Probability and statistics

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Abstract—**This document has solutions of problems from Probability and statistics.**

Download python codes from

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svn co https://github.com/krishnajakodali/
    summer20/trunk/prob_stat/prob/codes
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1 Probability

1.1 Examples

1.1.1 Problem 41: If a fair coin is tossed 10 times, find the probability of
(i) exactly six heads
(ii) at least six heads
(iii) at most six heads

1.1.2 Solution: Let X be the random variable denoting the number of times head is obtained when a coin is tossed n times.Then by Binomial distribution,

$$\Pr(X = 1) = p \tag{1.1.1}$$
$$\Pr(X = k) = {}^{n}C_k p^k (1 - p)^{n-k} \tag{1.1.2}$$
$$k = 0, \ldots, n \tag{1.1.3}$$

For the given problem, $n = 10$ and $p = 1 - p = \frac{1}{2}$ for a fair coin
(i)To calculate probability for exactly six heads subsititute k=6 in equation (1.1.3),

$$\Pr(X = 6) = {}^{10}C_6 \frac{1}{2}^{10} \tag{1.1.4}$$
$$= \frac{105}{512} \tag{1.1.5}$$

(ii)Using (1.1.3), Probability of obtaining atleast six heads is ,

$$\Pr(X \geq 6) = \Pr(X = 6) + \Pr(X = 7) + \Pr(X = 8) + \tag{1.1.6}$$
$$\Pr(X = 9) + \Pr(X = 10) \tag{1.1.7}$$
$$\implies \frac{1}{2}^{10} \left({}^{10}C_6 + {}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}\right) \tag{1.1.8}$$
$$= \frac{193}{512} \tag{1.1.9}$$

(iii)Probability of obtaining atmost six heads is ,

$$\Pr(X \leq 6) = 1 - \Pr(X \geq 6) + \Pr(X = 6) \tag{1.1.10}$$

Substituting (1.1.5) and (1.1.5),

$$\Pr(X \leq 6) = 1 - \frac{193}{512} + \frac{105}{512} \tag{1.1.11}$$
$$= \frac{53}{64} \tag{1.1.12}$$

The python code for the above problem is,

```
./prob/codes/exam41.py
```

Experimental probability is calculated using the number of heads obtained in each of the 1,000,000 random experiments of tossing of 10 coins. The code compares the experimental probability to the theoretical probability.As number of experiments increase, the experimental probability approaches the theoritical probability.

1.1.3 Problem 42: Ten eggs are drawn successively with replacement from a lot containing 10% defective eggs. Find the probability that there is at least one defective egg.

1.1.4 Solution: Let X be the random variable representing the number of defective eggs from the ten eggs picked.X follows binomial distribution. Probability that there is atleast one defective egg is

$$\Pr(X \geq 1) = 1 - \Pr(X = 0) \tag{1.1.13}$$

Substituting n=10,p= 0.1 and k=0 in equation (1.1.3),

$$\Pr(X \geq 1) = 1 - {}^{10}C_0 0.1^0 0.9^{10} \tag{1.1.14}$$
$$= 1 - \frac{9}{10}^{10} \tag{1.1.15}$$
$$= 0.6513215599 \tag{1.1.16}$$

The python code for the above problem is,

```
./prob/codes/exam42.py
```

1.1.5 Problem 43: Coloured balls are distributed in four boxes as shown in the following table:
A box is selected at random and then a ball is randomly drawn from the selected box. The colour of the ball is black, what is the probability that ball drawn is from the box III?

Box	Black	White	Red	Blue
I	3	4	5	6
II	2	2	2	2
III	1	2	3	1
IV	4	3	1	5

TABLE I: Distribution of the balls in the boxes

1.1.6 Solution: Given that a black ball is selected, the probability that it is picked from box III is the case of conditional probability expressed as

$$\Pr(III|B) = \frac{\Pr(III \cap B)}{\Pr(B)} \tag{1.1.17}$$

By the definition of conditional probability

$$\Pr(B|III) = \frac{\Pr(III \cap B)}{\Pr(III)} \tag{1.1.18}$$
$$\Pr(III \cap B) = \Pr(B|III)\Pr(III) \tag{1.1.19}$$

Also

$$\implies \Pr(B) = \Pr(I \cap B) + \Pr(II \cap B) \tag{1.1.20}$$
$$+ \Pr(III \cap B) + \Pr(IV \cap B) \tag{1.1.21}$$
$$\implies \Pr(B) = \Pr(B|I)\Pr(I) + \Pr(B|II)\Pr(II) + \tag{1.1.22}$$
$$\Pr(B|III)\Pr(III) + \Pr(B|IV)\Pr(IV) \tag{1.1.23}$$

Substituting (1.1.19) and (1.1.23) in (1.1.17), We obtain the Baye's theorm as stated in (1.1.24)

$$\Pr(III|B) = \frac{\Pr(B|III)\Pr(III)}{\Pr(B|I)\Pr(I) + \Pr(B|II)\Pr(II) + \Pr(B|III)\Pr(III} \tag{1.1.24}$$

