Assignment 8

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***Abstract*—This document lists out the axioms satisfied for a vector space.**

Download all latex-tikz codes from

https://github.com/EE20MTECH14019/EE5609/tree/master/Assignment_8

1 Problem

On $\mathbb{R}^n$ define two operations

$$\alpha \oplus \beta = \alpha - \beta \tag{1.0.1}$$

$$c \cdot \alpha = -c\alpha \tag{1.0.2}$$

The operations on the right are usual ones. Which of the axioms for a vector space are satisfied by $(\mathbb{R}^n, \oplus, \cdot)$?

2 Solution

Let $(\alpha, \beta, \gamma) \in \mathbb{R}^n$ and c, c_1, c_2 are scalars taken from the field $\mathbb{R}$ where the vector space is defined on. Table 0 lists the axioms satisfied and not satisfied for $(\mathbb{R}^n, \oplus, \cdot)$.

UNSATISTIFD	SATISFIED
Associativity of addition $\alpha \oplus (\beta \oplus \gamma) = \alpha - \beta + \gamma$ $(\alpha \oplus \beta) \oplus \gamma = \alpha - \beta - \gamma$ $\alpha \oplus (\beta \oplus \gamma) \neq (\alpha \oplus \beta) \oplus \gamma$	**Additive identity** $\alpha \oplus \beta = \alpha - \beta = \alpha$ Additive identity is β unique $\beta = (0, 0, ...0)$
Commutativity of addition $\alpha \oplus \beta = \alpha - \beta$ $\beta \oplus \alpha = \beta - \alpha$ $\alpha \oplus \beta \neq \beta \oplus \alpha$	**Additive inverse** $\alpha \oplus \alpha = \alpha - \alpha = 0$ Additive inverse is α
Scalar multiplication with field multiplication $(c_1c_2) \cdot \alpha = (-c_1c_2)\alpha$ $c_1 \cdot (c_2 \cdot \alpha) = c_1c_2\alpha$ $(c_1c_2) \cdot \alpha \neq c_1 \cdot (c_2 \cdot \alpha)$	
Identity element of scalar multiplication $1 \cdot \alpha = -\alpha = \alpha$ for $\alpha = (0, 0, ..., 0)$ $1 \cdot \alpha = -\alpha \neq \alpha \quad \forall \quad \alpha \neq (0, 0, ..., 0)$	
Distributivity of scalar multiplication w.r.t vector addition $c \cdot (\alpha \oplus \beta) = -c(\alpha - \beta)$ $c \cdot \alpha \oplus c \cdot \beta = -c\alpha - (-c\beta)$ $c \cdot (\alpha \oplus \beta) \neq c \cdot \alpha \oplus c \cdot \beta$	
Distributivity of scalar multiplication w.r.t field addition $(c_1 + c_2) \cdot \alpha = -(c_1 + c_2)\alpha$ $c_1 \cdot \alpha \oplus c_2 \cdot \beta = -c_1\alpha - (-c_2\beta)$ $(c_1 + c_2) \cdot \alpha \neq c_1 \cdot \alpha \oplus c_2 \cdot \beta$	

TABLE 0: Axioms of vector space $(\mathbb{R}^n, \oplus, \cdot)$