Lab. of Applied ElectroMagnetics

Chap. 2. Coulomb의 법칙, 전계의 세기

2.1 Coulomb 실험법칙

- Electricity : 고대 그리스 Electra(호박), 마찰 전기
- 1600년, 영국 물리학자. Gilbert, 유리, 유황, 호박, 금속, 나무, 돌, 물, 기름흡입.
- Coulomb. 프랑스 육군 기술자, 비틀림 저울로 전기량을 최초로 측정함.

Newton(100년전)
만유인력, 중력

m_1 — r — m_2

$$F = k\frac{m_1 \cdot m_2}{r^2}[N]$$

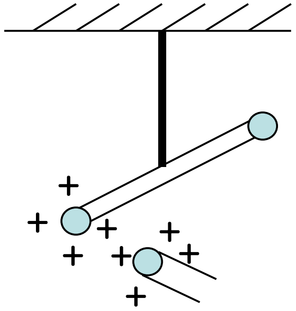
< 인력 >

Coulomb
정전기력

Q_1 — r — Q_2

$$F = k\frac{Q_1 \cdot Q_2}{r^2}[N]$$

< 인력, 척력 >



< Coulomb의 비틀림 저울 실험 >

$$\boxed{\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 \cdot Q_2}{r^2}\hat{a}_R[N]}$$

: Coulomb Force

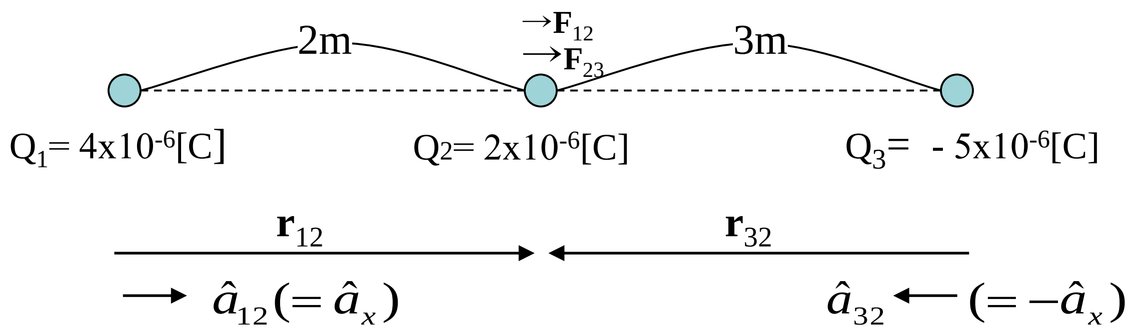
$$k = \frac{1}{4\pi\varepsilon_0} \cong 9\times10^9,$$

$$\varepsilon_0 = 8.854\times10^{-12} = \frac{1}{36\pi}\times10^{-9}[F/m]$$

: Permittivity

∴ Coulomb의 정전기력 : $\boxed{\mathbf{F} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q_1 \cdot Q_2}{r^2}\hat{a}_R[N]}$

(Ex) Coulomb Force :



$$\begin{cases} \mathbf{F}_{12} = 9\times10^9 \times \dfrac{4\times2\times10^{-12}}{2^2}\hat{a}_{21} = 1.8\times10^{-2}\hat{a}_x[N] \\ \mathbf{F}_{32} = 9\times10^9 \times \dfrac{2\times(-5)\times10^{-12}}{3^2}\hat{a}_{32} = -1\times10^{-2}(-\hat{a}_x) = 1\times10^{-2}\hat{a}_x[N] \end{cases}$$

$$\rightarrow \therefore \mathbf{F} = \mathbf{F}_{23} + \mathbf{F}_{23} = 2.8\times10^{-2}\hat{a}_x[N]$$

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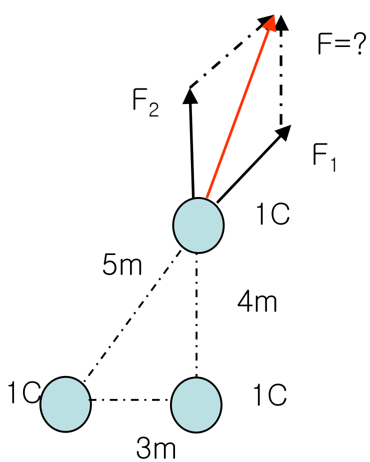
(Ex)



$$\left[\begin{array}{ll} \mathbf{r}_1 = 3\hat{a}_x + 4\hat{a}_y, & \mathbf{r}_2 = 4\hat{a}_y \\ \hat{a}_1 = \frac{3}{5}\hat{a}_x + \frac{4}{5}\hat{a}_y, & \hat{a}_2 = \hat{a}_y \end{array}\right]$$

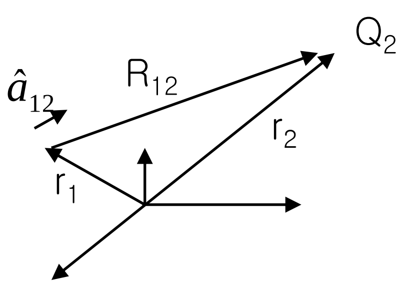
$$\mathbf{F}_1 = 9\times10^9 \times \frac{1}{5^2}\hat{a}_1 = \frac{9}{25}\times10^9\hat{a}_1[N]$$

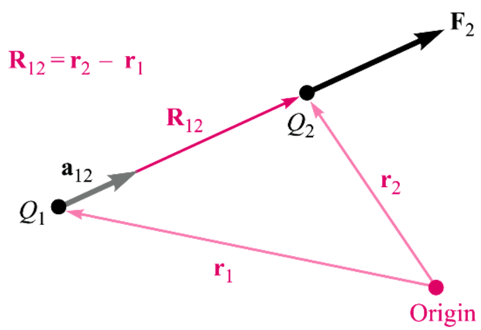
$$= \frac{9\times10^9}{25}\left(\frac{3}{5}\hat{a}_x + \frac{4}{5}\hat{a}_y\right) = \frac{3^2}{5^3}\times10^9\hat{a}_x + \frac{36}{5^3}\times10^9\hat{a}_y[N]$$

$$\mathbf{F}_2 = 9\times10^9 \times \frac{1}{4^2}\hat{a}_2 = \frac{9}{16}\times10^9\hat{a}_y[N]$$

$$\therefore \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \frac{27}{125}\times10^9\hat{a}_x + \left(\frac{36}{125} + \frac{9}{16}\right)\times10^9\hat{a}_y[N]$$