From table I,

$$\Pr(B|I) = \frac{1}{6} \tag{1.1.25}$$
$$\Pr(B|II) = \frac{1}{4} \tag{1.1.26}$$
$$\Pr(B|III) = \frac{1}{7} \tag{1.1.27}$$
$$\Pr(B|IV) = \frac{4}{13} \tag{1.1.28}$$
$$\Pr(I) = \Pr(II) = \Pr(III) = \Pr(IV) \tag{1.1.29}$$

Substituting the above values in equation (1.1.24),

$$\Pr(III|B) = \frac{156}{947} \tag{1.1.30}$$

The python code for the above problem is,

```
./prob/codes/exam43.py
```

1.1.7 Problem 44: Find the mean of the Binomial distribution B(4,$\frac{1}{3}$)

1.1.8 Solution: Let X be the random variable following Binomial distribution.Then,

$$\Pr(X = 1) = p, \tag{1.1.31}$$
$$\Pr(X = k) = {}^nC_k p^k (1-p)^{n-k}, \quad k = 0, \dots, n \tag{1.1.32}$$

Here $p = \frac{1}{3}$ and $n = 4$ Mean is given as

$$\bar{X} = \sum_{k=0}^{n} k \Pr(X = k) \tag{1.1.33}$$
$$\bar{X} = \sum_{k=0}^{4} k\,{}^4C_k p^k (1-p)^{4-k} \tag{1.1.34}$$
$$\bar{X} = \frac{1}{3} \tag{1.1.35}$$

The python code for the above problem is,

```
./prob/codes/exam44.py
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1.1.9 Problem 45: The probability of a shooter hitting a target is $\frac{3}{4}$. How many minimum number of times must he/she fire so that the probability of hitting the target at least once is more than 0.99?

1.1.10 Solution: Let X be the random variable representing the number of times the shooter hits the target. Let n be the total number of times that the shoter fires.Then

$$X\epsilon\{0, 1, 2....n\} \tag{1.1.36}$$

The shooter hits the target with a probability of $\frac{3}{4}$. X follows binomial distribution with parameters as n and $p = \frac{3}{4}$ Using equation (1.1.3) probability of hitting target atleast once is

$$\Pr(X \geq 1) = 1 - \Pr(X = 0) \tag{1.1.37}$$

$$\implies \Pr(X \geq 1) = 1 - {}^{n}C_{0}\frac{1}{4}^{n} \tag{1.1.38}$$

$$\implies 1 - \frac{1}{4}^{n} \geq 0.99 \tag{1.1.39}$$

$$\implies \frac{1}{4}^{n} \leq 0.01 \tag{1.1.40}$$

$$\implies n = 4 \tag{1.1.41}$$

The python code for the above problem is,

```
./prob/codes/exam45.py
```

1.1.11 Problem 46: A and B throw a die alternatively till one of them gets a '6' and wins the game. Find their respective probabilities of winning, if A starts first.

1.1.12 Solution: let X be the random variable representing the result when a die is thrown

$$X\epsilon\{0, 1, 2, 3, 4, 5, 6\} \tag{1.1.42}$$

All results are equally likely in a fair die. Hence

$$\Pr(X = 6) = \frac{1}{6} \tag{1.1.43}$$

$$\Pr(X \neq 6) = \frac{5}{6} \tag{1.1.44}$$

For A to win the game at k-th throw, A should throw a 6 in the k-th throw and both A and B must not throw a six in the preceding $(k - 1)$ throws.So probability of A winning after k thows is given as

$$\Pr(A_k) = \left(\frac{5}{6}\right)^{k-1}\frac{1}{6} \tag{1.1.45}$$

$$k\epsilon\{1, 2....\infty\} \tag{1.1.46}$$

So the total probability of A winning is given as

$$\Pr(A) = \sum_{k=0}^{\infty}\Pr(A_k) \tag{1.1.47}$$

$$\Pr(A) = \frac{1}{6} + \left(\frac{5}{6}\right)^{2}\frac{1}{6} + \left(\frac{5}{6}\right)^{4}\frac{1}{6} + ... \tag{1.1.48}$$

$$\Pr(A) = \frac{1}{6} \times \left(1 + \left(\frac{5}{6}\right)^{2} + \frac{5}{6}^{4} + ...\right) \tag{1.1.49}$$

$$\Pr(A) = \frac{1}{6} \times \frac{1}{1 - \left(\frac{5}{6}\right)^{2}} \tag{1.1.50}$$

$$\Pr(A) = \frac{6}{11} \tag{1.1.51}$$

Similarly probability of B winning is given as

$$\Pr(B) = \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^{3}\frac{1}{6} + \left(\frac{5}{6}\right)^{5}\frac{1}{6} + ... \tag{1.1.52}$$

$$\Pr(B) = \frac{1}{6} \times \frac{5}{6} \times \left(1 + \left(\frac{5}{6}\right)^{2} + \left(\frac{5}{6}\right)^{4} + ...\right) \tag{1.1.53}$$

$$\Pr(B) = \frac{5}{11} \tag{1.1.54}$$

The python code for the above problem is,

