Project *"Coil Gun Goes Pew Pew"*

Applications of Physics C E&M and Mechanics in
Coil Gun Engineering

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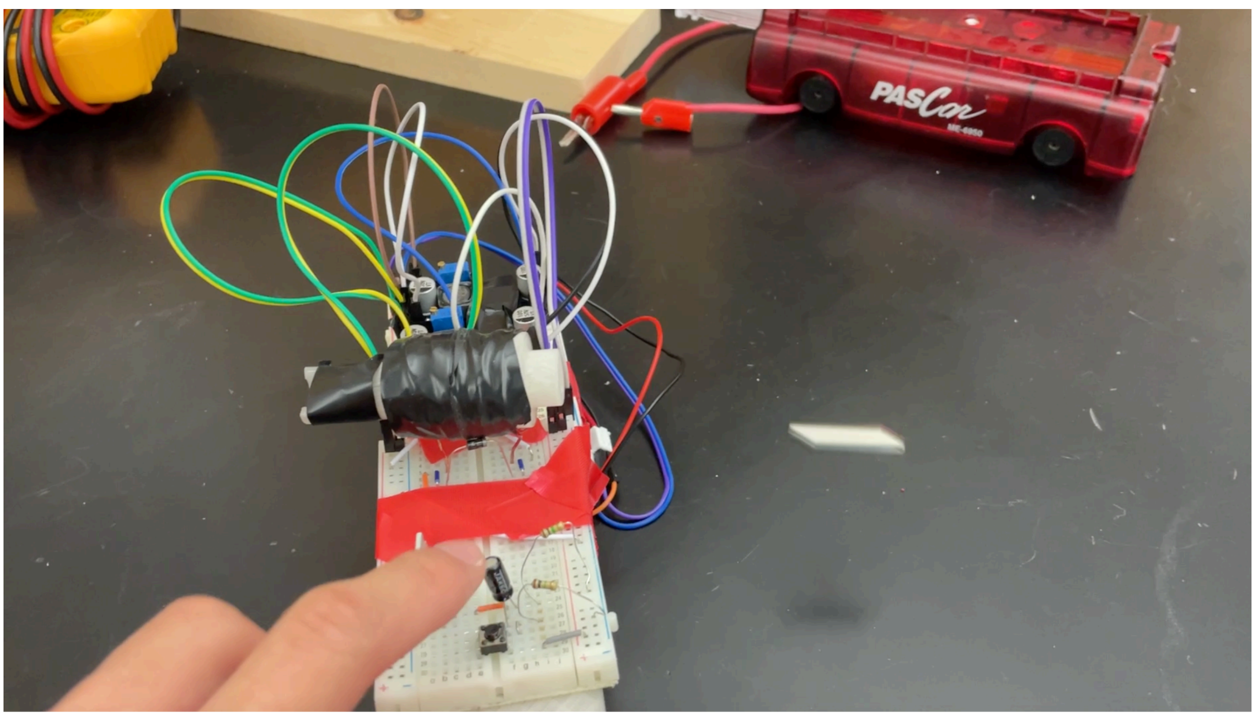
June 7, 2024[1]



[1] My birthday.

Abstract

The objective of my project is to design a coil gun prototype that can launch a magnetic rod with the push of a button. Additionally, I will expand the theoretical possibilities of this project through mathematical modeling and proofs; I intend for this project to be similar to U.S. missile launchers. To accomplish this, between May 20th and June 7th, 2024, I manufactured my coil gun and employed concepts from both areas of AP Physics C. For E&M, I used concepts in it to mathematically model the bullet within it and its circuitry components; while for Mechanics, its concepts helped me to calculate projectile range and adjustment of orientation through the use of a stepper motor that could be used in future enhancement of the engineered coil gun. By the end, we developed theoretical models and computer programs that describe the motion of a magnet within a solenoid along its axis of symmetry and as a projectile and created a coil gun that can launch a magnet decently well.

Acknowledgments

I would like to thank Mr. Buszka, who helped me, after one year of Physics C, find the "hidden curriculum" in physics and let me roam in his room after school to solve problems and talk with others. And Sourish, for reading through my paper halfway and giving useful suggestions. I would also like to thank David, Jennifer, Emma, and Arjun for looking at my paper and giving feedback. Lastly, I would like to thank the readers of this paper for their time, and I hope you enjoy reading my paper.

Contents

Device

Introduction

I aim to create a coil gun that will launch a cylindrical magnet with the press of a button. The inspiration for this project comes from modern warfare weapons which, ignoring the part where they are used in war and massacre, involve many complex systems and computations that relate to concepts in Physics C: E&M and Mechanics. In this project, I hope to effectively use our class time and make substantial improvements and progress with every day's progression.

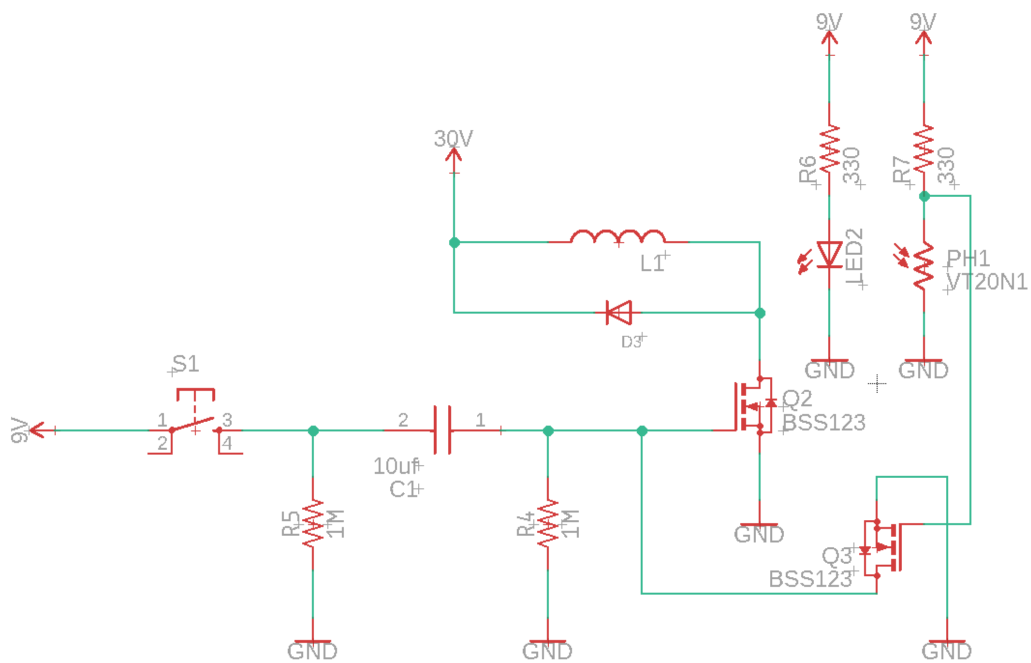
Circuitry

Introductory Considerations

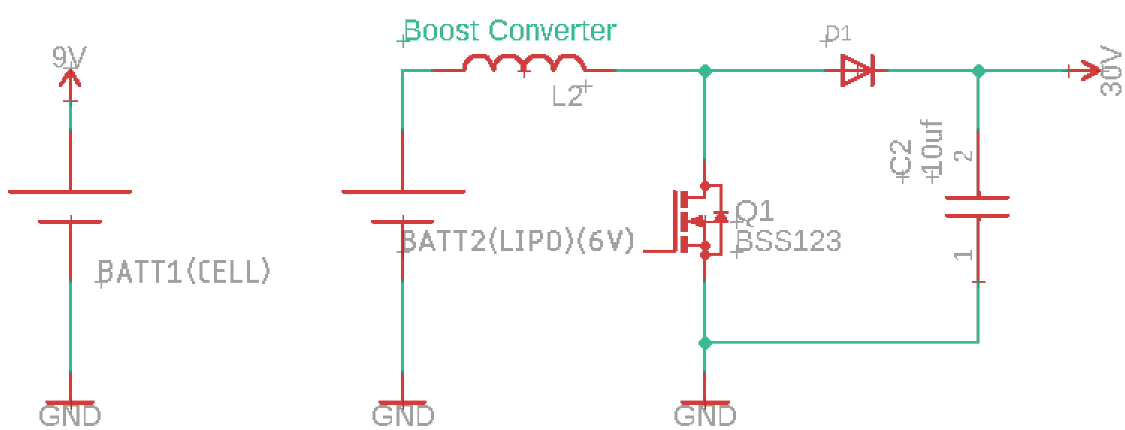
To design a coil gun, I need to consider the basic driving forces involved in launching the object. I first need a solenoid, which is essentially a coil of wires, to create a magnetic field while current flows through it. I will discuss calculations in-depth in the section *Theory*. Since there is a magnetic field and no moving charges, I need to use a permanent magnet to be launched through the coil. I also need a way to control when the gun launches the magnet, so I will use a button. Additionally, I will be dealing with high currents, so it is best to isolate ourselves from these potential differences and currents with a MOSFET. With these considerations noted, let us examine the circuit that I will build.

Schematic

I will attach the schematic file as a downloadable file in Appendix A. Underneath is attached a picture of the wiring diagrams.



