Matrix Theory (EE5609) Assignment-4

Prasanth Kumar Duba
EE20RESCH11008

Abstract—**This document contains the proof for the tangents to the circle at given points are parallel.**

Download all python codes from

https://github.com/EE20RESCH11008/Matrix−Theory/tree/master/Assignment−4/Code

and latex-tikz codes from

https://github.com/EE20RESCH11008/Matrix−Theory/tree/master/Assignment−4

1 Problem

Find the equations of the tangents to the circle

$$\mathbf{x}^T\mathbf{x} - \begin{pmatrix} 7 & 5 \end{pmatrix}\mathbf{x} + 18 = 0 \tag{1.0.1}$$

at the points $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$, showing that they are parallel.

2 Solution

Let

$$\mathbf{A} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{B} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{2.0.2}$$

The general equation of a second degree can be expressed as:

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.3}$$

Comparing 1.0.1 with 2.0.3

$$\mathbf{V} = \mathbf{I}, \mathbf{u} = \begin{pmatrix} \frac{-7}{2} \\ \frac{-5}{2} \end{pmatrix}, f = 18 \tag{2.0.4}$$

The Centre $\mathbf{c}$ and radius r are given by,

$$\mathbf{c} = -\mathbf{u} = \begin{pmatrix} \frac{7}{2} \\ \frac{5}{2} \end{pmatrix}, r = \sqrt{\mathbf{u}^T\mathbf{u} - f} = \sqrt{\frac{1}{2}} \tag{2.0.5}$$

The direction vector of the line joining $\mathbf{c}$ and $\mathbf{A}$ is:

$$\mathbf{n_1} = \begin{pmatrix} 3.5 \\ 2.5 \end{pmatrix} - \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} -0.5 \\ -0.5 \end{pmatrix} \tag{2.0.6}$$

The vector $\mathbf{n_1}$ is normal to the tangent drawn at$\mathbf{A}$. Thus the equation of the tangent can be written as:

$$\mathbf{n_1^T}(\mathbf{x} - \mathbf{A}) = 0 \tag{2.0.7}$$

$$\begin{pmatrix} 0.5 & 0.5 \end{pmatrix}\mathbf{x} = -3.5 \tag{2.0.8}$$

Similarly, the direction vector of the line joining $\mathbf{c}$ and $\mathbf{B}$ is:

$$\mathbf{n_2} = \begin{pmatrix} 3.5 \\ 2.5 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} \tag{2.0.9}$$

The vector $\mathbf{n_2}$ is normal to the tangent drawn at$\mathbf{B}$. Thus the equation of the tangent can be written as:

$$\mathbf{n_2^T}(\mathbf{x} - \mathbf{B}) = 0 \tag{2.0.10}$$

$$\begin{pmatrix} 0.5 & 0.5 \end{pmatrix}\mathbf{x} = 2.5 \tag{2.0.11}$$

The tangents will be parallel, if the normal vector of first tangent is scalar multiple of normal vector of second tangent.
From 2.0.6 and 2.0.9 we get,

$$\boxed{\mathbf{n_1} = -\mathbf{n_2}} \tag{2.0.12}$$

Hence, the two tangents are parallel.

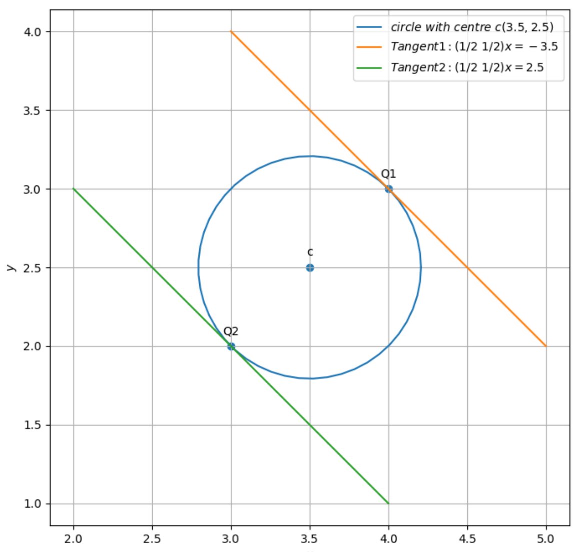

Fig. 0: Tangents to the circle at given points