

Matrix Theory (EE5609)

Assignment-5

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Abstract—This document contains traces the parabola when it's general second degree equation is given.

Download all python codes from

<https://github.com/EE20RESCH11008/Matrix-Theory/tree/master/Assignment-5/Code>

and latex-tikz codes from

<https://github.com/EE20RESCH11008/Matrix-Theory/tree/master/Assignment-5>

1 PROBLEM

Trace the parabola

$$16x^2 - 24xy + 9y^2 + 32x + 86y - 39 = 0 \quad (1.0.1)$$

2 SOLUTION

The general equation of a second degree can be expressed as:

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.1)$$

Comparing 1.0.1 with 2.0.1

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix} 16 & -12 \\ -12 & 9 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} 16 \\ 43 \end{pmatrix}, \quad f = -39 \quad (2.0.2)$$

2.1 Eigen Values:

The characteristic equation of \mathbf{V} is given as

$$|\lambda \mathbf{I} - \mathbf{V}| = 0 \quad (2.1.1)$$

$$\Rightarrow \begin{vmatrix} \lambda - 16 & 12 \\ 12 & \lambda - 9 \end{vmatrix} = 0 \quad (2.1.2)$$

$$\Rightarrow \lambda^2 - 25\lambda = 0 \quad (2.1.3)$$

The eigenvalues are the roots of the equation (2.0.5), which are as follows:

$$\lambda_1 = 0, \quad \lambda_2 = 25 \quad (2.1.4)$$

2.2 Eigen Vectors:

The eigen vector \mathbf{p} is defined as

$$\mathbf{V}\mathbf{p} = \lambda\mathbf{p} \quad (2.2.1)$$

$$\Rightarrow (\lambda \mathbf{I} - \mathbf{V})\mathbf{p} = 0 \quad (2.2.2)$$

For $\lambda_1 = 0$

$$(\lambda_1 \mathbf{I} - \mathbf{V}) = \begin{pmatrix} -16 & 12 \\ 12 & -9 \end{pmatrix} \xrightarrow[R_2 \leftarrow R_2 + 3R_1]{R_1 \leftarrow \frac{1}{4}R_1} \begin{pmatrix} -4 & 3 \\ 0 & 0 \end{pmatrix} \quad (2.2.3)$$

$$\Rightarrow \mathbf{p}_1 = \frac{1}{5} \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad (2.2.4)$$

For $\lambda_2 = 25$

$$(\lambda_2 \mathbf{I} - \mathbf{V}) = \begin{pmatrix} 9 & 12 \\ 12 & 1 \end{pmatrix} \xrightarrow[R_2 \leftarrow R_2 - 4R_1]{R_1 \leftarrow \frac{1}{3}R_1} \begin{pmatrix} 3 & 4 \\ 0 & 0 \end{pmatrix} \quad (2.2.5)$$

$$\Rightarrow \mathbf{p}_2 = \frac{1}{5} \begin{pmatrix} -4 \\ 3 \end{pmatrix} \quad (2.2.6)$$

2.3 Eigen Value Decomposition:

Using EVD, we can write

$$\mathbf{D} = \mathbf{P}\mathbf{V}\mathbf{P}^T \quad (2.3.1)$$

where,

$$\mathbf{P} = \frac{1}{5} \begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix} \quad (2.3.2)$$

$$\mathbf{D} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 25 \end{pmatrix} \quad (2.3.3)$$

2.4 Parabola

$$\text{Focal Length} = \left| \frac{2\eta}{\lambda_2} \right| \quad (2.4.1)$$

From (2.1.4) and (2.0.2)

$$\eta = \mathbf{p}_1^T \mathbf{u} = 44 \quad (2.4.2)$$

Substituting values from (2.4.2) and (2.1.4) in (2.4.1), we get

$$\text{Focal Length} = \left| \frac{88}{25} \right| = 3.52 \quad (2.4.3)$$

The standard equation of parabola is given by:

$$\mathbf{y}^T \mathbf{D} \mathbf{y} = -2\eta \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{y} \quad (2.4.4)$$

And the vertex \mathbf{c} is:

$$\begin{pmatrix} \mathbf{u}^T + \eta \mathbf{p}_1^T \\ \mathbf{v} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -f \\ \eta \mathbf{p}_1 - \mathbf{u} \end{pmatrix} \quad (2.4.5)$$

From (2.0.2) (2.4.1)(2.2.4),

$$\begin{pmatrix} \frac{212}{5} & \frac{391}{5} \\ 16 & -12 \\ -12 & 9 \end{pmatrix} \mathbf{c} = \begin{pmatrix} 39 \\ \frac{52}{5} \\ \frac{-39}{5} \end{pmatrix} \quad (2.4.6)$$

To find \mathbf{c} , perform row reduction on the augmented matrix as follows:

$$\begin{pmatrix} \frac{212}{5} & \frac{391}{5} & 39 \\ 16 & -12 & \frac{52}{5} \\ -12 & 9 & \frac{-39}{5} \end{pmatrix} \xrightarrow[R_1 \leftarrow -\frac{5}{212}R_1]{R_3 \leftarrow R_3 + \frac{3}{4}R_2} \begin{pmatrix} 1 & \frac{391}{212} & \frac{195}{212} \\ 16 & -12 & \frac{52}{5} \\ 0 & 0 & 0 \end{pmatrix} \quad (2.4.7)$$

$$\xrightarrow{R_2 \leftarrow R_2 - 16R_1} \begin{pmatrix} 1 & \frac{391}{212} & \frac{195}{212} \\ 0 & \frac{-2200}{53} & \frac{-1144}{265} \\ 0 & 0 & 0 \end{pmatrix} \quad (2.4.8)$$

$$\xrightarrow{R_2 \leftarrow -\frac{53}{2200}R_2} \begin{pmatrix} 1 & \frac{391}{212} & \frac{195}{212} \\ 0 & 1 & \frac{13}{125} \\ 0 & 0 & 0 \end{pmatrix} \quad (2.4.9)$$

$$\xrightarrow{R_1 \leftarrow R_1 - \frac{391}{212}R_2} \begin{pmatrix} 1 & 0 & \frac{4823}{6625} \\ 0 & 1 & \frac{13}{125} \\ 0 & 0 & 0 \end{pmatrix} \quad (2.4.10)$$

Hence,

$$\mathbf{c} = \begin{pmatrix} \frac{4823}{6625} \\ \frac{13}{125} \end{pmatrix} = \begin{pmatrix} 0.728 \\ 0.104 \end{pmatrix} \quad (2.4.11)$$

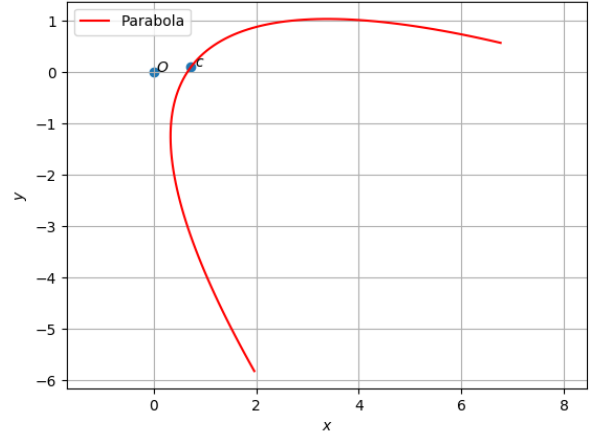


Fig. 0: Parabola with vertex c