EE5609: Matrix Theory Assignment-3

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Abstract—**This document contains a solution for showing that line AP bisects**

Download the python codes from latex-tikz codes from

1 PROBLEM

P is a point equidistant from two lines l and m intersecting at point A. Show that the line AP bisects the angle between them.

2 SOLUTION

1) Here, the following information is given:

$$\|\mathbf{P} - \mathbf{B}\| = \|\mathbf{P} - \mathbf{C}\| \qquad (2.0.1)$$

2) The lines PB is the perpendicular to line AB and PC is the perpendicular to line AC:

$$\left(\mathbf{P} - \mathbf{B}\right)^{T}\left(\mathbf{A} - \mathbf{B}\right) = 0 \qquad (2.0.2)$$

$$\left(\mathbf{P} - \mathbf{C}\right)^{T}\left(\mathbf{A} - \mathbf{C}\right) = 0 \qquad (2.0.3)$$

We need to prove that the line AP bisects the angle between them

$$\angle BAP \quad = \quad \angle CAP \qquad (2.0.4)$$

$$\implies \cos \angle BAP \quad = \quad \cos \angle CAP \qquad (2.0.5)$$

$$\frac{(\mathbf{A} - \mathbf{B})^{T}(\mathbf{A} - \mathbf{P})}{\|\mathbf{A} - \mathbf{B}\|\,\|\mathbf{A} - \mathbf{P}\|} \quad = \quad \frac{(\mathbf{A} - \mathbf{C})^{T}(\mathbf{A} - \mathbf{P})}{\|\mathbf{A} - \mathbf{C}\|\,\|\mathbf{A} - \mathbf{P}\|} \qquad (2.0.6)$$

This is reduced to

$$\implies \frac{(\mathbf{A} - \mathbf{B})^{T}}{\|\mathbf{A} - \mathbf{B}\|} \quad = \quad \frac{(\mathbf{A} - \mathbf{C})^{T}}{\|\mathbf{A} - \mathbf{C}\|} \qquad (2.0.7)$$

From (2.0.2) and (2.0.3),We can write that

$$\left(\mathbf{P} - \mathbf{B}\right)^{T}\left(\mathbf{A} - \mathbf{B}\right) = \left(\mathbf{P} - \mathbf{C}\right)^{T}\left(\mathbf{A} - \mathbf{C}\right) \qquad (2.0.8)$$

using (2.0.2) , the above equation reduces to

$$\left(\mathbf{A} - \mathbf{B}\right) = \left(\mathbf{A} - \mathbf{C}\right) \qquad (2.0.9)$$

By using (2.0.9) in (2.0.7),We can say that the line AP bisects the angle between them

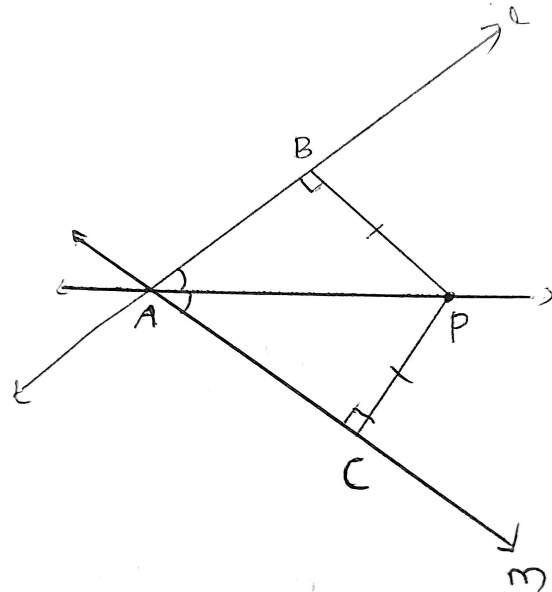$$\angle BAP \quad = \quad \angle CAP \qquad (2.0.10)$$

The Figure below shows the plot of given lines

Fig. 2: Plotting the lines