Probability And Statistics

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***Abstract*—A document implementing solutions to problems based on probability and statistics.**

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1 Question 1.1.21

1.1 Problem

1.1. An unbiased die is thrown twice. Let the event A be "odd number on the first throw" and B the event "odd number on the second throw". Check the independence of the events A and B.

The sample size = Total number of possibilities(S)=

$$\begin{pmatrix} \begin{Bmatrix}1 & 1\end{Bmatrix} & \begin{Bmatrix}1 & 2\end{Bmatrix} & \begin{Bmatrix}1 & 3\end{Bmatrix} & \begin{Bmatrix}1 & 4\end{Bmatrix} & \begin{Bmatrix}1 & 5\end{Bmatrix} & \begin{Bmatrix}1 & 6\end{Bmatrix} \\ \begin{Bmatrix}2 & 1\end{Bmatrix} & \begin{Bmatrix}2 & 2\end{Bmatrix} & \begin{Bmatrix}2 & 3\end{Bmatrix} & \begin{Bmatrix}2 & 4\end{Bmatrix} & \begin{Bmatrix}2 & 5\end{Bmatrix} & \begin{Bmatrix}2 & 6\end{Bmatrix} \\ \begin{Bmatrix}3 & 1\end{Bmatrix} & \begin{Bmatrix}3 & 2\end{Bmatrix} & \begin{Bmatrix}3 & 3\end{Bmatrix} & \begin{Bmatrix}3 & 4\end{Bmatrix} & \begin{Bmatrix}3 & 5\end{Bmatrix} & \begin{Bmatrix}3 & 6\end{Bmatrix} \\ \begin{Bmatrix}4 & 1\end{Bmatrix} & \begin{Bmatrix}4 & 2\end{Bmatrix} & \begin{Bmatrix}4 & 3\end{Bmatrix} & \begin{Bmatrix}4 & 4\end{Bmatrix} & \begin{Bmatrix}4 & 5\end{Bmatrix} & \begin{Bmatrix}4 & 6\end{Bmatrix} \\ \begin{Bmatrix}5 & 1\end{Bmatrix} & \begin{Bmatrix}5 & 2\end{Bmatrix} & \begin{Bmatrix}5 & 3\end{Bmatrix} & \begin{Bmatrix}5 & 4\end{Bmatrix} & \begin{Bmatrix}5 & 5\end{Bmatrix} & \begin{Bmatrix}5 & 6\end{Bmatrix} \\ \begin{Bmatrix}6 & 1\end{Bmatrix} & \begin{Bmatrix}6 & 2\end{Bmatrix} & \begin{Bmatrix}6 & 3\end{Bmatrix} & \begin{Bmatrix}6 & 4\end{Bmatrix} & \begin{Bmatrix}6 & 5\end{Bmatrix} & \begin{Bmatrix}6 & 6\end{Bmatrix} \end{pmatrix} \tag{1.1.1}$$

Event A is satisfied by the following possibilities =

$$\begin{pmatrix} \begin{Bmatrix}1 & 1\end{Bmatrix} & \begin{Bmatrix}1 & 2\end{Bmatrix} & \begin{Bmatrix}1 & 3\end{Bmatrix} & \begin{Bmatrix}1 & 4\end{Bmatrix} & \begin{Bmatrix}1 & 5\end{Bmatrix} & \begin{Bmatrix}1 & 6\end{Bmatrix} \\ \begin{Bmatrix}3 & 1\end{Bmatrix} & \begin{Bmatrix}3 & 2\end{Bmatrix} & \begin{Bmatrix}3 & 3\end{Bmatrix} & \begin{Bmatrix}3 & 4\end{Bmatrix} & \begin{Bmatrix}3 & 5\end{Bmatrix} & \begin{Bmatrix}3 & 6\end{Bmatrix} \\ \begin{Bmatrix}5 & 1\end{Bmatrix} & \begin{Bmatrix}5 & 2\end{Bmatrix} & \begin{Bmatrix}5 & 3\end{Bmatrix} & \begin{Bmatrix}5 & 4\end{Bmatrix} & \begin{Bmatrix}5 & 5\end{Bmatrix} & \begin{Bmatrix}5 & 6\end{Bmatrix} \end{pmatrix} \tag{1.1.2}$$

$$P(A) = \frac{18}{36} = \frac{1}{2} \tag{1.1.3}$$

Event B is satisfied by the following possibilities =

$$\begin{pmatrix} \begin{Bmatrix}1 & 1\end{Bmatrix} & \begin{Bmatrix}2 & 1\end{Bmatrix} & \begin{Bmatrix}3 & 1\end{Bmatrix} & \begin{Bmatrix}4 & 1\end{Bmatrix} & \begin{Bmatrix}5 & 1\end{Bmatrix} & \begin{Bmatrix}6 & 1\end{Bmatrix} \\ \begin{Bmatrix}1 & 3\end{Bmatrix} & \begin{Bmatrix}2 & 3\end{Bmatrix} & \begin{Bmatrix}3 & 3\end{Bmatrix} & \begin{Bmatrix}4 & 3\end{Bmatrix} & \begin{Bmatrix}5 & 3\end{Bmatrix} & \begin{Bmatrix}6 & 3\end{Bmatrix} \\ \begin{Bmatrix}1 & 5\end{Bmatrix} & \begin{Bmatrix}2 & 5\end{Bmatrix} & \begin{Bmatrix}3 & 5\end{Bmatrix} & \begin{Bmatrix}4 & 5\end{Bmatrix} & \begin{Bmatrix}5 & 5\end{Bmatrix} & \begin{Bmatrix}6 & 5\end{Bmatrix} \end{pmatrix} \tag{1.1.4}$$

$$P(B) = \frac{18}{36} = \frac{1}{2} \tag{1.1.5}$$

Event AB is satisfied by the following possibilities =

$$\begin{pmatrix} \begin{Bmatrix}1 & 1\end{Bmatrix} & \begin{Bmatrix}1 & 3\end{Bmatrix} & \begin{Bmatrix}1 & 5\end{Bmatrix} \\ \begin{Bmatrix}3 & 1\end{Bmatrix} & \begin{Bmatrix}3 & 3\end{Bmatrix} & \begin{Bmatrix}3 & 5\end{Bmatrix} \\ \begin{Bmatrix}5 & 1\end{Bmatrix} & \begin{Bmatrix}5 & 3\end{Bmatrix} & \begin{Bmatrix}5 & 5\end{Bmatrix} \end{pmatrix} \tag{1.1.6}$$

$$P(A \cap B) = \frac{9}{36} = \frac{1}{4} \tag{1.1.7}$$

$$\therefore P(A \cap B) = P(A).P(B) \tag{1.1.8}$$

Events A and B are independent.

2 Question 1.1.22

2.1 Problem

2.1. Three coins are tossed simultaneously. Consider the event E "three heads or three tails", F "at least two heads" and G "at most two heads". Of the pairs (E,F), (E,G) and (F,G), which are independent? which are dependent?