```
./prob/codes/exam46.py
```

In the above code 1000000 random outputs of a die are generated for A and B each.The probabilities are calculated using the total number of times A gets a six first and the total number of times B get a six first.

1.1.13 Problem 47: If a machine is correctly set up, it produces 90% acceptable items. If it is incorrectly set up, it produces only 40% acceptable items. Past experience shows that 80% of the set ups are correctly done. If after a certain set up, the machine produces 2 acceptable items, find the probability that the machine is correctly setup.

1.1.14 Solution: Let A: Event that machine produces 2 acceptable items
C: Event that a machine is correctly set up
I: Event that a machine is incorrectly set up
The probability that a machine that produced 2 acceptable items is correctly setup is

$$\Pr(C|A) \tag{1.1.55}$$

Using Bayes theorm (1.1.24),

$$\Pr(C|A) = \frac{\Pr(A|C)\Pr(C)}{\Pr(A|C)\Pr(C) + \Pr(A|I)\Pr(I)} \quad (1.1.56)$$

Probability of prducing two acceptable items when machine is correctly set up is

$$\Pr(A|C) = \left(\frac{9}{10}\right)^2 \quad (1.1.57)$$

Probability of prducing two acceptable items when machine is incorrectly set up is

$$\Pr(A|I) = \left(\frac{4}{10}\right)^2 \quad (1.1.58)$$

Also using the given data

$$\Pr(C) = \frac{8}{10} \quad (1.1.59)$$

$$\Pr(I) = 1 - \frac{8}{10} = \frac{2}{10} \quad (1.1.60)$$

Substituting the above values in (1.1.56)

$$\Pr(C|A) = \frac{81}{85} \quad (1.1.61)$$

The python code for the above problem is,

```
./prob/codes/exam47.py
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1.1.15 Problem 48: Find the probability of getting a head when a coin is tossed once. Also find the probability of getting a tail.