Schematic of main circuitry - inspired by Electroboom[2]

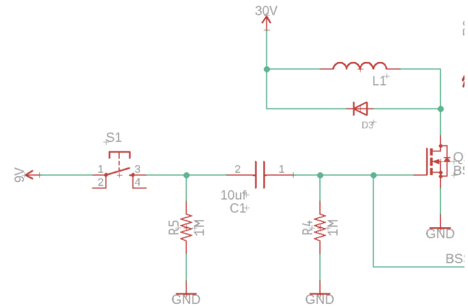


Schematic of power electronics

[2] As seen on https://www.electroboom.com/?p=101

Schematic Analysis

Main Circuitry

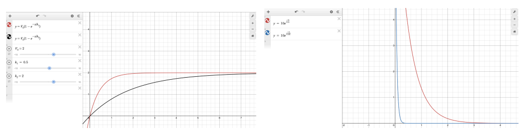


I will first examine the left half of the main schematic. I have a push button that once pressed allows charges to flow through the circuit into the capacitor and the N-channel MOSFET. A MOSFET is an electronic switch that comes in two types: P-channel and N-channel, where N-channel MOSFETs allow current to flow with an applied potential difference across its gate. Once the MOSFET's gate-source has a voltage flowing through, it allows the circuitry connected to it to start flowing, which completes the circuit between 30V and GND, allowing charges to flow through the inductor and thus generating a magnetic field in the orientation of the solenoid.

Another reason why we used a MOSFET instead of a traditional transistor that performs the same logic is that MOSFETs are typically rated for much higher voltages and amps than a transistor. More specifically, our MOSFET, the standard N-channel P30N06LE, is rated for 30 Amps and 60 volts.

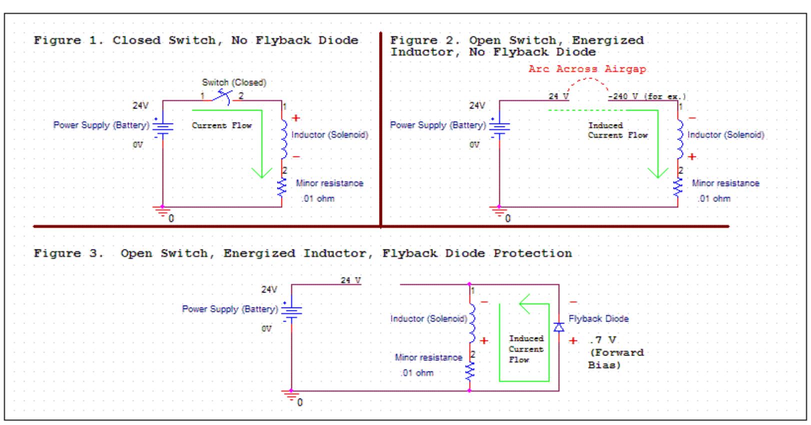
We have a capacitor that functions as a decoupling capacitor, essentially meaning that noise, or fluctuations in current in individual parts of the circuit, is separated from the button side and the MOSFET side. Additionally, this 10μF capacitor functions as a component similar to a timer, this is because the capacitor, which is connected to the circuit in series, prevents the flow of current once fully charged. According to the section *Theory: Core Principle*, a timer is something that we prefer in a coil gun's circuit.

The two 1M Ω resistors on both sides of the capacitor serve as bleeding resistors. These come into action when the push button is opened, and the circuit between the 9V battery and capacitor is broken. The resistors make sure that there are no excess charges stored in the capacitor and that on and after the charges don't flow into the MOSFET, but rather, into GND. This decision to use 1M stems from the fact that this dissipation does not need to take place rapidly because the operator of the gun may be interested in pressing the button multiple times, which means that the capacitor will be charged quickly again. It relates to the time constant principal in RC circuits, where $\tau = RC$.



RC circuit V(t) graphs. Left: charging. Right: discharging.

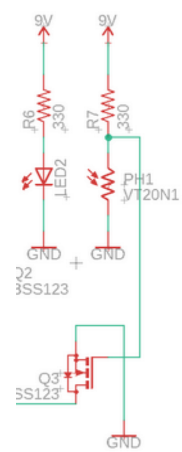
The diode connected in parallel to the inductor is used for flyback protection, which is when the current is switched off but the inductor resists that change by creating a large potential difference that damages electrical components, so I need a diode to allow the induced current to flow. A physical interpretation of a diode is that it only allows for current to flow in one direction.



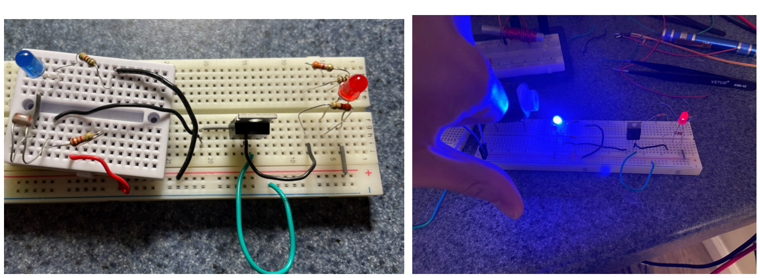
Demonstration of flyback protection diode. [3]

[3] Referenced https://en.wikipedia.org/wiki/Flyback_diode

The solenoid I made is a wire of 20 AWG, 9 meters in length, with 131 turns forming a solenoid with a radius of 7.35mm to 12.35mm and a width of 5mm. The inductance of a solenoid is given by the equation $L = \frac{N^2 \mu A}{l}$, where N is the number of turns of the coil, μ is the magnetic permeability in free space, I is the current flowing through our solenoid, and A is the area of the cross-section.[4] Additionally, the wire is rated 20 AWG, we know that the resistance per 1000m is 33.2 Ω[5], so the resistance of our circuit is $9m \ * \frac{33.2\Omega}{1000m} = 0.2988\ \Omega$. Thus, the inductance of our solenoid is $L = \frac{131^2 * (1.25663706 * 10^{-6}) * \pi * (0.00985)^2}{30/0.2988} = 6.5 * 10^{-8} H = 0.065\ \mu H$. Though I will not be using this in our theoretical calculations, it still serves as a handy reference for future expansion and development.



The right half of the main circuit functions as a bullet detector. This bullet detector functions by using a photoresistor and a LED that shines light onto it. Since in the dark a photoresistor acts as a resistor and under the light it acts as a wire, as the magnet is passing through the front of the coil gun's barrel (tube) Q3 will turn on allowing current to flow from the capacitor to GND instead of the Q2, which turns inhibits the current from flowing through the inductor and with the induced current changes the direction of the magnetic field within the solenoid, eventually dropping to none.



Bullet detector circuitry and testing

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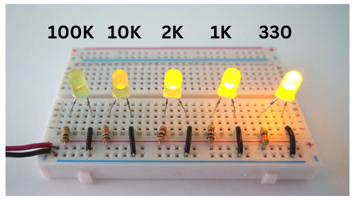
[4] Sourced from https://www.electricsolenoidvalves.com/blog/solenoid-coil-parameters-effects-measurement/
[5] Sourced from http://hyperphysics.phy-astr.gsu.edu/hbase/Tables/wirega.html

It is important to consider the resistor used in series with the LED, this is because I have to make sure that the current running through the LED does not exceed its rated value, and that I want to maximize this current so that the photoresistor is the least resistive when there is no object obstructing the LED from it. Therefore, I chose a 330 Ω resistor for this LED.



Resistorances corresponding to the brightness of the LEDs. [6]

Power Electronics

I have two power sources involved in this schematic, one which controls the logic and the other which is passed through the solenoid. The voltage that goes through the solenoid should be extremely high since I want to generate strong magnetic fields in the solenoid ($B = \mu In$ for a solenoid of infinite length).

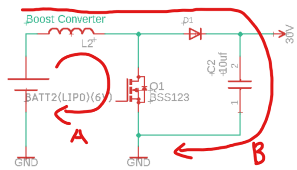
The logic voltage would be at a constant 9 volts, this is done because our MOSFETs have a gate voltage of 8-10V and a 9V battery is easier to embed into a possible rectangular hardware design, for example, a rectangular box container that houses the circuitry. Additionally, I considered that stepping down our voltage with homemade voltage dividers using resistors or linear regulators is easier than stepping up the voltage with a boost converter.

Boost Converters

I use a boost converter to boost two 3.7V lipo batteries connected in series to around 30V. However, during testing and real-world applications, it should be noted that the boost converter that I acquired sometimes unusually starts to decrease in potential difference.

[6] Photo retrieved from https://www.gsnetwork.com/led-resistor-values-for-current-limiting-resistor/

Additionally, in practice, I used two boost converters, which are rated for 4A, in parallel to supply enough amperage for our solenoid since essentially it is a short circuit.



Boost converter schematic with labels.

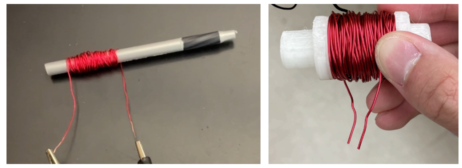
To interpret a boost converter, let us examine the circuit of a boost converter above. The MOSFET in the center acts as a switch, essentially allowing us to pass current through A when it is closed. Once current passes through loop A, the inductor L1 charges and stores energy. This can be expressed in the form $\frac{1}{2}LI^2$. Where L is the inductance, and I is the current passing through the inductor at a certain time frame. This energy is then converted to potential energy in the capacitor once the MOSFET, or switch, is opened, and charges flow through the diode into the capacitor and through loop B. These charges stored in the inductor thus are squished onto the plates of the capacitor which increase the total voltage of the capacitor by the ratio $V = \frac{Q}{C}$. Therefore, the boosted voltage is, surprisingly, the potential difference of the capacitor. [7]

To troubleshoot a boost converter, it is necessary to ask and observe the following questions: (proven extremely effective by experience and working on this project)

- Is the battery below the rated input voltage?
- Is there a short circuit?
- Are your leads on the ends of the boost converter fully in contact with the plates of the boost converter?
- Is your knob correctly adjusted with a flat-headed screwdriver?

