Matrix Theory (EE5609)
Assignment-4

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Abstract—**Proof for the tangents to the circle at given points are parallel.**

1 Problem

Find the equations of the tangents to the circle

$$\mathbf{x}^T\mathbf{x} - (7 \quad 5)\mathbf{x} + 18 = 0 \tag{1.0.1}$$

at the points $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$, showing that they are parallel.

2 Solution

The general equation of a second degree can be expressed as:

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.1}$$

Comparing 1.0.1 with 2.0.1

$$\mathbf{V} = \mathbf{I}, \mathbf{u} = \begin{pmatrix} \frac{-7}{2} \\ \frac{-5}{2} \end{pmatrix}, f = 18 \tag{2.0.2}$$

The vector equation of a line can be expressed as

$$\mathbf{x} = \mathbf{q} + \mu\mathbf{m} \tag{2.0.3}$$

2.1 Tangent-1

Given point

$$\mathbf{q_1} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \tag{2.1.1}$$

The direction vector of the line joining the point $\mathbf{q_1}$ and the centre $\mathbf{c}$ expressed as:

$$\mathbf{n_1} = \mathbf{q_1} - \mathbf{c} \tag{2.1.2}$$

This vector $\mathbf{n_1}$ is normal to the tangent drawn at $\mathbf{q_1}$

$$\mathbf{n_1} = \mathbf{q_1} + \mathbf{u} \tag{2.1.3}$$

where,

$$\mathbf{c} = -\mathbf{u} \tag{2.1.4}$$

The normal vector to the line is obtained as

$$\mathbf{n_1}^T = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix} \tag{2.1.5}$$

If $\mathbf{q_1}$ be a point on the line and $\mathbf{n_1}$ is the normal vector, the equation of the line can be expressed as

$$\mathbf{n_1}^T(\mathbf{x} - \mathbf{q_1}) = 0 \tag{2.1.6}$$

$$\implies \mathbf{n_1}^T\mathbf{x} = c \tag{2.1.7}$$

where

$$c = \mathbf{n_1}^T\mathbf{q_1} \tag{2.1.8}$$

From the equations 2.1.1 and 2.1.5,

$$\implies c = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 4 \\ 3 \end{pmatrix} = \frac{7}{2} \tag{2.1.9}$$

The line equation of Tangent-1 is:

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}\mathbf{x} = \frac{7}{2} \tag{2.1.10}$$

$$\implies \boxed{\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{x} = 7} \tag{2.1.11}$$

2.2 Tangent-2

Let us take,

$$\mathbf{q_2} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{2.2.1}$$

The direction vector of the line joining the point $\mathbf{q_2}$ and the centre $\mathbf{c}$ expressed as:

$$\mathbf{n_2} = \mathbf{q_2} - \mathbf{c} \qquad (2.2.2)$$

This vector $\mathbf{n_2}$ is normal to the tangent drawn at $\mathbf{q_2}$

$$\mathbf{n_2} = \mathbf{q_2} + \mathbf{u} \qquad (2.2.3)$$

The normal vector to the line is obtained as

$$\mathbf{n_2}^T = \begin{pmatrix} \frac{-1}{2} & \frac{-1}{2} \end{pmatrix} \qquad (2.2.4)$$

If $\mathbf{q_2}$ be a point on the line and $\mathbf{n_2}$ is the normal vector, the equation of the line can be expressed as

$$\mathbf{n_2}^T(\mathbf{x} - \mathbf{q_2}) = 0 \qquad (2.2.5)$$

$$\implies \mathbf{n_2}^T\mathbf{x} = c \qquad (2.2.6)$$

where

$$c = \mathbf{n_2}^T\mathbf{q_2} \qquad (2.2.7)$$

From the equations 2.2.1 and 2.2.4,

$$\implies c = \begin{pmatrix} \frac{-1}{2} & \frac{-1}{2} \end{pmatrix}\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \frac{-5}{2} \qquad (2.2.8)$$

The line equation of Tangent-2 is:

$$\begin{pmatrix} \frac{-1}{2} & \frac{-1}{2} \end{pmatrix}\mathbf{x} = \frac{-5}{2} \qquad (2.2.9)$$

$$\implies \boxed{\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{x} = 5} \qquad (2.2.10)$$

2.3 Result

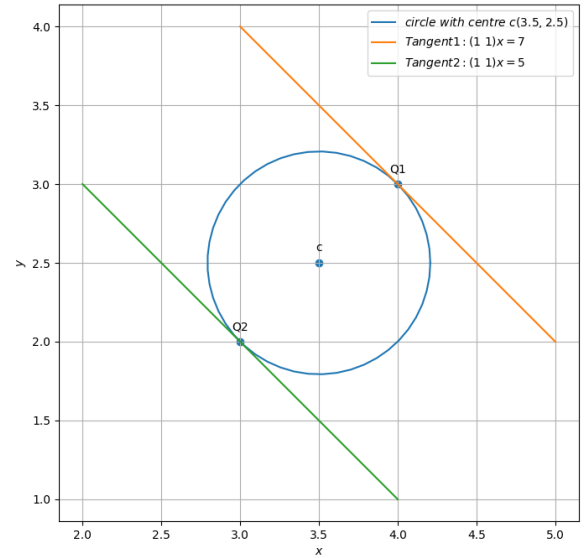From the equations 2.1.11 and 2.2.10, normal vector of Tangent-1 and Tangent-2 are equal. Hence, the two tangents are parallel.

Fig. 0: Tangents to the circle at given points