Lab. of Applied ElectroMagnetics

# Chap. 8 자기력, 자성체, 인덕턴스

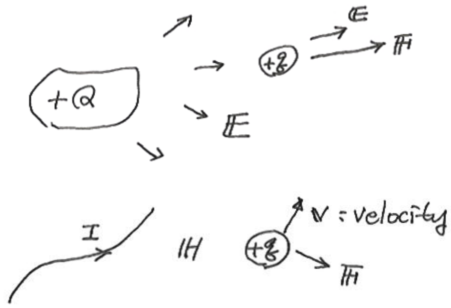
## 8.1 운동하는 전하에 작용하는 힘

- ( 전계 ) $\vec{E}$ Field 내의 Force :

$$\boxed{\vec{F} = q\vec{E}}$$

- ( 자계 ) $\vec{H}$ Field 내의 Force :

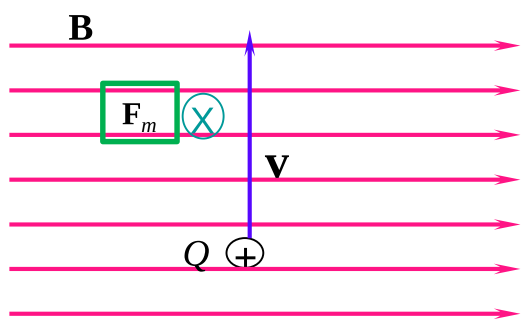
$$\boxed{\vec{F} = q\vec{v} \times \vec{B}}$$



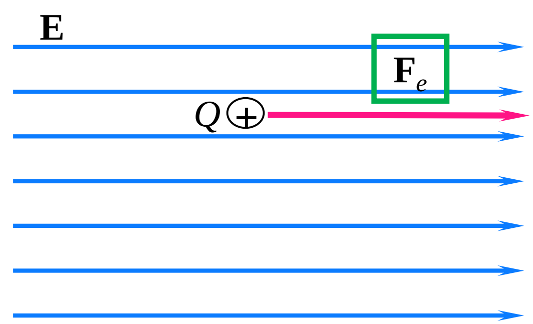

(i) $\vec{v} = 0 \;: \vec{F} = 0$

(ii) $|\vec{F}| = QvB\sin\theta, |\vec{F}| \propto |\vec{v}|$

(iii) 힘의 방향 ?



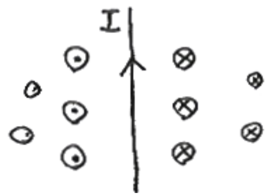

$\mathbf{F}_m = Q\,(\mathbf{v} \times \mathbf{B})$



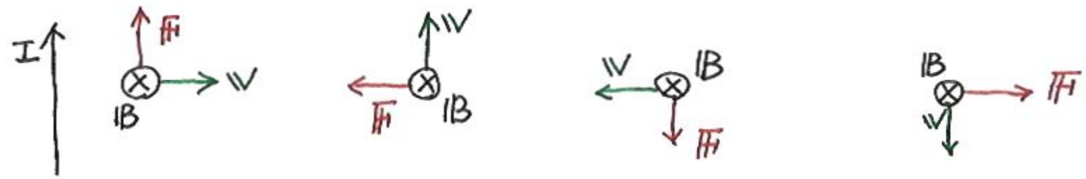

$\mathbf{F}_e = Q\mathbf{E}$

(Ex) :

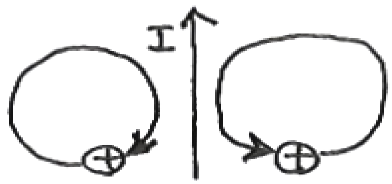
Field :



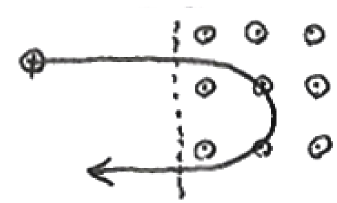

Force :



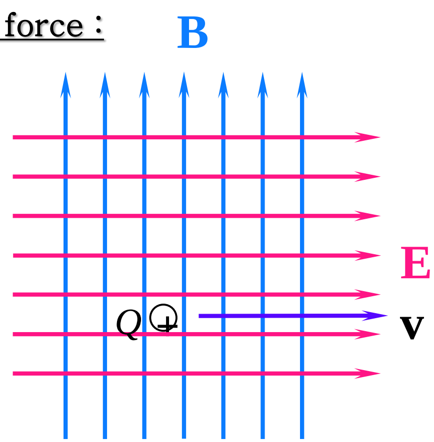

Motion :




<Magnetic Mirror>



- <u>Lorentz force :</u>



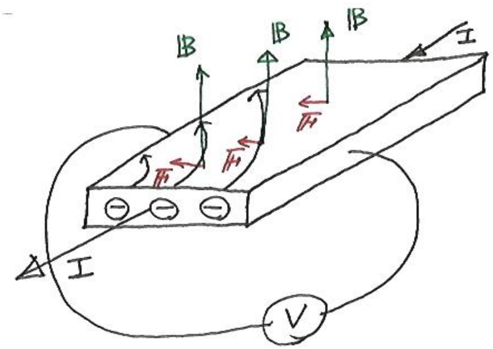

$\vec{E}$ & $\vec{B}$

$$\boxed{\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})}$$

→ Magnetron, Cyclotron, CRT Brown Tube 내의 전자의 운동경로

MHD 발전기, Plasma,

• Hall Effect와 자계의 측정 :

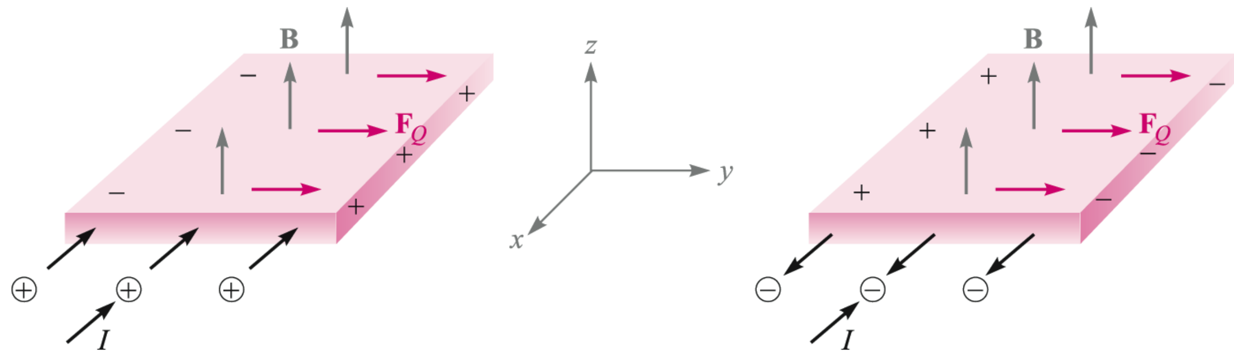



$$d\vec{F} = dQ\vec{v}\times\vec{B} = \rho dv\vec{v}\times\vec{B} = \rho\vec{v}\times\vec{B}dv = \vec{J}\times\vec{B}dv$$

$$\therefore \quad \boxed{\vec{F} = \vec{J}\times\vec{B}}$$

$\because$
- $\vec{J} = \rho\vec{v}$ : 대류전류밀도
- $dQ = \rho dv$ : 체적전하밀도
- $dv$ : 미소체적소

Hall Element : InSb
Hall Probe
Gauss Meter




(a) (b)

Lab. of Applied ElectroMagnetics

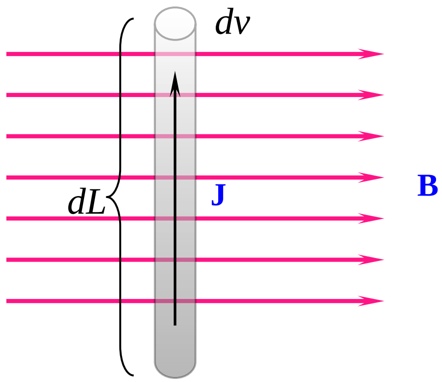
## 8.2 미소전류에 작용하는 힘

$$\boxed{\vec{F} = q\vec{v} \times \vec{B}}$$

$Q \to dQ$

$F \to dF$

$$dF = dQv \times B$$




$$d\mathbf{F} = dQ\,\mathbf{v} \times \mathbf{B} \qquad dQ = \rho_v dv$$

$$d\mathbf{F} = \rho_v dv\,\mathbf{v} \times \mathbf{B} \qquad \boxed{\mathbf{J} = \rho_v \mathbf{v}}$$

$$\boxed{d\mathbf{F} = \mathbf{J} \times \mathbf{B}\,dv}$$

• Current & Force :

$\boxed{d\mathbf{F} = \mathbf{J} \times \mathbf{B}\,dv}$ volume current density (3-D) $\mathbf{F} = \int_{\text{vol}} \mathbf{J} \times \mathbf{B}\,dv$

$\boxed{d\mathbf{F} = \mathbf{K} \times \mathbf{B}\,dS}$ surface current density (2-D) $\mathbf{F} = \int_S \mathbf{K} \times \mathbf{B}\,dS$

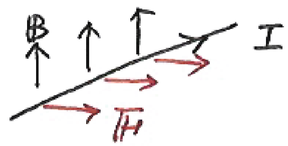
$\boxed{d\mathbf{F} = I\,d\mathbf{L} \times \mathbf{B}}$ filament current of length $dL$ (1-D) $\boxed{\mathbf{F} = I\mathbf{L} \times \mathbf{B}}$

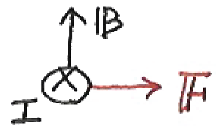
$$\vec{J}dv = \vec{K}d\vec{S} = Id\vec{L}$$

$$d\vec{F} = \vec{J} \times \vec{B}dv(3D) = \vec{K} \times \vec{B}d\vec{S}(2D) = Id\vec{L} \times \vec{B}(1D)$$

$$\to \vec{F} = \int_v \vec{J} \times \vec{B}dv = \int_S \vec{K} \times \vec{B}d\vec{S} = \oint Id\vec{L} \times \vec{B} = -I\oint \vec{B} \times d\vec{L}$$

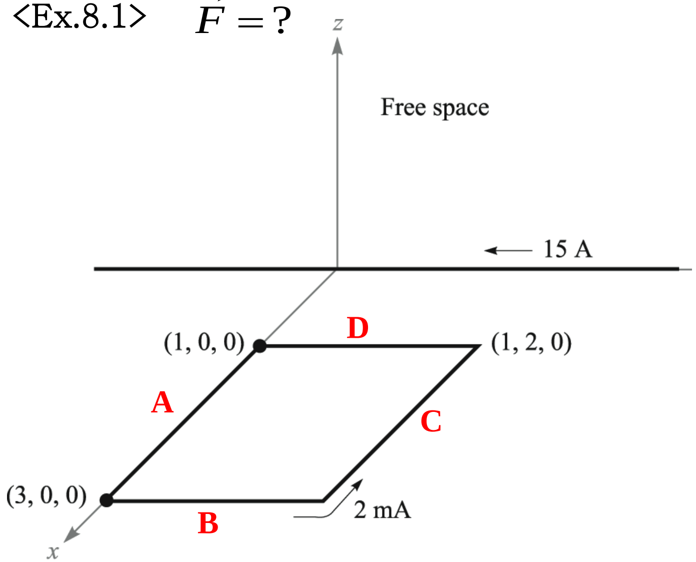
- IF : B가 균일하고 I가 직선전류일 경우




$$\boxed{\vec{F} = I\vec{L} \times \vec{B} = BIL \sin\theta}$$

<Ex.8.1> $\vec{F} = ?$




$$\mathbf{H} = \frac{I}{2\pi x}\mathbf{a}_z = \frac{15}{2\pi x}\mathbf{a}_z \text{ A/m}$$

$$\mathbf{B} = \mu_0\mathbf{H} = 4\pi \times 10^{-7}\mathbf{H} = \frac{3 \times 10^{-6}}{x}\mathbf{a}_z \text{ T}$$

$$\mathbf{F} = -I \oint \mathbf{B} \times d\mathbf{L}$$

$$\mathbf{F} = -2 \times 10^{-3} \times 3 \times 10^{-6} \left[ \underbrace{\int_{x=1}^{3} \frac{\mathbf{a}_z}{x} \times dx\,\mathbf{a}_x}_{A} + \underbrace{\int_{y=0}^{2} \frac{\mathbf{a}_z}{3} \times dy\,\mathbf{a}_y}_{B} + \underbrace{\int_{x=3}^{1} \frac{\mathbf{a}_z}{x} \times dx\,\mathbf{a}_x}_{C} + \underbrace{\int_{y=2}^{0} \frac{\mathbf{a}_z}{1} \times dy\,\mathbf{a}_y}_{D} \right]$$

$$= -6 \times 10^{-9} \left[ \ln x \Big|_1^3 \mathbf{a}_y + \frac{1}{3} y \Big|_0^2 (-\mathbf{a}_x) + \ln x \Big|_3^1 \mathbf{a}_y + y \Big|_2^0 (-\mathbf{a}_x) \right]$$

$$= -6 \times 10^{-9} \left[ (\ln 3)\mathbf{a}_y - \frac{2}{3}\mathbf{a}_x + \left(\ln \frac{1}{3}\right)\mathbf{a}_y + 2\mathbf{a}_x \right]$$

