Chap 11. 균일 평면파

• 용어 : Wave 전파, Propagation 전파, wave propagation → 전파전파(전번)

• 전파, 전파속도, 파장, 파동 임피던스, 위상전수, 감쇄정수

Poynting Vector, Poynting 정리, 전력 밀도

매질 경계에서의 전파의 반사 및 투과, 정재파비임피던스

11.1 자유공간 내에서의 전파

• 자유공간 : $J_S = 0, \quad \rho = 0, \quad \varepsilon = \varepsilon_0, \quad \mu = \mu_0$ $\nabla \times E = 0, \quad \nabla \times H = 0, \quad J = 0, \quad \rho = 0$

• $$\begin{bmatrix} \nabla \times H = J + \dfrac{\partial D}{\partial t} \\ \nabla \times E = -\dfrac{\partial B}{\partial t} \end{bmatrix} \rightarrow \begin{bmatrix} \nabla \times H = \varepsilon_0 \dfrac{\partial E}{\partial t} \\ \nabla \times E = -\mu_0 \dfrac{\partial H}{\partial t} \end{bmatrix}$$

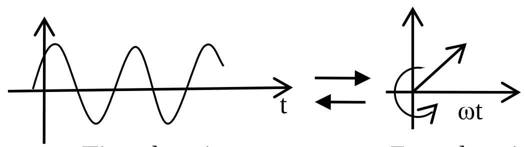
→ **E**의 변화 → **H** 발생, 변화 → **E** 발생

발생 위치가 u속도로 전파

⦿ Sinusoidal Variation과 Phasor

- Phasor → Phase + Vector
- $e^{j\omega t} = \cos\omega t + j\sin\omega t \quad \rightarrow \quad \begin{cases} \cos\omega t = \mathrm{Re}\{e^{j\omega t}\} \\ \sin\omega t = \mathrm{Im}\{e^{j\omega t}\} \end{cases}$

  - Frequency Domain Transformation



Time domain Freq. domain

시간에 대한 편미분 $\xrightarrow{\text{phasor}}$ × jω

$$(ex.)\ \frac{\partial E_x}{\partial t} = -\frac{1}{\varepsilon_0}\frac{\partial H_y}{\partial z} \Rightarrow j\omega E_{xs} = -\frac{1}{\varepsilon_0}\frac{\partial H_{ys}}{\partial z}$$

- $\mathrm{E} = E_x \hat{a}_x, \quad x$ 성분 $only$

$$E_x = E_{xyz}\cdot\cos(\omega t + \varphi) = \mathrm{Re}\{E_{xyz}\cdot e^{j\omega t + \varphi}\} = \mathrm{Re}\{\underbrace{E_{xyz}e^{j\varphi}}_{\text{Phasor}}\cdot e^{j\omega t}\}$$

E_x의 phasor : $\underline{E_{xs} = E_{xyz}e^{j\varphi}} \rightarrow \underline{E_x = E_{xs}\cdot e^{j\omega t}}$

- $$\frac{\partial E_x}{\partial t} = \frac{\partial}{\partial t}\{E_{xyz}\cdot\cos(\omega t+\varphi)\} = -\omega E_{xyz}\cdot\sin(\omega t+\varphi)$$
  $$= \mathrm{Re}\left[j\omega E_{xs}e^{j\omega t}\right]$$

- $$\begin{bmatrix} \nabla\times\mathrm{H} = \varepsilon_0\dfrac{\partial \mathrm{E}}{\partial t} \\ \nabla\times\mathrm{E} = -\mu_0\dfrac{\partial \mathrm{H}}{\partial t} \end{bmatrix} \longrightarrow \begin{bmatrix} \nabla\times\mathrm{H}_s = j\omega\varepsilon_0\mathrm{E}_s \\ \underline{\nabla\times\mathrm{E}_s = -j\omega\mu_0\mathrm{H}_s} \end{bmatrix} \quad \begin{matrix} \nabla\cdot\mathrm{E}_s = 0 \\ \nabla\cdot\mathrm{H}_s = 0 \end{matrix}$$

⦿ Helmholtz 방정식

- 자유공간에서의 Maxwell 방정식의 phasor form :

$$\begin{cases} \nabla\times \mathbf{H}_s = j\omega\varepsilon_0 \mathbf{E}_s & \text{---- ①} \\ \nabla\times \mathbf{E}_s = -j\omega\mu_0 \mathbf{H}_s & \text{---- ②} \\ \nabla\cdot \mathbf{E}_s = 0 & \text{---- ③} \\ \nabla\cdot \mathbf{H}_s = 0 & \text{---- ④} \end{cases}$$

- $$\left.\begin{array}{l} \nabla\times \mathbf{E}_s = -j\omega\mu_0 \mathbf{H}_s \quad (\because ②) \\ \nabla\times(\nabla\times \mathbf{E}_s) = \nabla\times(-j\omega\mu_0 \mathbf{H}_s) \end{array}\right\} (\nabla\times)$$

$$左 = \nabla\times(\nabla\times \mathbf{E}_s) = \nabla(\nabla\cdot \mathbf{E}_s) - \nabla^2 \mathbf{E}_s = \nabla^2 \mathbf{E}_s \quad (\because ③)$$

$$右 = -j\omega\mu_0(\nabla\times \mathbf{H}_s) = -j\omega\mu_0(j\omega\varepsilon_0 \mathbf{E}_s) = \omega^2\mu_0\varepsilon_0 \mathbf{E}_s \quad (\because ①)$$

$$\therefore \boxed{\nabla^2 \mathbf{E}_s = -\omega^2\mu_0\varepsilon_0 \mathbf{E}_s} \quad : \text{Helmholtz Equation}$$

⦿ 균일 평면파(Uniform Plane Wave)

- 직각 좌표계 x성분 : $\nabla^2 E_{xs} = -\omega^2\mu_0\varepsilon_0 E_{xs}$

$$\underbrace{\frac{\partial^2 E_{xs}}{\partial x} + \frac{\partial^2 E_{xs}}{\partial y^2}}_{\rightarrow \text{zero}} + \frac{\partial^2 E_{xs}}{\partial z^2} = -\omega^2\mu_0\varepsilon_0 E_{xs}$$

$$\frac{\partial^2 E_{xs}}{\partial z^2} = -\omega^2\mu_0\varepsilon_0 E_{xs}, \qquad E_{xs} = Ae^{-j\omega\sqrt{\mu_0\varepsilon_0}z}$$

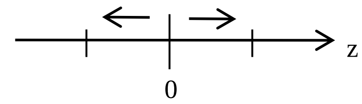
$$E_x = \mathrm{Re}\left[E_{xs}e^{j\omega t}\right] = A\cos\left\{\omega(t - z\sqrt{\mu_0\varepsilon_0})\right\}, \qquad z=0, t=0 \text{에서 } E_x = E_{x0} \text{라 두면}$$

$$\therefore \boxed{E_x = E_{x0}\cos\left[\omega(t - z\sqrt{\mu_0\varepsilon_0})\right]}$$

$$\therefore\ E_x = E_{x0}\cos\left[\omega(t - z\sqrt{\mu_0\varepsilon_0})\right]$$

- $\begin{bmatrix} E_x = E_{x0}\cdot\cos\left[\omega(t - z\sqrt{\mu_0\varepsilon_0})\right] \\ E_x' = E_{x0}'\cdot\cos\left[\omega(t + z\sqrt{\mu_0\varepsilon_0})\right] \end{bmatrix}$: pair solution

  ① 양방향으로 전파



  ② $\dfrac{z}{c},\ c = \dfrac{1}{\sqrt{\mu_0\varepsilon_0}} = u$: 빛의 속도로 전파

(i) 시간에 대하여 : 빛의 속도로 전파

ex.) $E_x = E_{x0}\cdot\cos\left[\omega(t - \dfrac{z}{c})\right]$

z = 0 : 서울 $\qquad E_x = E_{x0}\cdot\cos\omega t$

z ≈ 500km : 부산 $\qquad E_x = E_{x0}\cos\left[\omega(t - \dfrac{5\times10^5}{3\times10^8})\right] = E_{x0}\cos[\omega(t - 0.00167)]$

