Lab. of Applied ElectroMagnetics

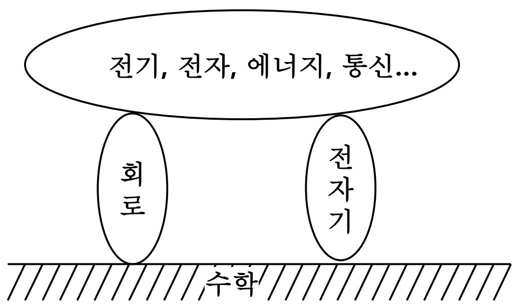
전자기학(Electro-Magnetics)

Engineering Electromagnetics

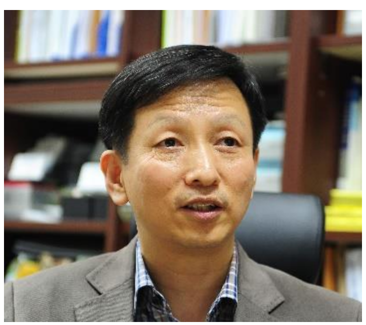
W. H. Hayt, Jr. and J. A. Buck

Lab. of Applied ElectroMagnetics

전자기학(Electro-Magnetics)

- 전기(Electricity) : $Q \xrightarrow{\text{How ?}} E$
  자기(Magnetism) : $I \xrightarrow{\text{How ?}} H$
- 수학 : 벡터, 미분/적분 방정식, Field / Flux Concept
- 

- 고전 전자기학(Classical Electromagnetics)
  - $\xrightarrow{\text{High Speed}}$ 상대성 이론(Relativistic Theory)
  - $\xrightarrow{\text{Small Size}}$ 양자역학(Quantam Theory)



- **박 관 수 (교내 2788, 010-9318-4412)**
  gspark@pusan.ac.kr, http://magnetics.pusan.ac.kr
  연구실 : 특공관 319호 / 실험실 : 특공관 307~308호
- 학위 : 서울대학교 전기공학과 학사/석사/박사
  현재 : 부산대학교 전기공학과 교수
- 1997~1999 : 미국 **Carnegie Mellon Univ.** 연구교수
- 2007~2008 : 미국 **MIT** 교환교수

- **교재 : Engineering Electromagnetics / 8th, Hayt, McGraw-Hill**

부교재 :

- Classical Electrodynamics, Jackson, Wiley
- Berkeley Physics Series II, Electromagnetics.
- Introduction to Electrodynamics, David J. Griffiths
- Elements of Electromagnetics, Sadicu, SciTech
- Electromagnetic Fields and Waves, Iskander, Prentice-Hall
- Electromagnetics (History, Theory, and Applications), R. S. Elliott, IEEE Press
- Electric & Magnetic Interactions, Chabay & Sherwood
- Electromagnetics, Kraus, McGraw-Hill
- Electromagnetic Field Theory, Markus Zahn, MIT Press
- Electromagnetic Fields and Energy, Haus &Melcher, Prentice-Hall

- Electro-Magnetics
- 전자기학(電磁氣學)
- 電氣 + 磁氣
- 氣 ??

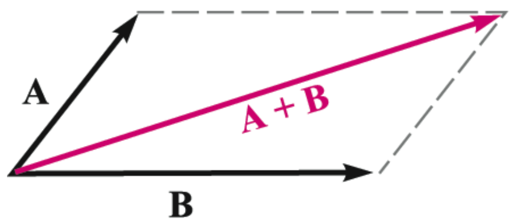
Lab. of Applied ElectroMagnetics

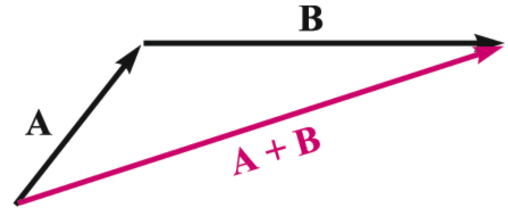
Maxwell Equation

	Differential	Integral
Gauss	$\nabla \cdot \mathbf{D} = \rho_v$	$\oint_S \mathbf{D} \cdot d\mathbf{s} = \int_{vol} \rho_v dv$
Faraday	$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	$\oint \mathbf{E} \cdot d\mathbf{l} = -\int_S \dfrac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$
	$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$
Ampere	$\nabla \times \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}$	$\oint \mathbf{H} \cdot d\mathbf{l} = I + \int_S \dfrac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{s}$

Chap. 1 Vector 해석

- Vector Addition : Associative Law: $\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$





- General Vector, B: $\boxed{\mathbf{B} = B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z}$

- Magnitude of B: $$|\mathbf{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

- Unit Vector in the Direction of B: $$\mathbf{a}_B = \frac{\mathbf{B}}{\sqrt{B_x^2 + B_y^2 + B_z^2}} = \frac{\mathbf{B}}{|\mathbf{B}|}$$

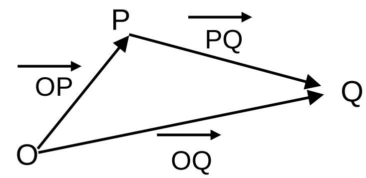
(Ex) Specify the unit vector extending from the origin toward the point $G(2, -2, -1)$

$$\mathbf{G} = 2\mathbf{a}_x - 2\mathbf{a}_y - \mathbf{a}_z \qquad |\mathbf{G}| = \sqrt{(2)^2 + (-2)^2 + (-1)^2} = 3$$

$$\mathbf{a}_G = \frac{\mathbf{G}}{|\mathbf{G}|} = \tfrac{2}{3}\mathbf{a}_x - \tfrac{2}{3}\mathbf{a}_y - \tfrac{1}{3}\mathbf{a}_z = 0.667\mathbf{a}_x - 0.667\mathbf{a}_y - 0.333\mathbf{a}_z$$

(Ex)

① 좌표 : P(1, 2, 3), Q(2, -2, 1)



② 좌표벡터 : $\vec{OP} = \hat{x} + 2\hat{y} + 3\hat{z}, \qquad \vec{OQ} = 2\hat{x} - 2\hat{y} + \hat{z},$

$$\vec{PQ} = \vec{OQ} - \vec{OP} = \hat{x} - 4\hat{y} - 2\hat{z}$$

$$\mathbf{r}_{pq} = \mathbf{r}_q - \mathbf{r}_p = (2-1)\hat{a}_x + (-2-2)\hat{a}_y + (1-3)\hat{a}_z = \hat{a}_x - 4\hat{a}_y - 2\hat{a}_z$$