1.1.16 Solution: Let X be the outcome of tossing a coin. Then,

$$X \epsilon \{H, T\} \quad (1.1.62)$$

$$\implies S = 2 \quad (1.1.63)$$

probability of getting a head is

$$\Pr(H) = \frac{H}{S} = \frac{1}{2} \quad (1.1.64)$$

Similarly probability of getting a tail is

$$\Pr(T) = \frac{T}{S} = \frac{1}{2} \quad (1.1.65)$$

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./prob/codes/exam48.py
```

1.1.17 Problem 49: A bag contains a red ball, a blue ball and a yellow ball, all the balls being of the same size.Kritika takes out a ball from the bag without looking into it. What is the probability that she takes out the (i) yellow ball?
(ii) red ball?
(iii) blue ball?

1.1.18 Solution: The sample size

$$S = 3 \quad (1.1.66)$$

(i)The number of yellow balls are

$$Y = 1 \quad (1.1.67)$$

The probability that a yellow ball is taken out is

$$\Pr(Y) = \frac{Y}{S} = \frac{1}{3} \quad (1.1.68)$$

(ii)The number of red balls are

$$R = 1 \quad (1.1.69)$$

The probability that a yellow ball is taken out is

$$\Pr(R) = \frac{R}{S} = \frac{1}{3} \quad (1.1.70)$$

(iii)The number of blue balls are

$$B = 1 \quad (1.1.71)$$

The probability that a yellow ball is taken out is

$$\Pr(B) = \frac{B}{S} = \frac{1}{3} \quad (1.1.72)$$

The python code for the distribution is

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./prob/codes/exam49.py
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1.1.19 Problem 50: Suppose we throw a die once. (i) What is the probability of getting a number greater than 4 ? (ii) What is the probability of getting a number less than or equal to 4 ?

1.1.20 Solution: Let X be the outcome of the dice. Then,

$$X \epsilon \{1, 2, 3, 4, 5, 6\} \quad (1.1.73)$$

$$\implies S = 6 \quad (1.1.74)$$

For a fair dice,

$$\Pr(X = k) = \begin{cases} \frac{1}{6} & k = 1, 2, 3, 4, 5, 6 \\ 0 & otherwise \end{cases} \quad (1.1.75)$$

(i)Probability of getting a number greater than 4 is

$$\Pr(X > 4) = \Pr(X = 5) + \Pr(X = 6) \quad (1.1.76)$$

$$\implies 2 \times \frac{1}{6} = \frac{1}{3} \quad (1.1.77)$$

(i)Probability of getting a number greater than 4 is

$$\Pr(X \leq 4) = 1 - \Pr(X > 4 =) = \frac{2}{3} \tag{1.1.78}$$

./prob/codes/exam50.py

1.2 Exercise

1.2.1 Problem 121: Gopi buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fish and 8 female fish.What is the probability that the fish taken out is a male fish?

1.2.2 Solution: The sample size is the total number of fishes in the tank

$$S = 5 + 8 = 13 \tag{1.2.1}$$

The number male fishes

$$M = 5 \tag{1.2.2}$$

The probability of gopi picking up a male fish is

$$\Pr(M) = \frac{M}{S} = \frac{5}{13} \tag{1.2.3}$$

The python code for the distribution is

./prob/codes/fish.py

The code checks how many times a male fish is picked out of the total times (taken as 100,000 in the given code) a fish is picked up from the tank with replacement.

1.2.3 Problem 122: A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 , and these are equally likely outcomes. What is the probability that it will point at
(i) 8 ?
(ii) an odd number?
(iii) a number greater than 2?
(iv) a number less than 9?

1.2.4 Solution: Let X be the random variable representing the number that arrow points

$$X \epsilon \{1, 2, 3, 4, 5, 6, 7, 8\} \tag{1.2.4}$$

Since all events are equally likely,

$$\Pr(X = x) = \begin{cases} \frac{1}{8} & x = 1, 2, 3, 4, 5, 6, 7, 8 \\ 0 & otherwise \end{cases} \tag{1.2.5}$$

(i)The probability that the outcome is 8 is

$$\Pr(X = 8) = \frac{1}{8} \tag{1.2.6}$$

(ii)Probability of occurance of odd numbers is

$$\Pr(X = 1) + \Pr(X = 3) + \Pr(X = 5) + \Pr(X = 7) \tag{1.2.7}$$

$$\implies 4 \times \frac{1}{8} = \frac{4}{8} = \frac{1}{2} \tag{1.2.8}$$

(iii)Probability of arrow pointing at a number greater than 2 is

$$\Pr(X > 2) = 1 - \Pr(X < 2) \tag{1.2.9}$$

$$\implies 1 - (\Pr(X = 1) + \Pr(X = 2)) \tag{1.2.10}$$

$$\implies 1 - \frac{2}{8} = \frac{3}{4} \tag{1.2.11}$$

(iv) Probability of arrow pointing at a value less than 9 is

$$\Pr(X < 9) = \frac{8}{8} = 1 \tag{1.2.12}$$

The python code for the distribution is

./prob/codes/chance.py

The above code checks occurance of each of these events when the arrow is spinned 100,000 times.

1.2.5 Problem 123: A die is thrown once. Find the probability of getting
(i) a prime number;
(ii) a number lying between 2 and 6;
(iii) an odd number.

1.2.6 Solution: Let X be the random variable representing the outcome when the dice is thrown

$$X \epsilon \{1, 2, 3, 4, 5, 6\} \tag{1.2.13}$$

Since all events are equally likely,

$$\Pr(X = x) = \begin{cases} \frac{1}{6} & x = 1, 2, 3, 4, 5, 6 \\ 0 & otherwise \end{cases} \tag{1.2.14}$$

(i)The probability that the outcome is a prime number is

$$\Pr(X = 2) + \Pr(X = 3) + \Pr(X = 5) \tag{1.2.15}$$

$$\implies 3 \times \frac{1}{6} = \frac{1}{2} \tag{1.2.16}$$

(ii)Probability of occurance of number between 2 and 6 is

$$\Pr(X = 3) + \Pr(X = 4) + \Pr(X = 5) \quad (1.2.17)$$

$$\implies 3 \times \frac{1}{6} = \frac{1}{2} \quad (1.2.18)$$

(iii)Probability of occurance of odd number is

$$\Pr(X = 1) + \Pr(X = 3) + \Pr(X = 5) \quad (1.2.19)$$

$$\implies 3 \times \frac{1}{6} = \frac{1}{2} \quad (1.2.20)$$

The python code for the distribution is

```
./prob/codes/dice123.py
```

The above code checks number of times each of the above events occur when the dice is thrown 100,000 times.

1.2.7 Problem 125: One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting (i) a king of red colour
(ii) a face card
(iii) a red face card
(iv) the jack of hearts
(v) a spade
(vi) the queen of diamonds

1.2.8 Solution: The sample size = total number of cards ina deck

$$S = 52 \quad (1.2.21)$$

(i) Number of kings of red color in a deck

$$E_1 = 2 \quad (1.2.22)$$

The probability of drawing a king of red colour

$$\Pr(E_1) = \frac{E_1}{S} = \frac{2}{52} \quad (1.2.23)$$

$$= \frac{1}{23} \quad (1.2.24)$$

(ii) Number of face cards in a deck

$$E_2 = 12 \quad (1.2.25)$$

The probability of drawing a face card is

$$\Pr(E_2) = \frac{E_2}{S} = \frac{12}{52} \quad (1.2.26)$$

$$= \frac{3}{13} \quad (1.2.27)$$

(iii) Number of face cards of red color in a deck

$$E_3 = 6 \quad (1.2.28)$$

The probability of drawing a red face card from the deck is

$$\Pr(E_3) = \frac{E_3}{S} = \frac{6}{52} \quad (1.2.29)$$

$$= \frac{3}{23} \quad (1.2.30)$$

(iv) Number of jacks of hearts in a deck

$$E_4 = 1 \quad (1.2.31)$$

The probability of drawing a jack of hearts is

$$\Pr(E_4) = \frac{E_4}{S} = \frac{1}{52} \quad (1.2.32)$$

(v) Number of spades in a deck

$$E_5 = 13 \quad (1.2.33)$$

The probability of drawing a spade is

$$\Pr(E_5) = \frac{E_5}{S} = \frac{13}{52} \quad (1.2.34)$$

$$= \frac{1}{4} \quad (1.2.35)$$

(v) Number of queens of diamond in a deck

$$E_6 = 1 \quad (1.2.36)$$

The probability of drawing a queen of diamond is

$$\Pr(E_6) = \frac{E_6}{S} = \frac{1}{52} \quad (1.2.37)$$

The python code for the distribution is