(In General)





$$\mathbf{a}_{12} = \frac{\mathbf{R}_{12}}{|\mathbf{R}_{12}|} = \frac{\mathbf{R}_{12}}{R_{12}} = \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|}$$

$$\boxed{\mathbf{F}_2 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2}\mathbf{a}_{12}}$$

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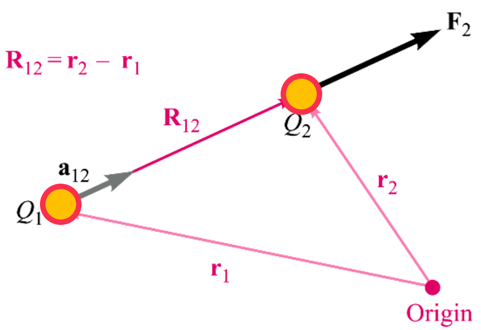
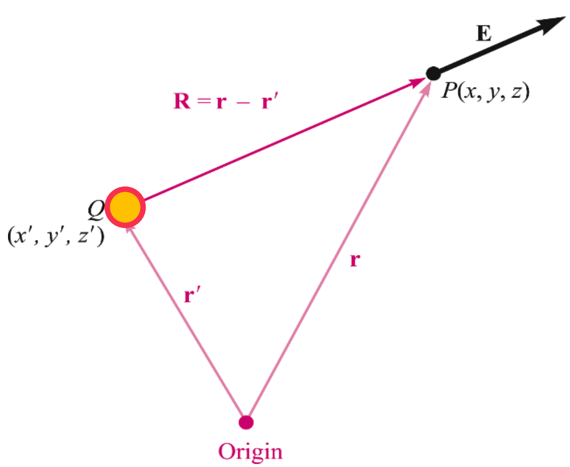
2.2 전계의 세기 (Electric Field Intensity)



$$\boxed{\mathbf{F}_2 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2}\mathbf{a}_{12}}$$

✓ 두 charge 간의 force

· Test charge Q_t : $\mathbf{F}_t = \dfrac{Q_1 Q_t}{4\pi\epsilon_0 R_{1t}^2}\mathbf{a}_{1t}$



$$\boxed{\mathbf{E}_1 = \frac{\mathbf{F}_t}{Q_1} = \frac{Q_1}{4\pi\epsilon_0 R_{1t}^2}\mathbf{a}_{1t}}$$

✓ 전하 주위의 공간의 장(field)
--> "field concept"

: <u>전계의 세기(Electric Field Intensity)</u>

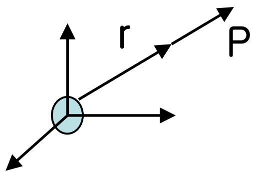
· Field Intensity / Flux Density

· 단위 : N/C = N·m/C·m = N·m/C · 1/m

≡ V/m, [N/C, V/m]

- 좌표계

(i) 구좌표계에서 : Q가 원점에 있을 경우, **P**(r, θ, Φ)



$(\mathbf{r} = r\hat{a}_r),$

$$\boxed{\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}\hat{a}_r}$$

(ii) 직각좌표계에서 : Q가 원점에 있을 경우, <u>P</u>(x, y, z)

$$\left[\begin{array}{l} \mathbf{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z \\ \hat{a}_r = \dfrac{x\hat{a}_x + y\hat{a}_y + z\hat{a}_z}{\sqrt{x^2+y^2+z^2}}, \end{array}\right]$$

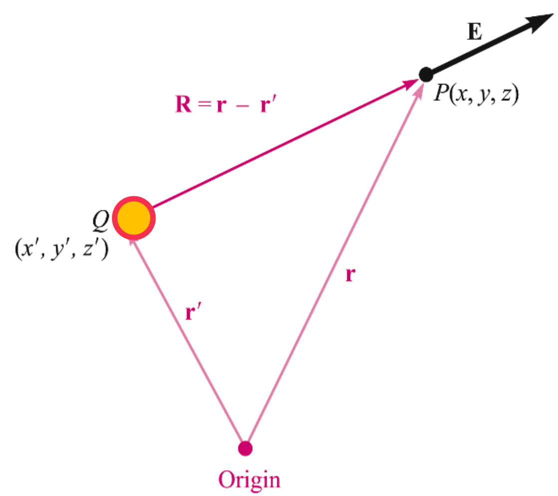
$$\boxed{\mathbf{E} = \frac{Q}{4\pi\varepsilon_0}\frac{x\hat{a}_x + y\hat{a}_y + z\hat{a}_z}{(x^2+y^2+z^2)^{2/3}}}$$

(iii) In General :

$$\mathbf{R} = \mathbf{r} - \mathbf{r}'$$

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{|\mathbf{r}-\mathbf{r}'|^2}\hat{a}_R$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{|\mathbf{r}-\mathbf{r}'|^3}(\mathbf{r}-\mathbf{r}')$$



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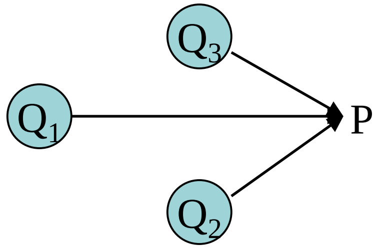
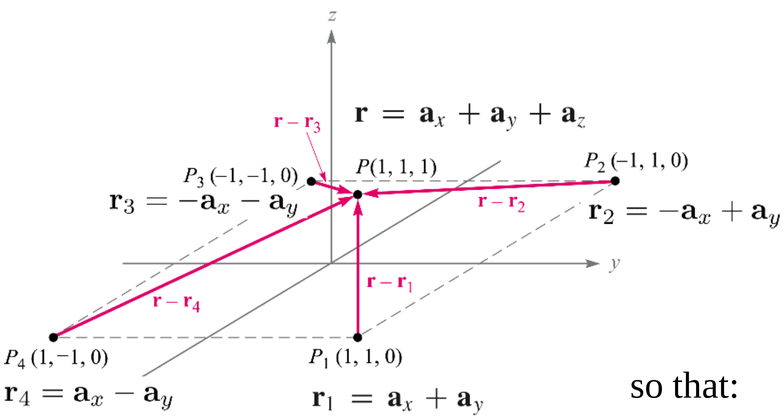
- 여러 개의 charge 에 의한 전계의 세기 : superposition



For n charges:

$$\mathbf{E}(\mathbf{r}) = \sum_{m=1}^{n} \frac{Q_m}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}_m|^2} \mathbf{a}_m$$