③ 벡터의 크기 : $|\vec{PQ}| = \sqrt{1^2 + (-4)^2(-2)^2} = \sqrt{21}$

④ 단위 벡터 : $\hat{a}_{\mathbf{r}} = \frac{\mathbf{r}_{pq}}{|\mathbf{r}_{pq}|} = \frac{\hat{a}_x - 4\hat{a}_y - \hat{a}_z}{\sqrt{1^2 + 4^2 + 2^2}} = \frac{1}{\sqrt{21}}\hat{a}_x - \frac{4}{\sqrt{21}}\hat{a}_y - \frac{2}{\sqrt{21}}\hat{a}_z$

$$\hat{a}_x, \ \hat{x}, \ \hat{i}, \ \hat{a}_R$$

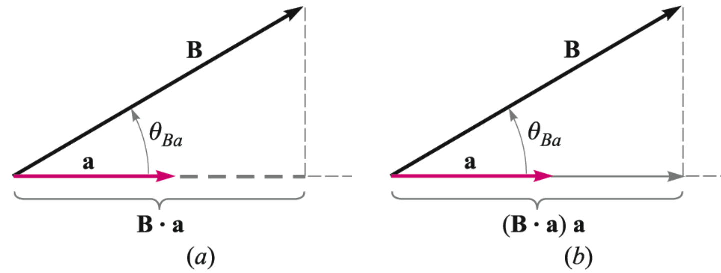
Lab. of Applied ElectroMagnetics

- Dot Product : $\boxed{\mathbf{A}\cdot\mathbf{B} = |\mathbf{A}|\,|\mathbf{B}|\cos\theta_{AB}}$

$$\mathbf{A} = A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z$$

$$\mathbf{B} = B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z$$

$$\boxed{\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z}$$



$$\mathbf{a}_x\cdot\mathbf{a}_y = \mathbf{a}_y\cdot\mathbf{a}_z = \mathbf{a}_x\cdot\mathbf{a}_z = 0$$

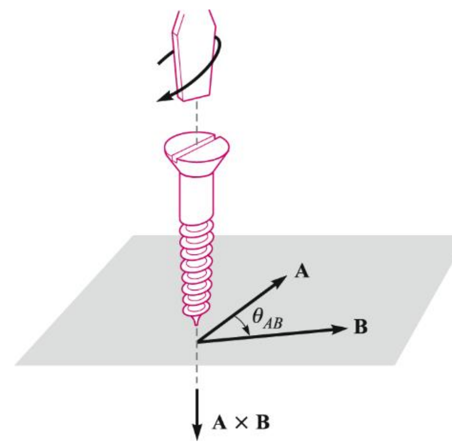
$$\mathbf{a}_x\cdot\mathbf{a}_x = \mathbf{a}_y\cdot\mathbf{a}_y = \mathbf{a}_z\cdot\mathbf{a}_z = 1$$

$$\boxed{\mathbf{A}\cdot\mathbf{A} = A^2 = |\mathbf{A}|^2}$$

- Cross Product : $\boxed{\mathbf{A}\times\mathbf{B} = \mathbf{a}_N|\mathbf{A}|\,|\mathbf{B}|\sin\theta_{AB}}$

$$\begin{aligned}\mathbf{A}\times\mathbf{B} = {} & A_xB_x\mathbf{a}_x\times\mathbf{a}_x + A_xB_y\mathbf{a}_x\times\mathbf{a}_y + A_xB_z\mathbf{a}_x\times\mathbf{a}_z \\ & + A_yB_x\mathbf{a}_y\times\mathbf{a}_x + A_yB_y\mathbf{a}_y\times\mathbf{a}_y + A_yB_z\mathbf{a}_y\times\mathbf{a}_z \\ & + A_zB_x\mathbf{a}_z\times\mathbf{a}_x + A_zB_y\mathbf{a}_z\times\mathbf{a}_y + A_zB_z\mathbf{a}_z\times\mathbf{a}_z \\ = {} & (A_yB_z - A_zB_y)\mathbf{a}_x + (A_zB_x - A_xB_z)\mathbf{a}_y + (A_xB_y - A_yB_x)\mathbf{a}_z\end{aligned}$$

$$\boxed{\mathbf{A}\times\mathbf{B} = \begin{vmatrix}\mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z\end{vmatrix}}$$



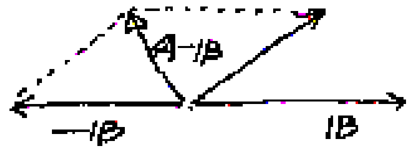
$$\mathbf{a}_x\times\mathbf{a}_y = \mathbf{a}_z$$
$$\mathbf{a}_y\times\mathbf{a}_z = \mathbf{a}_x$$
$$\mathbf{a}_z\times\mathbf{a}_x = \mathbf{a}_y$$

Lab. of Applied ElectroMagnetics

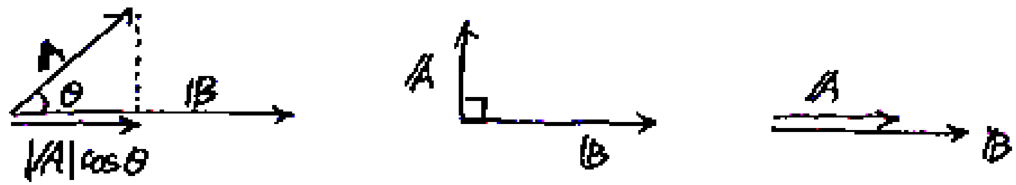
- <u>벡터 대수 :</u> $\mathbf{A} = A_x\hat{x} + A_y\hat{y}, \quad \mathbf{B} = B_x\hat{x} + B_y\hat{y}$

① $\mathbf{A}+\mathbf{B} = (A_x + B_x)\hat{x} + (A_y + B_y)\hat{y}$

② $\mathbf{A}+\mathbf{B} = \mathbf{A}+(\text{-}\mathbf{B})$

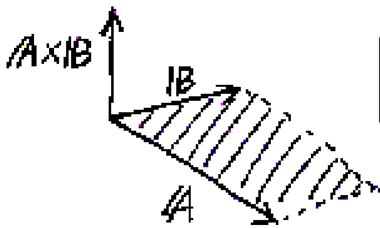


③ $\mathbf{A}\cdot\mathbf{B} = |\mathbf{A}||\mathbf{B}|\cos\theta = A_xB_x + A_yB_y + A_zB_z = \mathbf{B}\cdot\mathbf{A} \Rightarrow Scalar, \quad Projection$



④ $\mathbf{A}\times\mathbf{B} = |\mathbf{A}||\mathbf{B}|\sin\theta\,\hat{a}_N = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

(우수계/좌수계)



$\begin{cases} |\mathbf{A}\times\mathbf{B}|\text{: 면적} \\ \text{방향}: \mathbf{A}\times\mathbf{B} \perp \mathbf{A}\,\&\,\mathbf{B} \end{cases}$

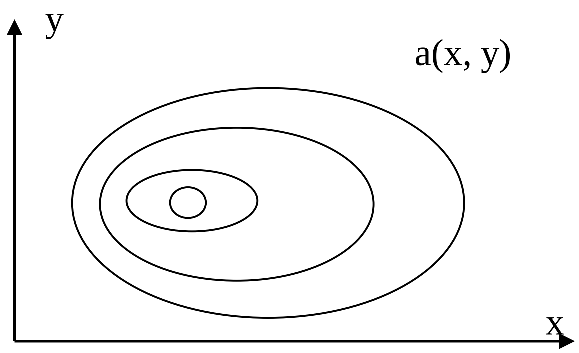
- **Vector Field :** $\mathbf{v} = v_x\mathbf{a}_x + v_y\mathbf{a}_y + v_z\mathbf{a}_z$

