

Matrix theory Assignment 1

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Abstract—This document explains the equation of the line passing through the point of intersection of the lines that has equal intercepts on the axes

Download all python codes from

<https://github.com/pavanmanesh/EE5609/tree/master/codes>

and latex-tikz codes from

<https://github.com/pavanmanesh/EE5609>

1 PROBLEM

Find the equation of the line passing through the point of intersection of the lines

$$\begin{aligned}(4 \ 7)\mathbf{x} &= 3 \\ (2 \ -3)\mathbf{x} &= -1\end{aligned}$$

that has equal intercepts on the axes

2 SOLUTION

Constructing the augmented matrix

$$\begin{pmatrix} 4 & 7 & 3 \\ 2 & -3 & -1 \end{pmatrix}$$

Transforming the matrix into row-echelon form

$$\begin{aligned} & \begin{pmatrix} 4 & 7 & 3 \\ 2 & -3 & -1 \end{pmatrix} \xrightarrow{R2 \leftarrow 2R2 - R1} \\ & \begin{pmatrix} 4 & 7 & 3 \\ 0 & -13 & -5 \end{pmatrix} \xrightarrow{R2 \leftarrow -R2/13, R1 \leftarrow R1/4} \\ & \begin{pmatrix} 1 & 7/4 & 3/4 \\ 0 & 1 & 5/13 \end{pmatrix} \xrightarrow{R1 \leftarrow R1 - 7/4 R2} \\ & \begin{pmatrix} 1 & 0 & 2/26 \\ 0 & 1 & 5/13 \end{pmatrix} \end{aligned} \quad (2.0.1)$$

Thus, The point of intersection is at point $(2/26, 5/13)$ i.e. $(0.07, 0.38)$

The equation of line in terms of vector notations can be written as

$$\mathbf{n}^T \mathbf{x} = b \quad \text{where} \quad \mathbf{n} = \begin{pmatrix} n_{11} \\ n_{12} \end{pmatrix} \quad (2.0.2)$$

or

$$\begin{pmatrix} n_{11} & n_{12} \end{pmatrix} \mathbf{x} = b \quad (2.0.3)$$

Let the intercepts be $\begin{pmatrix} a \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ b \end{pmatrix}$, respectively.

As the required line passes through the point of intersection $(2/26, 5/13)$ and the intercepts are equal, the intercepts can be written as: $\begin{pmatrix} 6/13 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 6/13 \end{pmatrix}$

When the line passes through $\begin{pmatrix} 6/13 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 6/13 \end{pmatrix}$, respectively, we get, upon substitution in (2.0.3):

$$6/13 n_{11} = b \quad \Rightarrow \quad n_{11} = \frac{b}{6/13}$$

$$6/13 n_{12} = b \quad \Rightarrow \quad n_{12} = -\frac{b}{6/13}$$

Therefore, the equation of first line is

$$\begin{aligned} & \begin{pmatrix} \frac{b}{6/13} & \frac{b}{6/13} \end{pmatrix} \mathbf{x} = b \\ \Rightarrow & \quad \quad \quad (1 \quad 1) \mathbf{x} = 6/13 \end{aligned} \quad (2.0.4)$$

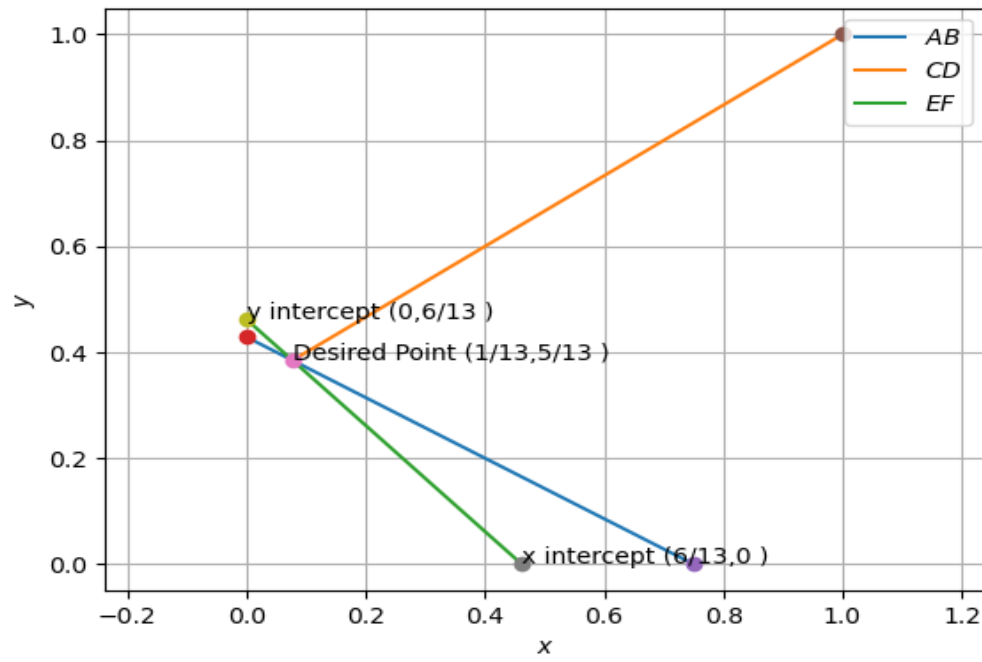


Fig. 0: The intercepts of the required line are equal