PI Game: Comparing the parametric and non-parametric frameworks in estimating optimal quantities for a bakery chain in Vilnius, Lithuania

Group 4 *

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Introduction

In operations and supply chain management, the newsvendor problem is a frequently used classical model. The newsvendor problem focuses on the optimal production capacity and balances the risk of surplus stocks and deficit stocks without knowing the actual demand. Based on statistical methods and historical data, newsvendor problems could be solved by improving estimates of the optimal production capacity.

The study of the Newsvendor Problem is important in optimization problems. Let us consider a bakery shop that needs to decide the number of breads to make. When the bakery produces more bread than needed, it may experience a cost of unsold bread being discounted at half price at the end of the day. Additionally, it may lose its potential sales from the customers if the bakery shop runs out of bread. The shop can solve this issue by determining the optimal order quantity that has the lowest expected cost. The supply of COVID-19 vaccines is another situation where the Newsvendor problem can be used. A shortage of vaccines may lead to a high death rate due to Covid-19, or there might be a waste of unused vaccines being discarded when the supply is more than needed. These risks can be tackled by identifying the optimal order quantity of vaccines. The Newsvendor problem can be explained mathematically using the following expression:

$$Q^* = F^{-1}\left(\frac{c_u}{c_o + c_u}\right)$$

The research article by Liu C., Letchford A.N., and Svetunkov in 2022 discusses that the expression includes c_u as the underage cost and c_o as the overage cost. The optimal order quantity is determined by taking the inverse function F^{-1} of the ratio of the underage cost to the total cost where F is the cumulative distribution function of the demand. Thus, $\frac{c_u}{c_o+c_u}$ is the target service level that the companies can aim for to maximize their expected profit.

Real-life examples that have been discussed come with their challenges. For example, in the food industry, the amount of raw materials affects the overall profit. With machine learning and historical data, it is possible to estimate the required quantity to maximize profit. Similarly, in hospitals, determining the optimal number of nurses for a day involves analyzing factors such as occupied beds to estimate the demand for nurses. Balancing underage and overage costs is crucial for determining the optimal quantity and calculating the lowest expected cost. Additionally, quantile regression can optimize inventory management when the demand is an uncertain variable.

A challenge with estimating the optimal quantity is that there may not be enough data available to compute the statistic of interest accurately. If the demand distribution is known, Monte Carlo simulations can be used in this case to generate samples from the PDF and compute the statistics. Another challenge may be that the demand distribution may be unknown. In that case, the non-parametric approach can be used to compute the optimal quantity by utilizing order statistics.

*Pooya Mers (2734368), Dinh Duy Phan (2802376), Rongyue Hua (2814202), Dan Ni Zhao (2811338), Zhenhua Yuan (2804277)

Lastly, suppose initial observations show that the distribution may follow a normal distribution. In that case, the Jarque-Bera test can be used to confirm the estimation by comparing the kurtosis and skewness of the sample data to the normal distribution.

The Newsvendor problem is not the only study where these methods can be applied. A similar study to the NVP is the Economic order quantity (EOQ). A research paper by Rahmadini et al. in 2023 investigates inventory control for pharmaceuticals using the EOQ. In the paper, Monte Carlo simulation is used to generate random inventory data, and the simulated data is used to compute the economic order quantity and its statistical properties. The bootstrap method is also used in the study, for example in a paper by Larin (2022) to compute the EOQ of aviation parts.

Continuing into the theoretical discussion about the Newsvendor Problem and its practical applications, the attention shifts to a specific case study: a bakery chain in Vilnius, Lithuania. This bakery confronts the challenge of determining the optimal daily production quantities to minimize waste and maximize profit. Due to the perishable nature of bakery products, excessive production results in significant waste, while insufficient production can lead to lost sales and customer dissatisfaction. By analyzing historical sales data and applying both parametric and non-parametric methods introduced in the course, to identify the optimal production quantities for this bakery chain.

This paper uses several methods that were discussed in the course. Firstly, the profit function and its variables are modified to fit the specific operations of the bakery. Then, the parametric and non-parametric order quantities are compared by using several metrics such as the Empirical service level (SL), and by utilizing the Monte Carlo simulation. Then, in response to the bakery's challenges, parametric and non-parametric methods are used to compute point estimates and intervals for efficient order quantities. Performing both approaches allows for a wider analysis of the bakery's production planning and can provide more options to the management, in case assumptions about the the demand distribution of the stores change.

Several conclusions were drawn during the investigations:

1. In general, if the true distribution is normal, the parametric estimator performs better.

2. When the service level (τ) is high and the sample size (n) is small, the parametric estimator produces much less error.

3. When the true distribution is unknown, the non-parametric estimator is more efficient.

Furthermore, to help the management in the operations of the bakery chain, point estimates and intervals for the optimal order quantities and optimal price changes to maintain profit levels were computed. The paper begins by looking into the properties of the profit function in the specific case of the bakery chain.

Theoretical Questions

The Profit Function and Optimal Order Quantity

The optimization of order quantities is crucial for improving operational efficiency and profitability, particularly for companies with perishable products. By determining the right quantities, companies can manage costs, minimize waste, increase profitability, and promote sustainability.

For companies, analyzing how changes in shipping costs affect the sensitivity of the optimal order quantity and the expected profit is essential. This analysis provides valuable insights into the impact of shipping cost variations on inventory management decisions and overall profitability.

The clearance price should be set lower than the combined cost of production and shipping cost. This is because the clearance price is usually set to sell excess inventory at a discounted rate to minimize losses. Additionally, if the clearance price exceeds the production and shipping costs, consumers may opt for alternative products with lower prices. Therefore, to remain competitive companies set their clearance price below their total cost.

