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MATES ED2MIT

Education and Training for Data Driven Maritime Industry

Tutorial D01

Statistical Data Analysis Basics: Data Structures, Statistical Characteristics

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Maritime Alliance for fostering the European Blue economy through a Marine Technology Skilling Strategy

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Outline

- Types of data
  - Quantitative data
  - Qualitative data
- Statistical characteristics
- Distributions
  - Normal distribution
- Measures of data dissimilarity
- Summary and takeaway

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Types of Data Sets

- Record
  - Relational records
  - Data matrix, e.g., numerical matrix, crosstabs
  - Document data: text documents: term-frequency vector
  - Transaction data
- Graph and network
  - World Wide Web
  - Social or information networks
  - Molecular Structures
- Ordered
  - Video data: sequence of images
  - Temporal data: time-series
  - Sequential Data: transaction sequences
  - Genetic sequence data
- Spatial, image and multimedia:
  - Spatial data: maps
  - Image data:
  - Video data:

	team	coach	play	ball	score	game	win	lost	timeout	season
Document 1	3	0	5	0	2	6	0	2	0	2
Document 2	0	7	0	2	1	0	0	3	0	0
Document 3	0	1	0	0	1	2	2	0	3	0

Cases/row/ events

TID	*Items*
1	**Bread, Coke, Milk**
2	**Beer, Bread**
3	**Beer, Coke, Diaper, Milk**
4	**Beer, Bread, Diaper, Milk**
5	**Coke, Diaper, Milk**

Concepts and Characteristics of Structured Data

- Dimensionality
  - Curse of dimensionality
- Sparsity
  - Only presence counts
- Resolution
  - Patterns depend on the scale
- Distribution
  - Centrality and dispersion

- Population: the whole set of a "universe"
- Sample: a sub-set of a population
- Parameter: an unknown "fixed" value of population characteristic
- Statistic: a known/calculable value of sample characteristic representing that of the population. E.g.

  μ = mean of population, X~ = mean of sample

Data Objects

- Data sets are made up of data objects
  - In a simple form data records
- A **data object** represents an entity.
- Examples:
  - sales database: customers, store items, sales
  - medical database: patients, treatments
  - university database: students, professors, courses
- Also called *samples* , *examples, instances, data points, objects, tuples*
- Data objects are described by **attributes**
- Database rows -> data objects; columns ->attributes

Attributes

- **Attribute (**or **dimensions, features, variables**): a data field, representing a characteristic or feature of a data object.
  - *E.g., customer _ID, name, address*
- Types:
  - Nominal
  - Binary
  - Numeric: quantitative, e.g. numerical/numbers
    - Interval-scaled
    - Ratio-scaled
  - Categorical e.g. Yes or No

Attribute Types

- **Nominal:** categories, states, or “names of things”
  - *Hair_color* = {*auburn, black, blond, brown, grey, red, white*}
  - marital status, occupation, ID numbers, zip codes
- **Binary (aka dummy)**
  - Nominal attribute with only 2 states (0 and 1)
  - <u>Symmetric binary</u>: both outcomes equally important
    - e.g., gender
  - <u>Asymmetric binary</u>: outcomes not equally important.
    - e.g., medical test (positive vs. negative)
    - Convention: assign 1 to most important outcome (e.g., HIV positive)
- **Ordinal**
  - Values have a meaningful order (ranking) but magnitude between successive values is not known.
  - *Size* = {*small, medium, large*}, grades, army rankings

Numeric Attribute Types

- Quantity (integer or real-valued)
- **Interval**
  - Measured on a scale of **equal-sized units**
  - Values have order
    - E.g., *temperature in C˚or F˚, calendar dates*
  - No true zero-point
- **Ratio**
  - Inherent **zero-point**
  - We can speak of values as being an order of magnitude larger than the unit of measurement (10 K˚ is twice as high as 5 K˚).
    - e.g., *temperature in Kelvin, length, counts, monetary quantities*

Discrete vs. Continuous Attributes

- **Discrete Attribute**
  - Has only a finite or countably infinite set of values
    - E.g., zip codes, profession, or the set of words in a collection of documents
  - Sometimes, represented as integer variables
  - Note: Binary attributes are a special case of discrete attributes
- **Continuous Attribute**
  - Has real numbers as attribute values
    - E.g., temperature, height, or weight
  - Practically, real values can only be measured and represented using a finite number of digits
  - Continuous attributes are typically represented as floating-point variables

Basic Statistical Descriptions of Data

- Motivation
  - To better understand the data: central tendency, variation and spread
- Data dispersion characteristics
  - median, max, min, quantiles, outliers, variance, etc.
- Numerical dimensions correspond to sorted intervals
  - Data dispersion: analyzed with multiple granularities of precision
  - Boxplot or quantile analysis on sorted intervals
- Dispersion analysis on computed measures
  - Folding measures into numerical dimensions
  - Boxplot or quantile analysis on the transformed cube

Central Tendency: Mean, Median, Mode

Measure	Advantages	Disadvantages
Mean (Sum of all values ÷ no. of values)	* Best known average * Exactly calculable * Make use of all data * Useful for statistical analysis	* Affected by extreme values * Can be absurd for discrete data (e.g. Family size = 4.5 person) * Cannot be obtained graphically
Median (middle value)	* Not influenced by extreme values * Obtainable even if data distribution unknown (e.g. group/aggregate data) * Unaffected by irregular class width * Unaffected by open-ended class	* Needs interpolation for group/ aggregate data (cumulative frequency curve) * May not be characteristic of group when: (1) items are only few; (2) distribution irregular * Very limited statistical use
Mode (most frequent value)	* Unaffected by extreme values * Easy to obtain from histogram * Determinable from only values near the modal class	* Cannot be determined exactly in group data * Very limited statistical use

Measuring the Central Tendency

- Mean (algebraic measure) (sample vs. population):

  Note: *n* is sample size and *N* is population size.

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \mu = \frac{\sum x}{N}$$

  - Weighted arithmetic mean:

$$\bar{x} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

  - Trimmed mean: chopping extreme values
- Median:
  - Middle value if odd number of values, or average of the middle two values otherwise
  - Estimated by interpolation (for *grouped data*):

$$median = L_1 + \left(\frac{n/2 - (\sum freq)l}{freq_{median}}\right) width$$

age	frequency
1–5	200
6–15	450
16–20	300
21–50	1500
51–80	700
81–110	44

- Mode
  - Value that occurs most frequently in the data
  - Unimodal, bimodal, trimodal
  - Empirical formula: $mean - mode = 3 \times (mean - median)$

Central Tendency – "Mean", $\bar{x}$ Example

- For individual observations
  X = {3,5,7,7,8,8,8,9,9,10,10,12}

$$X = \frac{\sum x_i}{n}$$

$\sum fx$ = 96; n = 12

Thus, $X = \frac{\sum x_i}{n}$ = 96/12 = 8

- The above observations can be organised into a frequency table and mean calculated on the basis of frequencies

x	3	5	7	8	9	1 0	1 2
f	1	1	2	3	2	2	1
Σf	3	5	1 4	2 4	1 8	2 0	1 2

$$X = \frac{\sum fx}{\sum f}$$

$\sum fx$ = 96; $\sum f$ = 12

Thus, $X = \frac{\sum fx}{\sum f}$ = 96/12 = 8

Variability (1)

- Indicates dispersion, spread, variation, deviation
- For single population or sample data:

$$\sigma^2 = \frac{\sum(x_i - \mu)^2}{n} \qquad s^2 = \frac{\sum(x_i - \bar{x})^2}{n-1}$$

where σ^2 and s^2 = population and sample variance respectively, x_i = individual observations, μ = population mean, $\bar{x}$ = sample mean, and n = total number of individual observations.

- The standard deviation is the square roots

$$\sigma = \sqrt{\frac{\sum(x_i - \mu)^2}{n}} \qquad s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$$

Variability (2)

- Why “measure of dispersion” important?
- Consider returns from two categories of shares:

  * Shares A (%) = {1.8, 1.9, 2.0, 2.1, 3.6}
  * Shares B (%) = {1.0, 1.5, 2.0, 3.0, 3.9}

  Mean A = mean B = 2.28%
  But different variability!
  Var(A) = 0.557, Var(B) = 1.367

  * Would you invest in category A shares or category B shares?

Variability (3)

- Coefficient of variation – COV – std. deviation as % of the mean:

$$COV = \frac{\sqrt{\dfrac{\sum(x_i - \bar{x})^2}{n}}}{\dfrac{\sum x_i}{n}} \times 100$$

- Could be a better measure compared to std. dev.

  COV(A) = 32.73%, COV(B) = 51.28%

Measuring the Dispersion of Data

- Quartiles, outliers and boxplots
  - **Quartiles**: Q_1 (25th percentile), Q_3 (75th percentile)
  - **Inter-quartile range**: IQR = $Q_3 – Q_1$
  - **Five number summary**: min, Q_1, median, Q_3, max
  - **Boxplot**: ends of the box are the quartiles; median is marked; add whiskers, and plot outliers individually
  - **Outlier**: usually, a value higher/lower than 1.5 x IQR
- Variance and standard deviation (*sample: s, population: σ)*
  - **Variance**: (algebraic, scalable computation)

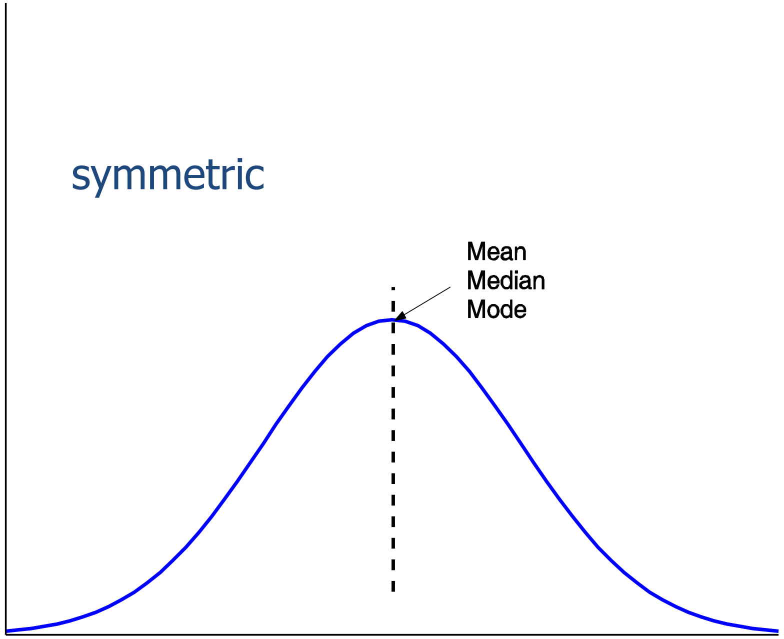
$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{1}{n-1}[\sum_{i=1}^{n}x_i^{\ 2} - \frac{1}{n}(\sum_{i=1}^{n}x_i)^2]$$

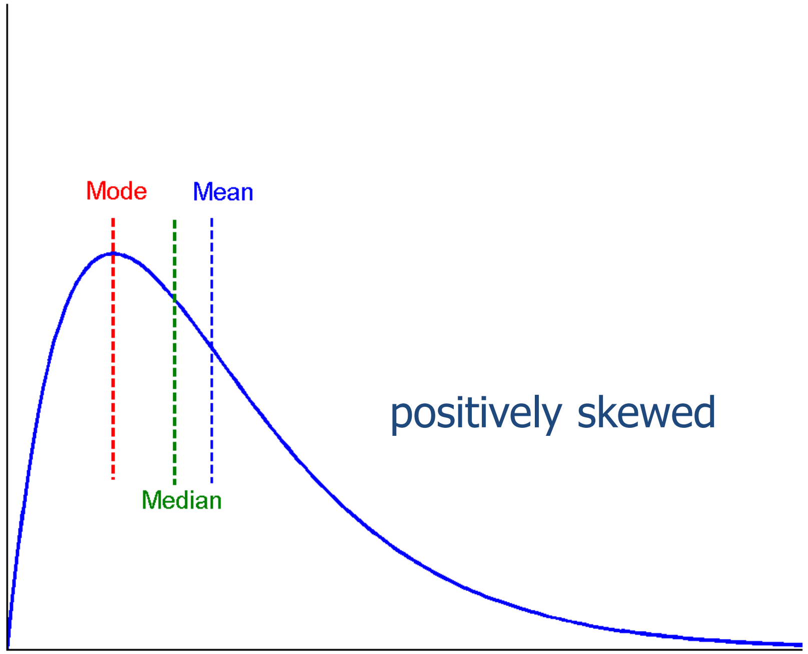
$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{n}(x_i - \mu)^2 = \frac{1}{N}\sum_{i=1}^{n}x_i^{\ 2} - \mu^2$$

