

EE5609: Matrix Theory

Assignment-3

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Abstract—This document contains a solution for showing that line AP bisects

Download the python codes from latex-tikz codes from

<https://github.com/pavanmanesh/EE5609/tree/master/Assignment3>

1 PROBLEM

P is a point equidistant from two lines l and m intersecting at point A. Show that the line AP bisects the angle between them.

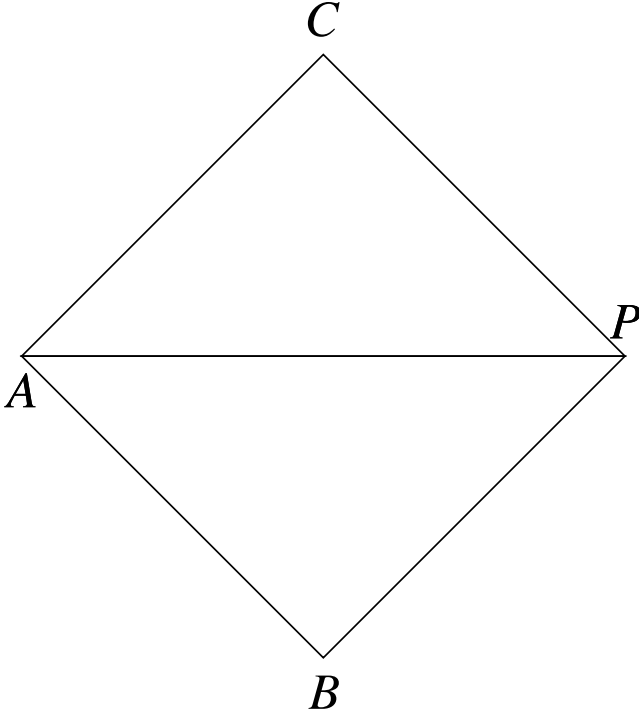


Fig. 0: figure

2 SOLUTION

1) Here, the following information is given:

$$\|\mathbf{P} - \mathbf{B}\| = \|\mathbf{P} - \mathbf{C}\| \quad (2.0.1)$$

2) The lines PB is the perpendicular to line AB and PC is the perpendicular to line AC:

$$(\mathbf{P} - \mathbf{B})^T (\mathbf{A} - \mathbf{B}) = 0 \implies \cos \angle PBA = 0 \quad (2.0.2)$$

$$(\mathbf{P} - \mathbf{C})^T (\mathbf{A} - \mathbf{C}) = 0 \implies \cos \angle PCA = 0 \quad (2.0.3)$$

We know that

$$\|\mathbf{P} - \mathbf{A}\|^2 = (\mathbf{P} - \mathbf{A})^T (\mathbf{P} - \mathbf{A}) \quad (2.0.4)$$

$$(\mathbf{P} - \mathbf{A})^T (\mathbf{P} - \mathbf{A}) = (\mathbf{P} - \mathbf{B} + \mathbf{B} - \mathbf{A})^T (\mathbf{P} - \mathbf{B} + \mathbf{B} - \mathbf{A}) \quad (2.0.5)$$

$$\begin{aligned} \|\mathbf{P} - \mathbf{A}\|^2 &= \|\mathbf{P} - \mathbf{B}\|^2 + \|\mathbf{B} - \mathbf{A}\|^2 \\ &\quad + 2 \|\mathbf{A} - \mathbf{P}\| \|\mathbf{B} - \mathbf{A}\| \cos \angle PBA \end{aligned} \quad (2.0.6)$$

using (2.0.2)

$$\implies \|\mathbf{P} - \mathbf{A}\|^2 = \|\mathbf{P} - \mathbf{B}\|^2 + \|\mathbf{B} - \mathbf{A}\|^2 \quad (2.0.7)$$