(Ex) Find **E** at *P*, using $\mathbf{E}(\mathbf{r}) = \sum_{m=1}^{n} \frac{Q_m}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}_m|^2} \mathbf{a}_m$



Find **E** at *P*, using $\mathbf{E}(\mathbf{r}) = \sum_{m=1}^{n} \frac{Q_m}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}_m|^2} \mathbf{a}_m$

where $\mathbf{a}_m = \dfrac{\mathbf{r} - \mathbf{r}_m}{|\mathbf{r} - \mathbf{r}_m|}$

Now: $Q/4\pi\epsilon_0 = 3 \times 10^{-9}/(4\pi \times 8.854 \times 10^{-12}) = 26.96\ \mathrm{V \cdot m}$

$|\mathbf{r} - \mathbf{r}_1| = 1$

$|\mathbf{r} - \mathbf{r}_2| = \sqrt{5}$

$|\mathbf{r} - \mathbf{r}_3| = 3$

$|\mathbf{r} - \mathbf{r}_4| = \sqrt{5}$

so that:

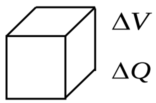
$$\mathbf{E} = 26.96 \left[\frac{\mathbf{a}_z}{1}\frac{1}{1^2} + \frac{2\mathbf{a}_x + \mathbf{a}_z}{\sqrt{5}}\frac{1}{(\sqrt{5})^2} + \frac{2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z}{3}\frac{1}{3^2} + \frac{2\mathbf{a}_y + \mathbf{a}_z}{\sqrt{5}}\frac{1}{(\sqrt{5})^2} \right]$$

$$= 6.82\mathbf{a}_x + 6.82\mathbf{a}_y + 32.8\mathbf{a}_z\ \mathrm{V/m}$$

2.3. 연속적인 체적전하 분포에 의한 전계의 세기

- Water : 1g/cm³, 연속체 표현.
  - microscopic – 물 분자, 물 밀도, 전자하나
  - macroscopic – 전하량, 전하밀도, (ex. 음극선관)

- <u>체적전하 밀도(Volume Charge Density)</u> ρ_v
  - 단위 : C/m^3
  - 의미 :



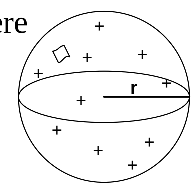
$$\Delta Q = \rho_v \Delta V \qquad \rho_v = \lim_{\Delta V \to 0} \frac{\Delta Q}{\Delta V}$$

$$Q = \int_{Vol} dQ = \int_{Vol} \rho_v dV$$: Total Charge

$$\mathbf{E} = \int_{vol} \frac{1}{4\pi\varepsilon_0} \cdot \frac{\rho_v(\mathbf{r}')dv'}{|\mathbf{r}-\mathbf{r}'|^2} \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}$$: 체적 전하에 의한 전계의 세기

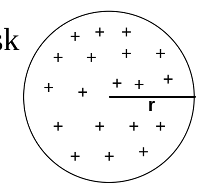
(Ex) 전하량 Q가 골고루 분포할 경우 각각 체적전하 밀도는 ?

- Sphere



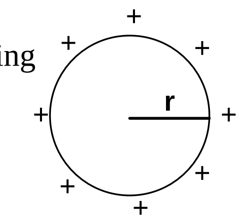
$$\rho_v = \frac{3Q}{4\pi r^3}[C/m^3]$$

- Disk



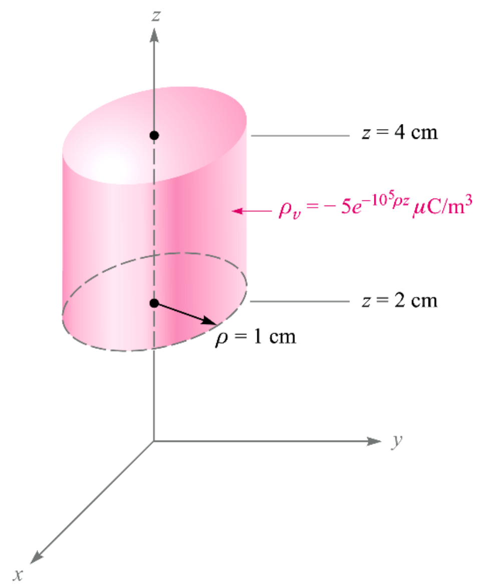
$$\rho_S = \frac{Q}{\pi r^2}[C/m^2]$$

- Ring



$$\rho_L = \frac{Q}{2\pi r}[C/m]$$

(Ex) Find the charge contained within a 2-cm length of the electron beam shown below, in which the charge density is $\rho_v = -5 \times 10^{-6} e^{-10^5 \rho z}$ C/m^2



$$Q = \int_{0.02}^{0.04} \int_{0}^{2\pi} \int_{0}^{0.01} -5 \times 10^{-6} e^{-10^5 \rho z} \rho \, d\rho \, d\phi \, dz$$

$$= \int_{0.02}^{0.04} \int_{0}^{0.01} -10^{-5} \pi e^{-10^5 \rho z} \rho \, d\rho \, dz$$

$$= \int_{0}^{0.01} \left(\frac{-10^{-5}\pi}{-10^5 \rho} e^{-10^5 \rho z} \rho \, d\rho \right)_{z=0.02}^{z=0.04}$$

$$= \int_{0}^{0.01} -10^{-5} \pi (e^{-2000\rho} - e^{-4000\rho}) d\rho$$

$$= \int_{0}^{0.01} -10^{-5} \pi (e^{-2000\rho} - e^{-4000\rho}) d\rho$$

$$= -10^{-10} \pi \left(\frac{e^{-2000\rho}}{-2000} - \frac{e^{-4000\rho}}{-4000} \right)_{0}^{0.01}$$

