Assignment 16

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***Abstract*—This document solves a problem on Jordan form of a complex matrix.**

Download all latex-tikz codes from

https://github.com/EE20MTECH14019/EE5609/tree/master/Assignment_16

1 Problem

How many possible Jordan forms are there for a 6×6 complex matrix with characteristic polynomial $(x+2)^4(x-1)^2$?

2 Explanation

From the characteristic polynomial,

$$(x+2)^4(x-1)^2 \tag{2.0.1}$$

We get the eigen values of the 6×6 complex matrix as,

$$\lambda_i = \{-2,-2,-2,-2,1,1\} \tag{2.0.2}$$

The minimal polynomial for a matrix with characteristic polynomial must have both $(x+2)$ and $(x-1)$ as factors and it must divide (2.0.1). Hence the minimal polynomial will be of the form

$$p = (x+2)^a(x-1)^b \ ,a\le 4, b\le 2 \tag{2.0.3}$$

So, there are 8 different possibilities for a minimal polynomial. Let us note that one minimal polynomial may correspond to more than one Jordan forms. The possible Jordan blocks associated with the eigen values -2 and 1 are given in the Table 1. From Table 1 , we can say that there are $5\times2 = 10$ possible Jordan forms. For example, the Jordan forms corresponding to minimal polynomial $p = (x+2)^2(x-1)$ built from the Jordan blocks can be,

$$\mathbf{J} = \begin{pmatrix} -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{2.0.4}$$

and

$$\mathbf{J} = \begin{pmatrix} -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{2.0.5}$$

3 Answer

Therefore 10 different Jordan forms are possible for a 6×6 complex matrix with characteristic polynomial $(x+2)^4(x-1)^2$.

Factor	Possible Jordan blocks
$(x+2)$	$$\begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} \qquad (2.0.6)$$ $$\begin{pmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} \qquad (2.0.7)$$ $$\begin{pmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & -2 \end{pmatrix}, \begin{pmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} \qquad (2.0.8)$$ $$\begin{pmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & -2 \end{pmatrix} \qquad (2.0.9)$$
$(x-1)$	$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad (2.0.10)$$ $$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \qquad (2.0.11)$$

TABLE 1: Possible Jordan Blocks