Matrix Theory (EE5609) Assignment-8

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***Abstract*—This document contains the solution to span of W which is a subspace of R^3.**

Download latex-tikz codes from

https://github.com/EE20RESCH11008/Matrix−Theory/tree/master/Assignment−8

1 Problem

Consider the subspaces $\mathbf{W_1}$ and $\mathbf{W_2}$ of $\mathbf{R}^3$ given by:

$$\mathbf{W_1} = \{(\mathbf{x},\mathbf{y},\mathbf{z}) \in \mathbf{R^3} : \mathbf{x}+\mathbf{y}+\mathbf{z} = \mathbf{0}\} \tag{1.0.1}$$

and

$$\mathbf{W_2} = \{(\mathbf{x},\mathbf{y},\mathbf{z}) \in \mathbf{R^3} : \mathbf{x}-\mathbf{y}+\mathbf{z} = \mathbf{0}\} \tag{1.0.2}$$

If $\mathbf{W}$ is a subspace of $\mathbf{R^3}$ such that

$$(i)\mathbf{W} \cap \mathbf{W_2} = \text{span}\{(0,1,1)\} \tag{1.0.3}$$

$$\tag{1.0.4}$$

$$(ii)\mathbf{W} \cap \mathbf{W_1}\text{is orthogonal to}\mathbf{W} \cap \mathbf{W_2} \tag{1.0.5}$$

with respect to the usual inner products of $\mathbf{R^3}$, then

1. $\mathbf{W}$ = span $\{(0,1,-1),(0,1,1)\}$
2. $\mathbf{W}$ = span $\{(1,0,-1),(0,1,-1)\}$
3. $\mathbf{W}$ = span $\{(1,0,-1),(0,1,1)\}$
4. $\mathbf{W}$ = span $\{(1,0,-1),(1,0,1)\}$

2 Solution

Using (1.0.1),

$$\mathbf{W_1} = \begin{pmatrix}1\\1\\1\end{pmatrix} \tag{2.0.1}$$

From (1.0.2),

$$\mathbf{W_2} = \begin{pmatrix}1\\-1\\1\end{pmatrix} \tag{2.0.2}$$

From (1.0.4), we can say that, both the subspaces $\mathbf{W}$ and $\mathbf{W_2}$ of $\mathbf{R^3}$ contains the column vector as follows: .

$$\mathbf{W} = \begin{pmatrix}0\\1\\1\end{pmatrix} \tag{2.0.3}$$

$$\mathbf{W_2} = \begin{pmatrix}0\\1\\1\end{pmatrix} \tag{2.0.4}$$

From (2.0.2) and (2.0.4),

$$\mathbf{W_2} = \begin{pmatrix}1 & 0\\-1 & 1\\1 & 1\end{pmatrix} \tag{2.0.5}$$

$$Rank(\mathbf{W_2}) = 2 \tag{2.0.6}$$

Since, rank $<$ 3 and the vectors are linearly independent they span a subspace of $\mathbf{R^3}$.

Consider the vector,

$$\begin{pmatrix}x\\y\\z\end{pmatrix} \in \mathbf{W} \cap \mathbf{W_1} \tag{2.0.7}$$

From (1.0.4) and (1.0.5),

The vector $\begin{pmatrix}x\\y\\z\end{pmatrix}$ is orthogonal to $\begin{pmatrix}0\\1\\1\end{pmatrix}$.

$$\implies \begin{pmatrix}x & y & z\end{pmatrix}.\begin{pmatrix}0\\1\\1\end{pmatrix} = 0 \tag{2.0.8}$$

$$\implies \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}0\\1\\-1\end{pmatrix} \tag{2.0.9}$$

Since, $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbf{W} \cap \mathbf{W_1}$:

From (2.0.1) and (2.0.9),

$$\mathbf{W_1} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{2.0.10}$$

Also from (2.0.3) and (2.0.9),

$$\boxed{\mathbf{W} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \\ 1 & -1 \end{pmatrix}} \tag{2.0.11}$$

Using (2.0.11),
The vectors linearly independent and rank($\mathbf{W}$)=2 (< 3), then the vector span subspace of $\mathbf{R^3}$.

Hence,

$$\boxed{\mathbf{W} = span\{(0, 1, -1), (0, 1, 1)\} \implies \mathbf{Ans : 1}} \tag{2.0.12}$$