

PERMANOVA

Eduard Szöcs

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How to

PERMANOVA

Eduard Szöcs

Institute for Environmental Sciences - University Koblenz-Landau

January 20, 2016

PERMANOVA

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How to

- ▶ Assumptions of MANOVA:
 - ▶ Independence of the sample units
 - ▶ Multivariate normality
 - ▶ Homogeneity of variance–covariance matrices
- ▶ Euclidean distance useful?
- ▶ Generally **not** met for ecological data sets!
- ▶ Need a robust method to handle complex data sets.

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Permutational Multivariate Analysis of Variance Using Distance Matrices

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Austral Ecology (2001) **26**, 32–46

A new method for non-parametric multivariate analysis of variance

MARTI J. ANDERSON

*Centre for Research on Ecological Impacts of Coastal Cities, Marine Ecology L
University of Sydney, New South Wales 2006, Australia*



Abstract Hypothesis-testing methods for multivariate data are needed to make rigorous tests about the effects of factors and their interactions in experiments. Analysis of variance

- ▶ Very influential paper in community ecology
 - ▶ Google Scholar: >4500 citations
- ▶ Non-parametric approach combined with ecological distance measures!

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Data set used in this lecture

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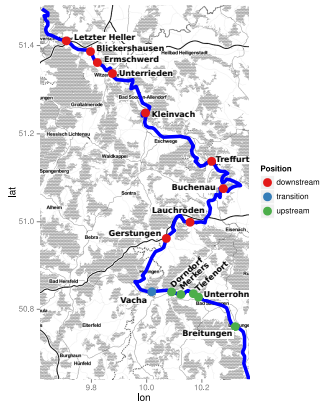
Summary

How to

Macroinvertebrate data from the River Werra¹

Aim : Effect of anthropogenic salinisation on macroinvertebrate communities

- ▶ upstream - downstream design
- ▶ salt brine discharge around Vacha
- ▶ Do the communities differ between up- and downstream?
- ▶ Not the original data (proprietary).



¹Bäthe, Jürgen, and Eckhard Coring. Biological Effects of Anthropogenic Salt-load on the Aquatic Fauna: A Synthesis of 17 Years of Biological Survey on the Rivers Werra and Weser. Limnologia - Ecology and Management of Inland Waters 41(2): 125-133.

First impression of the data

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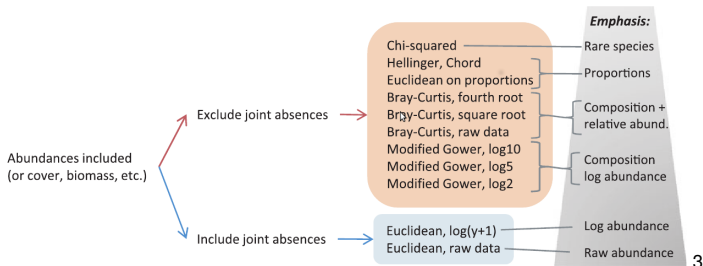
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How to

► NMDS Bray-Curtis-Distance and $x^{0.25}$ transformation.



³Anderson MJ, Crist TO, Chase JM, Vellend M, Inouye BD, Freestone AL, et al. Navigating the multiple meanings of beta diversity: a roadmap for the practicing ecologist. Ecology Letters. 2011;14(1):19–28.

First impression of the data - NMDS

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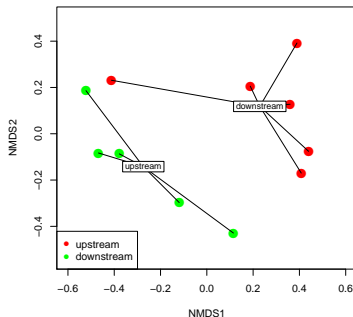
Assumptions

Summary

How to

- ▶ NMDS Bray-Curtis-Distance and $x^{0.25}$ transformation.

- ▶ upstream and downstream sites clearly separate in NMDS.
- ▶ Spread looks similar.
- ▶ Indication of a difference between upstream and downstream.



Recap: ANOVA

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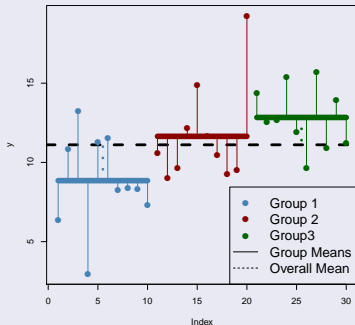
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How to

Question : How is univariate ANOVA calculated?

