EE5609: Matrix Theory
Assignment-2

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Abstract—**This document contains a solution for proving the determinant of the given matrix is zero.**

Download the python codes from

https://github.com/pavanmanesh/EE5609/blob/master/Assignment2/codes

and latex-tikz codes from

https://github.com/pavanmanesh/EE5609/tree/master/Assignment2

1 Problem

$$\begin{vmatrix} 0 & a & -b \\ -a & 0 & c \\ b & c & 0 \end{vmatrix} = 0 \tag{1.0.1}$$

2 Theorem for Homogeneous Systems

Theorem used for solving this problem:

$$A\mathbf{x} = \mathbf{0} \tag{2.0.1}$$

has a nontrivial solution if and only if $\left|A\right| = 0$ (i.e., A is singular).

3 Solution

Converting A into reduced echelon form:

$$\begin{pmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow R_2} \begin{pmatrix} -a & 0 & -c \\ 0 & a & -b \\ b & c & 0 \end{pmatrix} \tag{3.0.1}$$

$$\xleftrightarrow{R_3 \leftarrow a \times R_3 + b \times R_1} \begin{pmatrix} -a & 0 & -c \\ 0 & a & -b \\ 0 & ac & -bc \end{pmatrix} \tag{3.0.2}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - c \times R_2} \begin{pmatrix} -a & 0 & -c \\ 0 & a & -b \\ 0 & 0 & 0 \end{pmatrix} \tag{3.0.3}$$

Let A be the given matrix and x be a vector where x is

$$\mathbf{x} = \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} \tag{3.0.4}$$

From (2.0.1), we can write the augmented form in this way:

$$\begin{pmatrix} -a & 0 & -c & 0 \\ 0 & a & -b & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{3.0.5}$$

upon solving the above , the value of x:

$$\mathbf{x} = \begin{pmatrix} -\frac{c}{a}x3 \\ \frac{b}{a}x3 \\ x3 \end{pmatrix} \tag{3.0.6}$$

$$\mathbf{x} = \begin{pmatrix} -c \\ b \\ a \end{pmatrix} \frac{x3}{a} \tag{3.0.7}$$

There exists a nontrivial solution for the given matrix. From the theorem (2.0.1), the determinant is equal to 0.