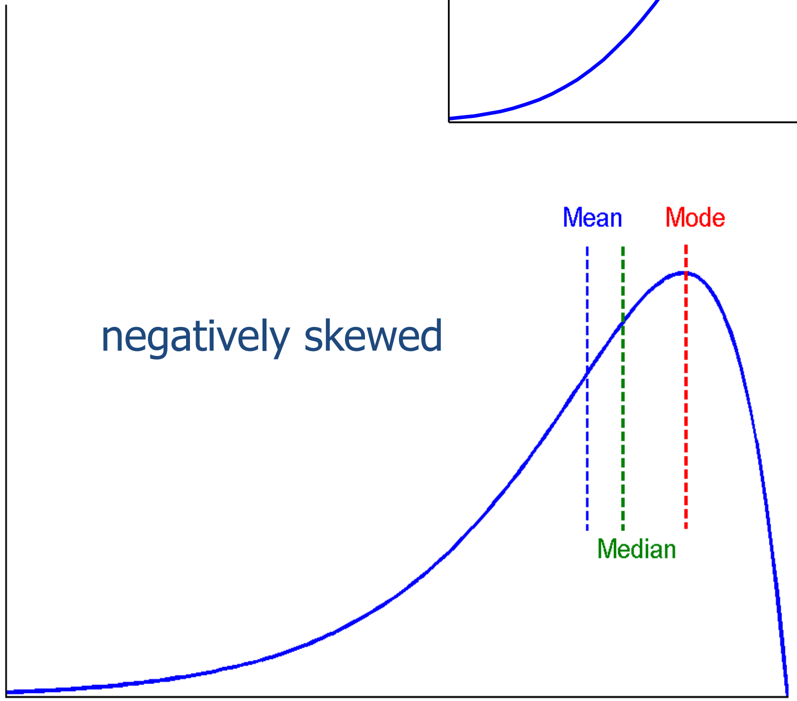
  - **Standard deviation** *s (or σ)* is the square root of variance s^2 *(or* σ^2*)*

Symmetric vs. Skewed Data

- Median, mean and mode of symmetric, positively and negatively skewed data





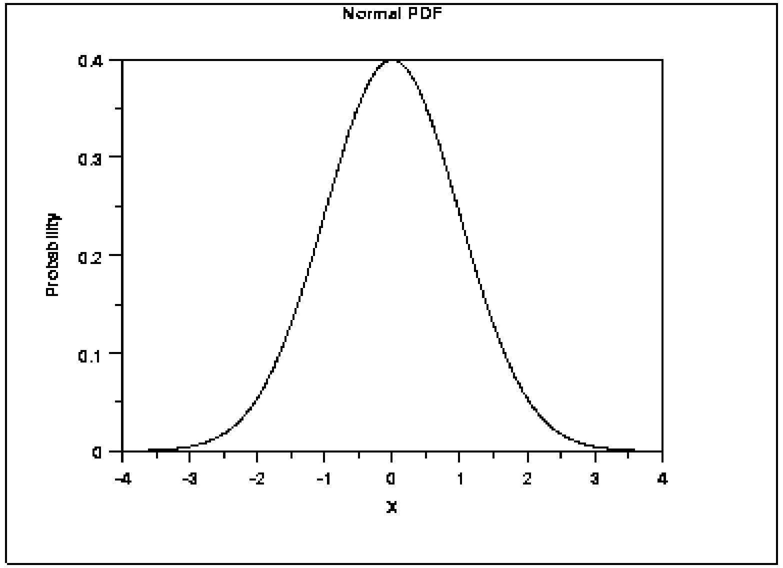


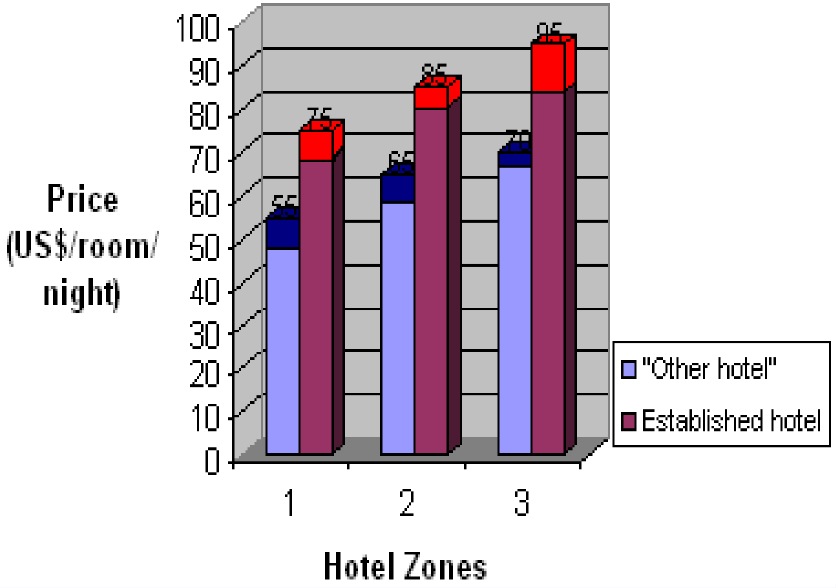
Probability Distribution

- Defined as of probability density function (pdf).
- Many types: Z, t, F, gamma, etc.
- “God-given” nature of the real world event.
- General form:

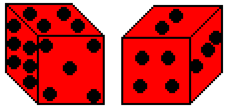
$$\int_a^b f(x)dx = Pr[a \le X \le b] \quad \text{(continuous)}$$

$$f(x) = Pr[X = x] \quad \text{(discrete)}$$



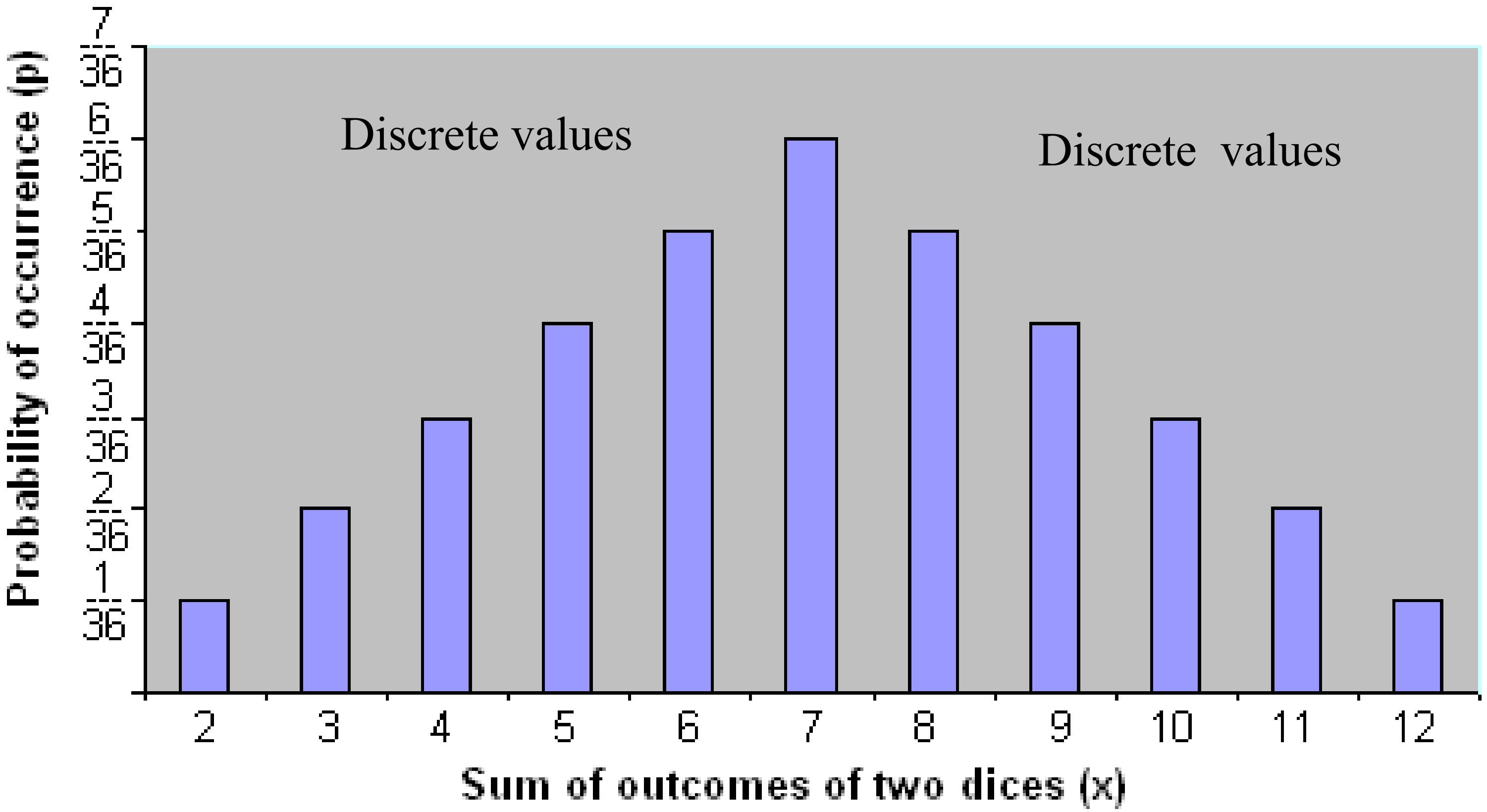


Probability Distribution – Dice example



Dice1 Dice2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

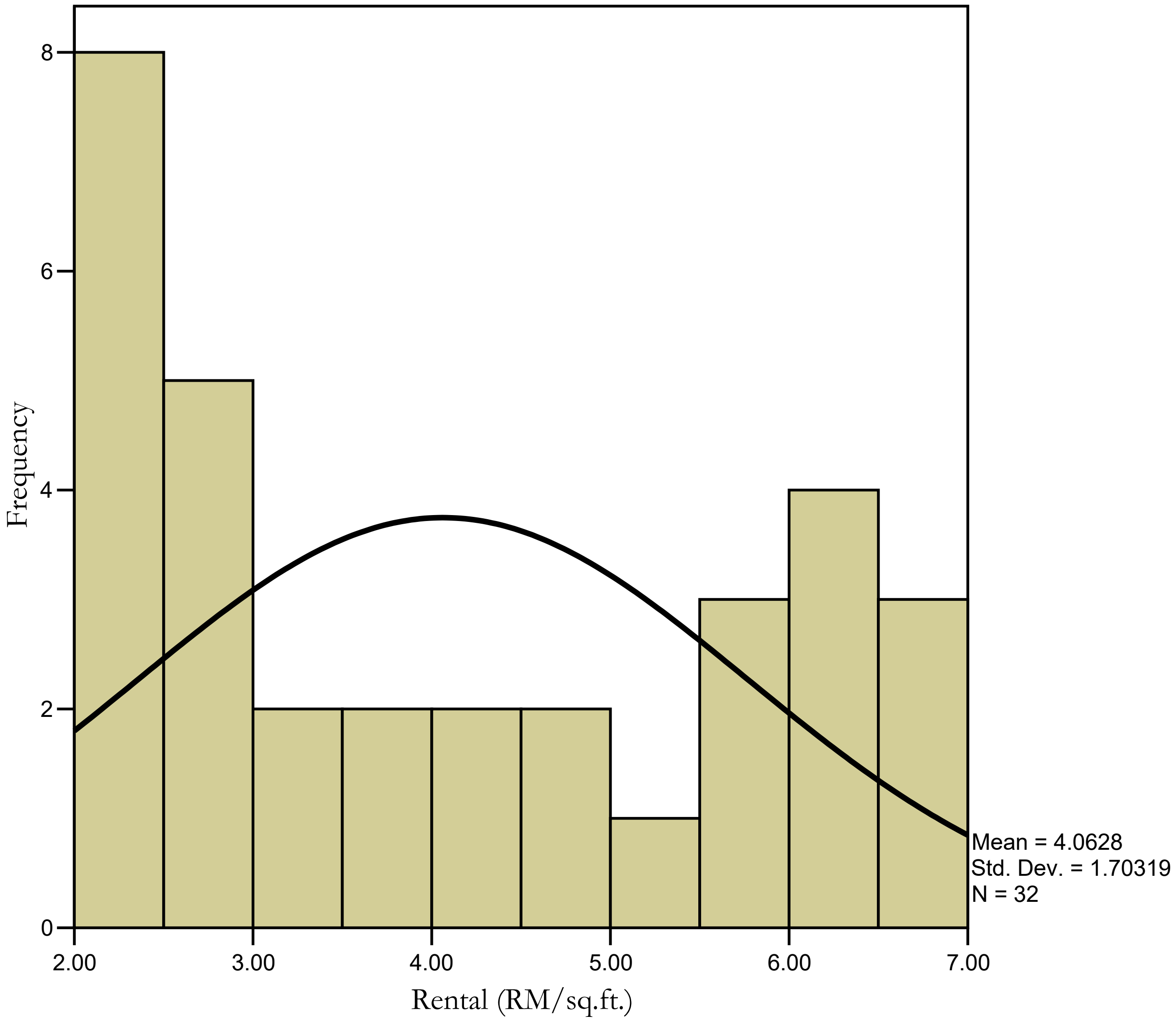
Probability Distribution – Dice bets probability