∴ t = 0, 서울 ⟶ t = 0.00167초 후 부산 도달

(ii) 공간(거리)에 대하여 : 정현적으로 분포

$$E_x = E_{x0}\cdot\cos\left[\omega(-\frac{z}{c})\right] = E_{x0}\cdot\cos\omega\frac{z}{c}$$

· sine 분포

· 파장 (Wave Length) : $\omega\dfrac{\lambda}{c} = 2\pi,\quad \lambda = \dfrac{2\pi c}{\omega} = \dfrac{c}{f} = \dfrac{3\times10^8}{f} = T\cdot c$

$$c = \lambda\cdot f,\quad \lambda = \frac{1}{f}c = Tc,\quad \omega = 2\pi f,\quad \omega T = 2\pi$$

· 진행파 (Traveling Wave) $\quad |E_x|$가 같은 부분 → (빛의속도로 진행)

$$t - \frac{z}{c} = const;\quad dt - \frac{1}{c}dz = 0,\quad \frac{dz}{dt} = c$$

- **E & H**

$$E_{xs} = Ae^{-j\omega\sqrt{\mu_0\varepsilon_0}\,z}, \quad H = \ ??$$

$$\nabla\times\mathbf{E}_s = -j\omega\mu_0\mathbf{H}_s, \quad \frac{\partial E_{xs}}{\partial z} = -j\omega\mu_0 H_{ys}$$

$$\therefore H_{ys} = -\frac{1}{j\omega\mu_0}\frac{\partial E_{xs}}{\partial z} = -\frac{1}{j\omega\mu_0}E_{x0}(-j\omega\sqrt{\mu_0\varepsilon_0})e^{-j\omega z/c} = E_{x0}\sqrt{\frac{\varepsilon_0}{\mu_0}}e^{-j\omega z/c}$$

$$\therefore \boxed{H_y = E_{x0}\sqrt{\frac{\varepsilon_0}{\mu_0}}\cos(t-\frac{z}{c})}$$

$$\cdot\ E_x = E_{x0}\cos\omega(t-z/c)$$

$$\therefore \boxed{\frac{E_x}{H_y} = \sqrt{\frac{\mu}{\varepsilon}}} :$$

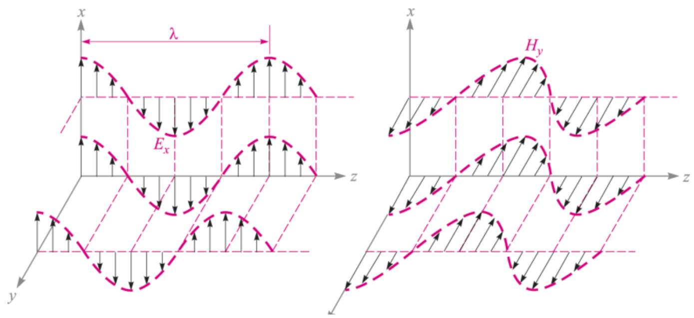
- E_x와 H_y는 서로 同相 (in-phase)
- 시간이 변하면 E_x, H_y는 변해도 그 비율은 항상 일정하게 같은 값

- **고유 임피던스 (Intrinsic Impedence) : η[Ω]**

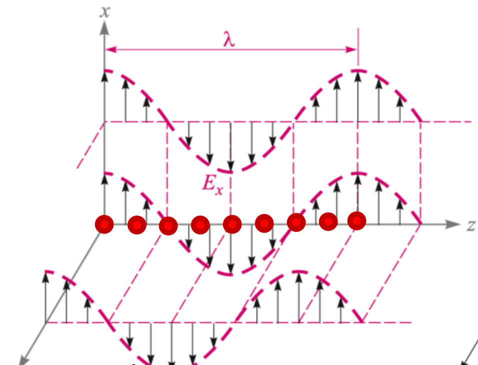
$$\boxed{\eta = \sqrt{\frac{\mu}{\varepsilon}}}\ (=\frac{E_x}{H_y}) :$$

- 일종의 매질 상수
- 자유공간에서 $\eta_0 = \sqrt{\mu_0/\varepsilon_0} = 377[\Omega] = 120\pi = 2\pi(60)!?$

- **균일 평면파**:
  - $z = z_0$에서 E_x, E_y는 모두 동일
  - 전파 진행 방향 ⊥ E, H (z ⊥ E ⊥ H)

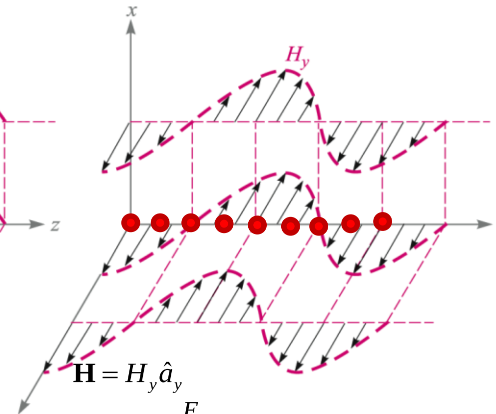


• 횡 전자파 (Transverse Electromotive Wave, TEM 파)



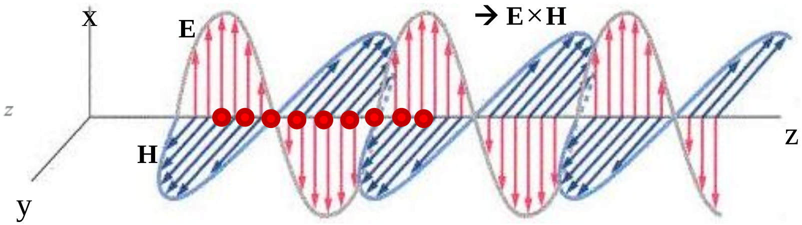
$\mathbf{E} = E_x \hat{a}_x$

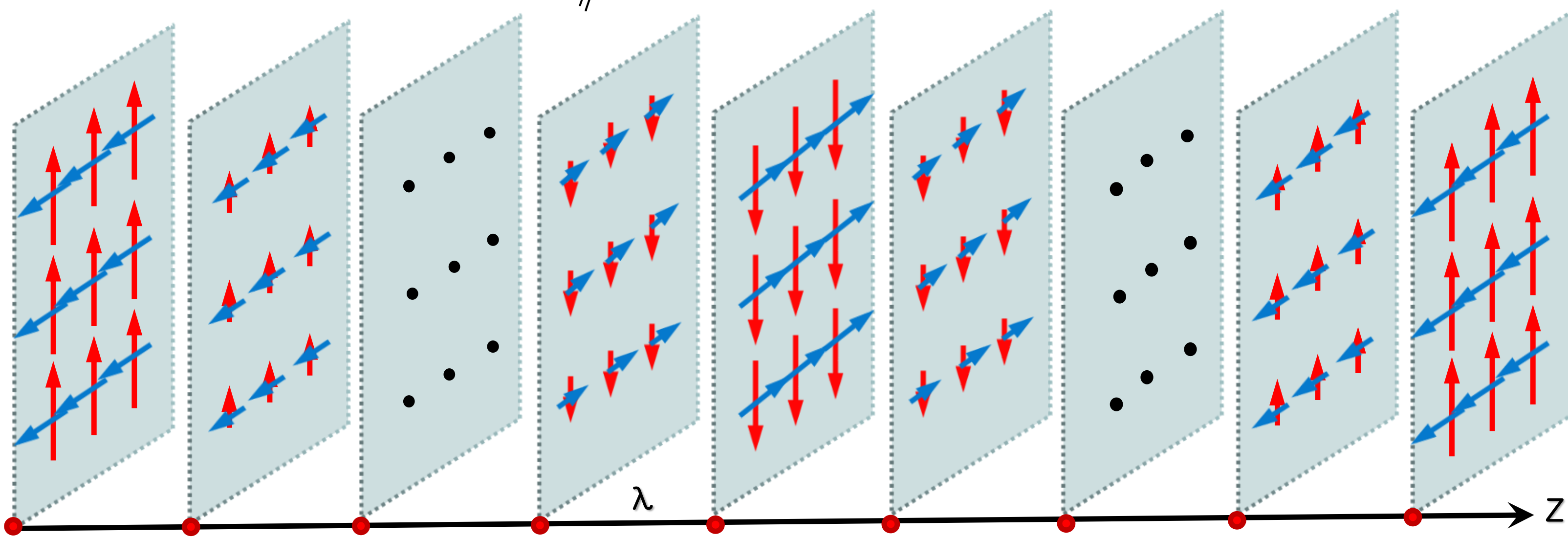
$\circ\ E_x = E_{x0} \cos \omega (t - z\sqrt{\omega\varepsilon})$



