Assignment 12

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***Abstract*—This document explains a proof in linear transformations.**

Download all latex-tikz codes from

https://github.com/EE20MTECH14019/EE5609/tree/master/Assignment_12

1 Problem

Let T be the linear operator on $\mathbb{R}^2$ defined by

$$T(x_1, x_2) = (-x_2, x_1) \tag{1.0.1}$$

Prove that for every real number c, the operator $(T - cI)$ is invertible.

2 Solution

From the equation (1.0.1), the matrix of T in standard order basis is,

$$\mathbf{T} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{2.0.1}$$

To find the invertibility of the operator $(\mathbf{T} - c\mathbf{I})$ for every real number c, let us start with

$$(\mathbf{T} - c\mathbf{I})(\mathbf{T} + c\mathbf{I}) \tag{2.0.2}$$

$$= \mathbf{T}^2 - c^2\mathbf{I} \tag{2.0.3}$$

Consider $\mathbf{T}^2$

$$\mathbf{T}^2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \implies \mathbf{T}^2 = -\mathbf{I} \tag{2.0.4}$$

Substituting equation (2.0.4) in (2.0.3),

$$(\mathbf{T} - c\mathbf{I})(\mathbf{T} + c\mathbf{I}) = -\left(1 + c^2\right)\mathbf{I} \tag{2.0.5}$$

As c is a real number, $c^2 \geq 0$ and hence factor $-\left(1 + c^2\right)$ is always non-zero. Therefore, from the equation (2.0.5),

$$(\mathbf{T} - c\mathbf{I})^{-1} = \frac{-1}{1 + c^2}(\mathbf{T} + c\mathbf{I}) \tag{2.0.6}$$

Hence the operator $(T - cI)$ is invertible and its inverse is given by the equation (2.0.6)