Solution: The sample size = Possible number of tosses=8

$$\begin{pmatrix} \{HHH\} & \{TTT\} & \{HHT\} & \{HTT\} & \{HTH\} & \{TTH\}\{THT\} & \{THH\} \end{pmatrix} \tag{2.1.1}$$

Favourable outcome for event E = three Heads (or) three Tails

$$\begin{pmatrix} \{HHH\} & \{TTT\} \end{pmatrix} \tag{2.1.2}$$

$$P(E) = \frac{1}{4} \tag{2.1.3}$$

Favourable outcome for event F = atleast two Heads

$$\left(\left\{HHT\right\} \quad \left\{HHH\right\} \quad \left\{HTH\right\} \quad \left\{THH\right\}\right) \tag{2.1.4}$$

$$P(F) = \frac{1}{2} \tag{2.1.5}$$

Favourable outcome for event G = atmost two Heads

$$\left(\left\{TTT\right\} \quad \left\{HHT\right\} \quad \left\{HTT\right\} \quad \left\{HTH\right\} \quad \left\{TTH\right\}\left\{THT\right\} \quad \left\{THH\right\}\right) \tag{2.1.6}$$

$$P(G) = \frac{7}{8} \tag{2.1.7}$$

Favourable outcome for event E∩F

$$\left(\left\{HHH\right\}\right) \tag{2.1.8}$$

$$P(E \cap F) = \frac{1}{8} \tag{2.1.9}$$

Favourable outcome for event F∩G

$$\left(\left\{HHH\right\} \quad \left\{HTH\right\} \quad \left\{THH\right\}\right) \tag{2.1.10}$$

$$P(F \cap G) = \frac{3}{8} \tag{2.1.11}$$

Favourable outcome for event E∩G

$$\left(\left\{TTT\right\}\right) \tag{2.1.12}$$

$$P(E \cap G) = \frac{1}{8} \tag{2.1.13}$$

From the above equations we see that

$$P(E \cap F) = P(E).P(F) \tag{2.1.14}$$
$$P(G \cap F) \neq P(G).P(F) \tag{2.1.15}$$
$$P(E \cap G) \neq P(E).P(G) \tag{2.1.16}$$

Hence only the pair (E,F) are independent events. The pairs (F,G) and (G,E) are dependent events.

3 Question 1.1.23

3.1 Problem

3.1. Prove that if E and F are independent events,then so are the events E and F'.
Solution: If two events E and F are independent,

$$P(E \cap F) = P(E).P(F) \tag{3.1.1}$$

We know that,

$$P(E \cap F') = P(E) - P(E \cap F) \tag{3.1.2}$$
$$P(E \cap F') = P(E) - P(E).P(F)\, From (4.1.1) \tag{3.1.3}$$
$$P(E \cap F') = P(E)(1 - P(F)) \tag{3.1.4}$$
$$P(E \cap F') = P(E)P(F') \tag{3.1.5}$$

∴ E and F' are independent events.

4 Question 1.1.24

4.1 Problem

4.1. If A and B are two independent events, then the probability of occurrence of at least one of A and B is given by 1- P(A')P(B').
Solution: If two events A and B are independent,

$$P(A \cap B) = P(A).P(B) \tag{4.1.1}$$

$P(\text{atleast one of A and B}) = P(A \cup B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{4.1.2}$$
$$P(A \cup B) = P(A) + P(B) - P(A).P(B) \tag{4.1.3}$$
$$P(A \cup B) = P(A) + P(B)(1 - P(A)) \tag{4.1.4}$$
$$P(A \cup B) = P(A) + P(B)P(A') \tag{4.1.5}$$
$$P(A \cup B) = 1 - P(A') + P(B)P(A') \tag{4.1.6}$$
$$P(A \cup B) = 1 - P(A')(1 - P(B)) \tag{4.1.7}$$
$$\therefore P(A \cup B) = 1 - P(A')P(B') \tag{4.1.8}$$

5 Question 1.1.25

5.1 Problem

5.1. A person has undertaken a construction job.The probabilities are 0.65 that there will

be strike, 0.80 that the construction job will be completed on time if there is no strike, and 0.32 that the construction job will be completed on time if there is a strike. Determine the probability that the construction job will be completed on time.
Solution: Let E denote the event of 'Having a strike'.
Let J denote the event 'Job is completed on time'.

$$P(E) = 0.65 \tag{5.1.1}$$
$$P(E') = 1 - P(E) = 0.35 \tag{5.1.2}$$

$$P(J) = P(E).P(\text{Job is completed on time with strike}) + P(E).P(\text{Job is completed on time without strike}) \tag{5.1.3}$$

$$P(J) = (0.65)(0.32) + (0.35)(0.80) \tag{5.1.4}$$

Therefore the probability that the construction job will be completed on time is found to be 0.488

6 Question 1.1.26

6.1 Problem

6.1. Bag I contains 3 red and 4 black balls while another Bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from Bag II.

Solution: Let E_1 and E_2 denote the events of selecting bag-1 and bag-2 respectively. Let A and B denote the events of selecting a red ball and black ball respectively. We need to find the probability of drawing a ball from bag-2 if it is red.i.e.$P\left(\frac{E_2}{A}\right)$.

$$P(E_1) = \frac{1}{2} \tag{6.1.1}$$
$$P(E_2) = \frac{1}{2} \tag{6.1.2}$$

$$P\left(\frac{A}{E_1}\right) = P(\text{Selecting red ball from bag1}) \tag{6.1.3}$$
$$P\left(\frac{A}{E_1}\right) = \frac{3}{7} \tag{6.1.4}$$

$$P\left(\frac{A}{E_2}\right) = P(\text{Selecting red ball from bag2}) \tag{6.1.5}$$
$$P\left(\frac{A}{E_2}\right) = \frac{5}{11} \tag{6.1.6}$$

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2).P\left(\frac{A}{E_2}\right)}{P(E_1).P\left(\frac{A}{E_1}\right) + P(E_2).P\left(\frac{A}{E_2}\right)} \tag{6.1.7}$$

$$P\left(\frac{E_2}{A}\right) = \frac{\frac{1}{2}\cdot\frac{5}{11}}{\frac{1}{2}\cdot\frac{3}{7} + \frac{1}{2}\cdot\frac{5}{11}} \tag{6.1.8}$$

which is found to be $\frac{35}{68}$.

7 Question 1.1.27

7.1 Problem

7.1. Given three identical boxes I, II and III, each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in the box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold?
Solution: Let B_1, B_2, B_3 denote the events of selecting box 1,box 2, box 3 respectively.
We need to find the probability that the other coin in the box is also gold if the 1^{st} coin is also gold.
Let G denote the event of '2^{nd} coin is gold'.

$$P\left(\frac{B_1}{G}\right) = \frac{P(B_1).P\left(\frac{G}{B_1}\right)}{P(B_1).P\left(\frac{G}{B_1}\right) + P(B_2).P\left(\frac{G}{B_2}\right) + P(B_3).P\left(\frac{G}{B_3}\right)} \tag{7.1.1}$$

$$P\left(\frac{B_1}{G}\right) = \frac{\frac{1}{3}.1}{\frac{1}{3}.1 + \frac{1}{3}.0 + \frac{1}{3}.\frac{1}{2}} \tag{7.1.2}$$

which is calculated to be 0.66

8 Question 1.1.28

8.1 Problem

8.1. Suppose that the reliability of a HIV test is specified as follows: Of people having HIV, 90% of the test detect the disease but 10% go undetected. Of people free of HIV, 99% of the test are judged HIV –ve but 1% are diagnosed as showing HIV +ve. From a large population of which only 0.1% have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV +ve. What is the probability that the person actually has HIV?

