Assignment 16

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Abstract—This document solves a problem on Jordan form of a complex matrix.

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1 PROBLEM

How many possible Jordan forms are there for a 6×6 complex matrix with characteristic polynomial $(x + 2)^4 (x - 1)^2$?

2 SOLUTION

Parameter	Description
A_M	Algebraic multiplicity of characteristic value λ in the characteristic polynomial, also equal to the size of Jordan block for that eigen value
G_M	Geometric multiplicity determines the number of Jordan sub-blocks in a Jordan block for λ.
$\mathbf{J}_{(x-\lambda)^k}$	Jordan block corresponding to the eigen value λ and k is the multiplicity of λ in the minimal polynomial determines size of largest Jordan sub-block.

TABLE 1: Parameters

Feature	Explanation
Characteristic Polynomial	$$(x+2)^4 (x-1)^2 \qquad (2.0.1)$$
Algebraic Multiplicity, A_M	$$\text{For } \lambda = -2, A_M = 4 \qquad (2.0.2)$$ $$\text{For } \lambda = 1, A_M = 2 \qquad (2.0.3)$$
Minimal Polynomial	$$p = (x+2)^a (x-1)^b \ , a \le 4, b \le 2 \qquad (2.0.4)$$
Possibilities of minimal polynomial	From equation (2.0.4), there are 8 different minimal polynomials possible.
Jordan Form	Jordan form built from Jordan blocks listed in table 3 will have the form, $$\mathbf{J} = \begin{pmatrix} -2 & * & 0 & 0 & 0 & 0 \\ 0 & -2 & * & 0 & 0 & 0 \\ 0 & 0 & -2 & * & 0 & 0 \\ 0 & 0 & 0 & -2 & * & 0 \\ 0 & 0 & 0 & 0 & 1 & * \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad (2.0.5)$$ where $*$ can be either 1 or 0
Number of possibilities of Jordan canonical forms	From Table 3, there are $5 \times 2 = 10$ different Jordan forms possible.
Jordan Form corresponding to $p = (x+2)^2 (x-1)$	One minimal polynomial can correspond to more than one Jordan forms. For example, minimal polynomial $p = (x+2)^2 (x-1)$ can correspond to two different Jordan forms namely, $$\mathbf{J} = \begin{pmatrix} -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{J} = \begin{pmatrix} -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$ $$(2.0.6)$$

TABLE 2: Parameters

Factor	Possible Jordan blocks	G_M
$(x+2)$	$\mathbf{J}_{(x+2)} = \begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}$	4
	$\mathbf{J}_{(x+2)^2} = \begin{pmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}$	3
	$\mathbf{J}_{(x+2)^2} = \begin{pmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & -2 \end{pmatrix}$	2
	$\mathbf{J}_{(x+2)^3} = \begin{pmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}$	2
	$\mathbf{J}_{(x+2)^4} = \begin{pmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & -2 \end{pmatrix}$	1
$(x-1)$	$\mathbf{J}_{(x-1)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	2
	$\mathbf{J}_{(x-1)^2} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$	1

TABLE 3: Possible Jordan Blocks