EE5609: Matrix Theory Assignment-8

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Abstract—**This document explains how to find a row-reduced matrix which is row equivalent to**

Download all latex-tikz codes from

https://github.com/pavanmanesh/EE5609/tree/master/Assignment8

and all python codes from

https://github.com/pavanmanesh/EE5609/tree/master/Assignment8/codes

1 Problem

Find a row-reduced matrix which is row equivalent to,

$$\mathbf{A} = \begin{pmatrix} 1 & -i \\ 2 & 2 \\ i & 1+i \end{pmatrix} \tag{1.0.1}$$

What are the solutions of Ax=0 ?

2 THEOREM

Let R be a row-reduced echelon matrix which is row equivalent to A. Then the systems

$$A\mathbf{x} = \mathbf{0}, R\mathbf{x} = \mathbf{0} \tag{2.0.1}$$

have the same solutions.

3 Solution

Step 1: Performing $R_2 \leftarrow R_2 - 2R_1$ and $R_3 \leftarrow R_3 - iR_1$ given by elementary matrix $\mathbf{E_{31}E_{21}}$ on the equation (1.0.1),

$$\mathbf{E_{31}E_{21}} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -i & 0 & 1 \end{pmatrix} \tag{3.0.1}$$

$$\mathbf{E_{31}E_{21}A} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -i & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -i \\ 2 & 2 \\ i & 1+i \end{pmatrix} \tag{3.0.2}$$

$$\implies \mathbf{A_1} = \mathbf{E_{31}E_{21}A} = \begin{pmatrix} 1 & -i \\ 0 & 2+2i \\ 0 & -i \end{pmatrix} \tag{3.0.3}$$

Step 2: Performing $R_2 \leftarrow \frac{1}{2+2i}R_2$ given by $\mathbf{A_1}$ on equation (3.0.3),

$$\mathbf{D_1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{4}(1-i) & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3.0.4}$$

$$\mathbf{D_1A_1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{4}(1-i) & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -i \\ 0 & 2+2i \\ 0 & -i \end{pmatrix} \tag{3.0.5}$$

$$\implies \mathbf{A_2} = \mathbf{D_1A_1} = \begin{pmatrix} 1 & -i \\ 0 & 1 \\ 0 & -i \end{pmatrix} \tag{3.0.6}$$

Step 3: Performing $R_3 \leftarrow R_3 + iR_2$ given by $\mathbf{E_{32}}$ on equation (3.0.6),

$$\mathbf{E_{32}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & i & 1 \end{pmatrix} \tag{3.0.7}$$

$$\mathbf{E_{32}A_2} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & i & 1 \end{pmatrix}\begin{pmatrix} 1 & i \\ 0 & 1 \\ 0 & -i \end{pmatrix} \tag{3.0.8}$$

$$\implies \mathbf{A_3} = \mathbf{E_{32}A_2} = \begin{pmatrix} 1 & -i \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \tag{3.0.9}$$

Step 4: Performing $R_1 \leftarrow R_1 + iR_2$ given by $\mathbf{E_{12}}$ on equation (3.0.9),

$$\mathbf{E_{12}} = \begin{pmatrix} 1 & i & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3.0.10}$$

$$\mathbf{E_{12}A_3} = \begin{pmatrix} 1 & i & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -i \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \tag{3.0.11}$$

$$\implies \mathbf{A_4} = \mathbf{E_{12}A_3} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \tag{3.0.12}$$

$\therefore$ Row-reduced matrix of $\mathbf{A}$ given by equation (1.0.1) is,

$$\mathbf{A} = \begin{pmatrix} 1 & -i \\ 2 & 2 \\ i & 1+i \end{pmatrix} \xleftrightarrow{RREF} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \tag{3.0.13}$$

From(2.0.1) and (3.0.13),the only solution of A$\mathbf{x}$ =0 is,

$$\mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{3.0.14}$$