Values of x are discrete (discontinuous)

Sum of lengths of vertical bars $\sum_{\text{all } x} p(X=x) = 1$

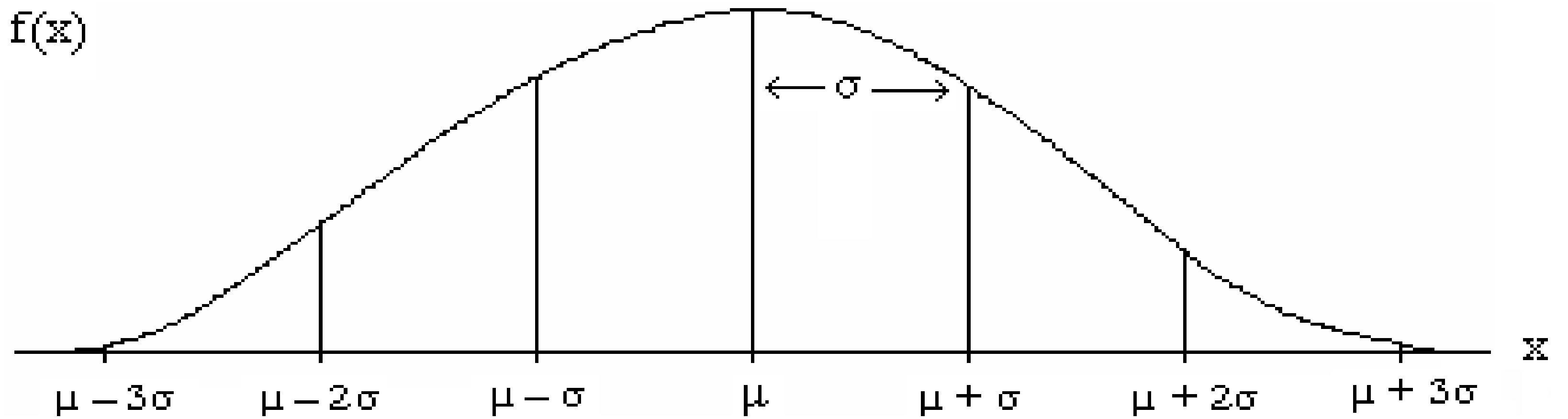
Probability Distribution – Discrete Distribution



- Many real world phenomena take a form of continuous random variable
- Can take any values between two limits (e.g. income, age, weight, price, rental, etc.)

Probability Distribution – Bell shaped

- Ideal distribution of a phenomena:



  - Bell-shaped, symmetrical
  - Has a function of

$$f(x) = \frac{1}{\sigma\sqrt{2\Pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

μ = mean of variable x

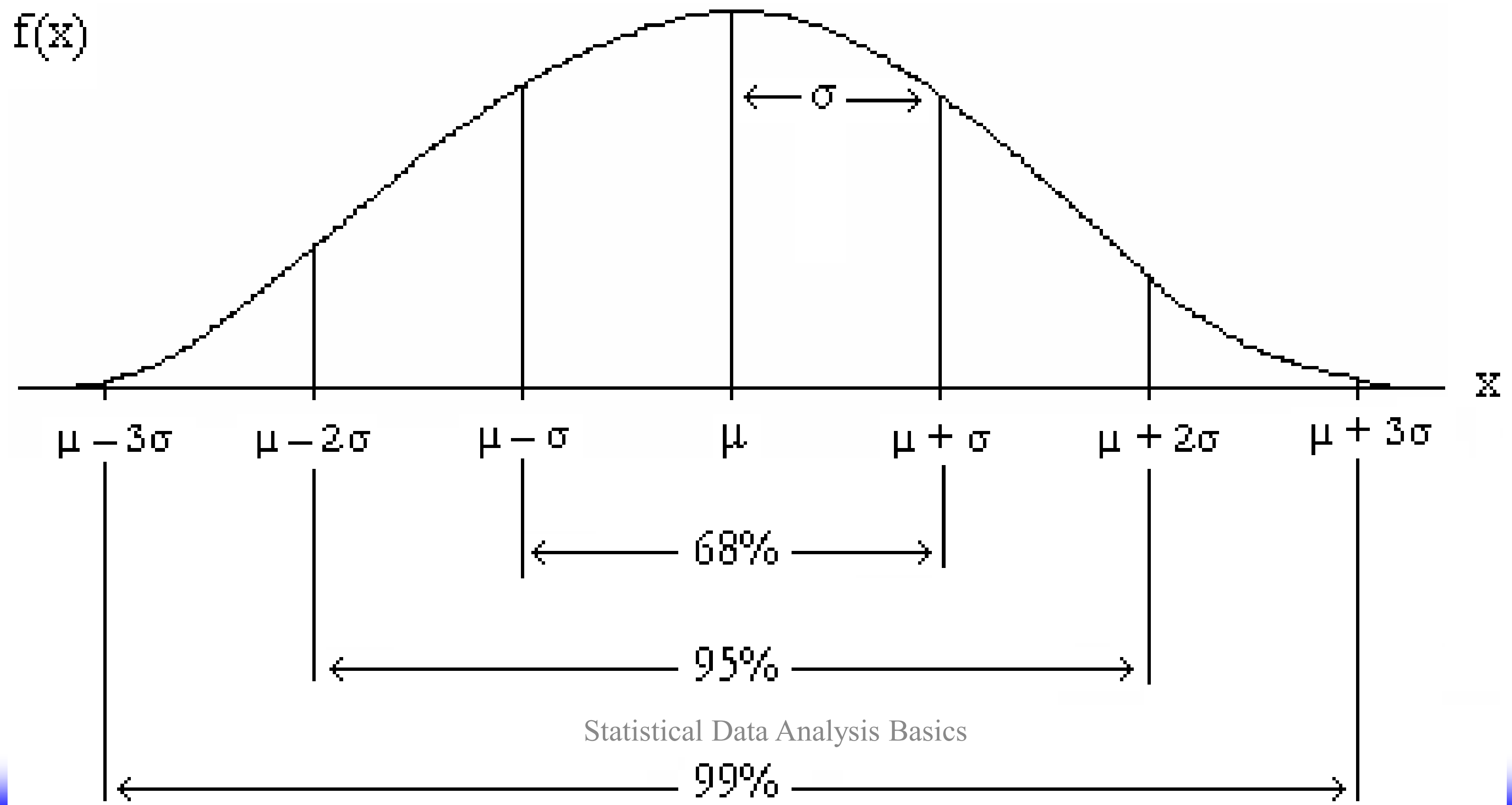
σ = std. dev. Of x

π = ratio of circumference of a circle to its diameter = 3.14

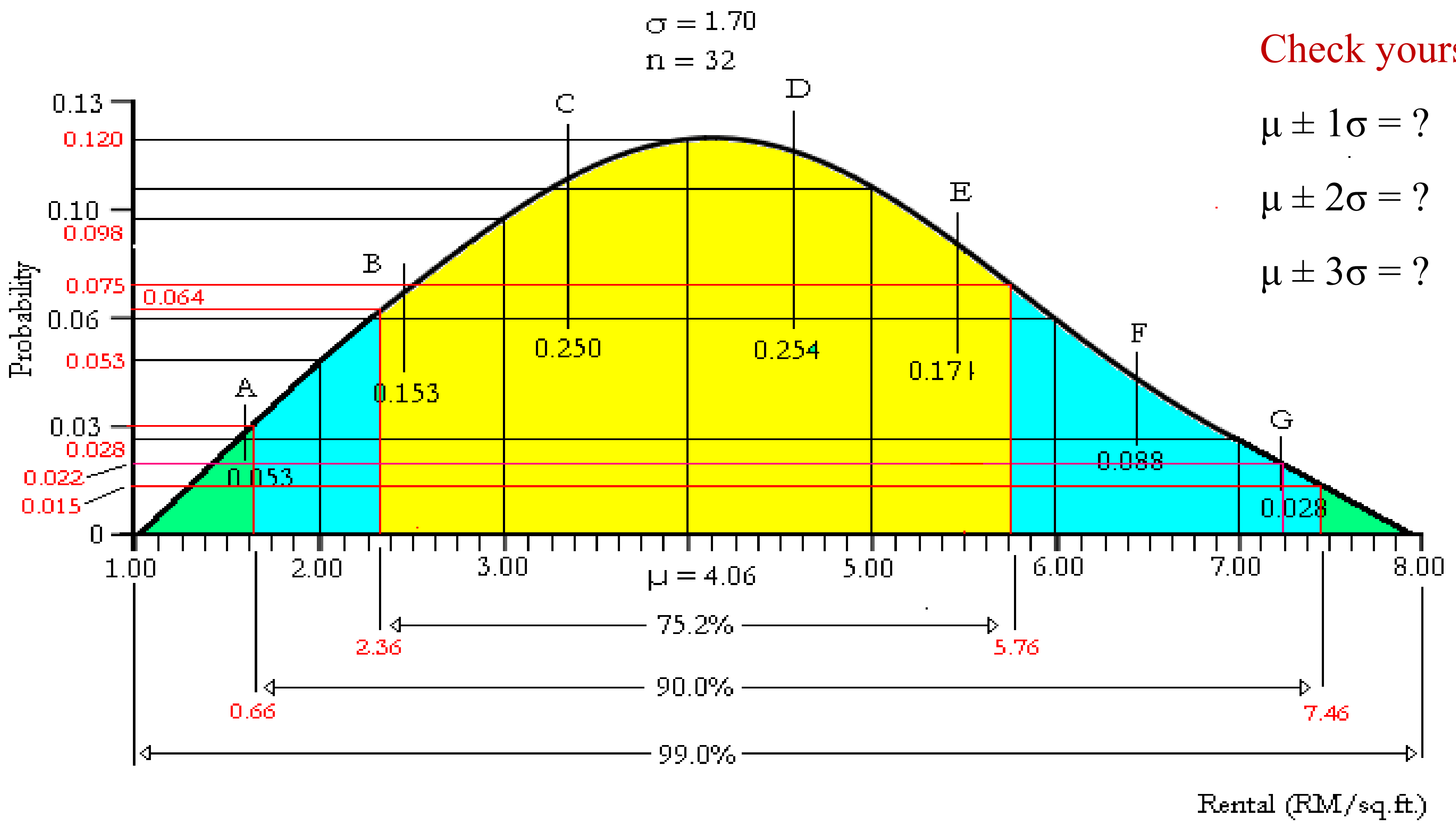
e = base of natural log = 2.71828

Properties of Normal Distribution Curve

- The normal (distribution) curve
  - From μ–σ to μ+σ: contains about 68% of the measurements (μ: mean, σ: standard deviation)
  - From μ–2σ to μ+2σ: contains about 95% of it
  - From μ–3σ to μ+3σ: contains about 99.7% of it



