ASSIGNMENT 3

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***Abstract*—This document proves that triangles on the same base and having equal areas lie between the same parallel lines.**

Download latex-tikz codes from

https://github.com/EE20MTECH14019/EE5609/tree/master/Assignment_3

1 Problem

Triangles on the same base (or equal bases) and having equal areas lie between the same parallels.

2 Explanation

Triangles $\triangle ABC$ and $\triangle ABD$ are two triangles lying on the same base AB such that area of $\triangle ABC$ is equal to area of $\triangle ABD$. Now we join C and D and and drop perpendiculars onto AB from C and D. Lets say the foot of the perpendiculars are E and F.

As $CE \perp AB$ and $DF \perp AB$, we can write that

$$(\mathbf{C}-\mathbf{E})^T(\mathbf{A}-\mathbf{B}) = 0 \tag{2.0.1}$$
$$(\mathbf{D}-\mathbf{F})^T(\mathbf{A}-\mathbf{B}) = 0 \tag{2.0.2}$$

From equations (2.0.1) and (2.0.2)

$$(\mathbf{C}-\mathbf{E}) = k(\mathbf{D}-\mathbf{F}) \tag{2.0.3}$$

Equation (2.0.3) shows that $CE \parallel DF$. Also given that

$$area(\triangle ABC) = area(\triangle ABD) \tag{2.0.4}$$
$$\implies \frac{1}{2}(\mathbf{AB}\times\mathbf{CA}) = \frac{1}{2}(\mathbf{AB}\times\mathbf{BD}) \tag{2.0.5}$$

Using the triangle law of vectors,

$$\implies (\mathbf{AB}\times(\mathbf{CE}+\mathbf{EA})) = (\mathbf{AB}\times(\mathbf{DF}+\mathbf{FB})) \tag{2.0.6}$$
$$\implies (\mathbf{AB}\times\mathbf{CE}) = (\mathbf{AB}\times\mathbf{DF}) \tag{2.0.7}$$

From equation (2.0.7), $CE{=}DF$.

As $CE \parallel DF$ and $CE{=}DF$, we could say that CDEF forms a parallelogram. And $CD \parallel EF$ as CD and EF are opposite sides of the parallelogram CDEF. From this, we can conclude that $CD \parallel AB$.

Hence $\triangle ABC$ and $\triangle ABD$ lie between the same parallels AB and CD.

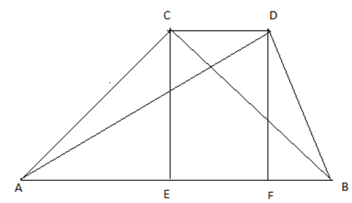

Fig. 0: Triangles of equal area and same base