Matrix Theory (EE5609) Assignment-3

Prasanth Kumar Duba
EE20RESCH11008

Abstract—**This document contains the proof on Quadrilateral.**

Download latex-tikz codes from

https://github.com/EE20RESCH11008/Matrix−Theory/tree/master/Assignment−3

1 Problem

Line segments AD and BC intersect at O and form $\triangle$OAB and $\triangle$ODC. $\angle B < \angle A$ and $\angle C < \angle D$. Show that $AD < BC$.

2 Solution

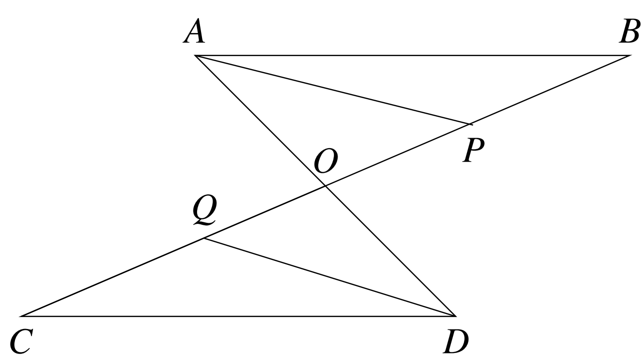

Fig. 1: Quadrilateral with $\angle B < \angle A$ and $\angle C < \angle D$

Case(i): Given that

$$\angle B < \angle A \tag{2.0.1}$$

Let P is a point on OB such that

$$\angle PAB = \angle OBA \tag{2.0.2}$$

From $\triangle PAB$,

$$\|\mathbf{A} - \mathbf{P}\| = \|\mathbf{B} - \mathbf{P}\| \tag{2.0.3}$$

$$\|\mathbf{P} - \mathbf{O}\| + \|\mathbf{A} - \mathbf{P}\| = \|\mathbf{P} - \mathbf{O}\| + \|\mathbf{B} - \mathbf{P}\| \tag{2.0.4}$$

$$\|\mathbf{P} - \mathbf{O}\| + \|\mathbf{A} - \mathbf{P}\| = \|\mathbf{B} - \mathbf{O}\| \tag{2.0.5}$$

In $\triangle OAP$,

$$\|\mathbf{A} - \mathbf{O}\| < \|\mathbf{P} - \mathbf{O}\| + \|\mathbf{A} - \mathbf{P}\| \tag{2.0.6}$$

$$\implies \|\mathbf{A} - \mathbf{O}\| < \|\mathbf{B} - \mathbf{O}\| \tag{2.0.7}$$

Case(ii): We have

$$\angle C < \angle D \tag{2.0.8}$$

Q is a point on OC such that

$$\angle QDC = \angle DCO \tag{2.0.9}$$

In $\triangle QCD$,

$$\|\mathbf{D} - \mathbf{Q}\| = \|\mathbf{C} - \mathbf{Q}\| \tag{2.0.10}$$

$$\|\mathbf{Q} - \mathbf{O}\| + \|\mathbf{D} - \mathbf{Q}\| = \|\mathbf{Q} - \mathbf{O}\| + \|\mathbf{C} - \mathbf{Q}\| \tag{2.0.11}$$

$$\|\mathbf{Q} - \mathbf{O}\| + \|\mathbf{D} - \mathbf{Q}\| = \|\mathbf{C} - \mathbf{O}\| \tag{2.0.12}$$

In $\triangle OQD$,

$$\|\mathbf{D} - \mathbf{O}\| < \|\mathbf{Q} - \mathbf{O}\| + \|\mathbf{D} - \mathbf{Q}\| \tag{2.0.13}$$

$$\implies \|\mathbf{D} - \mathbf{O}\| < \|\mathbf{C} - \mathbf{O}\| \tag{2.0.14}$$

From equations (2.0.7) and (2.0.14) we get,

$$\|\mathbf{A} - \mathbf{O}\|^2 + \|\mathbf{D} - \mathbf{O}\|^2 < \|\mathbf{B} - \mathbf{O}\|^2 + \|\mathbf{C} - \mathbf{O}\|^2 \tag{2.0.15}$$

$$\implies \|\mathbf{D} - \mathbf{A}\|^2 < \|\mathbf{C} - \mathbf{B}\|^2 \tag{2.0.16}$$

$$\implies AD < BC \tag{2.0.17}$$

Hence Proved.