Chap 3. Electric Flux

Chap 2 : Q, **F**, **E**

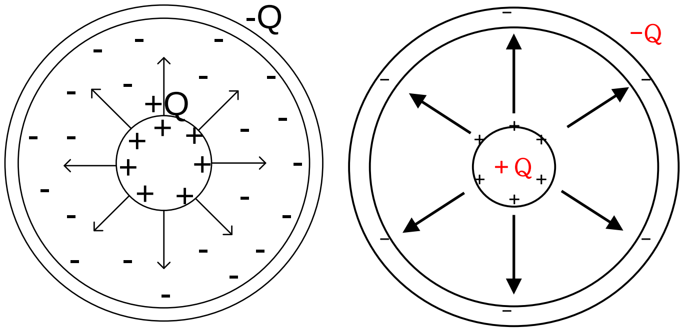
Chap 3 : Ψ, **D**, ε

$\nabla \cdot \mathbf{D} = \rho$

3. 1 전속 밀도

- 1837. Michael Faraday. London 왕립협회 원장

◎ 동심 금속 도체구 실험



① 내부 도체 + Q

② 매질 ε으로 채우고(2cm) 외부도체 씌움 : −Q, + Q 유도

③ 외부도체 접지 : + Q는 없어지고 −Q만 남음

④ 외부도체에 남겨진 | −Q | =내부도체의 | + Q |

⇒ 매질 ε의 종류에 관계없이 항상 동일.

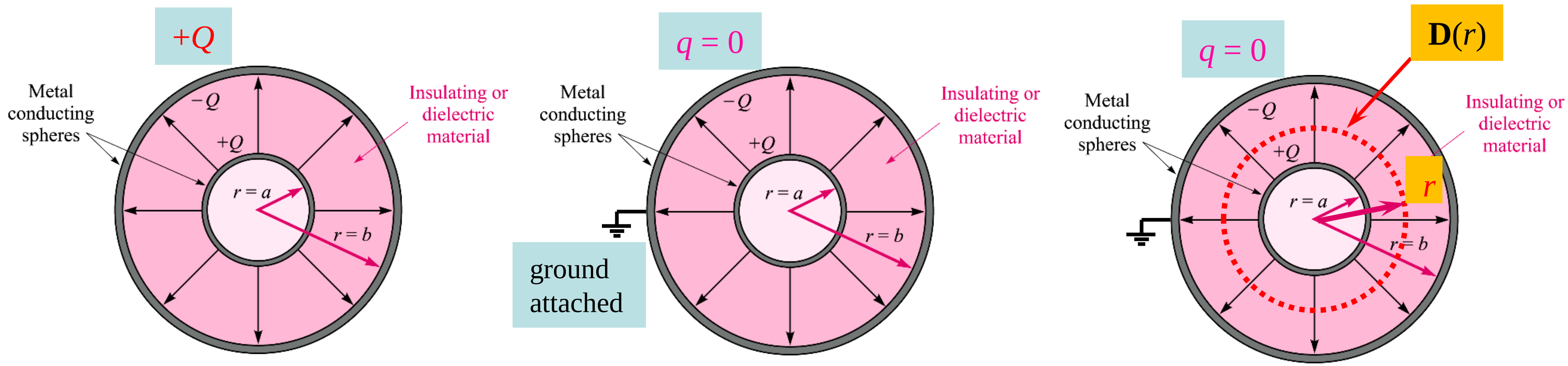
Electric Flux Ψ

◎ Gauss 법칙 :

$$\Psi = Q$$

Ψ : Electric Flux. [C]

Q : Electric Charge. [C]



◎ Ψ : Electric Flux. [C], 전속(량)

① 전하 Q에 의하여 발생

② 전하가 존재하면 항상 발생

③ 주위 매질에 상관없이 오직 전하량에만 비례

④ 전하량 Q와 같다. 단위도 동일[Coulomb, C]

⇒ Gauss' Law

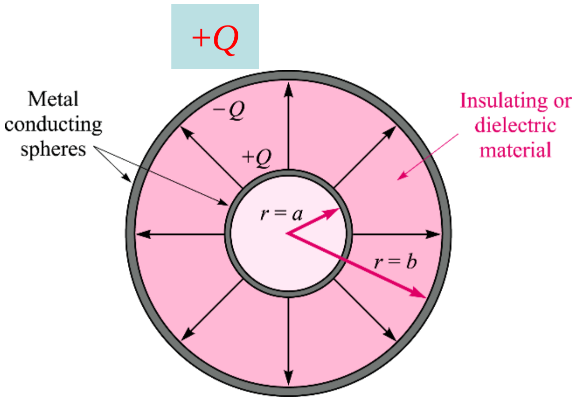
◎ D : Electric Flux Density. 전속밀도

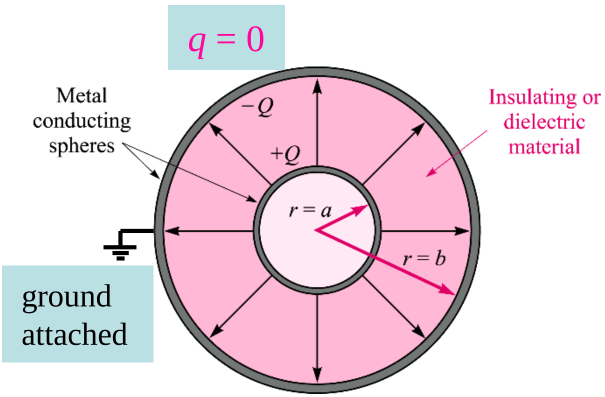
① 단위 면적당 Flux [C/m²]

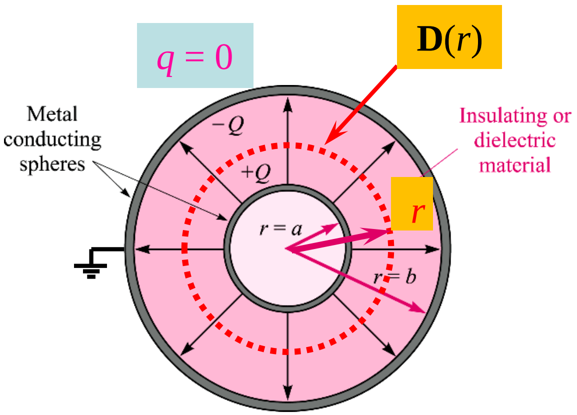
$$\boxed{\mathbf{D} = \frac{\Psi}{S}\hat{a}_N = \frac{Q}{S}\hat{a}_N}$$

② E 와 달리 주위 매질(유전체)에 상관없이
오직 전하 량에 만 비례

③ E 와의 관계는 ?