Solution: Let E and G denote the events that 'the selected person has HIV' and the 'test judges HIV +ve'. We need to find the probability that the person selected has HIV, if the test judges HIV +ve.i.e $P\left(\frac{E}{G}\right)$.

$$P(E) = 0.001 \tag{8.1.1}$$
$$P(E') = 1 - P(E) \tag{8.1.2}$$
$$P(E') = 0.999 \tag{8.1.3}$$

$$P\left(\frac{G}{E}\right) = P(\text{Test judges +ve if person has HIV}) \tag{8.1.4}$$
$$P\left(\frac{G}{E}\right) = 0.9 \tag{8.1.5}$$

$$P\left(\frac{G}{E'}\right) = P(\text{Test judges +ve if person doesn't have HIV}) \tag{8.1.6}$$
$$P\left(\frac{G}{E'}\right) = 0.01 \tag{8.1.7}$$

$$P\left(\frac{E}{G}\right) = \frac{P(E).P\left(\frac{G}{E}\right)}{P(E).P\left(\frac{G}{E}\right) + P(E').P\left(\frac{G}{E'}\right)} \tag{8.1.8}$$
$$P\left(\frac{E}{G}\right) = \frac{(0.001)(0.9)}{(0.001)(0.9) + (0.999)(0.01)} \tag{8.1.9}$$
$$\tag{8.1.10}$$

which is calculated to be 0.083

9 Question 1.1.29

9.1 Problem

9.1. In a factory which manufactures bolts, machines A, B and C manufacture respectively 25%, 35% and 40% of the bolts. Of their outputs, 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by the machine B?

Solution: Let A,B,C denotes the events that the bolt is manufactured from machines A,B and C respectively. Let D denote the event of obtaining a defective bolt. We need to find the probability that the bolt manufactured by machine B, if it is defective .i.e $P\left(\frac{B}{D}\right)$.

$$P(A) = 0.25 \quad P(B) = 0.35 \quad P(C) = 0.4 \tag{9.1.1}$$

$$P\left(\frac{D}{A}\right) = P(\text{Defective bolt obtained from A}) \tag{9.1.2}$$
$$P\left(\frac{D}{A}\right) = 0.05 \tag{9.1.3}$$

$$P\left(\frac{D}{B}\right) = P(\text{Defective bolt obtained from B}) \tag{9.1.4}$$
$$P\left(\frac{D}{B}\right) = 0.04 \tag{9.1.5}$$

$$P\left(\frac{D}{C}\right) = P(\text{Defective bolt obtained from C}) \tag{9.1.6}$$
$$P\left(\frac{D}{C}\right) = 0.02 \tag{9.1.7}$$

$$P\left(\frac{B}{D}\right) = \frac{P(B).P\left(\frac{D}{B}\right)}{P(A).P\left(\frac{D}{A}\right) + P(B).P\left(\frac{D}{B}\right) + P(C).P\left(\frac{D}{C}\right)} \tag{9.1.8}$$
$$P\left(\frac{B}{D}\right) = \frac{(0.35)(0.04)}{(0.25)(0.05) + (0.35)(0.04) + (0.4)(0.02)} \tag{9.1.9}$$

which is calculated to be $\frac{28}{69}$.

10 Question 1.1.30

10.1 Problem

10.1. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}, \frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}, \frac{1}{12}$, if he comes by train, bus and scooter respectively, but if he4 comes by other means of transport, then he will not be late. When he arrives, he is late.What is the probability that he comes by train?

Solution: Let T_1, T_2, T_3, T_4 denote the events where the doctor came by train,bus,scooter and other means of transport. Let L denote the event that the doctor arrives late. We need to find the probability that the doctor comes by train if he's late.i.e.$P\left(\frac{T_1}{L}\right)$.

$$P(T_1) = \frac{3}{10} \quad P(T_2) = \frac{1}{5} \quad P(T_3) = \frac{1}{10} \quad P(T_4) = \frac{2}{5} \tag{10.1.1}$$

$$P\left(\frac{L}{T_1}\right) = P(\text{Arrives late coming by train}) \tag{10.1.2}$$

$$P\left(\frac{L}{T_1}\right) = \frac{1}{4} \tag{10.1.3}$$

$$P\left(\frac{L}{T_2}\right) = P(\text{Arrives late coming by bus}) \tag{10.1.4}$$

$$P\left(\frac{L}{T_2}\right) = \frac{1}{3} \tag{10.1.5}$$

$$P\left(\frac{L}{T_3}\right) = P(\text{Arrives late coming by scooter}) \tag{10.1.6}$$

$$P\left(\frac{L}{T_3}\right) = \frac{1}{12} \tag{10.1.7}$$

$$P\left(\frac{L}{T_4}\right) = P(\text{Arrives late coming by other means}) \tag{10.1.8}$$

$$P\left(\frac{L}{T_4}\right) = 0 \tag{10.1.9}$$

$$P\left(\frac{T_1}{L}\right) = \frac{P(T_1).P\left(\frac{L}{T_1}\right)}{P(T_1).P\left(\frac{L}{T_1}\right) + P(T_2).P\left(\frac{L}{T_2}\right) + P(T_3).P\left(\frac{L}{T_3}\right) + P(T_4).P\left(\frac{L}{T_4}\right)} \tag{10.1.10}$$

$$P\left(\frac{T_1}{L}\right) = \frac{\frac{3}{40}}{\frac{18}{120}} \tag{10.1.11}$$

which is calculated to be 0.5

11 Question 1.2.21

11.1 Problem

11.1. A black and a red dice are rolled.
- Find the conditional probability obtaining a sum greater than 9, given that black die resulted in a 5.
- Find the conditional probability obtaining the sum 8, given that the red resulted in a number less than 4.

Solution:
- Let us take the first numbers to represent the black die and the second numbers to represent the red die.
  The sample size = Total number of possibilities(S)=
$$\begin{pmatrix} \begin{Bmatrix}1 & 1\end{Bmatrix} & \begin{Bmatrix}1 & 2\end{Bmatrix} & \begin{Bmatrix}1 & 3\end{Bmatrix} & \begin{Bmatrix}1 & 4\end{Bmatrix} & \begin{Bmatrix}1 & 5\end{Bmatrix} & \begin{Bmatrix}1 & 6\end{Bmatrix} \\ \begin{Bmatrix}2 & 1\end{Bmatrix} & \begin{Bmatrix}2 & 2\end{Bmatrix} & \begin{Bmatrix}2 & 3\end{Bmatrix} & \begin{Bmatrix}2 & 4\end{Bmatrix} & \begin{Bmatrix}2 & 5\end{Bmatrix} & \begin{Bmatrix}2 & 6\end{Bmatrix} \\ \begin{Bmatrix}3 & 1\end{Bmatrix} & \begin{Bmatrix}3 & 2\end{Bmatrix} & \begin{Bmatrix}3 & 3\end{Bmatrix} & \begin{Bmatrix}3 & 4\end{Bmatrix} & \begin{Bmatrix}3 & 5\end{Bmatrix} & \begin{Bmatrix}3 & 6\end{Bmatrix} \\ \begin{Bmatrix}4 & 1\end{Bmatrix} & \begin{Bmatrix}4 & 2\end{Bmatrix} & \begin{Bmatrix}4 & 3\end{Bmatrix} & \begin{Bmatrix}4 & 4\end{Bmatrix} & \begin{Bmatrix}4 & 5\end{Bmatrix} & \begin{Bmatrix}4 & 6\end{Bmatrix} \\ \begin{Bmatrix}5 & 1\end{Bmatrix} & \begin{Bmatrix}5 & 2\end{Bmatrix} & \begin{Bmatrix}5 & 3\end{Bmatrix} & \begin{Bmatrix}5 & 4\end{Bmatrix} & \begin{Bmatrix}5 & 5\end{Bmatrix} & \begin{Bmatrix}5 & 6\end{Bmatrix} \\ \begin{Bmatrix}6 & 1\end{Bmatrix} & \begin{Bmatrix}6 & 2\end{Bmatrix} & \begin{Bmatrix}6 & 3\end{Bmatrix} & \begin{Bmatrix}6 & 4\end{Bmatrix} & \begin{Bmatrix}6 & 5\end{Bmatrix} & \begin{Bmatrix}6 & 6\end{Bmatrix} \end{pmatrix} \tag{11.1.1}$$

  We need to find the probability of obtaining a sum greater than 9, given that the black die resulted in 5. Let F denote the event '5 appeared on black die' and E denote the event 'Sum of the numbers greater than 9'. We need to find $P\left(\frac{E}{F}\right)$.

