Assignment 8

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***Abstract*—This document checks the axioms satisfied by a given vector space.**

Download all latex-tikz codes from

https://github.com/EE20MTECH14019/EE5609/tree/master/Assignment_8

1 Problem

On $\mathbb{R}^n$ define two operations

$$\alpha \oplus \beta = \alpha - \beta \tag{1.0.1}$$

$$c \cdot \alpha = -c\alpha \tag{1.0.2}$$

The operations on the right are usual ones. Which of the axioms for a vector space are satisfied by $(\mathbb{R}^n, \oplus, \cdot)$?

2 Solution

Let $(\alpha, \beta, \gamma) \in \mathbb{R}^n$ and c, c_1, c_2 are scalars. Following are the axioms.

1) Associativity of addition

$$\alpha \oplus (\beta \oplus \gamma) = \alpha \oplus (\beta - \gamma) \tag{2.0.1}$$

$$= \alpha - \beta + \gamma \tag{2.0.2}$$

$$(\alpha \oplus \beta) \oplus \gamma = (\alpha - \beta) \oplus \gamma \tag{2.0.3}$$

$$= \alpha - \beta - \gamma \tag{2.0.4}$$

As $\alpha \oplus (\beta \oplus \gamma) \neq (\alpha \oplus \beta) \oplus \gamma$, $(\mathbb{R}^n, \oplus, \cdot)$ doesnot satisfy associativity.

2) Commutativity of addition

$$\alpha \oplus \beta = \alpha - \beta \tag{2.0.5}$$

$$\beta \oplus \alpha = \beta - \alpha \tag{2.0.6}$$

As $\alpha \oplus \beta \neq \beta \oplus \alpha$, $(\mathbb{R}^n, \oplus, \cdot)$ doesnot satisfy commutativity.

3) Additive identity

$$\alpha \oplus \beta = \alpha - \beta = \alpha \tag{2.0.7}$$

Hence a unique $\beta = (0, 0, ...0)$ is the additive identity of α.

4) Additive inverse

$$\alpha \oplus \alpha = \alpha - \alpha = 0 \tag{2.0.8}$$

Hence α is the additive inverse of itself.

5) Scalar multiplication with vector

$$(c_1c_2) \cdot \alpha = (-c_1c_2)\alpha \tag{2.0.9}$$

$$c_1 \cdot (c_2 \cdot \alpha) = c_1c_2\alpha \tag{2.0.10}$$

As $(c_1c_2) \cdot \alpha \neq c_1 \cdot (c_2 \cdot \alpha)$, $(\mathbb{R}^n, \oplus, \cdot)$ doesnot satisfy this axiom.

6) Identity element of scalar multiplication

$$1 \cdot \alpha = -\alpha = \alpha \text{ for } \alpha = (0, 0, ..., 0) \tag{2.0.11}$$

$$1 \cdot \alpha = -\alpha \neq \alpha \ \forall \ \alpha \neq (0, 0, ..., 0) \tag{2.0.12}$$

Hence there donot exists identity element of scalar multiplication for every α.

7) Distributivity of scalar multiplication with respect to vector addition

$$c \cdot (\alpha \oplus \beta) = -c(\alpha - \beta) \tag{2.0.13}$$

$$c \cdot \alpha \oplus c \cdot \beta = -c\alpha - (-c\beta) \tag{2.0.14}$$

As $c \cdot (\alpha \oplus \beta) \neq c \cdot \alpha \oplus c \cdot \beta$, $(\mathbb{R}^n, \oplus, \cdot)$ doesnot satisfy this axiom.

8) Distributivity of scalar multiplication with respect to vector addition

$$(c_1 + c_2) \cdot \alpha = -(c_1 + c_2)\alpha \tag{2.0.15}$$

$$c_1 \cdot \alpha \oplus c_2 \cdot \beta = -c_1\alpha - (-c_2\beta) \tag{2.0.16}$$

As $(c_1 + c_2) \cdot \alpha \neq c_1 \cdot \alpha \oplus c_2 \cdot \beta$, $(\mathbb{R}^n, \oplus, \cdot)$ doesnot satisfy this axiom.