Probability distribution – Standard Deviation



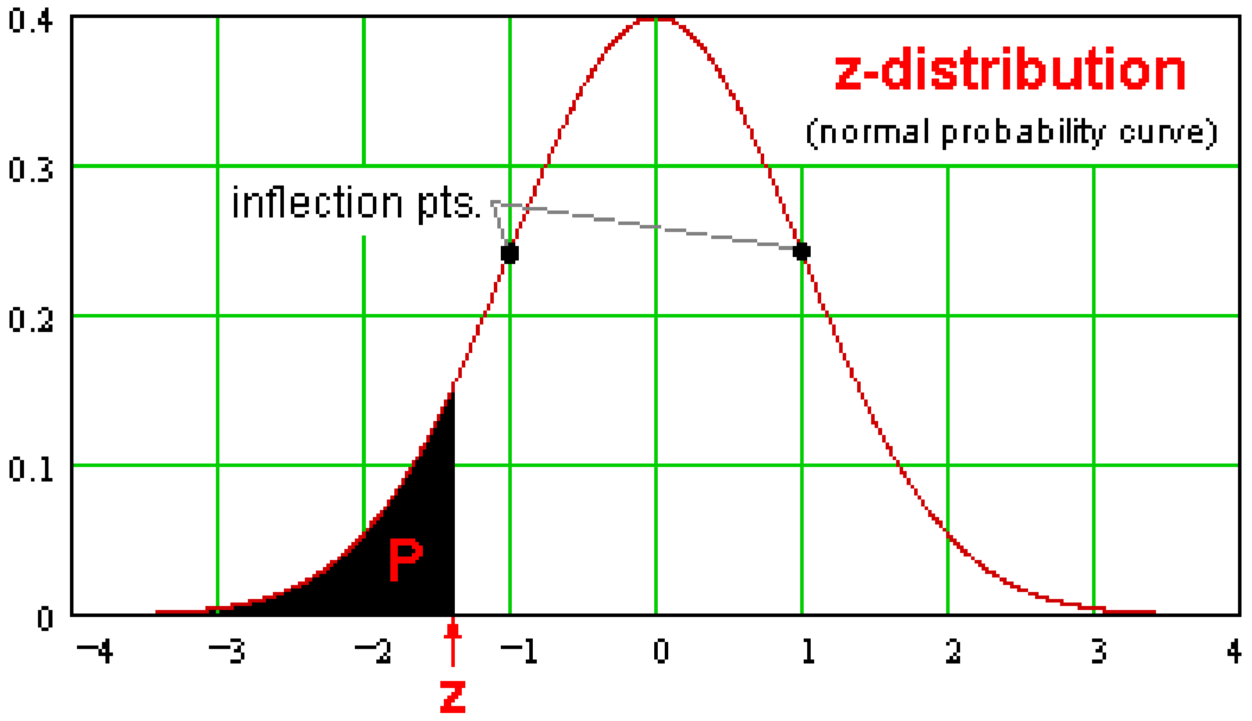
Check yourself

$\mu \pm 1\sigma = ?$ = ____% from total observation

$\mu \pm 2\sigma = ?$ = ____% from total observation

$\mu \pm 3\sigma = ?$ = ____% from total observation

Z-distribution



The z- is a N(0, 1) distribution, given by the equation:

$$f(z) = \frac{1}{2\pi} e^{\frac{-x^2}{2}}$$

The area within an interval (a,b) = normalcdf(a,b) =

$$\int_a^b e^{\frac{-z^2}{2}}\, dz$$

(It is not integratable algebraically.)

Z-Distribution

- $\phi(X=x)$ is given by area under curve
- Has no standard algebraic method of integration → $Z \sim N(0,1)$
- It is called “normal distribution” (ND)
- Standard reference/approximation of other distributions. Since there are various f(x) forming NDs, SND is needed
- To transform f(x) into f(z):

$$Z = \frac{x - \mu}{\sigma} \sim N(0, 1)$$

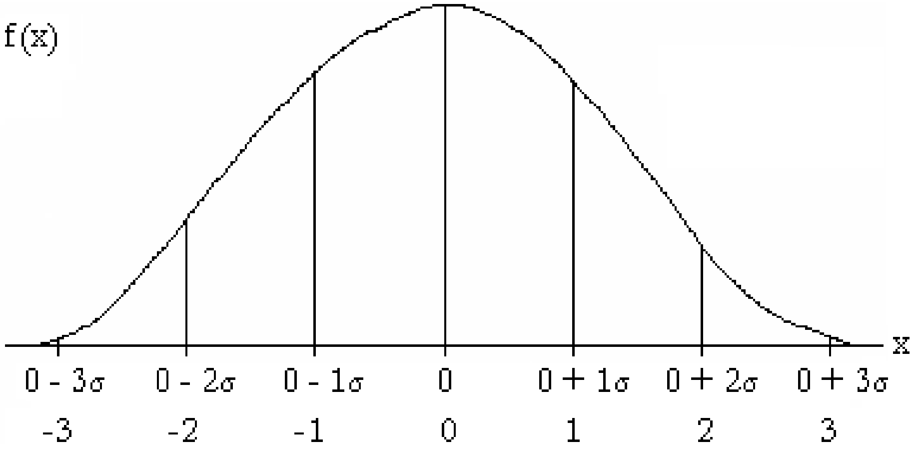
$$\text{E.g. } Z = \frac{160 - 155}{5.4} = 0.926$$

- Probability is such a way that:

  * Approx. 68% $-1 < z < 1$

  * Approx. 95% $-1.96 < z < 1.96$

  * Approx. 99% $-2.58 < z < 2.58$



Z-distribution (2)

- When X= μ, Z = 0, i.e.

$$Z = \frac{X-\mu}{\sigma} = \frac{\mu-\mu}{\sigma} = 0$$

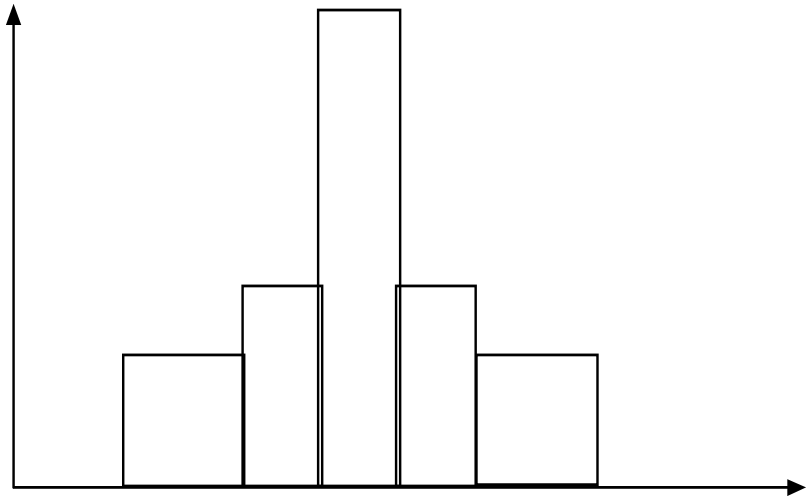
- When X = μ + σ, Z = 1
- When X = μ + 2σ, Z = 2
- When X = μ + 3σ, Z = 3 and so on.
- It can be proven that $P(X_1 < X < X_k) = P(Z_1 < Z < Z_k)$
- SND (standard normal distibution) shows the *probability to the right* of any particular value of Z.

Graphic Displays of Basic Statistical Descriptions

- **Histogram**: x-axis are values, y-axis repres. frequencies
- **Boxplot**: graphic display of five-number summary
- **Quantile plot**: each value x_i is paired with f_i indicating that approximately 100 f_i % of data are $\leq x_i$
- **Quantile-quantile (q-q) plot**: graphs the quantiles of one univariant distribution against the corresponding quantiles of another
- **Scatter plot**: each pair of values is a pair of coordinates and plotted as points in the plane

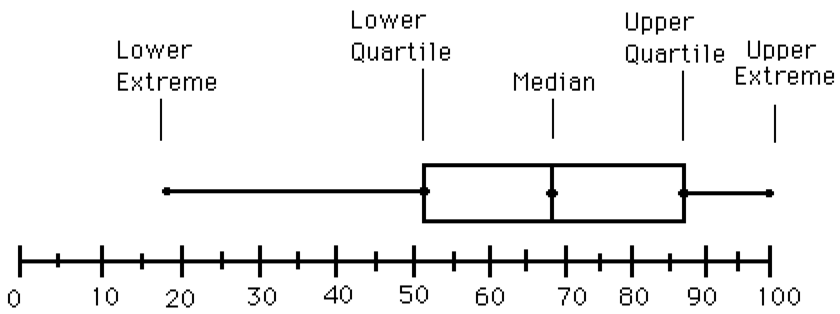
Histogram Analysis

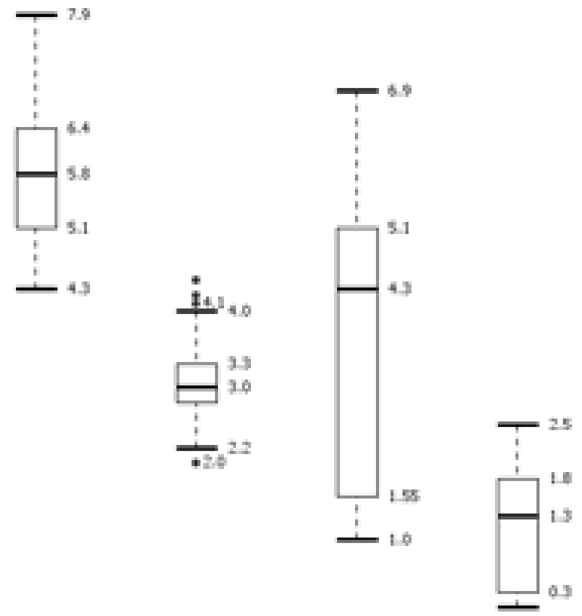
- Histogram: Graph display of tabulated frequencies, shown as bars
- It shows what proportion of cases fall into each of several categories
- Differs from a bar chart in that it is the *area* of the bar that denotes the value, not the height as in bar charts, a crucial distinction when the categories are not of uniform width
- The categories are usually specified as non-overlapping intervals of some variable. The categories (bars) must be adjacent



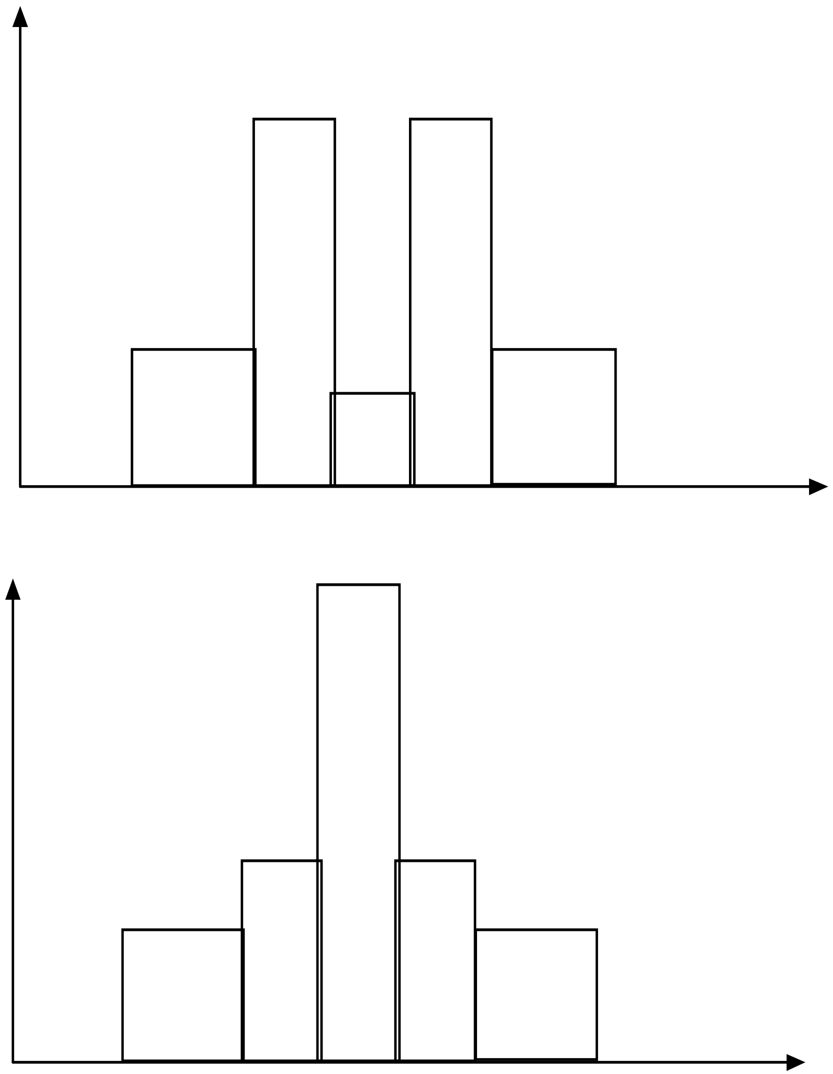
Boxplot Analysis

- **Five-number summary** of a distribution
  - Minimum, Q1, Median, Q3, Maximum
- **Boxplot**
  - Data is represented with a box
  - The ends of the box are at the first and third quartiles, i.e., the height of the box is IQR
  - The median is marked by a line within the box
  - Whiskers: two lines outside the box extended to Minimum and Maximum
  - Outliers: points beyond a specified outlier threshold, plotted individually