  The possibilities satisfying event E are
$$\begin{pmatrix} \begin{Bmatrix}4 & 6\end{Bmatrix} & \begin{Bmatrix}5 & 5\end{Bmatrix} & \begin{Bmatrix}5 & 6\end{Bmatrix} & \begin{Bmatrix}6 & 4\end{Bmatrix} & \begin{Bmatrix}6 & 5\end{Bmatrix} & \begin{Bmatrix}6 & 6\end{Bmatrix} \end{pmatrix} \tag{11.1.2}$$

$$P(E) = \frac{6}{36} = \frac{1}{6} \tag{11.1.3}$$

The possibilities satisfying event F are

$$\left(\begin{Bmatrix}5 & 1\end{Bmatrix} \quad \begin{Bmatrix}5 & 2\end{Bmatrix} \quad \begin{Bmatrix}5 & 3\end{Bmatrix} \quad \begin{Bmatrix}5 & 4\end{Bmatrix} \quad \begin{Bmatrix}5 & 5\end{Bmatrix} \quad \begin{Bmatrix}5 & 6\end{Bmatrix}\right) \tag{11.1.4}$$

$$P(F) = \frac{6}{36} = \frac{1}{6} \tag{11.1.5}$$

$$AlsoE \cap F = \left(\begin{Bmatrix}5 & 5\end{Bmatrix} \quad \begin{Bmatrix}5 & 6\end{Bmatrix}\right) \tag{11.1.6}$$

$$P(E \cap F) = \frac{2}{36} = \frac{1}{18} \tag{11.1.7}$$

$$P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)} \tag{11.1.8}$$

$$P\left(\frac{E}{F}\right) = \frac{\frac{2}{36}}{\frac{6}{36}} = \frac{1}{3} \tag{11.1.9}$$

- We need to find the probability of obtaining sum 8, given that the red die resulted in a number less than 4. Let F denote the event 'Number on red die is less than 4' and E denote the event 'Sum of the numbers is 8'. We need to find $P\left(\frac{E}{F}\right)$.

  The possibilities satisfying event E are

$$\left(\begin{Bmatrix}2 & 6\end{Bmatrix} \quad \begin{Bmatrix}3 & 5\end{Bmatrix} \quad \begin{Bmatrix}5 & 6\end{Bmatrix} \quad \begin{Bmatrix}4 & 4\end{Bmatrix} \quad \begin{Bmatrix}6 & 2\end{Bmatrix}\right) \tag{11.1.10}$$

$$P(E) = \frac{5}{36} \tag{11.1.11}$$

  The possibilities satisfying event F are

$$\begin{pmatrix} \begin{Bmatrix}1 & 1\end{Bmatrix} & \begin{Bmatrix}1 & 2\end{Bmatrix} & \begin{Bmatrix}1 & 3\end{Bmatrix} \\ \begin{Bmatrix}2 & 1\end{Bmatrix} & \begin{Bmatrix}2 & 2\end{Bmatrix} & \begin{Bmatrix}2 & 3\end{Bmatrix} \\ \begin{Bmatrix}3 & 1\end{Bmatrix} & \begin{Bmatrix}3 & 2\end{Bmatrix} & \begin{Bmatrix}3 & 3\end{Bmatrix} \\ \begin{Bmatrix}4 & 1\end{Bmatrix} & \begin{Bmatrix}4 & 2\end{Bmatrix} & \begin{Bmatrix}4 & 3\end{Bmatrix} \\ \begin{Bmatrix}5 & 1\end{Bmatrix} & \begin{Bmatrix}5 & 2\end{Bmatrix} & \begin{Bmatrix}5 & 3\end{Bmatrix} \\ \begin{Bmatrix}6 & 1\end{Bmatrix} & \begin{Bmatrix}6 & 2\end{Bmatrix} & \begin{Bmatrix}6 & 3\end{Bmatrix} \end{pmatrix} \tag{11.1.12}$$

$$P(F) = \frac{18}{36} = \frac{1}{2} \tag{11.1.13}$$

$$AlsoE \cap F = \left(\begin{Bmatrix}5 & 3\end{Bmatrix} \quad \begin{Bmatrix}6 & 2\end{Bmatrix}\right) \tag{11.1.14}$$

$$P(E \cap F) = \frac{2}{36} = \frac{1}{18} \tag{11.1.15}$$

$$P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)} \tag{11.1.16}$$

$$P\left(\frac{E}{F}\right) = \frac{\frac{2}{36}}{\frac{1}{2}} = \frac{1}{9} \tag{11.1.17}$$

12 Question 1.2.22

12.1 Problem

12.1. A fair die is rolled. Consider the events E = (1, 3, 5), F = (2, 3) and G = (2, 3, 4, 5) Find
- P(E/F) and P(F/E)
- P(E/G) and P(G/E)
- P((E ∪ F)/G) and P((E ∩ F)/G)

Solution:
- When a fair die is rolled the sample space is S = { 1,2,3,4,5,6 }

$$P(E) = \frac{3}{6} = \frac{1}{2} \tag{12.1.1}$$

$$P(F) = \frac{2}{6} = \frac{1}{3} \tag{12.1.2}$$

$$P(G) = \frac{4}{6} = \frac{2}{3} \tag{12.1.3}$$

$$AlsoE \cap F = \begin{Bmatrix}3\end{Bmatrix} \tag{12.1.4}$$

$$P(E \cap F) = \frac{1}{6} \tag{12.1.5}$$

$$P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)} \tag{12.1.6}$$

$$P\left(\frac{E}{F}\right) = \frac{\frac{1}{6}}{\frac{1}{3}} = \frac{1}{2} \tag{12.1.7}$$

$$P\left(\frac{F}{E}\right) = \frac{P(F \cap E)}{P(E)} \tag{12.1.8}$$

$$P\left(\frac{F}{E}\right) = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3} \tag{12.1.9}$$

- $$E \cap G = \left\{3, 5\right\} \tag{12.1.10}$$
$$P(E \cap G) = \frac{2}{6} = \frac{1}{3} \tag{12.1.11}$$
$$P\left(\frac{E}{G}\right) = \frac{P(E \cap G)}{P(G)} \tag{12.1.12}$$
$$P\left(\frac{E}{G}\right) = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2} \tag{12.1.13}$$
$$P\left(\frac{G}{E}\right) = \frac{P(G \cap E)}{P(G)} \tag{12.1.14}$$
$$P\left(\frac{G}{E}\right) = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3} \tag{12.1.15}$$
$$\tag{12.1.16}$$