The profit function, incorporating clearance price (p_L) and shipping cost (c_S), is given by:

$$\text{profit} = p \min(Q, Y) + p_L \max(0, Q - Y) - (c + c_S)Q$$

This equation considers the profit from selling the ordered quantity at the unit price, the profit from selling any excess inventory at the clearance price, and the total cost of production and shipping.

To calculate the optimal order quantity (Q^*), the following formulas are used:

$$Q = \Phi^{-1}(\text{ratio}; \mu, \sigma)$$

$$\text{ratio} = \frac{p + pL - c - cS}{p + pL}$$

This formula allows for adjustments in the optimal order quantity based on fluctuations in selling price, production cost, clearance price, and shipping cost.

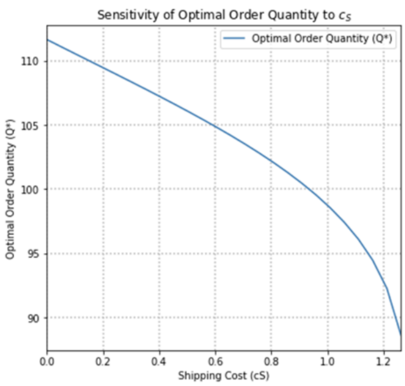
The expected profit is then derived by subtracting the total cost from the total profit for the order quantity and then subtracting the average profit while considering all potential demand variations. Using the following formulas:

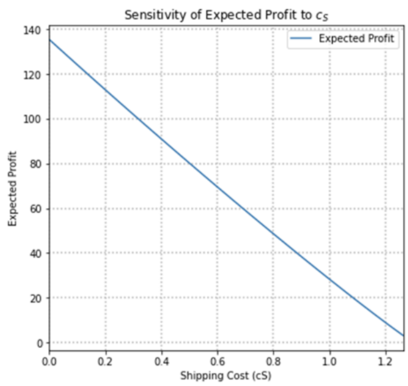
$$\text{Integral}_y = \int_{-\infty}^{Q} \Phi\left(\frac{y-\mu}{\sigma}\right) dy$$

$$\text{expected profit} = (p + p_L - c - c_S)Q - p \times \text{Integral}_y$$

Throughout sensitivity analysis, the shipping cost is varied from zero to five to calculate the optimal order quantity (Q^*) and the expected profit for each value of shipping cost. An increase in the shipping cost leads to a decrease in the optimal order quantity (Q^*). This is expected, as higher shipping cost reduces the profitability of holding excess inventory. Consequently, the predicted profit decreases as shipping costs increase, demonstrating the adverse effect of higher shipping expenses on profit margins.

These results highlight the importance of considering shipping costs in inventory management to maximize expected profits.





Parametric vs Non-parametric estimator

A simulation experiment was designed to compare the performance of parametric and nonparametric estimators. The experiment aimed to support Levi et al.'s theory (2015) on the potential inaccuracy of nonparametric estimators, particularly when the target service level is high (e.g., $\tau = 0.99$) and the sample size (n) is small.

The experiment samples are generated based on Monte Carlo simulation. In parametric estimation, parameters are estimated using sample data, and the specified quantiles are then calculated using the quantile function. In nonparametric estimation, the data is sorted and the quantile function is applied.

The Normal and Log-normal distributions are utilized for generating random samples. It is assumed that the true demand follows $N(115, 10)$ and $LN(6, 0.6)$. The sample sizes and target service levels are denoted by $n \in \{10, 50, 100, 200\}$ and $\tau \in \{0.01, 0.05, 0.1, 0.3, 0.5, 0.7, 0.9, 0.95, 0.99\}$ respectively.

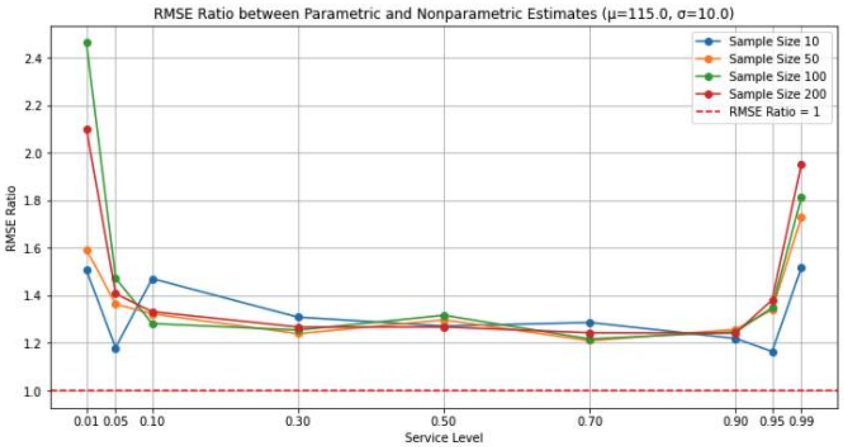
500 simulations allow for relatively accurate results while keeping computational costs low. Empirical root mean squared error (RMSE), Empirical service level (SL), and Empirical profit loss ratio (PLR) are obtained as rubrics.

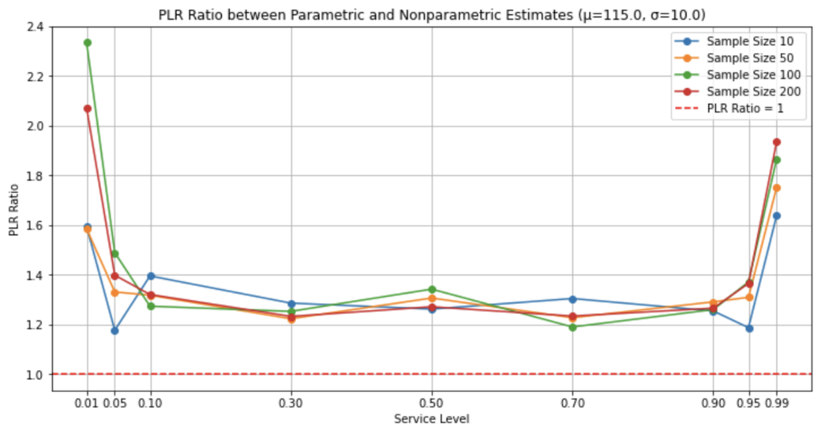
To facilitate a more intuitive comparison of the nonparametric and parametric estimate performance, RMSE ratios (np/p) and PLR ratios (np/p) were used as reference standards.

