

Assignment 16

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Abstract—This document solves a problem on Jordan form of a complex matrix.

Download all latex-tikz codes from

https://github.com/EE20MTECH14019/EE5609/tree/master/Assignment_16

$p = (x + 2)^2(x - 1)$ built from the Jordan blocks can be,

$$\mathbf{J} = \begin{pmatrix} -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (2.0.4)$$

1 PROBLEM

How many possible Jordan forms are there for a 6×6 complex matrix with characteristic polynomial $(x + 2)^4(x - 1)^2$?

and

$$\mathbf{J} = \begin{pmatrix} -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (2.0.5)$$

2 EXPLANATION

From the characteristic polynomial,

$$(x + 2)^4(x - 1)^2 \quad (2.0.1)$$

We get the eigen values of the 6×6 complex matrix as,

$$\lambda_i = \{-2, -2, -2, -2, 1, 1\} \quad (2.0.2)$$

The minimal polynomial for a matrix with characteristic polynomial p must have both $(x + 2)$ and $(x - 1)$ as factors and it must divide (2.0.1). Hence the minimal polynomial will be of form

$$p = (x + 2)^a(x - 1)^b, a \leq 4, b \leq 2 \quad (2.0.3)$$

So, there are 8 different possibilities for a minimal polynomial. Let us note that one minimal polynomial may correspond to more than one Jordan forms. The possible Jordan blocks associated with the eigen values -2 and 1 are given in the Table 1. From Table 1, we can say that there are $5 \times 2 = 10$ possible Jordan forms. For example, the Jordan forms corresponding to minimal polynomial

3 SOLUTION

For the given characteristic polynomial $(x + 2)^4(x - 1)^2$, 10 different Jordan forms are possible.

Factor	Possible Jordan blocks
$(x + 2)$	$\begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} \quad (2.0.6)$
	$\begin{pmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} \quad (2.0.7)$
	$\begin{pmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & -2 \end{pmatrix}, \begin{pmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} \quad (2.0.8)$
	$\begin{pmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & -2 \end{pmatrix} \quad (2.0.9)$
$(x - 1)$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (2.0.10)$
	$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \quad (2.0.11)$

TABLE 1: Possible Jordan Blocks