◎ Flux & Flux Density

$$\boxed{\Psi = Q}$$

$$D(r=a) = \frac{\Psi}{4\pi a^2} = \frac{Q}{4\pi a^2}$$

$$\mathbf{D}\Big|_{r=a} = \frac{Q}{4\pi a^2}\mathbf{a}_r \qquad \text{(inner sphere)}$$

$$\mathbf{D}\Big|_{r=b} = \frac{Q}{4\pi b^2}\mathbf{a}_r \qquad \text{(outer sphere)}$$

$$\boxed{\mathbf{D} = \frac{Q}{4\pi r^2}\mathbf{a}_r}$$ Coulombs/m^2, $(a \le r \le b)$

◎ E & D & Magterial

$$\boxed{\mathbf{D} = \frac{Q}{4\pi r^2}\mathbf{a}_r}$$ C/m^2 $(0 < r < \infty)$

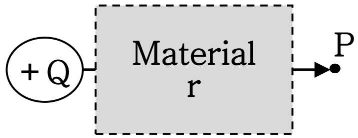
$$\boxed{\mathbf{E} = \frac{Q}{4\pi \epsilon_0 r^2}\mathbf{a}_r}$$ V/m $(0 < r < \infty)$

$$\boxed{\mathbf{D} = \epsilon_0 \mathbf{E}}$$ (free space only)

$$\boxed{\mathbf{D} = \int_{\text{vol}} \frac{\rho_v dv}{4\pi R^2}\mathbf{a}_R}$$

$$\boxed{\mathbf{E} = \int_{\text{vol}} \frac{\rho_v dv}{4\pi \epsilon_0 R^2}\mathbf{a}_R}$$ (free space only)

◎ E & D & Magterial



$\rightarrow$ E, $\leftarrow$ D] ?

- E : 매질 종류에 따라 달라짐.
- D : 매질 종류에 무관함.

① 공기(진공) : $\mathbf{E} = \dfrac{\mathbf{D}}{\varepsilon_0}$ $\boxed{\mathbf{D} = \varepsilon_0 \mathbf{E}}$ $\varepsilon_0 = 8.854 \times 10^{-12}$ 진공의 유전율

② 매질(유전체) : $\mathbf{E} = \dfrac{\mathbf{D}}{\varepsilon_0 \varepsilon_r}$ $\boxed{\mathbf{D} = \varepsilon \mathbf{E} = \varepsilon_0 \varepsilon_r \mathbf{E}}$ ε_r 비유전율(relative permittivity)

$\boxed{\varepsilon = \varepsilon_0 \varepsilon_r}$

- ε_0 : 진공 유전율. $\boxed{\dfrac{1}{36\pi \times 10^9} = 8.854 \times 10^{-12} [F/m]}$
- ε_r : 비 유전율.

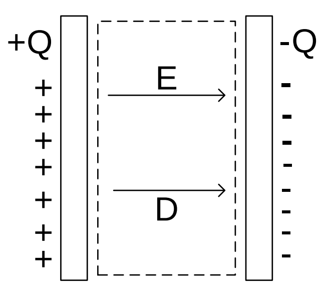
◎ 유전율(Permittivity) ε

• $\varepsilon = D/E$ E 에 대한 D의 비율. 단위전계에 대한 전속밀도의 비율. 전계의 세기에 대한 전속밀도 비율

$\mathbf{D} = \varepsilon \mathbf{E}$ — 동일한 전계의 세기를 인가했을 때 발생하는 전속밀도의 양. ε↑ : D↑(E 동일)

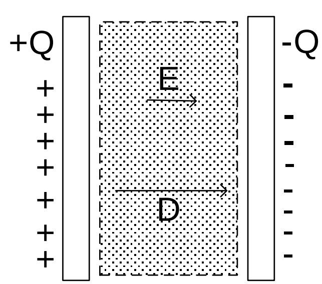
$\mathbf{E} = \dfrac{\mathbf{D}}{\varepsilon}$ — 동일한 전속밀도 일 경우 전계의 세기는 역비례. ε↑ : E↓(D 동일)

< 진공 중 >



$\varepsilon_0 \mathbf{E} = \mathbf{D}$

<유전체 내부>



$\varepsilon_0 \mathbf{E} = \mathbf{D}/\varepsilon_r$

• D는 Q가 일정하면 항상 동일.

• E가 크면 유전체 내부에 있는 전하가 받는 힘이 커진다.

(F = qE, > $F_{Threshold}$?)

[유전체에 전압인가 → 전계발생 → 유전체 내 q 발생시
힘 발생 → 임계치이상이면 절연파괴 : 고전압용 절연체 필요]

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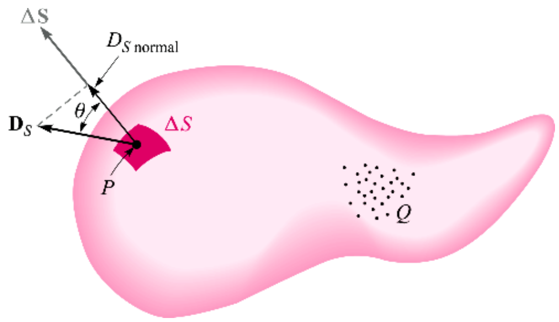
TABLE D.3 DIELECTRIC CONSTANT (ϵ_r) OF MATERIALS