Similarly

$$(\mathbf{P} - \mathbf{A})^T (\mathbf{P} - \mathbf{A}) = (\mathbf{P} - \mathbf{C} + \mathbf{C} - \mathbf{A})^T (\mathbf{P} - \mathbf{C} + \mathbf{C} - \mathbf{A}) \quad (2.0.8)$$

$$\begin{aligned} \|\mathbf{P} - \mathbf{A}\|^2 &= \|\mathbf{P} - \mathbf{C}\|^2 + \|\mathbf{C} - \mathbf{A}\|^2 \\ &\quad + 2 \|\mathbf{A} - \mathbf{P}\| \|\mathbf{C} - \mathbf{A}\| \cos \angle PCA \end{aligned} \quad (2.0.9)$$

using (2.0.3)

$$\implies \|\mathbf{P} - \mathbf{A}\|^2 = \|\mathbf{P} - \mathbf{C}\|^2 + \|\mathbf{C} - \mathbf{A}\|^2 \quad (2.0.10)$$

From (2.0.7) and (2.0.10) and substituting (2.0.1)

$$\|\mathbf{P} - \mathbf{C}\|^2 + \|\mathbf{C} - \mathbf{A}\|^2 = \|\mathbf{P} - \mathbf{B}\|^2 + \|\mathbf{B} - \mathbf{A}\|^2 \quad (2.0.11)$$

$$\implies \|\mathbf{C} - \mathbf{A}\|^2 = \|\mathbf{B} - \mathbf{A}\|^2 \implies \|\mathbf{C} - \mathbf{A}\| = \|\mathbf{B} - \mathbf{A}\| \quad (2.0.12)$$

We know that

$$\cos \angle BAP = \frac{(\mathbf{P} - \mathbf{A})^T(\mathbf{A} - \mathbf{B})}{\|\mathbf{P} - \mathbf{A}\| \|\mathbf{A} - \mathbf{B}\|} \quad (2.0.13)$$

$$= \frac{(\mathbf{P} - \mathbf{B} + \mathbf{B} - \mathbf{A})^T(\mathbf{A} - \mathbf{B})}{\|\mathbf{P} - \mathbf{A}\| \|\mathbf{A} - \mathbf{B}\|} \quad (2.0.14)$$

$$= \frac{(\mathbf{P} - \mathbf{B})^T(\mathbf{A} - \mathbf{B}) - \|\mathbf{B} - \mathbf{A}\|^2}{\|\mathbf{P} - \mathbf{A}\| \|\mathbf{A} - \mathbf{B}\|} \quad (2.0.15)$$

Using (2.0.2)

$$\cos \angle BAP = \frac{\|\mathbf{B} - \mathbf{A}\|}{\|\mathbf{P} - \mathbf{A}\|} \quad (2.0.16)$$

Similarly,

$$\cos \angle CAP = \frac{(\mathbf{P} - \mathbf{A})^T(\mathbf{A} - \mathbf{C})}{\|\mathbf{P} - \mathbf{A}\| \|\mathbf{A} - \mathbf{C}\|} \quad (2.0.17)$$

$$= \frac{(\mathbf{P} - \mathbf{C} + \mathbf{C} - \mathbf{A})^T(\mathbf{A} - \mathbf{C})}{\|\mathbf{P} - \mathbf{A}\| \|\mathbf{A} - \mathbf{C}\|} \quad (2.0.18)$$

$$= \frac{(\mathbf{P} - \mathbf{C})^T(\mathbf{A} - \mathbf{C}) - \|\mathbf{C} - \mathbf{A}\|^2}{\|\mathbf{P} - \mathbf{A}\| \|\mathbf{A} - \mathbf{C}\|} \quad (2.0.19)$$

Using (2.0.3)

$$\cos \angle CAP = \frac{\|\mathbf{C} - \mathbf{A}\|}{\|\mathbf{P} - \mathbf{A}\|} \quad (2.0.20)$$

Using (2.0.12) in (2.0.16) and (2.0.20),

$$\cos \angle BAP = \cos \angle CAP \quad (2.0.21)$$

$$\implies \angle BAP = \angle CAP \quad (2.0.22)$$

The line AP bisects the angle between them.