[7] Initial concepts from https://www.youtube.com/watch?v=vmNpsofY4-U

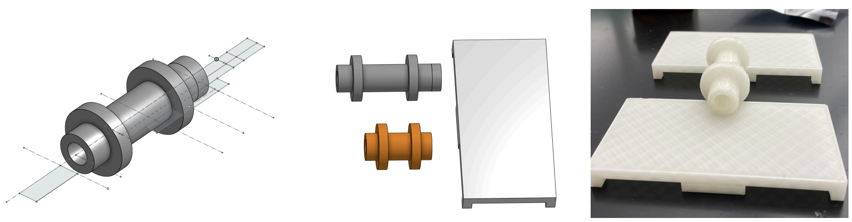
Device Construction



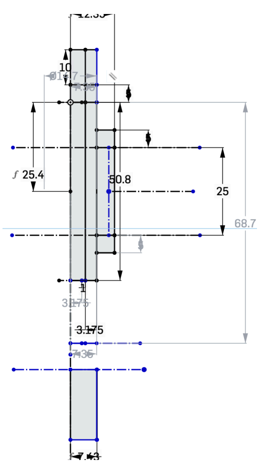
L: Prototype using a BIC pen that had a diameter of 70mm, slightly larger than the magnet's diameter allowing for early testing of the magnet in a solenoid. R: Wrapped wire around the 3D-printed solenoid.

3D Design

To create a solenoid that will generate an adequate magnetic field able to accelerate our magnet through and launch it, I designed a 3D print of a solenoid tube and some mounting plates for the structure that I will be using.



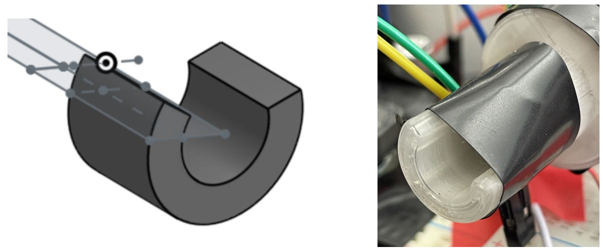
3D model of the tube and plate and printed board.[8]



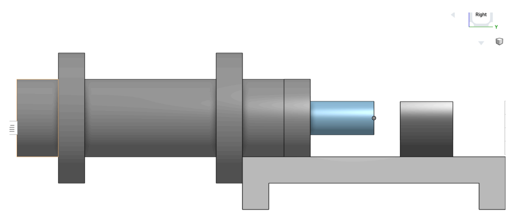
The tube in this design was constructed to the length of the magnet I used, which is 2 inches or 50.8mm. This is because I wanted to make sure that the magnet would be detected by the photogate-like contraption without it accelerating backward too much. Additionally, I allowed for a width of 5mm because I needed enough coils around the solenoid to generate a strong enough magnetic field. In consideration of the surface and frictional forces, I allowed for a 1 mm radial clearance between the magnet and the tube hole's inner surface.

[8] Please refer to this link for the online cad file

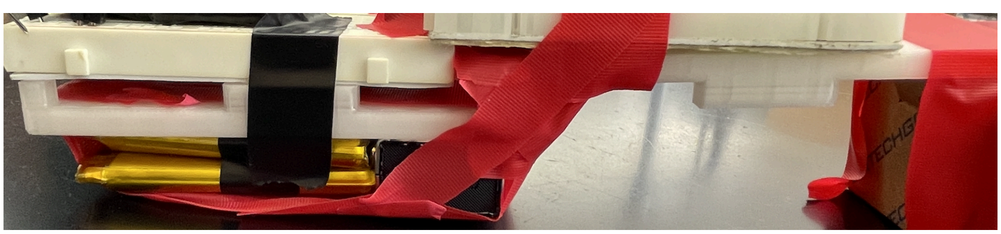
Moreover, I created an extension since when testing I realized that a lot of the actual magnet “hung” off the end of the launch location (torque on the end of the magnet due to gravity), which hindered the performance of the gun.



Extension to the tube. L: 3D design, extruded from 10mm by 3.202 mm rectangle and revolved 270 degrees. R: 3D print.



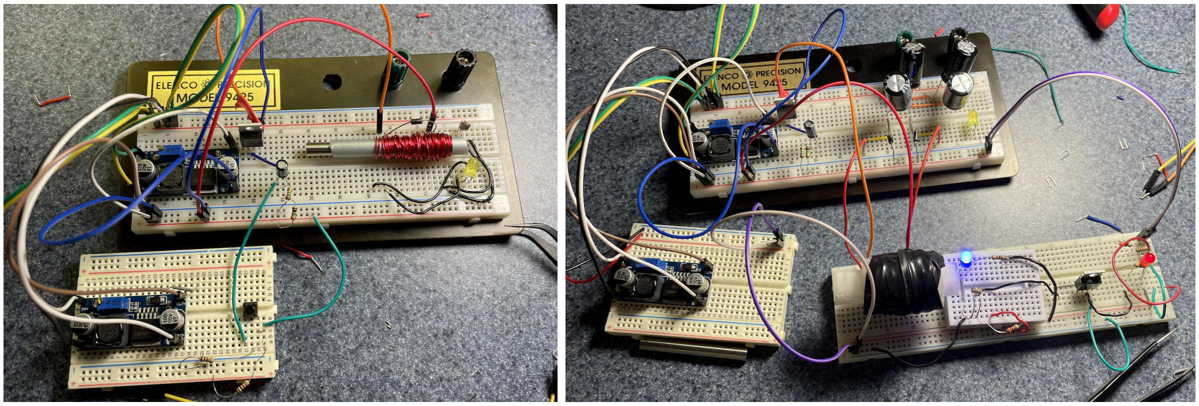
The plates were initially designed to hold the gun’s barrel perfectly from the ground - in a formation displayed on the right side. However, once the print came out and the solenoid was wrapped, I realized that this method would not work since the ends of the solenoid cannot connect to the breadboard without severe bending. Additionally, the connection of the barrel to the stands was extremely unfeasible. Therefore, I used the dimensions (w,l,h; 100mm, 50mm, 5mm) of the plates to our advantage and attached simplified and finalized breadboards to the top of the board.



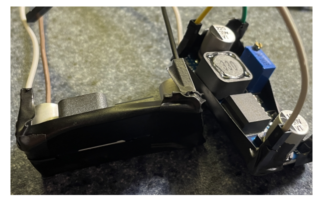
The plates are used in the design as a stand for the solenoid and battery containers.

Breadboarding

The breadboard mimicked the schematic at the top of this paper. Eventually, the breadboards would be converted into simplified and concise ones.

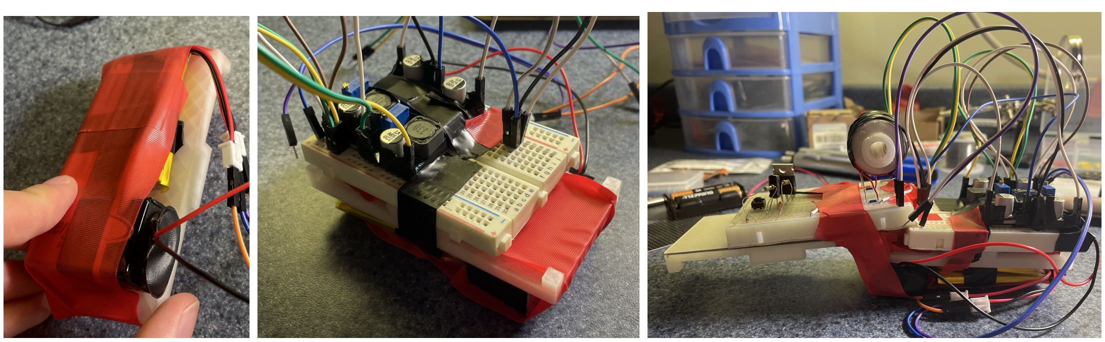


L: Prototype V1 with the prototype solenoid. R: Prototype V2 with 3D printed solenoid and bullet detection system.

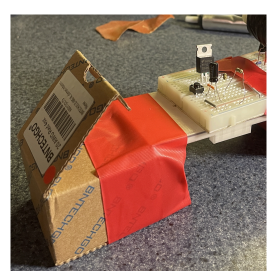


To simplify our breadboards, I first soldered jumper wires into the holes of the boost converters that represent in+, in-, out+, and out-. Next, I gently wrapped electrical tape around these solder joints to prevent electrical connections from forming between the boost converter and the surface the boost converter rests.

I attached our batteries to the top side of the plate and took advantage of the space there to tape everything together. To finalize the first section of the coil gun, I then attached the boost converter breadboard circuit to the opposite side of the plate mount.



L: Batteries M: Power electronics circuitry section R: All breadboard components taped and connected.

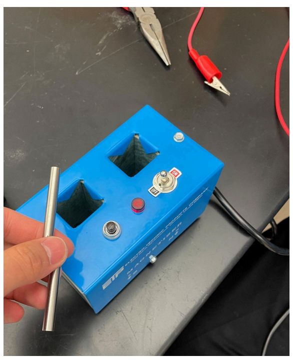


Finally, I added an ammo bay with the addition of a cardboard box to the end of the gun to not only function as a storage for magnets and components but to also balance the end of the coil gun board.

Setbacks

I experienced many setbacks during the construction of this device.

One to be highlighted is that our photoresistors from Amazon were extremely insensitive to light sources. This caused the entirety of the bullet detection system to fail. Measuring the potential difference between the resistor when the LED was on and the LED was off yielded values of 8.6V and 7.8V, which is a miniscule difference that would not trigger Q3. In the future, for future caution, it is necessary to review actual spec sheets and buy from more credible sources.



Another setback was during initial prototyping our magnets were demagnetized and created an illusionary monopole. However, with the brilliance of a magnetizer that uses strong solenoids to forcefully realign the magnetic spins within the magnet's atoms, I was able to remagnetize them.

Lastly, during initial prototyping, I did not notice that there was a nonconductive coating around the wiring I was using, which made the current unable to pass through the solenoid. Therefore, I used sandpaper to sand off the coating which allowed us to proceed with testing.