$\mathbf{H} = H_y \hat{a}_y$

$H_y = \dfrac{E_{x0}}{\eta} \cos(\omega t - \beta z)$

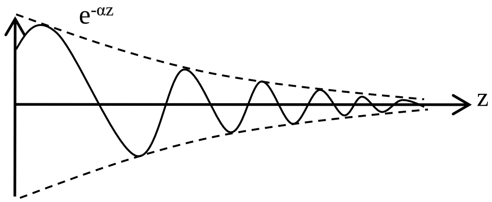




11.2 유전체 내의 전파

- 완전 유전체 : ε, μ
- $\nabla^2 \mathbf{E}_s = -\omega^2 \mu\varepsilon \mathbf{E}_s$ $\quad \dfrac{\partial^2 E_{xs}}{\partial z^2} = -\omega^2 \mu\varepsilon E_{xs}$, $\left[\begin{array}{l} E_{xs} = E_{x0} \cdot e^{-\alpha z} \cdot e^{-j\beta z} \ : \ phasor \ \ form \\ E_x = E_{x0} \cdot e^{-\alpha z} \cdot \cos(\omega t - \beta z) : e^{-\alpha z}\text{로 감소} \end{array} \right.$

- **감쇄 정수 (Attenuation Constant) : α**



· $e^{-\alpha z}$ 로 감쇄

- **위상 정수 (Phase Constant) : β**

· β : 1m당 위상 천이 양(Phase Shift), 위상이 어긋남

· $\omega \cdot T = \lambda \cdot \beta = 2\pi$ $\quad \boxed{\beta = \dfrac{2\pi}{\lambda}}$

- **전파 정수 (Propagation Constant) : γ**

· $\gamma = \alpha + j\beta$

· $E_{xs} = E_{x0} \cdot e^{-\alpha z} \cdot e^{-j\beta z} = E_{x0} \cdot e^{-(\alpha + j\beta)z} = E_{x0} e^{-\gamma z}$

즉 $E_{xs} = E_{x0} e^{-\gamma z}$

· $\dfrac{\partial^2 E_{xs}}{\partial z^2} = -\omega^2 \mu\varepsilon E_{xs}$ 에 대입하면

$$\gamma^2 E_{x0} e^{-\gamma z} = -\omega^2 \mu\varepsilon E_{x0} e^{-\gamma z}$$

$$\gamma^2 = -\omega^2 \mu\varepsilon, \qquad \therefore \boxed{\gamma = \pm j\omega\sqrt{\mu\varepsilon}}$$

$(\alpha = 0 : \text{무손실}$
$\beta = \omega\sqrt{\mu\varepsilon})$

$\boxed{\circ \ E_x = E_{x0} \cos\omega(t - z\sqrt{\omega\varepsilon})}$

$\boxed{\nabla^2 \mathbf{E}_s = -k^2 \mathbf{E}_s}$ $\quad \boxed{k = \omega\sqrt{\mu\epsilon} = k_0\sqrt{\mu_r \epsilon_r}}$

$\boxed{\dfrac{d^2 E_{xs}}{dz^2} = -k^2 E_{xs}}$ $\quad \boxed{jk = \alpha + j\beta}$

$\boxed{E_{xs} = E_{x0} e^{-jkz} = E_{x0} e^{-\alpha z} e^{-j\beta z}}$

$\boxed{E_x = E_{x0} e^{-\alpha z} \cos(\omega t - \beta z)}$

Complex permittivity $\boxed{\epsilon = \epsilon' - j\epsilon'' = \epsilon_0(\epsilon'_r - j\epsilon''_r)}$

Complex permeability $\mu = \mu' - j\mu'' = \mu_0(\mu'_r - j\mu''_r)$

Loss Tangent : ϵ''/ϵ'

$$k = \omega\sqrt{\mu(\epsilon' - j\epsilon'')} = \omega\sqrt{\mu\epsilon'}\sqrt{1 - j\frac{\epsilon''}{\epsilon'}}$$

$\boxed{jk = \alpha + j\beta}$

$$\alpha = \mathrm{Re}\{jk\} = \omega\sqrt{\frac{\mu\epsilon'}{2}}\left(\sqrt{1 + \left(\frac{\epsilon''}{\epsilon'}\right)^2} - 1\right)^{1/2}$$

$$\beta = \mathrm{Im}\{jk\} = \omega\sqrt{\frac{\mu\epsilon'}{2}}\left(\sqrt{1 + \left(\frac{\epsilon''}{\epsilon'}\right)^2} + 1\right)^{1/2}$$

- $E_x = E_{x0} \cos\omega(t - z\sqrt{\mu\varepsilon})$

< μ_r, ε_r 이 큰 매질 내에서는>

전파속도 : $c = \dfrac{1}{\sqrt{\mu\varepsilon}} = \dfrac{u}{\sqrt{\mu_r\varepsilon_r}}$: ① 전파속도가 느려진다.

파장 : $\lambda = \dfrac{c}{f} = \dfrac{u}{f}\cdot\dfrac{1}{\sqrt{\mu_r\varepsilon_r}} = \dfrac{\lambda_0}{\sqrt{\mu_r\varepsilon_r}}$: ② 파장이 짧아진다.

고유 임피던스 : $\eta = \sqrt{\dfrac{\mu}{\varepsilon}}$

- $\boxed{H_y = \dfrac{E_{x0}}{\eta}\cos(\omega t - \beta z)}$ $\left[\begin{array}{l} E_x \perp H_y,\ some\ phase \\ E_x \,\&\, H_y \perp \hat{z} \end{array}\right.$

<Ex.> 순수한 물속에서 300Mhz의 전파

· f = 3×10^{8}Hz, μ_r=1 ε_r =78

$$c = \frac{u}{\sqrt{\mu_r\varepsilon_r}} = 0.34\times10^8[m/s], \text{ 약 } 1/10\text{속도}(3\times10^8)$$

$$\lambda = \frac{u}{f} = 0.113[m], \text{ 약 } 1/10\text{파장}(3m)$$

$$\beta = \frac{2\pi}{\lambda} = 554[rad/m] = 80.6^\circ/in$$

$$E_{x0} = 0.1[V/m]\text{일 경우} \left[\begin{array}{l} E_x = 0.1\cos(6\pi\times10^8 t - 55.4z) \\ H_y = \dfrac{E_x}{\eta} = 2.34\times10^{-3}\cos(6\pi\times10^8 t - 55.4z) \end{array}\right.$$

< Ex 11.3 >

< Ex 11.4 >

- Phase Velocity : $\boxed{v_p = \dfrac{\omega}{\beta}}$ $\boxed{\lambda = \dfrac{2\pi}{\beta}}$ $\beta\lambda = 2\pi$

$$H_{ys} = \frac{E_{x0}}{\eta}e^{-\alpha z}e^{-j\beta z}$$

$$\boxed{\eta = \sqrt{\frac{\mu}{\epsilon' - j\epsilon''}} = \sqrt{\frac{\mu}{\epsilon'}}\frac{1}{\sqrt{1 - j(\epsilon''/\epsilon')}}}$$

