Chapter 0. From the Ground Up

Han-You Jeong

Networked Smart Systems Laboratory

Department of Electrical Engineering

Pusan National University

Contents

- Signals and Systems
- Signal Processing Applications
- Analog or Discrete
- Complex or Real

Definition of Signals and Systems

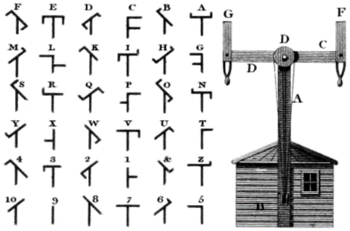
- **Signal**
  - A function of time, e.g., the voltages or currents encountered in circuits.

Smoke Signal

Voice Signal

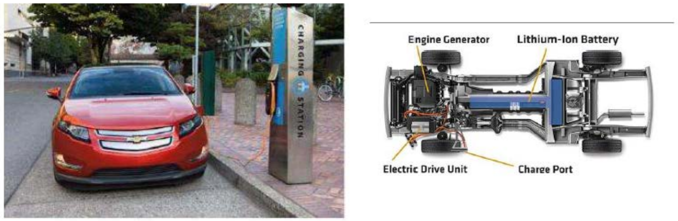
Image Signal

- **System**
  - Any device described by a mathematical model, e.g., the differential equations of an RLC circuit.