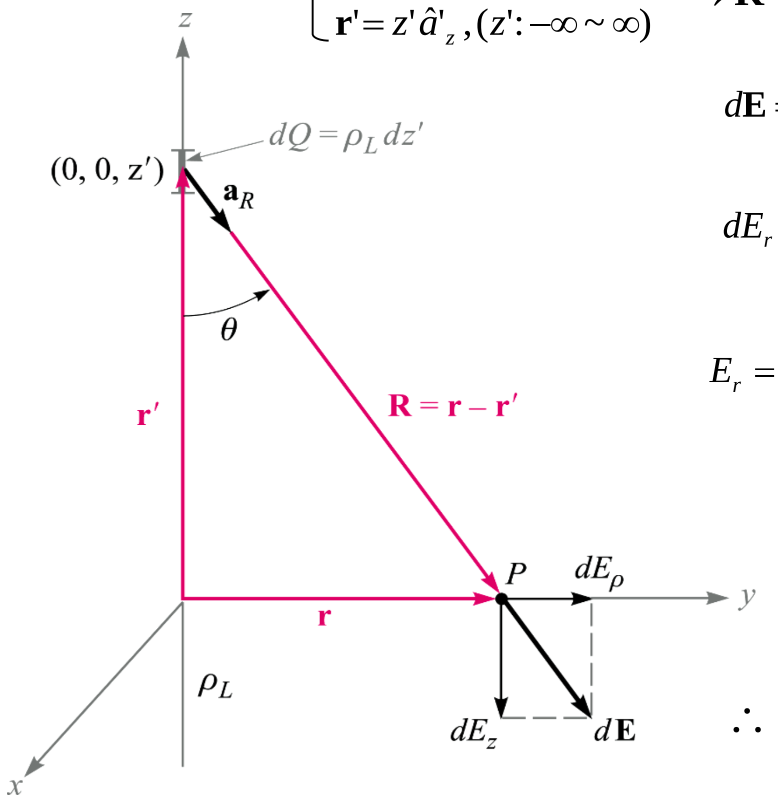
$$= -10^{-10} \pi \left(\frac{1}{2000} - \frac{1}{4000} \right)$$

$$= \frac{-\pi}{40} = 0.0785 \text{ pC}$$

2.4. 선전하에 의한 전계의 세기

- 선전하 밀도 (Line Charge Density) ρ_L [C/m]

- <u>무한 직선 전하에 의한 전계의 세기</u> :



$$\begin{cases} \mathbf{r} = r\hat{a}_r \\ \mathbf{r}' = z'\hat{a}'_z, (z' : -\infty \sim \infty) \end{cases} \rightarrow \mathbf{R} = \mathbf{r} - \mathbf{r}' = r\hat{a}_r - z'\hat{a}_z \qquad dQ = \rho_L dz'$$

$$d\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{\rho_L dz'}{(r^2+z'^2)^{3/2}}(r\hat{a}_r - z' = d\mathbf{E}_r + \cancel{d\mathbf{E}_z} = d\mathbf{E}_r = dE_r\hat{a}_r$$

$$dE_r = \frac{1}{4\pi\varepsilon_0}\frac{\rho_L r dz'}{(r^2+z'^2)^{3/2}}$$

$\hat{a}_z$ 성분은 상쇄되고
$\hat{a}_r$ 성분만 남음

$$E_r = \int dE_r = \int_{-\infty}^{\infty}\frac{1}{4\pi\varepsilon_0}\frac{\rho_L r}{(r^2+z'^2)^{3/2}}dz' = \underline{\frac{\rho_L r}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{1}{(r^2+z'^2)^{3/2}}dz'}$$

$\rightarrow$ ⓐ 적분 : next page

$$E_r = \frac{\rho_L r}{4\pi\varepsilon_0}\left(\frac{2}{r^2}\right) = \frac{1}{2\pi\varepsilon_0}\frac{\rho_L}{r}$$

$$\therefore \boxed{\mathbf{E} = \frac{1}{2\pi\varepsilon_0}\frac{\rho_L}{r}\hat{a}_r}$$

$$\begin{pmatrix} \text{점전하 :} & |\mathbf{E}| \propto \frac{1}{r^2} \\ \text{선전하 :} & |\mathbf{E}| \propto \frac{1}{r} \\ \text{면전하 :} & |\mathbf{E}| \propto const \end{pmatrix}$$

✓ 참고 : $E_r = \int dE_r = \int_{-\infty}^{\infty} \frac{1}{4\pi\varepsilon_0} \frac{\rho_L r}{(r^2 + z'^2)^{3/2}} dz' = \underline{\frac{\rho_L r}{4\pi\varepsilon_0} \int_{-\infty}^{\infty} \frac{1}{(r^2 + z'^2)^{3/2}} dz'}$

$\rightarrow$ ⓐ

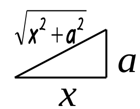
* ⓐ : $\int_{-\infty}^{\infty} \frac{dx}{(x^2 + a^2)^{3/2}}$ 계산 :

$$\begin{cases} x = a\cot\theta\,(put) \qquad dx = -a\,\mathrm{cosec}^2\theta d\theta = -a\dfrac{1}{\sin^2\theta} d\theta \\ (x^2 + a^2)^{3/2} = a^3(\cot^2\theta + 1)^{3/2} = a^3 \mathrm{cosec}^3\theta = a^3 \dfrac{1}{\sin^3\theta} \end{cases}$$

$$\therefore \int_{-\infty}^{\infty} \frac{dx}{(x^2 + a^2)^{3/2}} = \int_{-\infty}^{\infty} \frac{-a \cdot 1/\sin^2\theta}{a^3 \cdot 1/\sin^3\theta} d\theta = \int_{-\infty}^{\infty} -\frac{1}{a^2} \sin\theta d\theta$$

$$= -\frac{1}{a^2} \int_{-\infty}^{\infty} \sin\theta d\theta = \frac{1}{a^2} [\cos\theta]_{-\infty}^{\infty}$$

① :



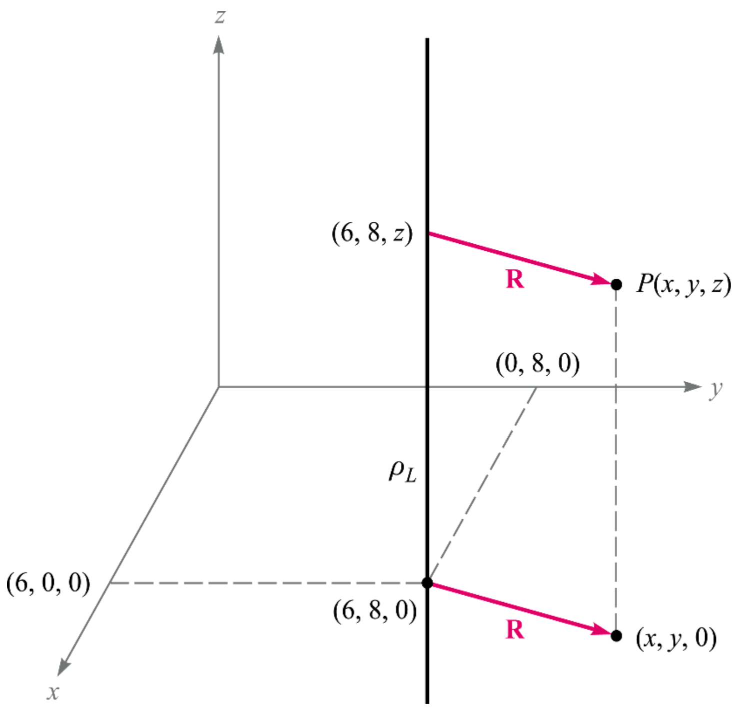
: ⓐ $= \frac{1}{a^2}\left[\frac{x}{\sqrt{x^2 + a^2}}\right]_{-\infty}^{\infty} = \frac{1}{a^2}[1 - (-1)] = \frac{2}{a^2}$