Device Application

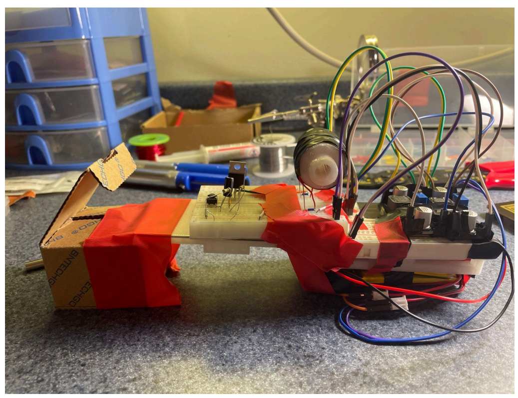
Once our device was fully built, I tested it with two differently-sized magnets with the same physical properties. In these tests, I measured the qualitative exit velocity and range of the magnets with a button press of the same time duration.


Left(Magnet 1): Length: 50.8mm D: 6.35mm | Right(Magnet 2, one used in initial design): Length: 25.4mm D: 6.35mm

The results were that magnet 1 experienced, qualitatively, a greater magnitude of acceleration than magnet 2, which was expected since the magnitude of acceleration is inversely proportional to the mass of the object, which in this case magnet 2 had twice of magnet 1's.

Furthermore, it is important to note that since I do not have the magnet detection circuit, I have to time the button press durations to an exact time that would not result in the magnet experiencing sinusoidal behavior within the solenoid.[9]

In conclusion, this circuitry satisfied our initial target of creating something that would launch a magnetic rod with a push of a button, with possible improvement points being present.

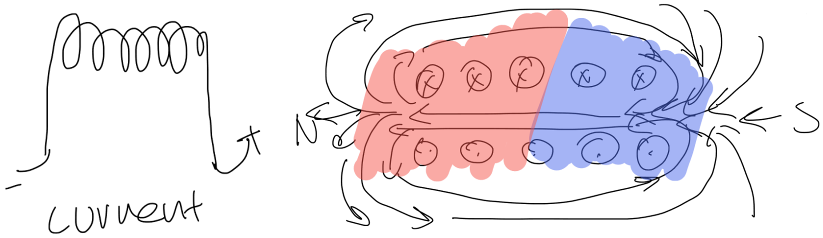


Final coil gun

[9] Reference Theory-Core Principle to learn more

Theory

Core Principle



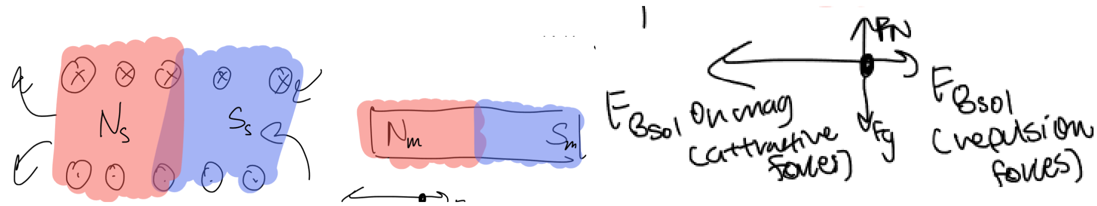
L: Solenoid R: A magnetic field directed towards the left is produced when current is passed from right to left.

Launch Mechanism

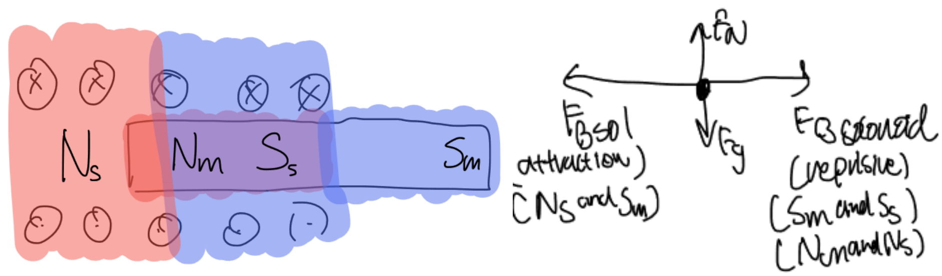
To launch a magnet, we need a force exerted on the magnet to accelerate it. Therefore, I will be using the force exerted on the magnet by the solenoid to accelerate it. Let us examine the scenario.

When current is first passed through our solenoid it creates a magnetic field that is directed to the left, which is justified through the right-hand rule with current passing into the page on the top and out of the page on the bottom. This magnetic field thus mimics a magnet, and we can label the corresponding sides of the solenoid north and south. Remembering how a magnet interacts with another physical magnet, we can conclude that this property will allow us to accelerate a physical magnet through the solenoid.

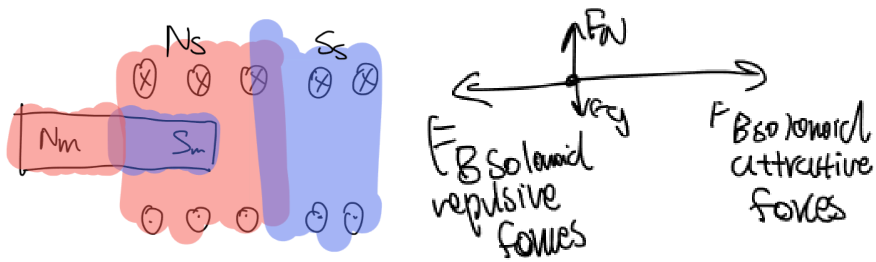
In order to visualize the motion and forces on the magnet through the solenoid, reference the table below that illustrates a magnet's motion next to a solenoid. In these drawings, I assume that the friction between the inner surface of the tube/barrel and the magnet is negligible, and the magnet itself doesn't induce any currents.



The imbalanced horizontal forces imply that the magnet accelerates toward the left

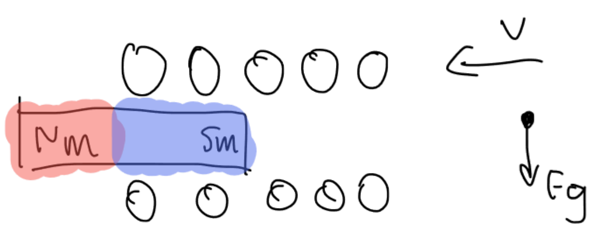


As the magnet reaches the center, the repulsive force between the magnet and the solenoids' ends increases, thus there is less acceleration towards the left.



Note that the magnet is still traveling left since it has a velocity towards the left. As the south side of the magnet passes into the north side of the solenoid, the magnet experiences a stronger force towards the right, which accelerates it to the right.

If I continue to allow current to pass through the wires, the magnet will experience oscillatory motion with decreasing amplitudes with the center of the solenoid being where the magnet's center ends. This is not what we want.



If I turned off the current as the south side of the magnet reaches the north side of the solenoid, ideally the magnetic field would reach zero and the magnet only experiences the force of gravity downwards, and possibly the normal force of the gun barrel, with a velocity directed to the left.

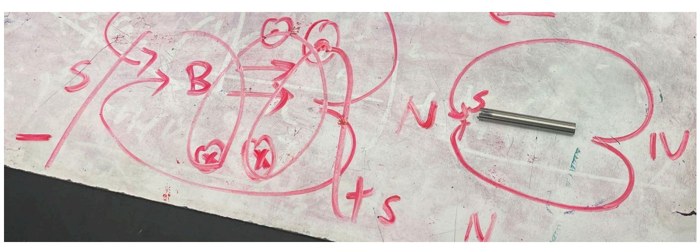
Physically, if the current is turned off through a switch, the solenoid, which is an inductor, will induce a current in the opposite direction, which creates a reserved magnetic field, thus mimicking a magnet turned around, which makes the south pole of the magnet overlay the new south pole of the solenoid. This overlay creates a repulsive force that accelerates the magnet to the left even more.

Mathematically, the induced current in the inductor/solenoid is found through $V = I_{induced}R = -L\frac{dI_{circuit}}{dt}$. This negative sign implies that the induced current will be opposite to the direction of the existing current. Additionally, the constant L/R is computationally smaller than 10^{-6}, which suggests that the magnitude of the induced magnetic field is less than the original one.[10]

Therefore, opening the circuit at the exact moment is extremely important for the functioning of this gun.

Even though correctly timing the amount of time the circuit is opened is important, the initial polarity of the solenoid, or the direction of how current flows through it, will not influence the effect of this gun. This is because through trial and error, I can identify which end of the magnet will allow it to be initially accelerated, not repealed, into the solenoid, allowing us to mark that end and use it for future endeavors.

We can also use this concept to map out the poles of the magnet, as seen below.



By drawing out the magnetic field and determining which end of the magnet is accelerated, we can determine the poles of the magnet, which may be helpful.

