# Matrix Theory (EE5609) Assignment-8

Prasanth Kumar Duba
EE20RESCH11008

*Abstract*—**This document contains the solution to span of W which is a subspace of $\mathbf{R^3}$.**

Download latex-tikz codes from

https://github.com/EE20RESCH11008/Matrix−Theory/tree/master/Assignment−8

## 1 Problem

Consider the subspaces $\mathbf{W_1}$ and $\mathbf{W_2}$ of $\mathbf{R}^3$ given by:

$$\mathbf{W_1} = \{(\mathbf{x}, \mathbf{y}, \mathbf{z}) \in \mathbf{R^3} : \mathbf{x} + \mathbf{y} + \mathbf{z} = \mathbf{0}\} \tag{1.0.1}$$

and

$$\mathbf{W_2} = \{(\mathbf{x}, \mathbf{y}, \mathbf{z}) \in \mathbf{R^3} : \mathbf{x} - \mathbf{y} + \mathbf{z} = \mathbf{0}\} \tag{1.0.2}$$

If $\mathbf{W}$ is a subspace of $\mathbf{R^3}$ such that

$$(i)\mathbf{W} \cap \mathbf{W_2} = span\{(0, 1, 1)\} \tag{1.0.3}$$

$$(ii)\mathbf{W} \cap \mathbf{W_1} \ \ is\ orthogonal\ to\ \ \mathbf{W} \cap \mathbf{W_2} \tag{1.0.4}$$

with respect to the usual inner products of $\mathbf{R^3}$, then

1. $\mathbf{W}$ = span $\{(0, 1, -1), (0, 1, 1)\}$
2. $\mathbf{W}$ = span $\{(1, 0, -1), (0, 1, -1)\}$
3. $\mathbf{W}$ = span $\{(1, 0, -1), (0, 1, 1)\}$
4. $\mathbf{W}$ = span $\{(1, 0, -1), (1, 0, 1)\}$

## 2 Solution

Using (1.0.1),

$$\mathbf{W_1} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \tag{2.0.1}$$

From (1.0.2),

$$\mathbf{W_2} = \begin{pmatrix} 1 & -1 & 1 \end{pmatrix} \tag{2.0.2}$$

From (1.0.3), we can say that, both the subspaces $\mathbf{W}$ and $\mathbf{W_2}$ of $\mathbf{R^3}$ contains the vector (0,1,1).

$$\mathbf{W} = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{W_2} = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix} \tag{2.0.4}$$

From (2.0.2) and (2.0.4),

$$\mathbf{W_2} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \tag{2.0.5}$$

$$Rank(\mathbf{W_2}) = 2 \tag{2.0.6}$$

Since, rank < 3 and the vectors (1,1,1) and (0,1,1) are linearly independent they span a subspace of $\mathbf{R^3}$. Consider the vector (a,b,c) $\in \mathbf{W} \cap \mathbf{W_1}$

$$\implies \mathbf{W_1} = \begin{pmatrix} 1 & 1 & 1 \\ a & b & c \end{pmatrix} \tag{2.0.7}$$

$$\implies \mathbf{W} = \begin{pmatrix} 0 & 1 & 1 \\ a & b & c \end{pmatrix} \tag{2.0.8}$$

From (1.0.4),
The vector (a,b,c) is orthogonal to (0,1,1).

$$\implies <(a, b, c), (0, 1, 1)>= 0 \tag{2.0.9}$$

$$\implies (a, b, c)^T.(0, 1, 1) = 0 \tag{2.0.10}$$

$$\implies (a, b, c) = (0, 1, -1) \tag{2.0.11}$$

From (2.0.7) and (2.0.11),

$$\mathbf{W_1} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix} \tag{2.0.12}$$

And from (2.0.8) and (2.0.11),

$$\boxed{\mathbf{W} = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix}} \tag{2.0.13}$$

The vectors (0,1,1) and (0,1,-1) linearly independent and the rank($\mathbf{W}$)=2 (< 3), then the vector span subspace of $\mathbf{R^3}$.

Hence,

$$\boxed{\mathbf{W} = span\{(0, 1, -1), (0, 1, 1)\} \implies \mathbf{Ans} : \mathbf{1}} \tag{2.0.14}$$