Assignment 16

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***Abstract*—This document solves a problem on Jordan form of a complex matrix.**

Download all latex-tikz codes from

https://github.com/EE20MTECH14019/EE5609/tree/master/Assignment_16

1 Problem

How many possible Jordan forms are there for a 6×6 complex matrix with characteristic polynomial $(x+2)^4(x-1)^2$?

2 Explanation

From the characteristic polynomial,

$$(x+2)^4(x-1)^2 \tag{2.0.1}$$

We get the eigen values of the 6×6 complex matrix as,

$$\lambda_i = \{-2,-2,-2,-2,1,1\} \tag{2.0.2}$$
$$\text{for } \lambda = -2, A_M = 4 \tag{2.0.3}$$
$$\text{for } \lambda = 1, A_M = 2 \tag{2.0.4}$$

The minimal polynomial for a matrix with characteristic polynomial must have both $(x+2)$ and $(x-1)$ as factors and it must divide (2.0.1). Hence the minimal polynomial will be of the form

$$p = (x+2)^a(x-1)^b \quad, a \leq 4, b \leq 2 \tag{2.0.5}$$

So, there are 8 different possibilities for a minimal polynomial. Let us note that one minimal polynomial may correspond to more than one Jordan forms. The possible Jordan blocks associated with the eigen values -2 and 1 are given in the Table 2. From Table 2 , we can say that there are $5\times2=10$ possible Jordan forms. For example, the Jordan forms corresponding to minimal polynomial $p=(x+2)^2(x-1)$ built from the Jordan blocks can be,

$$\mathbf{J} = \begin{pmatrix} -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{2.0.6}$$

and

$$\mathbf{J} = \begin{pmatrix} -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{2.0.7}$$

Parameter	Description
A_M	Algebraic multiplicity of λ in the characteristic polynomial, also equal to the size of Jordan block for that eigen value
G_M	Geometric multiplicity determines the number of Jordan sub-blocks in a Jordan block for λ.
$\mathbf{J}_{(x-\lambda)^k}$	Jordan block corresponding to the eigen value λ and k is the multiplicity of λ in the minimal polynomial determines size of largest Jordan sub-block.

TABLE 1: Parameters

Factor	Possible Jordan blocks
$(x+2)$	$$\mathbf{J}_{(x+2)} = \begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} \quad (G_M = 4) \tag{2.0.8}$$ $$\mathbf{J}_{(x+2)^2} = \begin{pmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} \quad (G_M = 3) \tag{2.0.9}$$ $$\mathbf{J}_{(x+2)^2} = \begin{pmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & -2 \end{pmatrix} \quad (G_M = 2) \tag{2.0.10}$$ $$\mathbf{J}_{(x+2)^3} = \begin{pmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} \quad (G_M = 2) \tag{2.0.11}$$ $$\mathbf{J}_{(x+2)^4} = \begin{pmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & -2 \end{pmatrix} \quad (G_M = 1) \tag{2.0.12}$$
$(x-1)$	$$\mathbf{J}_{(x-1)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (G_M = 2) \tag{2.0.13}$$ $$\mathbf{J}_{(x-1)^2} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \quad (G_M = 1) \tag{2.0.14}$$

TABLE 2: Possible Jordan Blocks

3 Answer

Therefore 10 different Jordan forms are possible for a 6×6 complex matrix with characteristic polynomial $(x+2)^4 (x-1)^2$.