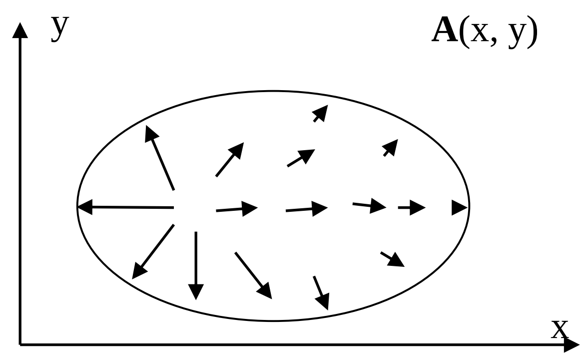
$$\mathbf{v}(\mathbf{r}) = v_x(\mathbf{r})\mathbf{a}_x + v_y(\mathbf{r})\mathbf{a}_y + v_z(\mathbf{r})\mathbf{a}_z \quad \text{where } r = (x,y,z)$$

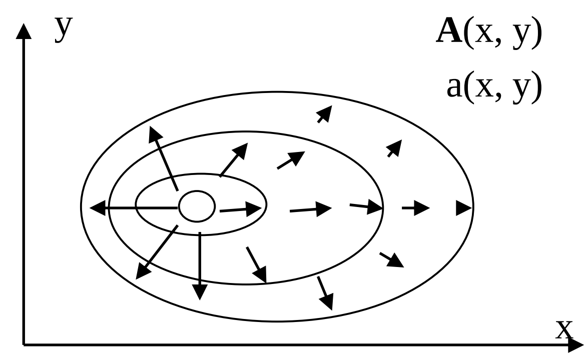
A vector field is a *function* defined in space that has magnitude and direction at all points:

