Assignment 15

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***Abstract*—This document solves a matrix problem using the property of eigen values.**

Download all latex-tikz codes from

https://github.com/EE20MTECH14019/EE5609/tree/master/Assignment_15

1 Problem

Let $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$. Then the smallest positive integer n such that $\mathbf{A}^n = \mathbf{I}$ is

2 Property of eigen values of A

Let $\mathbf{A}$ be an arbitary $n \times n$ matrix of complex numbers with eigen values $\lambda_1, \lambda_2, \ldots, \lambda_n$. Then the eigen values of kth power of $\mathbf{A}$, that is the eigen values of $\mathbf{A}^k$, for any positive integer k are $\lambda_1^k, \lambda_2^k, \ldots, \lambda_n^k$.

3 Solution

Let us calculate the eigen values of $\mathbf{A}$.

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix} \tag{3.0.1}$$

$$\det(\mathbf{A} - \lambda\mathbf{I}) = 0 \tag{3.0.2}$$

$$\begin{vmatrix} -\lambda & 1 \\ -1 & 1-\lambda \end{vmatrix} = 0 \tag{3.0.3}$$

$$-\lambda(1-\lambda) + 1 = 0 \tag{3.0.4}$$

$$\lambda^2 - \lambda + 1 = 0 \tag{3.0.5}$$

$$\implies \lambda = \frac{-1 \pm \sqrt{3}i}{2} \tag{3.0.6}$$

From the property 2, the eigen values of $\mathbf{A}^n$ are λ^n. Also as it is given that $\mathbf{A}^n = \mathbf{I}$,

$$\implies \lambda^n = 1 \tag{3.0.7}$$

$$\implies \left(\frac{-1 \pm \sqrt{3}i}{2}\right)^n = 1 \tag{3.0.8}$$

Clearly $n \neq 1$. For $n = 2$,

$$\left(\frac{-1 \pm \sqrt{3}i}{2}\right)^2 = \frac{-1 \mp \sqrt{3}i}{2} \tag{3.0.9}$$

For $n = 4$,

$$\left(\frac{-1 \pm \sqrt{3}i}{2}\right)^4 = \frac{-1 \pm \sqrt{3}i}{2} \tag{3.0.10}$$

For $n = 6$,

$$\left(\frac{-1 \pm \sqrt{3}i}{2}\right)^6 = 1 \tag{3.0.11}$$

Hence $n = 6$ is the smallest positive integer.