Control systems(EE2101)

EE19BTECH11016
S.Naga Chandra Mouli

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Find the transfer function,**G(s)** = $V_0(s)/V_i(s)$ *for each of the* networks shown below,solve the problem using mesh analysis ?

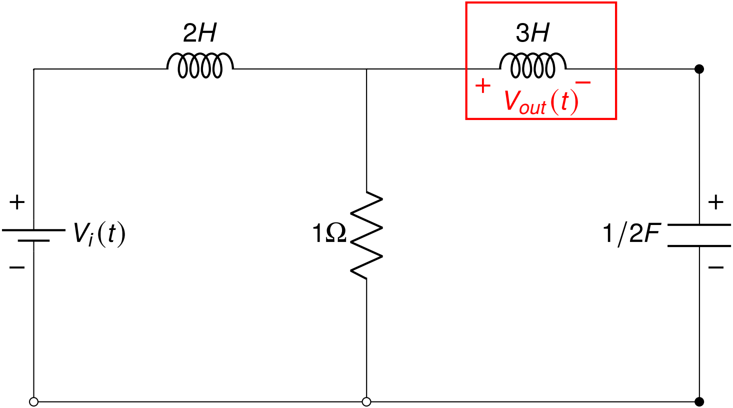

Figure: 1st circuit

by converting above circuit into frequency domain we get as below

consider the currents $I_1(s)$ *and* $I_2(s)$ *in loop*1 *and loop*2 respectively in clockwise direction

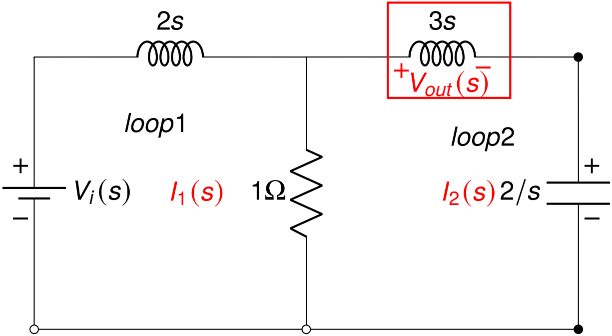

Figure: 1st circuit

- from loop1 we get

$$V_i(s) = 2sI_1(s) + 1(I_1(s) - I_2(s) \quad (1)$$
$$V_i(s) = (2s+1)I_1(s) - I_2(s) \quad (2)$$

- from loop2 we get

$$\mathrm{I}_2(s)(3s + 2/s + 1) - I_1(s) = 0$$
$$\mathrm{I}_2(s)(3s + 2/s + 1) = I_1(s)$$
$$\mathrm{V}_0(s) = 3sI_2(s)$$

by substituting $I_2(s)$ *in terms of* $I_1(s)$ *in equation*(1) *gives*
and substitute $I_2(s)$ *in terms of* $V_{out}(s)$ *we get*

$$V_i(s) = (2s+1)(3s+2/s+1)I_2(s) - I_2(s)$$
$$V_i(s) = [(2s+1)(3s+2/s+1) - 1]I_2(s)$$
$$V_i(s) = [(2s+1)(3s+2/s+1) - 1]\frac{V_{out}(s)}{3s}$$
$$\frac{V_{out}(s)}{V_i(s)} = \frac{3s^2}{6s^3+5s^2+4s+2}$$

$$\boxed{\mathbf{G(s)} = \frac{V_{out}(s)}{V_i(s)} = \frac{3s^2}{6s^3+5s^2+4s+2}} \tag{3}$$