- perfect dielectric $\epsilon'' = 0$ $\epsilon = \epsilon'$ $\alpha = 0$,
- Lossless Media : $\boxed{\beta = \omega\sqrt{\mu\epsilon'}}$

$$\boxed{E_x = E_{x0}\cos(\omega t - \beta z)}$$

$$\boxed{v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon'}} = \frac{c}{\sqrt{\mu_r\epsilon'_r}}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mu\epsilon'}} = \frac{1}{f\sqrt{\mu\epsilon'}} = \frac{c}{f\sqrt{\mu_r\epsilon'_r}} = \frac{\lambda_0}{\sqrt{\mu_r\epsilon'_r}}$$

$$\boxed{H_y = \frac{E_{x0}}{\eta}\cos(\omega t - \beta z)}$$

$$\boxed{\eta = \sqrt{\frac{\mu}{\epsilon}}}$$

⦿ 손실이 있는 유전체의 경우

- 도전율 σ, $\mathbf{J}_S = \sigma \mathbf{E}_S$
- $$\begin{cases} \nabla \times \mathbf{H}_s = \mathbf{J}_s + j\omega \mathbf{E}_s = (\sigma + j\omega\varepsilon)\mathbf{E}_s \\ \nabla \times \mathbf{E}_s = -j\omega\mu \mathbf{H}_s \end{cases}$$

$$\gamma^2 = (\sigma + j\omega\varepsilon)\, j\omega\mu$$

$$\boxed{\therefore \gamma = \pm j\omega\sqrt{\mu\varepsilon}\sqrt{1 - j\frac{\sigma}{\omega\varepsilon}} = \alpha + j\beta,}$$

$$\begin{cases} if \quad \sigma = 0 : \gamma = j\omega\sqrt{\mu\varepsilon} : \text{무손실} \\ \sigma \neq 0 : \alpha \neq 0, \text{ 감쇄} \end{cases}$$

- $$\begin{cases} E_{xs} = E_{x0} e^{-\alpha z} e^{-j\beta z} \\ H_{ys} = \dfrac{E_{x0}}{\eta} e^{-\alpha z} e^{j\beta z} \end{cases}$$

$$\boxed{\therefore \eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} \cdot \frac{1}{\sqrt{1 - j\sigma/\omega\varepsilon}}}$$

✓ E와 H는 not in-phase

<Ex.> 증류수 내의 15.9Ghz전파. $\omega = 10^{11}$rad/s, $\mu_r = 1$, $\varepsilon_r = 50$, $\sigma = 20$ ℧/m

$\gamma = 522 + j2402[\text{m}^{-1}]$

(i) $\alpha = 522[N_p/m]$: E_x와 H_y는 1/522m(≈0.2cm) 전파 후 e^{-1}(=0.368)배로 크기가 줆

→ 수중 radar ×, sonar 수중음파기, 대기 수분·강우 영향

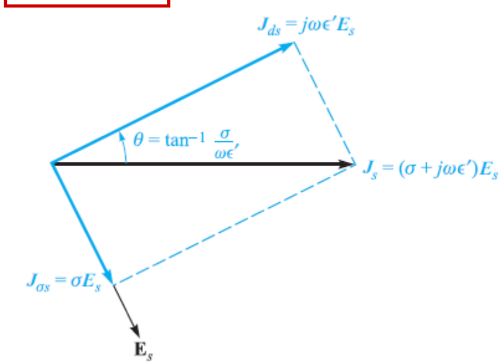
(ii) $\beta = 2420$[rad/m] : $\sigma = 0$ 일 때는 2360. 큰 차이 없음

(iii) $\lambda = 1.88$cm, 물에서는 2.6mm

(iv) $\eta = 49.6+j10.7[\Omega] = 50.8 \angle 12.2°$, E_x는 H_y보다 12.2° 앞서는 위상

⦿ Loss Tangent : $(\frac{\sigma}{\omega\varepsilon})$

- σ로 인한 Loss
- $\nabla \times \mathbf{H}_s = (\sigma + j\omega\varepsilon)\mathbf{E}_s = \mathbf{J}_{rs} + \mathbf{J}_{ds}$
- $\frac{\mathbf{J}_{rs}}{\mathbf{J}_{ds}} = \frac{\sigma}{j\omega\varepsilon}$: 변위 전류에 대한 전도전류의 비율
  서로 90° 위상차를 가짐. 비율은 항상 동일
  변위 전류는 전도 전류보다 90° 앞선다
- $\boxed{\tan\theta = \frac{\sigma}{\omega\varepsilon}}$: Loss Tangent (=1/Q)



- Loss Tangent가 작을 때(σ/ωε < 0.1)의 근사식

$$\gamma = j\omega\sqrt{\mu\varepsilon}\sqrt{1 - j\frac{\sigma}{\omega\varepsilon}} \fallingdotseq \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} + j\omega\sqrt{\mu\varepsilon}$$

$$\eta \fallingdotseq \sqrt{\frac{\mu}{\varepsilon}}\left(1 + j\frac{\sigma}{2\omega\varepsilon}\right)$$

< Ex 11.5 >

$$\nabla \times \mathbf{H}_s = j\omega(\epsilon' - j\epsilon'')\mathbf{E}_s = \omega\epsilon''\mathbf{E}_s + j\omega\epsilon'\mathbf{E}_s$$

$$\nabla \times \mathbf{H}_s = \mathbf{J}_s + j\omega\epsilon\mathbf{E}_s \qquad \mathbf{J}_s = \sigma\mathbf{E}_s$$

$$\boxed{\nabla \times \mathbf{H}_s = (\sigma + j\omega\epsilon')\mathbf{E}_s = \mathbf{J}_{\sigma s} + \mathbf{J}_{ds}}$$

$$\mathbf{J}_{\sigma s} = \sigma\mathbf{E}_s \qquad \mathbf{J}_{ds} = j\omega\epsilon'\mathbf{E}_s$$

$$\boxed{\epsilon'' = \frac{\sigma}{\omega}} \qquad \boxed{\frac{J_{\sigma s}}{J_{ds}} = \frac{\epsilon''}{j\epsilon'} = \frac{\sigma}{j\omega\epsilon'}}$$

- Loss Tangent : $\sigma/\omega\epsilon'$ $\qquad \boxed{\tan\theta = \frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega\epsilon'}}$

good dielectric $\quad \epsilon''/\epsilon' \ll 1,$

- Conductive Media : $\epsilon'' = \sigma/\omega,$

$$jk = j\omega\sqrt{\mu\epsilon'}\sqrt{1 - j\frac{\sigma}{\omega\epsilon'}}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots$$

$$\boxed{\alpha = \mathrm{Re}(jk) \doteq j\omega\sqrt{\mu\epsilon'}\left(-j\frac{\sigma}{2\omega\epsilon'}\right) = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon'}}}$$

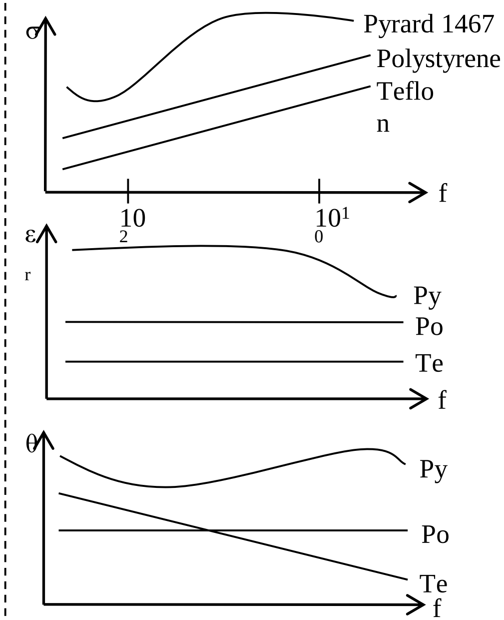
$$\boxed{\beta = \mathrm{Im}(jk) \doteq \omega\sqrt{\mu\epsilon'}\left[1 + \frac{1}{8}\left(\frac{\sigma}{\omega\epsilon'}\right)^2\right]}$$

$$\boxed{\beta \doteq \omega\sqrt{\mu\epsilon'}}$$

$$\eta \doteq \sqrt{\frac{\mu}{\epsilon'}}\left[1 - \frac{3}{8}\left(\frac{\sigma}{\omega\epsilon'}\right)^2 + j\frac{\sigma}{2\omega\epsilon'}\right]$$

$$\boxed{\eta \doteq \sqrt{\frac{\mu}{\epsilon'}}\left(1 + j\frac{\sigma}{2\omega\epsilon'}\right)}$$

