Matrix Theory (EE5609) Assignment-3

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***Abstract*—This document contains the proof on Quadrilateral.**

Download latex-tikz codes from

https://github.com/EE20RESCH11008/Matrix−Theory/tree/master/Assignment−3

1 Problem

Line segments AD and BC intersect at O and form △OAB and △ODC. $\angle B < \angle A$ and $\angle C < \angle D$. Show that $AD < BC$.

2 Solution

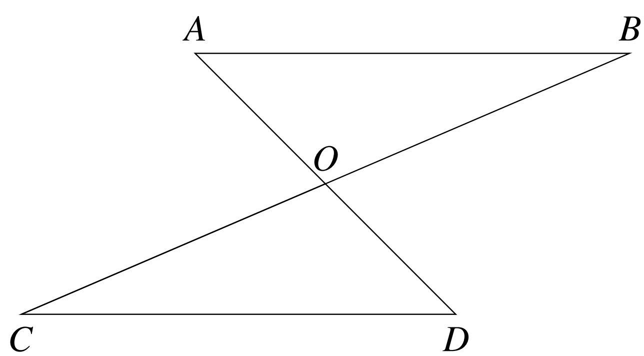

Fig. 1: Quadrilateral with $\angle B < \angle A$ and $\angle C < \angle D$

In $\triangle OAB$,

$$\frac{\|\mathbf{B}-\mathbf{O}\|}{Sin\mathbf{A}} = \frac{\|\mathbf{A}-\mathbf{O}\|}{Sin\mathbf{B}} = \frac{\|\mathbf{A}-\mathbf{B}\|}{Sin\mathbf{O}} \tag{2.0.1}$$

Let

$$\frac{\|\mathbf{B}-\mathbf{O}\|}{Sin\mathbf{A}} = \frac{\|\mathbf{A}-\mathbf{O}\|}{Sin\mathbf{B}} \tag{2.0.2}$$

$$\implies \frac{\|\mathbf{B}-\mathbf{O}\|}{\|\mathbf{A}-\mathbf{O}\|} = \frac{Sin\mathbf{A}}{Sin\mathbf{B}} = k \tag{2.0.3}$$

Since $\angle B < \angle A$ and $\angle A + \angle B + \angle O = 180$
For $0 < A < 180$,

$$Sin\mathbf{B} < Sin\mathbf{A} \implies k > 1 \tag{2.0.4}$$

$$\implies \|\mathbf{A}-\mathbf{O}\| = \frac{\|\mathbf{B}-\mathbf{O}\|}{k} \tag{2.0.5}$$

$$\implies \|\mathbf{A}-\mathbf{O}\| < \|\mathbf{B}-\mathbf{O}\| \tag{2.0.6}$$

Case(ii):
In $\triangle OCD$,

$$\frac{\|\mathbf{D}-\mathbf{O}\|}{Sin\mathbf{C}} = \frac{\|\mathbf{C}-\mathbf{O}\|}{Sin\mathbf{D}} = \frac{\|\mathbf{C}-\mathbf{D}\|}{Sin\mathbf{O}} \tag{2.0.7}$$

Consider,

$$\frac{\|\mathbf{D}-\mathbf{O}\|}{Sin\mathbf{C}} = \frac{\|\mathbf{C}-\mathbf{O}\|}{Sin\mathbf{D}} \tag{2.0.8}$$

$$\implies \frac{\|\mathbf{C}-\mathbf{O}\|}{\|\mathbf{D}-\mathbf{O}\|} = \frac{Sin\mathbf{D}}{Sin\mathbf{C}} = k \tag{2.0.9}$$

Since $\angle C < \angle D$ and $\angle C + \angle D + \angle O = 180$
For $0 < D < 180$,

$$Sin\mathbf{C} < Sin\mathbf{D} \implies k > 1 \tag{2.0.10}$$

$$\implies \|\mathbf{D}-\mathbf{O}\| = \frac{\|\mathbf{C}-\mathbf{O}\|}{k} \tag{2.0.11}$$

$$\implies \|\mathbf{D}-\mathbf{O}\| < \|\mathbf{C}-\mathbf{O}\| \tag{2.0.12}$$

From equations (2.0.6) and (2.0.12) we get,

$$\|\mathbf{A}-\mathbf{O}\| + \|\mathbf{D}-\mathbf{O}\| < \|\mathbf{B}-\mathbf{O}\| + \|\mathbf{C}-\mathbf{O}\| \tag{2.0.13}$$

$$\implies \|\mathbf{D}-\mathbf{A}\| < \|\mathbf{C}-\mathbf{B}\| \tag{2.0.14}$$

$$\implies AD < BC \tag{2.0.15}$$

Hence Proved.