Matrix theory Assignment 1

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Abstract—This document explains the equation of the line passing through the point of intersection of the lines that has equal intercepts on the axes

1 PROBLEM

Find the equation of the line passing through the point of intersection of the lines

$$(4\ 7)\mathbf{x} = 3$$
$$(2\ -3)\mathbf{x} = -1$$

that has equal intercepts on the axes

2 SOLUTION

The above two line equations can be expressed as the matrix equation

$$\begin{pmatrix} 4 & 7 \\ 2 & -3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \quad (2.0.1)$$

Constructing the augmented matrix

$$\begin{pmatrix} 4 & 7 & 3 \\ 2 & -3 & -1 \end{pmatrix} \quad (2.0.2)$$

Transforming the matrix into row-echelon form

$$\begin{pmatrix} 4 & 7 & 3 \\ 2 & -3 & -1 \end{pmatrix} \xleftrightarrow{R2\leftarrow 2R2-R1}$$

$$\begin{pmatrix} 4 & 7 & 3 \\ 0 & -13 & -5 \end{pmatrix} \xleftrightarrow{R2\leftarrow -R2/13, R1\leftarrow R1/4}$$

$$\begin{pmatrix} 1 & 7/4 & 3/4 \\ 0 & 1 & 5/13 \end{pmatrix} \xleftrightarrow{R1\leftarrow R1-7/4R1/4}$$

$$\begin{pmatrix} 1 & 0 & 2/26 \\ 0 & 1 & 5/13 \end{pmatrix} \quad (2.0.3)$$

The solution for x can be written as

$$\mathbf{x} = \begin{pmatrix} 2/26 \\ 5/13 \end{pmatrix} \quad (2.0.4)$$

Thus, The point of intersection is at point (2/26 , 5/13) i.e. (0.07, 0.38)
Let the equation of the line be

$$\mathbf{n}^T \mathbf{x} = c \implies \mathbf{x}^T \mathbf{n} = c \quad (2.0.5)$$

Let the intercepts be a,b on the x and y axis respectively. Then,

$$\begin{pmatrix} a & 0 \end{pmatrix} \mathbf{n} = c \quad (2.0.6)$$

$$\begin{pmatrix} 0 & b \end{pmatrix} \mathbf{n} = c \quad (2.0.7)$$

resulting in the matrix equation

$$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \mathbf{n} = c \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (2.0.8)$$

As the intercepts are equal,Let a=b

$$\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \mathbf{n} = c \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (2.0.9)$$

$$\mathbf{n} = \frac{c}{a} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (2.0.10)$$

As the line passes through point of intersection,We can use the equation (2.0.4) in equation (2.0.5) to find the value of c

$$c = \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} \frac{2}{26} \\ \frac{2}{26} \end{pmatrix} \quad (2.0.11)$$

$$\implies c = \frac{2}{26} + \frac{2}{26} \implies c = \frac{6}{13} \quad (2.0.12)$$

So, the equation of line can be written as

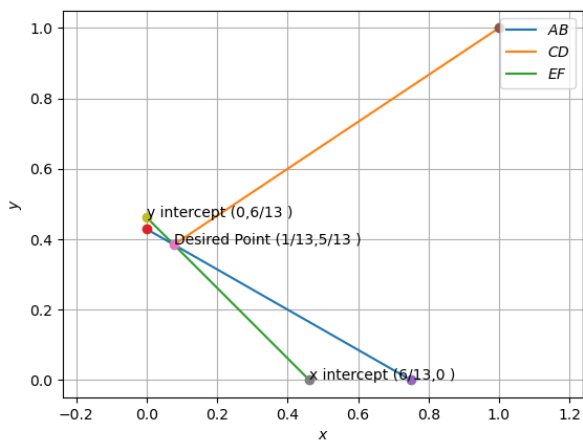$$\implies \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = \frac{6}{13} \quad (2.0.13)$$

Fig. 0: The intercepts of the required line are equal