- 재료에 따른 σ, ε_r, θ의 주파수에 의한 변화



<Ex.11.3> 순수한 물속에서 1 Mhz In water, $\mu_r = 1$ and at 1 MHz, $\epsilon_r' = 81$ $\epsilon'' \doteq 0$

$$\beta = \omega\sqrt{\mu\epsilon'} = \omega\sqrt{\mu_0\epsilon_0}\sqrt{\epsilon_r'} = \frac{\omega\sqrt{\epsilon_r'}}{c} = \frac{2\pi \times 10^6\sqrt{81}}{3.0 \times 10^8} = 0.19 \text{ rad/m}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{.19} = 33 \text{ m}$$ The wavelength in air would have been 300 m.

$$v_p = \frac{\omega}{\beta} = \frac{2\pi \times 10^6}{.19} = 3.3 \times 10^7 \text{ m/s}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon'}} = \frac{\eta_0}{\sqrt{\epsilon_r'}} = \frac{377}{9} = 42\ \Omega$$

$$E_x = 0.1\cos(2\pi 10^6 t - .19z) \text{ V/m}$$

$$H_y = \frac{E_x}{\eta} = (2.4 \times 10^{-3})\cos(2\pi 10^6 t - .19z) \text{ A/m}$$

<Ex.11.4> 순수한 물속에서 2.5 Ghz *microwave oven.* The permittivity values are $\epsilon_r' = 78$ and $\epsilon_r'' = 7$.

$$\alpha = \frac{(2\pi \times 2.5 \times 10^9)\sqrt{78}}{(3.0 \times 10^8)\sqrt{2}}\left(\sqrt{1 + \left(\frac{7}{78}\right)^2} - 1\right)^{1/2} = 21 \text{ Np/m}$$

$$\beta = 464 \text{ rad/m}$$

$$\lambda = 2\pi/\beta = 1.4 \text{ cm} \qquad \lambda_0 = c/f = 12 \text{ cm}$$

$$\eta = \frac{377}{\sqrt{78}}\frac{1}{\sqrt{1 - j(7/78)}} = 43 + j1.9 = 43\angle 2.6^\circ\ \Omega$$

<Ex.11.5> 손실매질에서 2.5 Ghz loss tangent $\epsilon''/\epsilon' = 7/78 = 0.09$ $\epsilon'' = \sigma/\omega$,

$$\alpha \doteq \frac{\omega\epsilon''}{2}\sqrt{\frac{\mu}{\epsilon'}} = \frac{1}{2}(7 \times 8.85 \times 10^{12})(2\pi \times 2.5 \times 10^9)\frac{377}{\sqrt{78}} = 21 \text{ cm}^{-1}$$

$$\beta \doteq (2\pi \times 2.5 \times 10^9)\sqrt{78}/(3 \times 10^8) = 464 \text{ rad/m}$$

$$\eta \doteq \frac{377}{\sqrt{78}}\left(1 + j\frac{7}{2 \times 78}\right) = 43 + j1.9$$

11.3 Poynting Vector와 전력

- 전자파와 전력
- 자계 에너지

$$\nabla\times\mathbf{H}=\mathbf{J}+\frac{\partial\mathbf{D}}{\partial t}\qquad(\because)\nabla\cdot(\mathbf{E}\times\mathbf{H})=-\mathbf{E}\cdot(\nabla\times\mathbf{H})+\mathbf{H}\cdot(\nabla\times\mathbf{E})$$

$$\mathbf{H}\cdot(\nabla\times\mathbf{E})-\nabla\cdot(\mathbf{E}\times\mathbf{H})=\mathbf{J}\cdot\mathbf{E}+\mathbf{E}\cdot\frac{\partial\mathbf{D}}{\partial t}$$

$$\mathbf{H}\cdot(-\frac{\partial\mathbf{B}}{\partial t})-\nabla\cdot(\mathbf{E}\times\mathbf{H})=\mathbf{J}\cdot\mathbf{E}+\mathbf{E}\cdot\frac{\partial\mathbf{D}}{\partial t}$$

$$-\nabla\cdot(\mathbf{E}\times\mathbf{H})=\mathbf{J}\cdot\mathbf{E}+\varepsilon\mathbf{E}\cdot\frac{\partial\mathbf{E}}{\partial t}+\mu\mathbf{H}\cdot\frac{\partial\mathbf{H}}{\partial t}\qquad(\because)\left[\begin{array}{l}\varepsilon\mathbf{E}\cdot\frac{\partial\mathbf{E}}{\partial t}=\frac{1}{2}\varepsilon\frac{\partial\mathbf{E}^2}{\partial t}=\frac{\partial}{\partial t}(\frac{\varepsilon}{2}\mathbf{E}^2)\\ \mu\mathbf{H}\cdot\frac{\partial\mathbf{H}}{\partial t}=\frac{\partial}{\partial t}(\frac{\mu}{2}\mathbf{H}^2)\end{array}\right.$$

$$-\nabla\cdot(\mathbf{E}\times\mathbf{H})=\mathbf{J}\cdot\mathbf{E}+\frac{\partial}{\partial t}(\frac{\varepsilon}{2}\mathbf{E}^2+\frac{\mu}{2}\mathbf{H}^2)$$

$$\boxed{\therefore\ -\oint_s(\mathbf{E}\times\mathbf{H})\cdot d\mathbf{S}=\int_v\mathbf{J}\cdot\mathbf{E}dv+\frac{\partial}{\partial t}\int_v(\frac{\varepsilon}{2}\mathbf{E}^2+\frac{\mu}{2}\mathbf{H}^2)dv}$$

Ohm 전력손실 　 전계 및 자계로 저축되는 에너지

$\boxed{\mathrm{P}=\mathbf{E}\times\mathbf{H}[watt/m^2]}$ **: Poynting Vector. 순간 전력 밀도**

$(E_x\hat{a}_x\times H_y\hat{a}_y=P_z\hat{a}_z)$ · 전력 흐름의 방향 $\perp$ **E, H**

$$\nabla\times\mathbf{H}=\mathbf{J}+\frac{\partial\mathbf{D}}{\partial t}$$

$$\mathbf{E}\cdot\nabla\times\mathbf{H}=\mathbf{E}\cdot\mathbf{J}+\mathbf{E}\cdot\frac{\partial\mathbf{D}}{\partial t}$$

$$\nabla\cdot(\mathbf{E}\times\mathbf{H})=-\mathbf{E}\cdot\nabla\times\mathbf{H}+\mathbf{H}\cdot\nabla\times\mathbf{E}$$

$$\mathbf{H}\cdot\nabla\times\mathbf{E}-\nabla\cdot(\mathbf{E}\times\mathbf{H})=\mathbf{J}\cdot\mathbf{E}+\mathbf{E}\cdot\frac{\partial\mathbf{D}}{\partial t}$$