From univariate...



$$SS_{total} = SS_{residual} + SS_{group}$$

$$F - ratio = \frac{SS_{group}}{SS_{residual}} \cdot \frac{df_{residual}}{df_{group}}$$

Recap: ANOVA

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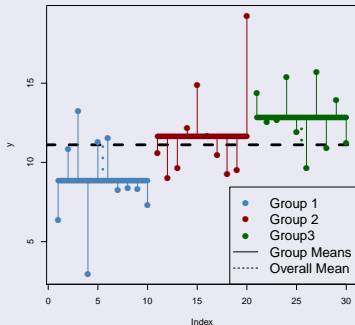
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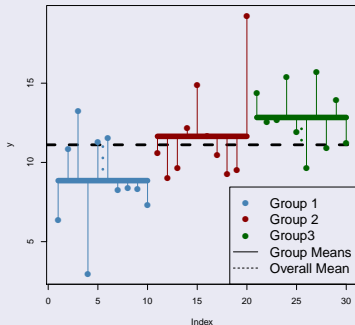
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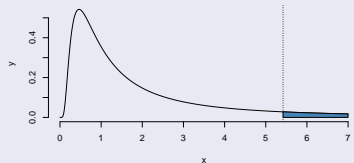
From univariate...



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F-distribution



Distance-based MANOVA

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Distance-based MANOVA is analogous!

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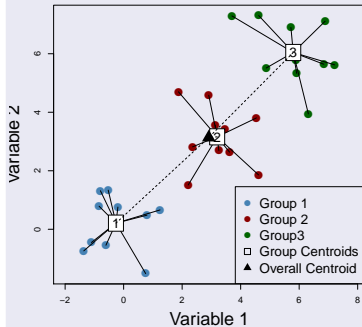
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Summary

How to

... to multivariate ANOVA



- ▶ Partitioning into variance components:

$$SS_{total} = SS_{group} + SS_{residual}$$

- ▶ **centroids**
- ▶ p-value by **permutations**

Variance Partitioning

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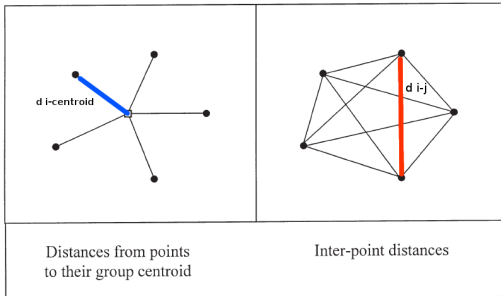
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How to

- ▶ We can use any **Distance Matrix** to partition the variance.
- ▶ Sum of squared distances from individual points to their centroid is equal to the sum of squared interpoint distances divided by the number of points.
- ▶ $\sum d_{i-\text{centroid}}^2 = \frac{1}{N} \sum d_{i-j}^2$



Variance Partitioning

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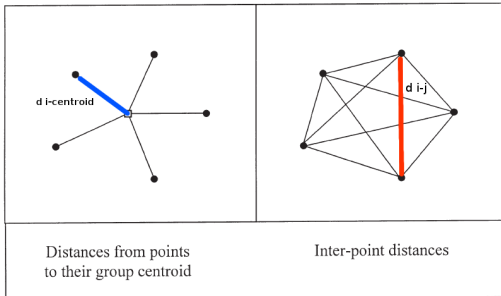
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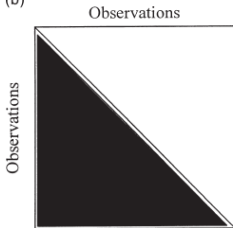
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How to

Like in univariate ANOVA variance can be partitioned:

(b)



$$SS_{total} = \frac{1}{N} \sum_{i=1}^{N-1} \sum_{j=i+1}^N d_{ij}^2$$

N = total number of observations

$$SS_{residual} = \frac{1}{n} \sum_{i=1}^{N-1} \sum_{j=i+1}^N d_{ij}^2 \epsilon_{ij}$$

n = number of observations per group

$$\epsilon_{ij} = \begin{cases} 1, & \text{if observations } i \text{ and } j \text{ are in the same group} \\ 0, & \text{otherwise} \end{cases}$$

$$SS_{group} = SS_{total} - SS_{residual}$$

$$(Pseudo-)F = \frac{SS_{group}}{SS_{residual}} \frac{N-a}{a-1}$$

a = no. groups

p-value is assessed via permutations.