Material	ϵ_r
Air	1.0006
Alcohol, ethyl	25
Asbestos fiber	4.8
Barium titanate	1200
Earth (dry)	7
Earth (moist)	15
Earth (wet)	30
Glass	4–10
Ice	4.2
Mica	5.4
Nylon	4
Paper	2–4
Polystyrene	2.56
Porcelain	6
Pyrex glass	5
Quartz	3.8
Rubber	2.5–3
Silica	3.8
Snow	3.3
Styrofoam	1.03
Teflon	2.1
Water (distilled)	81
Water (sea)	70

Material	ϵ'_R	ϵ''/ϵ'
Air	1.0005	
Alcohol, ethyl	25	0.1
Aluminum oxide	8.8	0.0006
Amber	2.7	0.002
Bakelite	4.74	0.022
Barium titanate	1200	0.013
Carbon dioxide	1.001	
Ferrite (NiZn)	12.4	0.00025
Germanium	16	
Glass	4–7	0.002
Ice	4.2	0.05
Mica	5.4	0.0006
Neoprene	6.6	0.011
Nylon	3.5	0.02
Paper	3	0.008
Plexiglas	3.45	0.03
Polyethylene	2.26	0.0002
Polypropylene	2.25	0.0003
Polystyrene	2.56	0.00005
Porcelain (dry process)	6	0.014
Pyranol	4.4	0.0005
Pyrex glass	4	0.0006
Quartz (fused)	3.8	0.00075
Rubber	2.5–3	0.002
Silica or SiO_2 (fused)	3.8	0.00075
Silicon	11.8	
Snow	3.3	0.5
Sodium chloride	5.9	0.0001
Soil (dry)	2.8	0.05
Steatite	5.8	0.003
Styrofoam	1.03	0.0001
Teflon	2.1	0.0003
Titanium dioxide	100	0.0015
Water (distilled)	80	0.04
Water (sea)		4
Water (dehydrated)	1	0
Wood (dry)	1.5–4	0.01

3.2 Gauss 의 법칙



" The electric flux passing through any closed surface is equal to the total charge enclosed by that surface"

$$\Delta\Psi = \text{flux crossing } \Delta S = D_{S,\text{norm}}\Delta S = D_S\cos\theta\,\Delta S = \mathbf{D}_S\cdot\Delta\mathbf{S}$$

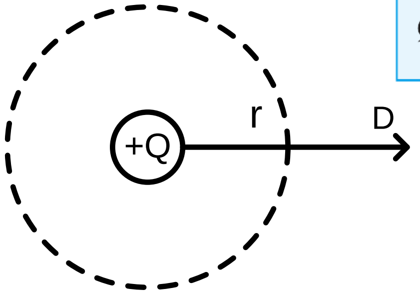
$$d\mathbf{S} = \mathbf{n}\,dS \qquad \Psi = \int d\Psi = \oint_{\substack{\text{closed}\\ \text{surface}}} \mathbf{D}_S\cdot d\mathbf{S}$$

◆ 폐곡면을 통과하는 총 전속량은 폐곡면 내의 총 전하량과 같다. $(\Psi = Q)$

- 폐곡면 내의 전하량 Q (3D) = 폐곡면 상의 전속량 Ψ (2D) $\left(\int_{2D} \mathbf{D}\cdot d\mathbf{S} = \int_{3D} \rho dv \right)$
- 폐곡면을 통과하는 Flux의 합은 폐곡면 내부의 전하의 합과 같다. $\boxed{\int \mathbf{D}\cdot d\mathbf{S} = Q}$

◆ Knowing Q, we need to solve for **D**, using Gauss' Law:

The solution is easy if we can choose a surface, S, over which to integrate (Gaussian surface) that satisfies the following two conditions:



$$\boxed{Q = \oint_S \mathbf{D}_S\cdot d\mathbf{S}}$$

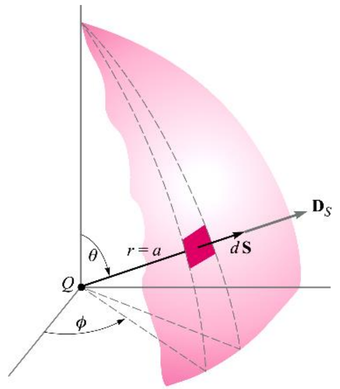
1. $\mathbf{D}_S$ is everywhere either normal or tangential to the closed surface, so that $\mathbf{D}_S\cdot d\mathbf{S}$ becomes either $D_S dS$ or zero, respectively.
2. On that portion of the closed surface for which $\mathbf{D}_S\cdot d\mathbf{S}$ is not zero, D_S = constant.

$$\oint_S \mathbf{D}_s\cdot d\mathbf{S} = \underbrace{\oint_S D_s\,dS}_{\text{Condition 1}} = \underbrace{D_s\oint_S dS}_{\text{Condition 2}} = Q$$

$$\boxed{D_s = \frac{Q}{\oint_S dS}}$$

3.2 Gauss 의 법칙의 응용예제 $\oint \mathbf{D}\cdot d\mathbf{S} = Q$

(1) Point Charge Field : Spherical surface of radius *a* surrounded the charge Q



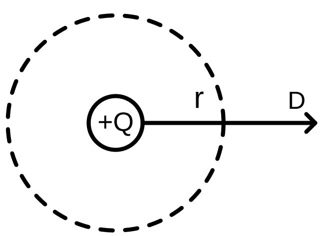
$$\mathbf{D} = \frac{Q}{4\pi r^2}\mathbf{a}_r \qquad \mathbf{D}_S = \frac{Q}{4\pi a^2}\mathbf{a}_r \qquad dS = r^2\sin\theta\, d\theta\, d\phi = a^2 \sin\theta\, d\theta\, d\phi$$

$$d\mathbf{S} = a^2 \sin\theta\, d\theta\, d\phi\ \mathbf{a}_r$$

$$\mathbf{D}_S\cdot d\mathbf{S} = \frac{Q}{4\pi a^2}a^2\sin\theta\, d\theta\, d\phi \mathbf{a}_r\cdot\mathbf{a}_r = \frac{Q}{4\pi}\sin\theta\, d\theta\, d\phi$$

$$\oint_S \mathbf{D}_S\cdot d\mathbf{S} = \int_{\phi=0}^{\phi=2\pi}\int_{\theta=\phi}^{\theta=\pi}\frac{Q}{4\pi}\sin\theta\, d\theta\, d\phi = \int_0^{2\pi}\frac{Q}{4\pi}(-\cos\theta)_0^{\pi} d\phi = \int_0^{2\pi}\frac{Q}{2\pi}d\phi = Q$$

