

EE5609: Matrix Theory

Assignment-8

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Abstract—This document explains how to find a row-reduced matrix which is row equivalent to

Download all latex-tikz codes from

<https://github.com/pavanmanesh/EE5609/tree/master/Assignment8>

and all python codes from

<https://github.com/pavanmanesh/EE5609/tree/master/Assignment8/codes>

$$\mathbf{R} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \quad (3.0.3)$$

Row-reduced matrix of \mathbf{A} is,

$$\mathbf{A} = \begin{pmatrix} 1 & -i \\ 2 & 2 \\ i & 1+i \end{pmatrix} \xrightarrow{RREF} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \quad (3.0.4)$$

From (2.0.1) and (3.0.4),

$$\mathbf{A}\mathbf{x} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \quad (3.0.5)$$

1 PROBLEM

Find a row-reduced matrix which is row equivalent to \mathbf{A} . What are the solutions of $\mathbf{A}\mathbf{x}=0$?

$$\mathbf{A} = \begin{pmatrix} 1 & -i \\ 2 & 2 \\ i & 1+i \end{pmatrix} \quad (1.0.1)$$

The solution of $\mathbf{A}\mathbf{x}=0$ is,

$$\mathbf{I}_2 \mathbf{x} = 0 \quad (3.0.6)$$

$$\implies \mathbf{x} = 0 \quad (3.0.7)$$

As \mathbf{I}_2 is invertible.

2 THEOREM

Let \mathbf{R} be a row-reduced echelon matrix which is row equivalent to \mathbf{A} . Then the systems

$$\mathbf{A}\mathbf{x} = \mathbf{0}, \mathbf{R}\mathbf{x} = \mathbf{0} \quad (2.0.1)$$

have the same solutions.

3 SOLUTION

On performing elementary row operations on (1.0.1),

$$\mathbf{R} = \mathbf{B}\mathbf{A} \quad (3.0.1)$$

where \mathbf{B} is the product of all elementary matrices. Reducing the given matrix, we get

$$\mathbf{B} = \begin{pmatrix} 1 & i & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & i & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{4}(1-i) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -i & 0 & 1 \end{pmatrix} \quad (3.0.2)$$