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}$$

$$-\mathbf{H}\cdot\frac{\partial\mathbf{B}}{\partial t}-\nabla\cdot(\mathbf{E}\times\mathbf{H})=\mathbf{J}\cdot\mathbf{E}+\mathbf{E}\cdot\frac{\partial\mathbf{D}}{\partial t}$$

$$-\nabla\cdot(\mathbf{E}\times\mathbf{H})=\mathbf{J}\cdot\mathbf{E}+\epsilon\mathbf{E}\cdot\frac{\partial\mathbf{E}}{\partial t}+\mu\mathbf{H}\cdot\frac{\partial\mathbf{H}}{\partial t}$$

$$\epsilon\mathbf{E}\cdot\frac{\partial\mathbf{E}}{\partial t}=\frac{\partial}{\partial t}\left(\frac{1}{2}\mathbf{D}\cdot\mathbf{E}\right)\qquad\mu\mathbf{H}\cdot\frac{\partial\mathbf{H}}{\partial t}=\frac{\partial}{\partial t}\left(\frac{1}{2}\mathbf{B}\cdot\mathbf{H}\right)$$

$$\boxed{-\nabla\cdot(\mathbf{E}\times\mathbf{H})=\mathbf{J}\cdot\mathbf{E}+\frac{\partial}{\partial t}\left(\frac{1}{2}\mathbf{D}\cdot\mathbf{E}\right)+\frac{\partial}{\partial t}\left(\frac{1}{2}\mathbf{B}\cdot\mathbf{H}\right)}$$

$$-\int_{\text{vol}}\nabla\cdot(\mathbf{E}\times\mathbf{H})\,dv=\int_{\text{vol}}\mathbf{J}\cdot\mathbf{E}\,dv+\int_{\text{vol}}\frac{\partial}{\partial t}\left(\frac{1}{2}\mathbf{D}\cdot\mathbf{E}\right)dv+\int_{\text{vol}}\frac{\partial}{\partial t}\left(\frac{1}{2}\mathbf{B}\cdot\mathbf{H}\right)dv$$

$$\boxed{-\oint_{\text{area}}(\mathbf{E}\times\mathbf{H})\cdot d\mathbf{S}=\int_{\text{vol}}\mathbf{J}\cdot\mathbf{E}\,dv+\frac{d}{dt}\int_{\text{vol}}\frac{1}{2}\mathbf{D}\cdot\mathbf{E}\,dv+\frac{d}{dt}\int_{\text{vol}}\frac{1}{2}\mathbf{B}\cdot\mathbf{H}\,dv}$$

total power flowing *out* of the volume $\boxed{\oint_{\text{area}}(\mathbf{E}\times\mathbf{H})\cdot d\mathbf{S}\quad\text{W}}$

Poynting vector, S $\boxed{\mathbf{S}=\mathbf{E}\times\mathbf{H}\quad\text{W/m}^2}$

- $\boxed{\mathrm{P} = \mathbf{E}\times\mathbf{H}[watt/m^2]}$

완전 유전체(무손실) 에서

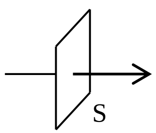
$$\begin{bmatrix} E_x = E_{x0}\cos(\omega t-\beta z) \\ H_y = \dfrac{E_{x0}}{\eta}\cos(\omega t-\beta z) \end{bmatrix} \qquad \mathrm{P}_z = \frac{E_{x0}^2}{\eta}\cos^2(\omega t-\beta z)$$

$$\mathrm{P}_{z,av} = f\cdot\int_0^{1/f}\frac{E_{x0}^2}{\eta}\cos^2(\omega t-\beta z)dt$$

$$= \frac{f}{2}\cdot\frac{E_{x0}^2}{\eta}\int_0^{1/f}[1+\cos(2\omega t-2\beta z)]dt$$

$$= \frac{f}{2}\cdot\frac{E_{x0}^2}{\eta}\left[t+\frac{1}{2\omega}\sin(2\omega t-2\beta z)\right]_0^{1/f}$$

$$\boxed{\therefore\ \mathrm{P}_{z,av} = \frac{1}{2}\frac{E_{x0}^2}{\eta}}$$



$$\mathrm{P}_{z,av} = \frac{1}{2}\frac{E_{x0}^2}{\eta}\cdot S$$

$$\mathrm{P}_{z,av} = \frac{E_{x0}^2}{\eta}$$: E_{x0}의 최대치는 실효치를 사용할 경우 :

✓ 손실이 있는 유전체의 경우

$$\mathrm{P}_{z,av} = \frac{1}{2}\frac{E_{x0}^2}{\eta}e^{-2\alpha z}\cos\theta_m$$

· E_x와 H_y는 θ_m 만큼의 위상차

· $\eta = \eta_m\angle\theta_m$, polar form

$$\boxed{\mathbf{S} = \mathbf{E}\times\mathbf{H} \quad \mathrm{W/m^2}}$$

$$E_x\mathbf{a}_x \times H_y\mathbf{a}_y = S_z\mathbf{a}_z$$

$$E_x = E_{x0}\cos(\omega t-\beta z)$$

$$H_y = \frac{E_{x0}}{\eta}\cos(\omega t-\beta z)$$

$$S_z = \frac{E_{x0}^2}{\eta}\cos^2(\omega t-\beta z)$$

$$E_x = E_{x0}e^{-\alpha z}\cos(\omega t-\beta z) \qquad \eta = |\eta|\angle\theta_\eta$$

$$H_y = \frac{E_{x0}}{|\eta|}e^{-\alpha z}\cos(\omega t-\beta z-\theta_\eta)$$

$$S_z = E_xH_y = \frac{E_{x0}^2}{|\eta|}e^{-2\alpha z}\cos(\omega t-\beta z)\cos(\omega t-\beta z-\theta_\eta)$$

$$\langle S_z\rangle = \frac{1}{T}\int_0^T\frac{1}{2}\frac{E_{x0}^2}{|\eta|}e^{-2\alpha z}[\cos(2\omega t-2\beta z-2\theta_\eta)+\cos\theta_\eta]\,dt$$

$$\boxed{\langle S_z\rangle = \frac{1}{2}\frac{E_{x0}^2}{|\eta|}e^{-2\alpha z}\cos\theta_\eta}$$

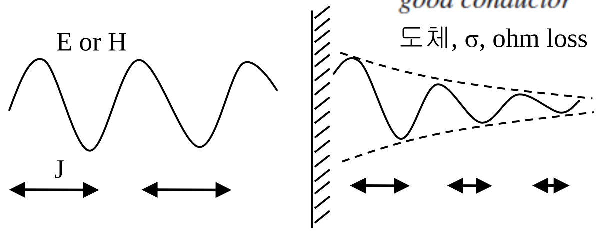
$$\boxed{\langle\mathbf{S}\rangle = \frac{1}{2}\mathrm{Re}(\mathbf{E}_s\times\mathbf{H}_s^*) \quad \mathrm{W/m^2}}$$

$$\mathbf{E}_s = E_{x0}e^{-j\beta z}\mathbf{a}_x$$

$$\mathbf{H}_s^* = \frac{E_{x0}}{\eta^*}e^{+j\beta z}\mathbf{a}_y = \frac{E_{x0}}{|\eta|}e^{j\theta}e^{+j\beta z}\mathbf{a}_y$$

$$\boxed{\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}}$$

11.4 양 도체 내의 전파 전파 : 표피효과

- 

good conductor

도체, σ, ohm loss

loss tangent $\frac{\sigma}{\omega\varepsilon} \gg 1$

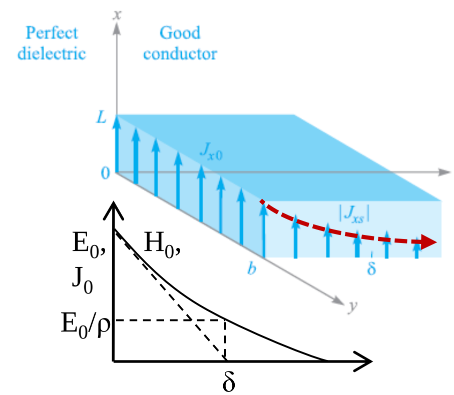
(ex.) 니크롬선. $\sigma = 10^6$, f = 100Mhz → 2×10^8

- 전파 정수 $\gamma = j\omega\sqrt{\mu\varepsilon}\sqrt{1-j\frac{\sigma}{\omega\varepsilon}} \approx j\omega\sqrt{\mu\varepsilon}\sqrt{1-j\frac{\sigma}{\omega\varepsilon}} \doteqdot (1+j)\sqrt{\pi f \mu\sigma}$

