EE5609: Matrix Theory
Assignment-3

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Abstract—**This document contains a solution for showing that line AP bisects**

Download the python codes from latex-tikz codes from

1 PROBLEM

P is a point equidistant from two lines l and m intersecting at point A. Show that the line AP bisects the angle between them.
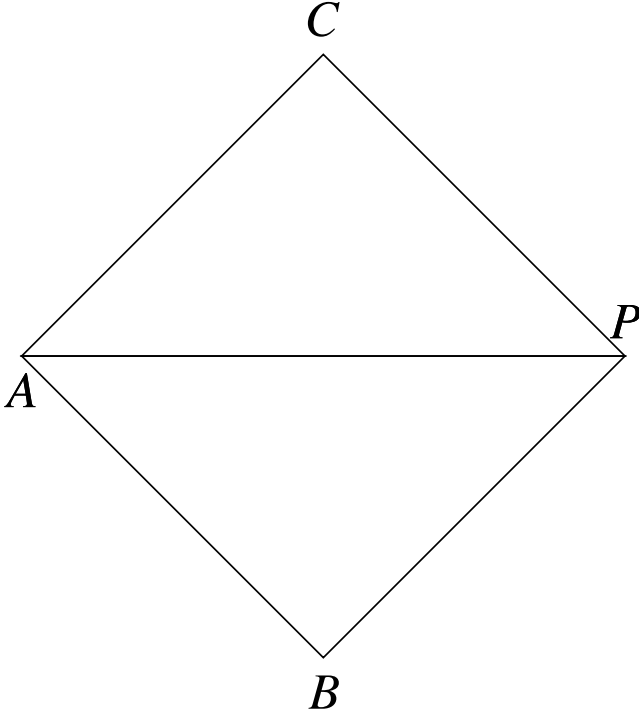
2 SOLUTION

Fig. 0: figure

1) Here, the following information is given:
$$\|\mathbf{P} - \mathbf{B}\| = \|\mathbf{P} - \mathbf{C}\| \tag{2.0.1}$$

2) The lines PB is the perpendicular to line AB and PC is the perpendicular to line AC:
$$\left(\mathbf{P} - \mathbf{B}\right)^T \left(\mathbf{A} - \mathbf{B}\right) = 0 \implies \cos \angle PBA = 0 \tag{2.0.2}$$

$$\left(\mathbf{P} - \mathbf{C}\right)^T \left(\mathbf{A} - \mathbf{C}\right) = 0 \implies \cos \angle PCA = 0 \tag{2.0.3}$$

We know that
$$\|\mathbf{P} - \mathbf{A}\|^2 = (\mathbf{P} - \mathbf{A})^T (\mathbf{P} - \mathbf{A}) \tag{2.0.4}$$

$$(\mathbf{P} - \mathbf{A})^T (\mathbf{P} - \mathbf{A}) = (\mathbf{P} - \mathbf{B} + \mathbf{B} - \mathbf{A})^T (\mathbf{P} - \mathbf{B} + \mathbf{B} - \mathbf{A}) \tag{2.0.5}$$

$$\|\mathbf{P} - \mathbf{A}\|^2 = \|\mathbf{P} - \mathbf{B}\|^2 + \|\mathbf{B} - \mathbf{A}\|^2 \\ +2 \|\mathbf{A} - \mathbf{P}\| \|\mathbf{B} - \mathbf{A}\| \cos \angle PBA \tag{2.0.6}$$

using (2.0.2)
$$\implies \|\mathbf{P} - \mathbf{A}\|^2 = \|\mathbf{P} - \mathbf{B}\|^2 + \|\mathbf{B} - \mathbf{A}\|^2 \tag{2.0.7}$$

Similarly
$$(\mathbf{P} - \mathbf{A})^T (\mathbf{P} - \mathbf{A}) = (\mathbf{P} - \mathbf{C} + \mathbf{C} - \mathbf{A})^T (\mathbf{P} - \mathbf{C} + \mathbf{C} - \mathbf{A}) \tag{2.0.8}$$

$$\|\mathbf{P} - \mathbf{A}\|^2 = \|\mathbf{P} - \mathbf{C}\|^2 + \|\mathbf{C} - \mathbf{A}\|^2 \\ +2 \|\mathbf{A} - \mathbf{P}\| \|\mathbf{C} - \mathbf{A}\| \cos \angle PCA \tag{2.0.9}$$

using (2.0.3)
$$\implies \|\mathbf{P} - \mathbf{A}\|^2 = \|\mathbf{P} - \mathbf{C}\|^2 + \|\mathbf{C} - \mathbf{A}\|^2 \tag{2.0.10}$$

From (2.0.7) and (2.0.10) and substituting (2.0.1)

$$\|\mathbf{P} - \mathbf{C}\|^2 + \|\mathbf{C} - \mathbf{A}\|^2 = \|\mathbf{P} - \mathbf{B}\|^2 + \|\mathbf{B} - \mathbf{A}\|^2 \tag{2.0.11}$$

$$\implies \|\mathbf{C} - \mathbf{A}\|^2 = \|\mathbf{B} - \mathbf{A}\|^2 \implies \|\mathbf{C} - \mathbf{A}\| = \|\mathbf{B} - \mathbf{A}\| \tag{2.0.12}$$

Using (2.0.1), (2.0.2) and (2.0.12) and by RHS(Right angle – Hypotenuse – Side) property of congruence we can say that:-

$$\triangle APB \cong \triangle APC \tag{2.0.13}$$

From (2.0.13),

$$\angle BAP \;=\; \angle CAP \tag{2.0.14}$$

From (2.0.14),We can say that the line AP bisects the angle between them.