

Matrix Theory (EE5609)

Assignment-8

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Abstract—This document contains the solution to span of \mathbf{W} which is a subspace of \mathbf{R}^3 .

Download latex-tikz codes from

<https://github.com/EE20RESCH11008/Matrix-Theory/tree/master/Assignment-8>

1 PROBLEM

Consider the subspaces \mathbf{W}_1 and \mathbf{W}_2 of \mathbf{R}^3 given by:

$$\mathbf{W}_1 = \{(\mathbf{x}, \mathbf{y}, \mathbf{z}) \in \mathbf{R}^3 : \mathbf{x} + \mathbf{y} + \mathbf{z} = \mathbf{0}\} \quad (1.0.1)$$

and

$$\mathbf{W}_2 = \{(\mathbf{x}, \mathbf{y}, \mathbf{z}) \in \mathbf{R}^3 : \mathbf{x} - \mathbf{y} + \mathbf{z} = \mathbf{0}\} \quad (1.0.2)$$

If \mathbf{W} is a subspace of \mathbf{R}^3 such that

$$(i) \mathbf{W} \cap \mathbf{W}_2 = \text{span}\{(0, 1, 1)\} \quad (1.0.3)$$

$$(ii) \mathbf{W} \cap \mathbf{W}_1 \text{ is orthogonal to } \mathbf{W} \cap \mathbf{W}_2 \quad (1.0.4)$$

with respect to the usual inner products of \mathbf{R}^3 , then

1. $\mathbf{W} = \text{span}\{(0, 1, -1), (0, 1, 1)\}$
2. $\mathbf{W} = \text{span}\{(1, 0, -1), (0, 1, -1)\}$
3. $\mathbf{W} = \text{span}\{(1, 0, -1), (0, 1, 1)\}$
4. $\mathbf{W} = \text{span}\{(1, 0, -1), (1, 0, 1)\}$

2 SOLUTION

Using (1.0.1),

$$\mathbf{W}_1 = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \quad (2.0.1)$$

From (1.0.2),

$$\mathbf{W}_2 = \begin{pmatrix} 1 & -1 & 1 \end{pmatrix} \quad (2.0.2)$$

From (1.0.3), we can say that, both the subspaces \mathbf{W} and \mathbf{W}_2 of \mathbf{R}^3 contains the vector $(0, 1, 1)$.

$$\mathbf{W} = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix} \quad (2.0.3)$$

$$\mathbf{W}_2 = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix} \quad (2.0.4)$$

From (2.0.2) and (2.0.4),

$$\mathbf{W}_2 = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad (2.0.5)$$

$$\text{Rank}(\mathbf{W}_2) = 2 \quad (2.0.6)$$

Since, $\text{rank} < 3$ and the vectors $(1, 1, 1)$ and $(0, 1, 1)$ are linearly independent they span a subspace of \mathbf{R}^3 .

Consider the vector $(a, b, c) \in \mathbf{W} \cap \mathbf{W}_1$

$$\Rightarrow \mathbf{W}_1 = \begin{pmatrix} 1 & 1 & 1 \\ a & b & c \end{pmatrix} \quad (2.0.7)$$

$$\Rightarrow \mathbf{W} = \begin{pmatrix} 0 & 1 & 1 \\ a & b & c \end{pmatrix} \quad (2.0.8)$$

From (1.0.4),

The vector (a, b, c) is orthogonal to $(0, 1, 1)$

$$\Rightarrow (a, b, c) \cdot (0, 1, 1)^T = 0 \quad (2.0.9)$$

$$\Rightarrow (a, b, c) = (0, 1, -1) \quad (2.0.10)$$

From (2.0.7) and (2.0.10),

$$\mathbf{W}_1 = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix} \quad (2.0.11)$$

And from (2.0.8) and (2.0.10),

$$\mathbf{W} = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix} \quad (2.0.12)$$

The vectors $(0, 1, 1)$ and $(0, 1, -1)$ linearly independent and the $\text{rank}(\mathbf{W})=2 (< 3)$, then the vector span subspace of \mathbf{R}^3 .

Hence,

$$\mathbf{W} = \text{span}\{(0, 1, -1), (0, 1, 1)\} \Rightarrow \text{Ans : 1} \quad (2.0.13)$$