

Matrix Theory (EE5609)

Assignment-4

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Abstract—Proof for the tangents to the circle at given points are parallel.

Download all python codes from

<https://github.com/EE20RESCH11008/Matrix-Theory/tree/master/Assignment-4/Code>

and latex-tikz codes from

<https://github.com/EE20RESCH11008/Matrix-Theory/tree/master/Assignment-4>

1 PROBLEM

Find the equations of the tangents to the circle

$$\mathbf{x}^T \mathbf{x} - (7 \ 5)\mathbf{x} + 18 = 0 \quad (1.0.1)$$

at the points $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$, showing that they are parallel.

2 SOLUTION

The general equation of a second degree can be expressed as:

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.1)$$

Comparing 1.0.1 with 2.0.1

$$\mathbf{V} = \mathbf{I}, \mathbf{u} = \begin{pmatrix} \frac{-7}{2} \\ \frac{-5}{2} \end{pmatrix}, f = 18 \quad (2.0.2)$$

The vector equation of a line can be expressed as

$$\mathbf{x} = \mathbf{q} + \mu \mathbf{m} \quad (2.0.3)$$

2.1 Tangent-1

Given point

$$\mathbf{q}_1 = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \quad (2.1.1)$$

The normal vector to the line is obtained as

$$\mathbf{n}_1 = \mathbf{q}_1 + \mathbf{u} \quad (2.1.2)$$

$$\mathbf{n}_1^T = \left(\frac{1}{2} \ \frac{1}{2} \right) \quad (2.1.3)$$

If \mathbf{q}_1 be a point on the line and \mathbf{n}_1 is the normal vector, the equation of the line can be expressed as

$$\mathbf{n}_1^T (\mathbf{x} - \mathbf{q}_1) = 0 \quad (2.1.4)$$

$$\Rightarrow \mathbf{n}_1^T \mathbf{x} = c \quad (2.1.5)$$

where

$$c = \mathbf{n}_1^T \mathbf{q}_1 \quad (2.1.6)$$

From the equations 2.1.1 and 2.1.3,

$$\Rightarrow c = \left(\frac{1}{2} \ \frac{1}{2} \right) \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \frac{7}{2} \quad (2.1.7)$$

The line equation of Tangent-1 is:

$$\left(\frac{1}{2} \ \frac{1}{2} \right) \mathbf{x} = \frac{7}{2} \quad (2.1.8)$$

$$\Rightarrow \boxed{(1 \ 1) \mathbf{x} = 7} \quad (2.1.9)$$

2.2 Tangent-2

Let us take,

$$\mathbf{q}_2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad (2.2.1)$$

The normal vector to the line is obtained as

$$\mathbf{n}_2 = \mathbf{q}_2 + \mathbf{u} \quad (2.2.2)$$

$$\mathbf{n}_2^T = \left(\frac{-1}{2} \ \frac{-1}{2} \right) \quad (2.2.3)$$

If \mathbf{q}_2 be a point on the line and \mathbf{n}_2 is the normal vector, the equation of the line can be expressed as

$$\mathbf{n}_2^T (\mathbf{x} - \mathbf{q}_2) = 0 \quad (2.2.4)$$

$$\implies \mathbf{n}_2^T \mathbf{x} = c \quad (2.2.5)$$

where

$$c = \mathbf{n}_2^T \mathbf{q}_2 \quad (2.2.6)$$

From the equations 2.2.1 and 2.2.3,

$$\implies c = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} = -\frac{5}{2} \quad (2.2.7)$$

The line equation of Tangent-2 is:

$$\begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} \end{pmatrix} \mathbf{x} = -\frac{5}{2} \quad (2.2.8)$$

$$\implies \boxed{\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 5} \quad (2.2.9)$$

2.3 Result

From the equations 2.1.9 and 2.2.9, normal vector of Tangent-1 and Tangent-2 are equal.

Hence, the two tangents are parallel.

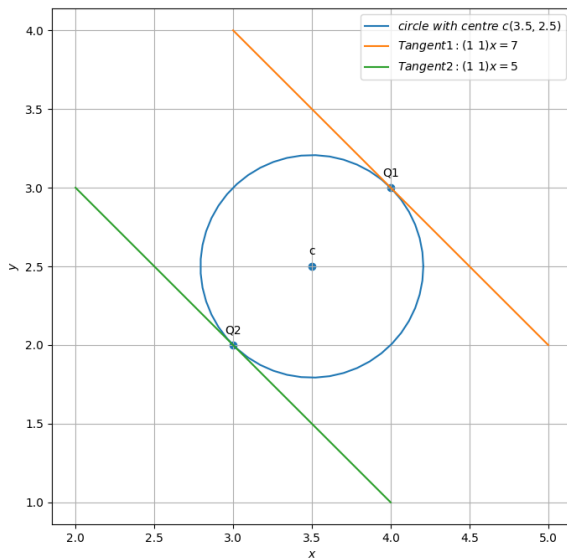


Fig. 0: Tangents to the circle at given points