[10] Reference *Device-Circuitry-Main Circuitry*

Bullet Modeling[11]

Nomenclature

S - symbol for solenoid

l - length of solenoid

R - radius of the solenoid

R_Ω - resistance of the wire of the solenoid

N - number of turns in the solenoid

n - expression for N/l

x_{old} - cylindrical magnetic leftmost face's distance from rightmost solenoid turn

x_{new} - cylindrical magnetic's leftmost face's distance from the origin

L - length of cylindrical magnetic

I - current passing through solenoid

I_{ind} - induced current through the solenoid

m - magnetic dipole moment of magnet

M - mass of magnet

a - acceleration of magnet

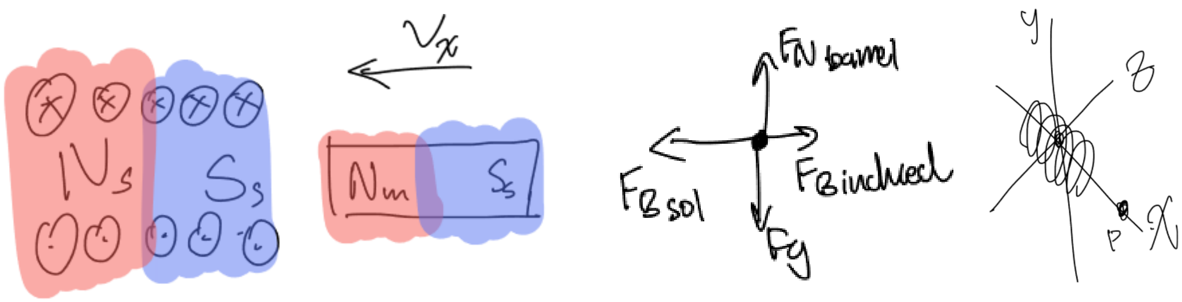
Motivation

In modern history, computational modeling and analysis allow for semi-accurate predictions of the future and how to systematically design weapons. For example, scientists need to model the forces on an ICBM before directing it to its landing target.[12]

[11] Work read during research: https://ntrs.nasa.gov/api/citations/19980227402/downloads/19980227402.pdf
[12] Learn more at https://www.lockheedmartin.com/

Using the same logic, we want to model the magnet's position inside a solenoid. This would allow us to calculate the bullet's initial velocity after being accelerated and perform calculations based on that.

In order to model the magnet's position on the **x-axis**, we need to identify the forces acting on it to use $F_{net} = Ma$. Let us assume the scenario above, but with an induced current, and we combine the $F_{B\ solenoid}$ together.



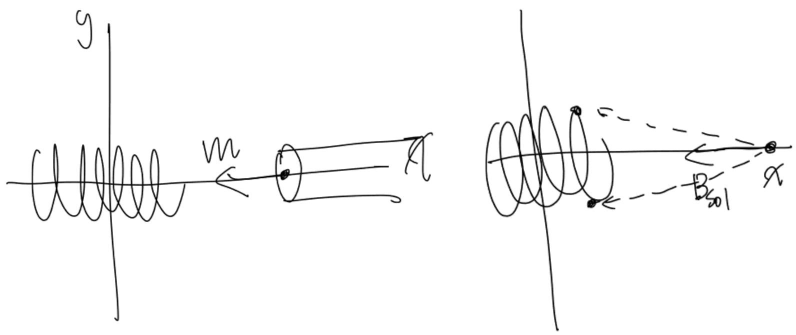
A visualization of the scenario.

Force of Solenoid on Magnet

To derive the force of the solenoid on the magnet, we first need to use equation $F_{Solenoid\ on\ Magnet\ m} = \nabla(m \cdot B)$ [13]. m is the magnetic dipole moment, defined by m = I*a[14], where I is the current surrounding a certain area. B is the vector for the magnetic field at a certain point. And ∇ is nabla or the gradient of the dot product between m and B. The gradient of a certain vector or expression is essentially the combination of partial derivatives of the x, y, and z-axis. Partial derivatives are derivatives, so they describe the change in the function over time concerning different planes.

[13] https://en.wikipedia.org/wiki/Force_between_magnets#Magnetic_force_due_to_non-uniform_magnetic_field
[14] https://en.wikipedia.org/wiki/Magnetic_moment

The magnetic dipole moment m is a vector that points directly perpendicular to the face of the magnetic cylinder.[15] I calculated this value to be 0.9602384464 amps per meter squared. Reference Appendix A for numerical calculations of this value for our magnet.



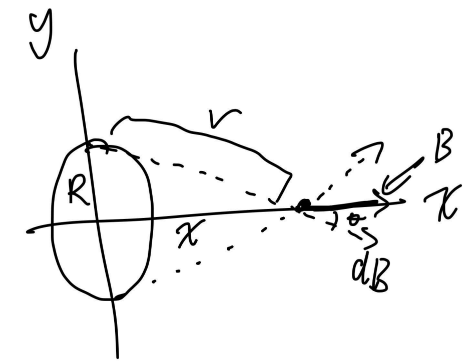
Left: Magnetic dipole moment m | Right: B_x from solenoid

One method to simplify this equation is to first consider m as a vector that is represented by $<m_x, m_y, m_z>$ and magnetic field B as a vector represented by $<B_x, B_y, B_z>$. The dot product $m \cdot B$ is equivalent to $m_xB_x + m_yB_y + m_zB_z$. If we assume that there are no edge effects (a very bold one, in fact), then this expression is just equivalent to $\mathbf{m_xB_x\,\hat{x} = mB_x\,\hat{x}}$ since the solenoid is positioned onto the x-axis. m_x would be equal to m since m is directly lying on the x-axis. The "$\hat{x}$" represents this value as a vector in the x direction.

Another method to simplify this equation is to simplify this dot product so that it calculates the magnitude of force, making $|m \cdot B| \Rightarrow mBcos(\theta)$. Since we are neglecting edge effects, B -> B_x. Moreover, $cos(\theta)$ is equal to 1 since the angle between the vectors is 0. Thus, our dot product's magnitude expression becomes $\mathbf{mB_x}$, which is identical in magnitude to the expression above.

Therefore, we need to solve for B_x of the solenoid. In this scenario, it is impractical to use the infinite length approximation of the magnitude of magnetic field within a solenoid ($B = \mu In$) since we are dealing with points outside a short, finite solenoid on its symmetry axis.

[15] Refer to https://web2.ph.utexas.edu/~vadim/Classes/2024s-u/Hfield.pdf page 5 to learn more on this.



Let us start with a single current loop and continue the derivation from there. From Biot-Savart Law, we know that

$$\overrightarrow{dB} = \frac{\mu_0 I \overrightarrow{dL} \times \hat{r}}{4\pi r^2}$$

In this case, let us neglect $\hat{r}$ since we know the direction of our final vector. Additionally, let us substitute r^2 for $x^2 + R^2$. Moreover, it is important to note that

$$B = \int dB \cos(\theta)$$

since all the vertical components along the symmetrical axis of the current circle are canceled out, only leaving the horizontal components. θ in this equation refers to the angle made by dB and the x-axis.

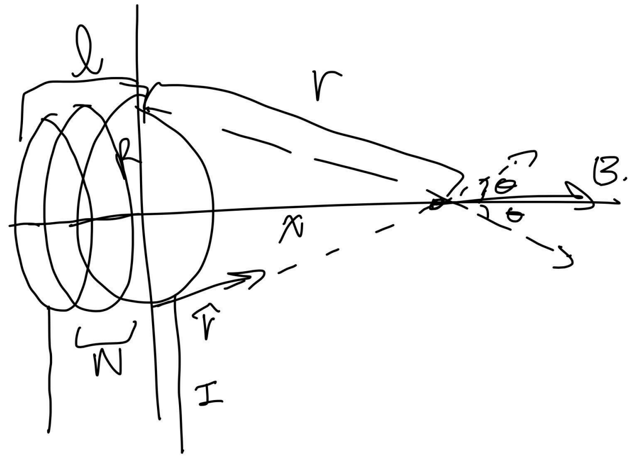
Thus, we have the equation

$$B = \frac{\mu_0 IR}{4\pi(x^2+R^2)} \int \cos(\theta) dl$$

$\cos(\theta)$ is equivalent to $R/\sqrt{x^2 + R^2}$ with some geometry, so the equation for B becomes

$$B = \frac{\mu_0 IR}{4\pi(x^2+R^2)^{\frac{3}{2}}} \int dl = \frac{\mu_0 IR(2\pi R)}{4\pi(x^2+R^2)^{\frac{3}{2}}} = \frac{\mu_0 IR^2}{2(x^2+R^2)^{\frac{3}{2}}}.$$

Let us now expand that into a finite solenoid's expression with a new scenario. Now we have *N* rings (turns of a solenoid) with a total distance of *l*. Additionally, it has current *I* passing through it.



[16]

The first observation we make is that this is essentially multiple rings spaced apart. Therefore, let us take the differential of B for a single ring for differentials of I.

$$dB = \frac{\mu_0 R^2}{2(x^2+R^2)^{\frac{3}{2}}} dI$$

We have done this because we know that a current creates a magnetic field around it, so an infinitely small amount of current will create an infinitely small amount of magnetic field. However, we need to express B in terms of dx so that we can take an integral that can represent a solenoid a distance x from it. To do this, we use a ratio $\frac{NI}{l} = \frac{dI}{dx}$ that implies $dI = nIdx$, where n is N/x. So, we can express dB in terms of dx, which yields us the equation

$$dB = \frac{\mu_0 R^2}{2(x^2+R^2)^{\frac{3}{2}}} nIdl = \frac{\mu_0 nI}{2} \frac{R^2}{(x^2+R^2)^{\frac{3}{2}}} dx$$

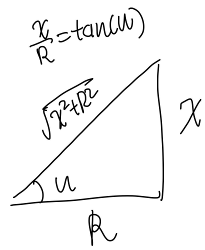
$$B = \int \frac{\mu_0 nIR^2}{2} \frac{1}{(x^2+R^2)^{\frac{3}{2}}} dx$$
.

Let us solve this integral with trigonometric substitution. First, we need to identify what we are substituting. In this scenario, let us do

[16] Initial inspiration from here. However, calculations and proofs are done by myself on paper.

$$x = R\tan(u)$$
$$dx = R\sec^2(u)\ du.$$

Additionally, here is a reference triangle that we will use in our derivations.