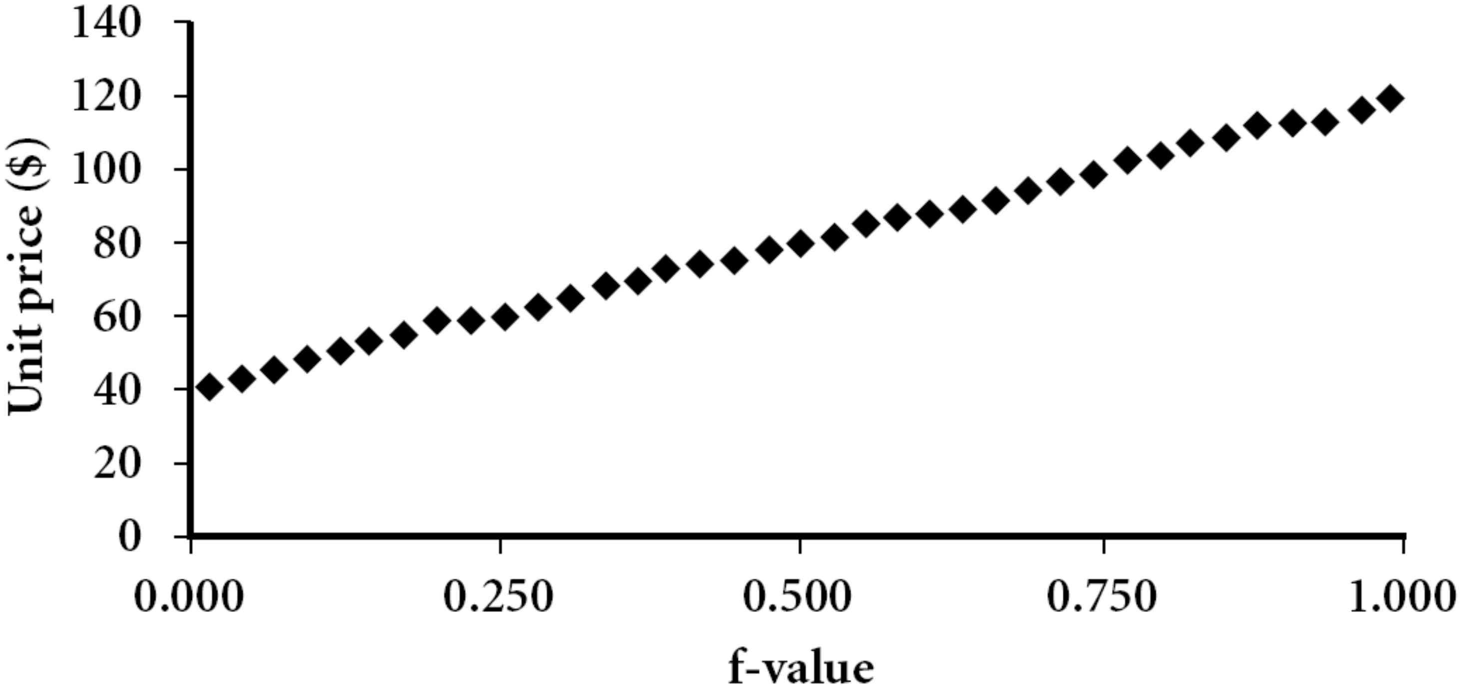
Histograms Often Tell More than Boxplots



- The two histograms shown in the left may have the same boxplot representation
  - The same values for: min, Q1, median, Q3, max
- But they have rather different data distributions

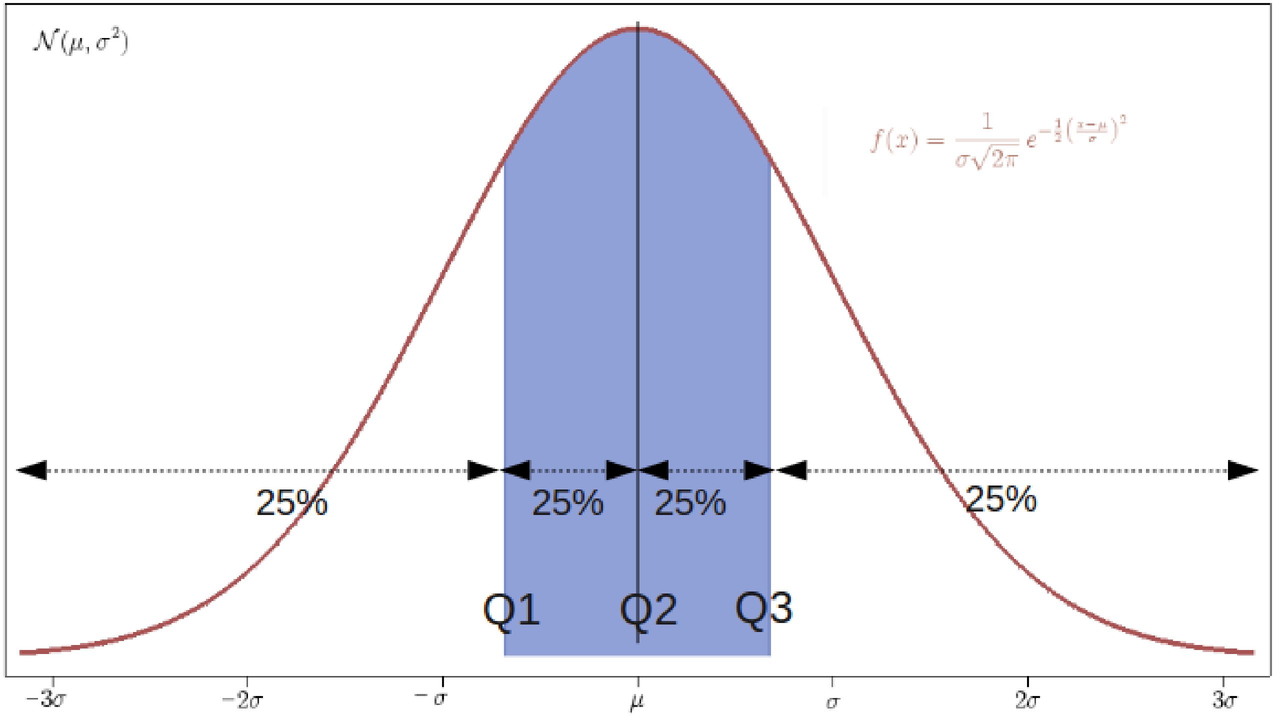
Quantile Plot

- Displays all of the data (allowing the user to assess both the overall behavior and unusual occurrences)
- Plots **quantile** information
  - For a data x_i data sorted in increasing order, f_i indicates that approximately 100 f_i% of the data are below or equal to the value x_i



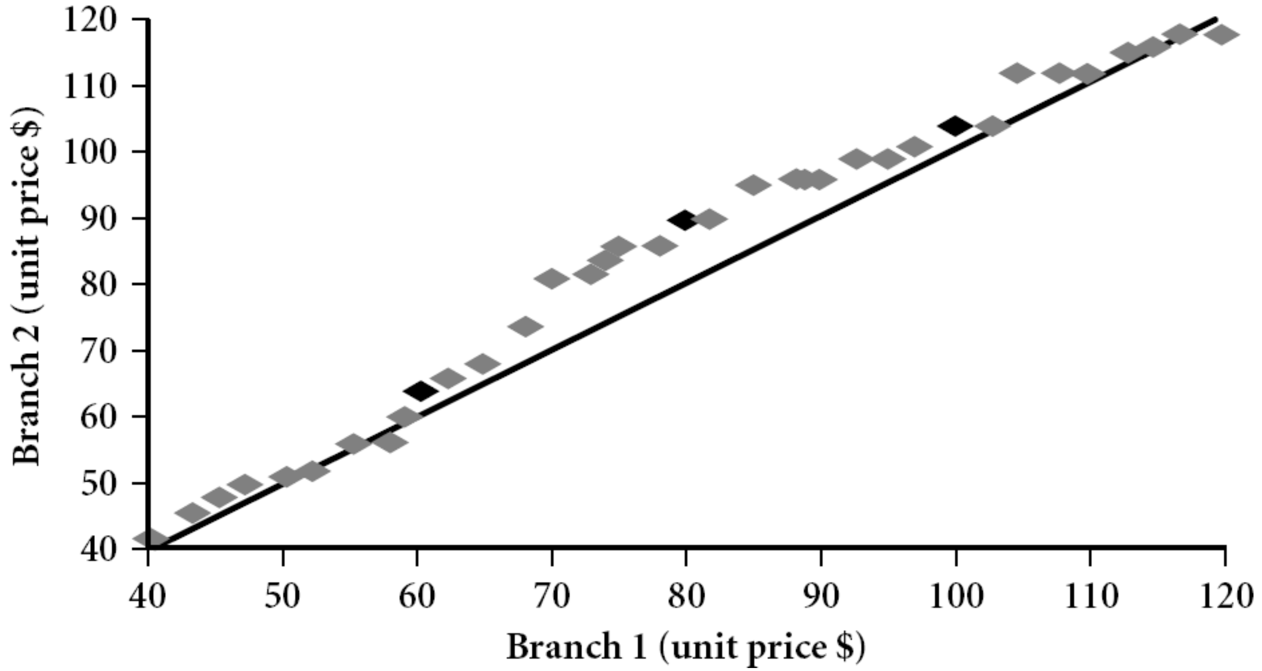
Quantile Plot – What is quartile?