- $$Let E \cup F = A = \left\{1, 2, 3, 5\right\} \tag{12.1.17}$$
$$P(A) = \frac{4}{6} = \frac{2}{3} \tag{12.1.18}$$
$$A \cap G = \left\{2, 3, 5\right\} \tag{12.1.19}$$
$$P(A \cap G) = \frac{3}{6} = \frac{1}{2} \tag{12.1.20}$$
$$P\left(\frac{A}{G}\right) = \frac{P(A \cap G)}{P(G)} \tag{12.1.21}$$
$$P\left(\frac{A}{G}\right) = \frac{\frac{3}{6}}{\frac{4}{6}} = \frac{3}{4} \tag{12.1.22}$$

$$Let E \cap F = B = \left\{3\right\} \tag{12.1.23}$$
$$P(B) = \frac{1}{6} \tag{12.1.24}$$
$$B \cap G = \left\{3\right\} \tag{12.1.25}$$
$$P(B \cap G) = \frac{1}{6} \tag{12.1.26}$$
$$P\left(\frac{B}{G}\right) = \frac{P(B \cap G)}{P(G)} \tag{12.1.27}$$
$$P\left(\frac{B}{G}\right) = \frac{\frac{1}{6}}{\frac{4}{6}} = \frac{1}{4} \tag{12.1.28}$$

13 Question 1.2.23

13.1 Problem

13.1. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that
- the youngest is a girl,
- at least one is a girl?

Solution: We need to find the probability that both the children are girls, given that the youngest is a girl. Let F denote the event 'youngest child is a girl' and E denote the event 'Both children are girls'. We need to find $P\left(\frac{E}{F}\right)$.

The possibilities satisfying event E are
$$\left(\left\{g \quad g\right\}\right) \tag{13.1.1}$$

$$P(E) = \frac{1}{4} \tag{13.1.2}$$

The possibilities satisfying event F are
$$\left(\left\{g \quad g\right\}, \left\{b \quad g\right\}\right) \tag{13.1.3}$$

$$P(F) = \frac{2}{4} = \frac{1}{2} \tag{13.1.4}$$
$$Also E \cap F = \left(\left\{g \quad g\right\}\right) \tag{13.1.5}$$
$$P(E \cap F) = \frac{1}{4} \tag{13.1.6}$$
$$P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)} \tag{13.1.7}$$
$$P\left(\frac{E}{F}\right) = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2} \tag{13.1.8}$$

14 Question 1.2.24

14.1 Problem

14.1. An instructor has a question bank consisting of 300 easy True / False questions, 200 difficult True / False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question?

Solution: Let A_1, A_2, B_1, B_2 denote easy T/F , easy MCQ, difficult T/F, difficult MCQ. We need to find the probability that a selected questionwill be easy, given that it is a MCQ.i.e.$P\left(\frac{(A_1+A_2)}{(A_2+B_2)}\right)$.

$$P(A_1 + A_2) = \frac{\text{Easy T/F + Easy MCQ}}{\text{Total questions}} \tag{14.1.1}$$

$$P(A_1 + A_2) = \frac{300 + 500}{300 + 200 + 500 + 400} = \frac{800}{1400} \tag{14.1.2}$$

$$P(A_1 + A_2) = \frac{4}{7} \tag{14.1.3}$$

$$P(A_2 + B_2) = \frac{\text{Difficult MCQ + Easy MCQ}}{\text{Total questions}} \tag{14.1.4}$$

$$P(A_1 + A_2) = \frac{400 + 500}{300 + 200 + 500 + 400} = \frac{900}{1400} \tag{14.1.5}$$

$$P(A_1 + A_2) = \frac{9}{14} \tag{14.1.6}$$

$$Also\,(A_1 + A_2) \cap (A_2 + B_2) = A_2 \tag{14.1.7}$$

$$P((A_1 + A_2) \cap (A_2 + B_2)) = P(A_2) = \frac{5}{14} \tag{14.1.8}$$

$$P\left(\frac{(A_1 + A_2)}{(A_2 + B_2)}\right) = \frac{P((A_1 + A_2) \cap (A_2 + B_2))}{P(A_2 + B_2)} \tag{14.1.9}$$

$$P\left(\frac{(A_1 + A_2)}{(A_2 + B_2)}\right) = \frac{\frac{5}{14}}{\frac{9}{14}} = \frac{5}{9}. \tag{14.1.10}$$

15 Question 1.2.25

15.1 Problem

15.1. Given that the two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'.
Solution:
When a dice is thrown the sample space = Total number of possibilities(S)

$$\begin{pmatrix} \begin{Bmatrix}1 & 1\end{Bmatrix} & \begin{Bmatrix}1 & 2\end{Bmatrix} & \begin{Bmatrix}1 & 3\end{Bmatrix} & \begin{Bmatrix}1 & 4\end{Bmatrix} & \begin{Bmatrix}1 & 5\end{Bmatrix} & \begin{Bmatrix}1 & 6\end{Bmatrix} \\ \begin{Bmatrix}2 & 1\end{Bmatrix} & \begin{Bmatrix}2 & 2\end{Bmatrix} & \begin{Bmatrix}2 & 3\end{Bmatrix} & \begin{Bmatrix}2 & 4\end{Bmatrix} & \begin{Bmatrix}2 & 5\end{Bmatrix} & \begin{Bmatrix}2 & 6\end{Bmatrix} \\ \begin{Bmatrix}3 & 1\end{Bmatrix} & \begin{Bmatrix}3 & 2\end{Bmatrix} & \begin{Bmatrix}3 & 3\end{Bmatrix} & \begin{Bmatrix}3 & 4\end{Bmatrix} & \begin{Bmatrix}3 & 5\end{Bmatrix} & \begin{Bmatrix}3 & 6\end{Bmatrix} \\ \begin{Bmatrix}4 & 1\end{Bmatrix} & \begin{Bmatrix}4 & 2\end{Bmatrix} & \begin{Bmatrix}4 & 3\end{Bmatrix} & \begin{Bmatrix}4 & 4\end{Bmatrix} & \begin{Bmatrix}4 & 5\end{Bmatrix} & \begin{Bmatrix}4 & 6\end{Bmatrix} \\ \begin{Bmatrix}5 & 1\end{Bmatrix} & \begin{Bmatrix}5 & 2\end{Bmatrix} & \begin{Bmatrix}5 & 3\end{Bmatrix} & \begin{Bmatrix}5 & 4\end{Bmatrix} & \begin{Bmatrix}5 & 5\end{Bmatrix} & \begin{Bmatrix}5 & 6\end{Bmatrix} \\ \begin{Bmatrix}6 & 1\end{Bmatrix} & \begin{Bmatrix}6 & 2\end{Bmatrix} & \begin{Bmatrix}6 & 3\end{Bmatrix} & \begin{Bmatrix}6 & 4\end{Bmatrix} & \begin{Bmatrix}6 & 5\end{Bmatrix} & \begin{Bmatrix}6 & 6\end{Bmatrix} \end{pmatrix} \tag{15.1.1}$$

We need to find the probability that the sum of the numbers on the dice is 4, given that the two numbers are different
Let F denote the event 'numbers are different' and E denote the event 'sum of the numbers is 4'. We need to find $P\left(\frac{E}{F}\right)$.

