EE5609: Matrix Theory Assignment-8

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***Abstract*—This document explains how to find a row-reduced matrix which is row equivalent to given matrix**

Download all latex-tikz codes from

https://github.com/pavanmanesh/EE5609/tree/master/Assignment8

and all python codes from

https://github.com/pavanmanesh/EE5609/tree/master/Assignment8/codes

1 Problem

Find a row-reduced matrix which is row equivalent to A.What are the solutions of Ax=0 ?

$$\mathbf{A} = \begin{pmatrix} 1 & -i \\ 2 & 2 \\ i & 1+i \end{pmatrix} \tag{1.0.1}$$

2 THEOREM

Let R be a row-reduced echelon matrix which is row equivalent to A. Then the systems

$$A\mathbf{x} = \mathbf{0}, R\mathbf{x} = \mathbf{0} \tag{2.0.1}$$

have the same solutions.

3 Solution

On performing elementary row operations on (1.0.1),

$$\mathbf{R} = \mathbf{B}\mathbf{A} \tag{3.0.1}$$

where $\mathbf{B}$ is the product of all elementary matrices. Reducing the given matrix, we get

$$\begin{aligned}\mathbf{B} &= (\mathbf{E_5 E_4 E_3 E_2 E_1}) \\ &= \begin{pmatrix} 1 & i & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -i & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{4}(1-i) & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &\quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -i & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{2}(1-i) & \frac{1}{4}(1+i) & 0 \\ \frac{1}{2}(-1+i) & \frac{1}{4}(1-i) & 0 \\ \frac{1}{2}(1-i) & \frac{1}{4}(-1-i) & 1 \end{pmatrix}\end{aligned} \tag{3.0.2}$$

$$\mathbf{R} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \tag{3.0.3}$$

Row-reduced matrix of $\mathbf{A}$ is,

$$\mathbf{A} = \begin{pmatrix} 1 & -i \\ 2 & 2 \\ i & 1+i \end{pmatrix} \xleftrightarrow{RREF} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \tag{3.0.4}$$

From(2.0.1) and (3.0.4),

$$A\mathbf{x} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \tag{3.0.5}$$

The solution of A$\mathbf{x}$ =0 is,

$$\mathbf{I_2 x} = 0 \tag{3.0.6}$$

$$\implies \mathbf{x} = 0 \tag{3.0.7}$$

As $\mathbf{I_2}$ is invertible.