- **Scalar & Vector**
  Scalar Field & Vector Field

< Scalar Field > < Vector Field >

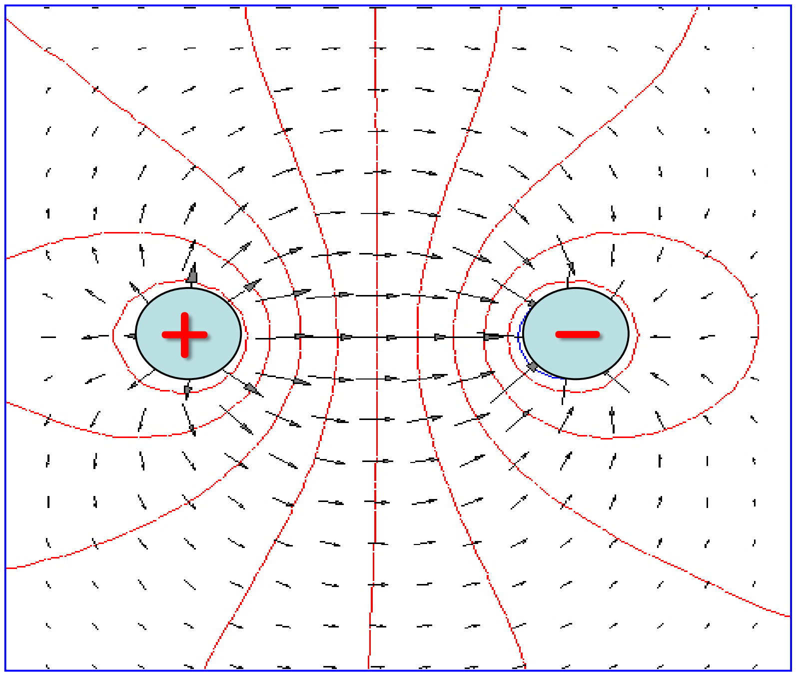




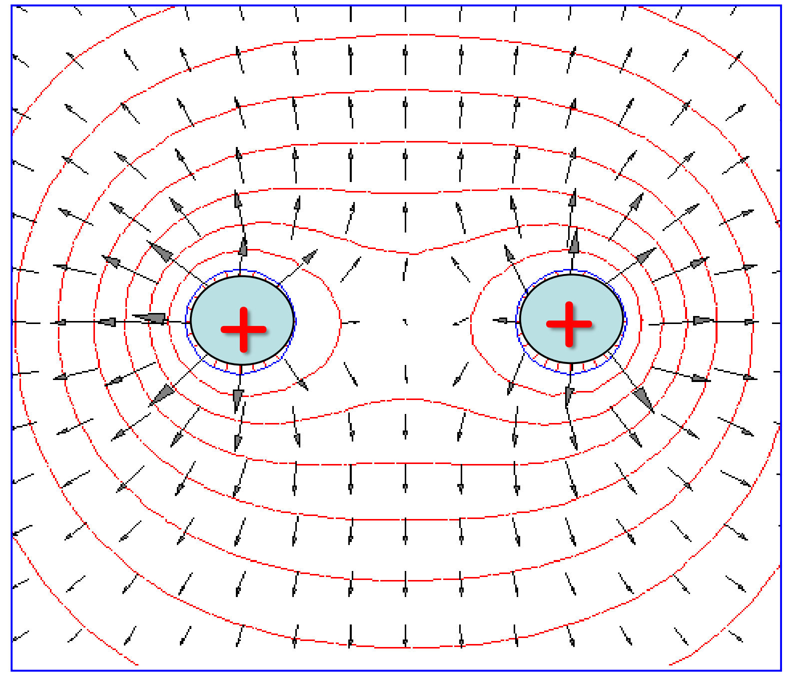


- **<u>전기장의 분포와 등 전위선</u>**

✓ (+)전하와 (–)전하가 존재할 때



✓ (+)전하와 (+)전하가 존재할 때

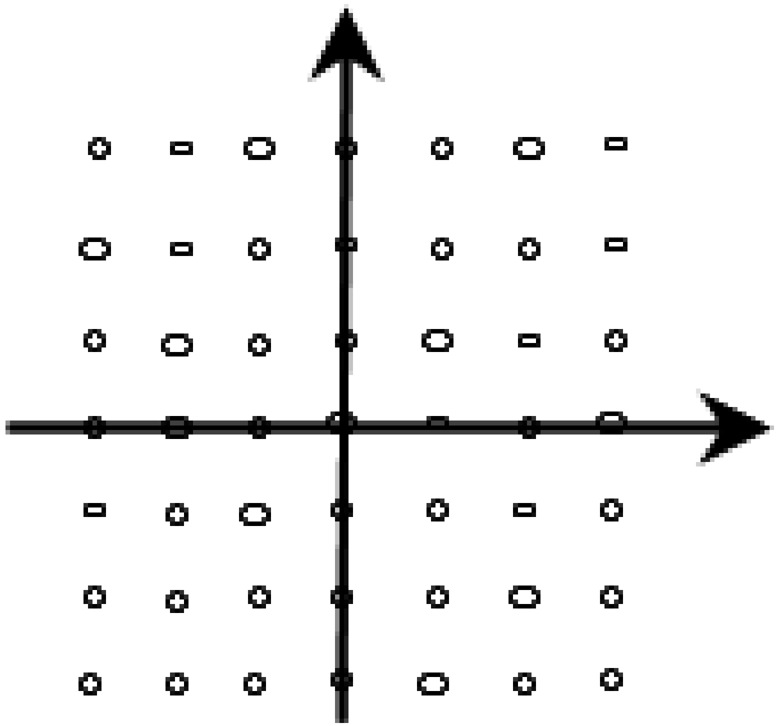


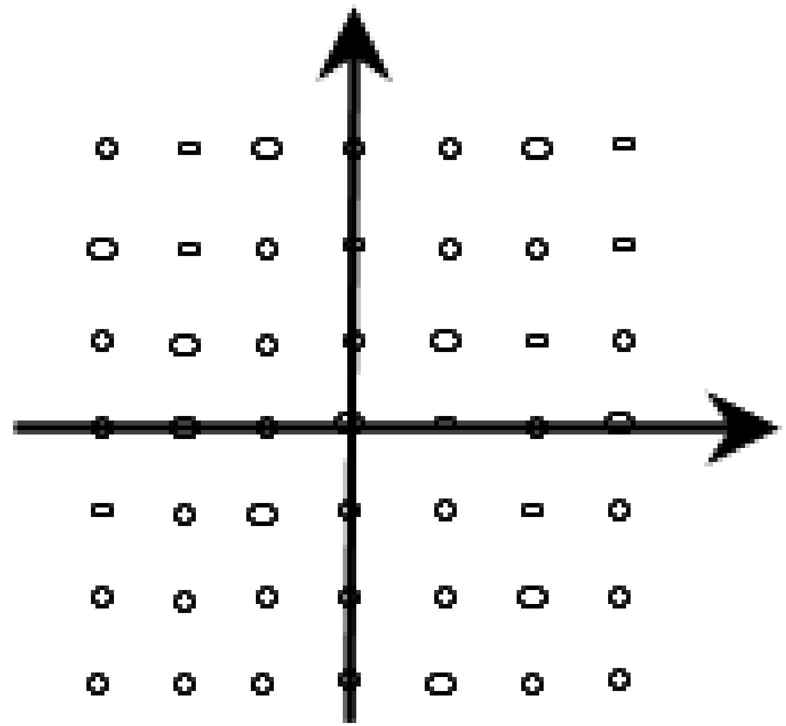
- 전기장 / 등 전위선 , Field (Flux) / Potential

(Scalar/Vector Field) 아래 좌표에 다음 벡터장을 그리시오. (ξ=[−3,−2,−1,0,1,2,3] , ψ=[−3,−2,−1,0,1,2,3])

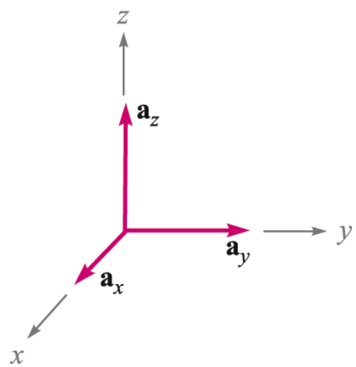
(1) 벡터장 $\vec{B}(x,y) = x\vec{a_x} + y\vec{a_y}$

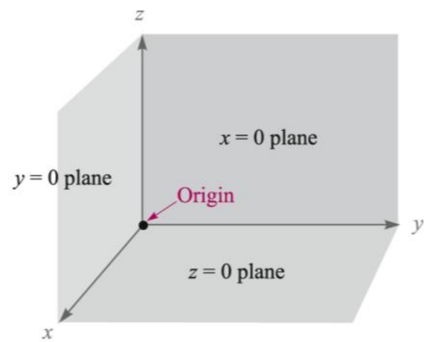
(2) 벡터장 $\vec{B}(x,y) = -y\vec{a_x} + x\vec{a_y}$

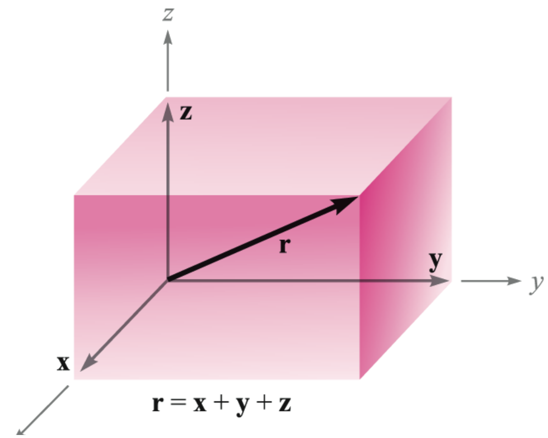


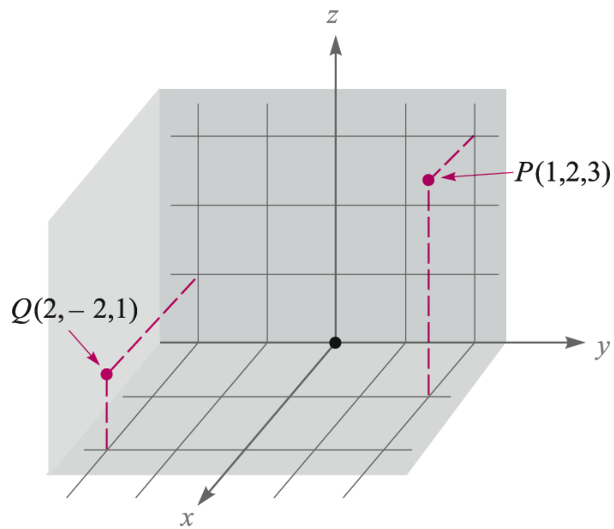


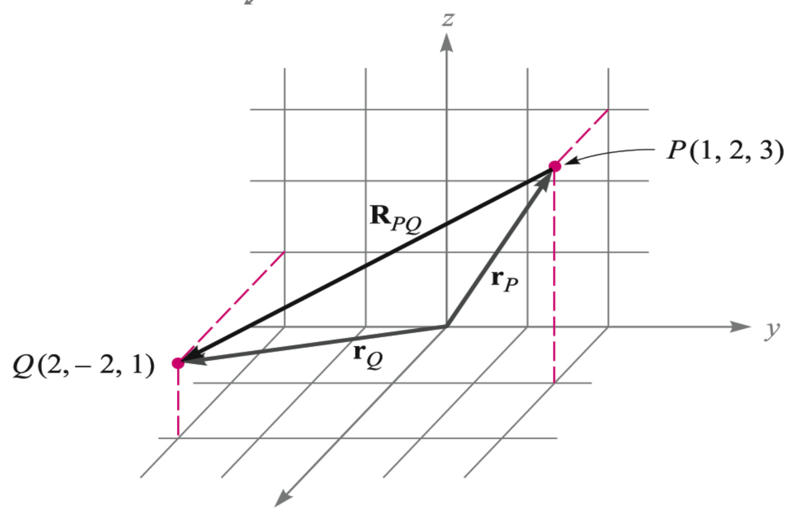
- Vectors in Coordinate System :