➢ 점전하 :



$$\mathbf{D} = D_r\cdot\hat{a}_r$$

$$\oint \mathbf{D}\cdot d\mathbf{S} = Q$$

$$4\pi r^2\cdot D_r = Q$$

$$\therefore D_r = \frac{Q}{4\pi a^2}$$

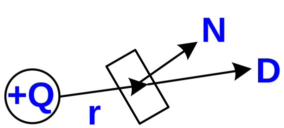
$$\boxed{\mathbf{D} = \frac{Q}{4\pi a^2}\hat{a}_r}$$

- 폐곡면 상에서 D vector 는
  ① 크기가 동일 , ② 방향은 $\hat{a}_r \;\rightarrow\; \mathbf{D} = D_r\hat{a}_r$
- 폐곡면의 표면적 : $4\pi r^2$, 폐곡면 내부 총 전하량 : + Q

✓ 매질에 무관한 양

cf. $\mathbf{E} = \mathbf{D}/\varepsilon_0 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r}\hat{a}_r$

✓ 구좌표계에서

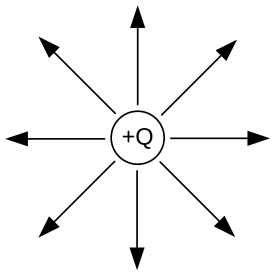


$$d\mathbf{S} = r^2\sin\theta d\theta d\phi\ \hat{a}_N \ ,\ \mathbf{D} = \frac{Q}{4\pi r^2}\hat{a}_r$$

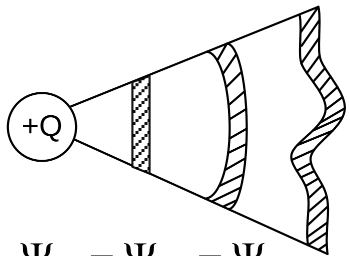
$$\oint \mathbf{D}\cdot d\mathbf{S} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\frac{Q}{4\pi r^2}\cdot r^2\sin\theta d\theta d\phi = \frac{Q}{4\pi}\int_0^{2\pi}\int_0^{\pi}\sin\theta d\theta d\phi = Q$$

◎ **폐곡면** 상의 Flux 계산 :

< Q에 의한 Flux >

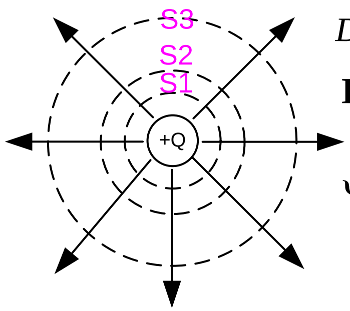


< Flux on the Surface >



$$\Psi_{S1} = \Psi_{S2} = \Psi_{S3}$$

< Equi = Flux Surface >



D_r이 같은 값 : $S_1,\ S_2,\ S_3$

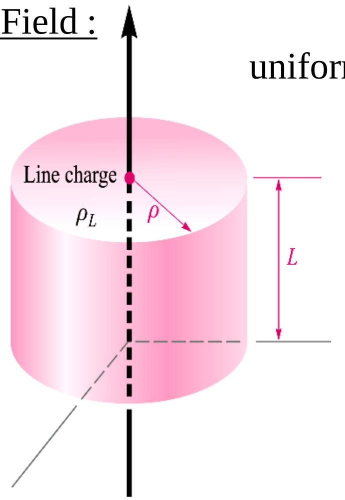
$$\mathbf{D} = D_r \cdot \hat{a}_r$$

$$\Psi_{S1} = \Psi_{S2} = \Psi_{S3}$$

(2) Line Charge Field :

$$\mathbf{D} = D_\rho \mathbf{a}_\rho$$

$$D_\rho = f(\rho)$$



uniform charge density ρ_L on the z axis, $-\infty < z < \infty$

We apply Gauss's law,

$$Q = \oint_{\text{cyl}} \mathbf{D}_S \cdot d\mathbf{S} = D_S \int_{\text{sides}} dS + 0 \int_{\text{top}} dS + 0 \int_{\text{bottom}} dS$$

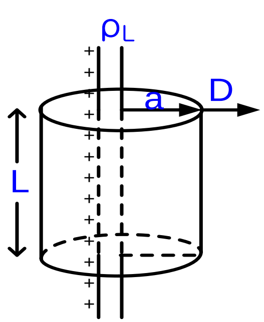
$$= D_S \int_{z=0}^{L} \int_{\phi=0}^{2\pi} \rho \, d\phi \, dz = D_S 2\pi\rho L$$

$$D_S = D_\rho = \frac{Q}{2\pi\rho L}$$

In terms of the charge density ρ_L, the total charge enclosed is $\quad Q = \rho_L L$

$$\boxed{D_\rho = \frac{\rho_L}{2\pi\rho}} \qquad E_\rho = \frac{\rho_L}{2\pi\epsilon_0\rho}$$

➢ <u>선전하 :</u>



- 원통에서

$$\oint \mathbf{D}\cdot d\mathbf{S} \begin{bmatrix} \text{윗면} : & \mathbf{D} \perp d\mathbf{S} & \therefore \mathbf{D}\cdot d\mathbf{S} = 0 \\ \text{아랫면} & \mathbf{D} \perp d\mathbf{S} & \therefore \mathbf{D}\cdot d\mathbf{S} = 0 \\ \text{옆면} : & \mathbf{D} // d\mathbf{S} & \therefore \mathbf{D} = D_r \hat{a}_r \end{bmatrix}$$

$$\oint \mathbf{D}\cdot d\mathbf{S} = \cancel{\int_{\text{윗면}}}^{0} + \cancel{\int_{\text{아랫면}}}^{0} + \int_{\text{옆면}} = \int_{\text{옆면}}$$