② $\begin{cases} z = -\infty \sim \infty \\ \theta = 0^+ \sim \pi^- \end{cases}$: ⓐ $= \frac{1}{a^2}[\cos\theta]_{\pi^-}^{0^+} = \frac{1}{a^2}[\cos 0^+ - \cos\pi^-] = \frac{2}{a^2}$

$\int \frac{a\,dx}{a^2 + x^2} = \tan^{-1}\frac{x}{a}$	$\int \frac{x\,dx}{x^2 + a^2} = \frac{1}{2}\ln(x^2 + a^2)$
$\int \frac{dx}{\sqrt{x^2 + a^2}} = \sinh^{-1}\frac{x}{a}$	$\int \frac{x\,dx}{\sqrt{x^2 + a^2}} = \sqrt{x^2 + a^2}$
$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2 + a^2}}$,	$\int \frac{x\,dx}{(x^2 + a^2)^{3/2}} = \frac{-1}{\sqrt{x^2 + a^2}}$

$$\mathbf{E} = \frac{1}{2\pi\varepsilon_0}\frac{\rho_L}{r}\hat{a}_r$$

(Ex) Off-Axis Line Charge



With the line displaced to (6,8), the field becomes:

$$\mathbf{E} = \frac{\rho_L}{2\pi\epsilon_0\sqrt{(x-6)^2+(y-8)^2}}\mathbf{a}_R$$

where

$$\mathbf{a}_R = \frac{\mathbf{R}}{|\mathbf{R}|} = \frac{(x-6)\mathbf{a}_x + (y-8)\mathbf{a}_y}{\sqrt{(x-6)^2+(y-8)^2}}$$

Finally:

$$\boxed{\mathbf{E} = \frac{\rho_L}{2\pi\epsilon_0}\frac{(x-6)\mathbf{a}_x + (y-8)\mathbf{a}_y}{(x-6)^2+(y-8)^2}}$$

(Ex) 수소원자의 만유인력과 정전기력 비교 :

수소원자의 ① 만유인력, ② Coulomb Force, ③ 전자의 평균회전 속도를 구하시오.

수소원자	양성자	전자
질량	1.6×10^{-27}kg	9.11×10^{-31}kg
전하량	1.6×10^{-19}C	-1.6×10^{-19}C

: 전자와 원자핵간의 평균거리

= 5.3×10^{-11}m

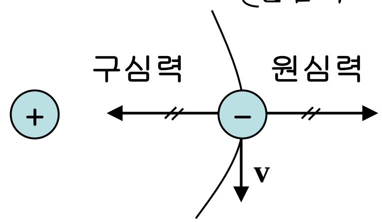
① 만유인력 : $F_{Newton} = G \cdot \dfrac{m_1 \cdot m_2}{r^2} =$

② Coulomb Force : $F_{Coulomb} = k\,\dfrac{Q_1\ Q_2}{r^2} =$

*** 총 F = $\mathbf{F_{newton}} + \mathbf{F_{coulomb}}$** =

*** 비교 :** $\dfrac{F_{coulomb}}{F_{Newton}} =$

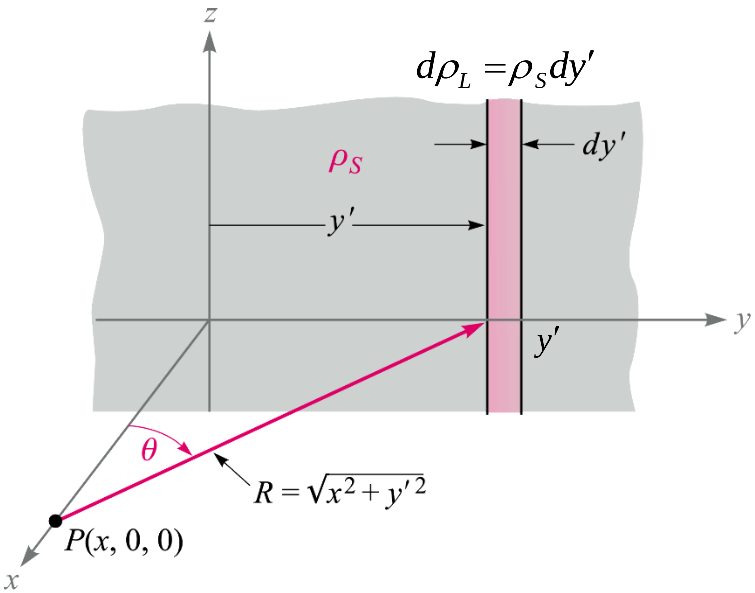
③ 전자의 평균회전 속도 :
$$\begin{cases} \text{구심력} = \\ \text{원심력} = \quad m\dfrac{v^2}{r} \end{cases}$$



$$\therefore v = \sqrt{\frac{r \cdot F}{m}} =$$

2.5. 판전하에 의한 전계의 세기

- 표면전하 밀도(Surface Charge Density) : ρ_s[C/m^2]
- 판전하에 의한 전계의 세기 : (무한 평판 전하)



y − z 평면전하, x축 P점에서의 전계의 세기

$$\mathbf{E} = E_x\hat{a}_x + E_y\hat{a}_y + E_z\hat{a}_z = E_x\hat{a}_x \quad \text{(무한평면, 대칭, 상쇄)}$$

$$dE_x = \frac{1}{2\pi\varepsilon_0}\frac{\rho_S dy'}{\sqrt{x^2 + y'^2}}\cos\theta$$

$$= \frac{1}{2\pi\varepsilon_0}\frac{\rho_S dy'}{\sqrt{x^2 + y'^2}}\frac{x}{\sqrt{x^2 + y'^2}} = \frac{\rho_S}{2\pi\varepsilon_0}\frac{xdy'}{x^2 + y'^2}$$

$$E_x = \frac{\rho_S}{2\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{xdy'}{x^2 + y'^2}$$

$$= \frac{\rho_S}{2\pi\varepsilon_0}[\tan^{-1}\frac{y'}{x}]_{-\infty}^{\infty} = \frac{\rho_S}{2\varepsilon_0}$$

$$\boxed{\therefore \mathbf{E} = \frac{\rho_S}{2\varepsilon_0}\hat{a}_N}$$

✓ 방향 : $\hat{a}_N$

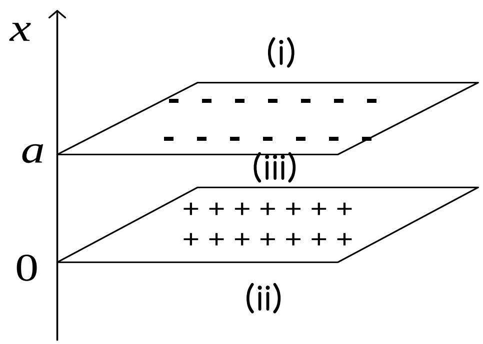
✓ $|\mathbf{E}| \propto r$, 항상 동일한 크기, 방향