The possibilities satisfying event E are

$$\left(\begin{Bmatrix}1 & 3\end{Bmatrix} \quad \begin{Bmatrix}3 & 1\end{Bmatrix} \quad \begin{Bmatrix}2 & 2\end{Bmatrix}\right) \tag{15.1.2}$$

$$P(E) = \frac{3}{36} = \frac{1}{12} \tag{15.1.3}$$

The possibilities satisfying event F are

$$\begin{pmatrix} \begin{Bmatrix}1 & 2\end{Bmatrix} & \begin{Bmatrix}1 & 3\end{Bmatrix} & \begin{Bmatrix}1 & 4\end{Bmatrix} & \begin{Bmatrix}1 & 5\end{Bmatrix} & \begin{Bmatrix}1 & 6\end{Bmatrix} \\ \begin{Bmatrix}2 & 1\end{Bmatrix} & \begin{Bmatrix}2 & 3\end{Bmatrix} & \begin{Bmatrix}2 & 4\end{Bmatrix} & \begin{Bmatrix}2 & 5\end{Bmatrix} & \begin{Bmatrix}2 & 6\end{Bmatrix} \\ \begin{Bmatrix}3 & 1\end{Bmatrix} & \begin{Bmatrix}3 & 2\end{Bmatrix} & \begin{Bmatrix}3 & 4\end{Bmatrix} & \begin{Bmatrix}3 & 5\end{Bmatrix} & \begin{Bmatrix}3 & 6\end{Bmatrix} \\ \begin{Bmatrix}4 & 1\end{Bmatrix} & \begin{Bmatrix}4 & 2\end{Bmatrix} & \begin{Bmatrix}4 & 3\end{Bmatrix} & \begin{Bmatrix}4 & 5\end{Bmatrix} & \begin{Bmatrix}4 & 6\end{Bmatrix} \\ \begin{Bmatrix}5 & 1\end{Bmatrix} & \begin{Bmatrix}5 & 2\end{Bmatrix} & \begin{Bmatrix}5 & 3\end{Bmatrix} & \begin{Bmatrix}5 & 4\end{Bmatrix} & \begin{Bmatrix}5 & 6\end{Bmatrix} \\ \begin{Bmatrix}6 & 1\end{Bmatrix} & \begin{Bmatrix}6 & 2\end{Bmatrix} & \begin{Bmatrix}6 & 3\end{Bmatrix} & \begin{Bmatrix}6 & 4\end{Bmatrix} & \begin{Bmatrix}6 & 5\end{Bmatrix} \end{pmatrix} \tag{15.1.4}$$

$$P(F) = \frac{30}{36} = \frac{5}{6} \tag{15.1.5}$$

$$AlsoE \cap F = \left(\begin{Bmatrix}1 & 3\end{Bmatrix} \quad \begin{Bmatrix}3 & 1\end{Bmatrix}\right) \tag{15.1.6}$$

$$P(E \cap F) = \frac{2}{36} = \frac{1}{18} \tag{15.1.7}$$

$$P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)} \tag{15.1.8}$$

$$P\left(\frac{E}{F}\right) = \frac{\frac{2}{36}}{\frac{2}{30}} = \frac{1}{15} \tag{15.1.9}$$

16 Question 1.2.26

16.1 Problem

16.1. Consider the experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 3'.
Solution: Let the event E be "coin shows a tail" and F the event "Atleast one die shows a 3".We need to find the probability of the coin showing a tail, given that atleast one die shows a 3.i.e.$P\left(\frac{E}{F}\right)$

Event E is satisfied by the following possibilities =

$$\left(\left\{1 \quad T\right\} \quad \left\{2 \quad T\right\} \quad \left\{4 \quad T\right\} \quad \left\{5 \quad T\right\}\right) \tag{16.1.1}$$

$$P(E) = P\left(\left\{1 \quad T\right\}\right) + P\left(\left\{2 \quad T\right\}\right) + P\left(\left\{4 \quad T\right\}\right) + P\left(\left\{5 \quad T\right\}\right) \tag{16.1.2}$$

$$P(E) = \left(\frac{1}{6}.\frac{1}{2}\right) + \left(\frac{1}{6}.\frac{1}{2}\right) + \left(\frac{1}{6}.\frac{1}{2}\right) + \left(\frac{1}{6}.\frac{1}{2}\right) \tag{16.1.3}$$

$$P(E) = \frac{4}{12} = \frac{1}{3} \tag{16.1.4}$$

Event F is satisfied by the following possibilities =

$$\left(\left\{3 \quad 1\right\} \quad \left\{3 \quad 2\right\} \quad \left\{3 \quad 3\right\} \quad \left\{3 \quad 4\right\} \quad \left\{3 \quad 5\right\} \quad \left\{3 \quad 6\right\} \quad \left\{6 \quad 3\right\}\right) \tag{16.1.5}$$

$$P(E) = P\left(\left\{3 \quad 1\right\}\right) + P\left(\left\{3 \quad 2\right\}\right) + P\left(\left\{3 \quad 3\right\}\right) + P\left(\left\{3 \quad 4\right\}\right) + P\left(\left\{3 \quad 5\right\}\right) + P\left(\left\{3 \quad 6\right\}\right) + P\left(\left\{6 \quad 3\right\}\right) \tag{16.1.6}$$

$$P(F) = \left(\frac{1}{6}.\frac{1}{6}\right) + \left(\frac{1}{6}.\frac{1}{6}\right) + \left(\frac{1}{6}.\frac{1}{6}\right) + \left(\frac{1}{6}.\frac{1}{6}\right) + \left(\frac{1}{6}.\frac{1}{6}\right) + \left(\frac{1}{6}.\frac{1}{6}\right) + \left(\frac{1}{6}.\frac{1}{6}\right) \tag{16.1.7}$$

$$P(F) = \frac{7}{36} \tag{16.1.8}$$

$$Also E \cap F = \phi \tag{16.1.9}$$

$$P(E \cap F) = 0 \tag{16.1.10}$$

$$P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)} = 0 \tag{16.1.11}$$

17 Question 1.2.27

17.1 Problem

17.1. If P(A) = 12 , P(B) = 0, then what is the value of P$\left(\frac{A}{B}\right)$?

Solution:

$$P(B) = 0 \implies B = \phi \tag{17.1.1}$$

$$S o A \cap B = \phi \tag{17.1.2}$$

$$P(A \cap B) = 0 \tag{17.1.3}$$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{0}{0} \tag{17.1.4}$$

$\therefore$ it is undefined.

18 Question 1.2.28

18.1 Problem

18.1. If A and B are events such that P(A/B) = P(B/A), then a) A B but A , B b) A = B c) A B = d) P(A) = P(B)

Solution: Since

$$P\left(\frac{A}{B}\right) = P\left(\frac{B}{A}\right) \tag{18.1.1}$$

$$\frac{P(A \cap B)}{P(A)} = \frac{P(B \cap A)}{P(B)} \tag{18.1.2}$$

$$\frac{P(A \cap B)}{P(A)} = \frac{P(A \cap B)}{P(B)} \tag{18.1.3}$$

$$\frac{1}{P(A)} = \frac{1}{P(B)} \tag{18.1.4}$$

$$\implies P(A) = P(B) \tag{18.1.5}$$

19 Question 1.2.29

19.1 Problem

19.1. If P(A) = $\frac{3}{5}$ and P(B) = $\frac{1}{5}$, find P (A $\cap$ B) if A and B are independent events.

Solution: Since A and B are independent events.

$$P(A \cap B) = P(A).P(B) = \frac{3}{25} \tag{19.1.1}$$

20 Question 1.2.30

20.1 Problem

20.1. Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.