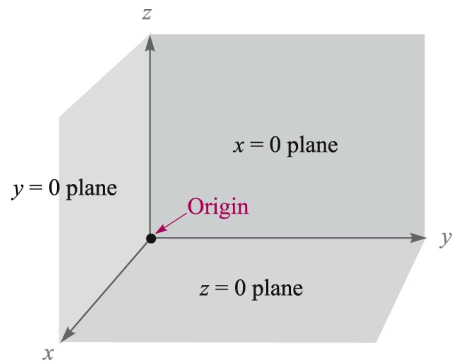


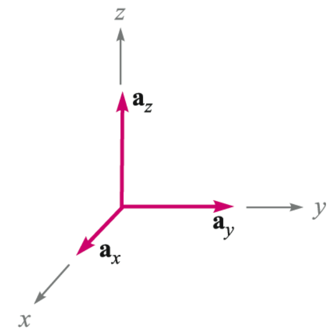
$$
\begin{aligned}
\mathbf{R}_{PQ} &= \mathbf{r}_Q - \mathbf{r}_P \\
&= (2-1)\mathbf{a}_x + (-2-2)\mathbf{a}_y + (1-3)\mathbf{a}_z \\
&= \mathbf{a}_x - 4\mathbf{a}_y - 2\mathbf{a}_z
\end{aligned}
$$

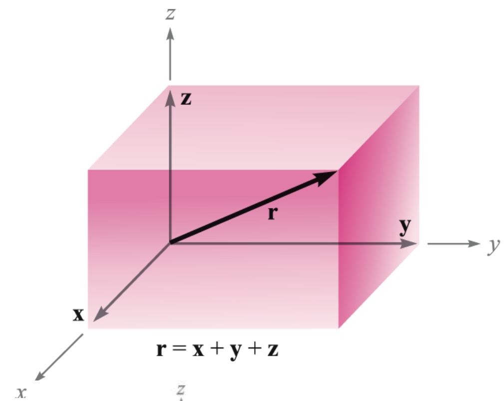
Lab. of Applied ElectroMagnetics

- Cartesian :

$P(x,y,z)$

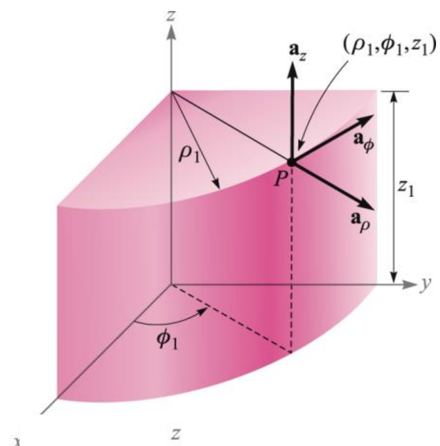


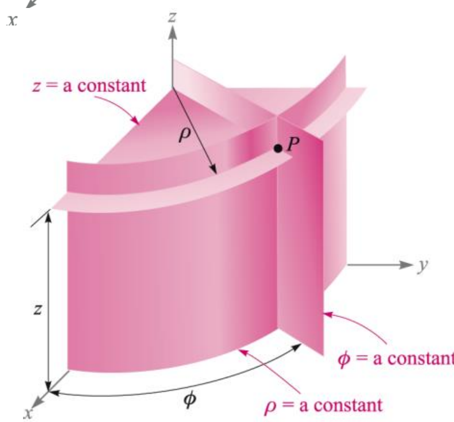




- Cylindrical :

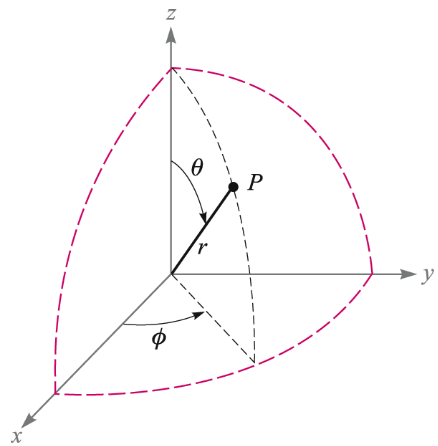
$P(\rho,\phi,z)$

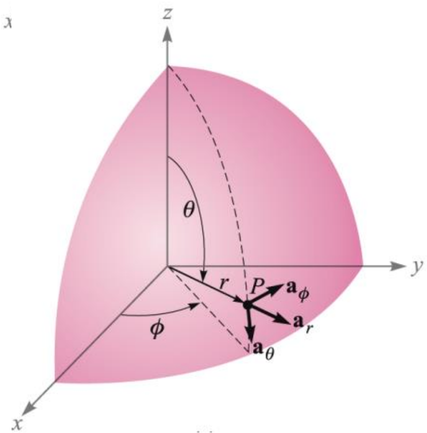


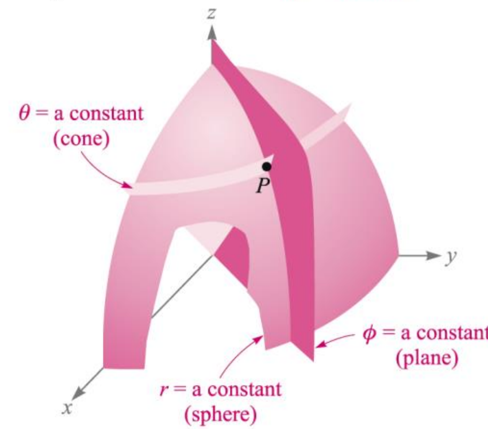


- Spherical :