$\int \frac{adx}{a^2 + x^2} = \tan^{-1}\frac{x}{a}$	$\int \frac{xdx}{x^2 + a^2} = \frac{1}{2}\ln(x^2 + a^2)$
$\int \frac{dx}{\sqrt{x^2 + a^2}} = \sinh^{-1}\frac{x}{a}$	$\int \frac{xdx}{\sqrt{x^2 + a^2}} = \sqrt{x^2 + a^2}$
$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2 + a^2}},$	$\int \frac{xdx}{(x^2 + a^2)^{3/2}} = \frac{-1}{\sqrt{x^2 + a^2}}$

Lab. of Applied ElectroMagnetics

$$\therefore \mathbf{E} = \frac{\rho_S}{2\varepsilon_0}\hat{a}_N$$

- 두 무한 평판에 의한 전계의 세기(Condenser)



(i) $x > a$: $\mathbf{E}_+ = \frac{\rho_S}{2\varepsilon_0}\hat{a}_x, \quad \mathbf{E}_- = -\frac{\rho_S}{2\varepsilon_0}\hat{a}_x,$

$\rightarrow \mathbf{E} = \mathbf{E}_+ + \mathbf{E}_- = \mathbf{0}$

(ii) $x < 0$: $\mathbf{E}_+ = -\frac{\rho_S}{2\varepsilon_0}\hat{a}_x, \quad \mathbf{E}_- = \frac{\rho_S}{2\varepsilon_0}\hat{a}_x,$

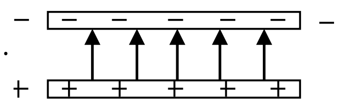
$\rightarrow \mathbf{E} = \mathbf{E}_+ + \mathbf{E}_- = \mathbf{0}$

(iii) $0 < x < a$: $\mathbf{E}_+ = \frac{\rho_S}{2\varepsilon_0}\hat{a}_x, \quad \mathbf{E}_- = \frac{\rho_S}{2\varepsilon_0}\hat{a}_x,$

$$\boxed{\rightarrow \mathbf{E} = \frac{\rho_S}{\varepsilon_0}\hat{a}_x}$$

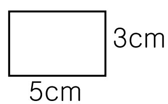
$\therefore$ 즉, 내부 전계는 $\frac{\rho_S}{\varepsilon_0}\hat{a}_x$, 외부 전계는 zero

전계 차폐, 전계 집중.



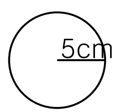
◎ 전하 / 전하밀도.

(Ex1)



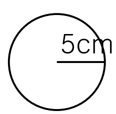
, $\rho_S = 10^{\;3}(xy)[C/m^2],$ $\quad Q = \int_0^{0.05}\int_0^{0.03} 10^{\;3} xydxdy = \sim$

(Ex2)



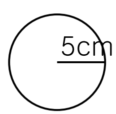
원판 , $\rho_S = 10^{\;3}(0.05 - r)[C/m^2],$ $\quad Q = \int_{r=0}^{0.05}\int_{\varphi=0}^{2\pi} 10^{\;3}(0.05 \quad r)rdrd\varphi = \sim$

(Ex3)



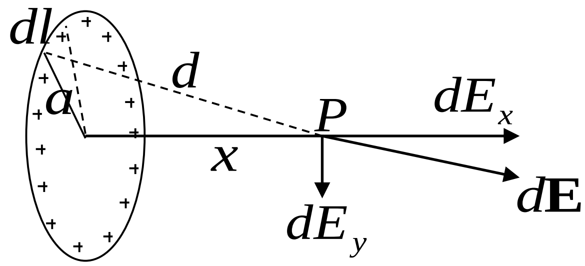
원판 , $Q = 1[mC], \qquad \rho_S = ?$

(Ex4)



구 , $Q = 1[\mu C], \qquad \rho_S = ?$

(Ex) 원형 선전하에 의한 전계의 세기



$$\begin{cases} Q = \rho_L 2\pi a \\ \rho_L [C/m],\ d = \sqrt{x^2 + a^2} \end{cases}$$

$$dQ = \rho_L dl = \rho_L a \cdot d\phi$$

$$d\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dQ}{d^2} \hat{a}_R$$

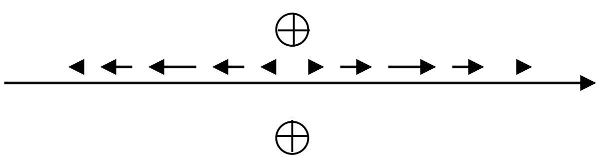
$$dE_x = |d\mathbf{E}| \cos\theta = |d\mathbf{E}| \frac{x}{d}$$

$$dE_x = \frac{1}{4\pi\varepsilon_0} \frac{\rho_L a d\phi}{d^2} \frac{x}{d} = \frac{1}{4\pi\varepsilon_0} \frac{\rho_L a x d\phi}{(a^2 + x^2)^{3/2}}$$

$$E_x = \int dE_x = \frac{1}{4\pi\varepsilon_0} \frac{\rho_L a x}{(a^2 + x^2)^{3/2}} \int d\phi$$

$$= \frac{1}{2\varepsilon_0} \frac{\rho_L a x}{(a^2 + x^2)^{3/2}}$$

$$\boxed{\therefore \mathbf{E} = \frac{1}{2\varepsilon_0} \frac{\rho_L a x}{(a^2 + x^2)^{3/2}} \hat{a}_x}$$