Variance Partitioning

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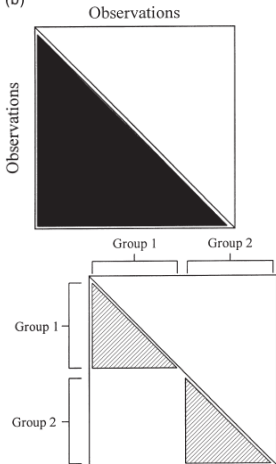
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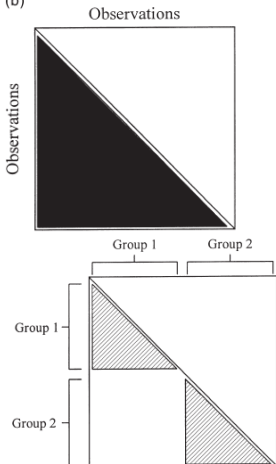
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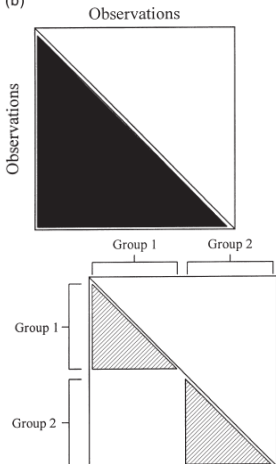
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p-values using permutations

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- ▶ Cannot use Fisher's F-ratio

- ▶ normal distribution?
- ▶ euclidean distance?

- ▶ Instead use **permutations**

- ▶ shuffle data randomly
- ▶ compute F-Ratio (F_{perm})
- ▶ repeat many times

- ▶ compare **F of randomized data** with **original F**.

- ▶
$$p = \frac{\text{No. of } F_{perm} \geq F}{\text{No. of permutations} + 1}$$

p-values using permutations

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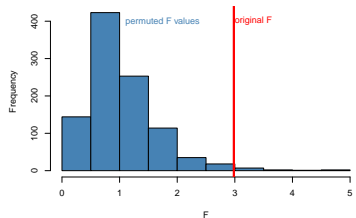
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- ▶
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Histogramm of permuted F values



Permutations (II)

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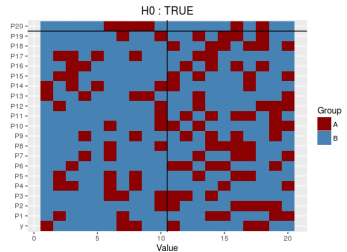
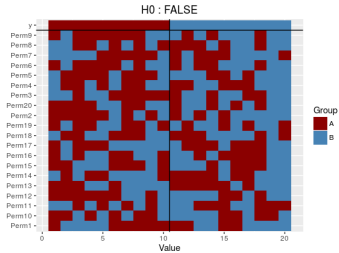
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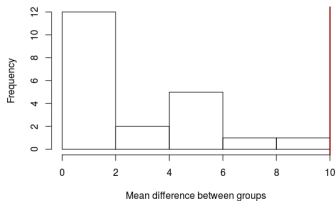
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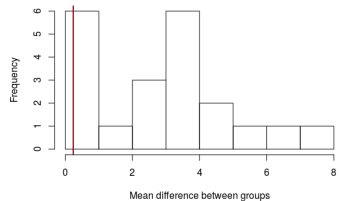
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Hist. of differences between groups



Hist. of differences between groups



Assumptions of PERMANOVA

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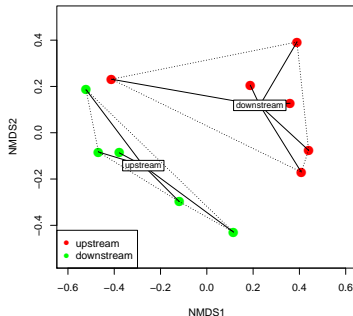
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How to

- ▶ equal **dispersions**
- ▶ Visual inspection
- ▶ Multivariate analogue to Levene's test available.⁴
- ▶ Multivariate Dispersion
 - ▶ β -diversity
 - ▶ functional diversity
 - ▶ see literature folder



⁴ Anderson, M. J. 2006. Distance Based Tests for Homogeneity of Multivariate Dispersions. Biometrics 62 (1): 245-253.

Assumptions of PERMANOVA

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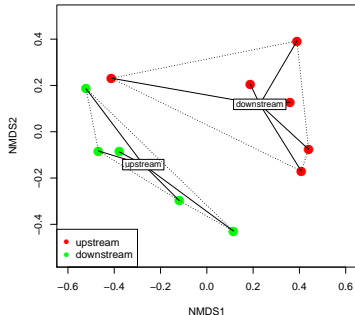
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PERMANOVA is a

- ▶ flexible (any distance measure) and
 - ▶ easy (analogue to univariate Anova)
- tool for ecologists.

However,

- ▶ non-parametric does no mean assumption free.

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Lets do it in R!

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use **R**!