Optical Telegraphy

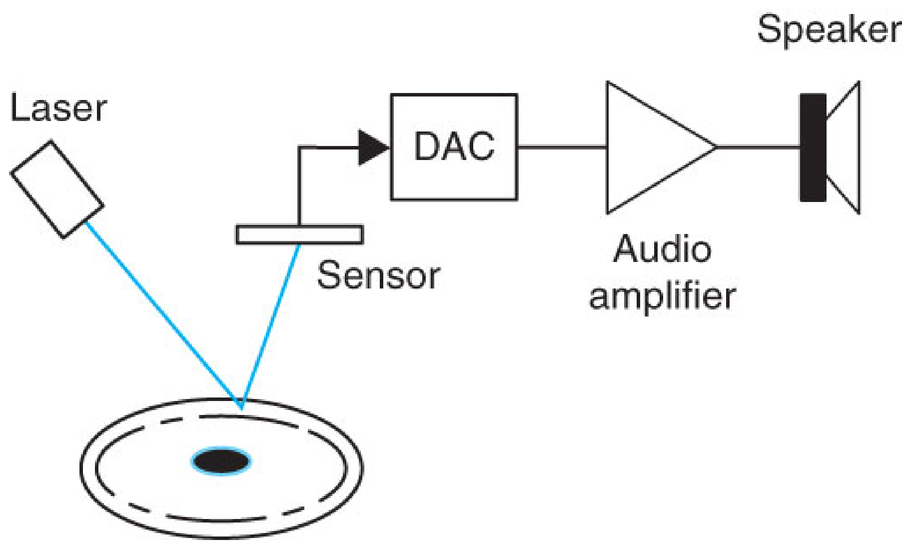
Smartphone



Plug-in Hybrid Vehicle

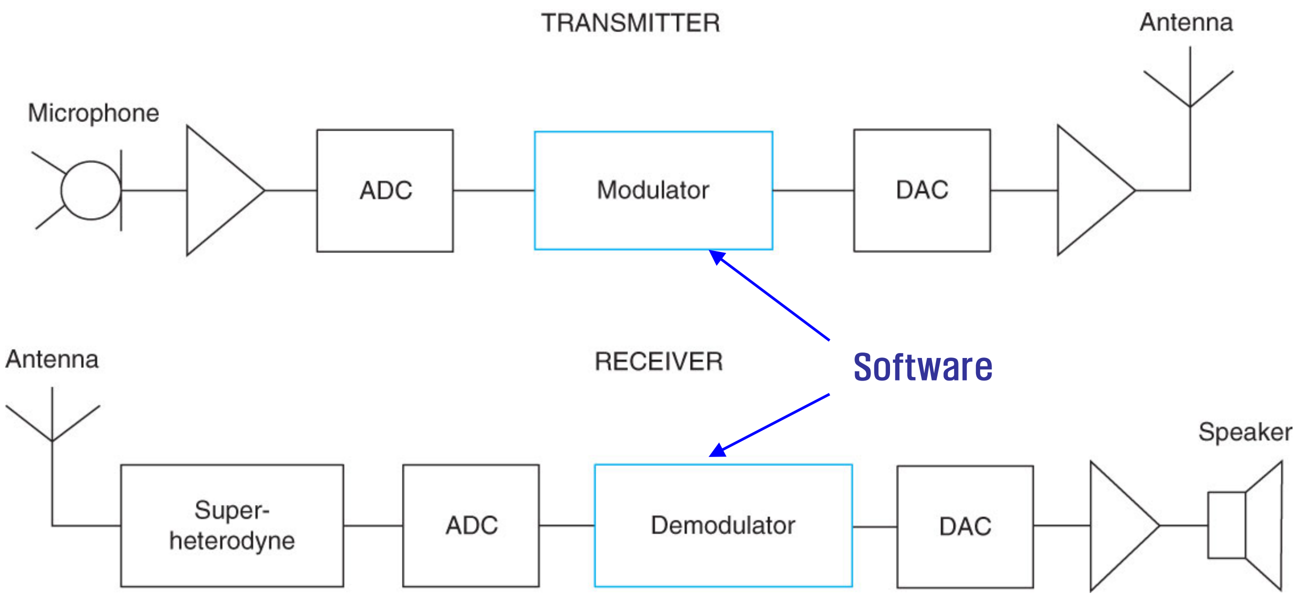
CD Player

- CD Player
  - Pits and bumps on CD correspond to ones and zeros, respectively.
  - Control issues
    - **Rotate the disc at different speeds** depending on the location of track
    - **Focus a laser and a lens system** to read the pits and bumps on the disc.
    - **Move the laser** to follow the track being read.



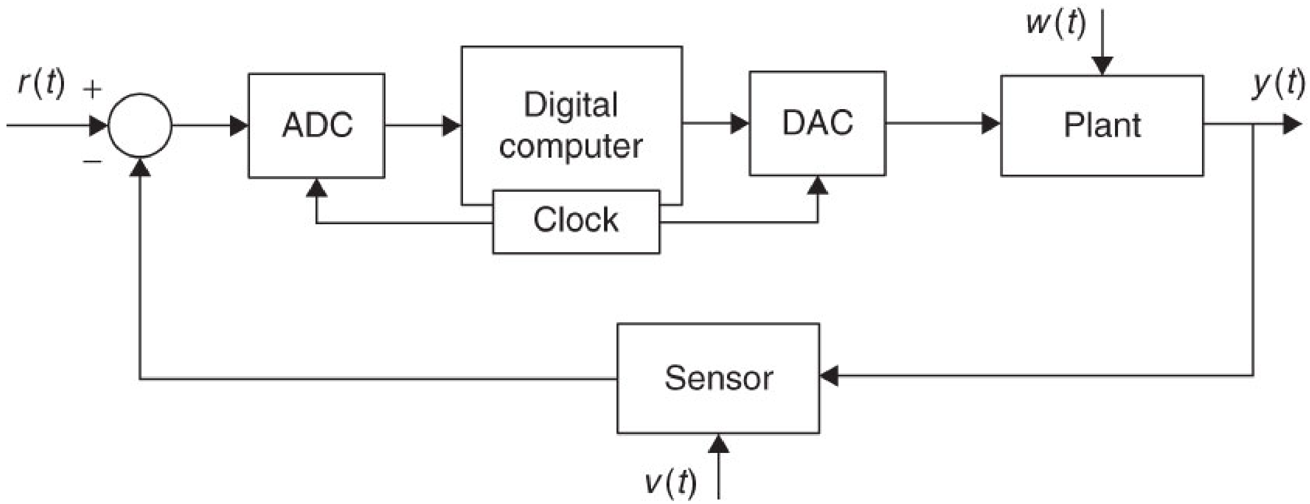
SDR and CR

- **Software Defined Radio (SDR)** and **Cognitive Radio (CR)**
  - In SDR, some of the radio functions typically implemented in hardware are **reconfigured by changing the software**.
  - CR improves the RF spectrum efficiency by **reusing a spectrum when the primary users are not active**.



