Matrix Theory (EE5609) Assignment-6

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***Abstract*—This document contains the QR decomposition.**

Download latex-tikz codes from

https://github.com/EE20RESCH11008/Matrix−Theory/tree/master/Assignment−6

1 Problem

Find QR decomposition of

$$\mathbf{V} = \begin{pmatrix} 16 & -12 \\ -12 & 9 \end{pmatrix} \tag{1.0.1}$$

1.1 QR Decomposition

Let, the column vectors of $\mathbf{V}$ be $\mathbf{v_1}$ and $\mathbf{v_2}$:

$$\mathbf{v_1} = \begin{pmatrix} 16 \\ -12 \end{pmatrix} \tag{1.1.1}$$

$$\mathbf{v_2} = \begin{pmatrix} -12 \\ 9 \end{pmatrix} \tag{1.1.2}$$

To find $\mathbf{Q} = \begin{pmatrix} \mathbf{u_1} & \mathbf{u_2} \end{pmatrix}$, we will orthonormalize the columns of $\mathbf{V}$ using Gram-Schmidt method:

$$\mathbf{u_1} = \frac{\mathbf{v_1}}{k_1} \tag{1.1.3}$$

$$k_1 = \|\mathbf{v_1}\| = \sqrt{16^2 + (-12)^2} = 20 \tag{1.1.4}$$

$$\implies \mathbf{u_1} = \frac{1}{20}\begin{pmatrix} 16 \\ -12 \end{pmatrix} = \begin{pmatrix} \frac{4}{5} \\ \frac{-3}{5} \end{pmatrix} \tag{1.1.5}$$

$$\mathbf{u_2} = \frac{\mathbf{v_2} - r_1\mathbf{u_1}}{\|\mathbf{v_2} - r_1\mathbf{u_1}\|} \tag{1.1.6}$$

$$r_1 = \frac{\mathbf{u_1}^T\mathbf{v_2}}{\|\mathbf{u_1}\|^2} = \frac{\begin{pmatrix} \frac{4}{5} & \frac{-3}{5} \end{pmatrix}\begin{pmatrix} -12 \\ 9 \end{pmatrix}}{(\frac{4}{5})^2 + (\frac{-3}{5})^2} = -15 \tag{1.1.7}$$

From (1.1.2) (1.1.5) and (1.1.7),

$$\mathbf{u_2} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{1.1.8}$$

$$k_2 = \mathbf{u_2}^T\mathbf{v_2} = 0 \tag{1.1.9}$$

The QR decomposition is given as:

$$\begin{pmatrix} \mathbf{v_1} & \mathbf{v_2} \end{pmatrix} = \begin{pmatrix} \mathbf{u_1} & \mathbf{u_2} \end{pmatrix}\begin{pmatrix} k_1 & r_1 \\ 0 & k_2 \end{pmatrix} \tag{1.1.10}$$

Where,

$$\mathbf{Q} = \begin{pmatrix} \mathbf{u_1} & \mathbf{u_2} \end{pmatrix} \tag{1.1.11}$$

$$\mathbf{R} = \begin{pmatrix} k_1 & r_1 \\ 0 & k_2 \end{pmatrix} \tag{1.1.12}$$

From (1.1.5) (1.1.8) and (1.1.11)

$$\mathbf{Q} = \begin{pmatrix} \frac{4}{5} & 0 \\ \frac{-3}{5} & 0 \end{pmatrix} = \begin{pmatrix} \frac{4}{5} \\ \frac{-3}{5} \end{pmatrix} \tag{1.1.13}$$

From (1.1.4) (1.1.7) (1.1.9) and (1.1.12)

$$\mathbf{R} = \begin{pmatrix} 20 & -15 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 20 & -15 \end{pmatrix} \tag{1.1.14}$$

Substituting the values of (1.1.5) (1.1.8) (1.1.4) (1.1.9) and (1.1.7) in (1.1.10) We get,

$$\boxed{\mathbf{V} = \begin{pmatrix} \frac{4}{5} & 0 \\ \frac{-3}{5} & 0 \end{pmatrix}\begin{pmatrix} 20 & -15 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \frac{4}{5} \\ \frac{-3}{5} \end{pmatrix}\begin{pmatrix} 20 & -15 \end{pmatrix}} \tag{1.1.15}$$