EE5609: Matrix Theory Assignment-3

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Abstract—**This document contains a solution for showing that line AP bisects**

Download the python codes from latex-tikz codes from

https://github.com/pavanmanesh/EE5609/tree/master/Assignment3

1 Problem

P is a point equidistant from two lines l and m intersecting at point A. Show that the line AP bisects the angle between them.

2 Solution

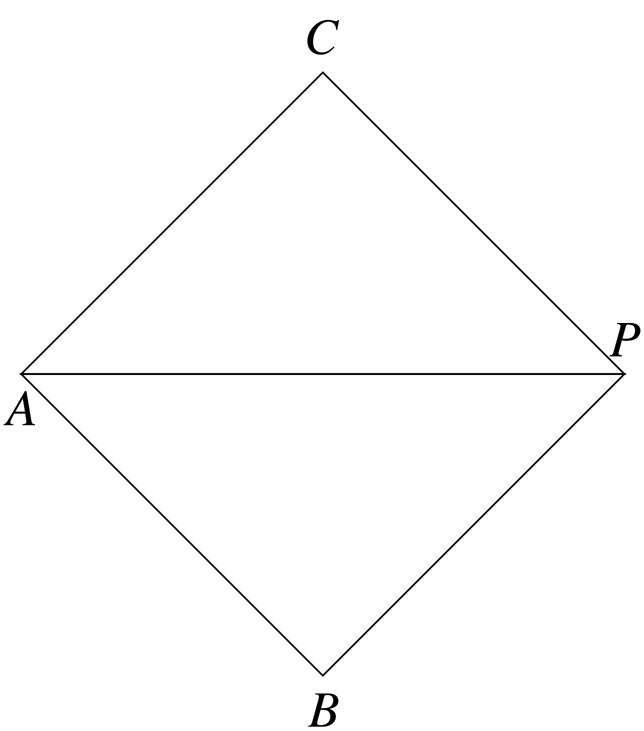

Fig. 0: figure

1) Here, the following information is given:
$$\|\mathbf{P}-\mathbf{B}\| = \|\mathbf{P}-\mathbf{C}\| \tag{2.0.1}$$
2) The lines PB is the perpendicular to line AB and PC is the perpendicular to line AC:
$$\left(\mathbf{P}-\mathbf{B}\right)^T\left(\mathbf{A}-\mathbf{B}\right) = 0 \implies \cos\angle PBA = 0 \tag{2.0.2}$$
$$\left(\mathbf{P}-\mathbf{C}\right)^T\left(\mathbf{A}-\mathbf{C}\right) = 0 \implies \cos\angle PCA = 0 \tag{2.0.3}$$

We know that
$$\|\mathbf{P}-\mathbf{A}\|^2 = (\mathbf{P}-\mathbf{A})^T(\mathbf{P}-\mathbf{A}) \tag{2.0.4}$$
$$(\mathbf{P}-\mathbf{A})^T(\mathbf{P}-\mathbf{A}) = (\mathbf{P}-\mathbf{B}+\mathbf{B}-\mathbf{A})^T(\mathbf{P}-\mathbf{B}+\mathbf{B}-\mathbf{A}) \tag{2.0.5}$$
$$\begin{aligned}\|\mathbf{P}-\mathbf{A}\|^2 &= \|\mathbf{P}-\mathbf{B}\|^2 + \|\mathbf{B}-\mathbf{A}\|^2 \\ &+2\|\mathbf{A}-\mathbf{P}\|\|\mathbf{B}-\mathbf{A}\|\cos\angle PBA\end{aligned} \tag{2.0.6}$$
using (2.0.2)
$$\implies \|\mathbf{P}-\mathbf{A}\|^2 = \|\mathbf{P}-\mathbf{B}\|^2 + \|\mathbf{B}-\mathbf{A}\|^2 \tag{2.0.7}$$
Similarly
$$(\mathbf{P}-\mathbf{A})^T(\mathbf{P}-\mathbf{A}) = (\mathbf{P}-\mathbf{C}+\mathbf{C}-\mathbf{A})^T(\mathbf{P}-\mathbf{C}+\mathbf{C}-\mathbf{A}) \tag{2.0.8}$$
$$\begin{aligned}\|\mathbf{P}-\mathbf{A}\|^2 &= \|\mathbf{P}-\mathbf{C}\|^2 + \|\mathbf{C}-\mathbf{A}\|^2 \\ &+2\|\mathbf{A}-\mathbf{P}\|\|\mathbf{C}-\mathbf{A}\|\cos\angle PCA\end{aligned} \tag{2.0.9}$$
using (2.0.3)
$$\implies \|\mathbf{P}-\mathbf{A}\|^2 = \|\mathbf{P}-\mathbf{C}\|^2 + \|\mathbf{C}-\mathbf{A}\|^2 \tag{2.0.10}$$

