FINDING THE EQUATION OF CIRCLE

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February 15, 2019

Question

If the line 2x+3y+1=0 and 3x-y-4=0 lie along the diameters of a circle of circumference 10π then the equation of the circle is ?

Solution

given equations are

$$\begin{bmatrix} 2 & 3 \end{bmatrix} x = -1$$
$$\begin{bmatrix} 3 & -1 \end{bmatrix} x = 4$$

and circumference of circle is 10π

by solving we will get the intersection point as $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

We know that the circumference of the circle is 2πr As the given circumference is 10π

$$2\pi r = 10\pi$$

therefore

radius of given circle is (r=5)

we know that the equation of circle with centre

$$C = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\|(X - C)\|^2 = r^2$$

$$(X - C)^T(X - C) = r^2$$

$$X^T X - 2C^T X = r^2 - C^T C$$

by substituting the centre C equation of the circle is

$$X^T X - \begin{bmatrix} 1 & -1 \end{bmatrix} X = r^2 - \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

therefore

$$x^2 + y^2 - 2x + 2y - 23 = 0$$

is the required circle equation

Extra

x=$r\cos(\Theta)$ +h y=$\sin(\Theta)$ +k

by differentiating slope of circle at Θ is

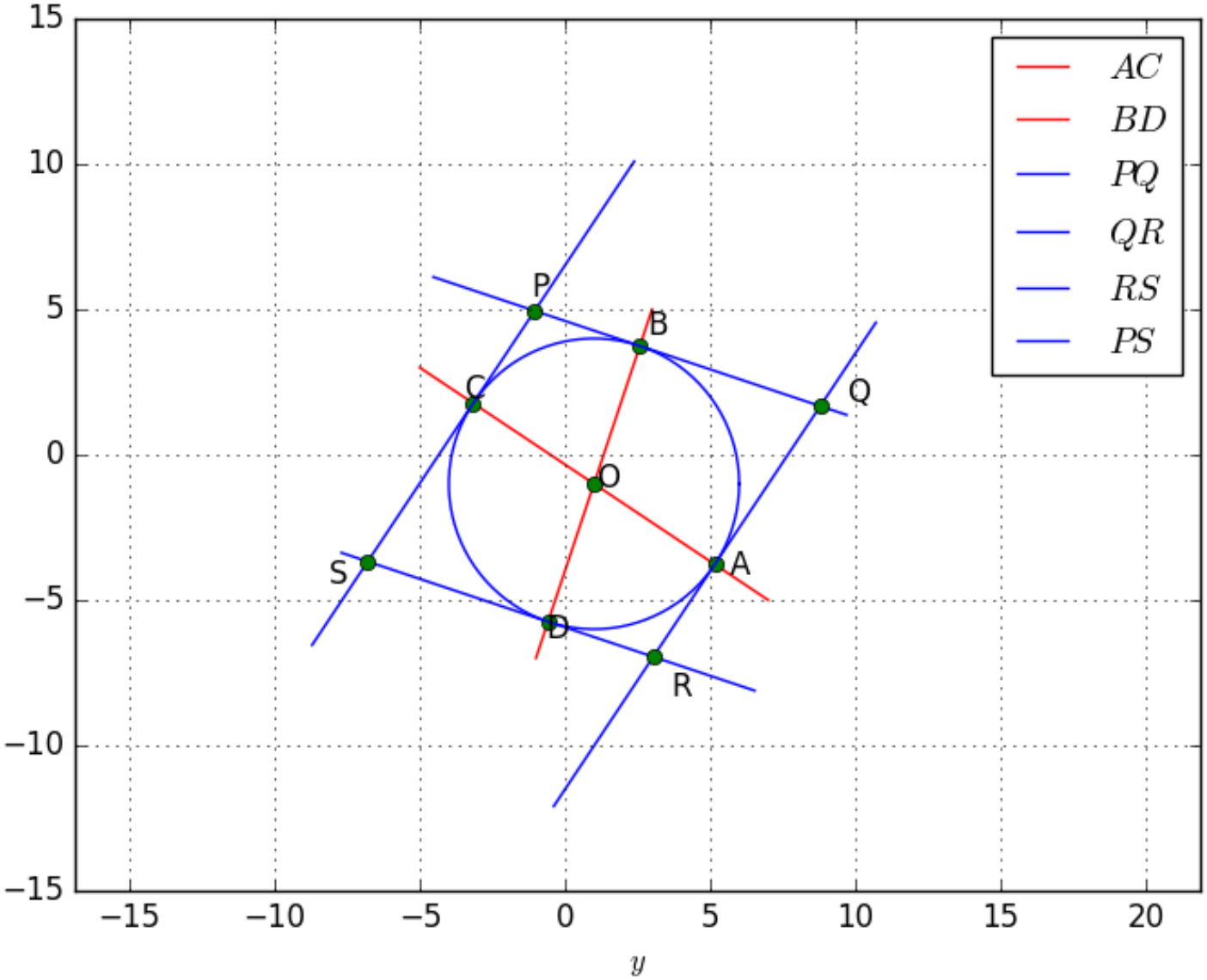
m = -$\cot\Theta$

as we know the values of Θ by the given line equations we will get the slopes of tangents at 4 intersection points now we will find tangent equations from slope and point

we will get the 4 intersection points of tangents

now we can find the area of one triangle by $(base * height) \div 2$

now the area of quadrilateral is 2*area of triangle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