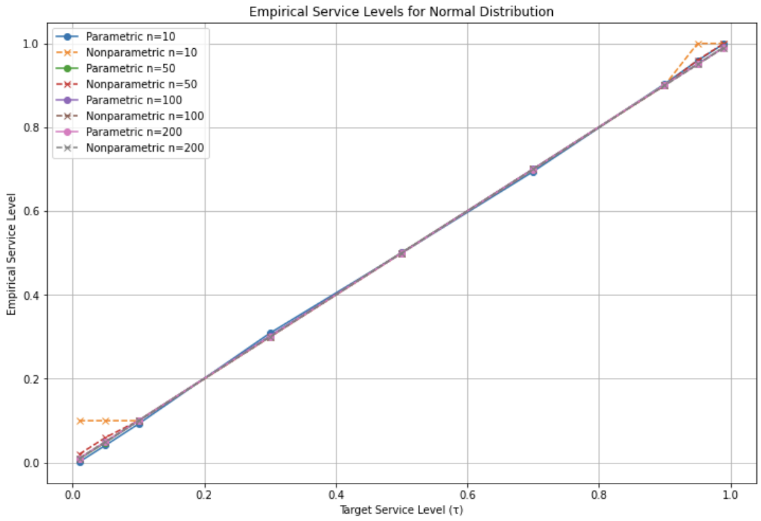
For Normal distribution, it has been observed that the RMSE and PLR ratios consistently exceed 1. This suggests that the error and the loss ratio of the nonparametric estimates are larger than those of the parametric estimates. Therefore, the parametric estimates generally demonstrate higher levels of accuracy compared to the nonparametric estimates.

At extreme service levels ($\tau = 0.01$, $\tau = 0.05$ or $\tau = 0.99$, $\tau = 0.95$) and small sample sizes ($n = 10$), the ratios fluctuate significantly as they approach the extremes, suggesting that the error and loss ratios of the nonparametric estimates are substantial at this point, which leads to low confidence in the estimates.

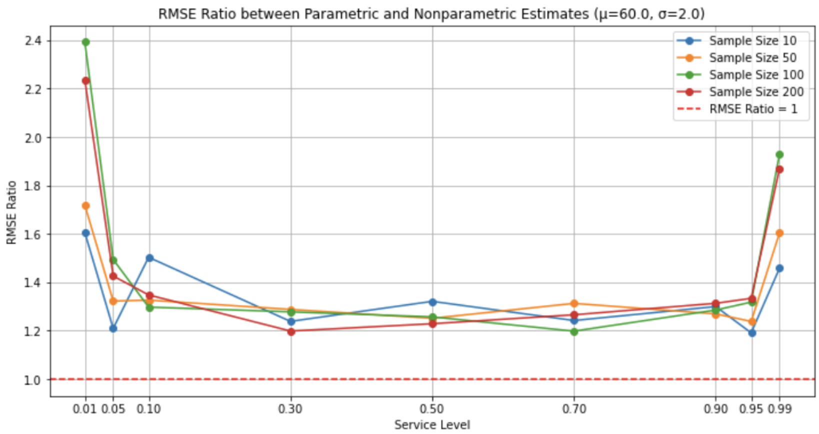
The graph depicts the empirical service level and the target service level. The empirical service level for estimations closely approximates the target service level, with deviations observed for the empirical service level of nonparametric estimation (n = 10) mainly at the extremes of the x-axis.

