Assignment 7

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Abstract—**This document explains the process of finding the distance between a given point and a plane using Singular Value Decomposition.**

Download all latex-tikz codes from

https://github.com/EE20MTECH14019/EE5609/tree/master/Assignment_7

1 Problem

Let $\mathbb{F}$ be the field of complex numbers. Are the following two systems of linear equations equivalent? If so, express each equation in each system as a linear combination of equations in other system. First system of equations:

$$2x_1 + (-1+i)x_2 + x_4 = 0 \tag{1.0.1}$$

$$3x_2 - 2ix_3 + 5x_4 = 0 \tag{1.0.2}$$

The second system of equations:

$$(1+\frac{i}{2})x_1 + 8x_2 - ix_3 - x_4 = 0 \tag{1.0.3}$$

$$\frac{2}{3}x_1 - \frac{1}{2}x_2 + x_3 + 7x_4 = 0 \tag{1.0.4}$$

2 Solution

Let $\mathbf{R_1}$ and $\mathbf{R_2}$ be the reduced row echelon forms of the augumented matrices of the following systems of homogeneous equations respectively.

$$\mathbf{AX} = \mathbf{0} \tag{2.0.1}$$

$$\mathbf{BX} = \mathbf{0} \tag{2.0.2}$$

Where $\mathbf{A}$ and $\mathbf{B}$ as follows

$$\mathbf{A} = \begin{pmatrix} 2 & -1+i & 0 & 1 \\ 0 & 3 & -2i & 5 \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{B} = \begin{pmatrix} 1+\frac{i}{2} & 8 & -i & -1 \\ \frac{2}{3} & \frac{-1}{2} & 1 & 7 \end{pmatrix} \tag{2.0.4}$$

On performing elementary row operations on (2.0.3),

$$\mathbf{R_1} = \mathbf{CA} \tag{2.0.5}$$

where $\mathbf{C}$ is the product of all elementary matrices. Reducing the first system of linear equations, we get,

$$\mathbf{C} = \begin{pmatrix} 1 & \frac{1-i}{2} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{pmatrix} \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.6}$$

$$\mathbf{R_1} = \begin{pmatrix} 1 & 0 & \frac{-1-i}{3} & \frac{4}{3} - \frac{5i}{6} \\ 0 & 1 & \frac{-2i}{3} & \frac{5}{3} \end{pmatrix} \tag{2.0.7}$$

On performing elementary row operations on (2.0.4),

$$\mathbf{R_2} = \mathbf{DA} \tag{2.0.8}$$

where $\mathbf{D}$ is the product of all elementary matrices. Reducing the second system of linear equations, we get,

$$\mathbf{D} = \begin{pmatrix} \frac{4}{5}(1-\frac{i}{2}) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{-2}{3} & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \frac{-6(143+43i)}{4909} \end{pmatrix} \begin{pmatrix} 1 & \frac{16(-2+i)}{5} \\ 0 & 1 \end{pmatrix} \tag{2.0.9}$$

$$\mathbf{R_2} = \begin{pmatrix} 1 & 0 & \frac{6702}{4909} - \frac{708i}{4909} & \frac{46620}{4909} - \frac{1998i}{4909} \\ \\ 0 & 1 & \frac{-2(441+472i)}{4909} & \frac{-2(3283+1332i)}{4909} \end{pmatrix} \tag{2.0.10}$$

From the equations (2.0.7) and (2.0.10), we can say that

$$\mathbf{R_1} \neq \mathbf{R_2} \tag{2.0.11}$$

Hence the given systems of linear equations are not equivalent.