```
./prob/codes/cards125.py
```

1.2.9 Problem 126: Five cards—the ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random.
(i) What is the probability that the card is the queen?
(ii) If the queen is drawn and put aside, what is the probability that the second card picked up is (a) an ace? (b) a queen?

1.2.10 Solution: (i)The sample size is equal to number of cards

$$S = 5 \quad (1.2.38)$$

number of queens in the cards are

$$Q = 1 \quad (1.2.39)$$

The probability that a queen is picked is

$$\Pr(Q) = \frac{Q}{S} = \frac{1}{5} \tag{1.2.40}$$

(ii) After a queen is drawn and put aside, the new sample space is

$$S' = 4 \tag{1.2.41}$$

(a)number of aces in the remaining cards are

$$A = 1 \tag{1.2.42}$$

The probability that an ace is picked is

$$\Pr(A) = \frac{A}{S'} = \frac{1}{4} \tag{1.2.43}$$

(a)number of queens in the remaining cards are

$$Q' = 0 \tag{1.2.44}$$

The probability that a queen is picked from the remaining cards is

$$\Pr(Q') = \frac{Q'}{S'} = 0 \tag{1.2.45}$$

The python code below calculates the above probabilities for 100000 picks

```
./prob/codes/cards126.py
```

1.2.11 Problem 127: 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.

1.2.12 Solution: The sample size

$$S = 132 + 12 = 144 \tag{1.2.46}$$

The number of good pens is

$$G = 132 \tag{1.2.47}$$

The probability of taking out a good pen is

$$\Pr(G) = \frac{G}{S} = \frac{132}{144} \tag{1.2.48}$$

$$= \frac{11}{12} \tag{1.2.49}$$

The python code for the above solution is

```
./prob/codes/pens127.py
```

1.2.13 Problem 128: (i) A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective?
(ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective ?

1.2.14 Solution: (i)The sample size

$$S = 20 \tag{1.2.50}$$

The number of defective bulbs is

$$D = 4 \tag{1.2.51}$$

The probability of drawing a defective bulb is

$$\Pr(D) = \frac{D}{S} = \frac{4}{20} \tag{1.2.52}$$

$$= \frac{1}{5} \tag{1.2.53}$$

(ii)After drawing a non defective bulb The new sample size

$$S' = 19 \tag{1.2.54}$$

The number of non-defective bulbs in remaining lot is

$$N = 20 - 4 - 1 = 15 \tag{1.2.55}$$

The probability of drawing a non-defective bulb is

$$\Pr(N) = \frac{N}{S'} = \frac{15}{19} \tag{1.2.56}$$

The python code for the above solution is

```
./prob/codes/exer128.py
```

1.2.15 Problem 129: A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears (i) a two-digit number (ii) a perfect square number (iii) a number divisible by 5.

1.2.16 Solution: (i)The sample size

$$S = 90 \tag{1.2.57}$$

(i)number of discs bearing a two digit number is

$$T = 81 \tag{1.2.58}$$

The probability of drawing a disc bearing two digit number is

$$\Pr(T) = \frac{T}{S} = \frac{81}{90} \tag{1.2.59}$$
$$= \frac{9}{10} \tag{1.2.60}$$

(ii)number of discs bearing a perfect square is

$$Sq = 9 \tag{1.2.61}$$

The probability of drawing a disc bearing perfect square is

$$\Pr(Sq) = \frac{Sq}{S} = \frac{9}{90} \tag{1.2.62}$$
$$= \frac{1}{10} \tag{1.2.63}$$

(iii)number of discs bearing number divisible by 5 is

$$F = 18 \tag{1.2.64}$$

The probability of drawing a disc bearing number divisible by 5 is

$$\Pr(F) = \frac{F}{S} = \frac{18}{90} \tag{1.2.65}$$
$$= \frac{1}{5} \tag{1.2.66}$$

The python code for the above solution is

