Matrix theory Assignment 1

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***Abstract*—This document explains the equation of the line passing through the point of intersection of the lines that has equal intercepts on the axes**

Download all python codes from

https://github.com/pavanmanesh/EE5609/tree/master/codes

and latex-tikz codes from

https://github.com/pavanmanesh/EE5609

1 Problem

Find the equation of the line passing through the point of intersection of the lines

$$(4\ 7)\mathbf{x} = 3$$
$$(2\ -3)\mathbf{x} = -1$$

that has equal intercepts on the axes

2 Solution

Constructing the augmented matrix

$$\begin{pmatrix} 4 & 7 & 3 \\ 2 & -3 & -1 \end{pmatrix}$$

Transforming the matrix into row-echelon form

$$\begin{pmatrix} 4 & 7 & 3 \\ 2 & -3 & -1 \end{pmatrix} \xleftrightarrow{R2 \leftarrow 2R2 - R1}$$
$$\begin{pmatrix} 4 & 7 & 3 \\ 0 & -13 & -5 \end{pmatrix} \xleftrightarrow{R2 \leftarrow -R2/13, R1 \leftarrow R1/4}$$
$$\begin{pmatrix} 1 & 7/4 & 3/4 \\ 0 & 1 & 5/13 \end{pmatrix} \xleftrightarrow{R1 \leftarrow R1 - 7/4R1/4}$$
$$\begin{pmatrix} 1 & 0 & 2/26 \\ 0 & 1 & 5/13 \end{pmatrix} \tag{2.0.1}$$

Thus, The point of intersection is at point (2/26 , 5/13) i.e. (0.07, 0.38)
The equation of line in terms of vector notations can be written as

$$\mathbf{n}^T\mathbf{x} = b \qquad \text{where} \qquad \mathbf{n} = \begin{pmatrix} n_{11} \\ n_{12} \end{pmatrix} \tag{2.0.2}$$

or

$$\begin{pmatrix} n_{11} & n_{12} \end{pmatrix}\mathbf{x} = b \tag{2.0.3}$$

Let the intercepts be $\begin{pmatrix} a \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ b \end{pmatrix}$, respectively.
As the required line passes through the point of intersection (2/26, 5/13) and the intercepts are equal, the intercepts can be written as:
$\begin{pmatrix} 6/13 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 6/13 \end{pmatrix}$
When the line passes through $\begin{pmatrix} 6/13 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 6/13 \end{pmatrix}$, respectively, we get, upon substitution in (2.0.3):

$$6/13 n_{11} = b \implies n_{11} = \frac{b}{6/13}$$
$$6/13 n_{12} = b \implies n_{12} = -\frac{b}{6/13}$$

Therefore, the equation of first line is

$$\begin{pmatrix} \frac{b}{6/13} & \frac{b}{6/13} \end{pmatrix}\mathbf{x} = b$$

$\implies$

$$\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{x} = 6/13 \tag{2.0.4}$$

Plot of the two lines: