf{S}=Q$$

$$2L\cdot D_N=\rho_S\cdot L$$

$$\mathbf{D}=\frac{\rho_S}{2}\hat{a}_N$$

$$\boxed{\mathbf{E}=\frac{\rho_S}{2\varepsilon_0}\hat{a}_N}$$

$$V=-\int \mathbf{E}\cdot d\mathbf{L}$$

$$=-\int\frac{\rho_S}{2\varepsilon_0}dr$$

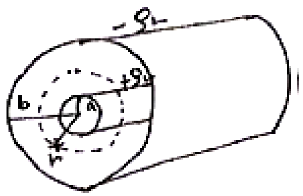
$$\boxed{=\frac{\rho_S}{2\varepsilon_0}\cdot r}$$

④ 체적전하 :

$$\mathbf{E}=\int_{Vol}\frac{1}{4\pi\varepsilon_0}\cdot\frac{\rho_V}{r^2}dv\,\hat{a}_r$$

$$V=\int_{Vol}\frac{1}{4\pi\varepsilon_0}\cdot\frac{\rho_V(\mathbf{r}')dv'}{|\mathbf{r}-\mathbf{r}'|}$$

⑤ 동축 케이블 :



$$a<r<b:E_r=\frac{1}{2\pi\varepsilon_0}\cdot\frac{\rho_L}{r}$$

r=b : V=0 – 밖 케이블 접지시.

$(Q\Leftrightarrow\rho_s)$

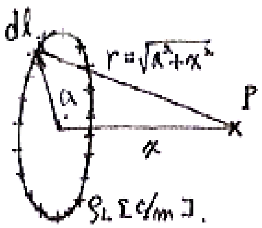
$$V=\int_b^r E_r dr=-\int_b^r\frac{1}{2\pi\varepsilon_0}\frac{\rho_L}{r}dr$$

$$=\frac{-\rho_L}{2\pi\varepsilon_0}[\ln r]_b^r=\frac{-\rho_L}{2\pi\varepsilon_0}(\ln r-\ln b)$$

$$\boxed{=\frac{-\rho_L}{2\pi\varepsilon_0}\ln\frac{b}{r}}$$

Lab. of Applied ElectroMagnetics

⑥ Ring 전하 :



$$dq = \rho_L dl$$

$$\text{o}\, dV_P = \frac{dq}{4\pi\varepsilon_0 r} = \frac{\rho_L dl}{4\pi\varepsilon_0 \sqrt{a^2+x^2}}$$

$$V_P = \oint dV_P = \int_0^{2\pi a} \frac{\rho_L dl}{4\pi\varepsilon_0 \sqrt{a^2+x^2}}$$

$$= \frac{\rho_L}{4\pi\varepsilon_0 \sqrt{a^2+x^2}} \int_0^{2\pi a} dl$$

$$\boxed{= \frac{\rho_L}{2\varepsilon_0 \sqrt{a^2+x^2}} [V]}$$

$$* \; x = 0 : V_{P_0} = \frac{\rho_L}{2\varepsilon_0} [V]$$

※ cf $\begin{pmatrix} V \rightarrow \vec{E} \\ \vec{E} \rightarrow V \end{pmatrix}$

$$\text{o}\, \mathbf{E} = -\nabla V_P = -\frac{\partial V_P}{\partial x} \hat{a}_x$$

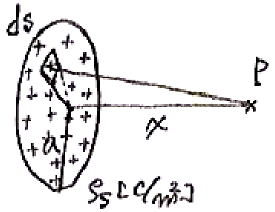
$$= -\frac{\rho_L}{2\varepsilon_0} \frac{\partial}{\partial x} \left(\frac{a}{\sqrt{a^2+x^2}} \right) \hat{a}_x$$

$$= -\frac{\rho_L}{2\varepsilon_0} \left(-\frac{1}{2} \frac{2xa}{(a^2+x^2)^{3/2}} \right) \hat{a}_x$$

$$\boxed{= -\frac{\rho_L}{2\varepsilon_0} \frac{xa}{(a^2+x^2)^{3/2}} \hat{a}_x [V/m]}$$

$$* \; x = 0 : \mathbf{E} = 0$$

⑦ Disk 전하 :



$$dq = \rho_S dS$$

$$\text{o}\, E_Z = \cdots = \frac{\rho_S}{2\varepsilon_0} \left(1 - \frac{x}{\sqrt{a^2+x^2}} \right)$$

o$z = \infty$에서 $V = 0$이라 하면 $x = h$에서 V :

$$V = -\int_\infty^h \frac{\rho_S}{2\varepsilon_0} \left(1 - \frac{x}{\sqrt{a^2+x^2}} \right) dx = -\frac{\rho_S}{2\varepsilon_0} [x - \sqrt{a^2+x^2}]_\infty^h = -\frac{\rho_S}{2\varepsilon_0} [h - \sqrt{a^2+h^2}]$$

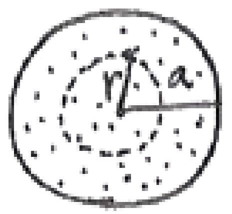
$$\boxed{= \frac{\rho_S}{2\varepsilon_0} [\sqrt{a^2+h^2} - h]}$$

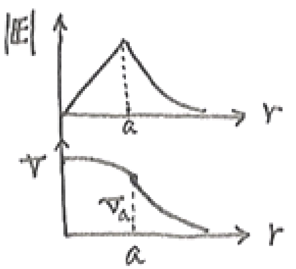
$$* \; h = 0 : V = \frac{\rho_S}{2\varepsilon_0} a$$

Lab. of Applied ElectroMagnetics

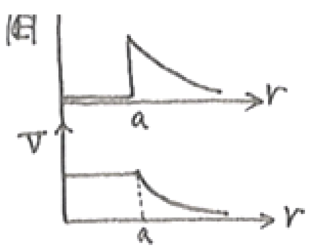
⑧ 구전하 :

ρ [c/m³].





cf. 금속구



(i) r<a : $\oint \vec{D}\cdot d\vec{S} = Q$

$$Q = \frac{4}{3}\pi\ r^3 \cdot \rho_v$$

$$\cancel{4\pi}\, r^2 \cdot E_r = \frac{\cancel{4\pi}}{3\varepsilon_0} r^3 \cdot \rho_v$$

$$E_r = \frac{\rho_v}{3\varepsilon_0}\cdot r\hat{a}_r\ [V/m]$$

(ii) r>a : $Q = \frac{4}{3}\pi\ r^3 \cdot \rho_v$

$$\vec{E} = \frac{1}{\cancel{4\pi}\varepsilon_0}\cdot\frac{\cancel{\frac{4}{3}\pi}a^3\cdot\rho_v}{r^2}\hat{a}_r$$

$$= \frac{\rho_v}{3\varepsilon_0}\cdot\frac{a^3}{r^2}\ [V/m]$$

(i) r>a : $V = -\int_\infty^r \frac{\rho_v a^3}{3\varepsilon_0 r'^2}dr'$

$$= -\frac{\rho_v a^3}{3\varepsilon_0}\int_\infty^r \frac{dr'}{r'^2}$$

$$= \frac{\rho_v}{3\varepsilon_0}\cdot\frac{a^3}{r}\ [V]$$

(ii) r<a : $r = a$ 때 $V_a = \frac{\rho_v}{3\varepsilon_0}\cdot\frac{a^3}{a} = \frac{\rho_v a^2}{3\varepsilon_0}\ [V]$

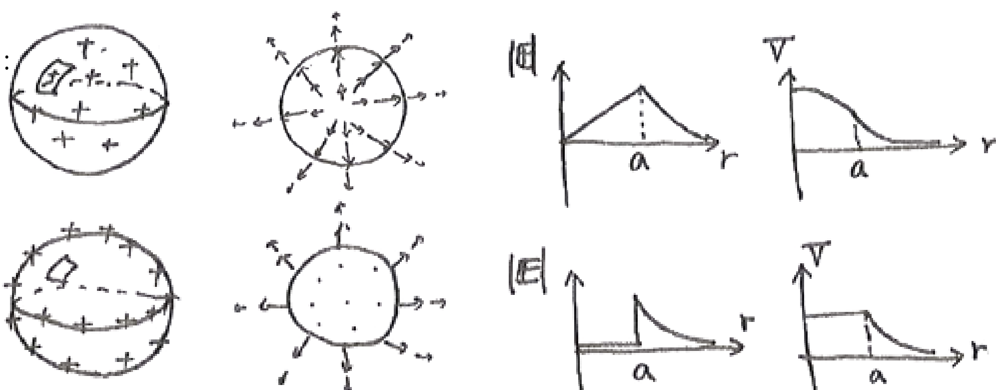
$$V_m = -\int_a^r \frac{\rho_v r'}{3\varepsilon_0}dr' = -\frac{\rho_v}{3\varepsilon_0}\left[\frac{r'^2}{2}\right]_a^r$$

$$= \frac{\rho_v}{3\varepsilon_0}\left[\frac{r^2}{2} - \frac{a^2}{2}\right] = \frac{\rho_v}{6\varepsilon_0}(a^2 - r^2)$$

$$\therefore\ V = V_{ra} + V_a = \frac{\rho}{6\varepsilon_0}(3a^2 - r^2)\ [V]$$

비금속구 :

금속구 :

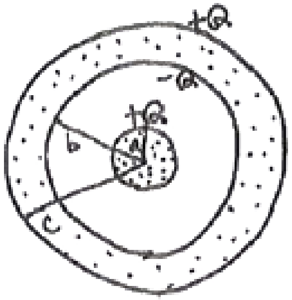


⑨ 동심도체구 : 정전유도

< 내구, 외구안, 외구밖>

$$\begin{cases} \text{(i) 내부도체에 } +Q : & +Q, & -Q, & +Q \\ \text{(ii) 외부도체에 } +Q : & 0, & 0, & +Q \\ \text{(iii) 내부 } +Q\text{, 접지} : & +Q, & -Q, & 0 \end{cases}$$

$V_a = ?$



Case (i) :

$a \le r < b$: $E_r = \dfrac{Q}{4\pi\varepsilon_0 r^2}$

$r \ge c$: $E_r = \dfrac{+Q-Q+Q}{4\pi\varepsilon_0 r^2} = \dfrac{Q}{4\pi\varepsilon_0 r^2}$ 이므로

전압은

$b \le r < c$: 등전위, $V_c = -\int_\infty^c E_r dr = -\dfrac{Q}{4\pi\varepsilon_0}\int_\infty^c \dfrac{dr}{r^2} = \dfrac{Q}{4\pi\varepsilon_0 c}\,[V]$

$a \le r < b$ 에서의 전위차 : $V_{ab} = -\int_b^a E_r dr = -\dfrac{Q}{4\pi\varepsilon_0}\int_b^a \dfrac{dr}{r^2} = \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{1}{a}-\dfrac{1}{b}\right)$

$r = a$ 에서의 전위 : $V_a = \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{1}{a}-\dfrac{1}{b}+\dfrac{1}{c}\right)[V]$

Case (iii) :

$r \ge c$: $E_r = 0$

$a \le r < b$: $E_r = \dfrac{Q}{4\pi\varepsilon_0 r^2}$ 이므로

전압은 $V_a = -\int_b^a E_r dr = \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{1}{a}-\dfrac{1}{b}\right)[V]$