- 원통 내부의 전하량 : $\rho_L \cdot L$

$$\begin{cases} \oint \mathbf{D}\cdot d\mathbf{S} = Q \\ 2\pi r \cdot \cancel{L} \cdot D_r = \rho_L \cdot \cancel{L} \\ D_r = \dfrac{\rho_L}{2\pi r} \end{cases}$$

$$\boxed{\therefore \mathbf{D} = \frac{\rho_L}{2\pi r}\hat{a}_r} \qquad \underline{\mathbf{E} = \frac{1}{2\pi\varepsilon_0}\cdot\frac{\rho_L}{r}\hat{a}_r}$$

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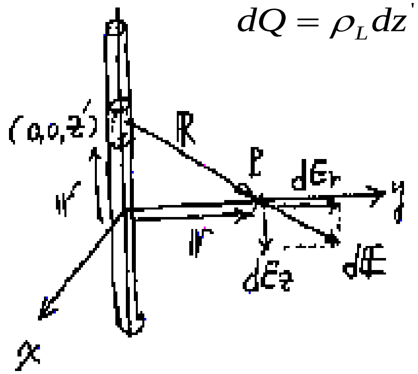
✓ 비교

2.4. 선전하에 의한 전계의 세기



$$dQ = \rho_L dz'$$

$$\mathbf{r} = r\hat{a}_r \quad \mathbf{r}' = z'\hat{a}'_z \,, (z' : -\infty \sim \infty) \quad \rightarrow \mathbf{R} = \mathbf{r} - \mathbf{r}' = r\hat{a}_r - z'\hat{a}_z$$

$$d\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{\rho_L dz'}{(r^2 + z'^2)^{3/2}}(r\hat{a}_r - z'\hat{a}_z) \quad = d\mathbf{E}_r + \cancel{d\mathbf{E}_z} = d\mathbf{E}_r = dE_r\hat{a}_r$$

$$dE_r = \frac{1}{4\pi\varepsilon_0}\frac{\rho_L r dz'}{(r^2 + z'^2)^{3/2}}$$

$$E_r = \int dE_r = \int_{-\infty}^{\infty}\frac{1}{4\pi\varepsilon_0}\frac{\rho_L r}{(r^2 + z'^2)^{3/2}}dz' = \frac{\rho_L r}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{1}{(r^2 + z'^2)^{3/2}}dz'$$

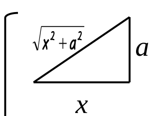
$$\int_{-\infty}^{\infty}\frac{dx}{(x^2 + a^2)^{3/2}} \qquad x = a\cot\theta\,(put) \qquad dx = -a\,\mathrm{cosec}^2\theta d\theta = -a\frac{1}{\sin^2\theta}d\theta$$

$$(x^2 + a^2)^{3/2} = a^3(\cot^2\theta + 1)^{3/2} = a^3\mathrm{cosec}^3\theta = a^3\frac{1}{\sin^3\theta}$$

$$\therefore \int_{-\infty}^{\infty}\frac{dx}{(x^2 + a^2)^{3/2}} = \int_{-\infty}^{\infty}\frac{-a\cdot 1/\sin^2\theta}{a^3\cdot 1/\sin^3\theta}d\theta = \int_{-\infty}^{\infty} -\frac{1}{a^2}\sin\theta d\theta \quad = -\frac{1}{a^2}\int_{-\infty}^{\infty}\sin\theta d\theta = \frac{1}{a^2}[\cos\theta]_{-\infty}^{\infty}$$

$$\therefore \; E_r = \frac{\rho_L r}{4\pi\varepsilon_0}\left(\frac{2}{r^2}\right) = \frac{1}{2\pi\varepsilon_0}\frac{\rho_L}{r}$$

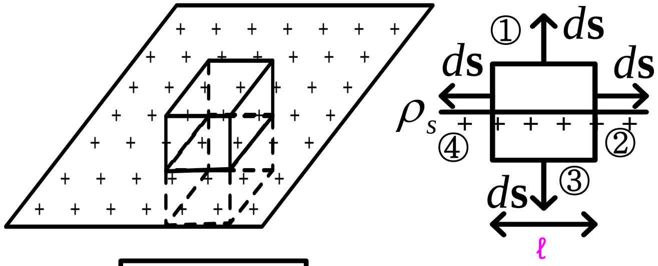
$$\boxed{\mathbf{E} = \frac{1}{2\pi\varepsilon_0}\frac{\rho_L}{r}\hat{a}_r}$$

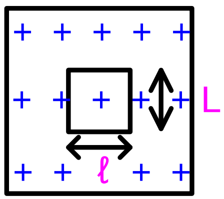


$$\frac{1}{a^2}\left[\frac{x}{\sqrt{x^2 + a^2}}\right]_{-\infty}^{\infty} = \frac{1}{a^2}[1 - (-1)] = \frac{2}{a^2}$$

$$\begin{bmatrix} z = -\infty \sim \infty \\ \theta = 0^+ \sim \pi^- \end{bmatrix} \quad \frac{1}{a^2}[\cos\theta]_{\pi^-}^{0^+} = \frac{1}{a^2}[\cos 0^+ - \cos\pi^-] = \frac{2}{a^2}$$

(3) 면전하 :





- 육면체에서

$$\oint \mathbf{D}\cdot d\mathbf{S} \begin{bmatrix} \text{②와④} : & \mathbf{D}\perp d\mathbf{S} & \therefore \mathbf{D}\cdot d\mathbf{S}=0 \\ \text{①과③} : & \mathbf{D}\,//\,d\mathbf{S} & \therefore \mathbf{D}=D_n\hat{a}_N \end{bmatrix}$$

$$\oint \mathbf{D}\cdot d\mathbf{S} = \int_{①} + \cancelto{0}{\int_{②}} + \int_{③} + \cancelto{0}{\int_{④}} = \int_{①} + \int_{③}$$