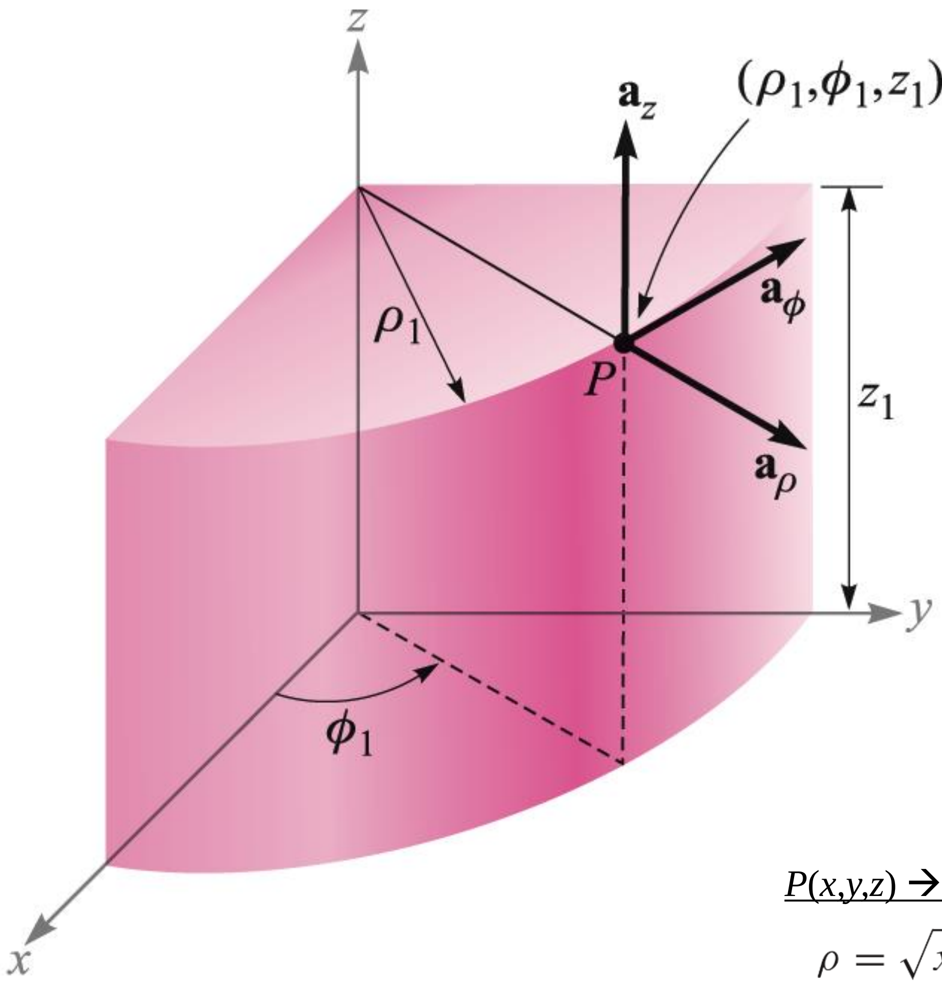
$P(r,\theta,\phi)$







- Transformation :



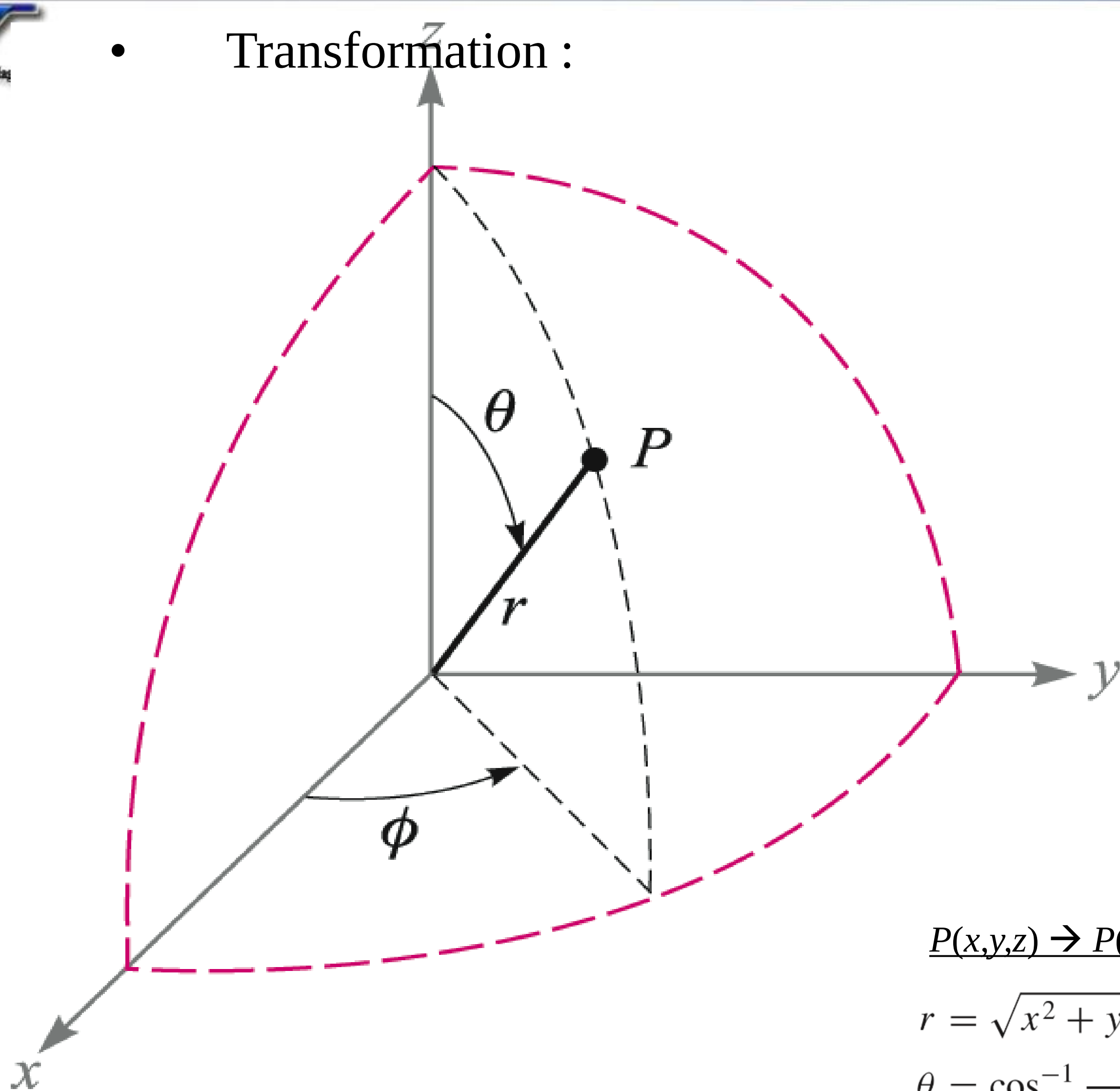
<u>$P(x,y,z) \rightarrow P(\rho,\phi,z)$</u>

$$\rho = \sqrt{x^2 + y^2} \quad (\rho \geq 0)$$
$$\phi = \tan^{-1}\frac{y}{x}$$
$$z = z$$

<u>$P(\rho,\phi,z) \rightarrow P(x,y,z)$</u>

$$x = \rho\cos\phi$$
$$y = \rho\sin\phi$$
$$z = z$$

- Transformation :



$P(x,y,z) \rightarrow P(r,\theta,\phi)$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \cos^{-1} \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$\phi = \tan^{-1} \frac{y}{x}$$

