Matrix theory Assignment 1

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Abstract—**This document explains the equation of the line passing through the point of intersection of the lines that has equal intercepts on the axes**

1 PROBLEM

Find the equation of the line passing through the point of intersection of the lines

$$(4\ 7)\mathbf{x} = 3$$
$$(2\ \text{-}3)\mathbf{x} = \text{-}1$$

that has equal intercepts on the axes

2 SOLUTION

Constructing the augmented matrix

$$\begin{pmatrix} 4 & 7 & 3 \\ 2 & -3 & -1 \end{pmatrix}$$

Transforming the matrix into row-echelon form

$$\begin{pmatrix} 4 & 7 & 3 \\ 2 & -3 & -1 \end{pmatrix} \xleftarrow{R2 \leftarrow 2R2 - R1}$$

$$\begin{pmatrix} 4 & 7 & 3 \\ 0 & -13 & -5 \end{pmatrix} \xleftarrow{R2 \leftarrow -R2/13, R1 \leftarrow R1/4}$$

$$\begin{pmatrix} 1 & 7/4 & 3/4 \\ 0 & 1 & 5/13 \end{pmatrix} \xleftarrow{R1 \leftarrow R1 - 7/4R1/4}$$

$$\begin{pmatrix} 1 & 0 & 2/26 \\ 0 & 1 & 5/13 \end{pmatrix} \quad (2.0.1)$$

Thus, The point of intersection is at point (2/26 , 5/13) i.e. (0.07, 0.38)
let the equation of the line be

$$\mathbf{n}^T\mathbf{x} = c \implies \mathbf{x}^T\mathbf{n} = c \quad (2.0.2)$$

Let the intercepts be a,b on the x and y axis respectively. Then,

$$\begin{pmatrix} a & 0 \end{pmatrix}\mathbf{n} = c \quad (2.0.3)$$

$$\begin{pmatrix} 0 & b \end{pmatrix}\mathbf{n} = c \quad (2.0.4)$$

As the required line passes through the point of intersection (2/26, 5/13) and the intercepts are equal, the intercepts can be written as:

$$\begin{pmatrix} 6/13 & 0 \end{pmatrix}\mathbf{n} = c \quad (2.0.5)$$

$$\begin{pmatrix} 0 & 6/13 \end{pmatrix}\mathbf{n} = c \quad (2.0.6)$$

resulting in the matrix equation

$$\begin{pmatrix} 6/13 & 0 \\ 0 & 6/13 \end{pmatrix}\mathbf{n} = c\begin{pmatrix} 1 \\ 1 \end{pmatrix} where\, \mathbf{n} = \begin{pmatrix} n_{11} \\ n_{12} \end{pmatrix} \quad (2.0.7)$$

$$\implies \begin{pmatrix} 6/13 & 0 \\ 0 & 6/13 \end{pmatrix}\begin{pmatrix} n_{11} \\ n_{12} \end{pmatrix} = c\begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (2.0.8)$$

upon substitution we get,

$$\frac{6}{13}n_{11} = c \implies n_{11} = \frac{c}{6/13} \quad (2.0.9)$$

$$\frac{6}{13}n_{12} = c \implies n_{12} = \frac{c}{6/13} \quad (2.0.10)$$

So, the equation of line can be written as

$$\mathbf{n}^T\mathbf{x} = c \quad (2.0.11)$$

$$\begin{pmatrix} n_{11} & n_{12} \end{pmatrix}\mathbf{x} = c \quad (2.0.12)$$

$$\begin{pmatrix} \frac{c}{6/13} & \frac{c}{6/13} \end{pmatrix}\mathbf{x} = c \quad (2.0.13)$$

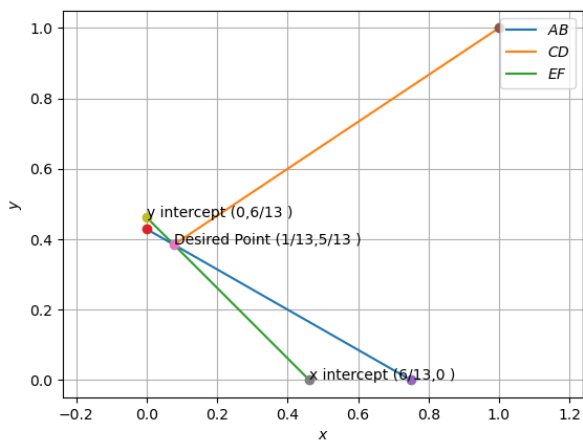$$\implies \begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{x} = 6/13 \quad (2.0.14)$$

Fig. 0: The intercepts of the required line are equal