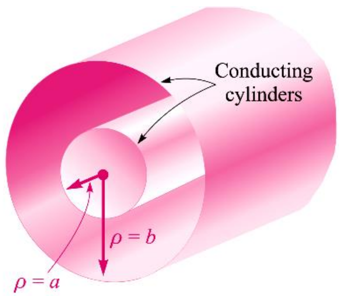
- 육면체 내부 전하량 : $\rho_S \cdot l \cdot L$

$$\begin{cases} \oint \mathbf{D}\cdot d\mathbf{S} = Q \\ D_n \cdot \cancel{l}\cdot\cancel{L} + D_n \cdot \cancel{l}\cdot\cancel{L} = \rho_L \cdot \cancel{l}\cdot\cancel{L} \\ D_n = \rho_S/2 \end{cases}$$

$$\boxed{\therefore \mathbf{D} = \frac{\rho_S}{2}\hat{a}_N}$$

$$\underline{\mathbf{E} = \frac{\rho_S}{2\varepsilon_0}\hat{a}_N}$$

(4) Coaxial Transmission Line :



Surface charge of density ρ_S exists on the outer surface of the inner cylinder.

A ρ-directed field is expected, and this should vary only with ρ (like a line charge).
In the cylindrical Gaussian surface of length L and of radius ρ, where $a < \rho < b$:

$$\oint_S \mathbf{D}_S \cdot d\mathbf{S} = \int_0^L \int_0^{2\pi} D_S\, \mathbf{a}_\rho \cdot \underbrace{\mathbf{a}_\rho\, \rho\, d\phi\, dz}_{d\mathbf{S}} = 2\pi\rho D_S L = Q$$

$$Q = \int_{\rho=a} \rho_S\, dS = \int_0^L \int_0^{2\pi} \rho_S\, a\, d\phi\, dz = 2\pi a L \rho_S$$

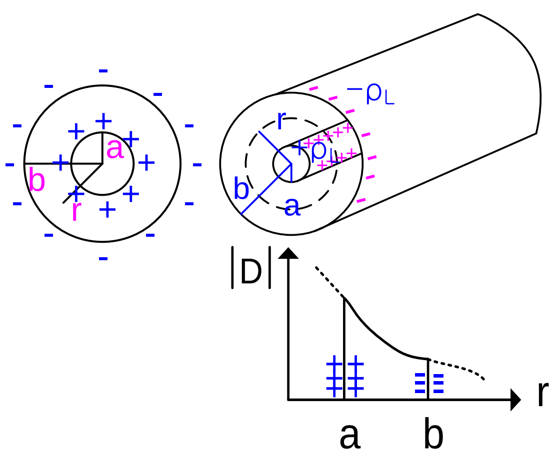
$$\boxed{\mathbf{D}(\rho) = \frac{Q}{2\pi\rho L}\mathbf{a}_\rho = \frac{a\rho_S}{\rho}\mathbf{a}_\rho}$$

$$Q_{\text{outer cyl}} = -2\pi a L \rho_{S,\text{inner cyl}} \qquad 0 = D_S 2\pi\rho L \qquad (\rho > b)$$

$$2\pi b L \rho_{S,\text{outer cyl}} = -2\pi a L \rho_{S,\text{inner cyl}} \qquad D_S = 0 \qquad (\rho > b)$$

$$\mathbf{E} = \frac{a\rho_S}{\epsilon_0 \rho}\mathbf{a}_\rho \ \text{V/m} \qquad (a < \rho < b)$$

➢ **동축케이블 :**



① $r < a \ : \ Q = 0. \quad \therefore \mathbf{D} = \mathbf{0}$

② $r > b \ : \ Q = \rho_L \cdot L - \rho_L \cdot L = 0 \quad \therefore \mathbf{D} = \mathbf{0}$

③ $a < r < b \ : \ \oint \mathbf{D} \cdot d\mathbf{S} = Q$

$$2\pi r \cancel{L} \cdot D_r = \rho_L \cdot \cancel{L}$$

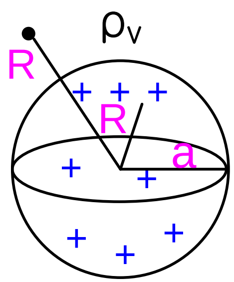
$$D_r = \frac{\rho_L}{2\pi r}$$

$$\boxed{\therefore \mathbf{D} = \frac{\rho_L}{2\pi r}\hat{a}_r}$$

• 전계는 두 원통 사이에만 존재한다.

• 동축케이블 , 동축콘덴서 (Coexial capacitor)

(5) Charged Sphere (Non-Conductor)

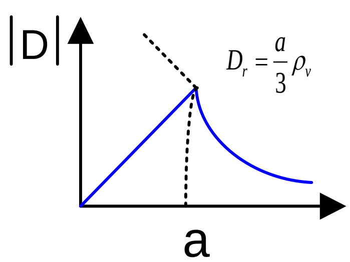


① $R < a$: 구 내부의 전속밀도

$$\begin{cases} \oint \mathbf{D}\cdot d\mathbf{S} = 4\pi R^2 \cdot D_r \\ Q = \frac{4}{3}\pi R^3 \rho_v \end{cases}$$

$$\therefore \begin{cases} \oint \mathbf{D}\cdot d\mathbf{S} = Q \\ \cancel{4\pi} R^2 \cdot D_r = \frac{\cancel{4}}{3}\cancel{\pi} R^{\cancel{3}} \rho_v \\ D_r = \frac{\rho_v}{3} R \end{cases}$$

$$\boxed{\therefore \mathbf{D} = \frac{\rho_v}{3} R\, \hat{a}_R}$$



② $R > a$: 구 외부의 전속밀도

$$\begin{cases} \oint \mathbf{D}\cdot d\mathbf{S} = 4\pi R^2 \cdot D_r \\ Q = \frac{4}{3}\pi R^3 \rho_v \end{cases}$$

$$\therefore \begin{cases} \oint \mathbf{D}\cdot d\mathbf{S} = Q \\ \cancel{4\pi} R^2 \cdot D_r = \frac{\cancel{4}}{3}\cancel{\pi} R^{3} \rho_v \\ D_r = \frac{a^3 \rho_v}{3R^2} \end{cases}$$

$$\boxed{\therefore \mathbf{D} = \frac{a^3 \rho_v}{3} \cdot \frac{1}{R^2}\, \hat{a}_R}$$