```
./prob/codes/exer129.py
```

1.2.17 Problem 130: A child has a die whose six faces show the letters as given below: The die is thrown once. What is the probability of getting (i) A? (ii) D?

1.2.18 Solution: The sample size= total faces of a die

$$S = 6 \tag{1.2.67}$$

(i)number of faces on which letter A appears

$$A = 2 \tag{1.2.68}$$

The probability of getting an A

$$\Pr(A) = \frac{A}{S} = \frac{2}{6} \tag{1.2.69}$$
$$= \frac{1}{3} \tag{1.2.70}$$

(ii)number of faces on which letter D appears

$$D = 1 \tag{1.2.71}$$

The probability of getting an A

$$\Pr(D) = \frac{D}{S} = \frac{1}{6} \tag{1.2.72}$$

The python code for the above solution is

```
./prob/codes/exer130.py
```

2 Statistics

2.1 Exercise

2.1.1 Problem 32: A study was conducted to find out the concentration of sulphur dioxide in the air in parts per million (ppm) of a certain city. The data obtained for 30 days is as follows: (i)Make a grouped frequency distribution table for this data with class intervals as 0.00-0.04, 0.04-0.08, and so on.
(ii) For how many days, was the concentration of sulphur dioxide more than 0.11 parts per million?

0.03	0.08	0.04
0.16	0.02	0.18
0.11	0.12	0.22
0.08	0.1	0.09
0.11	0.05	0.01
0.08	0.09	0.17
0.05	0.06	0.2
0.08	0.13	0.07
0.01	0.06	0.18
0.07	0.07	0.04

TABLE II: Concentrations of sulphur dioxide in air in ppm for 30 days

2.1.2 Solution: Least value=0.01
Greatest value=0.23
class interval=0.04
The grouped frequency distribution table III is construced using the python code

```
./stat/codes/exer32.py
```

From the table II, The sulphur dioxide concentration was greater than 0.11ppm for 8 days.

2.1.3 Problem 33: Three coins were tossed 30 times simultaneously. Each time the number of heads occurring was noted down as follows:
Prepare a frequency distribution table for the data given above.

Class	frequency
0.0-0.04	4
0.04-0.08	9
0.08-0.12	9
0.12-0.16	2
0.16-0.2	4
0.2-0.24	2

TABLE III: Grouped frequency distribution table for the data in II

0	1	2	2	1
1	3	1	1	2
3	0	0	1	1
2	3	1	3	0
2	0	1	2	1
2	3	2	2	0

TABLE IV: Number of heads obtained when 3 coins were tossed 30 times

2.1.4 Solution: The possible values are 0,1,2,3. The frequency distribution table is given in table V. The frequency distribution table V is constructed

no of heads	Frequency
0	6
1	10
2	9
3	5
Total	30

TABLE V: Frequency distribution table for the data in IV

using the python code

```
./stat/codes/exer33.py
```

2.1.5 Problem 34: The value of π upto 50 decimal places is given below: 3.14159265358979323846264338327950288419716939937510
(i)Make a frequency distribution of the digits from 0 to 9 after the decimal point.
(ii) What are the most and the least frequently occurring digits?

2.1.6 Solution: The 50 digits appearing after the decimal point of π are tabulated as follows:
The frequency distribution table for the digits 0-9

1	4	1	5	9	2	6	5	3	5
8	9	7	9	3	2	3	8	4	6
2	6	4	3	3	8	3	2	7	9
5	0	2	8	8	4	1	9	7	1
6	9	3	9	9	3	7	5	1	0

TABLE VI: The 50 decimal places of π

is given in table VII

no of heads	Frequency
0	2
1	5
2	5
3	8
4	4
5	5
6	4
7	4
8	5
9	8
Total	50

TABLE VII: Frequency distribution table for the numbers in VI

From the table VII,
0 is the least frequent digit
3,9 are the most frequent digits.
The python code for above problem is

```
./stat/codes/exer34.py
```

2.1.7 Problem 35: Thirty children were asked about the number of hours they watched TV programmes in the previous week. The results were found as follows:
(i) Make a grouped frequency distribution table for this data,taking class width 5 and one of the class intervals as 5-10.
(ii) How many children watched television for 15 or more hours a week?

2.1.8 Solution: Maximum number of hours=17
class interval=5
Since one of the class intervals must be 5-10 the

1	6	2	3	5	12
5	8	4	8	10	3
4	12	2	8	15	1
17	6	3	2	8	5
9	6	8	7	14	12

TABLE VIII: Number of hours 30 children spent watching TV in a week

class intervals are taken as 0-5,5-10,10-15,15-20. The grouped frequency distribution table IX is con-

Class	frequency
0-5	10
5-10	13
10-15	5
15-20	2
Total:	30

TABLE IX: Grouped frequency distribution table for the data in TableVIII

strucd using the python code