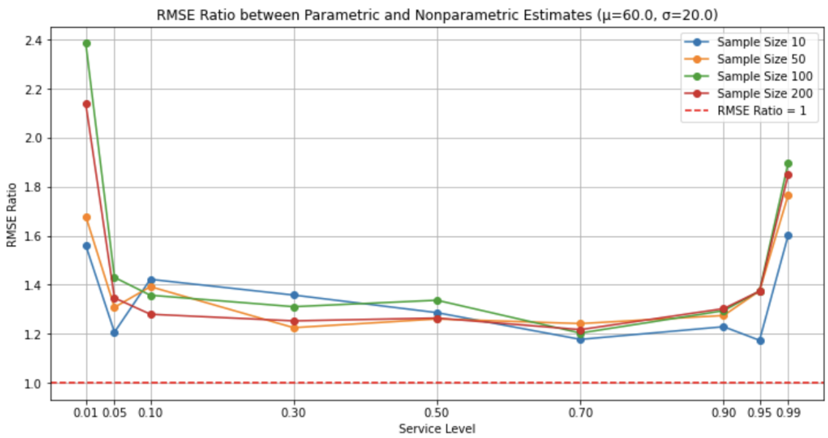


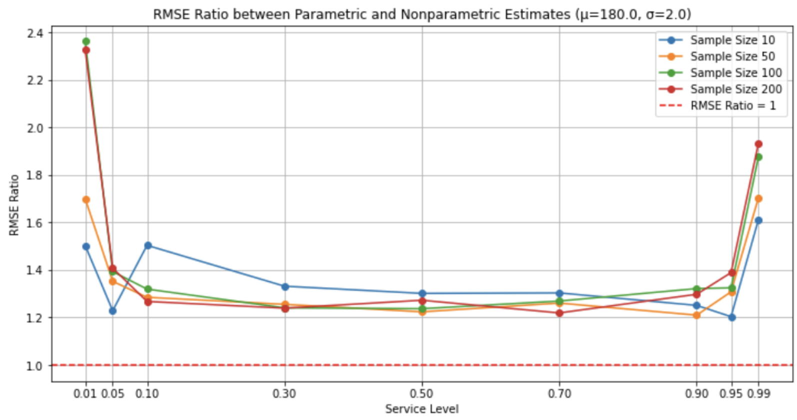


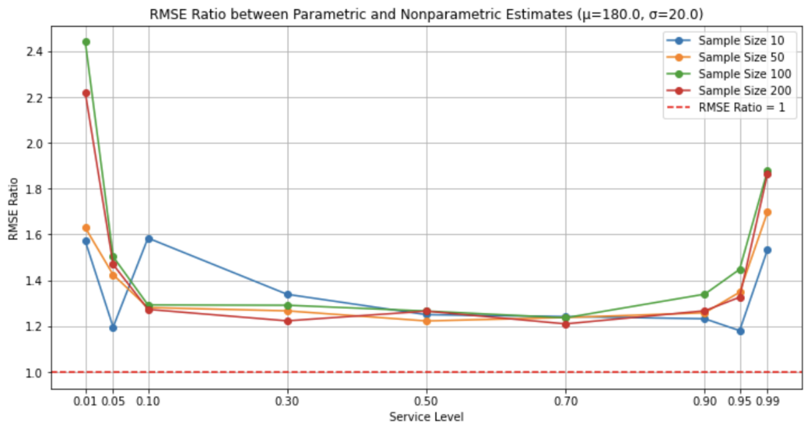


By selecting different parameter values, the results can be expanded further. The Normal distributions $N(\mu, \sigma^2)$ may take values of $(\mu, \sigma) \in \{(60, 180), (2, 20)\}$. An examination of various parameter values for μ and σ suggests that the overall distribution of the three indicator performance graphs remains consistent. The ratios and service level comparison are approximately similar in size and do not change significantly.



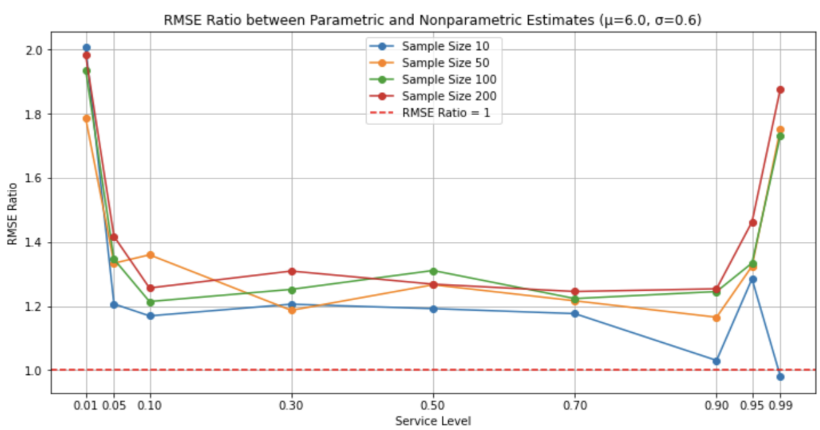


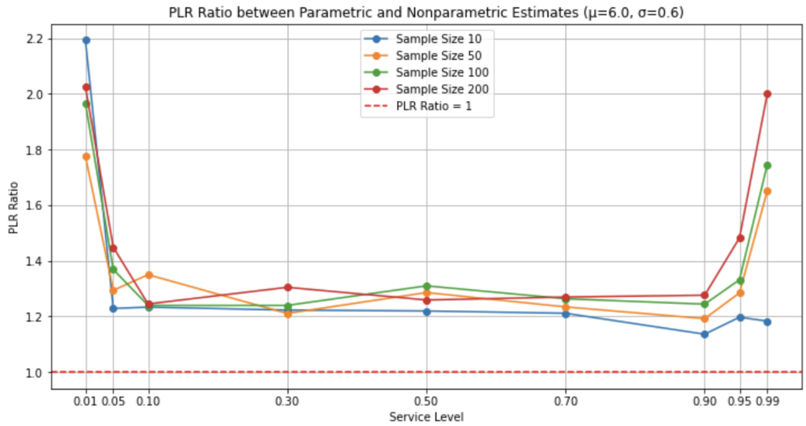


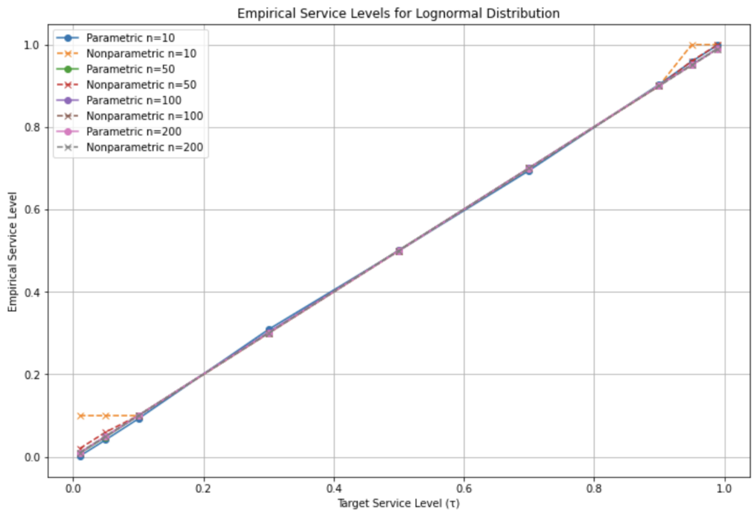


In the case of the Log-normal distribution, the findings were comparable, with the only distinction being that the RMSE ratios were reversed (i.e. RMSE ratios below 1) for small sample sizes and high levels of service. This suggests that, under these conditions, the nonparametric estimates were more precise than the parametric estimates.

