EE5609: Matrix Theory Assignment-2

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***Abstract*—This document contains a solution for proving the determinant of the given matrix is zero.**

Download the python codes from

https://github.com/pavanmanesh/EE5609/blob/master/Assignment2/codes

and latex-tikz codes from

https://github.com/pavanmanesh/EE5609/tree/master/Assignment2

1 PROBLEM

$$\begin{vmatrix} 0 & a & -b \\ -a & 0 & c \\ b & c & 0 \end{vmatrix} = 0 \tag{1.0.1}$$

2 PROPERTIES

Properties used for solving this problem:

$$A\mathbf{x} = \mathbf{0} \tag{2.0.1}$$

A has a zero eigen value if x has a nontrivial solution.

3 SOLUTION

Converting A into reduced echelon form:

$$\begin{pmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow R_2} \begin{pmatrix} -a & 0 & -c \\ 0 & a & -b \\ b & c & 0 \end{pmatrix} \tag{3.0.1}$$

$$\xleftrightarrow{R_3 \leftarrow a \times R_3 + b \times R_1} \begin{pmatrix} -a & 0 & -c \\ 0 & a & -b \\ 0 & ac & -bc \end{pmatrix} \tag{3.0.2}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - c \times R_2} \begin{pmatrix} -a & 0 & -c \\ 0 & a & -b \\ 0 & 0 & 0 \end{pmatrix} \tag{3.0.3}$$

Let A be the given matrix and x be a vector where x is

$$\mathbf{x} = \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} \tag{3.0.4}$$

From (2.0.1), we can write the augmented form in this way:

$$\begin{pmatrix} -a & 0 & -c & 0 \\ 0 & a & -b & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{3.0.5}$$

upon solving the above , the value of x:

$$\mathbf{x} = \begin{pmatrix} -\frac{c}{a}x3 \\ \frac{b}{a}x3 \\ x3 \end{pmatrix} \tag{3.0.6}$$

$$\mathbf{x} = \begin{pmatrix} -c \\ b \\ a \end{pmatrix} \tag{3.0.7}$$

From (2.0.1),We can write that

$$\begin{pmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{pmatrix} \begin{pmatrix} -c \\ b \\ a \end{pmatrix} = \mathbf{0} \tag{3.0.8}$$

So,one of the eigen value is equal to 0.say

$$\lambda_1 = 0 \tag{3.0.9}$$

We know that the

$$\det(A) = \lambda_1 \lambda_2 \cdots \lambda_n \tag{3.0.10}$$

From (3.0.9), $\det(A) = 0$