The integral *without the constants*, with the substitution of $\frac{\sqrt{x^2+R^2}}{R} = sec(u)$, becomes

$$\int \frac{1}{(x^2+R^2)^{\frac{3}{2}}} dx = \int \frac{1}{(R\sec(u))^3} R\sec^2(u)\ du = \frac{1}{R^2} \int \frac{1}{\sec(u)}\ du.$$

Solving this integral, and substituting $sin(u) = \frac{x}{\sqrt{x^2+R^2}}$,

$$\frac{1}{R^2} \int \frac{1}{\sec(u)}\ du = \frac{1}{R^2} \int \cos(u)\ du = \frac{1}{R^2}\sin(u) = \frac{1}{R^2} \frac{x}{\sqrt{x^2+R^2}}.$$

Lastly, substituting this solution of the integral without constants back into our expression of *B*, and evaluating it with the bounds *x* and *x-l*, which represent the distance from the nearest loop and furthest loop, we obtain the expression

$$\frac{1}{R^2} \frac{\mu_0 n I R^2}{2(x^2+R^2)^{\frac{3}{2}}} \frac{x}{\sqrt{x^2+R^2}} \Bigg|_{x-l}^{x}.$$

Hence, for a point distance x away from the right side and on the symmetrical axis (the x axis) of a solenoid,

$$B_{solenoid\ x} = \frac{\mu_0 n I}{2} \left[\frac{x}{\sqrt{x^2+R^2}} - \frac{x-l}{\sqrt{(x-l)^2+R^2}} \right].$$

However, since we are dealing with motion, using this equation would assume the origin, or frame of reference, is the point intersecting the axis of symmetry and the left face of the cylindrical magnet, which would make calculations difficult.

So, we have to move our origin somewhere that is not moving, let us say the center of the solenoid. Therefore, if I have our magnet a distance of *x* from the origin, and still have the solenoid with a length of *l*, our expression for the magnitude of *B* would now become

$$B_{solenoid} = \frac{\mu_0 n I}{2}\left[\frac{x+l/2}{\sqrt{(x+l/2)^2+R^2}} - \frac{x-l/2}{\sqrt{(x-l/2)^2+R^2}}\right].$$

To approximate the value the *entire* magnet experiences, we need to take the average of *B* across the magnet with length L, so we have to take another integral, which is

$$\frac{1}{(x-L)-x}\int_x^{x+L} \frac{\mu_0 n I}{2}\left[\frac{x+l/2}{\sqrt{(x+l/2)^2+R^2}} - \frac{x-l/2}{\sqrt{(x-l/2)^2+R^2}}\right].$$

Solving this integral by splitting apart the expression and then u-substituting with $u = (x \pm l/2)^2$, we arrive at the expression for the average magnetic of magnetic field experienced for the magnetic cylinder as

$$\frac{1}{L}\frac{\mu_0 n I}{2}\left[\sqrt{(x+L+l/2)^2+R^2} - \sqrt{(x+l/2)^2+R^2} - \sqrt{(x+L-l/2)^2+R^2} + \sqrt{(x-l/2)^2+R^2}\right].$$

Since $F_{S\,on\,m} = \nabla(m \cdot B) =< \frac{\partial}{\partial x}mB_x, \frac{\partial}{\partial y}mB_y, \frac{\partial}{\partial z}mB_z >= \frac{\partial}{\partial x}mB_x = \frac{d}{dx}mB_x$, we take the derivative by applying the chain rule of our found expression from above and find the force on the magnet along the symmetrical axis (x-axis) of the solenoid as

$$\begin{aligned}F_{S\,on\,m} &= m\frac{d}{dx}B_x \\ &= \frac{m\mu_0 n I}{2L}\frac{d}{dx}\left[\sqrt{(x+L+l/2)^2+R^2} - \sqrt{(x+l/2)^2+R^2} - \sqrt{(x+L-l/2)^2+R^2} + \sqrt{(x-l/2)^2+R^2}\right] \\ &= \frac{m\mu_0 n I}{2L}\left[\frac{x+L+l/2}{\sqrt{(x+L+l/2)^2+R^2}} - \frac{x+l/2}{\sqrt{(x+l/2)^2+R^2}} - \frac{x+L-l/2}{\sqrt{(x+L-l/2)^2+R^2}} + \frac{x-l/2}{\sqrt{(x-l/2)^2+R^2}}\right].\end{aligned}$$

Force of Induced Current on Magnet[17]

While the magnet moves through the solenoid, there is a current induced because there is a change in magnetic flux. This is due to Faraday's law, which states that $V_{induced} = -\frac{d\phi_B}{dt}$. Additionally, Lenz's law states that this current is created in the direction that opposes the change in magnetic flux, therefore, the induced magnetic field points outwards, or in the opposite direction of the magnetic field of the solenoid. Therefore, there is not only the force of the solenoid on the magnet but also the force of the induced current's magnetic field on the magnet as well.

Derby and Olbert solved exact solutions to model the axial and radial magnetic fields of a cylindrical magnet. In the paper below, I will refer to these solutions as B_p and B_x for radial and axial, respectively. More specifically, B_p and B_x are essentially a set of equations that represent the equations and solutions for given inputs.

To find the equation, we use the derived equation from the previous section and plug in I_{ind} for I; where $I_{ind} = -\frac{d\phi_B}{R_\Omega dt}$. We will only find the magnitude of this induced current since we can qualitatively predict the direction of force in the final steps. So, the expression we want to solve is now $\frac{d\phi_B}{R_\Omega dt}$.

We will first apply a chain rule expansion to $\frac{d\phi_B}{dt}$, which would yield us $\frac{d\phi_B}{dx}\frac{dx}{dt}$, this simplifies to $v_x \frac{d\phi_B}{dx}$. By Maxwell's equations, the total magnetic flux in a contained volume in space is 0, or there are no monopoles. Therefore, $d\phi_B = B_\rho\,(x,\ z)\ 2\pi r dx$. For readers interested

[17] This section takes HEAVY inspiration from Derby and Olbert. Learn more at https://arxiv.org/pdf/0909.3880.

in the derivation, please refer to Derby and Olbert's paper starting at equation (19). Plugging this into our expression, we have $I_{ind} = v_x \frac{d\phi_B}{dx} = v_x B_\rho(R, x)\, 2\pi r$.

Lastly, plugging this expression into our derived expression from the previous section, it yields

$$\begin{aligned} F_{induced\ on\ m} &= \frac{m\mu_0 nI}{2L}\left[\frac{x+L+l/2}{\sqrt{(x+L+l/2)^2+R^2}} - \frac{x+l/2}{\sqrt{(x+l/2)^2+R^2}} - \frac{x+L-l/2}{\sqrt{(x+L-l/2)^2+R^2}} + \frac{x-l/2}{\sqrt{(x-l/2)^2+R^2}}\right] \\ &= \frac{m\mu_0 n\,(v_x B_p(R,x)2\pi R)}{2L}\left[\frac{x+L+l/2}{\sqrt{(x+L+l/2)^2+R^2}} - \frac{x+l/2}{\sqrt{(x+l/2)^2+R^2}} - \frac{x+L-l/2}{\sqrt{(x+L-l/2)^2+R^2}} + \frac{x-l/2}{\sqrt{(x-l/2)^2+R^2}}\right] \\ &= v_x\frac{m\mu_0 n B_p(R,x)\pi R}{L}\left[\frac{x+L+l/2}{\sqrt{(x+L+l/2)^2+R^2}} - \frac{x+l/2}{\sqrt{(x+l/2)^2+R^2}} - \frac{x+L-l/2}{\sqrt{(x+L-l/2)^2+R^2}} + \frac{x-l/2}{\sqrt{(x-l/2)^2+R^2}}\right]. \end{aligned}$$

Unfortunately, this method only models the force of the induced current from the magnet on the axis of symmetry, not considering the effects.

To truly consider edge effects, a preferred method, which researchers Derby and Olbert applied, would be to solve for the force of the magnet on the induced current by employing the equation $F_{B\ on\ i} = i\, l \times B$, where l is a vector representing the direction of current and length, and B is the vector of the magnetic field. They then reason with Newton's third law and conclude that the opposite of this force is the force the induced current has on the magnet.

Combination

Once we have both the forces, we use $F_{net} = Ma$ to finalize our equation. Thus, plugging in $F_{solenoid\ on\ magnet}$ - $F_{induced\ current\ on\ magnet}$ as F_{net}, we have the expression

$$\begin{aligned} F_{net} &= \frac{m\mu_0 nI}{2L}\left[\frac{x+L+l/2}{\sqrt{(x+L+l/2)^2+R^2}} - \frac{x+l/2}{\sqrt{(x+l/2)^2+R^2}} - \frac{x+L-l/2}{\sqrt{(x+L-l/2)^2+R^2}} + \frac{x-l/2}{\sqrt{(x-l/2)^2+R^2}}\right] - \\ &\quad v_x\frac{m\mu_0 n B_p(R,x)\pi R}{L}\left[\frac{x+L+l/2}{\sqrt{(x+L+l/2)^2+R^2}} - \frac{x+l/2}{\sqrt{(x+l/2)^2+R^2}} - \frac{x+L-l/2}{\sqrt{(x+L-l/2)^2+R^2}} + \frac{x-l/2}{\sqrt{(x-l/2)^2+R^2}}\right] \\ &= \frac{m\mu_0 n}{L}(1 - 2v_x B_p(R,x)\pi R)\left[\frac{x+L+l/2}{\sqrt{(x+L+l/2)^2+R^2}} - \frac{x+l/2}{\sqrt{(x+l/2)^2+R^2}} - \frac{x+L-l/2}{\sqrt{(x+L-l/2)^2+R^2}} + \frac{x-l/2}{\sqrt{(x-l/2)^2+R^2}}\right]. \end{aligned}$$

This expression neglects the edge effects of the solenoid, which may not be the most accurate but serves as a baseline in approximating the forces on the magnet.