$P(r,\theta,\phi) \rightarrow P(x,y,z)$

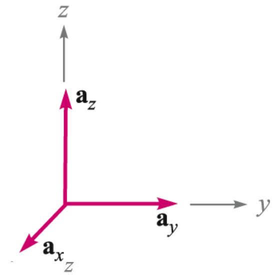
$$x = r \sin\theta \cos\phi$$

$$y = r \sin\theta \sin\phi$$

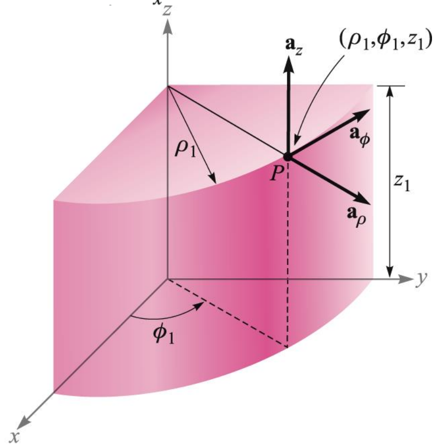
$$z = r \cos\theta$$

- Transformation :

P(*x*,*y*,*z*)



P(ρ,ϕ,*z*)



<u>*P*(*x*,*y*,*z*) → *P*(ρ,ϕ,*z*)</u>

$$\rho = \sqrt{x^2 + y^2} \quad (\rho \geq 0)$$

$$\phi = \tan^{-1} \frac{y}{x}$$

$$z = z$$

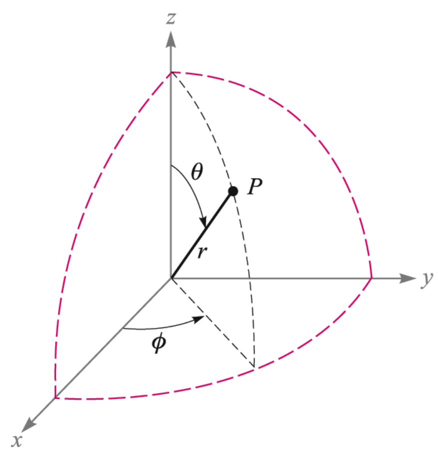
<u>*P*(ρ,ϕ,*z*) → *P*(*x*,*y*,*z*)</u>

$$x = \rho \cos \phi$$

$$y = \rho \sin \phi$$

$$z = z$$

P(*r*,θ,ϕ)



<u>*P*(*x*,*y*,*z*) → *P*(*r*,θ,ϕ)</u>

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \cos^{-1} \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$\phi = \tan^{-1} \frac{y}{x}$$

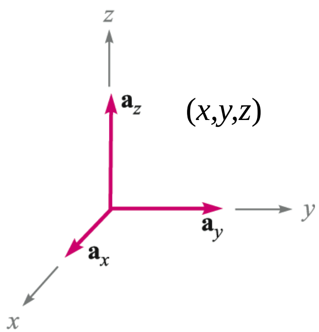
<u>*P*(*r*,θ,ϕ) → *P*(*x*,*y*,*z*)</u>

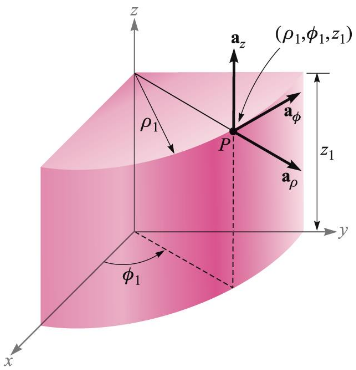
$$x = r \sin \theta \cos \phi$$

$$y = r \sin \theta \sin \phi$$

$$z = r \cos \theta$$

- Cartecian ←→ Cylindrical :





	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x \cdot$	$\cos\phi$	$-\sin$	0
$\mathbf{a}_y \cdot$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z \cdot$	0	0	0

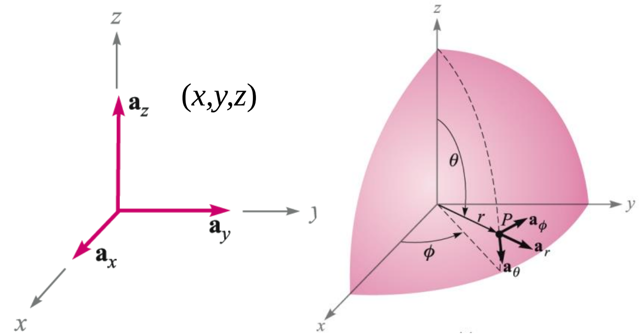
(Ex) Transform the vector, $\mathbf{B} = y\mathbf{a}_x - x\mathbf{a}_y + z\mathbf{a}_z$ into cylindrical coordinates:

$$B_\rho = \mathbf{B} \cdot \mathbf{a}_\rho = y(\mathbf{a}_x \cdot \mathbf{a}_\rho) - x(\mathbf{a}_y \cdot \mathbf{a}_\rho) = y\cos\phi - x\sin\phi = \rho\sin\phi\cos\phi - \rho\cos\phi\sin\phi = 0$$

$$B_\phi = \mathbf{B} \cdot \mathbf{a}_\phi = y(\mathbf{a}_x \cdot \mathbf{a}_\phi) - x(\mathbf{a}_y \cdot \mathbf{a}_\phi) = -y\sin\phi - x\cos\phi = -\rho\sin^2\phi - \rho\cos^2\phi = -\rho$$

$$\mathbf{B} = -\rho\mathbf{a}_\phi + z\mathbf{a}_z \quad \Leftarrow \quad \mathbf{B} = y\mathbf{a}_x - x\mathbf{a}_y + z\mathbf{a}_z$$

- Cartecian ←→ Spherical :



	$\mathbf{a}_r$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x \cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\mathbf{a}_y \cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\mathbf{a}_z \cdot$	$\cos\theta$	$-\sin\theta$	0

(Ex) Transform the vector, $\mathbf{G} = (xz/y)\mathbf{a}_x$ into spherical coordinates :

$$G_r = \mathbf{G}\cdot\mathbf{a}_r = \frac{xz}{y}\mathbf{a}_x\cdot\mathbf{a}_r = \frac{xz}{y}\sin\theta\cos\phi = r\sin\theta\cos\theta\frac{\cos^2\phi}{\sin\phi}$$

$$G_\theta = \mathbf{G}\cdot\mathbf{a}_\theta = \frac{xz}{y}\mathbf{a}_x\cdot\mathbf{a}_\theta = \frac{xz}{y}\cos\theta\cos\phi = r\cos^2\theta\frac{\cos^2\phi}{\sin\phi}$$

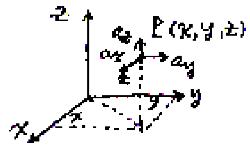
$$G\phi = \mathbf{G}\cdot\mathbf{a}_\phi = \frac{xz}{y}\mathbf{a}_x\cdot\mathbf{a}_\phi = \frac{xz}{y}(-\sin\phi) = -r\cos\theta\cos\phi$$

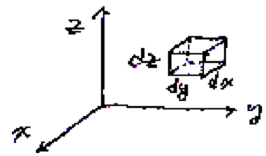
$$\mathbf{G} = r\cos\theta\cos\phi\,(\sin\theta\cot\phi\,\mathbf{a}_r + \cos\theta\cot\phi\,\mathbf{a}_\theta - \mathbf{a}_\phi) \quad \Leftarrow \quad \mathbf{G} = (xz/y)\mathbf{a}_x$$