- Probability density of a normal distribution, with quartiles shown.
- The area below the red curve is the same in the intervals (-∞,Q1), (Q1,Q2), (Q2,Q3), and (Q3,+∞).
- Quartiles are used in boxplot
- The only 2-quantile is called the median



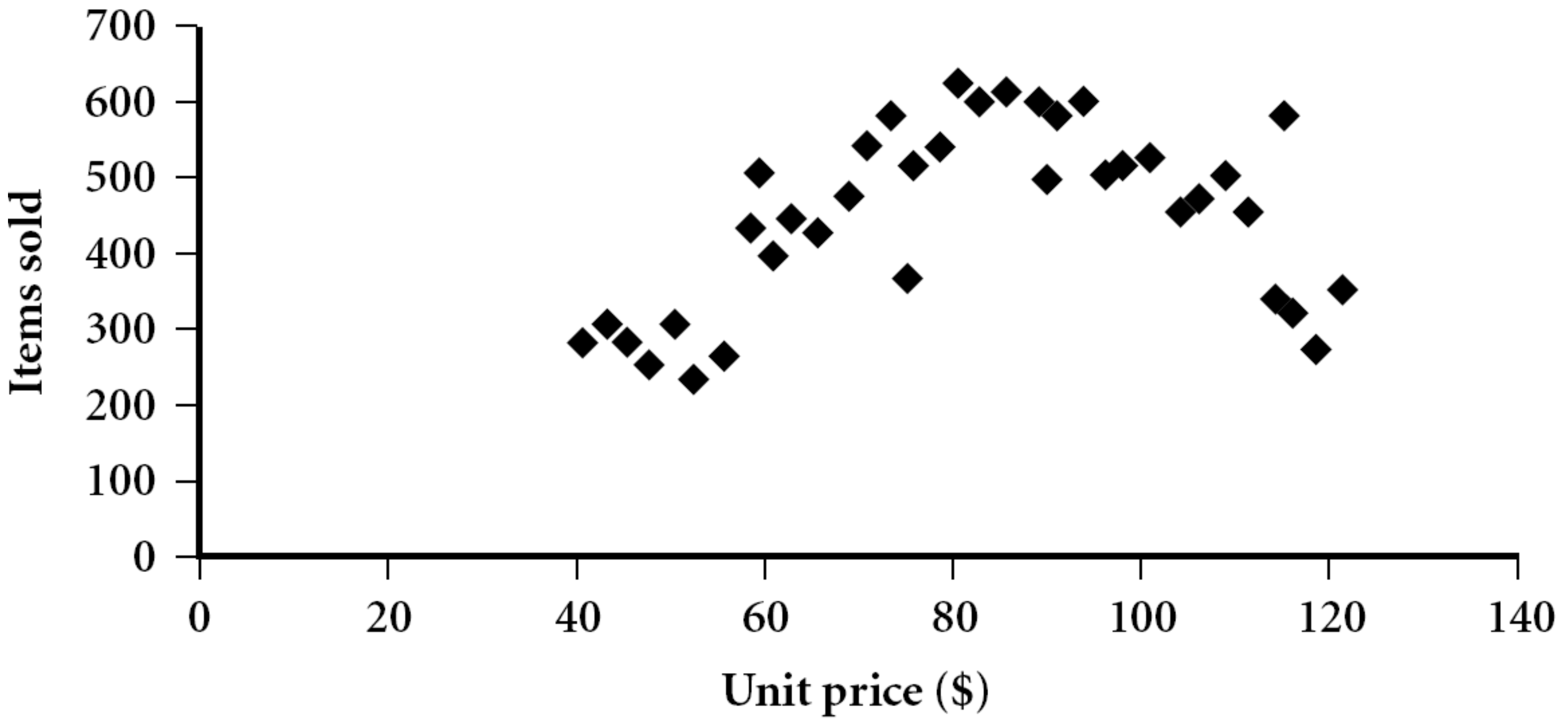
Quantile-Quantile (Q-Q) Plot

- Graphs the quantiles of one univariate distribution against the corresponding quantiles of another
- View: Is there is a shift in going from one distribution to another?
- Example shows unit price of items sold at Branch 1 vs. Branch 2 for each quantile. Unit prices of items sold at Branch 1 tend to be lower than those at Branch 2.

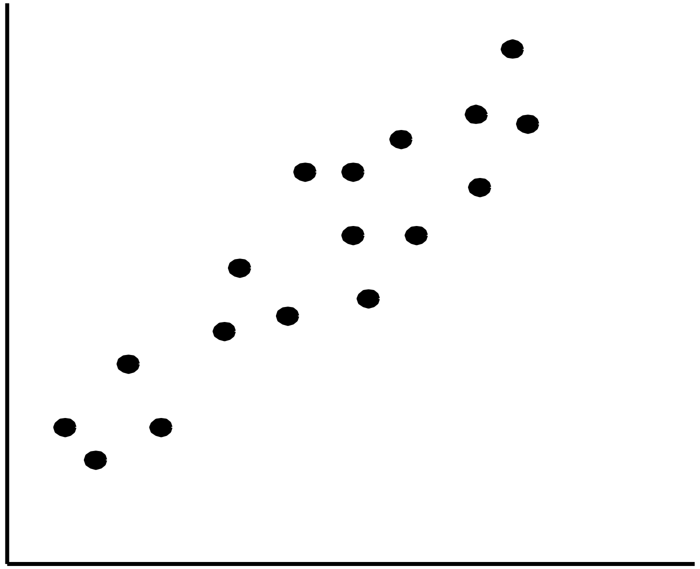


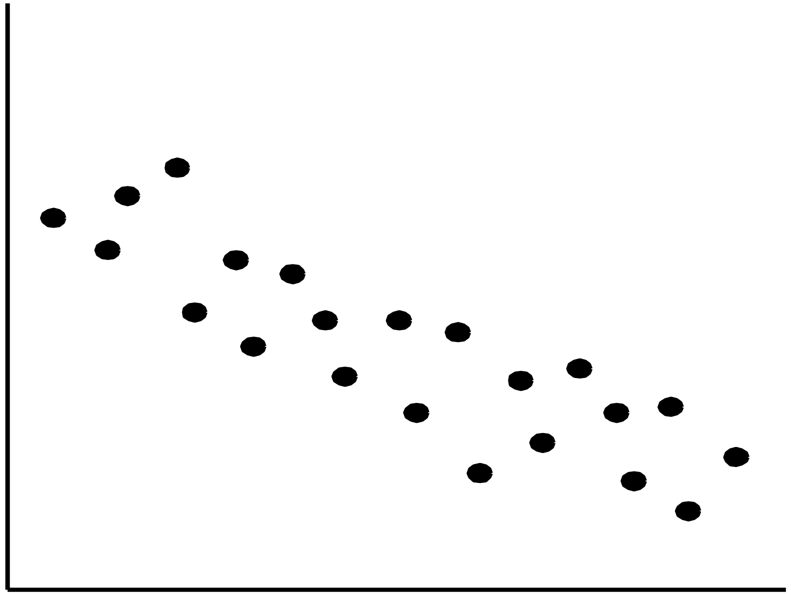
Scatter plot

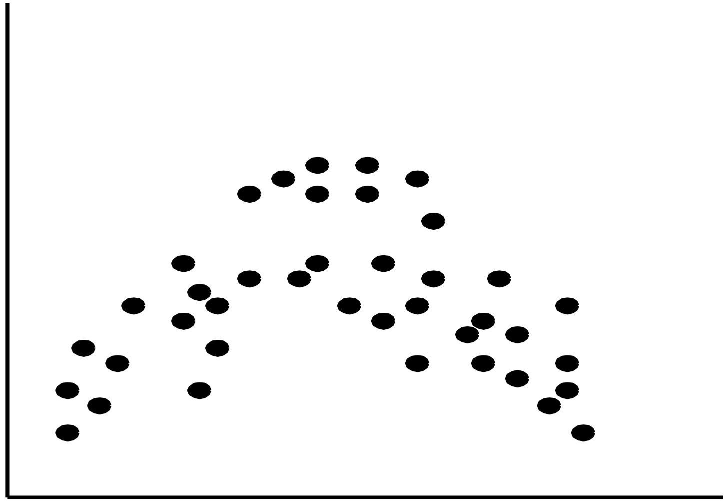
- Provides a first look at bivariate data to see clusters of points, outliers, etc
- Each pair of values is treated as a pair of coordinates and plotted as points in the plane



Positively and Negatively Correlated Data

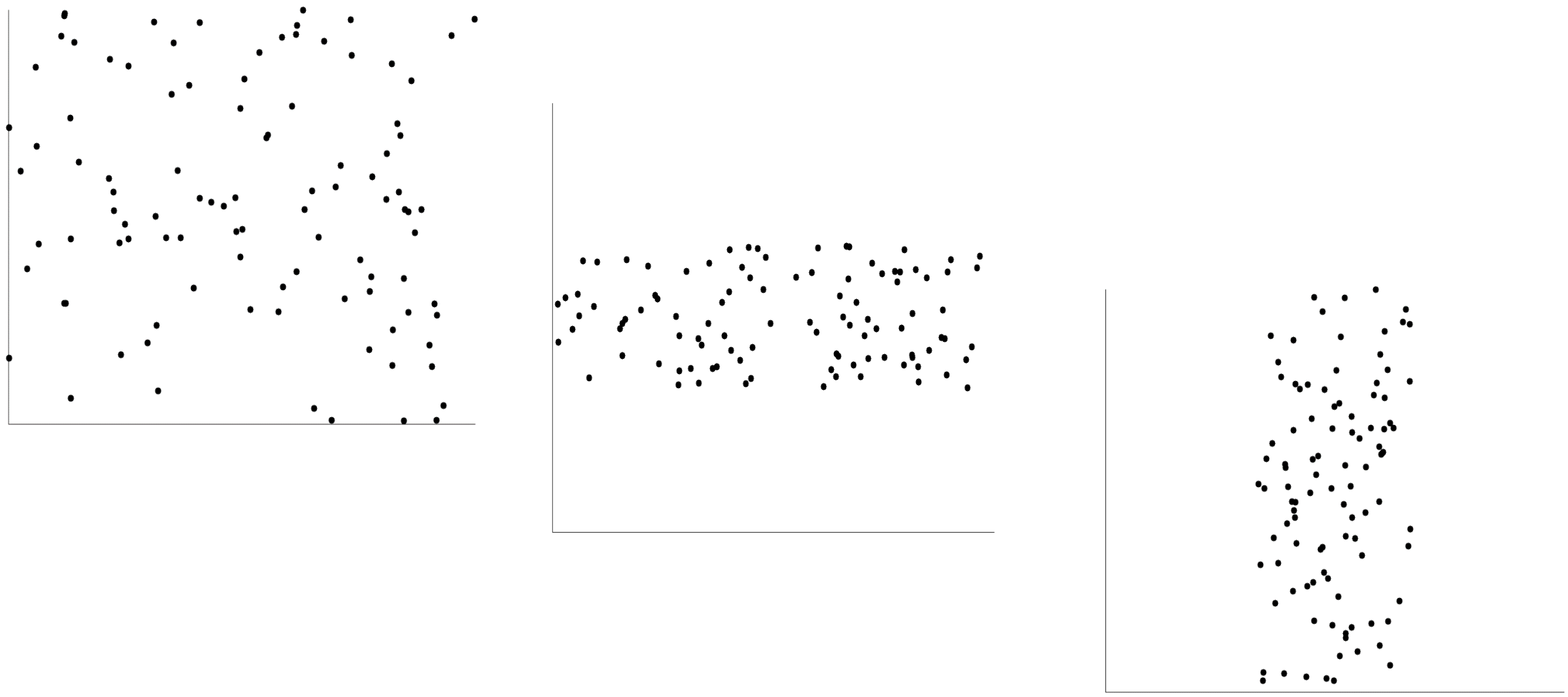






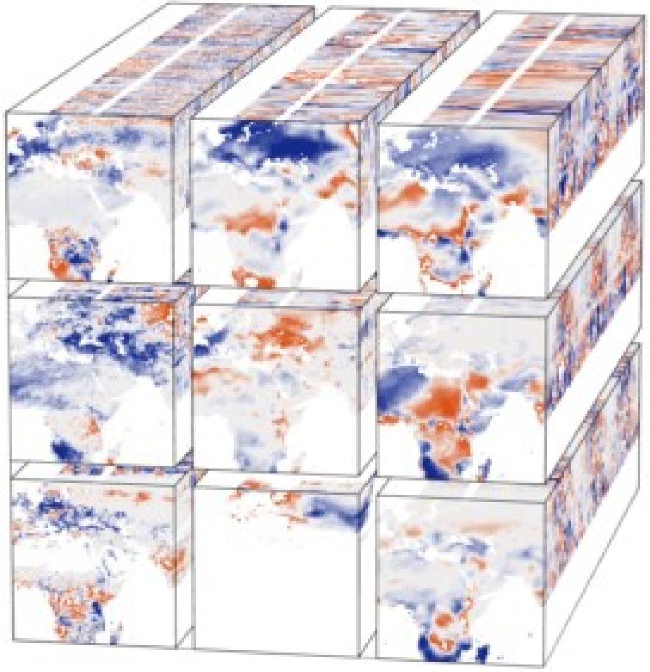
- The left half fragment is positively correlated
- The right half is negative correlated

Uncorrelated Data



Data Cube

- A 3-D visualization technique where hierarchical information is displayed as nested semi-transparent cubes
- The outermost cubes correspond to the top level data, while the subnodes or the lower level data are represented as smaller cubes inside the outermost cubes, and so on
- Multi-dimensional data cubes



[ref] Earth Data Cube https://deepcube-h2020.eu/technology/earth-system-data-cube/