$$= \underline{-8\mathbf{a}_x \text{ nN}}$$

segment 별 force
A, C →Zero
D>B

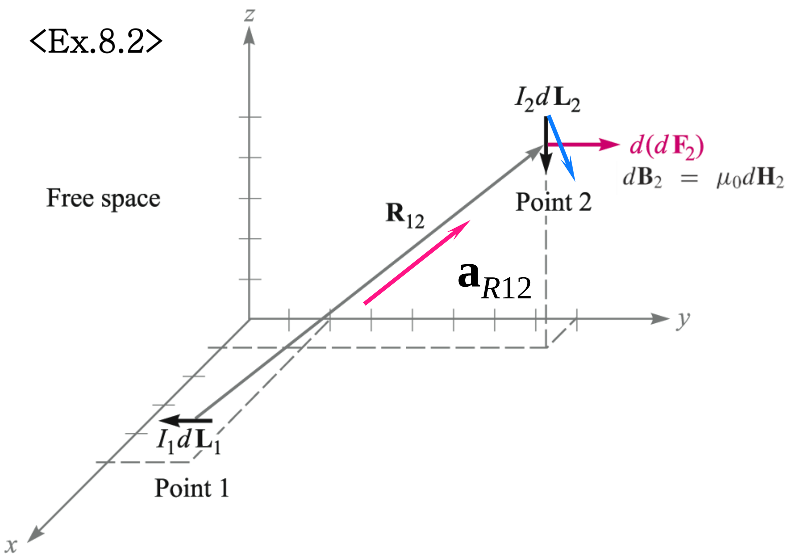
# 8.3 미소전류소 사이에 작용하는 힘


$$d\mathbf{H}_2 = \frac{I_1 d\mathbf{L}_1 \times \mathbf{a}_{R12}}{4\pi R_{12}^2} \qquad d\mathbf{B}_2 = \mu_0 d\mathbf{H}_2$$

$$d(d\mathbf{F}_2) = I_2 d\mathbf{L}_2 \times d\mathbf{B}_2$$

$$\boxed{d(d\mathbf{F}_2) = \mu_0 \frac{I_1 I_2}{4\pi R_{12}^2} d\mathbf{L}_2 \times (d\mathbf{L}_1 \times \mathbf{a}_{R12})}$$

<Ex.8.2>




Given: $I_1 d\mathbf{L}_1 = -3\mathbf{a}_y \mathrm{A \cdot m}$ at $P_1(5, 2, 1)$

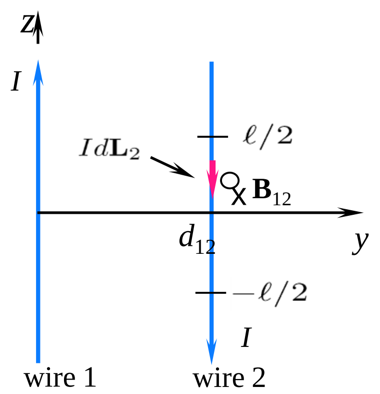
$I_2 d\mathbf{L}_2 = -4\mathbf{a}_z \mathrm{A \cdot m}$ at $P_2(1, 8, 5)$

Then $\mathbf{R}_{12} = -4\mathbf{a}_x + 6\mathbf{a}_y + 4\mathbf{a}_z$

$$d(d\mathbf{F}_2) = \frac{4\pi 10^{-7}}{4\pi} \frac{(-4\mathbf{a}_z) \times [(-3\mathbf{a}_y) \times (-4\mathbf{a}_x + 6\mathbf{a}_y + 4\mathbf{a}_z)]}{(16 + 36 + 16)^{1.5}}$$
$$= 8.56\mathbf{a}_y \text{ nN}$$

# ➢Force between two line current

- Easy Way :



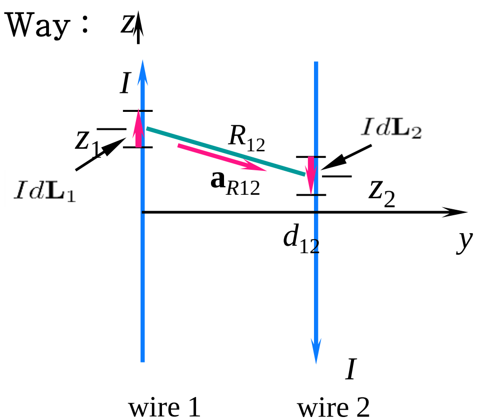

$$\mathbf{B}_{12} = -\frac{\mu_0 I}{2\pi d_{12}}\,\mathbf{a}_x \qquad I d\mathbf{L}_2 = -I\,dz_2\,\mathbf{a}_z$$

$$\mathbf{B}_{12} = -\frac{\mu_0 I}{2\pi d_{12}}\,\mathbf{a}_x \qquad I d\mathbf{L}_2 = -I\,dz_2\,\mathbf{a}_z$$

$$d\mathbf{F}_2 = I d\mathbf{L}_2 \times \mathbf{B}_{12} = -I\,dz_2\,\mathbf{a}_z \times \left(-\frac{\mu_0 I}{2\pi d_{12}}\,\mathbf{a}_x\right) = \frac{\mu_0 I^2 dz_2}{2\pi d_{12}}\,\mathbf{a}_y$$

$$\mathbf{F}_2 = \int_{-\ell/2}^{\ell/2} I d\mathbf{L}_2 \times \mathbf{B}_{12} = \boxed{\frac{\mu_0 I^2 \ell}{2\pi d_{12}}\,\mathbf{a}_y}$$

- Hard Way :




$$I d\mathbf{L}_1 = I\,dz_1\,\mathbf{a}_z \qquad I d\mathbf{L}_2 = -I\,dz_2\,\mathbf{a}_z$$

$$R_{12} = \sqrt{(z_2 - z_1)^2 + d_{12}^2} \qquad \mathbf{a}_{R12} = \frac{d_{12}\,\mathbf{a}_y + (z_2 - z_1)\mathbf{a}_z}{\sqrt{(z_2 - z_1)^2 + d_{12}^2}}$$

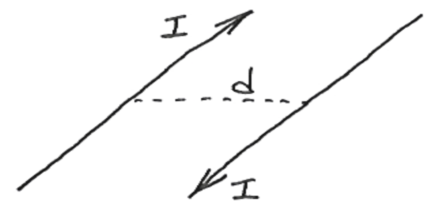
$$\mathbf{F}_2 = \mu_0 \frac{I^2}{4\pi} \int_{-\ell/2}^{\ell/2} \left[\int_{-\infty}^{\infty} \frac{d_{12}\,\mathbf{a}_y + (z_2 - z_1)\,\mathbf{a}_z}{[(z_2 - z_1)^2 + d_{12}^2]^{3/2}} \times dz_1 \mathbf{a}_z\right] \times (-dz_2\,\mathbf{a}_z)$$

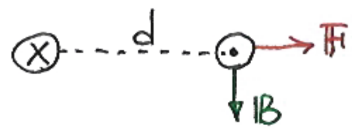
$$\mathbf{F}_2 = \mu_0 \frac{I^2 d_{12}}{4\pi}\,\mathbf{a}_y \int_{-\ell/2}^{\ell/2} \int_{-\infty}^{\infty} \frac{dz_1\,dz_2}{[(z_2 - z_1)^2 + d_{12}^2]^{3/2}}$$

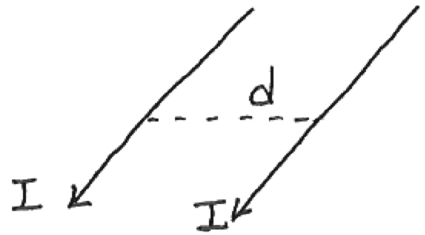
$$\int_{-\infty}^{\infty} \frac{dz_1}{[(z_2 - z_1)^2 + d_{12}^2]^{3/2}} = \left.\frac{z_1 - z_2}{d_{12}^2\,[(z_2 - z_1)^2 + d_{12}^2]^{1/2}}\right|_{z_1=-\infty}^{z_1=\infty} = \frac{2}{d_{12}^2}$$

$$\mathbf{F}_2 = \mu_0 \frac{I^2 d_{12}}{4\pi}\,\mathbf{a}_y \int_{-\ell/2}^{\ell/2} \frac{2}{d_{12}^2}\,dz_2 = \boxed{\frac{\mu_0 I^2 \ell}{2\pi d_{12}}\,\mathbf{a}_y}$$

## (Ex) 긴 선전류에 작용하는 힘



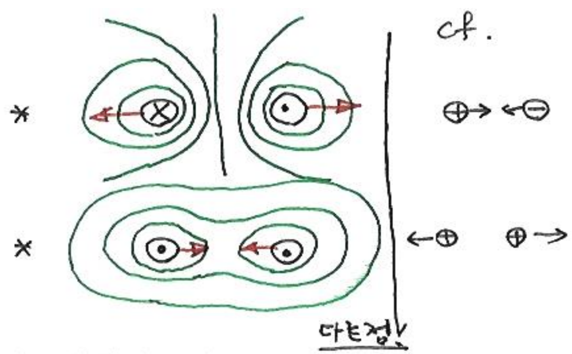




$$\vec{H} = \frac{I}{2\pi d}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi d}$$

$$\vec{F} = BIL\sin\theta = \frac{\mu_0 I}{2\pi d}\cdot I \cdot L$$

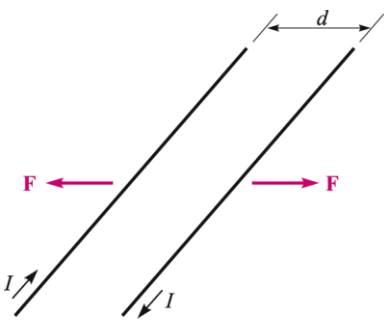



$$d(d\mathbf{F}_2) = \mu_0 \frac{I_1 I_2}{4\pi R_{12}^2} d\mathbf{L}_2 \times (d\mathbf{L}_1 \times \mathbf{a}_{R12})$$

$$\mathbf{F}_2 = \mu_0 \frac{I_1 I_2}{4\pi} \oint \left[ d\mathbf{L}_2 \times \oint \frac{d\mathbf{L}_1 \times \mathbf{a}_{R12}}{R_{12}^2} \right]$$

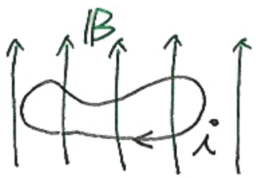
$$= \mu_0 \frac{I_1 I_2}{4\pi} \oint \left[ \oint \frac{\mathbf{a}_{R12} \times d\mathbf{L}_1}{R_{12}^2} \right] \times d\mathbf{L}_2$$

$$\boxed{F = \frac{\mu_0 I_1 I_2}{2\pi d}}$$



# 8.4 폐회로에 작용하는 힘과 회전력

## • 균일자기장 내의 힘

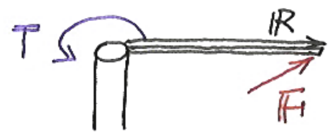



$$\vec{F} = -I\oint \vec{B}\times d\vec{L} = -I\vec{B}\times\oint d\vec{L}$$

$$= 0 \ (\because \nabla\cdot\vec{J} = 0,\ \int \vec{J}\cdot d\vec{S} = 0\ ),\quad \underline{\underline{\vec{F} = 0}}$$

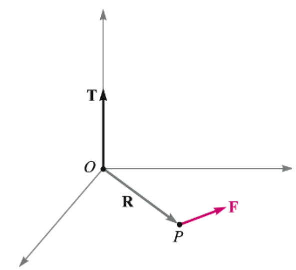
→ force는 Zero, 회전력만 작용

✓ Torque, Moment ( 회전력, 힘의 능률 ) $\boxed{\mathbf{T} = \mathbf{R}\times\mathbf{F}}$



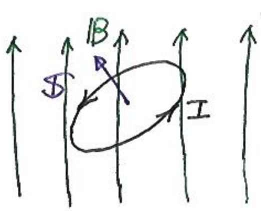

$\vec{T}$ 의 방향은 ? : 

$\vec{T}$ 의 크기는 ? : $\left|\vec{T}\right| \propto \vec{R},\ \vec{F},\ \sin\theta$




## • 균일자기장 내의 회전력

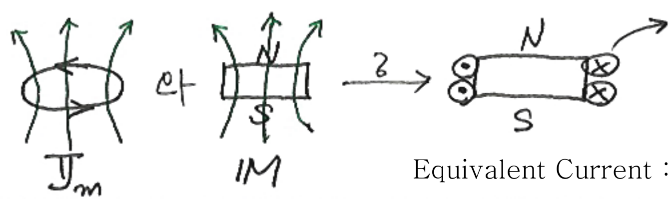
$\vec{S}$ 의 크기, 방향 정의




$$d\vec{T} = \underline{Id\vec{S}\times\vec{B}} \qquad \boxed{\vec{T} = I\vec{S}\times\vec{B}}$$

$$\left.\begin{aligned} &\vec{T} \text{ 의 크기 : } \quad \vec{S}\uparrow : \vec{T}\uparrow, \quad \vec{B}\uparrow, \vec{I}\uparrow : \vec{T}\uparrow \\ &\vec{T} \text{ 의 방향 : } \quad \vec{S} \,//\, \vec{B} : \vec{T} = 0 \quad \vec{S}\perp\vec{B} : \vec{T} \rightarrow Max \end{aligned}\right\} \rightarrow \vec{S} \,//\, \vec{B} \text{ 되도록 작용}$$