Once we have F_{net}, we combine this with *Ma* to yield

$$Ma = \frac{m\mu_0 n}{L}(1 - 2v_x B_p(R,x)\pi R)\left[\frac{x+L+l/2}{\sqrt{(x+L+l/2)^2+R^2}} - \frac{x+l/2}{\sqrt{(x+l/2)^2+R^2}} - \frac{x+L-l/2}{\sqrt{(x+L-l/2)^2+R^2}} + \frac{x-l/2}{\sqrt{(x-l/2)^2+R^2}}\right]$$
$$a = \frac{m\mu_0 n}{ML}(1 - 2v_x B_p(R,x)\pi R)\left[\frac{x+L+l/2}{\sqrt{(x+L+l/2)^2+R^2}} - \frac{x+l/2}{\sqrt{(x+l/2)^2+R^2}} - \frac{x+L-l/2}{\sqrt{(x+L-l/2)^2+R^2}} + \frac{x-l/2}{\sqrt{(x-l/2)^2+R^2}}\right]$$
$$\ddot{x} = \frac{m\mu_0 n}{ML}(1 - 2\dot{x} B_p(R,x)\pi R)\left[\frac{x+L+l/2}{\sqrt{(x+L+l/2)^2+R^2}} - \frac{x+l/2}{\sqrt{(x+l/2)^2+R^2}} - \frac{x+L-l/2}{\sqrt{(x+L-l/2)^2+R^2}} + \frac{x-l/2}{\sqrt{(x-l/2)^2+R^2}}\right].$$

Do note that the dots above the variables *x* represent the order of differentiation of position. Essentially, x dot dot is equivalent to $\frac{d^2x}{dt^2}$ and x dot is $\frac{dx}{dt}$.

To simplify our expression, we will let

$$C(x) = \frac{x+L+l/2}{\sqrt{(x+L+l/2)^2+R^2}} - \frac{x+l/2}{\sqrt{(x+l/2)^2+R^2}} - \frac{x+L-l/2}{\sqrt{(x+L-l/2)^2+R^2}} + \frac{x-l/2}{\sqrt{(x-l/2)^2+R^2}}$$
$$\therefore\ \ddot{x} = \frac{m\mu_0 n}{ML}(1 - 2\dot{x} B_p(R,x)\pi R)C(x).$$

This second-order differential equation is fairly reminiscent of simple harmonic motion, where

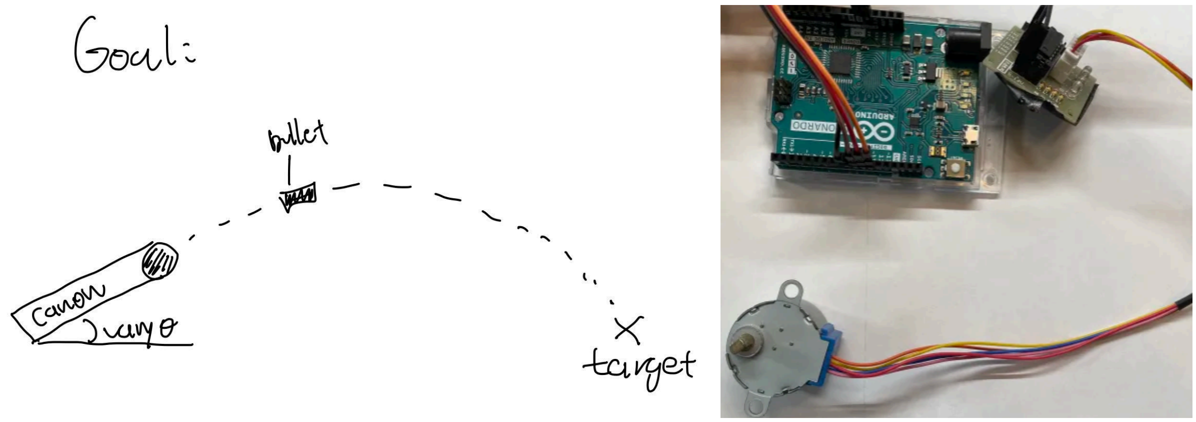
$$F_{net} = kx$$
$$m\ddot{x} = kx$$
$$x(t) = A\cos(wt + \phi).$$

So, we can reason that our differential equation would also yield a solution that involves sinusoidal functions, which does match up with our observations if I let the current continuously flow through the solenoid.

Another method in visualizing our second-order differential equation may be to use Euler's method to approximate solutions with given initial values, such as our initial position in time.[18]

[18] Referenced Sun and Xing 2023. Source.

Canon Modeling: Mechanics



L: Desired scenario with canon and target. R: Stepper motor model 28BYJ-48 and controller.

Motivation and Planning

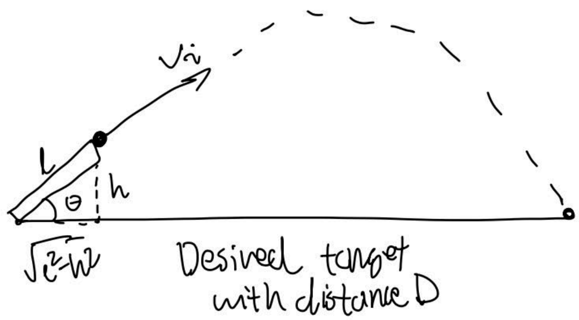
Modern ICBMs are often launched from silos. However, since I do not have a consistent propulsion system like an engine on a magnet, I will stick to more primal canons as inspiration for our modeling. In the medieval ages, these weapons were the ones that caused kingdoms to fall and were considered machines of mass destruction. Therefore, it is in our interest to model a canon-like structure that involves the coil gun.

In all models that involve a trajectory of an object, there requires the launch velocity vector. Theoretically, I can solve for the initial velocity by taking the derivative of our expression for x(t) of the magnet in the solenoid, and plugging in time t for which we stop the current from flowing. Let us designate this value to a variable v_i.

After finding v_i, we can vary the launch angle concerning the ground to hit different target distances by attaching a string on one end of the platform and letting a motor above the platform wind the string by rotating certain angle measures.

Modeling

Trajectory



Scenario with annotations

To calculate the range of the bullet and estimate where it would land, we will employ concepts from kinematics.

We first start by writing out the minimum equations in an ideal scenario where air resistance is negligible.

$$a_x = 0$$
$$a_y = -g$$
$$v_y(t) = \int_0^t a_y(t)dt = -gt + v_i \sin(\theta)$$
$$s_y(t) = \int_0^t v_y(t)dt = -0.5gt^2 + v_i \sin(\theta)t + h$$
$$s_x(t) = v_i \cos(\theta) + \sqrt{l^2 - h^2}$$

Next, we will solve $s_y(t)$ for our flight time. Since this equation is quadratic and hard to simplify, we will employ the quadratic equation, this yields an expression of

$$h = l\sin(\theta)$$
$$t = \frac{-v_i \sin(\theta) \pm \sqrt{(v_i \sin(\theta))^2 + gh}}{-g} = \frac{v_i \sin(\theta) \pm \sqrt{(v_i \sin(\theta))^2 + gh}}{g}$$

Since $x < \sqrt{x^2 + a}$, where x is an expression in interest and a is a constant, we can rule the second solution involving the minus sign to be invalid since that would produce a negative *t* value.

Therefore, we have the flight time T equation as

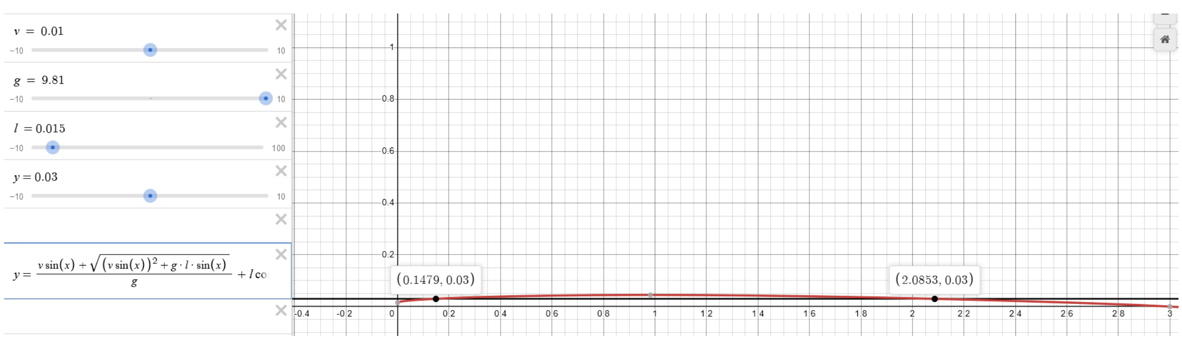
$$t = \frac{v_i \sin(\theta) + \sqrt{(v_i \sin(\theta))^2 + gl\sin(\theta)}}{g}.$$

To find an expression for the range *D*, we plug this value into our $s_x(t)$ function to yield

$$D = \frac{v_i \sin(\theta) + \sqrt{(v_i \sin(\theta))^2 + gl\sin(\theta)}}{g} v_i \cos(\theta) + l\cos(\theta).$$

We need to solve for θ that will give us a range *D* to the target. However, pure computation by hand will become extremely difficult. Therefore, we will employ a computer program written in Python that solves our rearranged equations for the solution angle.

This computer program, located in Appendix B, is extremely accurate, as seen below in a graphical comparison with the output.