Computer-Controlled Systems

- **Computer-Controlled Systems**
  - Control systems are **feedback systems** where the response of a system is changed to make it follow a desirable behavior.
  - **The sensor acts as a transducer** whenever the output of the plant is of a different type than the reference.



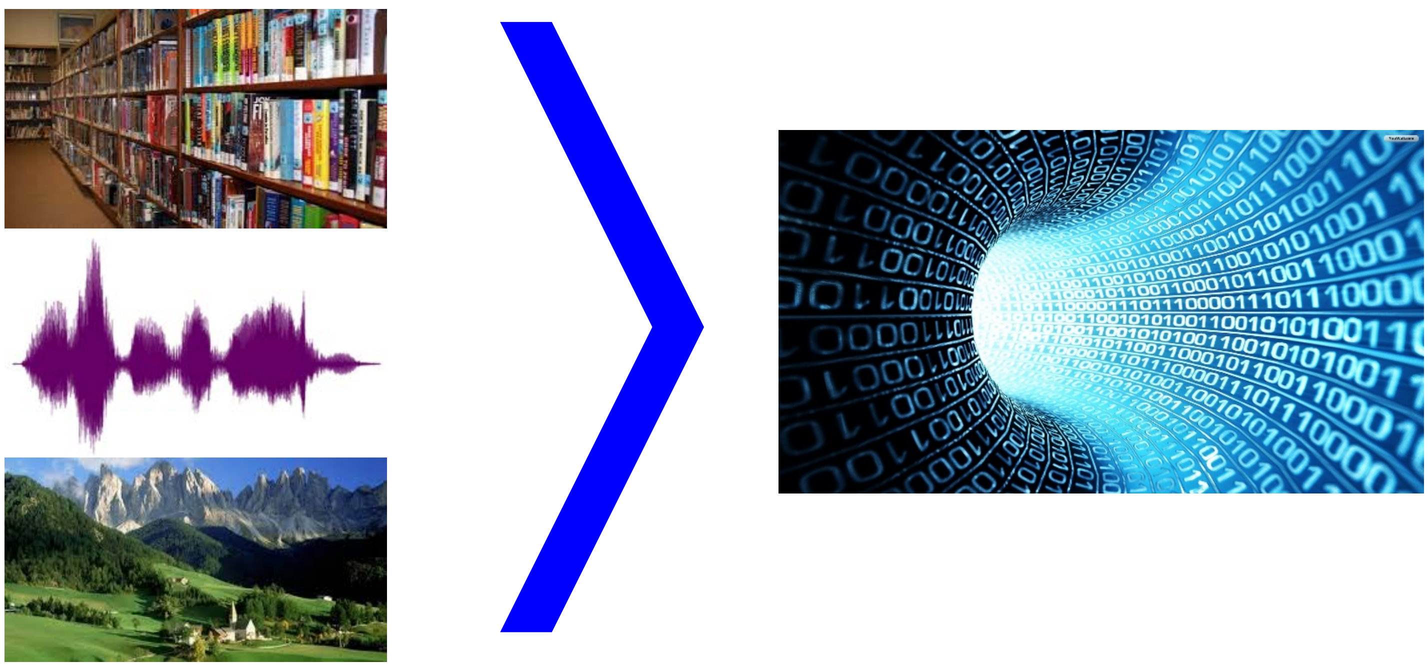
Analog or Discrete

- **Analog** or **continuous-time signals** $x(t)$
  - A function of one or more continuously changing variables.
  - Dealt by **(infinitesimal) calculus**, e.g., differentiation/integration.

- **Discrete-time signals** $x[n]$
  - A sequence of measurements typically made at **uniform times**.
  - Dealt by **finite calculus** where differentiation and integration can be done only approximately by difference and summation, respectively.

Digital Revolution

- **Digital Revolution**
  - Transforms all kinds of information, e.g. texts, voice, image, video, etc., into a sequence of binary digits, i.e. 0 and 1.