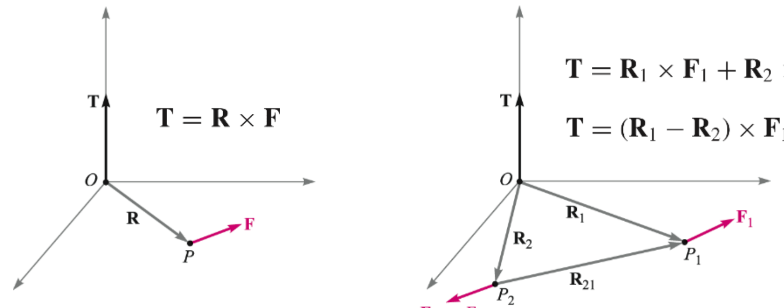
✓ Loop Current
& Magnet





Equivalent Current : $\vec{J}_m = \nabla\times\vec{M}$

- Torque in a Loop Current

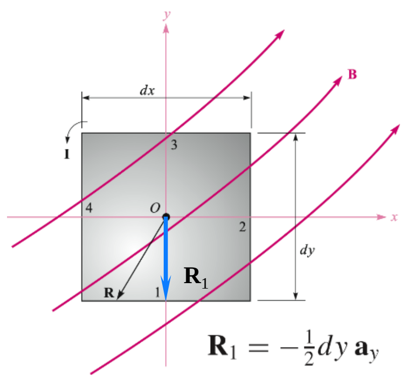



(a) (b)

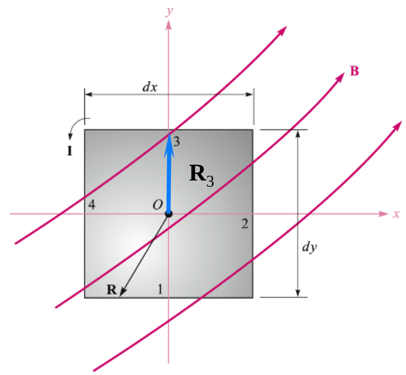
$$\mathbf{T} = \mathbf{R} \times \mathbf{F}$$

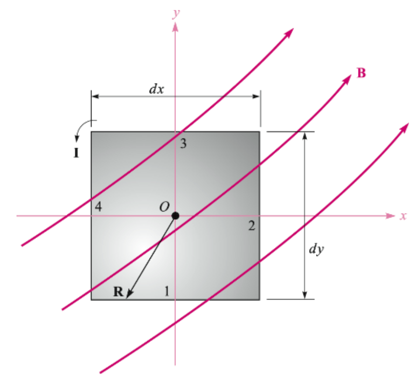
$$\mathbf{T} = \mathbf{R}_1 \times \mathbf{F}_1 + \mathbf{R}_2 \times \mathbf{F}_2 \qquad \text{since } \mathbf{F}_2 = -\mathbf{F}_1 :$$

$$\mathbf{T} = (\mathbf{R}_1 - \mathbf{R}_2) \times \mathbf{F}_1 = \mathbf{R}_{21} \times \mathbf{F}_1$$




$$\mathbf{R}_1 = -\tfrac{1}{2} dy\, \mathbf{a}_y$$




$$d\mathbf{F}_1 = I\, dx\, \mathbf{a}_x \times \mathbf{B}_0 \;= I\, dx(B_{0y}\mathbf{a}_z - B_{0z}\mathbf{a}_y)$$

$$d\mathbf{T}_1 = \mathbf{R}_1 \times d\mathbf{F}_1$$
$$= -\tfrac{1}{2} dy\, \mathbf{a}_y \times I\, dx(B_{0y}\mathbf{a}_z - B_{0z}\mathbf{a}_y)$$
$$= -\tfrac{1}{2} dx\, dy\, I\, B_{0y}\mathbf{a}_x$$

$$d\mathbf{T}_3 = \mathbf{R}_3 \times d\mathbf{F}_3 \;= \tfrac{1}{2} dy\, \mathbf{a}_y \times (-I\, dx\, \mathbf{a}_x \times \mathbf{B}_0)$$
$$= -\tfrac{1}{2} dx\, dy\, I B_{0y}\mathbf{a}_x = d\mathbf{T}_1 \;\text{(!)}$$

$$d\mathbf{T}_1 + d\mathbf{T}_3 = -dx\, dy\, I B_{0y}\mathbf{a}_x$$

$$d\mathbf{T}_2 + d\mathbf{T}_4 = dx\, dy\, I B_{0x}\mathbf{a}_y$$

$$d\mathbf{T} = I\, dx\, dy(B_{0x}\mathbf{a}_y - B_{0y}\mathbf{a}_x)$$

$$d\mathbf{T} = I\, dx\, dy(\mathbf{a}_z \times \mathbf{B}_0) \qquad d\mathbf{S} = dx dy\, \mathbf{a}_z$$

$$\boxed{d\mathbf{T} = I\, d\mathbf{S} \times \mathbf{B}}$$

$$\boxed{d\mathbf{T} = I\, d\mathbf{S} \times \mathbf{B}}$$

$$\boxed{d\mathbf{m} = I\, d\mathbf{S}}$$

$$\boxed{d\mathbf{T} = d\mathbf{m} \times \mathbf{B}}$$

$$\boxed{\mathbf{T} = I\mathbf{S} \times \mathbf{B} = \mathbf{m} \times \mathbf{B}}$$

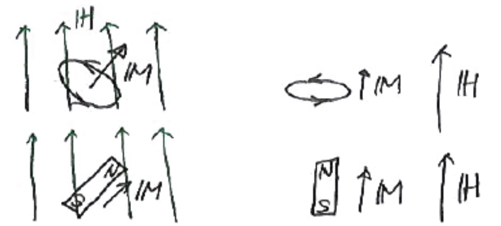
## ➢ Dipole Moment (쌍극자 모멘트)

( 전계 ) 전기 쌍극자 모멘트 : $\boxed{\vec{P}}$

$$d\vec{T} = d\vec{P} \times \vec{E} \qquad (\,*\; d\vec{P} = dQ\vec{d}\,)$$

( 자계 ) 자기 쌍극자 모멘트 : $\boxed{\vec{m}}$

$$d\vec{T} = d\vec{m} \times \vec{B} \qquad (\,*\; d\vec{m} = Id\vec{S}\,)$$



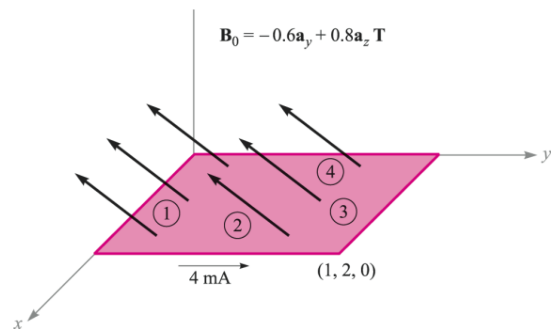
✓ 나침반
✓ MRI

• 균일 자계내에서의 자기 쌍극자 모멘트

$$\boxed{\vec{T} = I\vec{S} \times \vec{B} = \vec{m} \times \vec{B}}$$

< Ex 8.3 > 정방형 Loop Current의 Force, Torque 계산

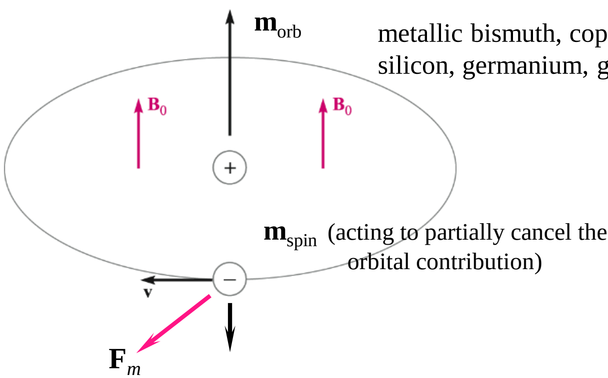
$$\mathbf{T} = 4 \times 10^{-3}[(1)(2)\mathbf{a}_z] \times (-0.6\mathbf{a}_y + 0.8\mathbf{a}_z) = 4.8\mathbf{a}_x \text{ mN}\cdot\text{m}$$



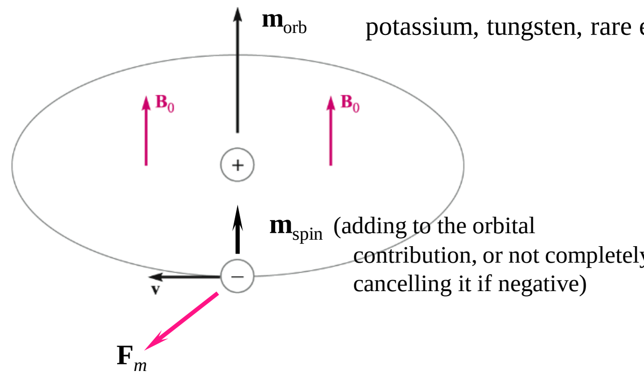

< Ex 8.4  Torque 계산

# 8.5 자성체의 성질

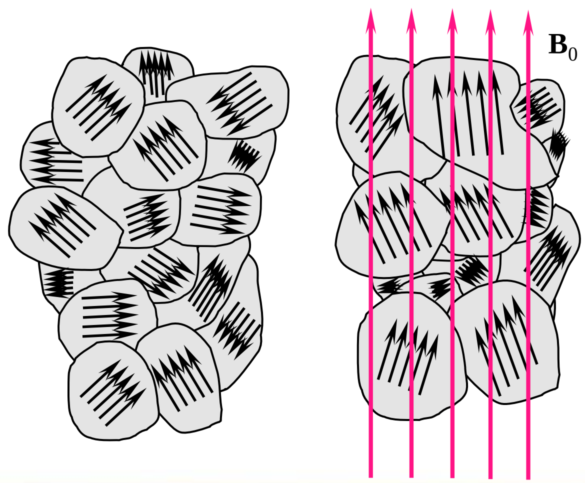
- 자성체,  매질,  분자,  원자,  전자
- 자성체
  - 반 자성체( Dia Magnetism ), Cu
  - 상 자성체( Para Magnetism ), Al
  - 강 자성체( Ferro Magnetism ), Fe
    - 연 자성체( Soft ) : Fe
    - 경 자성체( Hard ) : 영구자석
  - 역강 자성체( Anti Ferro Magnetism )
  - 페리 자성체( Ferri Ferro Magnetism )
  - 초상 자성체( Super Para Magnetism )




metallic bismuth, copper, gold, silicon, germanium, graphite, sulfur.




potassium, tungsten, rare earth elements

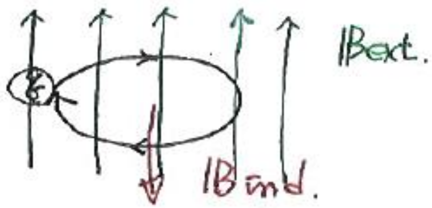



iron, nickel, cobalt (room temperature), gadolinium, dysprosium (low temperature).

※ 전자의 공전 → 반 자성체 성질
전자의 자전 → 상 자성체 성질
모든 물질에는 두 가지 다 공존함
어느 것이 큰가에 따라 결정

## (1) 반 자성체 ( Dia-Magnetic Material )

• 자계 내에서 운동하는 전하 :  Lentz Force :  $\vec{F} = q\vec{v} \times \vec{B}$




q 의 회전 운동 → Current → $\vec{B}_{ind}$ 발생
$\vec{B}_{ind}$ 의 방향 : $-\vec{B}_{ext}$ 방향

$\vec{B}_{total} = \vec{B}_{ext} + \vec{B}_{ind} < \vec{B}_{ext}$   즉 매질로 인해 자계가 줄어드는 효과

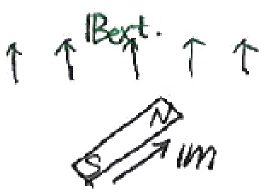

* 외부자계는 전자의 공전을 방해한다.
* 모든 물체는 전자의 공전운동이 있으므로 반자성적 성질이 있다.

비스무스 : 0.99999986, 수소, 헬륨, 탄소, 황산
구리,금,은,실리콘,게르마늄

## (2) 상 자성체 ( Para-Magnetic Material )

• 전자의 자전 :

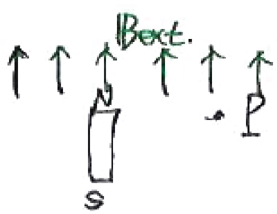

하나의 자석




$\begin{cases} W = -\vec{m} \cdot \vec{B} = -mB\cos\theta \\ \vec{T} = \vec{m} \times \vec{B} = mB\sin\theta \end{cases}$

Θ = 0°때 : // → W 최소 , $\vec{T} = 0$ (stable)
Θ = 90°때 : ⊥ → W 최대 , $\vec{T}$ 최대

→ Θ = 0°로, 즉 외부자계방향으로 정렬
→ 외부자계를 증대시키는 결과




$\vec{B}_{total} = \vec{B}_{ext} + \vec{B}_{ind} > \vec{B}_{ext}$

모멘트 정렬로 인하여 자계가 커진다.   $\mu > 1$

원자 내의 전자의 자전운동은 외부자계의 영향으로
정렬되어 자계의 크기를 증가 시킨다.

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## (3) 강 자성체 ( Ferro-Magnetic Material )

- 철은 왜 자석에 달라 붙을 까?  Al은?  Cu 는?  ($\mu_r > 1000$ !!! , *Why* ?)

- 1906년 Pierre Weiss 의 분자장 이론: 가설. Heisenberg 양자역학 → Exchange Energy로 분자장 이론을 설명
  - ① 자발자화( Spontaneous Magnetization ). (영구히 자화) ?
  - ② 자구( Magnetic Domain ). 존재. (서로반발) ?