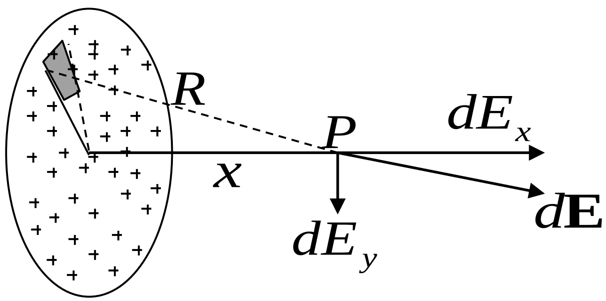


① x = 0 : **E = 0**

② x = ? : **|E| = max**

Lab. of Applied ElectroMagnetics

(Ex) 원형 판전하에 의한 전계의 세기



$$\begin{cases} dQ = \rho_S ds = \rho_S r d\phi dr \\ \mathbf{R} = x\hat{a}_x - r\hat{a}_r \qquad R = \sqrt{x^2 + r^2} \end{cases}$$

$$dE_x = |d\mathbf{E}|\cos\theta = |d\mathbf{E}|\frac{x}{R} = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{R^2}\frac{x}{R}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{x\rho_S dS}{R^3}$$

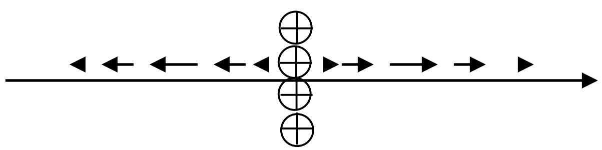
$$\boxed{\therefore \mathbf{E} = \frac{\rho_S}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{x^2 + a^2}}\right)\hat{a}_x}$$

$$E_x = \int dE_x = \int_{r=0}^{a}\int_{\phi=0}^{2\pi}\frac{1}{4\pi\varepsilon_0}\frac{x\rho_S r d\phi dr}{(x^2 + r^2)^{3/2}}$$

$$= \frac{x\rho_S}{4\pi\varepsilon_0}\int_{\phi=0}^{2\pi} d\phi \int_{r=0}^{a}\frac{rdr}{(x^2 + r^2)^{3/2}}$$

$$= \frac{x\rho_S}{2\varepsilon_0}\left[\frac{-1}{\sqrt{x^2 + r^2}}\right]_{r=0}^{a}$$

$$= \frac{\rho_S}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{x^2 + a^2}}\right)$$



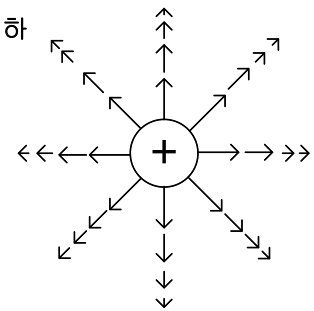
- If $a \to \infty$: 무한원판 : $\mathbf{E} = \dfrac{\rho_S}{2\varepsilon_0}\hat{a}_x$, 무한평면
- If $x \to 0$: at center : $\mathbf{E} = \dfrac{\rho_S}{2\varepsilon_0}\hat{a}_x$, 무한원판?
- If $x \to \dfrac{a}{\sqrt{3}}$ $\quad \mathbf{E}' = \dfrac{1}{2}\mathbf{E} = \dfrac{1}{2}\dfrac{\rho_S}{2\varepsilon_0}\hat{a}_x$
- If $x \to \infty$: $|\mathbf{E}| \to 0$

2.6 전계의 묘사

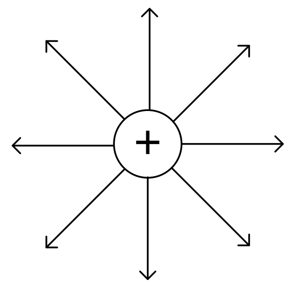
- 용어 : 전속선(electric flux line), 방향선(directional line)
  유선(stream line), 전력선(line of electric force)

- 전계 (Electric Field) 의 표현방법

• 한 선전하



Meaning Vector Field



Shape to draw

$$*\mathbf{E} = \frac{1}{2\pi\varepsilon_0}\frac{\rho_S}{r}\hat{a}_r$$

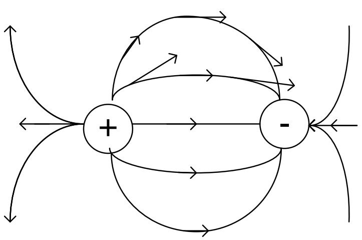
$$\begin{cases} \mathbf{E} \text{ 의 방향 : } \hat{a}_r \\ |\,\mathbf{E}\,| \propto \dfrac{1}{r} \end{cases}$$

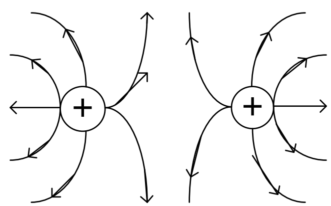
* 약속 :

- E 벡터의 방향 : line의 접선방향
- E 벡터의 크기 : line의 밀도

⊕ ⟶ ⊖

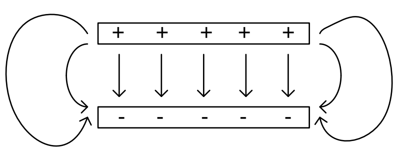
• 두 선전하

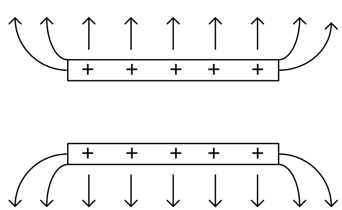




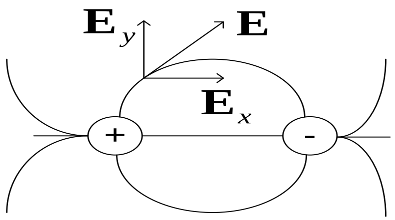
✓ 균일한 전기장 얻기(크기,방향)
fringing field

• 두 평판전하





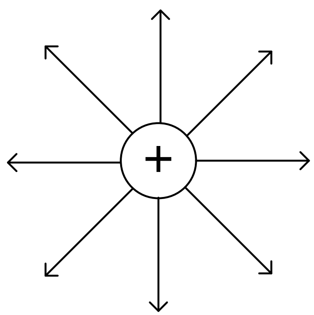
• 전력선 방정식



접선관계이므로 $\quad$ $\mathbf{E}$의 방향 : $\hat{a}_r$

$$\frac{E_y}{E_x} = \frac{\Delta y}{\Delta x} \Rightarrow \frac{dy}{dx}$$

(Ex) 무한 선전하 전력선 방정식 $\quad \rho_L = 2\pi\varepsilon_0 [C/m], \quad r = \sqrt{x^2 + y^2}, \quad \hat{a}_r = \dfrac{x\hat{a}_x + y\hat{a}_y}{\sqrt{x^2+y^2}}$



$$\mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\hat{a}_r = \frac{1}{r}\hat{a}_r = \frac{x}{x^2+y^2}\hat{a}_x + \frac{y}{x^2+y^2}\hat{a}_y \equiv E_x\hat{a}_x + E_y\hat{a}_y$$

