

# ASSIGNMENT 4

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**Abstract**—This document finds the equation of circle passing through three given points.

Download all python codes from

[https://github.com/EE20MTECH14019/EE5609/tree/master/Assignment\\_4/Codes](https://github.com/EE20MTECH14019/EE5609/tree/master/Assignment_4/Codes)

and latex-tikz codes from

[https://github.com/EE20MTECH14019/EE5609/tree/master/Assignment\\_4](https://github.com/EE20MTECH14019/EE5609/tree/master/Assignment_4)

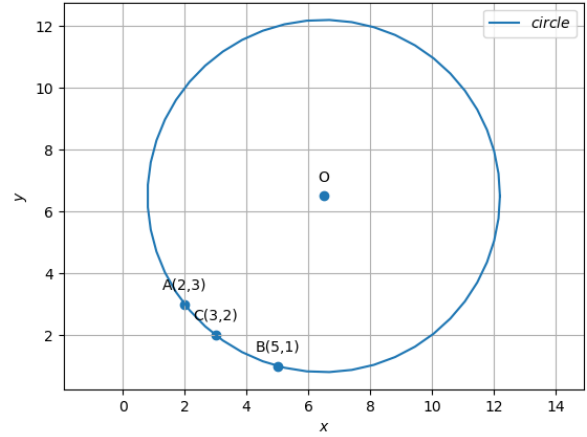


Fig. 0: Circle passing through the points A,B,C with center O

## 1 PROBLEM

Find the equation of the circle that passes through the points  $\begin{pmatrix} 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 5 \\ 1 \end{pmatrix}$

## 2 SOLUTION

The general equation of circle is represented as

$$\mathbf{x}^T \mathbf{x} - 2\mathbf{c}^T \mathbf{x} + f = 0 \quad (2.0.1)$$

where  $\mathbf{c}$  is the center of the circle. Substituting the given points in the equation (2.0.1), we obtain

$$2 \begin{pmatrix} 2 & 3 \end{pmatrix} \mathbf{c} - f = 13 \quad (2.0.2)$$

$$2 \begin{pmatrix} 3 & 2 \end{pmatrix} \mathbf{c} - f = 13 \quad (2.0.3)$$

$$2 \begin{pmatrix} 5 & 1 \end{pmatrix} \mathbf{c} - f = 36 \quad (2.0.4)$$

can be expressed in matrix form as

$$\begin{pmatrix} 4 & 6 & -1 \\ 6 & 4 & -1 \\ 10 & 2 & -1 \end{pmatrix} \begin{pmatrix} \mathbf{c} \\ f \end{pmatrix} = \begin{pmatrix} 13 \\ 13 \\ 36 \end{pmatrix} \quad (2.0.5)$$

The augmented matrix for (2.0.5) can be row

reduced as follows

$$\begin{pmatrix} 4 & 6 & -1 & 13 \\ 6 & 4 & -1 & 13 \\ 10 & 2 & -1 & 26 \end{pmatrix} \quad (2.0.6)$$

$$\begin{matrix} R_3 \leftarrow 4R_3 - 10R_1 \\ R_2 \leftarrow 4R_2 - 6R_1 \end{matrix} \begin{pmatrix} 4 & 6 & -1 & 13 \\ 0 & -20 & 2 & -26 \\ 0 & -52 & 6 & -26 \end{pmatrix} \quad (2.0.7)$$

$$\begin{matrix} R_3 \leftarrow 5R_3 - 13R_2 \end{matrix} \begin{pmatrix} 4 & 6 & -1 & 13 \\ 0 & -20 & 2 & -26 \\ 0 & 0 & 4 & 208 \end{pmatrix} \quad (2.0.8)$$

$$\begin{matrix} R_2 \leftarrow 2R_2 - R_3 \\ R_1 \leftarrow 4R_1 + R_3 \end{matrix} \begin{pmatrix} 16 & 24 & 0 & 260 \\ 0 & -40 & 0 & -260 \\ 0 & 0 & 4 & 208 \end{pmatrix} \quad (2.0.9)$$

$$\begin{matrix} R_1 \leftarrow 5R_1 + 3R_2 \end{matrix} \begin{pmatrix} 80 & 0 & 0 & 520 \\ 0 & -40 & 0 & -260 \\ 0 & 0 & 4 & 208 \end{pmatrix} \quad (2.0.10)$$

$$\begin{matrix} R_2 \leftarrow \frac{R_2}{-20}, R_3 \leftarrow \frac{R_3}{4} \\ R_1 \leftarrow \frac{R_1}{40} \end{matrix} \begin{pmatrix} 2 & 0 & 0 & 13 \\ 0 & 2 & 0 & 13 \\ 0 & 0 & 1 & 52 \end{pmatrix} \quad (2.0.11)$$

From the matrix (2.0.11),

$$\mathbf{c} = \begin{pmatrix} 13/2 \\ 13/2 \end{pmatrix} \quad (2.0.12)$$

$$k = 52 \quad (2.0.13)$$

$$r = \sqrt{\|\mathbf{c}\|^2 - f} = 11 \quad (2.0.14)$$