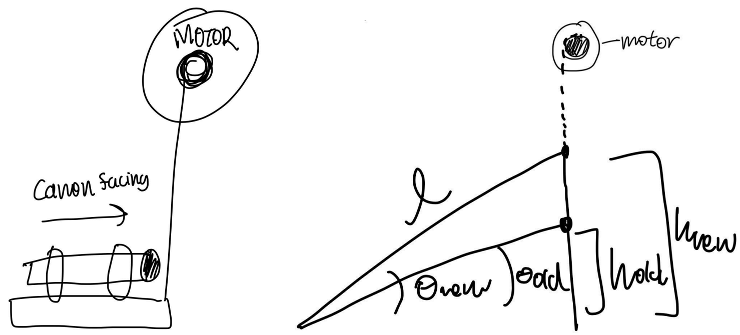


Plot of our graph and solution comparison. There are two solutions for a distance of 0.03m - 0.1479rad and 2.065rad. A graphical simulation can be found here: https://www.desmos.com/calculator/h8hjh78fi4

```
(.venv) PS C:\Users\Eddie Tang\PythonProjects\stocks\stock test 1> python test.py
0.14794111779090802 8.476401665866806
(.venv) PS C:\Users\Eddie Tang\PythonProjects\stocks\stock test 1>
```

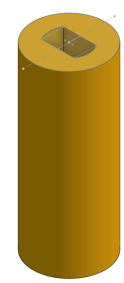
Computer output up to more than 7 significant figures.
Left output in radians and Right output in degrees.
Simulation ran on the same constants as graphical simulation. Find the constants in Appendix B's code.

Canon launch angle



L: Motor and coil gun contraption. R: Diagram of change in height vs change in angle of the board.

I will assume that our coil gun is pointing in the direction specified in the left diagram above and is on a platform of length l. Additionally, the motor is directly above the left vertical edge of the platform and has a string with a minimal mass attached to it. Therefore, for a specific θ that the board forms with the horizontal, its relationship with its height from the horizontal is $sin(\theta) = h/l$.



For a θ_{new} and θ_{old} of the platform with respect to the ground, we want to be able to calculate how much the motor needs to turn. Therefore, it is essentially solving for an equation that represents $\Delta\theta_{motor}$ in terms of θ_{new} and θ_{old} of the platform.

First, we want to substitute h for $R\theta_{motor}$, where R is the radius of the motor knob mount, as pictured to the right in a 3D cad we created for the specific motor. In this design, it is 10mm.

Then, we can easily solve for the expression:

$$\begin{aligned}
\sin(\theta_{board}) &= h/l \rightarrow \sin(\theta_{board}) = R\theta_{motor}/l \\
R\theta_{motor}/l &= \sin(\theta_{board}) \\
\theta_{motor} &= \frac{l\sin(\theta_{board})}{R} \\
\Delta\theta_{motor} &= \frac{l\sin(\theta_{board\ new})}{R} - \frac{l\sin(\theta_{board\ old})}{R}.
\end{aligned}$$

Theoretically, to combine both models and programs, we will take the solved angle from the equation for distance in the previous section and plug that value for θ_{new} to achieve a cohesive target-angle system.

The final version of this code is available in Appendix B. We tested the rotateMotor function and it qualitatively worked extremely well, rotating the motor approximately the amount of degrees the user inputted in the Serial monitor.

Conclusion

Our project was extremely fruitful. Since many projects define success in their terms, I will allow the reader to make the judgment themselves. With that being said, I accomplished my goal of creating a coil gun that will launch a magnet with the push of a button. Moreover, I developed theoretical models and computer programs that describe the motion of a magnet within a solenoid along its axis of symmetry and the motion of a projectile accelerated through my gun to a certain target. Additionally, I learned a lot about many different concepts that I previously hadn't touched on before, which is fruitful in our terms. Lastly, I managed my time extremely well, spacing out the workload and ensuring that I had enough time to create our prototype, solve equations, and write this research report.

There are only a few setbacks that I would like to reflect on, one being how I could have improved my model of the magnet inside the solenoid by making computer programs to simulate and solve the equation and considering edge effects in our calculations. Furthermore, I did not manage to create a magnet detection system with the photoresistor since there were issues with that. So, in the future, I would buy components from more trustworthy suppliers. Overall, I resolved issues persistently and effectively, which was also rewarding.

Appendix A

Schematic diagram file for the coil gun (V6.4.2024)

schematic.sch

Magnetic Dipole Moment Calculations

To calculate the magnetic dipole moment of a magnet, we need to employ the equation

$$m = \frac{B_r V}{\mu_0}.$$ [19]

Where

- B_r is the magnet's residual flux density
- V is the magnet's volume
- and μ_o is the magnetic permeability of free space

We can calculate V easily by measuring the magnet, which yields us

$$V = (\pi(\frac{0.25}{2})^2) * 2\, inches = 0.981747 in^3 = 1.60879 * 10^{-5} m^3$$

For B_r, I had to determine the MMPA class of our magnet's material, which in this case is Alnico. To do this, I had to look up the maximum service temperature, density, and more on a reference sheet located on the sixth page of this pdf of our model A5RC025X200 magnet and compare it with values on a table in the Wikipedia page containing many values.

[19] Formula from https://www.kjmagnetics.com/blog.asp?p=dipole

MMPA class	IEC code ref.	by weight (Fe comprises remainder)					Max. energy product, $(BH)_{max}$		Residual induction, B_r		Coercive force, H_c		Intrinsic coercive force, H_{ci}		Density		Tensile strength		Transverse modulus of rupture		HRC	Thermal expansion coefficient (10^{-6} per °C)	Electrical resistivity, at 20 °C (μΩ·cm)	Reversible temp. coefficient, (% per °C)			Curie temp.		Max. service temp.	
		Al	Ni	Co	Cu	Ti	(MGOe)	(kJ/m³)	(gauss)	(mT)	(Oe)	(kA/m)	(Oe)	(kA/m)	(lb/in³)	(g/cm³)	(psi)	(MPa)	(psi)	(MPa)				Near B_r	Near max. energy prod.	Near H_c	(°C)	(°F)	(°C)	(°F)
Isotropic cast AlNiCo																														
Alnico 1	R1-0-1	12	21	5	3	-	1.4	11.1	7200	720	470	37	480	38	0.249	6.9	4000	28	14000	97	45	12.6	75							
Alnico 2	R1-0-4	10	19	13	3	-	1.7	13.5	7500	750	560	45	580	46	0.256	7.1	3000	21	7000	48	45	12.4	65	-0.03	-0.02	-0.02	810	1490	450	840
Alnico 3	R1-0-2	12	25	-	3	-	1.35	10.7	7000	700	480	38	500	40	0.249	6.9	12000	83	23000	158	45	13.0	60							
Anisotropic cast AlNiCo																														

[20]

Something that I noticed is that our magnet has a density two times less than the density given in the specification sheet, which is quite strange. To specify, our measured mass was 0.25 pounds, or 0.011339809 kg, which means that our density is 0.255102 lb/in^3.

Additionally, the temperature specifications on the spec sheet do not match up with what the table lists for Alnico 5, which is a bit suspicious.

Thus, this density difference and the max service temperature of the magnet's model in this spec sheet leads us to believe that the magnet's material is MMPA class 2.

This means that the B_r is 0.750 T.

μ_o is $4\pi \times 10^{-7}$.

Therefore,

$$m = \frac{0.75*1.61\times10^{-5}}{4\pi\times10^{-7}} = 0.9602384\ A \cdot m^2$$

[20] Wikipedia page can be found here: https://en.wikipedia.org/wiki/Alnico

Appendix B

Motor Control Code

```
//Includes the Arduino Stepper Library - a library that contains the raw
code to run stepper motors
#include <Stepper.h>

// Defines the number of steps per rotation
const int stepsPerRevolution = 2038;
//15cm
const float l = 0.015
// 10mm radius
const float R = 0.01

//variables used to keep track of angle of the board with respect to the
ground
float thetaOld = 0;
float thetaNew = 0;

// Stepper class
// IN1-IN3-IN2-IN4 - specific pin sequence that is used by the
microcontroller to control the stepper's movement
Stepper myStepper = Stepper(stepsPerRevolution, 8, 10, 9, 11);


void setup() {
  Serial.begin(9600);
}

void loop() {
  //wait for user input
  if (Serial.available() > 0) {
    //converts the user inputted value to a number then converts that to
radians
    String msg = Serial.readString();
    float angle = msg.toFloat();
    float angleRad = (PI/180.0) * angle;
```

```
    thetaNew = angleRad;
    rotateBoard(thetaNew, thetaOld);
  }
}
void rotateBoard(float angleNew, float angleOld){
  float thetaMotor = (l/R)*(sin(angleNew)-sin(angleOld));
  rotateMotor(thetaMotor);
  thetaOld = thetaNew;
}

void rotateMotor(float theta){
  float steps = theta * (stepsPerRevolution/(2.0*PI));
  myStepper.setSpeed(5);
  int intsteps = round(steps);
  //.step can only take in integers
  //myStepper.step tells the stepper motor to run until intsteps amount of
steps is turned
  myStepper.step(intsteps);
}
```

Root Calculator Program

```
import numpy as np
from scipy.optimize import fsolve[21]

# Init values (in a scenario)
vi = 0.01   # initial velocity (m/s)
g = 9.81  # gravitational acceleration  at the surface of earth
l = 0.015  # length (m)
D = 0.030  # distance (m)


def Dis(theta):
    return (
        (
            (
                vi * np.sin(theta)
```

[21] fsolve is a function from the scipy library which uses Hybrd root solving algorithm that involves a jacobian. Learn more at https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.fsolve.html

```
                + np.sqrt((vi * np.sin(theta)) ** 2 + g * l *
np.sin(theta))
            )
            / g
        )
        + l * np.cos(theta)
        - D
    )

initial_guess = 0.52  # 0.52 rad ~ 30 deg

# set factor to 0.1 and epsfcn to 0.1 because our function is very
confined to a small interval (both are related to step lengths)
(solution,) = fsolve(Dis, initial_guess, factor=0.1, epsfcn=0.1)

theta_solution = solution
theta_solution_degrees = np.degrees(theta_solution)

print(theta_solution, theta_solution_degrees)
```