Continuous and Discrete Representations

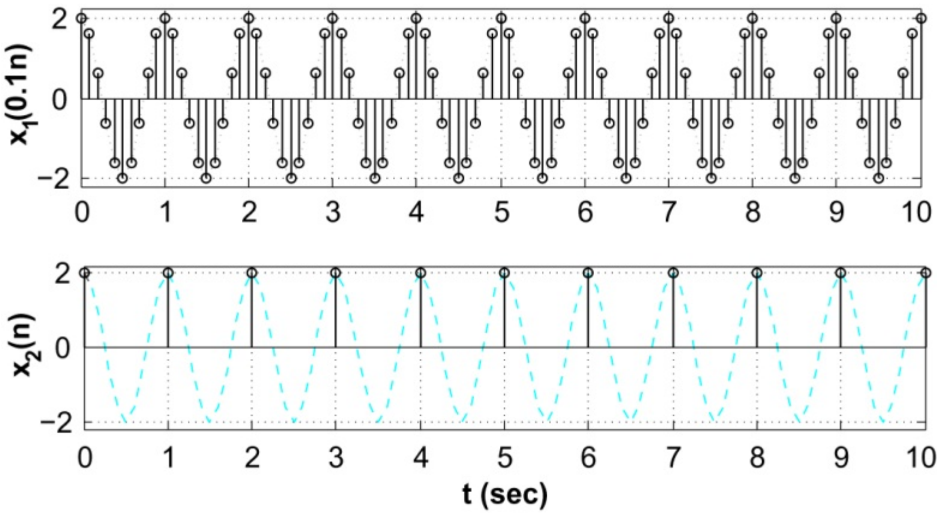
- **Sampling** or **discretization** of an analog signal (Ch. 7)

$$x[n] = x(nT_s) = x(t)_{|t=nT_s}$$

or

$$\{\cdots x(-T_s)\ \ x(0)\ \ x(T_s)\ \ x(2T_s)\cdots\}$$

$$\{\cdots x[-1]\ \ x[0]\ \ x[1]\ \ x[2]\cdots\}$$



Examples of sampling

- Inherent discrete-time signals

Example of discrete-time signal : Stock market

Calculus vs Finite Calculus

- Derivatives vs. Finite Differences
  - **Derivative operator** measures rate of change of an analog signal

$$D[x(t)] = \frac{dx(t)}{dt} = \lim_{h \to 0} \frac{x(t+h) - x(t)}{h}$$

  - **Forward finite-difference operator** measures the change in the signal from one sample to the next

$$\Delta[x(nT_s)] = x((n+1)T_s) - x(nT_s) \quad \Rightarrow \quad \frac{dx(t)}{dt}\Big|_{t=nT_s} = \lim_{T_s \to 0} \frac{\Delta[x(nT_s)]}{T_s}$$

- Integrals vs. Summations
  - Integration is the opposite of differentiation

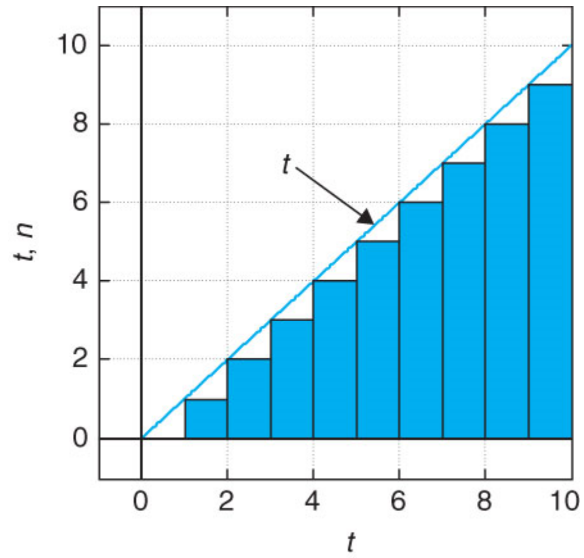
$$I(t) = \int_{t_0}^{t} x(\tau)\, d\tau \quad \Leftrightarrow \quad \frac{dI(t)}{dt} = \lim_{h \to 0} \frac{I(t) - I(t-h)}{h} = \lim_{h \to 0} \frac{1}{h} \int_{t-h}^{t} x(\tau)\, d\tau$$

$$\approx \lim_{h \to 0} \frac{x(t) + x(t-h)}{2} = x(t) \qquad D[D^{-1}[x(t)]] = x(t).$$