- Bitter. 자구 관측. 자구 : Ferrite : 0.8 ~1 $\mu m$
  Sm-Co : 2 $\mu m$
  NdFeB : 0.5 $\mu m$

※ 자석의 역사
전자석/영구자석
말굽자석/페라이트자석/희토류자석-AlNiCo,Sm-Co,NdFeB
→(일본 Smitomo, 미국 Quench)

- 강자성체의
  - 자기포화 ( Magnetic Saturation )
  - 히스테리시스 ( Magnetic Hysteresis )

- 연자성체 ( Soft ) / 경자성체(Hard)

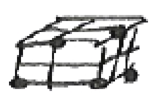
- Curie 온도 : Fe(769°), Co(1127°), Ni(358°)
  실온에의 강자성체 - Fe, Co, Ni only

## (4) 역 반 강 자성체 ( Anti Ferro-Magnetic Material ) MnO, ↑↓ ↑↓

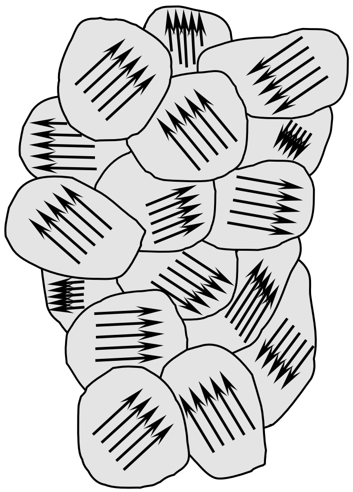
## (5) 페라이트 ( Ferrite ) ↑↓ ↑↓ 자철광( $Fe_2O_3$, $Fe_3O_4$ ), 니켈페라이트, 니켈아연 페라이트

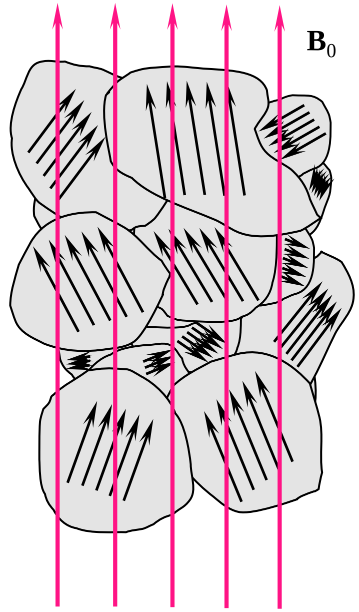
Ceramic Magnet, Ferrospinels = Ferrite, $\sigma$小, $\mu$大 고주파기기에 사용

## (6) 초상자성체 ( Super para-Magnetic Material ) 자기테이프

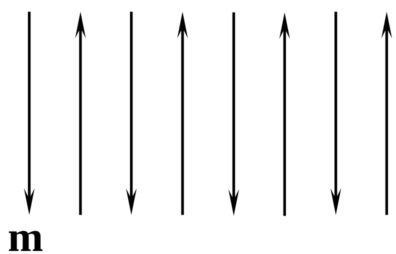


✓ Ferro-Magnetic Material

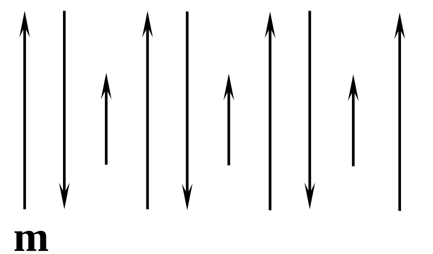





✓ Anti Ferro-Magnetic Material , MnO




✓ Ferrite , $Fe_2O_3$, $Fe_3O_4$



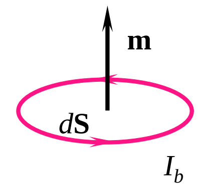
## ➢ Magnetic Material Summary

| Classification | Magnetic Moments | B Values | Comments |
|---|---|---|---|
| Diamagnetic | $\mathbf{m}_{\text{orb}} + \mathbf{m}_{\text{spin}} = 0$ | $B_{\text{int}} < B_{\text{appl}}$ | $B_{\text{int}} \doteq B_{\text{appl}}$ |
| Paramagnetic | $\mathbf{m}_{\text{orb}} + \mathbf{m}_{\text{spin}} = \text{small}$ | $B_{\text{int}} > B_{\text{appl}}$ | $B_{\text{int}} \doteq B_{\text{appl}}$ |
| Ferromagnetic | $\lvert\mathbf{m}_{\text{spin}}\rvert \gg \lvert\mathbf{m}_{\text{orb}}\rvert$ | $B_{\text{int}} \gg B_{\text{appl}}$ | Domains |
| Antiferromagnetic | $\lvert\mathbf{m}_{\text{spin}}\rvert \gg \lvert\mathbf{m}_{\text{orb}}\rvert$ | $B_{\text{int}} \doteq B_{\text{appl}}$ | Adjacent moments oppose |
| Ferrimagnetic | $\lvert\mathbf{m}_{\text{spin}}\rvert \gg \lvert\mathbf{m}_{\text{orb}}\rvert$ | $B_{\text{int}} > B_{\text{appl}}$ | Unequal adjacent moments oppose; low $\sigma$ |
| Superparamagnetic | $\lvert\mathbf{m}_{\text{spin}}\rvert \gg \lvert\mathbf{m}_{\text{orb}}\rvert$ | $B_{\text{int}} > B_{\text{appl}}$ | Nonmagnetic matrix; recording tapes |

## 8.6 자화 및 투자율

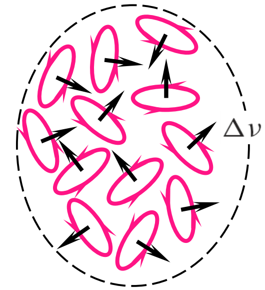
- 자화 $\vec{M}$ ( magnetization )

$$\mathbf{m} = I_b d\mathbf{S}$$ , 속박전류

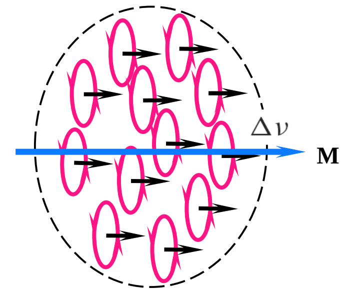

$$\mathbf{m}_{\text{total}} = \sum_{i=1}^{n\Delta\nu} \mathbf{m}_i$$

자화( 또는 자화의 세기 ) = 단위 체적당 자기쌍극자 모멘트




$$\mathbf{M} = \lim_{\Delta\nu \to 0} \frac{1}{\Delta\nu} \sum_{i=1}^{n\Delta\nu} \mathbf{m}_i$$

$$\mathbf{M} = n\mathbf{m} = nI_b d\mathbf{S}$$

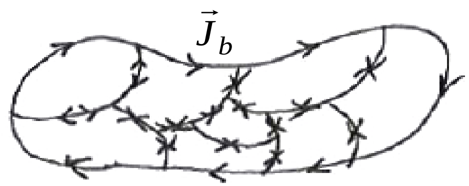



- 속박 전류 밀도 : $\vec{J}_b$

$$I_b = \oint \vec{J}_b \cdot d\vec{S} = \oint \vec{M} \cdot d\vec{L}$$

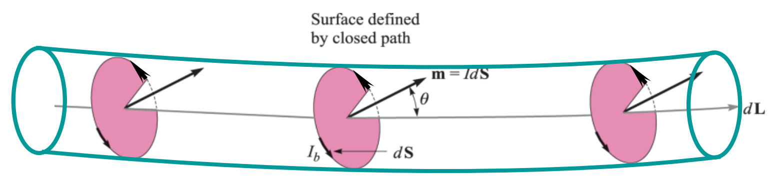
$$\oint \vec{M} \cdot d\vec{L} = \oint_s \vec{J}_b \cdot d\vec{S}$$

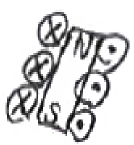
$$\oint \nabla \times \vec{M} \cdot d\vec{S} = \oint_s \vec{J}_b \cdot d\vec{S}$$




가운데는 서로 상쇄, 경계만 남음

$$\therefore \quad \boxed{\nabla \times \vec{M} = \vec{J}_b}$$




M  $\vec{J}_b$



$$I_B = \oint \mathbf{M} \cdot d\mathbf{L}$$

$$I = \oint \mathbf{H} \cdot d\mathbf{L}$$

- $\underline{\vec{B}, \vec{M}, \vec{H}}$

자유공간에서 : $\nabla \times \vec{H}_T = \vec{J}_T = \vec{J}_s + \vec{J}_b$

$$\vec{J}_b = \nabla \times \vec{M}$$

$$\vec{H}_T = \frac{1}{\mu_0}\vec{B}_T \qquad \vec{B}_T = \mu_0 \vec{H}_T$$

$$\therefore \nabla \times \frac{1}{\mu_0}\vec{B}_T = \vec{J}_s + \nabla \times \vec{M}$$

$$\nabla \times \left(\frac{\vec{B}_T}{\mu_0} - \vec{M}\right) = \vec{J}_s$$

$$\nabla \times \vec{H}_s = \vec{J}_s$$

$$\vec{H}_s = \frac{\vec{B}_T}{\mu_0} - \vec{M} \qquad \therefore \boxed{\vec{B}_T = \mu_0(\vec{H}_s + \vec{M})}$$

$$\boxed{\vec{D} = \varepsilon_0 \vec{E} + \vec{P}}$$

Conduction Current: $I = \int_s \mathbf{J} \cdot d\mathbf{S}$

Bound Current: $I_B = \int_s \mathbf{J}_B \cdot d\mathbf{S}$

Total Current(= $I$+$I_B$) : $I_T = \int_s \mathbf{J}_T \cdot d\mathbf{S}$

*


$$\vec{J}_s \rightarrow \vec{H}_s \qquad \left\{ \begin{array}{l} \nabla \times \vec{H}_s = \vec{J}_s \\ \nabla \times \vec{H}_T = \vec{J}_s + \nabla \times \vec{M} \end{array} \right.$$

$$\vec{H}_T = \vec{H}_s + f(\vec{M})$$

$$\left\{ \begin{array}{l} \vec{B}_T = \mu_0(\vec{H}_s + \vec{M}) \\ \vec{H}_T = \vec{H}_s + \vec{M} \end{array} \right.$$

• 자화율 ( Magnetic Susceptibility ) : $\chi_m$

$$\boxed{\vec{M} = \chi_m \vec{H}_s} \qquad \chi_m = \frac{\vec{M}}{\vec{H}_s} \qquad \vec{M} \,//\, \vec{H}_s \ \ ?$$ ( 등방성 / 이방성 )

• 투자율 ( Magnetic Permeability ) : $\mu$

$$\boxed{\vec{B}_T = \mu \vec{H}_s} \qquad \vec{B} = \mu_0(\vec{H}_s + \vec{M}) = \mu_0(\vec{H}_s + \chi_m \vec{H}_s) = \mu_0(1+\chi_m)\vec{H}_s = \mu_0 \mu_r \vec{H}_s$$

$$\mu = \mu_0 \mu_r \qquad \mu_r = 1 + \chi_m$$

(Ex 8.5) Given a ferrite material that we shall specify to be operating in a linear mode with $B = 0.05$ T, let us assume $\mu_r = 50$, and calculate values for $\chi_m$, $M$, and $H$.

Because $\mu_r = 1 + \chi_m$, we have $\chi_m = \mu_r - 1 = 49$

Now: $B = \mu_r \mu_0 H$ so that…. $H = \dfrac{0.05}{50 \times 4\pi \times 10^{-7}} = 796$ A/m

The magnetization is $\chi_m H$, or 39000 A/m

• Anisotropic Media : Permeability tensor

$$\mathbf{B} = \bar{\bar{\mu}}\mathbf{H} \qquad \begin{aligned} B_x &= \mu_{xx}H_x + \mu_{xy}H_y + \mu_{xz}H_z \\ B_y &= \mu_{yx}H_x + \mu_{yy}H_y + \mu_{yz}H_z \\ B_z &= \mu_{zx}H_x + \mu_{zy}H_y + \mu_{zz}H_z \end{aligned}$$

➢ Sum

$$\begin{cases} \text{Vacuum} : & \mu_r = 1, \quad \chi_m = 0, \quad \vec{M} = 0 \\ \text{Dia} : & \mu_r < 1, \quad \chi_m < 0, \quad \vec{M} < 0, \ (\chi_m : -0.00005 \sim 0) \\ \text{Para} : & \mu_r > 1, \quad \chi_m > 0, \quad \vec{M} > 0, \ (\chi_m : -0.0001 \sim 0.01) \\ \text{Ferro} : & \mu_r >> 1, \quad \chi_m >> 0, \quad \vec{M} >> 0, \ (\chi_m : 100 \sim 100\text{만}) \end{cases}$$

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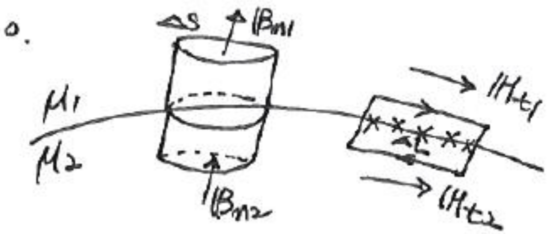
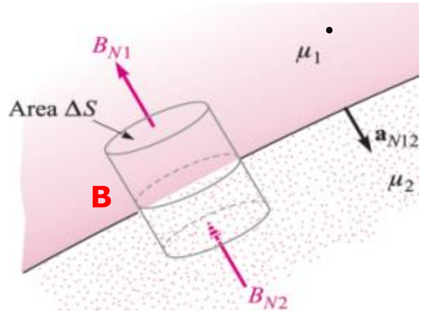
## 8.7 자계 경계조건