The lognormal distribution has thicker tails than the normal distribution, which increases the probability of extreme values. In small samples, parametric estimation methods may inaccurately estimate these extremes, while nonparametric estimation provides more accurate results by directly utilizing ordination. This makes nonparametric estimation more robust when dealing with unknown or irregular data distributions or when outliers are present.







According to the simulation design, it was noted that the parametric estimator typically performs better than the non-parametric estimator for both Normal and Log-normal distributions. This is particularly evident for the Normal distribution, especially when aiming for a high target service level (e.g. $\tau = 0.99$) with a small sample size (n).

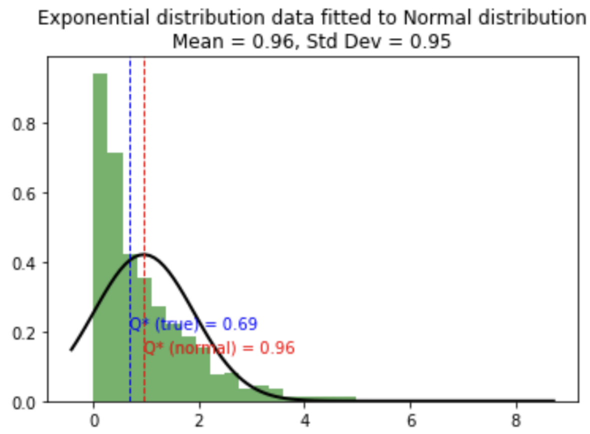
Analysis of Parametric Estimator Performance Under Incorrect Demand Distribution Assumptions

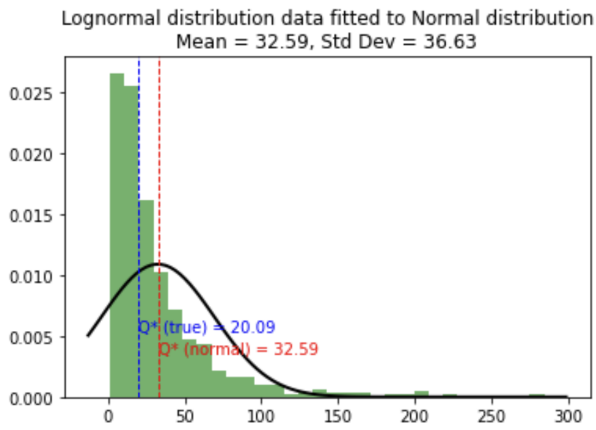
In order to analyze the weakness of the parameter estimator when the predicted distribution is incorrectly identified from the actual demand distribution., it is demonstrated that reprocessing the samples to fit a normal distribution in a real-world problem may lead to suboptimal order quantities. Based on the Monte Carlo simulation study, the following are comparative plots and analysis of fitting the data generated from the three distributions (lognormal, Poisson, and exponential) to a normal distribution.

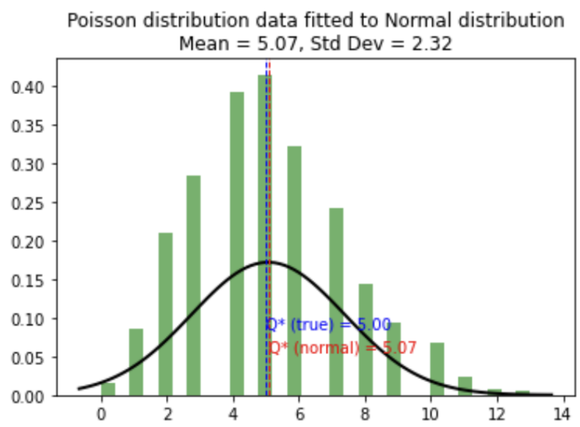
Since the lognormal distribution produces data that are normally distributed after a logarithmic transformation, it can be better fitted to a normal distribution in some cases. However, even so, the data from the lognormal distribution is significantly right skewed, and the mean and standard deviation of the data may not reflect well the true distributional characteristics of the data after fitting to a normal distribution and the information in the long-tailed part may be lost after fitting to the normal distribution.

While attempting to fit the Poisson distribution to a normal distribution, errors may occur due to the discreteness of Poisson data and the continuity of the normal distribution. Consequently, the parametric estimator could overestimate the optimal order quantity compared to the true distribution.

The exponential distribution is not a good fit for a normal distribution due to its highly right-skewed nature. When fitting exponentially distributed data to a normal distribution, the resulting mean and standard deviation may not effectively represent the data's characteristics. The shapes of exponential and normal distributions differ significantly, resulting in less effective fitting, particularly in the tails. Therefore, the true optimal order quantity is significantly lower than the one estimated using the normal distribution.







The results show that parametric estimators based on normal distributions may lead to inaccurate order quantities when confronted with distributions that are significantly skewed or have large kurtosis, such as exponential and lognormal distributions. In contrast, nonparametric estimators do not depend on distributional assumptions and provide more accurate estimates of optimal order quantities.

These findings support Levi et al. (2015) that fitting non-normal demand data to a normal distribution may lead to suboptimal order quantities. To obtain more accurate order quantity estimates, the appropriate distribution should be chosen for fitting based on the real demand data.