Approximation of Integrals by Sums

- The area of a triangle of base of 10 and height of 10

$$\int_0^{10} t \; dt = \frac{t^2}{2} \Big|_{t=0}^{10} = 50.$$



- Approximation of the area
  - By pulses of width 1 and height nT_s

$$\sum_{n=0}^{9} p[n] = \sum_{n=0}^{9} n = 0 + 1 + 2 + \cdots 9 = 0.5\left[\sum_{n=0}^{9} n + \sum_{k=9}^{0} k\right]$$

$$= 0.5\left[\sum_{n=0}^{9} n + \sum_{n=0}^{9} (9-n)\right] = \frac{9}{2}\sum_{n=0}^{9} 1 = \frac{10 \times 9}{2} = 45$$

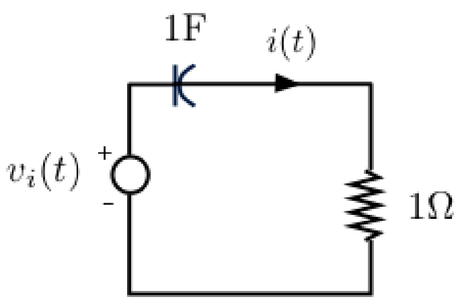
Differential and Difference Equations

- A **differential equation** characterizes the way the system responds to inputs over time.
  - Most systems are characterized by **nonlinear, time-dependent coefficient differential equations**.
  - Solutions are obtained by an **analog computer** consisting of op-amps, resistors, capacitors, voltage sources, and relays.
    - The drawback is the **storage of the solution** which is difficult to record.
  - Laplace transform (Ch. 3)

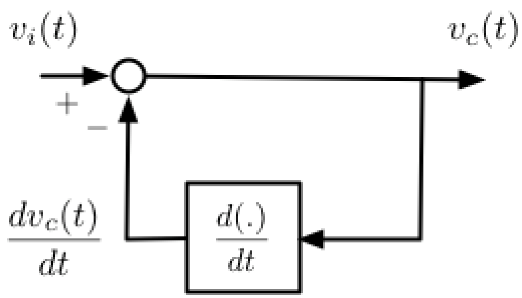
- A digital computer can easily solve the **difference equation** which **approximates a differential equation** using the trapezoidal rule.
  - Z-transform (Ch. 9)

Example - RC Circuit

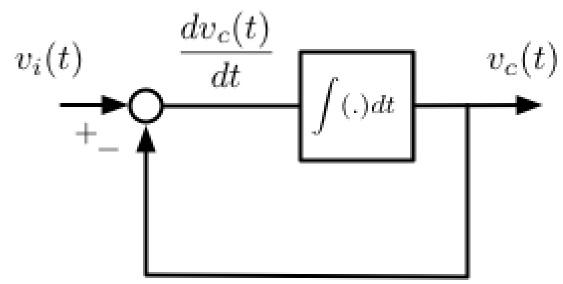
- RC circuit
  - A first-order linear with constant-coefficient differential equation
  - **Differential equation** using **differentiator/integrator**