Solution: Let the event A be ”first card is black” and B the event ”second card is black if first card is black”. Let E be the event ”both the cards are black”.

Since there are 26 black cards in a pack of 52 cards,

$$P(A) = \frac{26}{52} = \frac{1}{2} \tag{20.1.1}$$

If the first card drawn is black 51 cards remain out of which 25 are black cards.

$$P(B) = \frac{25}{51} \tag{20.1.2}$$

$$\therefore P(E) = P(A).P(B) \tag{20.1.3}$$

$$P(E) = \frac{25}{51} \times \frac{1}{2} = \frac{25}{102} \tag{20.1.4}$$

21 Question 2.1.21

21.1 Problem

21.1. The heights (in cm) of 9 students of a class are as follows: 155 160 145 149 150 147 152 144 148 .Find the median of this data.
Solution: Let N be the no. of observations = 9
Arranging the heights in ascending order we get:
144,145,147,148,149,150,152,155,160

$$\text{Median} = \left(\frac{N+1}{2}\right)^{th} term = 5^{th} term \tag{21.1.1}$$

$\therefore$ the median is 149.

22 Question 2.1.22

22.1 Problem

22.1. The points scored by a Kabaddi team in a series of matches are as follows: 17, 2, 7, 27, 15, 5, 14, 8, 10, 24, 48, 10, 8, 7, 18, 28 .Find the median of the points scored by the team.
Solution: Arranging the points scored by the team in ascending order we get:
2,5,7,7,8,8,10,10,14,15,17,18,24,27,28,48
Let N be the no. of observations = 16

$$\text{Median} = \frac{\left(\frac{N}{2}\right)^{th}\text{ value} + \left(\frac{N}{2}+1\right)^{th}\text{ value}}{2} \tag{22.1.1}$$

$$\text{Median} = \frac{(8)^{th}\text{ value} + (9)^{th}\text{ value}}{2} \tag{22.1.2}$$

$$\text{Median} = \frac{10+14}{2} \tag{22.1.3}$$

$$\text{Median} = 12 \tag{22.1.4}$$

23 Question 2.1.23

23.1 Problem

23.1. Find the mode of the following marks (out of 10) obtained by 20 students:4, 6, 5, 9, 3, 2, 7, 7, 6, 5, 4, 9, 10, 10, 3, 4,7, 6, 9, 9
Solution:

Mark	Frequency
2	1
3	2
4	3
5	2
6	3
7	3
9	4
10	2

TABLE I: Marks obtained by students

As we can see from table I, 9 occurs the maximum number of times.
Thus Mode = 9.

24 Question 2.1.24

24.1 Problem

24.1. Consider a small unit of a factory where there are 5 employees : a supervisor and four labourers. The labourers draw a salary of —5,000 per month each while the supervisor gets —15,000 per month. calculate the mean, median and mode of the salaries of this unit of the factory.
Solution:

a) Finding Mean

$$\text{Mean salary} = \frac{\text{Supervisor's salary} + 4 \times \text{Labourer's salary}}{5} \tag{24.1.1}$$

$$\text{Mean salary} = \frac{15000 + 5000 + 5000 + 5000 + 5000}{5} \tag{24.1.2}$$

$\therefore$ the mean salary is 7000.

b) Finding Median
We need to arrange the salaries in ascending order.Thus we get 5000,5000,5000,5000,15000
Let N = no.of employees = 5

$$\text{Median} = \left(\frac{N+1}{2}\right)^{th} term = 3^{rd} value \tag{24.1.3}$$

$\therefore$ the median is 5000.

c) Finding Mode
Mode is the highest occurring frequency of the distribution. 5000 is the most repeating salary.
$\therefore$ Modal salary is 5000.

25 Question 2.2.21

25.1 Problem

25.1. 100 surnames were picked randomly from a telephone directory and the frequency distribution of the number of letters in english alphabet in surnames were obtained as follows.Determine the median number of letters in the surname and also find the modal size of the surnames. Also find the mean.

No.of.letters	Surnames	Cum.Freq
001-004	6	6
004-007	30	36
007-010	40	76
10-13	16	92
13-16	4	96
16-19	4	100

TABLE II: Frequency distribution of the number of letters in surname

Solution: The following python code computes the mean, median and mode.

```
codes/statistics/exercises/q21.py
```

$$\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h \tag{25.1.1}$$

$$n = \sum f_i = 100 \implies \frac{n}{2} = 50 \tag{25.1.2}$$

$$\tag{25.1.3}$$

$\therefore$ 7-10 is the median class.
Here l is the lower limit of the median class = 7
h is the classinterval =3
cf is the cumulative frequency of the class before median class = 36
f is the frequency of the median class

No.of.letters	f_i	x_i	$f_i x_i$
001-004	6	2.5	15
004-007	30	5.5	165
007-010	40	8.5	340
10-13	16	11.5	184
13-16	4	14.5	58
16-19	4	17.5	70

TABLE III: Frequency distribution of the number of letters in surname

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} \tag{25.1.4}$$

$$\text{Mean} = \frac{832}{100} \tag{25.1.5}$$

$$\text{Mean} = 8.32 \tag{25.1.6}$$

$$\text{Median} = 7 + \frac{50 - 36}{40} \times 3 \tag{25.1.7}$$

$$\text{Median} = 7 + 1.05 = 8.05 \tag{25.1.8}$$

Hence median no. of letters is 8.05

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h \tag{25.1.9}$$

Modal class is the interval with the highest frequency = 7-10, where
l is lower limit of the modal class = 7
h is the class interval = 4-1=3
f_1 is the frequency of the modal class = 40
f_0 is frequency of the class before modal class = 30

f_2 is frequency of the class after modal class = 16

$$\text{Mode} = 7 + \frac{40 - 30}{80 - 30 - 16} \times 3 \qquad (25.1.10)$$

which is calculated as 7.88

26 Question 2.2.22

26.1 Problem

26.1. Given the frequency distribution of the weights of 30 students of a class, find the median weight of the class. **Solution:** The following python code computes the mean, median and mode.

```
codes/statistics/exercises/q22.py
```

$$\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h \qquad (26.1.1)$$

$$\text{n} = \sum f_i = 100 \implies \frac{n}{2} = 50 \qquad (26.1.2)$$

$$(26.1.3)$$

$\therefore$ 55-60 is the median class.
Here l is the lower limit of the median class = 55
h is the classinterval =5
cf is the cumulative frequency of the class before median class = 13
f is the frequency of the median class

$$\text{Median} = 55 + \frac{15 - 13}{6} \times 5 \qquad (26.1.4)$$

$$\text{Median} = 55 + 1.67 = 56.67 \qquad (26.1.5)$$

Hence median weight is 56.67

27 Question 2.2.23

27.1 Problem

27.1. The following distribution gives the daily income of 50 workers in a factory. Convert this distribution to a less than type cumulative frequency distribution and draw its ogive. **Solution:**
The following python code generates the required ogive.

```
./codes/statistics/exercises/q23.py
```

weight	no.of.student	Cum.Freq
40-45	2	2
45-50	3	5
50-55	8	13
55-60	6	19
60-65	6	25
65-70	3	28
70-75	2	30
total	30	

TABLE IV: Frequency distribution of the weights of students

Dailywages	workers	Cum.Freq
100-120	12	12
120-140	14	12+14=26
140-160	8	26+8=34
160-180	6	34+6=40
180-200	10	40+10=50
Total		50