```
./stat/codes/exer35.py
```

From the table IX, 2 children watched TV for more than or equal to 15 hours a week.

2.1.9 Problem 36: A company manufactures car batteries of a particular type. The lives (in years) of 40 such batteries were recorded as follows:

Construct a grouped frequency distribution table

2.6	3	3.7	3.2	2.2
4.1	3.5	4.5	3.5	2.3
3.2	3.4	3.8	3.2	4.6
3.7	2.5	4.4	3.4	3.3
2.9	3	4.3	2.8	3.5
3.2	3.9	3.2	3.2	3.1
3.7	3.4	4.6	3.8	3.2
2.6	3.5	4.2	2.9	3.6

TABLE X: Lives of 40 batteries

for this data, using class intervals of size 0.5 starting from the interval 2 - 2.5.

2.1.10 Solution: Starting interval=2-2.5
class interval=0.5
Greatest value=4.6
Therefor end interval=4.5-5

The grouped frequency distribution table XI is

Class	frequency
2.0-2.5	2
2.5-3.0	6
3.0-3.5	14
3.5-4.0	11
4.0-4.5	4
4.5-5.0	3
Total	40

TABLE XI: Grouped frequency distribution table for car-battery lives

constructed using the python code

```
./stat/codes/exer36.py
```

2.1.11 Problem 37: A survey conducted by an organisation for the cause of illness and death among the women between the ages 15 - 44 (in years) worldwide, found the following figures (in %):

(i) Represent the information given above

Causes	**Fatality rate(%)**
Reproductive health cond.	31.8
Neuropsychiatric cond.	25.4
Injuries	12.4
Cardiovascular conditions	4.3
Respiratory conditions	4.1
Other causes	22

TABLE XII: Illness and fatality rate amongst women

graphically.
(ii) Which condition is the major cause of women's ill health and death worldwide?
(iii) Try to find out, with the help of your teacher, any two factors which play a major role in the cause in (ii) above being the major cause.

2.1.12 Solution: (i) The bar graph representing the above data is shown in figure 1

(ii)Reproductive health conditions is the major cause of women's illness and death.

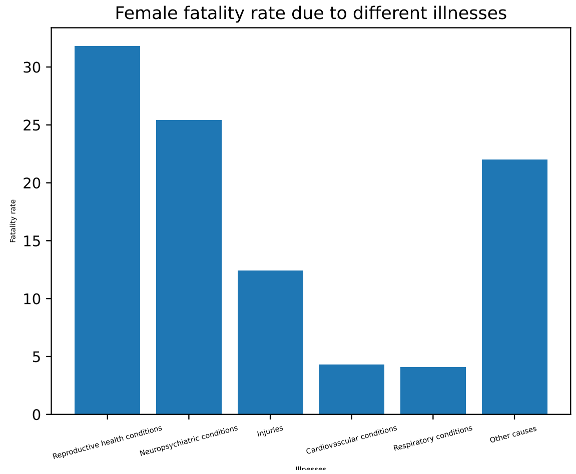

Fig. 1: Illnesses and respective fatality rates amongst women

(iii)Lack of awareness and understanding about reproductive health results in high female fatality rate due to reproductive health conditions.
The python code used to generate the bar graph 1 is

```
./stat/codes/exer37.py
```

2.1.13 Problem 38: The following data on the number of girls (to the nearest ten) per thousand boys in different sections of Indian society is given below.
(i) Represent the information above by a bar

Section	**girls/1000 boys**
Scheduled caste (SC)	940
Scheduled Tribe (ST)	970
Non-SC/ST	920
Backward districts	950
Non-backward district	920
Rural	930
Urban	910

TABLE XIII: Number of girls per 1000 boys

graph.
(ii) In the classroom discuss what conclusions can be arrived at from the graph.

2.1.14 Solution: The bar graph representing the above data is shown in figure 2
The python code for the above problem is

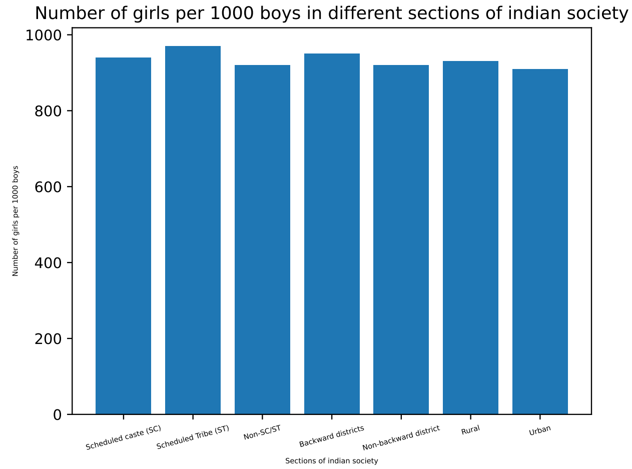

Fig. 2: Number of girls per 1000 boys

```
./stat/codes/exer38.py
```

2.1.15 Problem 39: Given below are the seats won by different political parties in the polling outcome of a state assembly elections:
(i) Draw a bar graph to represent the polling

Political party	**Seats won**
A	75
B	55
C	37
D	29
E	10
F	37

TABLE XIV: Illness and fatality rate amongst women

results.
(ii) Which political party won the maximum number of seats?

2.1.16 Solution: (i) The bar graph representing the above data is shown in figure 3
(ii)From the graph in 3, Political party A won the maximum seats The python code used to generate the bar graph for the above problem is