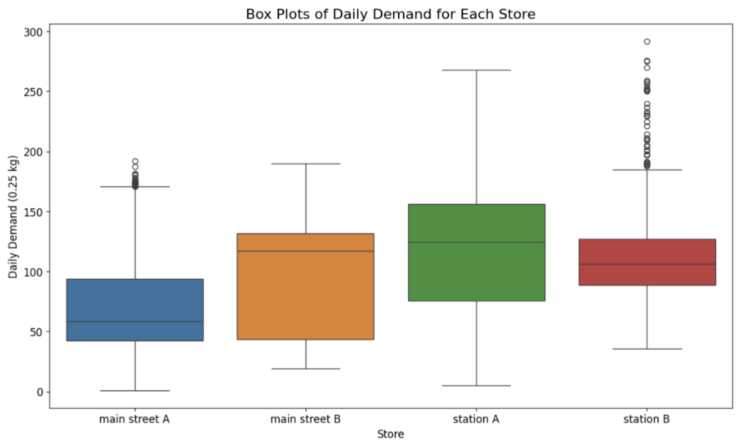
Empirical Questions

Analytics of The Bakery Data

To understand the statistical properties of each store, the mean, variance, and number of demand observations can be calculated. Station A has the highest mean and variance for demand, while Main Street A has the largest recorded data.

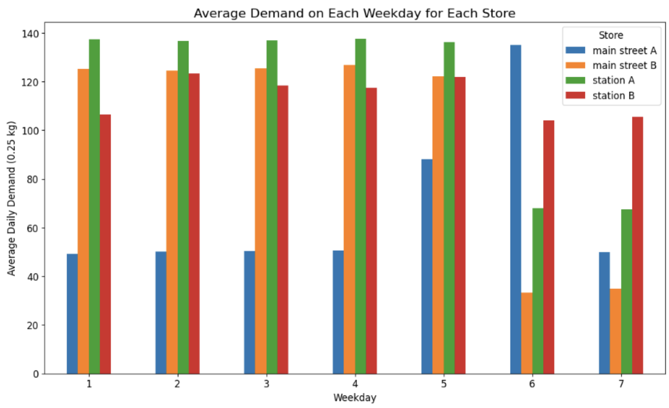
Mean, Variance, and Count of all stores

	Mean	Variance	Count
Weekdays	-	-	2941
Main street A	67.63	1736.78	2941
Main street B	99.09	1890.89	486
Station A	117.24	2786.03	2110
Station B	113.92	1666.71	694

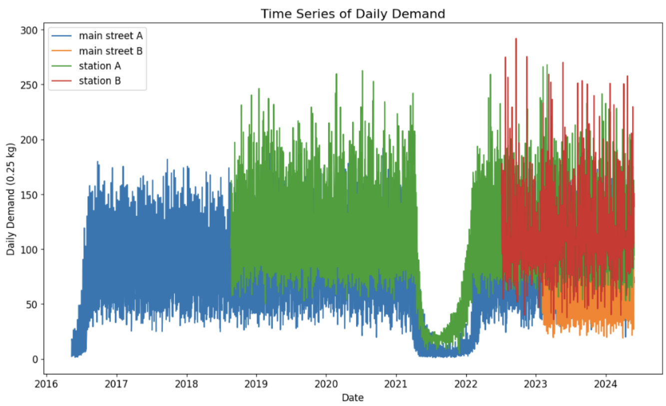


To analyze the data using probability functions, it is useful to know the approximate demand distribution of different stores. The daily demand can be visualized in two ways: firstly by plotting the demand based on frequency; and secondly by calculating the average demand on each weekday. The former method can be used to make a guess about the distribution, while the latter visualizes the daily demand pattern for each store.

An initial observation shows that Main Street A may follow a normal distribution, and Station B may follow a log-normal distribution. Main Street B and Station A seem to have a bimodal demand distribution.



The average demand plot yields interesting results about the daily demand patterns of the stores. There is a significant decline in demand at Station A and Main Street B during the weekend, due to fewer people passing these shops while commuting to work, as claimed in the background information. The demand at Main Street A is higher on Fridays and reaches its peak on Saturdays, then declines significantly starting on Sundays. The demand at Station B(located near old-town) is generally consistent, though it is slightly higher from Tuesday until Friday, possibly due to a higher presence of locals and tourists.



The Time Series plot shows the changes in the daily demand in the past few years. The shops at Main Street A, Station A, Station B, and Main Street B opened in 2016, 2018, 2022, and 2023 respectively. In 2021, the demand for the products declined sharply due to COVID-19, but it increased again steadily, reaching its original level in 2022.

The plots reinforce and illustrate the claims made in the background information about the opening dates, locations, and demand changes during COVID-19.

Estimated Short-run Optimal Quantities

The profit function $\Pi(Q, Y, c, p)$ considers the profit from selling the ordered quantity Q at the unit price p, the profit from selling excess inventory at the clearance price p_L, and the total cost of producing and shipping:

$$\Pi(Q, Y, c, p) = p * \min(Q, Y) + p_L * \max(0, Q - Y) - (c + c_S) * Q$$

The optimal order quantity Q^* is affected by the shipping cost c_S and the clearance price p_L, showing how the changes in these parameters affect the profitability and highlighting the importance of using the most appropriate method based on data distribution when estimating the optimal order quantities for profitability.

As stated above, the frequency plots for the demand distributions of Main Street A and Station B indicate that they may follow a normal and a log-normal distribution respectively. For this reason, these two stores are suitable candidates for analysis to compare the parametric and non-parametric framework.

As a visual estimate is not enough to confirm the best distribution of the data, a goodness-of-fit test must be conducted to see whether the properties of sample data match the properties of the assumed true distribution. An appropriate test here is the Jarque-Bera test, which checks whether the kurtosis and skewness of the sample data match the Normal distribution. Additionally, since it is estimated that some data may instead follow the log-normal distribution, the sample data must be modified in order to use the JB test. If the sample data is indeed log-normally distributed, taking the logarithm of all the data points would convert them to normally distributed data.

