Matrix Theory (EE5609) Assignment-7

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Abstract—**This document contains proof for the Matrix Multiplication.**

Download the latex-tikz codes from

https://github.com/EE20RESCH11008/Matrix−Theory/tree/master/Assignment−7

1 Problem

If $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are matrices over the field $\mathbf{F}$ such that the products $\mathbf{BC}$ and $\mathbf{A}(\mathbf{BC})$ are defined, then so are the products $\mathbf{AB}, (\mathbf{AB})\mathbf{C}$ and

$$\mathbf{A}(\mathbf{BC}) = (\mathbf{AB})\mathbf{C} \tag{1.0.1}$$

2 Solution

Let the matrices with the dimensions:

$$\mathbf{B} = m \times n \tag{2.0.1}$$

Since $\mathbf{BC}$ defined,

$$\mathbf{C} = n \times p. \tag{2.0.2}$$

Also $\mathbf{A}(\mathbf{BC})$ defined,

$$\mathbf{A} = l \times m. \tag{2.0.3}$$

From (2.0.1) and (2.0.3),

$$\mathbf{AB} = l \times n. \tag{2.0.4}$$

From (2.0.2) and (2.0.4)

$$(\mathbf{AB})\mathbf{C} = l \times p. \tag{2.0.5}$$

From (2.0.4) and (2.0.5),

Thus the products $\mathbf{AB}$ and $(\mathbf{AB})\mathbf{C}$ are exists.

In $\mathbf{A}(\mathbf{BC})$ any element i, j is given by:

$$[\mathbf{A}(\mathbf{BC})]_{ij} = \sum_s \mathbf{A}_{is}(\mathbf{BC})_{sj} \tag{2.0.6}$$
$$= \sum_s \mathbf{A}_{is} \sum_t \mathbf{B}_{st}\mathbf{C}_{tj} \tag{2.0.7}$$
$$= \sum_s \sum_t \mathbf{A}_{is}\mathbf{B}_{st}\mathbf{C}_{tj} \tag{2.0.8}$$
$$= \sum_t \sum_s \mathbf{A}_{is}\mathbf{B}_{st}\mathbf{C}_{tj} \tag{2.0.9}$$
$$= \sum_t \sum_s (\mathbf{A}_{is}\mathbf{B}_{st})\mathbf{C}_{tj} \tag{2.0.10}$$
$$= \sum_t (\mathbf{AB})_{it}\mathbf{C}_{tj} \tag{2.0.11}$$
$$= [(\mathbf{AB})\mathbf{C}]_{ij} \tag{2.0.12}$$

$$\implies [\mathbf{A}(\mathbf{BC})]_{ij} = [(\mathbf{AB})\mathbf{C}]_{ij}. \tag{2.0.13}$$

Hence it is proved that,

$$\boxed{[\mathbf{A}(\mathbf{BC})] = [(\mathbf{AB})\mathbf{C}]} \tag{2.0.14}$$

3 Example:

Let us consider the matrices,

$$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 1 & 2 & 0 \\ 3 & 4 & 1 \end{pmatrix} \mathbf{C} = \begin{pmatrix} 1 & 1 & 1 & 2 \\ 1 & 0 & 1 & 3 \\ 1 & 2 & 1 & 1 \end{pmatrix} \tag{3.0.1}$$

$$\mathbf{BC} = \begin{pmatrix} 1 & 2 & 0 \\ 3 & 4 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 2 \\ 1 & 0 & 1 & 3 \\ 1 & 2 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 1 & 3 & 8 \\ 8 & 5 & 8 & 19 \end{pmatrix} \tag{3.0.2}$$

$$\mathbf{A}(\mathbf{BC}) = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 3 & 1 & 3 & 8 \\ 8 & 5 & 8 & 19 \end{pmatrix} \tag{3.0.3}$$
$$= \begin{pmatrix} 11 & 6 & 11 & 27 \\ 30 & 17 & 30 & 73 \end{pmatrix} \tag{3.0.4}$$

$$\mathbf{AB} = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 3 & 4 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 6 & 1 \\ 11 & 16 & 3 \end{pmatrix} \tag{3.0.5}$$

$$(\mathbf{AB})\mathbf{C} = \begin{pmatrix} 4 & 6 & 1 \\ 11 & 16 & 3 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 2 \\ 1 & 0 & 1 & 3 \\ 1 & 2 & 1 & 1 \end{pmatrix} \tag{3.0.6}$$

$$= \begin{pmatrix} 11 & 6 & 11 & 27 \\ 30 & 17 & 30 & 73 \end{pmatrix} \tag{3.0.7}$$

From (3.0.4) and (3.0.7),

$$\boxed{[\mathbf{A}(\mathbf{BC})] = [(\mathbf{AB})\mathbf{C}]}$$