```
./stat/codes/exer39.py
```

2.1.17 Problem 40: The length of 40 leaves of a plant are measured correct to one millimetre, and the obtained data is represented in the following

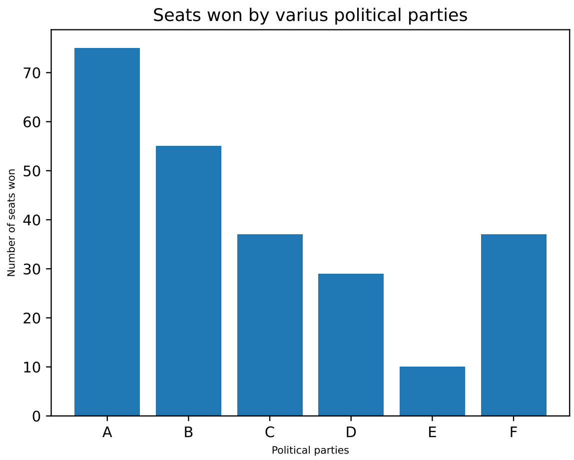

Fig. 3: Seats won by different political parties

table:
(i) Draw a histogram to represent the given data.

Length (mm)	No. of leaves
118-126	3
127-135	5
136-144	9
145-153	12
154-162	5
163-171	4
172-180	2

TABLE XV: Lengths of 40 leaves in mm

[Hint: First make the class intervals continuous]
(ii) Is there any other suitable graphical representation for the same data?
(iii) Is it correct to conclude that the maximum number of leaves are 153 mm long?
why?

2.1.18 Solution: (i)To draw a histogram, the data must be made continuous.

$$Gap/2 = \frac{127 - 126}{2} = \frac{1}{2} = 0.5 \tag{2.1.1}$$

$$\tag{2.1.2}$$

So we add 0.5 to every upperclass limit and subtract 0.5 from every lower class limit to obtain continous grouped frequency distribution table as shown in TableXVI The histogram is shown in figure 4 The below python code was used o generate the

Length (mm)	No. of leaves
117.5-126.5	3
126.5-135.5	5
135.5-144.5	9
144.5-153.5	12
153.5-162.5	5
162.5-171.5	4
171.5-180.5	2

TABLE XVI: Illness and fatality rate amongst women

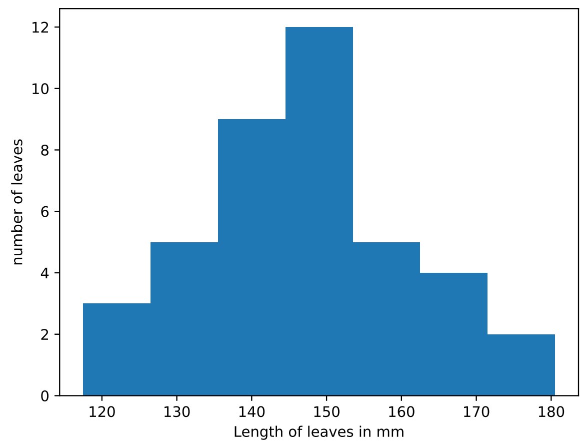

Fig. 4: Length of leaves in mm

histogram 4

```
./stat/codes/exer40.py
```

(ii)The same data can also be represented using a frequency polygon.
(iii)No, the maximum leaves have lengths ranging from 144.5mm to 153.5mm and might not be equal to 153mm.

2.1.19 Problem 41: The following table gives the life times of 400 neon lamps:
(i) Represent the given information with the help of a histogram.
(ii) How many lamps have a life time of more than 700 hours?

2.1.20 Solution: (i)The data is already continuous. The histogram is created using the following python code

Life time in hrs	**No. of lamps**
300-400	14
400-500	56
500-600	60
600-700	86
700-800	74
800-900	62
900-1000	48
Total	**400**

TABLE XVII: Lives of neon lamps

```
./stat/codes/exer41.py
```

The histogram is shown in figure 5 (ii)184 lamps

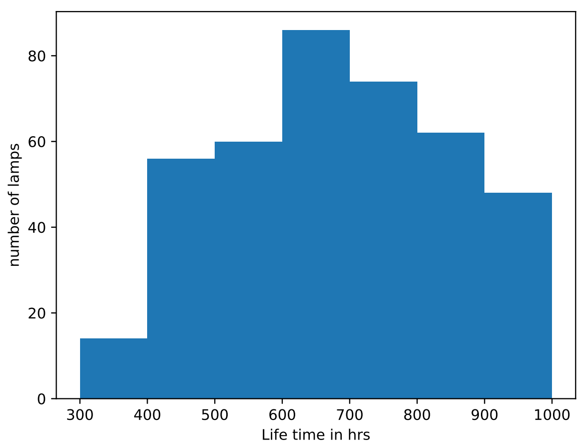

Fig. 5: Lives of neon lamps

have a life time of more than 700 hours.