Similarity and Dissimilarity

- **Similarity**
  - Numerical measure of how alike two data objects are
  - Value is higher when objects are more alike
  - Often falls in the range [0,1]
- **Dissimilarity** (e.g., distance)
  - Numerical measure of how different two data objects are
  - Lower when objects are more alike
  - Minimum dissimilarity is often 0
  - Upper limit varies
- **Proximity** refers to a similarity or dissimilarity

Data Matrix and Dissimilarity Matrix

- Data matrix
  - n data points with p dimensions
  - Two modes

$$\begin{bmatrix} x_{11} & \cdots & x_{1f} & \cdots & x_{1p} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{i1} & \cdots & x_{if} & \cdots & x_{ip} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{n1} & \cdots & x_{nf} & \cdots & x_{np} \end{bmatrix}$$

- Dissimilarity matrix
  - n data points, but registers only the distance
  - A triangular matrix
  - Single mode

$$\begin{bmatrix} 0 & & & & \\ d(2,1) & 0 & & & \\ d(3,1) & d(3,2) & 0 & & \\ : & : & : & & \\ d(n,1) & d(n,2) & \cdots & \cdots & 0 \end{bmatrix}$$

Proximity Measure for Nominal Attributes

- Can take 2 or more states, e.g., red, yellow, blue, green (generalization of a binary attribute)
- Method 1: Simple matching
  - *m*: # of matches, *p*: total # of variables

$$d(i,j) = \frac{p-m}{p}$$

- Method 2: Use a large number of binary attributes
  - creating a new binary attribute for each of the *M* nominal states

Proximity Measure for Binary Attributes

- A contingency table for binary data

		Object j		
		1	0	sum
Object i	1	q	r	$q+r$
	0	s	t	$s+t$
	sum	$q+s$	$r+t$	p

- Distance measure for symmetric binary variables:

$$d(i, j) = \frac{r+s}{q+r+s+t}$$

- Distance measure for asymmetric binary variables:

$$d(i, j) = \frac{r+s}{q+r+s}$$

- Jaccard coefficient (*similarity* measure for *asymmetric* binary variables):

$$sim_{Jaccard}(i, j) = \frac{q}{q+r+s}$$

  - Note: Jaccard coefficient is the same as "coherence":

$$coherence(i, j) = \frac{sup(i, j)}{sup(i) + sup(j) - sup(i, j)} = \frac{q}{(q+r) + (q+s) - q}$$

Dissimilarity between Binary Variables

- Example

Name	Gender	Fever	Cough	Test-1	Test-2	Test-3	Test-4
Jack	M	Y	N	P	N	N	N
Mary	F	Y	N	P	N	P	N
Jim	M	Y	P	N	N	N	N

  - Gender is a symmetric attribute
  - The remaining attributes are asymmetric binary
  - Let the values Y and P be 1, and the value N 0

$$d(jack, mary) = \frac{0+1}{2+0+1} = 0.33$$

$$d(jack, jim) = \frac{1+1}{1+1+1} = 0.67$$

$$d(jim, mary) = \frac{1+2}{1+1+2} = 0.75$$

Standardizing Numeric Data

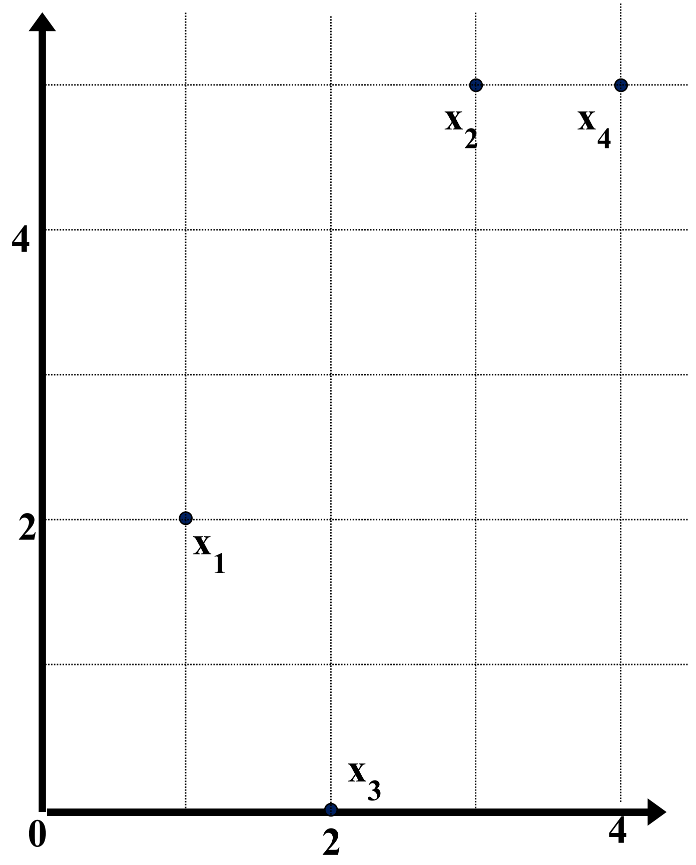
- Z-score: $$z = \frac{x - \mu}{\sigma}$$
  - X: raw score to be standardized, μ: mean of the population, σ: standard deviation
  - the distance between the raw score and the population mean in units of the standard deviation
  - negative when the raw score is below the mean, "+" when above
- An alternative way: Calculate the mean absolute deviation

$$s_f = \frac{1}{n}(|x_{1f} - m_f| + |x_{2f} - m_f| + \ldots + |x_{nf} - m_f|)$$

where $$m_f = \frac{1}{n}(x_{1f} + x_{2f} + \ldots + x_{nf}).$$

  - standardized measure (*z-score*): $$z_{if} = \frac{x_{if} - m_f}{s_f}$$
- Using mean absolute deviation is more robust than using standard deviation

Example: Matrix and Dissimilarity Matrix



Data Matrix

point	attribute1	attribute2
x1	1	2
x2	3	5
x3	2	0
x4	4	5

Dissimilarity Matrix

(with Euclidean Distance)

	x1	*x2*	*x3*	*x4*
x1	0			
x2	3.61	0		
x3	5.1	5.1	0	
x4	4.24	1	5.39	0

Distance on Numeric Data: Minkowski Distance

- *Minkowski distance*: A general definition for distance measure

$$d(i,j) = \sqrt[h]{|x_{i1} - x_{j1}|^h + |x_{i2} - x_{j2}|^h + \cdots + |x_{ip} - x_{jp}|^h}$$

where $i = (x_{i1}, x_{i2}, \ldots, x_{ip})$ and $j = (x_{j1}, x_{j2}, \ldots, x_{jp})$ are two p-dimensional data objects, and h is the order (the distance so defined is also called L-h norm)

- Properties
  - $d(i, j) > 0$ if $i \neq j$, and $d(i, i) = 0$ (Positive definiteness)
  - $d(i, j) = d(j, i)$ (Symmetry)
  - $d(i, j) \leq d(i, k) + d(k, j)$ (Triangle Inequality)
- A distance that satisfies these properties is a metric

Special Cases of Minkowski Distance

- $h = 1$: Manhattan (city block, L_1 norm) distance
  - E.g., the Hamming distance: the number of bits that are different between two binary vectors

$$d(i,j) = |x_{i_1} - x_{j_1}| + |x_{i_2} - x_{j_2}| + \ldots + |x_{i_p} - x_{j_p}|$$

- $h = 2$: (L_2 norm) Euclidean distance

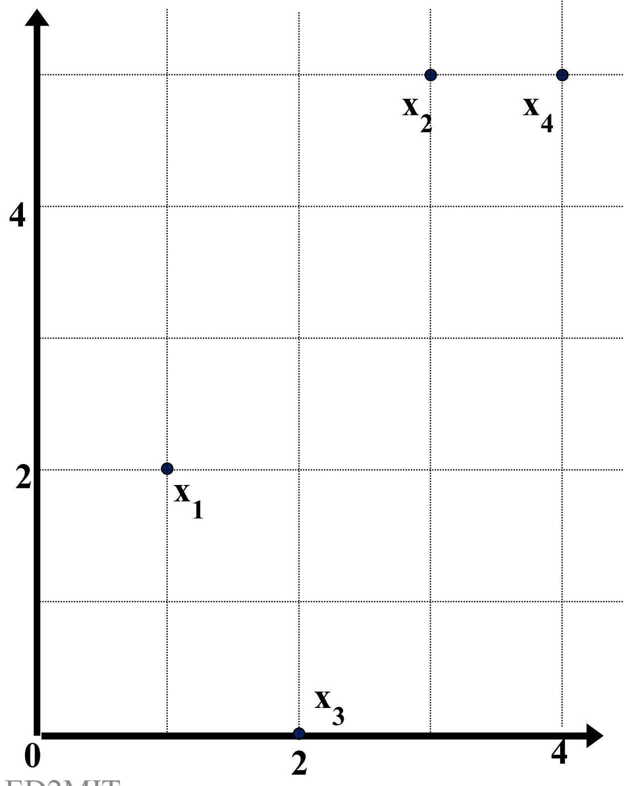
$$d(i,j) = \sqrt{(|x_{i_1} - x_{j_1}|^2 + |x_{i_2} - x_{j_2}|^2 + \ldots + |x_{i_p} - x_{j_p}|^2)}$$

- $h \to \infty$. "supremum" (L_{max} norm, L_∞ norm) distance.
  - This is the maximum difference between any component (attribute) of the vectors

$$d(i,j) = \lim_{h \to \infty} \left(\sum_{f=1}^{p} |x_{if} - x_{jf}|^h \right)^{\frac{1}{h}} = \max_{f}^{p} |x_{if} - x_{jf}|$$

Example: Minkowski Distance

point	attribute 1	attribute 2
x1	1	2
x2	3	5
x3	2	0
x4	4	5



Dissimilarity Matrices

Manhattan (L_1)

L	x1	x2	x3	x4
x1	0			
x2	5	0		
x3	3	6	0	
x4	6	1	7	0

Euclidean (L_2)

L2	x1	x2	x3	x4
x1	0			
x2	3.61	0		
x3	2.24	5.1	0	
x4	4.24	1	5.39	0

Supremum

L_∞	x1	x2	x3	x4
x1	0			
x2	3	0		
x3	2	5	0	
x4	3	1	5	0

Ordinal Variables

- An ordinal variable can be discrete or continuous
- Order is important, e.g., rank
- Can be treated like interval-scaled
  - replace x_{if} by their rank $r_{if} \in \{1, \ldots, M_f\}$
  - map the range of each variable onto [0, 1] by replacing *i*-th object in the *f*-th variable by

$$z_{if} = \frac{r_{if} - 1}{M_f - 1}$$

  - compute the dissimilarity using methods for interval-scaled variables

Attributes of Mixed Type

- A database may contain all attribute types
  - Nominal, symmetric binary, asymmetric binary, numeric, ordinal
- One may use a weighted formula to combine their effects

$$d(i,j) = \frac{\sum_{f=1}^{p} \delta_{ij}^{(f)} d_{ij}^{(f)}}{\sum_{f=1}^{p} \delta_{ij}^{(f)}}$$

  - *f* is binary or nominal:

    $d_{ij}^{(f)} = 0$ if $x_{if} = x_{jf}$, or $d_{ij}^{(f)} = 1$ otherwise
  - *f* is numeric: use the normalized distance
  - *f* is ordinal
    - Compute ranks r_{if} and
    - Treat z_{if} as interval-scaled

$$z_{if} = \frac{r_{if} - 1}{M_f - 1}$$

Cosine Similarity

- A **document** can be represented by thousands of attributes, each recording the *frequency* of a particular word (such as keywords) or phrase in the document.