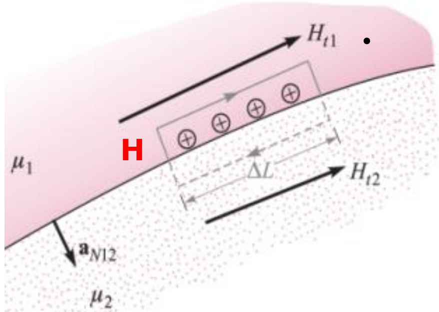

$$\oint \vec{B}\cdot d\vec{S} = 0$$

$$\vec{B}_{n1}\cdot \cancel{\Delta S} - \vec{B}_{n2}\cdot \cancel{\Delta S} = 0$$

$$\mu_1 \vec{H}_{n1} = \mu_2 \vec{H}_{n2}$$

$$\boxed{\vec{B}_{n1} = \vec{B}_{n2}}$$

$$\boxed{\vec{H}_{n2} = \frac{\mu_1}{\mu_2}\vec{H}_{n1}}$$




$$\oint \vec{H}\cdot d\vec{L} = I$$

$$\vec{H}_{t1}\cdot \cancel{\Delta L} - \vec{H}_{t2}\cdot \cancel{\Delta L} = k\cdot \cancel{\Delta L}$$

$$\vec{B}_{t1}/\mu_1 - \vec{B}_{t2}/\mu_2 = k$$

$$\boxed{\vec{H}_{t1} - \vec{H}_{t2} = k}$$

$$\boxed{\vec{B}_{t2} = \frac{\mu_2}{\mu_1}\vec{B}_{t1}}$$

(부도체: k=0)

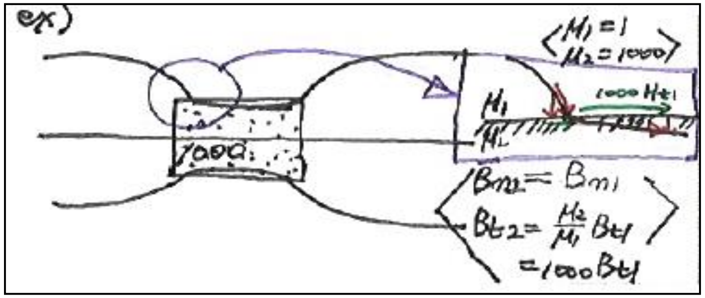
$$\boxed{\vec{H}_{t1} = \vec{H}_{t2}}$$

- Sum

$$\left[\begin{array}{ll} \boxed{\vec{B}_{n1} = \vec{B}_{n2}} & \vec{B}_{t2} = \frac{\mu_1}{\mu_2}\vec{B}_{t1} \\ \vec{H}_{n2} = \frac{\mu_1}{\mu_2}\vec{H}_{n1} & \boxed{\vec{H}_{t1} = \vec{H}_{t2}} \end{array}\right]$$

$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

$$\vec{B} = \mu\,\vec{H} \qquad (?)$$



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# 8.8 자기회로

✓ 전기회로 : 전압, 전기저항, 전류
자기회로 : 기자력, 자기저항, 자속(자속밀도)

## < Electricity >

- Electro-Motive Force(emf) :

$$\mathbf{E} = -\nabla V \qquad V_{AB} = \int_A^B \mathbf{E} \cdot d\mathbf{L}$$

- Electric Current :

$$\mathbf{J} = \sigma \mathbf{E} \qquad I = \int_S \mathbf{J} \cdot d\mathbf{S}$$

- Electric Resistance :

$$V = IR \qquad R = \frac{d}{\sigma S}$$

- Electric Circuit :

$$\oint \mathbf{E} \cdot d\mathbf{L} = 0$$

## < Magnetism >

- Magneto-Motive Force(mmf) :

$$\mathbf{H} = -\nabla V_m \qquad V_{mAB} = \int_A^B \mathbf{H} \cdot d\mathbf{L}$$

- Magnetic Flux (Density) :

$$\mathbf{B} = \mu \mathbf{H} \qquad \Phi = \int_S \mathbf{B} \cdot d\mathbf{S}$$

- Magnetic Reluctance :

$$\boxed{V_m = \Phi \Re} \qquad \boxed{\Re = \frac{d}{\mu S}}$$

- Magnetic Circuit :

$$\oint \mathbf{H} \cdot d\mathbf{L} = I_{\text{total}} \qquad \boxed{\oint \mathbf{H} \cdot d\mathbf{L} = NI}$$

| 전기회로 | 자기회로 |
| --- | --- |

## 전기회로

$\vec{E} = -\nabla V$

$V_{AB} = \int_A^B \vec{E} \cdot d\vec{L}$

$\vec{J} = \sigma \vec{E}$ , $\sigma$ : 도전율

$\boxed{I = \int_A \vec{J} \cdot d\vec{S}}$

$\boxed{V = IR}$

$\boxed{R = \frac{V}{I}}$

$= \frac{\ell}{\sigma S}$


$\sigma[S/m] =$ 6.17×10⁷, 은

- $6.17\times10^{7}$, 은
- $5.8\times10^{7}$, 동
- $4.1\times10^{7}$, 금
- $3.8\times10^{7}$, Al
- $1.03\times10^{7}$, 철
- $6.17\times10^{-10}$, 도자기
- $2\times10^{-13}$, 다이아몬드

$\boxed{\oint \vec{E} \cdot d\vec{L} = 0}$

## 자기회로

$\vec{H} = -\nabla V_m$ $V_m$ = 기자력(Magnetic Motive Force, mmf) [AT]

$V_{mAB} = \int_A^B \vec{H} \cdot d\vec{L}$

$\vec{B} = \mu \vec{E}$ , $\mu$ : 투자율

$\boxed{\Phi = \int_S \vec{B} \cdot d\vec{S}}$

$\boxed{V_m = \Phi R_m}$

$\boxed{R_m = \frac{V_m}{\Phi}}$ $R_m$ = Reluctance [AT/Wb]

$= \frac{\ell}{\mu S}$

$P = \frac{1}{R_m}$ = Permeance

$\mu[H/m] =$

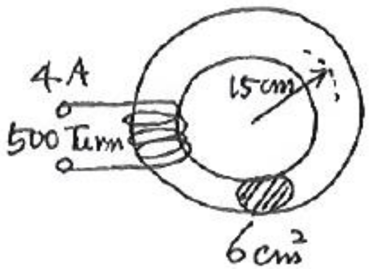
- Fe : 1000~4000
- 철분 : 100
- Supermalloy : 10만

→자기누설, 비선형 자기회로

$\boxed{\oint \vec{H} \cdot d\vec{L} = NI}$

$\oint \vec{H} \cdot d\vec{L} = NI$

< Ex. 본문 > 공심 토로이드(Air-Core Toroid)




① 기자력 : $V_m = N \cdot I = 500 \times 4 = 2000$ [A·T]

② 자기저항 :


$$R_m = \frac{\ell}{\mu S} = \frac{2\pi \times 0.15}{4\pi \times 10^{-7} \times 6 \times 10^{-4}} = 1.25 \times 10^{9} \, [A \cdot T / Wb]$$

③ 자속 : $\Phi = \dfrac{V_m}{R_m} = \dfrac{2000}{1.25 \times 10^{9}} = 1.6 \times 10^{-6} \, [Wb]$

자속밀도 : $B = \dfrac{\Phi}{S} = 2.67 \times 10^{-3} \, [Wb/m^2 = T] = 26.7 \, [G]$

③ $H = \dfrac{B}{\mu} = \underline{\underline{2120 \, [AT/m]}}$

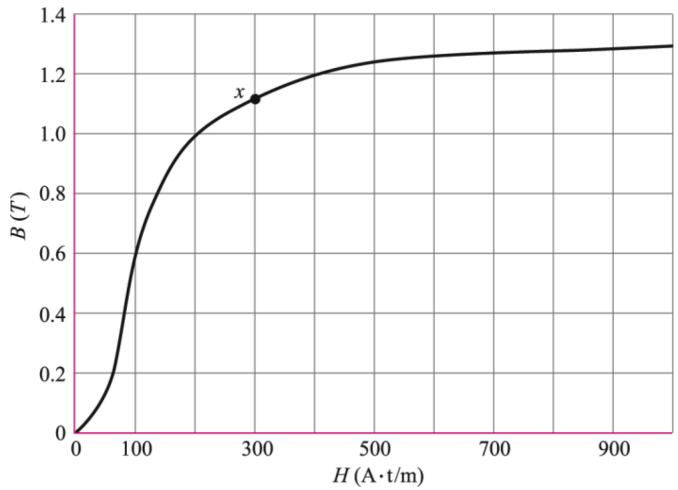
✓ Ampere 주회법칙 $H_\phi = \dfrac{NI}{2\pi r} = \underline{\underline{2120 [AT/m]}}$

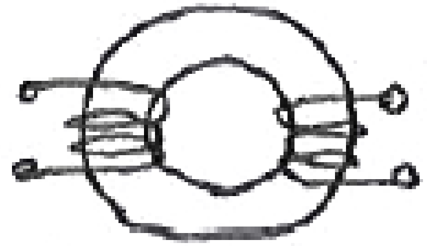
$$\int \vec{H} \cdot d\vec{L} = NI$$

$$H_\phi \cdot 2\pi r = NI$$

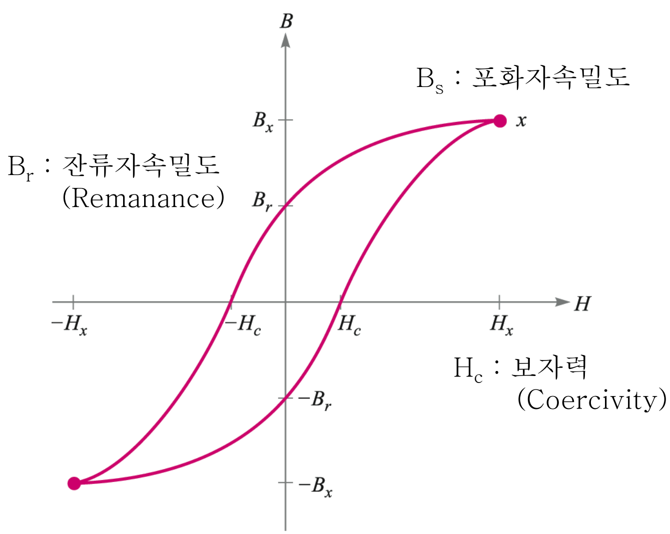
## ◎ 자기포화현상 ( Magnetic Saturation )

< B-H Curve >




## ◎ 자기히스테리시스현상 ( Magnetic Hysteresis )

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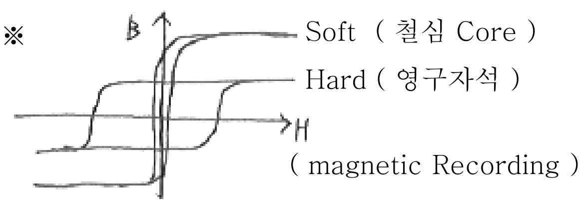
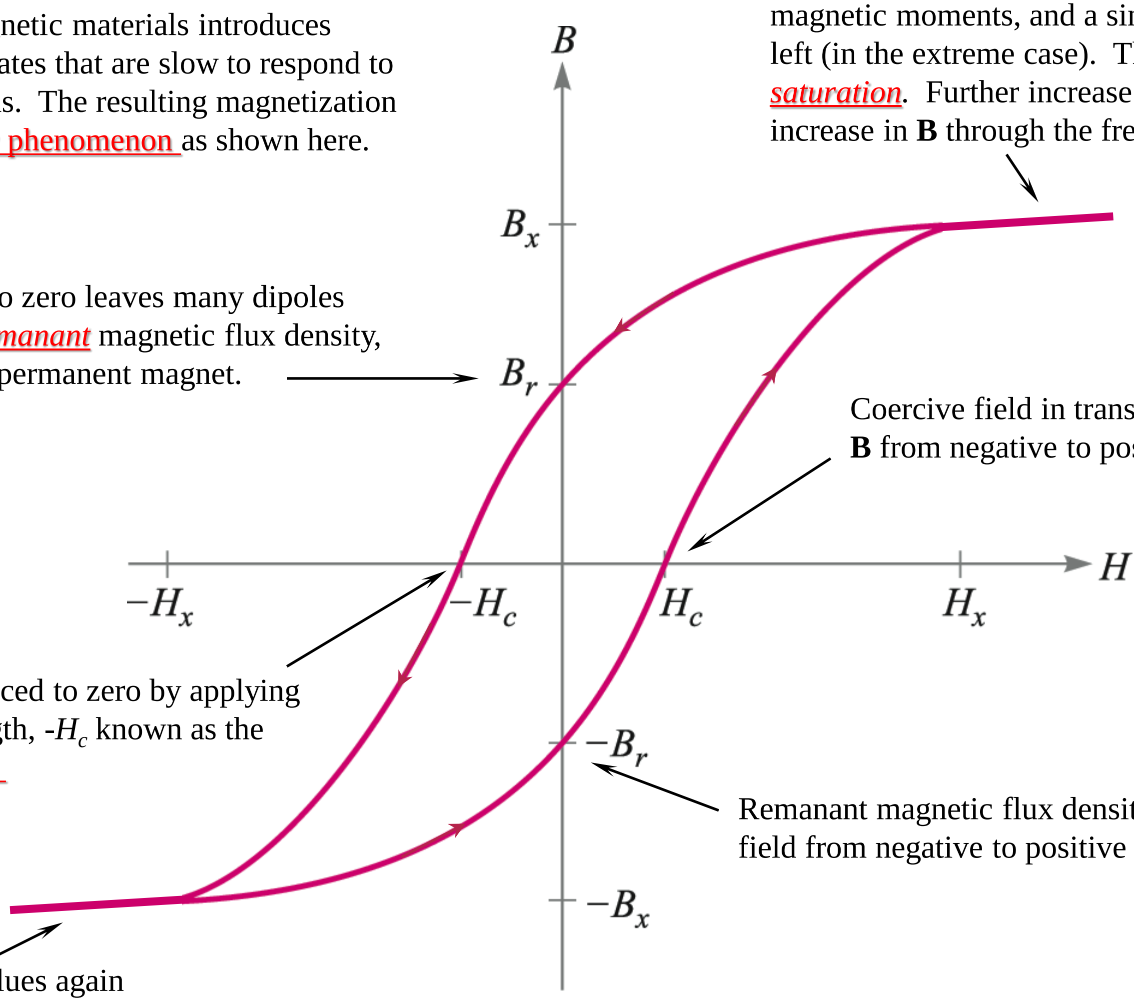


# Magnetic Hysteresis

Domain wall shifting in ferromagnetic materials introduces semi-permanent magnetization states that are slow to respond to changes in applied magnetic fields. The resulting magnetization curve demonstrates the *hysteresis phenomenon* as shown here.