$\gamma = \alpha + j\beta$

$$\boxed{\alpha = \beta = \sqrt{\pi f \mu \sigma}}$$

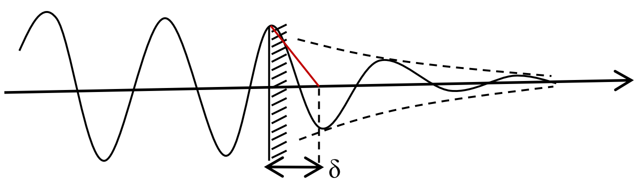
양 도체에서 μ, σ, f 에 관계없이 α = β



⦿ Skin Depth (표피효과, Depth of Penetration)

$$E_x = E_{x0}e^{-z\sqrt{\pi f \mu\sigma}}\cos\left(\omega t - z\sqrt{\pi f \mu\sigma}\right)$$

$$J_x = \sigma E_x = \sigma E_{x0}e^{-z\sqrt{\pi f \mu\sigma}}\cos\left(\omega t - z\sqrt{\pi f \mu\sigma}\right)$$



$z = \frac{1}{\sqrt{\pi f \mu\sigma}}$ 일 때 크기는 $e^{-1} = 0.368$ 배로 감소

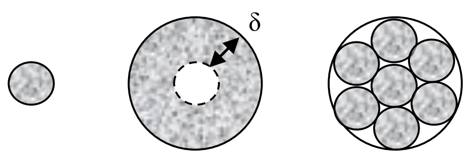
$$\boxed{\delta = \frac{1}{\sqrt{\pi f \mu\sigma}} = \frac{1}{\alpha} = \frac{1}{\beta}}$$

skin depth,:

: 1/α = 1/β = δ

E또는 H의 크기가 36.8%로 줄어드는 깊이

전력은 $e^{-2\alpha z}$로 감소



도체 굵기, Wave Guide, 전기 차폐

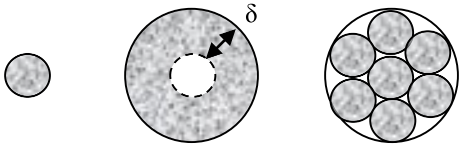
* 주파수가 커질수록 δ↓. 표면.

* 전자파와 에너지는 도체 내부로 전송되지 않는다.

$$\boxed{\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}}$$

(Ex.) Cu : $\mu_r = 1$, $\sigma = 5.0\times 10^7$ $\delta_{Cu} = \dfrac{0.066}{\sqrt{f}} \; (f \propto \dfrac{1}{\delta})$

도체 굵기, Wave Guide, 전기 차폐



- 60Hz : δ = 8.53mm, 일반 전력용은 5mm이하로!
- 1MHz, 방송파 : δ = 0.0661mm, 전류는 표면에만 존재, 中空도체 사용
- 10GHz, 마이크로파 : δ = 0.661um, 전류는 표면에만 존재, 유리에 은을 흡착시켜 사용, 우주선, 우주복

(Ex.) Sea Water, $\varepsilon_r = 80$, $\sigma = 4$, 표면에서 1V/m의 E가 1μV/m되는 깊이는 ? (1/100만)

- 1kHz : x = 13.8/α =13.8/0.13 = 106m
- 10kHz : x = 35m
- 100kHz : x = 11m
- 1MkHz : x = 3.5m

*σ » ωε이므로

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} = 0.13\,N_p/m$$

$$E/E_0 = 10^{-6} = e^{-\alpha x}$$

$$\therefore x = (6/\alpha)\log e = 13.8/\alpha$$

⦿ 양도체 내의 전파속도 및 파장

- · 파장 : λ = 2π/β = 2πδ
- · 전파속도 : u = fλ = f·2πδ = ωδ

$$\alpha = \beta = \frac{1}{\delta} = \sqrt{\pi f \mu \sigma}$$

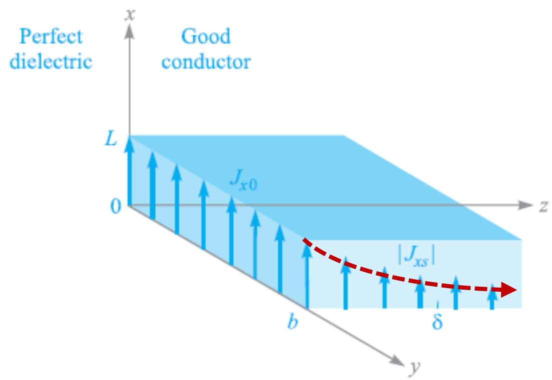
$$\beta = \frac{2\pi}{\lambda} \qquad \lambda = 2\pi\delta \qquad v_p = \frac{\omega}{\beta}$$

$$\boxed{v_p = \omega\delta}$$

(Ex.) For copper at 60 Hz, $\lambda = 5.36$ cm and $v_p = 3.22$ m/s,

In free space, of course, a 60 Hz λ = 3110mile, u = C = 3×10^8m/s

- Power Loss :



$$I = \int_0^\infty \int_0^b J_x \, dy \, dz \qquad J_x = J_{x0} e^{-z/\delta} \cos\left(\omega t - \frac{z}{\delta}\right)$$

$$J_{xs} = J_{x0} e^{-z/\delta} e^{-jz/\delta} = J_{x0} e^{-(1+j)z/\delta}$$

$$I_s = \int_0^\infty \int_0^b J_{x0} e^{-(1+j)z/\delta} dy \, dz = J_{x0} b e^{-(1+j)z/\delta} \left.\frac{-\delta}{1+j}\right|_0^\infty = \frac{J_{x0} b\delta}{1+j}$$

$$I = \frac{J_{x0} b\delta}{\sqrt{2}} \cos\left(\omega t - \frac{\pi}{4}\right) \qquad J' = \frac{J_{x0}}{\sqrt{2}} \cos\left(\omega t - \frac{\pi}{4}\right)$$

$$P_{Li}(t) = \frac{1}{\sigma}(J')^2 bL\delta = \frac{J_{x0}^2}{2\sigma} bL\delta \cos^2\left(\omega t - \frac{\pi}{4}\right)$$

$$\boxed{P_L = \frac{1}{4\sigma} J_{x0}^2 bL\delta}$$

good conductor , H_y, associated with E_x $\quad \epsilon'' = \sigma/\omega$,

$$\boxed{\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon'}}} \qquad \sigma \gg \omega\epsilon', \quad \eta = \sqrt{\frac{j\omega\mu}{\sigma}} \qquad \boxed{\eta = \frac{\sqrt{2}\angle 45^\circ}{\sigma\delta} = \frac{(1+j)}{\sigma\delta}}$$

$$E_x = E_{x0} e^{-z/\delta} \cos\left(\omega t - \frac{z}{\delta}\right) \qquad H_y = \frac{\sigma\delta E_{x0}}{\sqrt{2}} e^{-z/\delta} \cos\left(\omega t - \frac{z}{\delta} - \frac{\pi}{4}\right)$$

$$\langle S_z \rangle = \frac{1}{2} \frac{\sigma\delta E_{x0}^2}{\sqrt{2}} e^{-2z/\delta} \cos\left(\frac{\pi}{4}\right) \qquad \boxed{\langle S_z \rangle = \frac{1}{4}\sigma\delta E_{x0}^2 e^{-2z/\delta}}$$

$$P_L = \int_{\text{area}} \langle S_z \rangle da = \int_0^b \int_0^L \left.\frac{1}{4}\sigma\delta E_{x0}^2 e^{-2z/\delta}\right|_{z=0} dx \, dy = \frac{1}{4}\sigma\delta bLE_{x0}^2 \qquad J_{x0} = \sigma E_{x0}$$

$$\boxed{P_L = \frac{1}{4\sigma}\delta bLJ_{x0}^2}$$

A round copper wire of 1 mm radius and 1 km length

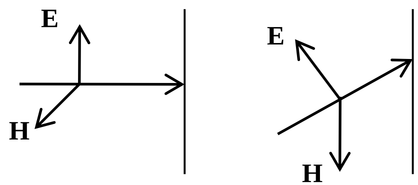
$$\boxed{R = \frac{L}{\sigma S} = \frac{L}{2\pi a\sigma\delta}} \qquad R_{dc} = \frac{10^3}{\pi 10^{-6}(5.8 \times 10^7)} = 5.48\ \Omega$$

