Matrix Theory (EE5609)
Assignment-8

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Abstract—**This document contains the solution to span of W which is a subspace of $\mathbf{R}^3$.**

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1 PROBLEM

Consider the subspaces $\mathbf{W_1}$ and $\mathbf{W_2}$ of $\mathbf{R}^3$ given by:

$$\mathbf{W_1} = \{(\mathbf{x}, \mathbf{y}, \mathbf{z}) \in \mathbf{R}^3 : \mathbf{x} + \mathbf{y} + \mathbf{z} = \mathbf{0}\} \quad (1.0.1)$$

and

$$\mathbf{W_2} = \{(\mathbf{x}, \mathbf{y}, \mathbf{z}) \in \mathbf{R}^3 : \mathbf{x} - \mathbf{y} + \mathbf{z} = \mathbf{0}\} \quad (1.0.2)$$

If $\mathbf{W}$ is a subspace of $\mathbf{R}^3$ such that

$$(i)\mathbf{W} \cap \mathbf{W_2} = span\{(0, 1, 1)\} \quad (1.0.3)$$

$$(ii)\mathbf{W} \cap \mathbf{W_1} \text{ is orthogonal to } \mathbf{W} \cap \mathbf{W_2} \quad (1.0.4)$$

with respect to the usual inner products of $\mathbf{R}^3$, then
1. $\mathbf{W} = $ span $\{(0, 1, -1), (0, 1, 1)\}$
2. $\mathbf{W} = $ span $\{(1, 0, -1), (0, 1, -1)\}$
3. $\mathbf{W} = $ span $\{(1, 0, -1), (0, 1, 1)\}$
4. $\mathbf{W} = $ span $\{(1, 0, -1), (1, 0, 1)\}$

2 SOLUTION

Using (1.0.1),

$$\mathbf{W_1} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (2.0.1)$$

From (1.0.2),

$$\mathbf{W_2} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \quad (2.0.2)$$

From (1.0.3), we can say that, both the subspaces $\mathbf{W}$ and $\mathbf{W_2}$ of $\mathbf{R}^3$ contains the column vector as follows: .

$$\mathbf{W} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \quad (2.0.3)$$

$$\mathbf{W_2} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \quad (2.0.4)$$

From (2.0.2) and (2.0.4),

$$\mathbf{W_2} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \\ 1 & 1 \end{pmatrix} \quad (2.0.5)$$

$$Rank(\mathbf{W_2}) = 2 \quad (2.0.6)$$

Since, rank < 3 and the vectors are linearly independent they span a subspace of $\mathbf{R}^3$.

Consider the vector,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbf{W} \cap \mathbf{W_1} \quad (2.0.7)$$

From (1.0.3) and (1.0.4),

The vector $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ is orthogonal to $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$.

$$\implies \begin{pmatrix} x & y & z \end{pmatrix} . \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = 0 \quad (2.0.8)$$

$$\implies \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \quad (2.0.9)$$

Since, $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbf{W} \cap \mathbf{W_1}$:

From (2.0.1) and (2.0.9),

$$\mathbf{W_1} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad (2.0.10)$$

Also from (2.0.3) and (2.0.9),

$$\boxed{\mathbf{W} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \\ 1 & -1 \end{pmatrix}} \qquad (2.0.11)$$

Using (2.0.11),
The vectors linearly independent and rank($\mathbf{W}$)=2 (< 3), then the vector span subspace of $\mathbf{R^3}$.

Hence,

$$\boxed{\mathbf{W} = span\{(0, 1, -1), (0, 1, 1)\} \implies \mathbf{Ans : 1}}$$
$$(2.0.12)$$