Increasing **H** to high positive values lines up all magnetic moments, and a single domain is left (in the extreme case). The core is thus in *saturation*. Further increase in **H** leads to an increase in **B** through the free space permeability




Decreasing the applied **H** field to zero leaves many dipoles still aligned, and we have the *remanant* magnetic flux density, $B_r$. The material has become a permanent magnet.

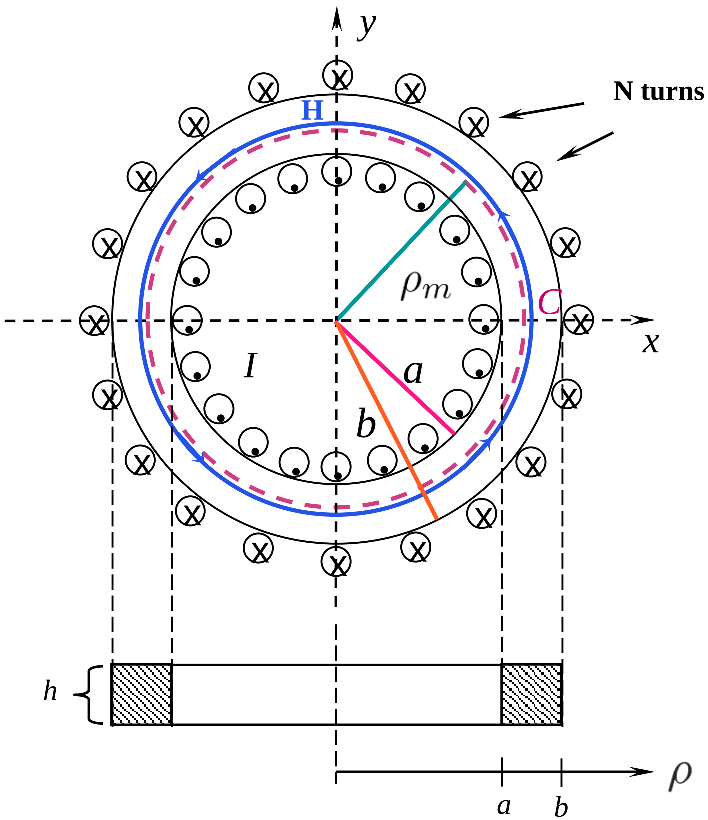
Coercive field in transistioning **B** from negative to positive values

The remanant flux density is reduced to zero by applying an opposing magentic field strength, $-H_c$ known as the *coercive field (or coercive force).*

Remanant magnetic flux density, for *increasing* **H** field from negative to positive values

Increasing **H** to high negative values again leads to saturation

- Application of Reluctance: Toroidal Coil




(1) H :

$$\Re = \frac{d}{\mu S} \approx \frac{2\pi\rho_m}{\mu(b-a)h}$$

$$\Phi = \frac{V_m}{\Re} = \frac{NI\mu(b-a)h}{2\pi\rho_m}$$

$$H = \frac{B}{\mu} = \frac{\Phi}{\mu S} = \frac{NI}{2\pi\rho_m}$$

$$\oint_C \mathbf{H}\cdot d\mathbf{L} = 2\pi\rho_m H_\phi = I_{encl} = NI \qquad \boxed{H_\phi = \frac{NI}{2\pi\rho_m}\ \text{A/m}}$$

(2) Φ :

$$\mathbf{B} = B_\phi \mathbf{a}_\phi = \frac{NI}{2\pi\rho}\mathbf{a}_\phi$$

$$\Phi = \int\!\!\int \mathbf{B}\cdot d\mathbf{S} = \int_0^h \int_a^b \frac{\mu NI}{2\pi\rho}\, d\rho\, dz \boxed{= \frac{\mu NIh}{2\pi}\ln\left(\frac{b}{a}\right)}$$

$$\left(\because \ln\left(\frac{b}{a}\right) = 2\left[\left(\frac{b/a-1}{b/a+1}\right) + \frac{1}{3}\left(\frac{b/a-1}{b/a+1}\right)^3 + \frac{1}{5}\left(\frac{b/a-1}{b/a+1}\right)^5 + \ldots\ldots\right]\right)$$

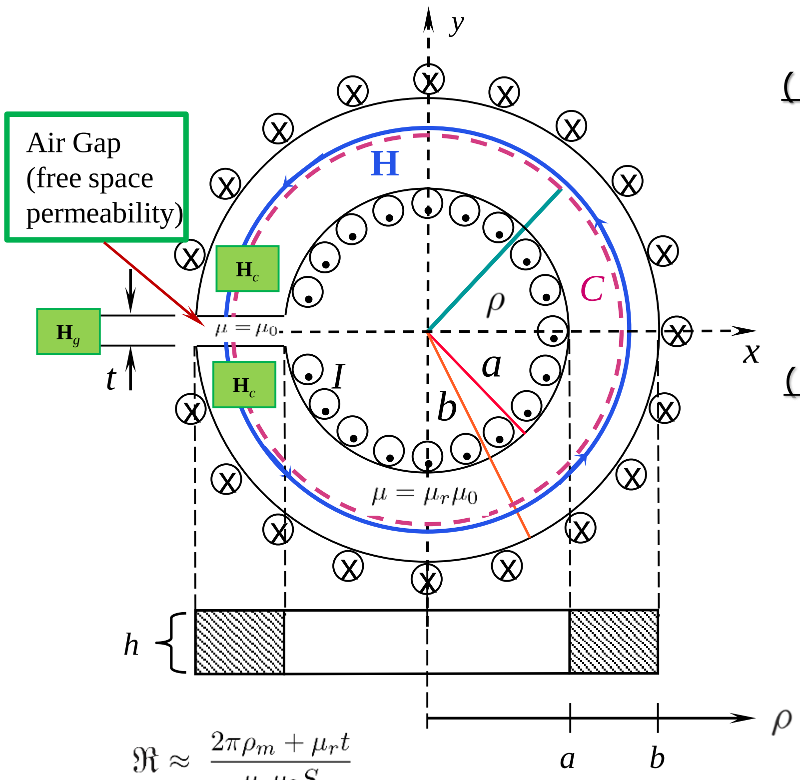
$$= \frac{\mu NIh}{2\pi}\, 2\left[\underbrace{\underbrace{\left(\frac{b/a-1}{b/a+1}\right)}_{0.33}}_{\text{Uniform field approximation}} + \underbrace{\underbrace{\frac{1}{3}\left(\frac{b/a-1}{b/a+1}\right)^3}_{1.2\times10^{-2}} + \underbrace{\frac{1}{5}\left(\frac{b/a-1}{b/a+1}\right)^5}_{8.2\times10^{-4}} + \ldots\ldots}_{\text{Correction terms}}\right]$$

$$\Phi = \frac{\mu NI(b-a)h}{2\pi\rho_m} \qquad \rho_m = \frac{1}{2}(b+a)$$

$$\Phi = \frac{\mu NIh}{2\pi}\left[\frac{2(b-a)}{(b+a)}\right] \boxed{= \frac{\mu NIh}{2\pi}\left[\frac{2(b/a-1)}{(b/a+1)}\right]}$$

$$\boxed{= \frac{\mu NIh}{2\pi}\left[\frac{2(b/a-1)}{(b/a+1)}\right]} \qquad \text{error } \approx 4\%$$

- Application of Reluctance: Toroidal Coil with a Gap




$$\Re \approx \frac{2\pi\rho_m + \mu_r t}{\mu_r\mu_0 S}$$

$$B = \frac{NI}{S} \approx \left[\frac{\mu_r\mu_0}{2\pi\rho_m + \mu_r t}\right] NI$$

$$NI \approx \left[\frac{2\pi\rho_m}{\mu_r\mu_0} + \frac{t}{\mu_0}\right] B \qquad H_g = \frac{B}{\mu_0}$$

given unknown

must satisfy Ampere's Circuital Law: $NI \approx 2\pi\rho_m H_c + t H_g$

If not, try again with a corrected value for B, over as many iterations as needed.

(1) H :

$$\Re = \frac{2\pi\rho_m - t}{\mu S} + \frac{t}{\mu_0 S} = \frac{2\pi\rho_m + (\mu_r - \cancel{1})t}{\mu_r\mu_0 S}$$

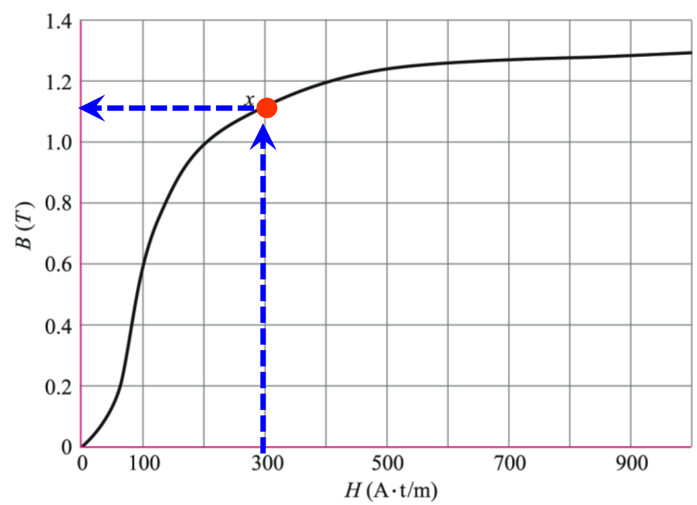
$$\approx \frac{2\pi\rho_m + \mu_r t}{\mu_r\mu_0 S} \qquad \mu_r >> 1$$

$$B_c = B_g = B = \frac{NI}{\Re S} \approx \left[\frac{\mu_r\mu_0}{2\pi\rho_m + \mu_r t}\right] NI$$

(2) Φ :

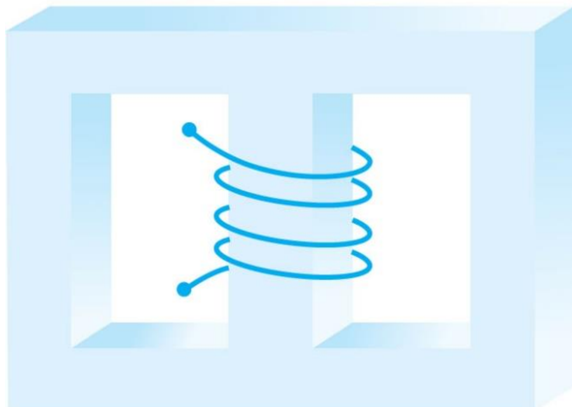
Core : $H_c = \dfrac{B}{\mu_r\mu_0} = \dfrac{NI}{2\pi\rho_m + \mu_r t}$

Gap: $H_g = \dfrac{B}{\mu_0} = \dfrac{\mu_r NI}{2\pi\rho_m + \mu_r t} = \mu_r H_c$

