Assignment 16

Yenigalla Samyuktha
EE20MTECH14019

***Abstract*—This document solves a problem on Jordan form of a complex matrix.**

Download all latex-tikz codes from

```
https://github.com/EE20MTECH14019/EE5609/
    tree/master/Assignment_16
```

1 Problem

How many possible Jordan forms are there for a 6×6 complex matrix with characteristic polynomial $(x+2)^4 (x-1)^2$?

2 Solution

Parameter	**Description**
A_M	Algebraic multiplicity of characteristic value λ in the characteristic polynomial, also equal to the size of Jordan block for that eigen value
G_M	Geometric multiplicity determines the number of Jordan sub-blocks in a Jordan block for λ.
$\mathbf{J}_{(x-\lambda)^k}$	Jordan block corresponding to the eigen value λ and k is the multiplicity of λ in the minimal polynomial determines size of largest Jordan sub-block.

TABLE 1: Parameters

<table>
<tr><th>Feature</th><th>Explanation</th></tr>
<tr><td>Characteristic Polynomial</td><td>$$(x+2)^4 (x-1)^2 \tag{2.0.1}$$</td></tr>
<tr><td>Algebraic Multiplicity, A_M</td><td>$$\text{For } \lambda = -2, A_M = 4 \tag{2.0.2}$$ $$\text{For } \lambda = 1, A_M = 2 \tag{2.0.3}$$</td></tr>
<tr><td>Minimal Polynomial</td><td>$$p = (x+2)^a (x-1)^b \ , a \le 4, b \le 2 \tag{2.0.4}$$</td></tr>
<tr><td>Possibilities of minimal polynomial</td><td>From equation (2.0.4), there are 8 different minimal polynomials possible.</td></tr>
<tr><td>Jordan Form</td><td>$$\mathbf{J} = \begin{pmatrix} -2 & * & 0 & 0 & 0 & 0 \\ 0 & -2 & * & 0 & 0 & 0 \\ 0 & 0 & -2 & * & 0 & 0 \\ 0 & 0 & 0 & -2 & * & 0 \\ 0 & 0 & 0 & 0 & 1 & * \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{2.0.5}$$ where $*$ can be either 1 or 0</td></tr>
<tr><td>Number of possibilities of Jordan canonical forms</td><td>From Table 3, there are $5 \times 2 = 10$ different Jordan forms possible.</td></tr>
<tr><td>Jordan Form corresponding to $p = (x+2)^2 (x-1)$</td><td>One minimal polynomial can correspond to more than one Jordan forms. For example, minimal polynomial $p = (x+2)^2 (x-1)$ can correspond to two different Jordan forms namely, $$\mathbf{J} = \begin{pmatrix} -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{J} = \begin{pmatrix} -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{2.0.6}$$</td></tr>
</table>

TABLE 2: Parameters

Factor	**Possible Jordan blocks**	G_M
$(x+2)$	$\mathbf{J}_{(x+2)} = \begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}$	4
	$\mathbf{J}_{(x+2)^2} = \begin{pmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}$	3
	$\mathbf{J}_{(x+2)^2} = \begin{pmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & -2 \end{pmatrix}$	2
	$\mathbf{J}_{(x+2)^3} = \begin{pmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}$	2
	$\mathbf{J}_{(x+2)^4} = \begin{pmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & -2 \end{pmatrix}$	1
$(x-1)$	$\mathbf{J}_{(x-1)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	2
	$\mathbf{J}_{(x-1)^2} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$	1

TABLE 3: Possible Jordan Blocks