Matrix Theory (EE5609) Assignment-8

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***Abstract*—This document contains the solution to span of W which is a subspace of $\mathbf{R^3}$.**

Download latex-tikz codes from

https://github.com/EE20RESCH11008/Matrix−Theory/tree/master/Assignment−8

1 Problem

Consider the subspaces $\mathbf{W_1}$ and $\mathbf{W_2}$ of $\mathbf{R}^3$ given by:

$$\mathbf{W_1} = \{(\mathbf{x}, \mathbf{y}, \mathbf{z}) \in \mathbf{R^3} : \mathbf{x} + \mathbf{y} + \mathbf{z} = \mathbf{0}\} \tag{1.0.1}$$

and

$$\mathbf{W_2} = \{(\mathbf{x}, \mathbf{y}, \mathbf{z}) \in \mathbf{R^3} : \mathbf{x} - \mathbf{y} + \mathbf{z} = \mathbf{0}\} \tag{1.0.2}$$

If $\mathbf{W}$ is a subspace of $\mathbf{R^3}$ such that

$$(i)\mathbf{W} \cap \mathbf{W_2} = span\{(0, 1, 1)\} \tag{1.0.3}$$

$$(ii)\mathbf{W} \cap \mathbf{W_1} \text{ } is \text{ } orthogonal \text{ } to \text{ } \mathbf{W} \cap \mathbf{W_2} \tag{1.0.4}$$

with respect to the usual inner products of $\mathbf{R^3}$, then

1. $\mathbf{W}$ = span $\{(0, 1, -1), (0, 1, 1)\}$
2. $\mathbf{W}$ = span $\{(1, 0, -1), (0, 1, -1)\}$
3. $\mathbf{W}$ = span $\{(1, 0, -1), (0, 1, 1)\}$
4. $\mathbf{W}$ = span $\{(1, 0, -1), (1, 0, 1)\}$

2 Solution

From (1.0.1),

$$\mathbf{W_1} = \{(\mathbf{x}, \mathbf{y}, \mathbf{z}) = (\mathbf{x}, \mathbf{y}, -\mathbf{x} - \mathbf{y})\} \tag{2.0.1}$$

$$\implies \mathbf{W_1} = span \text{ } \{(1, 0, -1), (0, 1, -1)\} \tag{2.0.2}$$

From (1.0.2),

$$\mathbf{W_2} = \{(\mathbf{x}, \mathbf{y}, \mathbf{z}) = (\mathbf{x}, \mathbf{y}, -\mathbf{x} + \mathbf{y})\} \tag{2.0.3}$$

$$\implies \mathbf{W_2} = span \text{ } \{(1, 0, -1), (0, 1, 1)\} \tag{2.0.4}$$

From (1.0.3),

$$\boxed{\mathbf{W} \cap \mathbf{W_2} \subseteq \mathbf{W} \implies \mathbf{W} = \{(0, 1, 1)\}} \tag{2.0.5}$$

Assume $(\mathbf{x}, \mathbf{y}, \mathbf{z}) \in \mathbf{W} \cap \mathbf{W_1} \subseteq (W)$

Using (1.0.4),

$$\{(\mathbf{x}, \mathbf{y}, \mathbf{z})\} \text{ } is \text{ } orthogonal \text{ } to \text{ } \mathbf{W} \cap \mathbf{W_2} \tag{2.0.6}$$

From (2.0.5),

$$\{(\mathbf{x}, \mathbf{y}, \mathbf{z})\} \text{ } orthogonal \text{ } to \text{ } \{(0, 1, 1)\} \tag{2.0.7}$$

$$\implies (\mathbf{x}, \mathbf{y}, \mathbf{z})(0, 1, 1)^T = 0 \tag{2.0.8}$$

$$\implies \mathbf{y} + \mathbf{z} = 0 \tag{2.0.9}$$

$$\implies \mathbf{y} = -\mathbf{z}. \tag{2.0.10}$$

Now,

$$\mathbf{W} \cap \mathbf{W_1} \subseteq \mathbf{W} = \{(\mathbf{x}, \mathbf{y}, \mathbf{z}) : (\mathbf{x}, \mathbf{y}, -\mathbf{y})\} \tag{2.0.11}$$

$$\boxed{\implies \mathbf{W} = \{(0, 1, -1)\}or\{(1, 1, -1)\}} \tag{2.0.12}$$

Finally from the (2.0.5) and (2.0.12) equations,

$\mathbf{W}$ can be given by:

$$\boxed{\mathbf{W} = span\{(0, 1, -1), (0, 1, 1)\} \implies \mathbf{Ans : 1}} \tag{2.0.13}$$