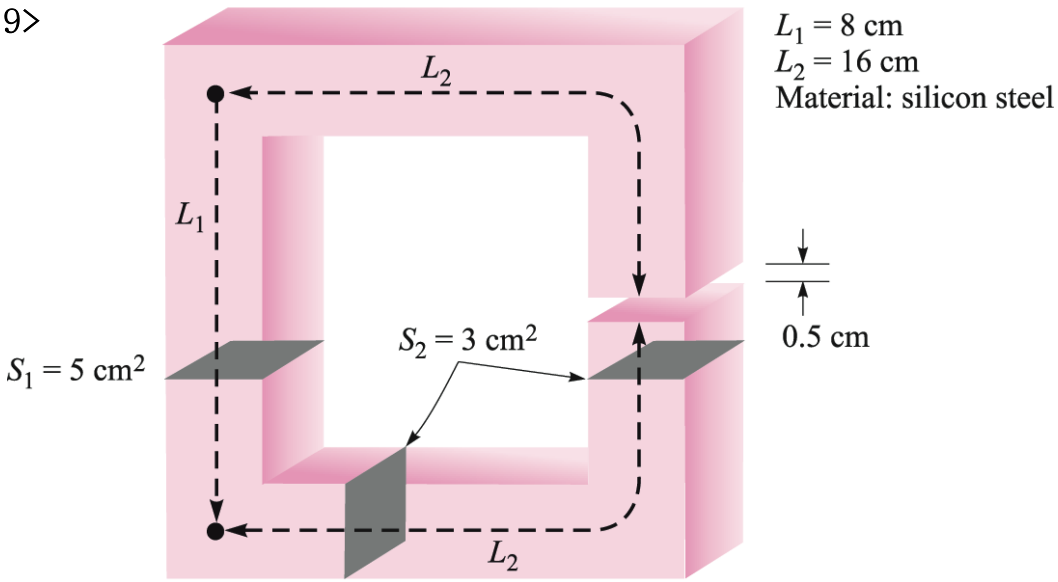


$\oint \vec{H} \cdot d\vec{L} = NI$





< 응용 8.9>



# 8.9 자성체에서의 포텐셜 에너지와 힘

- 전계 : $W_E = \frac{1}{2}\int_v \vec{D}\cdot\vec{E}dv$

  자계 : $W_H = \frac{1}{2}\int_v \vec{B}\cdot\vec{H}dv = \frac{1}{2}\int_v \mu H^2 dv = \frac{1}{2}\int_v \frac{B^2}{\mu}dv$

$$dW_H = \vec{F}\cdot d\vec{L} = \frac{1}{2}\frac{B^2}{\mu}\cdot dL\cdot S \qquad \therefore \boxed{F = \frac{B^2}{2\mu_0}\cdot S}$$

Pressure : $P = \frac{B^2}{2\mu_0}$

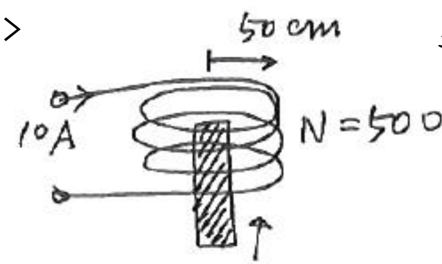
Force : $F = P\cdot S$

Energy : $W = \int F\cdot d\ell$

- Virtual Work :


* 비선형에서는 ?

< Ex. >




코일 중심에서 $H = 5\times 10^3, (= \frac{NI}{2a})$

$B = \mu_0 H = 63G$

$$P = \frac{B^2}{2\mu_0} = 15.7N/m^2 \cong 1.57kg/m^2$$

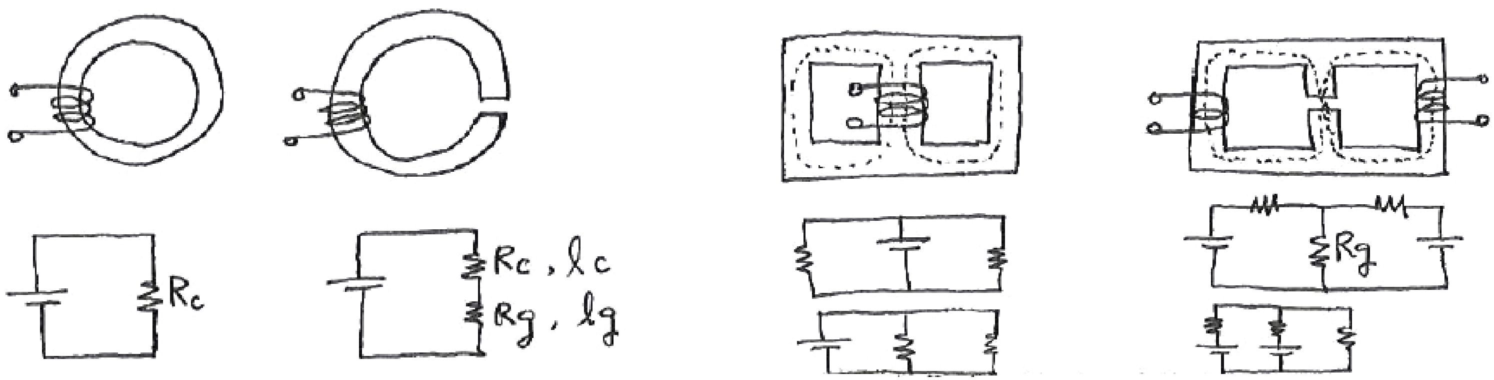
$$F = P\times S = 1.57kg/m^2 \times \pi(0.5)^2 \cong 1.23kg$$

◎ $\boxed{V_m = N \cdot I = R \cdot \Phi = H \cdot L}$

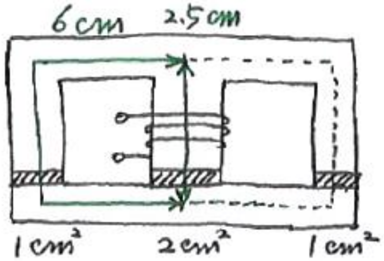
선형 : $N \cdot I = R \cdot \Phi$

비선형 : $N \cdot I = H \cdot L$

◎ 자기회로 종류



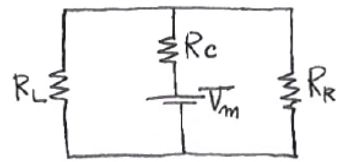
<Ex>

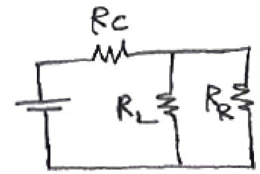



$I = \underline{\underline{8\ mA}}$

$\mu = 0.006\ [H/m]$

$N = 1500$




$$\begin{cases} R_L = \dfrac{\ell_L}{\mu S_L} = 10 \times 10^4 \\ R_c = \dfrac{\ell_c}{\mu S_c} = 2.1 \times 10^4 \end{cases}$$

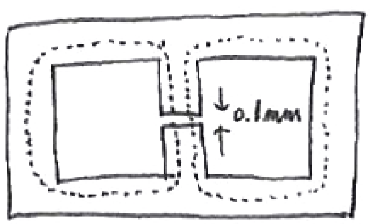
$: R_T = R_{C+} \dfrac{R_L}{2} = 7.1 \times 10^4$ $\qquad V_m = N \cdot I = 12$

$$\begin{cases} \Phi_C = \dfrac{V_m}{R_T} = 1.69 \times 10^{-4} \\ \Phi_L = \Phi_R = \dfrac{\Phi_C}{2} = 0.845 \times 10^{-4} \end{cases}$$

$B_C = \dfrac{\Phi_C}{S_C} = 0.845\ [T]$

$B_L = B_R = \dfrac{\Phi_L}{S_L} = 0.55\ [T]$

<Ex> 윗 예제 중심 Core 에 공극 0.1mm 가 있을 경우 같은 자속을 내기 위해 흘려야 하는 전류 값은?




$$R_g = \frac{\ell_g}{\mu_0 S_g} = 38.78 \times 10^4$$

$$\therefore\ R'_T = R_T + R_g = 7.1 \times 10^4 + 38.78 \times 10^4 = 46.9 \times 10^4$$

$\Phi = 1.69 \times 10^{-4}$ 를 내어야 하므로

$$V_m = N \cdot I = R'_T\, \Phi = 46.9 \times 10^4 \times 1.69 \times 10^{-4} = 79.26\ [AT]$$

$$\therefore\ I = \frac{V_m}{N} = \frac{79.26}{1500} = \underline{\underline{52.84\ [mA]}}$$

* gap 0.1mm 때문에 전류는 6.6배 증가

$\oint \vec{H} \cdot d\vec{L} = NI$



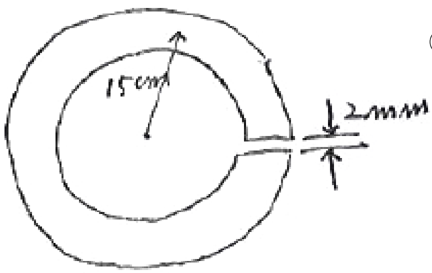
< Ex. > Find M

A Toroid having an iron core of square section (Fig.) and permeability is wound with N(closed space) turns of wire carrying a current I.
Find the magnitude of the magnetization M everywhere inside the iron. ( Wisconsin )

< Ex. > A cylindrical soft iron rod of length L and diameter d is bent into a circular shape of radius R leaving a gap where the two ends of the rod almost meet. The gap spacing S is constant over the face of the ends of the rod. Assume S<<d, d<<R, N turns of wire are wrapped tightly around the iron rod and a current I is passed through the wire. The relative permeability of the iron is $\mu_r$. Neglecting fringing, What is the magnetic field B in the gap?

(MIT)

< Ex. 본문 > 비선형 + 공극문제.    Core에서 1 [T] 를 유지하기 위해 흘려야 할 전류 값을 구하시오.

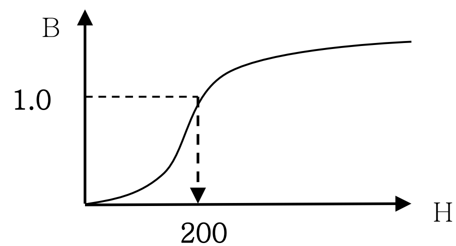



① 공극에서 : $R_a = \dfrac{\ell_a}{\mu S} = 2.65 \times 10^6 [AT/Wb]$

$\Phi = B \cdot S = 1 \times 6 \times 10^{-4} = 6 \times 10^{-4} [Wb]$

$V_{m,a} = \Phi \cdot R_a = \underline{1590 [AT]}$

② 철심에서 : $B_s = 1\ [T] \xrightarrow{\text{B-H Curve}} H_s = 200\ [At/m]$




$\ell_s = 0.3\pi$

$V_{m,s} = H_s \cdot R_s = \underline{188 [AT]}$

$\therefore\ V_{m,Total} = V_{m,a} + V_{m,s} = 1778\ [AT]$

$= NI$

$\therefore\ I = \dfrac{V_m}{N} = \dfrac{1778}{500} = \underline{3.56 [A]}$

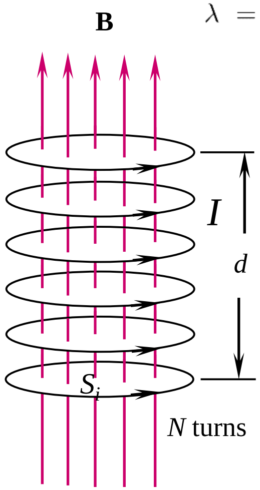
✓ Leakage, Fringing : $\dfrac{\mu_{iron}}{\mu_{air}} = 4000$  $\dfrac{\sigma_{iron}}{\sigma_{air}} = 10^{15}$

# 8.10 자기인덕턴스 및 상호 인덕턴스

• **Flux Linkage :**

$$\Phi_i = \int_{S_i} \mathbf{B}_i \cdot d\mathbf{S}_i$$ Flux, Weber [Wb]

$$\lambda = \sum_{i=1}^{N} \Phi_i$$ Flux linkage, Weber-turns [Wb-t] $\quad \lambda = N\Phi = N\int_S \mathbf{B}\cdot d\mathbf{S}$




$$\Phi_i = \int_{S_i} \mathbf{B}_i \cdot d\mathbf{S}_i \qquad B = \mu n I = \frac{\mu N I}{d}$$

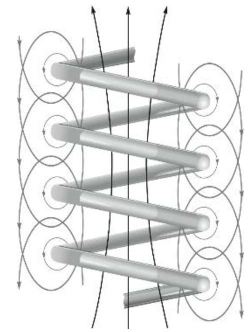
$$\lambda = \sum_{i=1}^{N} \Phi_i = N\Phi = N\int_S \mathbf{B}\cdot d\mathbf{S} = NBS = \frac{\mu N^2 I S}{d}$$

$$\lambda = \frac{\mu N^2 I S}{d}$$

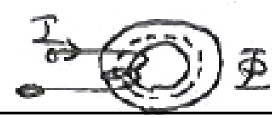
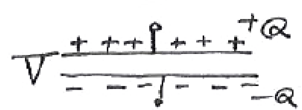
$$\boxed{L \equiv \frac{\lambda}{I} = N^2 \frac{\mu S}{d}}$$

The units of inductance : Wb-t/A = *Henry* [H].

$$\boxed{C = \frac{\epsilon S}{d}}$$



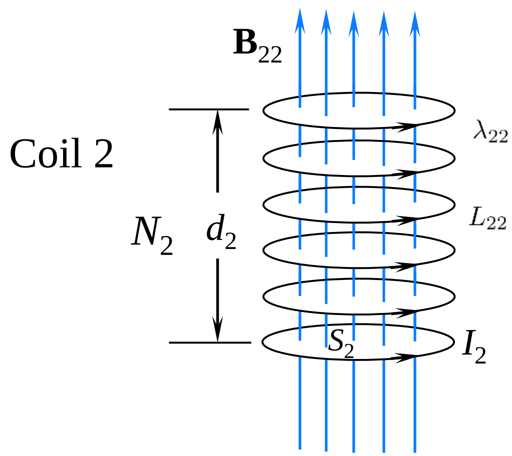
*fringing fields*

| | | | |
|---|---|---|---|
| 인덕턴스 L : ( 전류 → 자속 ) | $L = \dfrac{\Phi}{I}$ |  | $[Henry,\ H \equiv \dfrac{Wb}{A}]$ |
| 정전용량 C : ( 전하 → 전압 ) | $C = \dfrac{Q}{V}$ |  | $[Farad,\ F \equiv \dfrac{C}{V}]$ |
| 저　　항 R : ( 전압 → 전류 ) | $R = \dfrac{V}{I}$ |  | $[Ohm,\ \Omega \equiv \dfrac{V}{A}]$ |

℧ !