◎ 좌표계(Coordinate System)

(1) 직각 좌표계

P(x, y, z)

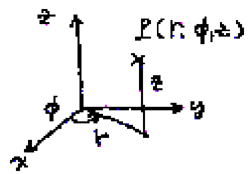


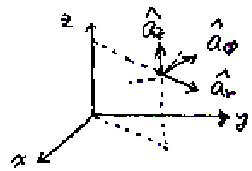


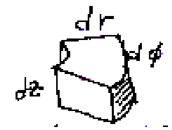
$$dV = dx \cdot dy \cdot dz$$
$$dS = dx \cdot dy$$

(2) 원통 좌표계

P(r, Φ, z)



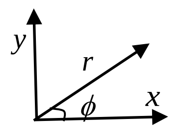




$$dV = rdr \cdot d\phi \cdot dz$$
$$dS = rd\phi \cdot dz$$

* 좌표변환 : (x, y, z) $\rightleftarrows$ (r, Φ, z)

$$\begin{bmatrix} x = r\cos\phi \\ y = r\sin\phi \\ z = z \end{bmatrix}$$

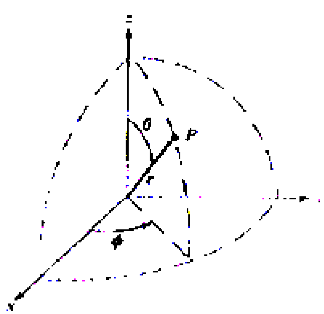


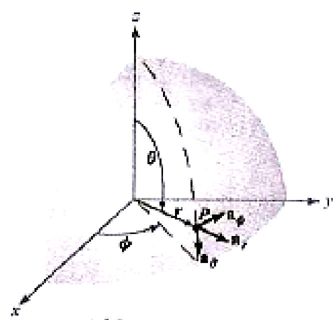
$$\begin{bmatrix} r = \sqrt{x^2 + y^2} \\ \phi = \tan^{-1}\dfrac{y}{x} \\ z = z \end{bmatrix}$$

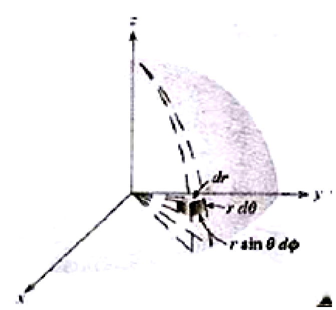
$$\hat{a}_x \cdot \hat{a}_x = 1$$
$$\hat{a}_x \cdot \hat{a}_r = \cos\phi$$
$$\hat{a}_x \cdot \hat{a}_\phi = -\sin\phi$$
$$\hat{a}_x \cdot \hat{a}_z = 0$$

(3) 구 좌표계

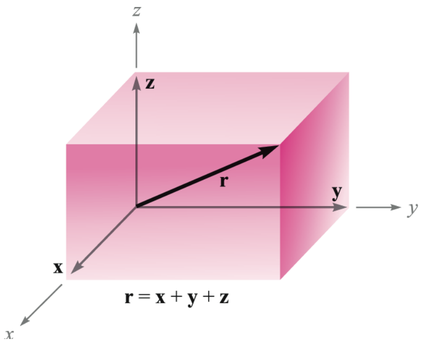
P(r, θ, Φ)

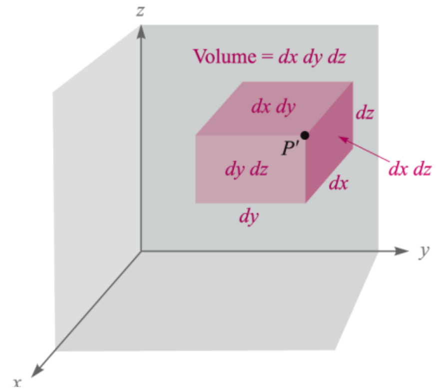




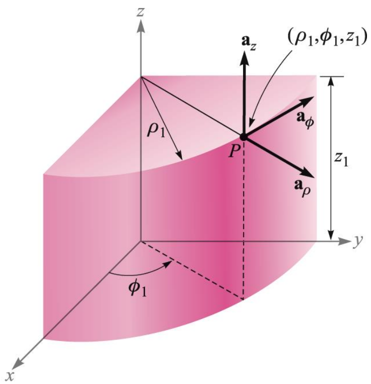


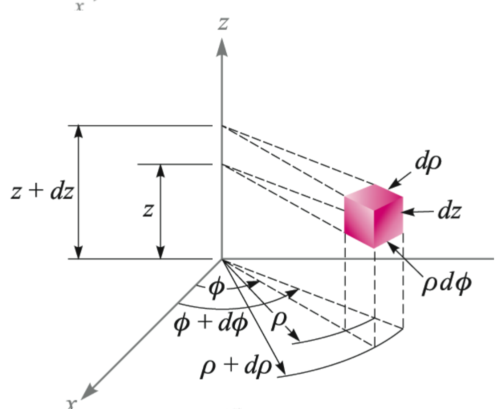
- Differential Volume :



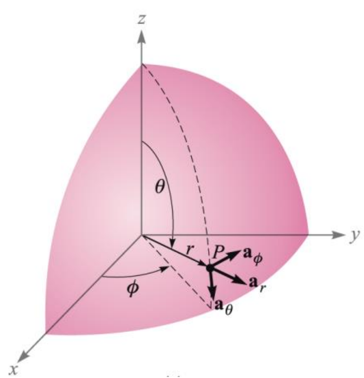


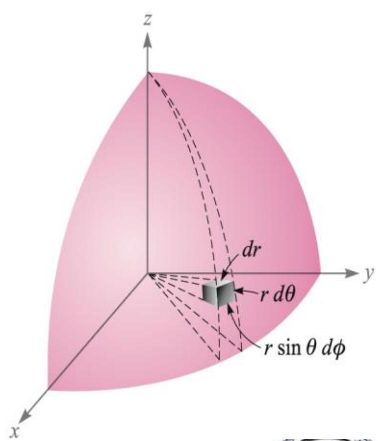
$$dV = dx\, dy\, dz$$





$$dV = \rho\, d\rho\, d\phi\, dz$$





$$dV = r^2 \sin\theta\ dr\, d\theta\, d\phi$$

Lab. of Applied ElectroMagnetics

Maxwell Equation

	Differential	Integral
Gauss	$\nabla \cdot \mathbf{D} = \rho_v$	$\oint_S \mathbf{D} \cdot d\mathbf{s} = \int_{vol} \rho_v dv$
Faraday	$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	$\oint \mathbf{E} \cdot d\mathbf{l} = -\int_S \dfrac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$
	$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$
Ampere	$\nabla \times \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}$	$\oint \mathbf{H} \cdot d\mathbf{l} = I + \int_S \dfrac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{s}$