Matrix theory Assignment 1

M Pavan Manesh

***Abstract*—This document explains the equation of the line passing through the point of intersection of the lines that has equal intercepts on the axes**

Download all python codes from

https://github.com/pavanmanesh/EE5609/tree/master/codes

and latex-tikz codes from

https://github.com/pavanmanesh/EE5609

1 Problem

Find the equation of the line passing through the point of intersection of the lines

$$(4\ 7)\mathbf{x} = 3$$
$$(2\ -3)\mathbf{x} = -1$$

that has equal intercepts on the axes

2 Solution

The above two line equations can be expressed as the matrix equation

$$\begin{pmatrix} 4 & 7 \\ 2 & -3 \end{pmatrix}\mathbf{x} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \tag{2.0.1}$$

Constructing the augmented matrix

$$\begin{pmatrix} 4 & 7 & 3 \\ 2 & -3 & -1 \end{pmatrix}$$

Transforming the matrix into row-echelon form

$$\begin{pmatrix} 4 & 7 & 3 \\ 2 & -3 & -1 \end{pmatrix} \xleftrightarrow{R2 \leftarrow 2R2 - R1}$$
$$\begin{pmatrix} 4 & 7 & 3 \\ 0 & -13 & -5 \end{pmatrix} \xleftrightarrow{R2 \leftarrow -R2/13, R1 \leftarrow R1/4}$$
$$\begin{pmatrix} 1 & 7/4 & 3/4 \\ 0 & 1 & 5/13 \end{pmatrix} \xleftrightarrow{R1 \leftarrow R1 - 7/4R1/4}$$
$$\begin{pmatrix} 1 & 0 & 2/26 \\ 0 & 1 & 5/13 \end{pmatrix} \tag{2.0.2}$$

The solution for x can be written as

$$\mathbf{x} = \begin{pmatrix} 2/26 \\ 5/13 \end{pmatrix} \tag{2.0.3}$$

Thus, The point of intersection is at point (2/26 , 5/13) i.e. (0.07, 0.38)
Let the equation of the line be

$$\mathbf{n}^T\mathbf{x} = c \implies \mathbf{x}^T\mathbf{n} = c \tag{2.0.4}$$

Let the intercepts be a,b on the x and y axis respectively. Then,

$$\begin{pmatrix} a & 0 \end{pmatrix}\mathbf{n} = c \tag{2.0.5}$$

$$\begin{pmatrix} 0 & b \end{pmatrix}\mathbf{n} = c \tag{2.0.6}$$

resulting in the matrix equation

$$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}\mathbf{n} = c\begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{2.0.7}$$

As the intercepts are equal,Let a=b

$$\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}\mathbf{n} = c\begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{2.0.8}$$

$$\mathbf{n} = \frac{c}{a}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{2.0.9}$$

As the line passes through point of intersection,We can use the equation (2.0.3) in equation (2.0.4) to find the value of c

$$c = \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} \frac{2}{26} \\ \frac{2}{26} \end{pmatrix} \tag{2.0.10}$$

$$\implies c = \frac{2}{26} + \frac{2}{26} \implies c = \frac{6}{13} \tag{2.0.11}$$

So, the equation of line can be written as

$$\implies \begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{x} = 6/13 \tag{2.0.12}$$

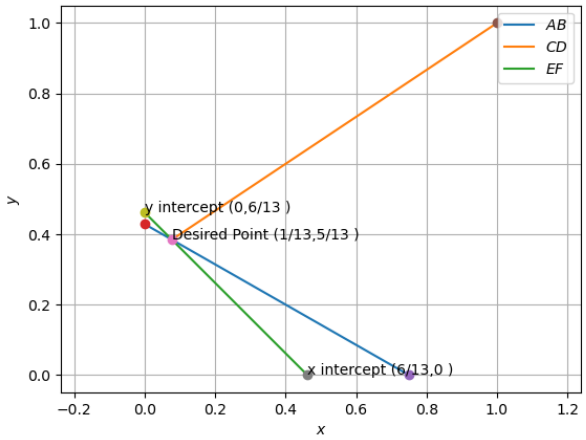

Fig. 0: The intercepts of the required line are equal