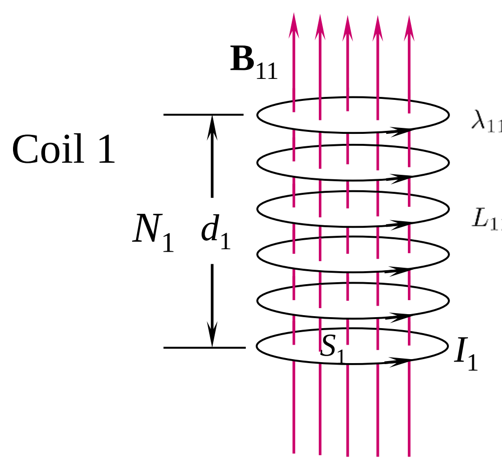
# ◎ 상호 인덕턴스( Mutual Inductance )

## <self inductance>




$$\lambda_{22} = N_2\Phi_{22} = N_2\int_{S_2}\mathbf{B}_{22}\cdot d\mathbf{S}_2$$

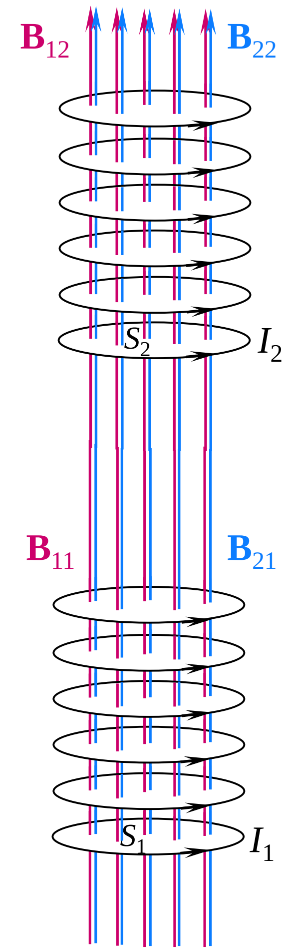
$$L_{22} = \frac{\lambda_{22}}{I_2} = N_2^2\,\frac{\mu_2 S_2}{d_2}$$




$$\lambda_{11} = N_1\Phi_{11} = N_1\int_{S_1}\mathbf{B}_{11}\cdot d\mathbf{S}_1$$

$$L_{11} = \frac{\lambda_{11}}{I_1} = N_1^2\,\frac{\mu_1 S_1}{d_1}$$

## <mutual inductance>




$$\lambda_{12} = N_2\Phi_{12} = N_2\int_{S_2}\mathbf{B}_{12}\cdot d\mathbf{S}_2$$

$$\boxed{M_{12} \equiv \frac{\lambda_{12}}{I_1}}$$

$$\lambda_{21} = N_1\Phi_{21} = N_1\int_{S_1}\mathbf{B}_{21}\cdot d\mathbf{S}_1$$

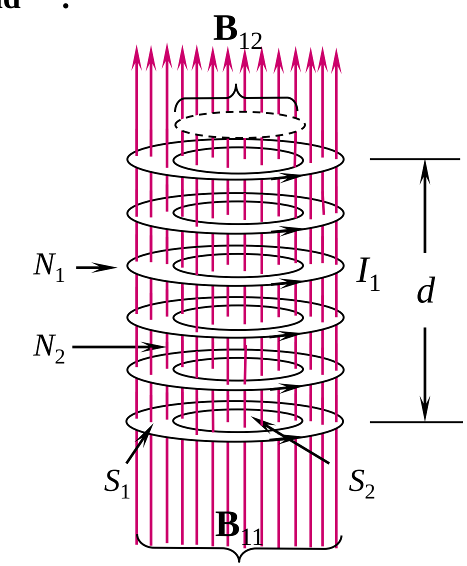
$$\boxed{M_{21} \equiv \frac{\lambda_{21}}{I_2}}$$

$$\boxed{M_{12} = M_{21}}$$

$\mathbf{B}_{ij}$ arising from coil $i$ evaluated within coil $j$

Red : generated by Coil 1,
Blue : generated by Coil 2

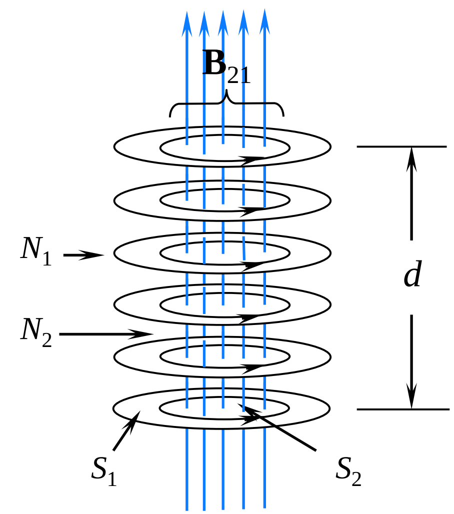
**<Ex> Concentric Solenoid :**




$$\mathbf{B}_{12} = \frac{\mu N_1 I_1}{d}\mathbf{a}_z$$

$$\lambda_{12} = N_2 \Phi_{12} = N_2 \frac{\mu N_1 I_1}{d} S_2$$

$$M_{12} = \frac{\lambda_{12}}{I_1} = N_1 N_2 \frac{\mu S_2}{d}$$




$$\mathbf{B}_{21} = \frac{\mu N_2 I_2}{d}\mathbf{a}_z$$

$$\lambda_{21} = N_1 \Phi_{21} = N_1 \frac{\mu N_2 I_2}{d} S_2$$

$$M_{21} = \frac{\lambda_{21}}{I_2} = N_1 N_2 \frac{\mu S_2}{d} = M_{12}$$

$$\boxed{M_{12} = M_{21}}$$

# Energy and Energy Density in the Magnetic Field

Energy in the electric field within volume *v* :

$$W_E = \int_v \frac{1}{2}\mathbf{D}\cdot\mathbf{E}\,dv \quad \mathrm{J}$$

Energy density in the electric field :

$$w_e = \frac{1}{2}\mathbf{D}\cdot\mathbf{E} \;=\; \underbrace{\frac{1}{2}\epsilon E^2}_{\text{valid for isotropic media}} \quad \mathrm{J/m^3}$$

Energy in the magnetic field within volume *v* :

$$W_M = \int_v \frac{1}{2}\mathbf{B}\cdot\mathbf{H}\,dv \quad \mathrm{J}$$

Energy density in the magnetic field :

$$w_m = \frac{1}{2}\mathbf{B}\cdot\mathbf{H} \;=\; \underbrace{\frac{1}{2}\mu H^2}_{\text{valid for isotropic media}} \quad \mathrm{J/m^3}$$

- ✓ restricted to *linear* media (in which permittivity and permeability are constant with field strength).
- ✓ Deriving the magnetic energy relation is very complicated, so we will not do it here.

## ◎ 에너지와 인덕턴스

- 전계 : $W_C = \frac{1}{2}CV^2$

  자계 : $\boxed{W_H = \frac{1}{2}LI^2}$

- $L = \frac{2W_H}{I^2} = \frac{1}{I^2}\int_v \vec{B}\cdot\vec{H}dv = \frac{1}{I^2}\int_v \vec{H}\cdot(\nabla\times\vec{A})dv = \frac{1}{I^2}[\int_v \nabla\cdot(\vec{A}\times\vec{H})dv + \int_v \vec{A}\cdot(\nabla\times\vec{H})dv]$

  $= \frac{1}{I^2}[\oint_s(\vec{A}\times\vec{H})ds + \int_v \vec{A}\cdot\vec{J}dv]$ $\quad (\because \nabla\cdot(\vec{A}\times\vec{H}) \equiv \vec{H}\cdot(\nabla\times\vec{A}) - \vec{A}\cdot(\nabla\times\vec{H}))$

  발산정리

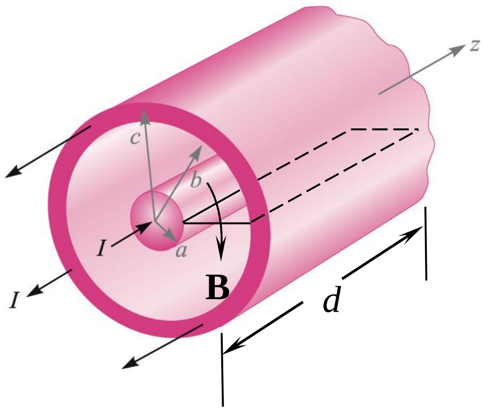
$\therefore$ $\boxed{L = \frac{1}{I^2}\int_v \vec{A}\cdot\vec{J}dv}$

(i) $L = \frac{1}{I^2}\int_v \vec{A}\cdot\vec{J}dv = \frac{1}{I^2}\int_v(\int_v \frac{\mu\vec{J}}{4\pi R}dv)\cdot\vec{J}dv = \frac{1}{I^2}\oint(\oint\frac{\mu Id\vec{L}}{4\pi R})\cdot Id\vec{L}$

$= \frac{\mu}{4\pi}\oint\oint\frac{d\vec{L}}{R}\cdot d\vec{L}$ : L 은 기하학적 구조와 재질의 함수

(ii) $L = \frac{1}{I^2}\int_v \vec{A}\cdot\vec{J}dv = \frac{1}{I}\oint\vec{A}\cdot d\vec{L} = \frac{1}{I}\int_S(\nabla\times\vec{A})\cdot d\vec{S} = \frac{1}{I}\int_S\vec{B}\cdot d\vec{S} = \frac{\Phi}{I}$

<Ex> 동축 케이블



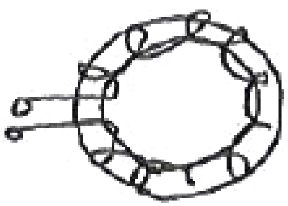

$$H_\phi = \frac{I}{2\pi\rho} \quad (a < \rho < b) \qquad \mathbf{B} = \mu_0 \mathbf{H} = \frac{\mu_0 I}{2\pi\rho}\mathbf{a}_\phi$$

$$\lambda = \Phi = \int_S \mathbf{B}\cdot d\mathbf{S} = \int_0^d \int_a^b \frac{\mu_0 I}{2\pi\rho}\mathbf{a}_\phi \cdot d\rho\, dz\, \mathbf{a}_\phi = \frac{\mu_0 I d}{2\pi}\ln\frac{b}{a}$$

$$L = \frac{\Phi}{I} = \frac{\mu_0 \ell}{2\pi}\ln\frac{b}{a} \quad : [\mathrm{H}]$$

$$\boxed{L/\ell = \frac{\mu_0}{2\pi}\ln\frac{b}{a}} \quad : [\mathrm{H/m}]$$

<Ex> 토로이드

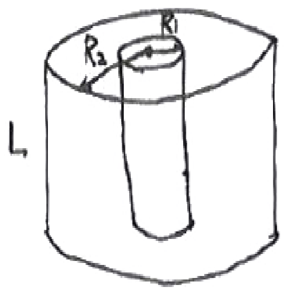


$$B_\Phi = \frac{\mu_0 N I}{2\pi\ r}$$

$$\Phi = B \cdot S = \frac{\mu_0 N I S}{2\pi\ r_0}$$

$$\boxed{L = \frac{N\Phi}{I} = \frac{\mu_0 N^2 S}{2\pi\ r_0}}$$

< Ex. 8.9> 동축 솔레노이드




- $n_1 = \dfrac{N}{L}, \qquad \vec{H}_1 = \begin{cases} n_1 I_1 \hat{a}_z & (0 < r < R_1) \\ 0 & (r > R_1) \end{cases}$

$$\vec{H}_2 = \begin{cases} n_2 I_2 \hat{a}_z & (0 < r < R_2) \\ 0 & (r > R_2) \end{cases}$$

- $\left\langle \begin{array}{ll} \Phi_{12} = \mu_0 n_1 I_1 \pi R_1^2 & \Phi_{21} = \mu_0 n_2 I_2 \pi R_1^2 \\ M_{12} = \mu_0 n_1 n_2 \pi R_1^2 & M_{21} = \mu_0 n_1 n_2 \pi R_1^2 = M_{12} \end{array} \right\rangle$

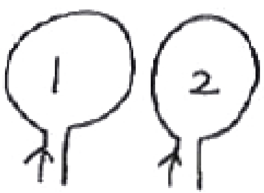
(ex) $R_1$ = 2 cm : $n_1$ = 50 T/cm
$R_2$ = 3 cm : $n_2$ = 80 T/cm → $\begin{cases} L_1 = 39.4 \text{ mH/m} \\ L_2 = 277.0 \text{ mH/m} \\ M_{12} = 63.0 \text{ mH/m} = M_{21} \end{cases}$

## ◎ 자기 인덕턴스( Self Inductance )


$$\underline{\underline{L = \frac{N\Phi}{I}:}}$$

I 에 의하여 발생하는 자속의 양

## ◎ 상호 인덕턴스( Mutual Inductance )




$$\underline{\underline{M_{12} = \frac{N_2 \Phi_{12}}{I_1}:}}$$

$I_1$ 에 의하여 발생하는 자속 중
Coil 2 에 쇄교되는 자속의 양

$$M_{12} = \frac{1}{I_1 I_2}\int_v (\vec{B} \cdot \vec{H})dv = \frac{1}{I_1 I_2}\int_v (\mu\vec{H} \cdot \vec{H})dv = M_{21}$$

$$\therefore \quad M_{12} = M_{21}$$