

ASSIGNMENT 3

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Abstract—This document proves that triangles on the same base and having equal areas lie between the same parallel lines.

Download latex-tikz codes from

https://github.com/EE20MTECH14019/EE5609/tree/master/Assignment_3

1 PROBLEM

Triangles on the same base (or equal bases) and having equal areas lie between the same parallels.

2 EXPLANATION

Triangles $\triangle ABC$ and $\triangle ABD$ are two triangles lying on the same base AB such that area of $\triangle ABC$ is equal to area of $\triangle ABD$. Now we join C and D and drop perpendiculars onto AB from C and D . Let's say the foot of the perpendiculars are E and F .

As $CE \perp AB$ and $DF \perp AB$, we can write that

$$(\mathbf{C} - \mathbf{E})^T(\mathbf{A} - \mathbf{B}) = 0 \quad (2.0.1)$$

$$(\mathbf{D} - \mathbf{F})^T(\mathbf{A} - \mathbf{B}) = 0 \quad (2.0.2)$$

From equations (2.0.1) and (2.0.2)

$$(\mathbf{C} - \mathbf{E}) = (\mathbf{D} - \mathbf{F}) \quad (2.0.3)$$

Equation (2.0.3) shows that $CE \parallel DF$. Also given that

$$\text{area}(\triangle ABC) = \text{area}(\triangle ABD) \quad (2.0.4)$$

$$\Rightarrow \frac{1}{2}(\mathbf{AB} \times \mathbf{CA}) = \frac{1}{2}(\mathbf{AB} \times \mathbf{BD}) \quad (2.0.5)$$

$$\Rightarrow (\mathbf{AB} \times (\mathbf{CE} + \mathbf{EA})) = (\mathbf{AB} \times (\mathbf{DF} + \mathbf{FB})) \quad (2.0.6)$$

$$\Rightarrow (\mathbf{AB} \times \mathbf{CE}) = (\mathbf{AB} \times \mathbf{DF}) \quad (2.0.7)$$

From equation (2.0.7), $CE = DF$.

As $CE \parallel DF$ and $CE = DF$, we could say that $CDEF$ forms a parallelogram. And $CD \parallel EF$ as CD and

EF are opposite sides of the parallelogram $CDEF$. From this, we can conclude that $CD \parallel AB$.

Hence $\triangle ABC$ and $\triangle ABD$ lie between the same parallels AB and CD .

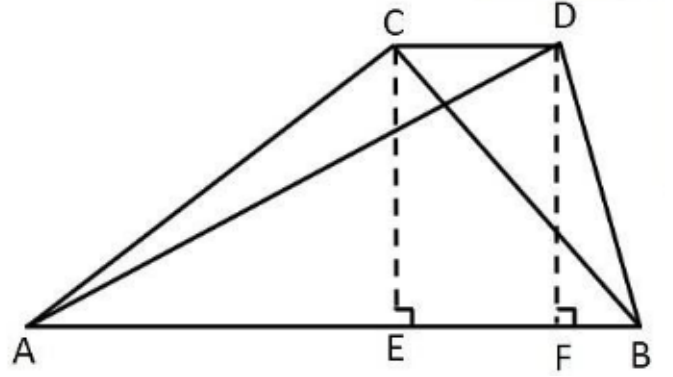


Fig. 0: Triangles of equal area and same base