- **Jarque–Bera (JB) test:**

  $$JB = n\left(\frac{(\hat{s}_n - 0)^2}{6} + \frac{(\hat{k}_n - 3)^2}{24}\right) \sim \chi^2_2, \quad \text{as } n \to \infty,$$

  where $\hat{s}_n$ is the sample skewness, $\hat{k}_n$ is the sample kurtosis, and χ^2_2 is the chi-squared distribution with 2 degrees of freedom.

The JB test can then be conducted on the converted data, and if the test shows that the result follows a Normal distribution, the original data can then be assumed to follow a log-normal distribution. The parametric and non-parametric point estimates and confidence intervals can then be computed by:

$$\hat{Q}^P_n(G_{\hat{\theta}_n}; c, p) = G^{-1}\left(\frac{p-c}{p} \mid \hat{\theta}_n\right)$$

(p.16, NPV_MLE_handout)

$$\hat{Q}^{NP}_n(c, p) = \inf\left\{y \in \mathbb{R} : \hat{F}_n(y) \geq \frac{p-c}{p}\right\}$$

(p.8 last lecture)

It is important to consider the significant decrease in demand during COVID-19. Including data from that period is not necessary in computing short-run estimates for the optimal order quantities. Those data would lower the estimates, without providing useful information in the short run because it is highly unlikely that a new pandemic will occur in the next few days. Thus, only the data from 31/05/2022 onwards is considered in the computational process. The results are as follows:

Parametric Estimates and Intervals

Location	Day	Point Estimate	95% Confidence Interval
Main Street A	Friday	94.52	[93.51, 95.53]
Main Street A	Saturday	138.39	[135.68, 141.09]
Main Street A	Sunday	41.23	[38.58, 43.87]
Station B	Friday	90.70	[83.11, 98.29]
Station B	Saturday	98.03	[95.97, 100.09]
Station B	Sunday	99.59	[97.37, 101.80]

Nonparametric Estimates and Intervals

Location	Day	Point Estimate	95% Confidence Interval
Main Street A	Friday	94.65	[92.82, 95.31]
Main Street A	Saturday	139.09	[135.90, 141.36]
Main Street A	Sunday	43.84	[39.39, 45.88]
Station B	Friday	94.72	[77.75, 101.82]
Station B	Saturday	98.44	[95.71, 99.66]
Station B	Sunday	101.12	[98.58, 102.75]

There are some similarities between parametric and nonparametric methods. First, both methods aim to make inferences about the population parameters based on sample data and provide a measure of uncertainty. The two tables show that both parametric and nonparametric methods are used to provide point estimates that are a single value from the sample to serve as the best estimate of what the true population parameter maybe and 95% confidence intervals that are a range of values that is likely to contain a population parameter with a certain level of confidence. For example, the best estimate for the optimal quantity of traditional food to prepare is 41.23, and we are 95% confident that this true value lies between the interval of 38.58 and 43.87 on Sunday at Main Street A by parametric methods or the best guess for the order quantity is 98.44, and we are 95% confident that this true value lies between 95.71 and 99.66 on Saturday at Station B by nonparametric approaches. Additionally, parametric and nonparametric approaches are also used in the hypothesis testing. For example, the parametric methods include a t-test which is the test for the difference between the means of two independent groups while there is a Krukal-Wallis test which is the test for comparing the medians of three or more groups for nonparametric methods.

However, there are some differences between them. Parametric methods are statistical techniques that assume the data follows a known probability distribution such as Normal distribution allowing for more efficient statistical tests when the assumptions hold true while nonparametric methods do not rely on specific assumptions about the distribution of the population, and they are also referred to as distribution-free methods. This can be observed from task 4 which demonstrates that fitting non-normal distributions including Lognormal, Poisson, and Exponential to a Normal distribution can lead to suboptimal order quantities through a parametric approach which is in contrast to nonparametric methods that do not assume normality can provide more accurate estimates for skewed and heavy-tailed distributions. Moreover, the results obtained from the two tables also show that the difference in distributional assumptions of these two methods results in different point estimates and confidence intervals.

Furthermore, the outputs generated through nonparametric methods are robust to outliers and skewed data, making them more reliable when dealing with noisy data which is in contrast to the results by parametric approaches that can be easily affected by outliers and lead to misleading results. As shown in task 3, nonparametric methods can provide more accurate estimates for heavy-tailed distributions.

Another notable point is that the computation for nonparametric approaches is often slower than the parametric ones. This can be observed from tasks 3 and 4 such that the lack of simplifying the specific distributions requires more computational effort for nonparametric approaches as opposed to parametric methods with specific distributional assumptions leading to simple computation.

In conclusion, both parametric and nonparametric methods have their benefits and drawbacks.

Parametric methods are effective under the right distributional assumptions, but they can lead to inaccurate results if the conditions are not satisfied. In contrast, nonparametric methods offer great flexibility and robustness to outliers but this approach is computationally intensive. The findings from both tasks 3 and 4 highlight the importance of selecting the most appropriate methods from the data distribution when it comes to estimating the optimal order quantity for the calculation of profit that is outlined in task 2 to ensure the accuracy in demand estimation, optimal inventory management, and maximum profits considering shipping and clearance costs.

Optimal Price-changes to Maintain Profit-levels

Identifying how much to increase the prices in the next 6 months to maintain a similar profit level after a 25% increase in cost can provide necessary recommendations for the manager of the stores at Main Street A and Station B. Since the exact current demands are not known we assume that the current demands for the stores that are located at Main Street A and Station B are 100 to simplify computations. This allows the calculation of the optimal price increase using the profit function.