RC Circuit



Differentiator



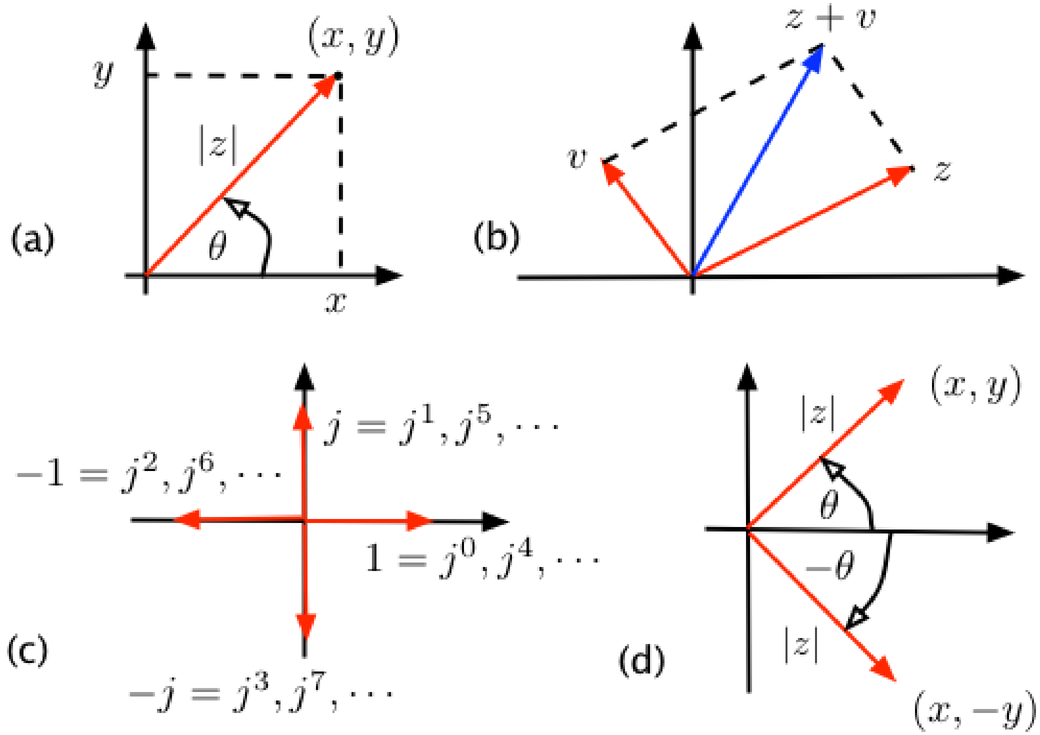
Integrator

Differentiator $$v_i(t) = v_c(t) + \frac{dv_c(t)}{dt}$$

Integrator $$v_c(t) = \int_0^t [v_i(\tau) - v_c(\tau)]d\tau + v_c(0) \qquad t \geq 0$$

Complex or Real

- Time-dependent signals are characterized by means of **frequency** and **damping**.
  - $s = \sigma + j\Omega$: analog signals in the Laplace transform
  - $z = re^{j\omega}$: discrete-time signals in the Z-transform



Euler’s Identity & Sinusoidal Function

- **Euler’s Identity**

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

- **Sinusoidal Function**

$$\cos(\theta) = \mathcal{R}e[e^{j\theta}] = \frac{e^{j\theta} + e^{-j\theta}}{2} \qquad \sin(\theta) = \mathcal{I}m[e^{j\theta}] = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$\cos^2(\theta) = \left[\frac{e^{j\theta} + e^{-j\theta}}{2}\right]^2 = \frac{1}{4}[2 + e^{j2\theta} + e^{-j2\theta}] = \frac{1}{2} + \frac{1}{2}\cos(2\theta)$$

$$\sin^2(\theta) = 1 - \cos^2(\theta) = \frac{1}{2} - \frac{1}{2}\cos(2\theta)$$

$$\sin(\theta)\cos(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}\,\frac{e^{j\theta} + e^{-j\theta}}{2} = \frac{e^{j2\theta} - e^{-j2\theta}}{4j} = \frac{1}{2}\sin(2\theta)$$

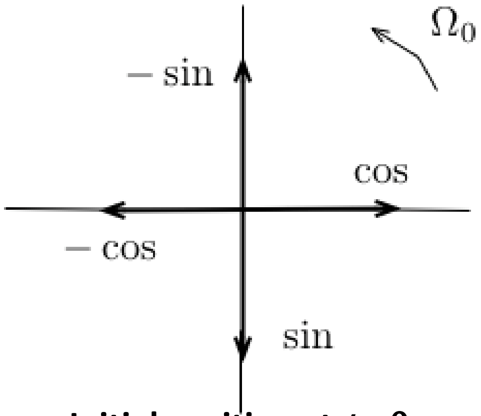
Phasors and Sinusoidal Steady-State

- **Sinusoid** $x(t) = A\cos(\Omega_0 t + \psi) \qquad -\infty < t < \infty$
  - A: amplitude, $\Omega_0 = 2\pi f_0$: frequency (rad/sec), ψ : phase (rad)