From (2.0.7) and (2.0.10) and substituting (2.0.1)

$$\|\mathbf{P}-\mathbf{C}\|^2+\|\mathbf{C}-\mathbf{A}\|^2=\|\mathbf{P}-\mathbf{B}\|^2+\|\mathbf{B}-\mathbf{A}\|^2 \tag{2.0.11}$$

$$\implies \|\mathbf{C}-\mathbf{A}\|^2=\|\mathbf{B}-\mathbf{A}\|^2 \implies \|\mathbf{C}-\mathbf{A}\|=\|\mathbf{B}-\mathbf{A}\| \tag{2.0.12}$$

We know that

$$\|\mathbf{P}-\mathbf{B}\|^2=(\mathbf{P}-\mathbf{B})^T(\mathbf{P}-\mathbf{B}) \tag{2.0.13}$$

$$(\mathbf{P}-\mathbf{B})^T(\mathbf{P}-\mathbf{B})=(\mathbf{P}-\mathbf{A}+\mathbf{A}-\mathbf{B})^T(\mathbf{P}-\mathbf{A}+\mathbf{A}-\mathbf{B}) \tag{2.0.14}$$

$$\begin{aligned}\|\mathbf{P}-\mathbf{B}\|^2&=\|\mathbf{P}-\mathbf{A}\|^2+\|\mathbf{A}-\mathbf{B}\|^2\\&+2\,\|\mathbf{P}-\mathbf{A}\|\,\|\mathbf{A}-\mathbf{B}\|\cos\angle PAB\end{aligned} \tag{2.0.15}$$

Similarly

$$\|\mathbf{P}-\mathbf{C}\|^2=(\mathbf{P}-\mathbf{C})^T(\mathbf{P}-\mathbf{C}) \tag{2.0.16}$$

$$(\mathbf{P}-\mathbf{C})^T(\mathbf{P}-\mathbf{C})=(\mathbf{P}-\mathbf{A}+\mathbf{A}-\mathbf{C})^T(\mathbf{P}-\mathbf{A}+\mathbf{A}-\mathbf{C}) \tag{2.0.17}$$

$$\begin{aligned}\|\mathbf{P}-\mathbf{C}\|^2&=\|\mathbf{P}-\mathbf{A}\|^2+\|\mathbf{A}-\mathbf{C}\|^2\\&+2\,\|\mathbf{P}-\mathbf{A}\|\,\|\mathbf{A}-\mathbf{C}\|\cos\angle PAC\end{aligned} \tag{2.0.18}$$

From (2.0.1),Equating (2.0.15) and (2.0.18)

$$\begin{aligned}&\|\mathbf{P}-\mathbf{A}\|^2+\|\mathbf{A}-\mathbf{B}\|^2+2\,\|\mathbf{P}-\mathbf{A}\|\,\|\mathbf{A}-\mathbf{B}\|\cos\angle PAB=\\&\|\mathbf{P}-\mathbf{A}\|^2+\|\mathbf{A}-\mathbf{C}\|^2+2\,\|\mathbf{P}-\mathbf{A}\|\,\|\mathbf{A}-\mathbf{C}\|\cos\angle PAC\end{aligned} \tag{2.0.19}$$

Using (2.0.12)

$$\begin{aligned}&2\,\|\mathbf{P}-\mathbf{A}\|\,\|\mathbf{A}-\mathbf{B}\|\cos\angle PAB=\\&2\,\|\mathbf{P}-\mathbf{A}\|\,\|\mathbf{A}-\mathbf{C}\|\cos\angle PAC\end{aligned} \tag{2.0.20}$$

$$\implies \cos\angle PAB=\cos\angle PAC$$

The line AP bisects the angle between them.