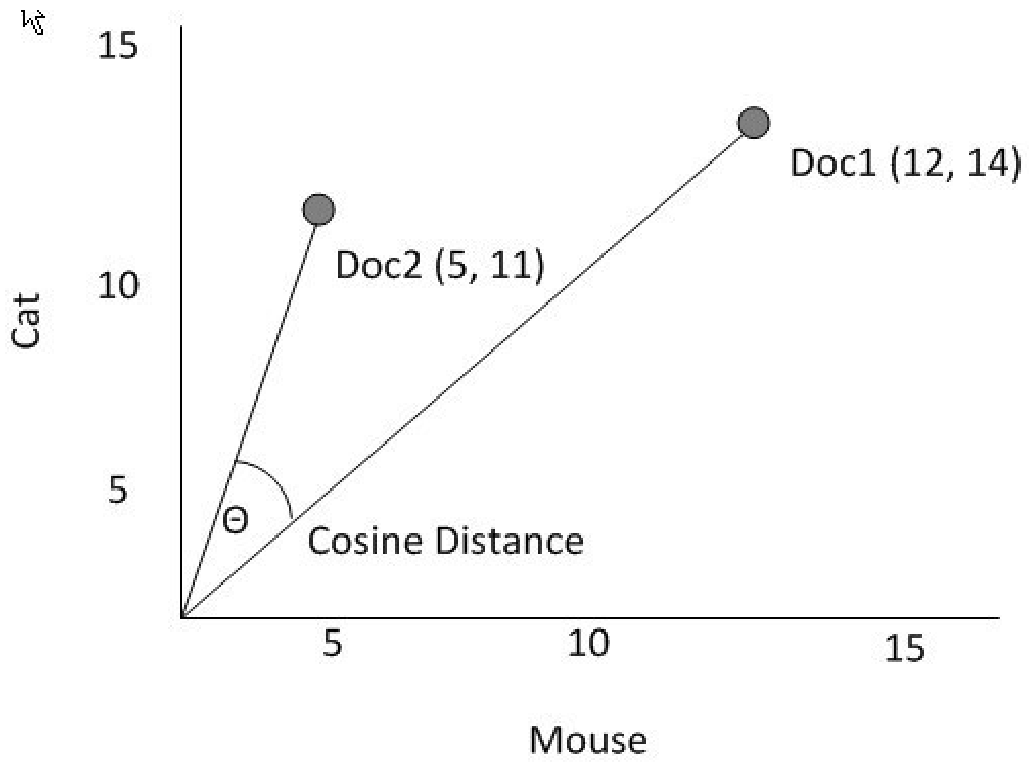
Document	*team*	*coach*	*hockey*	*baseball*	*soccer*	*penalty*	*score*	*win*	*loss*	*season*
Document1	5	0	3	0	2	0	0	2	0	0
Document2	3	0	2	0	1	1	0	1	0	1
Document3	0	7	0	2	1	0	0	3	0	0
Document4	0	1	0	0	1	2	2	0	3	0

- Other vector objects: gene features in micro-arrays, …
- Applications: information retrieval, biologic taxonomy, gene feature mapping, ...
- Cosine measure: If d_1 and d_2 are two vectors (e.g., term-frequency vectors), then

$$\cos(d_1, d_2) = (d_1 \bullet d_2) / ||d_1||\ ||d_2|| ,$$

where $\bullet$ indicates vector dot product, $||d||$: the length of vector d

Cosine Distance and Similarity



- Cosine distance is the angle subtended at the origin between the two documents. A value of 0 degrees represents identical documents and 90 degrees dissimilar documents.
  - Note that this distance is based on the relative frequency of words in a document. A document with, say, twice as many occurrences of all words compared to another document will be regarded as identical.
  - (example code https://nickgrattan.wordpress.com/)

Example: Cosine Similarity

- $\cos(d_1, d_2) = (d_1 \bullet d_2) / ||d_1||\ ||d_2||$,
  where $\bullet$ indicates vector dot product, $||d|$: the length of vector d

- Ex: Find the **similarity** between documents 1 and 2.

d_1 **=** (5, 0, 3, 0, 2, 0, 0, 2, 0, 0)
d_2 **=** (3, 0, 2, 0, 1, 1, 0, 1, 0, 1)

$d_1 \bullet d_2 = 5*3+0*0+3*2+0*0+2*1+0*1+0*1+2*1+0*0+0*1 = 25$
$||d_1|| = (5*5+0*0+3*3+0*0+2*2+0*0+0*0+2*2+0*0+0*0)^{0.5} = (42)^{0.5} = 6.481$
$||d_2|| = (3*3+0*0+2*2+0*0+1*1+1*1+0*0+1*1+0*0+1*1)^{0.5} = (17)^{0.5} = 4.12$
$\cos(d_1, d_2) = 0.94$

Summary and Takeaway

- Data attribute types: nominal, binary, ordinal, interval-scaled, ratio-scaled
- Many types of data sets, e.g., numerical, text, graph, Web, image.
- Gain insight into the data by:
  - Basic statistical data description: central tendency, dispersion, graphical displays
  - Measure data similarity
- Statistical data analysis is a first step of data preprocessing.
- Many methods have been developed but still an active area of research.

Practice Part – Investigating Selected Dataset

- Use self-study exercises provided for this course both in Python and RapidMiner
- Investigate and visualize dataset characteristics

Test yourself – Data Statistics

- Test yourself with exercises below

Test yourself

Q1: Calculate the min and std. variance of the following data:

PRICE - RM '000	130	137	128	390	140	241	342	143
SQ. M OF FLOOR	135	140	100	360	175	270	200	170

Q2: Calculate the mean price of the following low-cost houses, in various localities across the country:

PRICE - RM '000 (x)	36	37	38	39	40	41	42	43
NO. OF LOCALITIES (f)	3	14	10	36	73	27	20	17

Test yourself

Q3: From a sample information, a population of housing estate is believed have a “normal” distribution of X ~ (155, 45).

What is the general adjustment to obtain a Standard Normal Distribution of this population?

Q4: Consider the following ROI for two types of investment:

A: 3.6, 4.6, 4.6, 5.2, 4.2, 6.5

B: 3.3, 3.4, 4.2, 5.5, 5.8, 6.8

Decide which investment you would choose.

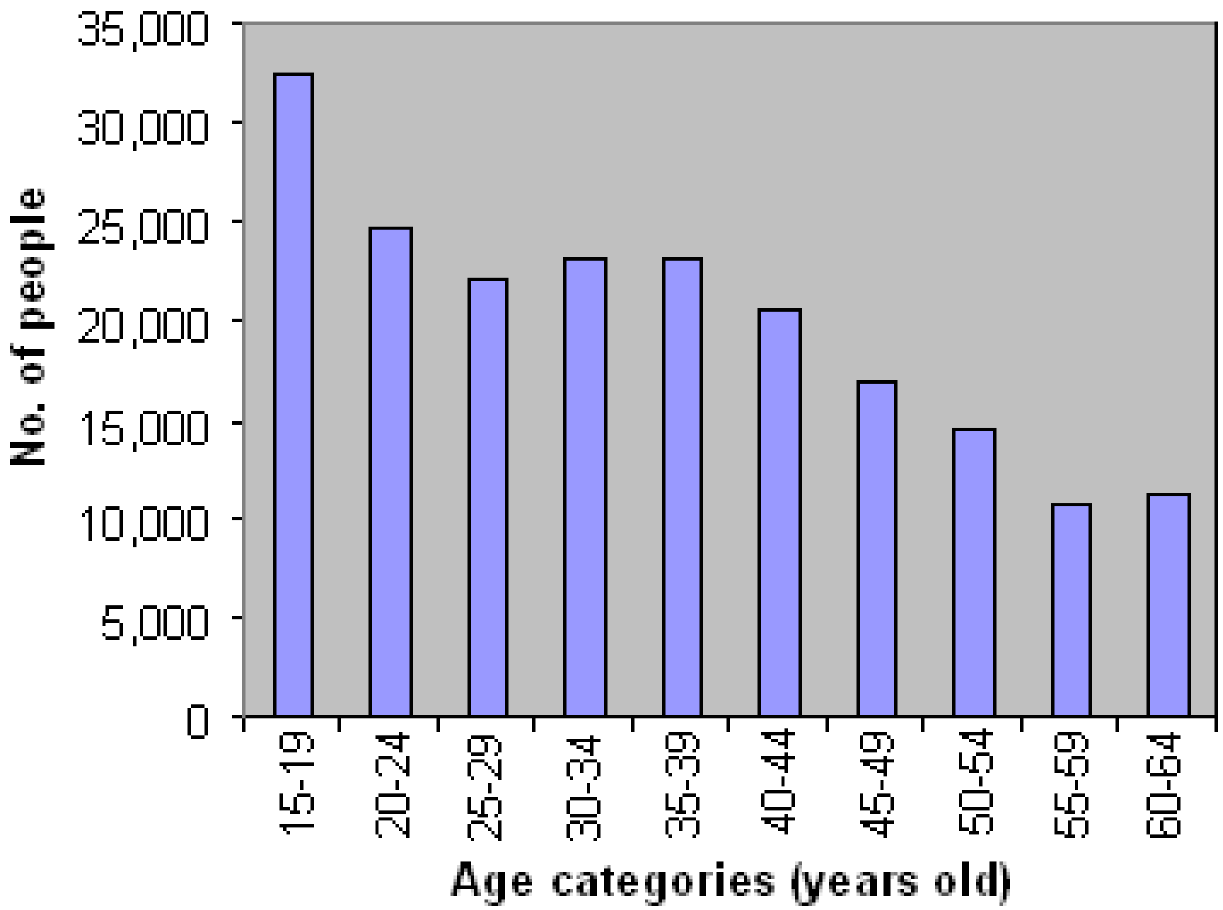
Test yourselves!

Q5: Find:

ϕ(AGE > "30-34")

ϕ(AGE ≤ 20-24)

ϕ("35-39"≤ AGE < "50-54")



Test yourself

Q6: You are asked by a property marketing manager to ascertain whether or not *distance to work* and *distance to the city* are "equally" important factors influencing people's choice of house location.

You are given the following data for the purpose of testing:

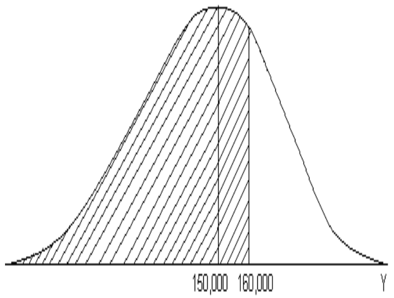
Explore the data as follows:

- Create histograms for both *distances*. Comment on the shape of the histograms. What is you conclusion?
- Construct scatter diagram of both *distances*. Comment on the output.
- Explore the data and give some analysis.
- Set a hypothesis that means of both distances are the same. Make your conclusion.

Questions: Normal distribution (1)

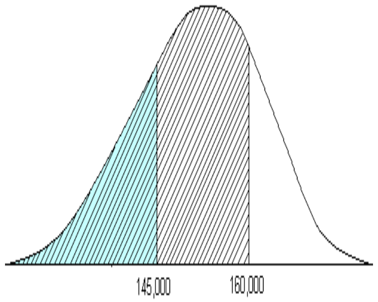
Your sample found that the mean price of "affordable" homes in Johor Bahru, Y, is RM 155,000 with a variance of RM 3.8×10^7. On the basis of a <u>normality</u> assumption, how sure are you that:

(a) The mean price is really ≤ RM 160,000
(b) The mean price is between RM 145,000 and 160,000



Answer (a):

$$P(Y \le 160{,}000) = P\left(Z \le \frac{160{,}000 - 155{,}000}{\sqrt{3.8\times10^7}}\right)$$
$$= P(Z \le 0.811)$$
$$= 0.1867$$



Using Z-table, the required probability is:
1-0.1867 = 0.8133

Always remember: to convert to SND, subtract the mean and divide by the std. dev.

Questions: Normal distribution (2)

Answer (b):

$$Z_1 = \frac{X_1 - \mu}{\sigma} = \frac{145{,}000 - 155{,}000}{\sqrt{3.8 \times 10^7}} = -1.622$$

$$Z_2 = \frac{X_2 - \mu}{\sigma} = \frac{160{,}000 - 155{,}000}{\sqrt{3.8 \times 10^7}} = 0.811$$

$P(Z_1<-1.622)=0.0455$; $P(Z_2>0.811)=0.1867$

$\therefore P(145{,}000<Z<160{,}000)$

$= P(1-(0.0455+0.1867)$

$= 0.7678$

Questions: Normal distribution (3)

You are told by a property consultant that the average rental for a shop house in Johor Bahru is RM 3.20 per sq. After searching, you discovered the following rental data:

2.20, 3.00, 2.00, 2.50, 3.50,3.20, 2.60, 2.00, 3.10, 2.70

What is the probability that the rental is greater than RM 3.00?