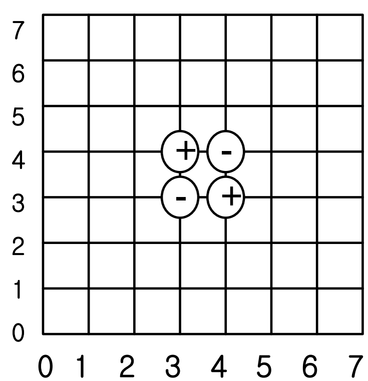
전력선 방정식 : $\dfrac{dy}{dx} = \dfrac{E_y}{E_x} = \dfrac{y}{x}$, 즉 $\dfrac{dy}{y} = \dfrac{dx}{x}$

적분하면 $\ln y = \ln x + C_1 = \ln x + \ln c = \ln cx$

$$\therefore \underline{y = cx}$$

점 P(−2, 7, 10)에서 :
7 = −2C $\quad \therefore$ C = −3.5
$\therefore$ y = −3.5x

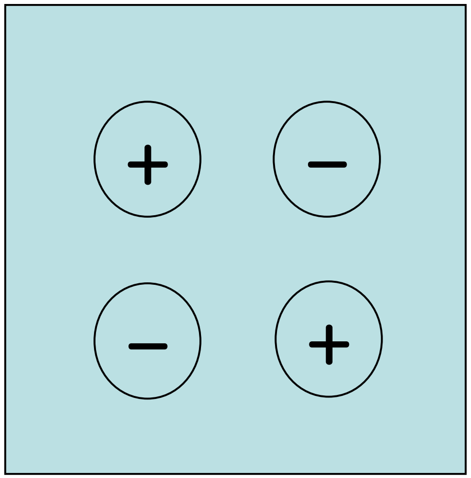
(H .W.)

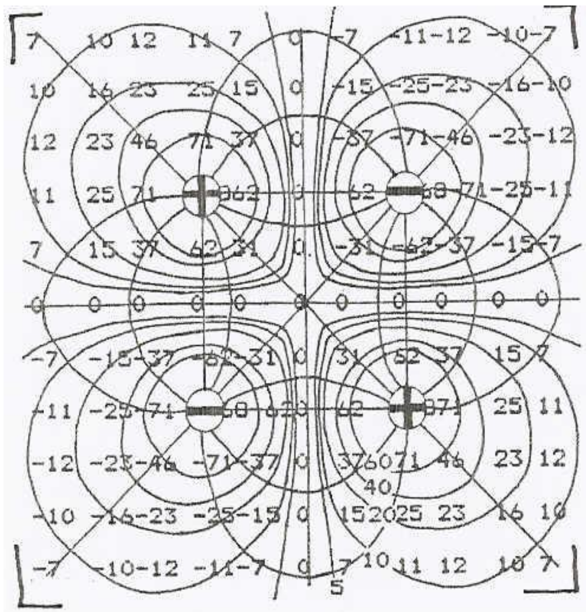


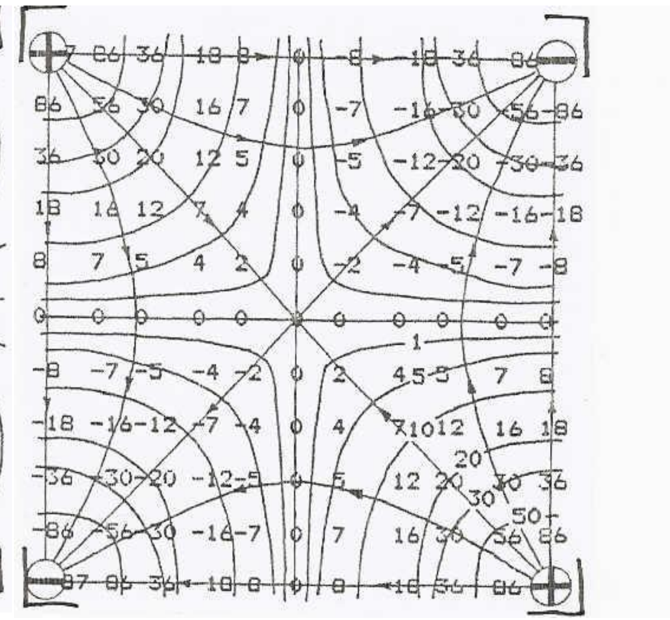
격자점에서 전계의 세기 E를 구하고 그림으로 그리시오.

Vector이므로 크기와 방향을 구할 것.
(C++ , VB, Excel, 계산기, Matlab. …)

q=1[nC]







◎ Sum : Coulomb's Law, Electric Field Intensity

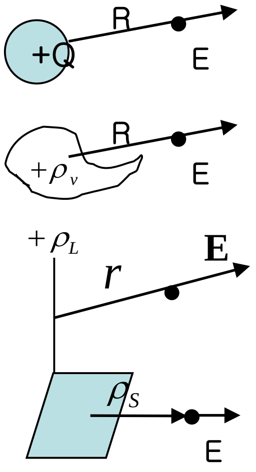
$$Q \xrightarrow{\text{Coulomb Force}} \mathbf{F} \xrightarrow{\text{Electic Field}} \mathbf{E}$$

- ① **Point Charge Q[C] :** $\mathbf{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{R^2}\hat{a}_R$

  ② **Volume Charge ρ_V[C/m³] :** $\mathbf{E} = \displaystyle\int_{vol}\dfrac{1}{4\pi\varepsilon_0}\dfrac{\rho_V(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^2}dV'\cdot\hat{a}_{\mathbf{r}-\mathbf{r}'}$

  ③ **Line Charge ρ_L[C/m] :** $\mathbf{E} = \dfrac{1}{2\pi\varepsilon_0}\dfrac{\rho_L}{R^2}\hat{a}_R$

  ④ **Plane Charge ρ_S[C/m²] :** $\mathbf{E} = \dfrac{\rho_S}{2\varepsilon_0}\hat{a}_N$



- **Charge**
  Charge Density

- 원형 선전하
  원형 판전하