ASSIGNMENT 3

YENIGALLA SAMYUKTHA
EE20MTECH14019

Abstract—**This document proves that triangles on the same base and having equal areas lie between the same parallel lines.**

Download latex-tikz codes from

https://github.com/EE20MTECH14019/EE5609/tree/master/Assignment_3

1 Problem

Triangles on the same base (or equal bases) and having equal areas lie between the same parallels.

2 Explanation

Consider $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are three points of a triangle $\triangle ABC$, $\mathbf{D}, \mathbf{B}, \mathbf{C}$ are three points of another triangle $\triangle DBC$, both triangles having same base BC and $\mathbf{B}$ at origin,

$$\mathbf{A} - \mathbf{B} = \mathbf{A} \tag{2.0.1}$$
$$\mathbf{C} - \mathbf{B} = \mathbf{C} \tag{2.0.2}$$
$$\mathbf{D} - \mathbf{B} = \mathbf{D} \tag{2.0.3}$$

The area of the triangle $\triangle ABC$ is,

$$Area\left(\triangle ABC\right) = \frac{1}{2}\left\|\left(\mathbf{A} - \mathbf{B}\right) \times \left(\mathbf{C} - \mathbf{B}\right)\right\| \tag{2.0.4}$$

Substituting (2.0.1) and (2.0.2) in (2.0.4),

$$\implies Area\left(\triangle ABC\right) = \frac{1}{2}\left\|\left(\mathbf{A} \times \mathbf{C}\right)\right\| \tag{2.0.5}$$

The area of the triangle $\triangle DBC$ is,

$$Area\left(\triangle ABC\right) = \frac{1}{2}\left\|\left(\mathbf{D} - \mathbf{B}\right) \times \left(\mathbf{C} - \mathbf{B}\right)\right\| \tag{2.0.6}$$

Substituting (2.0.2) and (2.0.3) in (2.0.6),

$$\implies Area\left(\triangle DBC\right) = \frac{1}{2}\left\|\left(\mathbf{D} \times \mathbf{C}\right)\right\| \tag{2.0.7}$$

Given the area of $\triangle ABC$ is equal to the area $\triangle DBC$, from equations (2.0.5) and (2.0.7)

$$\frac{1}{2}\left\|\left(\mathbf{A} \times \mathbf{C}\right)\right\| = \frac{1}{2}\left\|\left(\mathbf{D} \times \mathbf{C}\right)\right\| \tag{2.0.8}$$

In the above equation (2.0.8), the resultant cross product vectors of L.H.S and R.H.S are parallel. We can write equation (2.0.8) as,

$$\mathbf{B} \times \left(\mathbf{A} \times \mathbf{C}\right) = k\left(\mathbf{B} \times \left(\mathbf{D} \times \mathbf{C}\right)\right) \tag{2.0.9}$$

$$\implies \mathbf{A}\left(\mathbf{B}^T\mathbf{C}\right) - \mathbf{C}\left(\mathbf{B}^T\mathbf{A}\right) = k\left(\mathbf{D}\left(\mathbf{B}^T\mathbf{C}\right) - \mathbf{C}\left(\mathbf{B}^T\mathbf{D}\right)\right) \tag{2.0.10}$$

As we considered $\mathbf{B} = 0$ without the loss of generality, we can write (2.0.10) as,

$$\left(\mathbf{A} - \mathbf{D}\right) = k_1\left(\mathbf{B} - \mathbf{C}\right) \tag{2.0.11}$$

where k and k_1 are constants. From the equation (2.0.11), we can say that $AD \parallel BC$. Hence the two triangles $\triangle ABC$ and $\triangle DBC$ lie between the same parallels AD and BC

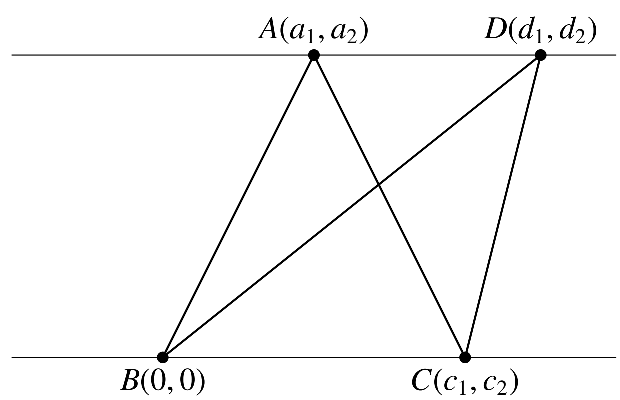

Fig. 0: $\triangle ABC$ and $\triangle DBC$ with BC as common base