1 MHz $\quad R = \dfrac{10^3}{2\pi 10^{-3}(5.8 \times 10^7)(0.066 \times 10^{-3})} = 41.5\ \Omega$

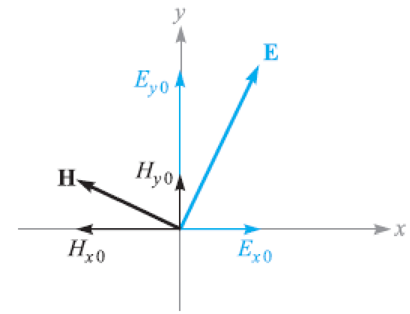
Lab. of Applied ElectroMagnetics

11.5 전자파의 편파 (Wave polarization)

- 임의 방향으로 입사되는 전파의 전파



$$\begin{cases} \mathbf{E}_s = (E_{x0}\mathbf{a}_x + E_{y0}\mathbf{a}_y)e^{-\alpha z}e^{-j\beta z} \\ \mathbf{H}_s = [H_{x0}\mathbf{a}_x + H_{y0}\mathbf{a}_y]\, e^{-\alpha z}e^{-j\beta z} \end{cases}$$



$$\langle \mathbf{S}_z \rangle = \frac{1}{2}\mathrm{Re}\{\mathbf{E}_s \times \mathbf{H}_s^*\} = \frac{1}{2}\mathrm{Re}\{E_{x0}H_{y0}^*(\mathbf{a}_x \times \mathbf{a}_y) + E_{y0}H_{x0}^*(\mathbf{a}_y \times \mathbf{a}_x)\}e^{-2\alpha z}$$

$$= \frac{1}{2}\mathrm{Re}\left\{\frac{E_{x0}E_{x0}^*}{\eta^*} + \frac{E_{y0}E_{y0}^*}{\eta^*}\right\} e^{-2\alpha z}\mathbf{a}_z$$

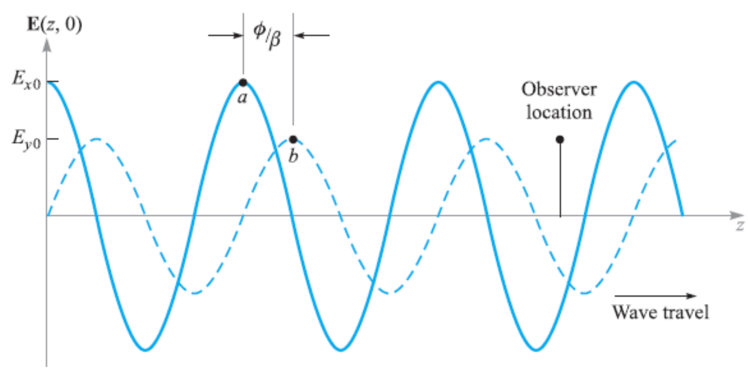
$$= \frac{1}{2}\mathrm{Re}\left\{\frac{1}{\eta^*}\right\}(|E_{x0}|^2 + |E_{y0}|^2)e^{-2\alpha z}\mathbf{a}_z \text{ W/m}^2$$

✓ x 및 y방향으로 평판되는 두 개의 균일 평면파의 합성으로 볼 수 있다.

- 위상차 $\phi(<\pi/2)$. E_{x0}/E_{y0}

· 비손실 매질에서

$$\begin{cases} \mathbf{E}_s = (E_{x0}\mathbf{a}_x + E_{y0}e^{j\phi}\mathbf{a}_y)e^{-j\beta z} \quad \text{(in phasor form)} \\ \mathbf{E}(z,t) = E_{x0}\cos(\omega t - \beta z)\mathbf{a}_x + E_{y0}\cos(\omega t - \beta z + \phi)\mathbf{a}_y \end{cases}$$



* t = 0일 경우 $\mathbf{E}(z,0) = E_{x0}\cos(\beta z)\mathbf{a}_x + E_{y0}\cos(\beta z - \phi)\mathbf{a}_y$

* $t \neq 0$일 경우 $z = z_1$ 지점에서 **E** vector의 종점 연결

- 타원편파 (elliptical polarization) : 일반적
- 원편파 (circular polarization) $\begin{cases} E_{x0} = E_{y0} = E_0 \\ \phi = \pm\pi/2 \end{cases}$

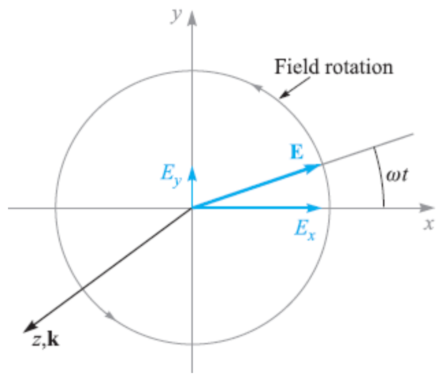
- Another special case of elliptical polarization : $E_{x0} = E_{y0} = E_0$, $\varphi = \pm\pi/2$. → Circular Polarization.

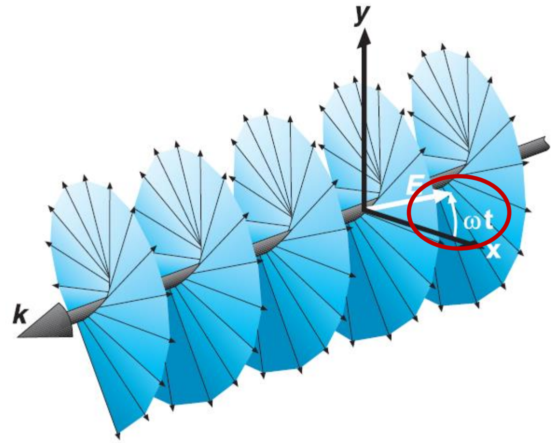
- 원편파에서 $\mathbf{E}(z,t) = E_0[\cos(\omega t - \beta z)\mathbf{a}_x + \cos(\omega t - \beta z \pm \pi/2)\mathbf{a}_y]$

$$= E_0[\cos(\omega t - \beta z)\mathbf{a}_x \mp \sin(\omega t - \beta z)\mathbf{a}_y]$$

$\phi = \frac{\pi}{2}$ 이면 $\boxed{\mathbf{E}(0,t) = E_0[\cos(\omega t)\mathbf{a}_x - \sin(\omega t)\mathbf{a}_y]}$ 좌회전 원편파. *left circular polarization* (l.c.p.)

$\phi = -\frac{\pi}{2}$ 이면 $\boxed{\mathbf{E}(0,t) = E_0[\cos(\omega t)\mathbf{a}_x + \sin(\omega t)\mathbf{a}_y]}$ 우회전 원편파. *right circular polarization* (r.c.p.)





- 원편파의 phasor form :

$$\mathbf{E}(z,t) = \mathrm{Re}\left\{E_0 e^{j\omega t} e^{-j\beta z}\left[\mathbf{a}_x + e^{\pm j\pi/2}\mathbf{a}_y\right]\right\} \qquad e^{\pm j\pi/2} = \pm j \text{ 이므로}$$

$\boxed{\mathbf{E}_s = E_0(\mathbf{a}_x \pm j\mathbf{a}_y)e^{-j\beta z}}$ + : 좌회전 원편파, − : 우회전 원편파

* z → −z :

$\boxed{\mathbf{E}_s = E_0(\mathbf{a}_x \pm j\mathbf{a}_y)e^{+j\beta z}}$ + : 우회전 원편파, − : 좌회전 원편파

연습 : 3, 5, 7, 11, 15, 21, 22, 23, 24, 26, 30