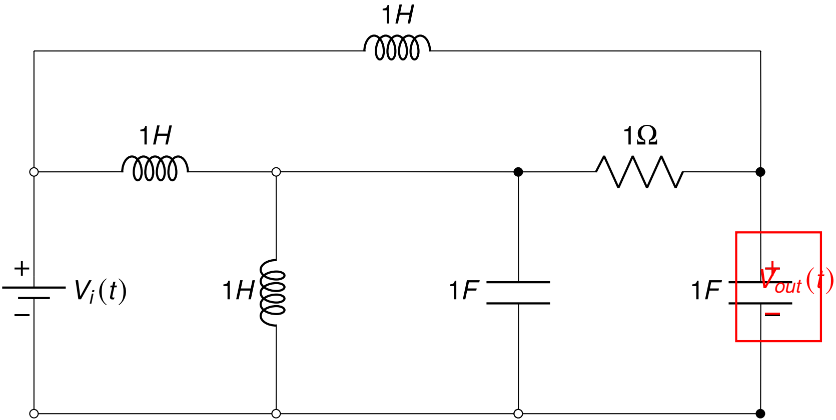

Figure: 2nd circuit

by simplifying parallel combination of s and 1/s we get

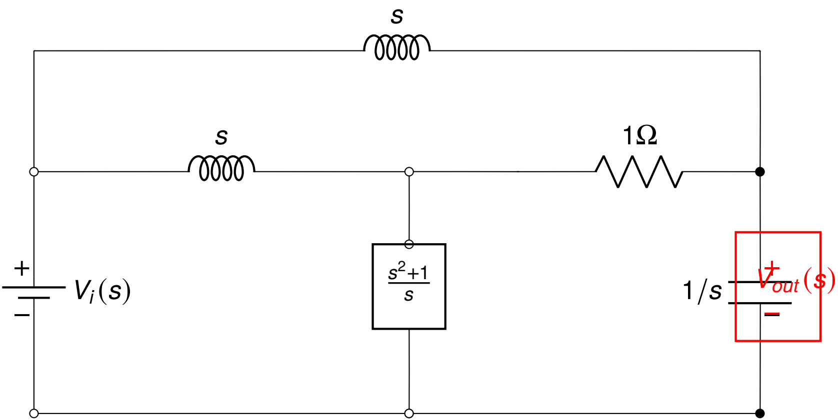

Figure: 2nd circuit

by converting above circuit into frequency domain we get as below

consider the currents $I_1(s)$ *and* $I_2(s)$ *and* $I_3(s)$ *in loop*1 and loop2 and loop3 respectively in clockwise direction

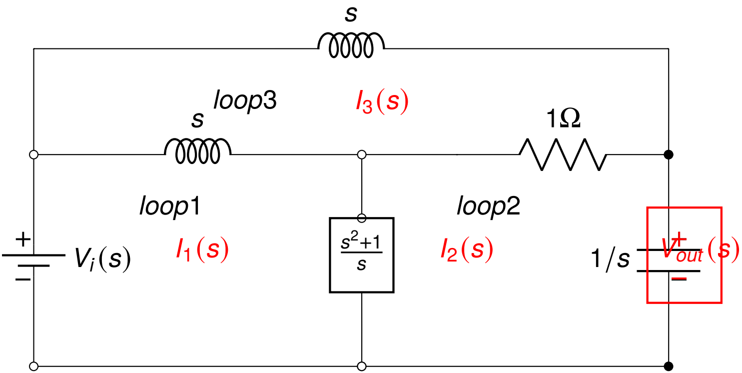

Figure: 2nd circuit

- by solving loop equations we get

$$(s+\frac{s}{s^2+1})I_1(s)-(\frac{s}{s^2+1})I_2(s)-sI_3(s)=V_i(s)$$

$$-(\frac{s}{s^2+1})I_1(s)+(\frac{s}{s^2+1}+1+\frac{1}{s})I_2(s)-I_3(s)=0$$

$$-sI_1(s)-I_2(s)+(2s+1)I_3(s)=0$$

$$\mathrm{V}_{out}(s) = \frac{I_2(s)}{s}$$

$$\begin{bmatrix} s+\frac{s}{s^2+1} & -\frac{s}{s^2+1} & -s \\ -\frac{s}{s^2+1} & \frac{s}{s^2+1}+1+\frac{1}{s} & -1 \\ -s & -1 & 2s+1 \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \\ I_3(s) \end{bmatrix} = \begin{bmatrix} V_i(s) \\ 0 \\ 0 \end{bmatrix}$$

From Cramer's Rule,

$$\mathrm{I}_1(s) = \frac{\Delta_1}{\Delta}, I_2(s) = \frac{\Delta_2}{\Delta}$$

$$\mathrm{I}_3(s) = \frac{\Delta_3}{\Delta}$$

$$\Delta_2 = \begin{vmatrix} s + \dfrac{s}{s^2+1} & V_i(s) & -s \\ -\dfrac{s}{s^2+1} & 0 & -1 \\ -s & 0 & 2s+1 \end{vmatrix}$$

$$= \mathrm{V}_i(s)\left[s\left(\frac{2s+1}{s^2+1}+1\right)\right]$$

$$\Delta_2 = sV_i(s)\frac{s^2+2s+2}{s^2+1}$$

$$\Delta = \begin{vmatrix} s + \dfrac{s}{s^2+1} & -\dfrac{s}{s^2+1} & -s \\ -\dfrac{s}{s^2+1} & \dfrac{s}{s^2+1} + 1 + \dfrac{1}{s} & -1 \\ -s & -1 & 2s+1 \end{vmatrix}$$

$$\Delta = \frac{s^4 + 2s^3 + 3s^2 + 3s + 2}{s^2+1}$$

$$\mathrm{I}_2(s) = \frac{\Delta_2}{\Delta}$$

$$\mathrm{V}_{out}(s) = \frac{I_2(s)}{s}$$

$$\mathrm{I}_2(s) = \frac{V_i(s)s(s^2+2s+2)}{s^4+2s^3+3s^2+3s+2}$$

$$\mathrm{V}_{out}(s) = \frac{V_i(s)(s^2+2s+2)}{s^4+2s^3+3s^2+3s+2}$$

$$\frac{V_{out}(s)}{V_i(s)} = \frac{(s^2+2s+2)}{s^4+2s^3+3s^2+3s+2}$$

$$\boxed{\mathbf{G(s)} = \frac{(s^2+2s+2)}{s^4+2s^3+3s^2+3s+2}}$$

The End