Sum :

$$\mathbf{D} = \frac{\rho_v}{3} r\hat{a}_r \ , \ r < a$$

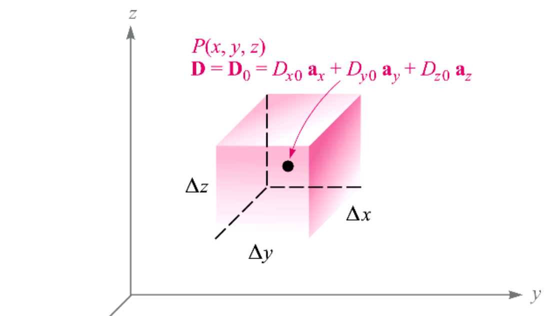
$$\mathbf{D} = \frac{a^3 \rho_v}{3} \cdot \frac{1}{r^2}\hat{a}_r \ , \ r > a$$

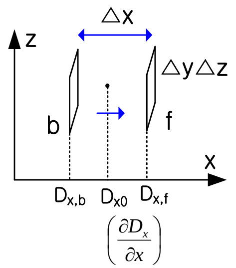
3.4 미소체적소

• 대칭성이 없는 경우 : $|\mathbf{D}| = \text{Constant}$ 폐곡면을 구할 수 없는 경우. $\oint \mathbf{D}\cdot d\mathbf{S}$ 계산?

• Gauss's Law : $\boxed{\Psi = Q}$
$$\begin{cases} \text{적분형} : & \oint \mathbf{D}\cdot d\mathbf{S} = Q \\ \text{미분형} : & \nabla\cdot\mathbf{D} = \rho \end{cases}$$

• 미소체적에서의 $\oint \mathbf{D}\cdot d\mathbf{S}$: $\oint_S \mathbf{D}\cdot d\mathbf{S} = \int_{\text{front}} + \int_{\text{back}} + \int_{\text{left}} + \int_{\text{right}} + \int_{\text{top}} + \int_{\text{bottom}}$





$$D_{x,\text{front}} \doteq D_{x0} + \frac{\Delta x}{2} \times \text{rate of change of } D_x \text{ with } x$$
$$\doteq D_{x0} + \frac{\Delta x}{2}\frac{\partial D_x}{\partial x}$$

✓ $\int_f + \int_b$:
$$\int_{\text{front}} \doteq \left(D_{x0} + \frac{\Delta x}{2}\frac{\partial D_x}{\partial x}\right)\Delta y\,\Delta z$$
$$\int_{\text{back}} \doteq \left(-D_{x0} + \frac{\Delta x}{2}\frac{\partial D_x}{\partial x}\right)\Delta y\,\Delta z$$

$$\int_{\text{front}} + \int_{\text{back}} \doteq \frac{\partial D_x}{\partial x}\Delta x\,\Delta y\,\Delta z$$

✓ minus sign because D_{x0} is *inward* flux through the back surface. $\because \hat{a}_N = -\hat{a}_x$

$$\int_{\text{front}} + \int_{\text{back}} \doteq \frac{\partial D_x}{\partial x}\Delta x\,\Delta y\,\Delta z$$

$$\int_{\text{right}} + \int_{\text{left}} \doteq \frac{\partial D_y}{\partial y}\Delta x\,\Delta y\,\Delta z$$

$$\int_{\text{top}} + \int_{\text{bottom}} \doteq \frac{\partial D_z}{\partial z}\Delta x\,\Delta y\,\Delta z$$

$$\boxed{\oint_S \mathbf{D}\cdot d\mathbf{S} \doteq \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right)\overbrace{\Delta x\,\Delta y\,\Delta z}^{\Delta v} \quad (= Q, \text{ by Gauss' Law})}$$

$$\therefore \oint_S \mathbf{D}\cdot d\mathbf{S} = \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right)\Delta x \Delta y \Delta z = \int_V \nabla\cdot\mathbf{D}\,dv$$

$$\boxed{\therefore \int_V \nabla\cdot\mathbf{D}\,dv = \oint_S \mathbf{D}\cdot d\mathbf{S}}$$

: Divergence Theorem
(3D ⟶ 2D)

Divergence and Maxwell's First Equation

The divergence of the vector flux density **A** *is the outflow of flux from a small closed surface per unit volume as the volume shrinks to zero.*

$$\text{div}\,\mathbf{A} = \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right) = \lim_{\Delta v \to 0} \frac{\oint_S \mathbf{A} \cdot d\mathbf{S}}{\Delta v}$$

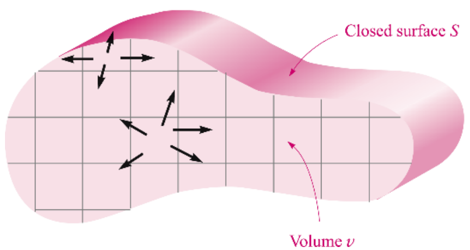
$$\left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right) = \lim_{\Delta v \to 0} \frac{\oint_S \mathbf{D} \cdot d\mathbf{S}}{\Delta v} = \text{div}\,\mathbf{D}$$

$$\therefore \oint_S \mathbf{D} \cdot d\mathbf{S} = \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right)\Delta x \Delta y \Delta z = \int_V \nabla \cdot \mathbf{D} dv \qquad \boxed{\therefore \int_V \nabla \cdot \mathbf{D} dv = \oint_S \mathbf{D} \cdot d\mathbf{S}}$$