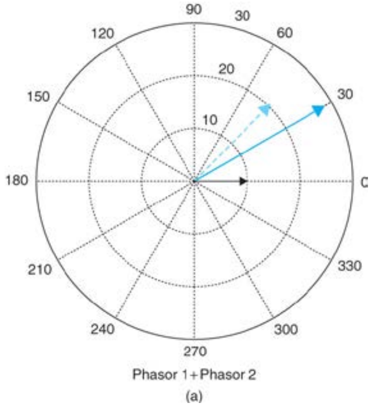
- The **phasor** is a **complex number** characterized by the **amplitude** and the **phase** of cosine signal of a certain frequency Ω_0

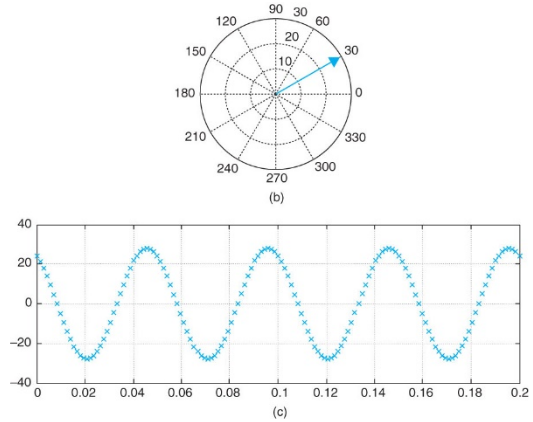
$$V = Ae^{j\psi} = A\cos(\psi) + jA\sin(\psi) = A\angle\psi$$

$$v(t) = \mathcal{R}e[Ve^{j\Omega_0 t}] = \mathcal{R}e[Ae^{j(\Omega_0 t+\psi)}] = A\cos(\Omega_0 t + \psi)$$

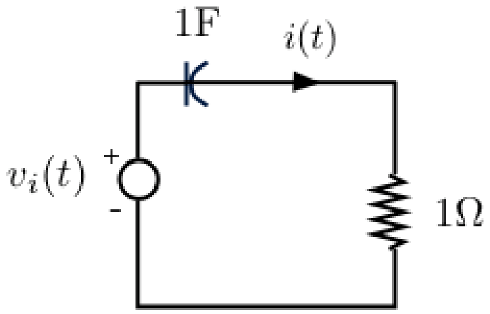


Initial position at t=0





Example – RC Circuit



$$v_c(t) = \mathcal{R}e\left[V_c e^{j\Omega_0 t}\right] \qquad V_c = Ce^{j\psi}$$

$$\frac{dv_c(t)}{dt} = \frac{d\mathcal{R}e[V_c e^{j\Omega_0 t}]}{dt} = \mathcal{R}e\left[V_c \frac{de^{j\Omega_0 t}}{dt}\right] = \mathcal{R}e\left[j\Omega_0 V_c e^{j\Omega_0 t}\right]$$

$$v_i(t) = \mathcal{R}e\left[V_i e^{j\Omega_0 t}\right] \text{ where } V_i = Ae^{j0}$$

Differential Equations:

$$v_i(t) = \frac{dv_c(t)}{dt} + v_c(t)$$

$$\mathcal{R}e\left[V_c(1 + j\Omega_0)e^{j\Omega_0 t}\right] = \mathcal{R}e\left[Ae^{j\Omega_0 t}\right]$$

$$V_c = \frac{A}{1 + j\Omega_0} = \frac{A}{\sqrt{1 + \Omega_0^2}} e^{-j\tan^{-1}(\Omega_0)} = Ce^{j\psi}$$

Sinusoidal Steady-State Response:

$$v_c(t) = \mathcal{R}e\left[V_c e^{j\Omega_0 t}\right] = \frac{A}{\sqrt{1 + \Omega_0^2}} \cos(\Omega_0 t - \tan^{-1}(\Omega_0)) \qquad \frac{V_c}{V_i} = \frac{1}{1 + j\Omega_0}$$

Thank You