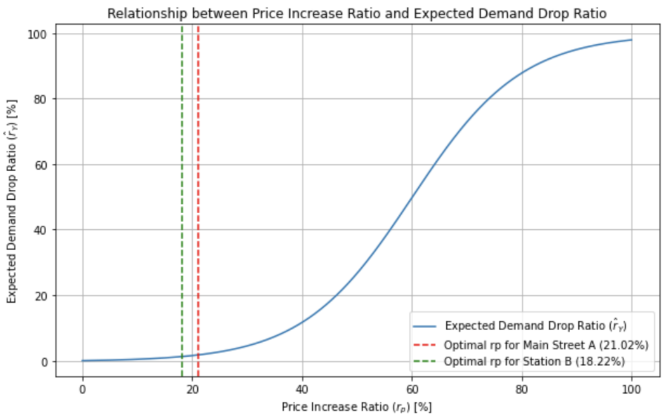
The relationship between price increases and demand decreases can help the manager make informed decisions about how much to raise prices to ensure that the stores maintain their profitability at the same level despite the rising costs.

This relationship can be explained by the following function:

$$r_Y = \left\{ \left[1 + \exp\left(6 - \frac{r_p}{10}\right) \right]^{-1} - 0.0025 \right\} \times 100$$

This equation shows how the expected demand drop r_Y is influenced by the price increase ratio r_p. For example, an increase in the price by 50% can lead to a drop of approximately 26.6441% in the expected demand.

The following plot can help to visualize the impact of the price increases on demand and highlight the optimal price increase ratios for maintaining the current profit levels.



The X-axis represents the percentage increase in price and the Y-axis shows the percentage decrease in expected demand as a result of the price increase.

The blue curve shows the expected demand drop ratio as a function of the price increase ratio. It shows a non-linear relationship, where small price increases have a smaller impact on the demand while larger price increases have a larger effect on demand.

The red dashed line indicates the optimal price increase ratio for the store that is located at Main Street A at 21.02%. This means that the store at Main Street A should increase prices by 21.02% to counterbalance the 25% cost increase resulting in a manageable demand drop. The corresponding demand drop due to this price increase can be observed where the red dashed line intersects the blue curve.

The green dashed line shows the optimal price increase ratio for the store that is located at Station B at 18.22%. This means this store at Station B should increase prices by 18.22% to manage cost increases resulting in a smaller demand drop. The intersection between the green dashed line and the blue curve illustrates the corresponding drop because of this price increase.

Suggestions and recommendations for the manager:

Implementing the optimal price increase by 21.02% for Main Street A and 18.22% for Station B helps these stores counterbalance the 25% increase in costs over the next 6 months while maintaining their similar profit levels.

The plot visually demonstrates the demand sensitivity to price changes. For example, an increase in price beyond the optimal point of 21.02% would lead to a higher demand drop, and this approach therefore provides a clear guide to which level of price should be increased to maintain profitability for these stores.

Alternative strategies for the manager:

Value addition that is to improve the product quality by offering additional services, or creating special offers to justify the price increases to customers.

Promotions and marketing by implementing targeted marketing campaigns to maintain customer interest and loyalty despite the price increase.

Regular review of economic factors such as inflation, supply chain disruptions, and market competition to make some proper adjustments for product prices can maintain the profitability of these stores.

Critical Review

Evaluation of Methodology, Future Research

Throughout the paper, several assumptions and restrictions were made to apply the methods discussed in the course. It is important to address these limitations to evaluate how the results and/or methods of the study may change if the limitations are not met.

In the theoretical section of the paper, the limitations of parametric and nonparametric estimation are investigated separately. Parametric estimation can be highly biased for real demand distributions with skewed or heavy tails, while nonparametric methods require large sample sizes to achieve sufficient accuracy.

In the empirical section, there are some limitations to parametric and nonparametric methods. One of them is that parametric approaches can lead to suboptimal order quantities if the assumption of demand distribution does not hold. This strong assumption of parametric methods also leads to less flexibility. Furthermore, the parametric approaches are likely to lead to wrong results due to their sensitivity to outliers and skewed data. Nonparametric methods also have their inconveniences such as their requirement for a larger sample size to achieve the same level of statistical power, and flexibility in the distributional assumptions, which result in a greater computational effort and thus lead to a slower process.

During our analysis, we made several assumptions. One key assumption is the demand for independence. The demand independence assumes that daily demand is independently and identically distributed (i.i.d.). Additionally, we assume fixed costs and price stability, meaning that costs, shipping costs, prices, and clearance prices are constant.

Parametric approaches in the empirical tasks focus on strong assumptions of the demand distribution. This is essential for achieving more accurate estimates under the right conditions and simplifying the computation process due to the specific distribution with lower datasets.

A key learning point in the comparison between Monte Carlo simulation and self-help methods is that Monte Carlo simulation is useful for evaluating the performance of an estimator in a variety of scenarios, whereas the self-help method provides robust estimates of the distribution of statistics without relying on parametric assumptions.

For future research, more data could be collected from the bakery stores. Additional information about the sales patterns given price changes, consumer preferences, and business expenses would allow a more complete analysis of the operations of the bakery chain

References

Larin, D. (2022). "Optimizing the Level of Slow Moving Aviation Spare Parts Inventory with Economic Order Quantity Model Using Bootstrap Method". In: *10.1007/978-3-030-96196-1*$_2$1.

Liu, C., A. N. Letchford, and I. Svetunkov (2022). "Newsvendor problems: An integrated method for estimation and optimization". In: *European Journal of Operational Research* 300.2, pp. 590–601.

Rahmadini, N. (2023). "Inventory Control of Vaccine Products in Pharmaceutical Company Using The Economic Order Quantity Model and Monte Carlo Simulation". In: *International Journal of Global Operations Research* 4, pp. 229–234. DOI: 10.47194/ijgor.v4i4.257.