✓ Del operator (*vector differential operator*) : $\boxed{\nabla = \frac{\partial}{\partial x}\mathbf{a}_x + \frac{\partial}{\partial y}\mathbf{a}_y + \frac{\partial}{\partial z}\mathbf{a}_z}$

$$\nabla \cdot \mathbf{D} = \left(\frac{\partial}{\partial x}\mathbf{a}_x + \frac{\partial}{\partial y}\mathbf{a}_y + \frac{\partial}{\partial z}\mathbf{a}_z\right) \cdot (D_x \mathbf{a}_x + D_y \mathbf{a}_y + D_z \mathbf{a}_z) = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \text{div}\,\mathbf{D}$$

$$\boxed{\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_{\text{vol}} \nabla \cdot \mathbf{D}\, dv}$$: Divergence Theorem



The integral of the normal component of any vector field over a closed surface is equal to the integral of the divergence of this vector field throughout the volume enclosed by the closed surface.

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = Q = \int_{\text{vol}} \rho_v dv = \int_{\text{vol}} \nabla \cdot \mathbf{D}\, dv$$

$$\boxed{\text{div}\,\mathbf{D} = \nabla \cdot \mathbf{D} = \rho_v}$$: Gauss' Law

Gauss' Law : $\boxed{\mathrm{div}\,\mathbf{D} = \nabla\cdot\mathbf{D} = \rho_v}$

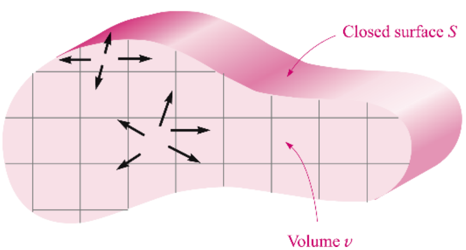
- by definition $Q = \int_V \rho_V dv$ $\int_V \nabla\cdot\mathbf{D}dv = \int \rho_V dv$
- $\boxed{\oint \mathbf{D}\cdot d\mathbf{S} = Q}$: Gauss's Law (적분형) ➢ 표면에서의 D 총합 = 체적 내의 전하량 총합

$\boxed{\nabla\cdot\mathbf{D} = \rho_V}$: Gauss's Law (미분형)

✓ Del operator (*vector differential operator*) $\boxed{\nabla = \frac{\partial}{\partial x}\mathbf{a}_x + \frac{\partial}{\partial y}\mathbf{a}_y + \frac{\partial}{\partial z}\mathbf{a}_z}$

$$\nabla\cdot\mathbf{D} = \left(\frac{\partial}{\partial x}\mathbf{a}_x + \frac{\partial}{\partial y}\mathbf{a}_y + \frac{\partial}{\partial z}\mathbf{a}_z\right)\cdot(D_x\mathbf{a}_x + D_y\mathbf{a}_y + D_z\mathbf{a}_z) = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \mathrm{div}\,\mathbf{D}$$

✓ Divergence Expressions in the Three Coordinate Systems



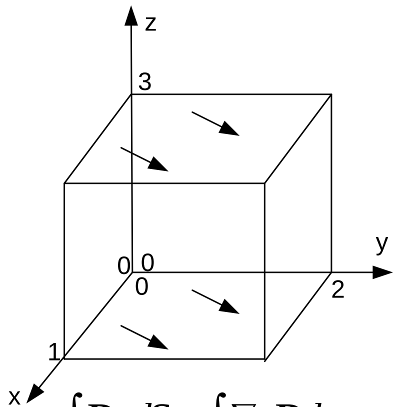
$$\mathrm{div}\,\mathbf{D} = \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right) \quad \text{(rectangular)}$$

$$\mathrm{div}\,\mathbf{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z} \quad \text{(cylindrical)}$$

$$\mathrm{div}\,\mathbf{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, D_\theta) + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial \phi} \quad \text{(spherical)}$$

Lab. of Applied ElectroMagnetics

(Ex) $\mathbf{D} = 2xy\,\hat{a}_x + x^2\,\hat{a}_y$, $\oint_S \mathbf{D}\cdot d\mathbf{S} \overset{?}{=} \int_V \nabla\cdot\mathbf{D}dv$



$$\int_S \mathbf{D}\cdot d\mathbf{S} = \int_v \nabla\cdot\mathbf{D}dv$$

$$左 = \oint \mathbf{D}\cdot d\mathbf{S} = \int_f + \int_b + \int_r + \int_l + \cancel{\int_t + \int_b}^{\,0} \quad (\Theta\, D_z = 0)$$

$$\begin{cases} \int_f = \int_{z=0}^{3}\int_{y=0}^{2}[\mathbf{D}]_{x=1}\cdot(dydz\,\hat{a}_x) \\ \int_b = \int_{z=0}^{3}\int_{y=0}^{2}\cancel{[\mathbf{D}]_{x=0}}\cdot(-dydz\,\hat{a}_x) \\ \cancel{\int_r} = \int_{z=0}^{3}\int_{y=0}^{2}[\mathbf{D}]_{y=0}\cdot(-dxdz\,\hat{a}_y) \\ \cancel{\int_l} = \int_{z=0}^{3}\int_{x=0}^{2}[\mathbf{D}]_{y=2}\cdot(dxdz\,\hat{a}_y) \end{cases}$$

$$= \int_0^3\int_0^2 D_x\big|_{x=1} dydz = \int_0^3\int_0^2 2ydydz = \int_0^3 4dz = \underline{12\ [C]}$$

$$右 = \int_v \nabla\cdot\mathbf{D}dv \ ,\quad \nabla\cdot\mathbf{D} = 2y + 0 = 2y$$

$$\begin{cases} = \int_{z=0}^{3}\int_{y=0}^{2}\int_{x=0}^{1}(2y)dxdydz \\ = \int_{z=0}^{3}\int_{y=0}^{2}(2y)dydz \\ = \int_{z=0}^{3}(4)dz \ = \underline{12\ [C]} \end{cases}$$

$$\therefore\ 左 = 右$$