TABLE V: Wages obtained by workers

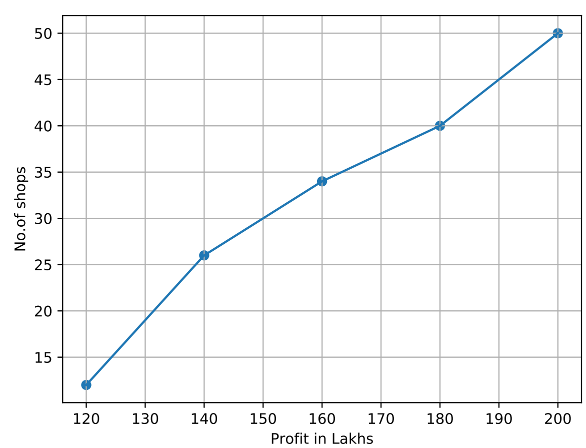

Fig. 1: Ogive of Q.23

28 Question 2.2.24

28.1 Problem

28.1. Given the weights of 35 students of a class as less than cumulative distribution find the median weight of the class.
Solution: The following python code computes the median .

wages	workers
Less than 120	12
Less than 140	26
Less than 160	34
Less than 180	40
Less than 200	50

TABLE VI: Wages obtained using less than cumulative frequency

codes/statistics/exercises/q24.py

Weight	No.of.student
<38	0
<40	3
<42	5
<44	9
<46	14
<48	28
<50	32
<52	35

TABLE VII: Frequency distribution of the weights of students

$$\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h \tag{28.1.1}$$

$$\text{n} = \sum f_i = 100 \implies \frac{n}{2} = 50 \tag{28.1.2}$$

$$\tag{28.1.3}$$

∴ 46-48 is the median class.
Here l is the lower limit of the median class = 46
h is the classinterval =2
cf is the cumulative frequency of the class before median class = 14
f is the frequency of the median class =14

$$\text{Median} = 46 + \frac{17.5 - 14}{14} \times 2 \tag{28.1.4}$$

$$\text{Median} = 46 + 0.5 = 46.5 \tag{28.1.5}$$

Hence median weight is 46.5

weight	No.of.student	Cum.Freq
0-38	0	0
38-40	3	3
40-42	2	5
42-44	4	9
44-46	5	14
46-48	14	28
48-50	4	32
50-52	3	35

TABLE VIII: Frequency distribution of the weights of students

29 Question 2.2.25

29.1 Problem

29.1. Given the production yield per hectare of wheat of 100 farms of a village. Change the distribution to more than type distribution and draw its ogive. **Solution:** The following python code generates the required ogive.

./codes/statistics/exercises/q25.py

Prodn.Yield	No.of.farms
50-55	2
55-60	8
60-65	12
65-70	24
70-75	38
75-80	16
total	100

TABLE IX: production yield per hectare of wheat of 100 farms

30 Question 2.2.26

30.1 Problem

30.1. Give five examples of data that you can collect in your daily life.
Solution:
a) Weights of students of a class
b) No. of plants in a locality
c) No. of boys and girls in a class
d) Runs scored in a match
e) Rainfall in the city in the past decade

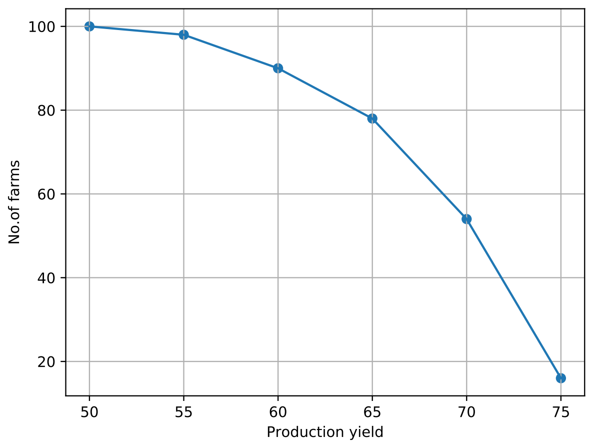

Fig. 2: Ogive of Q.25

Prodn.yield	No.of.farms
More than 50	100
More than 55	100-2=98
More than 60	98-8=90
More than 65	90-12=78
More than 70	78-24=54
More than 75	54-38=16

TABLE X: production yield using more than cumulative frequency

31 Question 2.2.27

31.1 Problem

31.1. For the examples given in the previous question classify them as primary data or secondary data
Solution: Primary data is the data collected by ourselves and secondary data is the information gathered from other sources.

a) Weights of students of a class - Primary
b) No. of plants in a locality - Primary
c) No. of boys and girls in a class - Primary
d) Runs scored in a match - Secondary
e) Rainfall in the city in the past decade - Secondary

32 Question 2.2.28

32.1 Problem

32.1. The blood groups of 30 students of Class VIII are recorded as follows: A, B, O, O, AB, O, A, O, B, A, O, B, A, O, O, A, AB, O, A, A, O, O, AB, B, A, O, B, A, B, O. Represent this data in the form of a frequency distribution table. Which is the most common, and which is the rarest, blood group among these students?
Solution: As we can see from table XI, the most common blood group is O and the rarest blood group is AB.

BloodGroup	No.of.Student
A	9
B	6
AB	3
O	12
Total	30

TABLE XI: production yield per hectare of wheat of 100 farms

33 Question 2.2.29

33.1 Problem

33.1. The distance (in km) of 40 engineers from their residence to their place of work were found as follows: 5,3,10,20, 25, 11, 13, 7, 12, 31,19, 10, 12, 17, 18, 11, 32, 17, 16, 2,7,9,7,8,3,5,12, 15, 18 ,3,12, 14, 2,9,6, 15, 15, 7,6, 12.Construct a grouped frequency distribution table with class size 5 for the data given above taking the first interval as 0-5 (5 not included). What main features do you observe from this tabular representation?
Solution: Since the minimum =2 and maximum = 32, we take the intervals as 0-5,5-10 and so on upto 30-35. It can be observed that there are very few engineers whose homes are at more than or equal to 20km distance from their work place and most engineers have their workplace at distance of 0km to 15km from their homes.

34 Question 2.2.30

34.1 Problem

34.1. The relative humidity (in %) of a certain city for a month of 30 days was as follows:98.1,98.6,99.2,90.3,86.5,95.3,92.9,96.3,94.2,95.1, 89.2,92.3,97.1,93.5,92.7,95.1,97.2,93.3,95.2,97.3, 96.2,92.1,84.9,90.2,95.7,98.3,97.3,96.1,92.1,89.0. Construct a grouped frequency distribution table with classes 84 - 86, 86 - 88, etc. Which

Distance(km)	No.of.engineers
0-5	5
005-10	11
10-15	11
15-20	9
20-25	1
25-30	1
30-35	2
Total	40

TABLE XII: distance of engineers from their residence to their place of work

month or season do you think this data is about? What is the range of this data?
Solution: Since the minimum =84.9 and maximum = 99.2, we take the intervals as 84-86,86-88 and so on upto 98-100. As the relative humidity is high, the data is about rainy season.

$$\text{Range} = \text{Maximum value - Minimum value} \quad (34.1.1)$$

$$\text{Range} = 99.2 - 84.9 \quad (34.1.2)$$

$$\text{Range} = 14.3 \quad (34.1.3)$$

RelativeHumidity	NO.of.days
84-86	1
86-88	1
88-90	2
90-92	2
92-94	7
94-96	6
96-98	7
98-100	4
Total	30

TABLE XIII: Relative humidity in %