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JEE 2018 through Python

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***Abstract*—This manual is a collection of math problems from the JEE 2018 mains paper. These problems can be solved using the Python scripts in the JEE 2016 manual. This will give the student enough practice in Python programming.**

1. If $\alpha, \beta \in \mathbf{C}$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, thean $\alpha^{101} + \beta^{107}$ is equal to:
   (1) -1
   (2) 0
   (3) 1
   (4) 2
2. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series
   $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$
   If $B - 2A = 100\lambda$, then λ is equal to:
   (1) 232
   (2) 248
   (3) 464
   (4) 496
3. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is:
   (1) 6
   (2) $\frac{7}{2}$
   (3) 4
   (4) $\frac{9}{2}$
4. Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x - \frac{1}{x}$, $x \in \mathbf{R} - \{-1, 0, 1\}$. If $h(x) = \frac{f(x)}{g(x)}$, then the local minimum value of $h(x)$ is :
   (1) 3
   (2) -3
   (3) $-2\sqrt{2}$
   (4) $2\sqrt{2}$
5. Let $g(x) = cosx^2$, $f(x) = \sqrt{x}$, and $\alpha, \beta (\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve $y = (g \circ f)(x)$ and the lines $x = \alpha, x = \beta$ and $y = 0$, is :
   (1) $\frac{1}{2}(\sqrt{3} - 1)$
   (2) $\frac{1}{2}(\sqrt{3} + 1)$
   (3) $\frac{1}{2}(\sqrt{3} - \sqrt{2})$
   (4) $\frac{1}{2}(\sqrt{2} - 1)$
6. A straight line through a fixed point $(2, 3)$ intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle $OPRQ$ is completed, then the locus of R is:
   (1) $3x + 2y = 6$
   (2) $2x + 3y = xy$
   (3) $3x + 2y = xy$
   (4) $3x + 2y = 6xy$
7. Let the orthocentre and centroid of a triangle be $A(-3, 5)$ and $B(3, 3)$ respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is :
   (1) $\sqrt{10}$
   (2) $2\sqrt{10}$
   (3) $3\sqrt{\frac{5}{2}}$
   (4) $\frac{3\sqrt{5}}{2}$
8. If the tangent at $(1, 7)$ to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of c is :
   (1) 195
   (2) 185
   (3) 85
   (4) 95
9. Tangent and normal are drawn at $P(16, 16)$ on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is :
   (1) $\frac{1}{2}$
   (2) 2
   (3) 3
   (4) $\frac{4}{3}$
10. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q. If these tangents intersect at the point $T(0, 3)$ then the area (in sq. units) of $\triangle PTQ$ is:
    (1) $45\sqrt{5}$

(2) $54\sqrt{3}$
(3) $60\sqrt{3}$
(4) $36\sqrt{5}$

11. If sum of all the solutions of the equation $8\cos x \cdot \left(\cos\left(\frac{\pi}{6}+x\right)\cdot\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$ in $[0,\pi]$ is $k\pi$, then k is equal to:
(1) $\frac{2}{3}$
(2) $\frac{13}{9}$
(3) $\frac{8}{9}$
(4) $\frac{20}{9}$

12. The Boolean expression $\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to:
(1) $\sim p$
(2) p
(3) q
(4) $\sim q$

13. If the tangent at $(1,7)$ to the curve $x^2 = y-6$ touches the circle $x^2+y^2+16x+12y+c=0$ then the value of c is :
(1) 85
(2) 95
(3) 195
(4) 185

14. Let $S = \{x \in \mathbf{R} : x \geqslant 0$ and $2\,|\sqrt{x}-3\,| + \sqrt{x}(\sqrt{x}-6)+6=0\}$. Then S :
(1) contains exactly two elements.
(2) contains exactly four elements.
(3) is an empty set.
(4) contains exactly one element.

15. Two sets A and B are as under:
$A = \{(a,b) \in \mathbf{R}\times\mathbf{R} : |a-5| < 1 \; and \; |b-5| < 1\}$;
$B = \{(a,b) \in \mathbf{R}\times\mathbf{R} : 4(a-6)^2+9(b-5)^2 \le 36\}$.
Then :
(1) $A \cap B = \phi$ (an empty set)
(2) neither $A \subset B$ nor $B \subset A$
(3) $B \subset A$
(4) $A \subset B$

16. Let $S = \{t \in \mathbf{R} : f(x) = |x-\pi| \cdot (e^{|x|}-1) sin\,|x|$ is not differentiable at $t\}$. Then the set S is equal to:
(1) $\{\pi\}$
(2) $\{0,\pi\}$
(3) ϕ (an empty set)
(4) $\{0\}$

17. $\lim\limits_{x\to 0} \frac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$ equals:
(1) $\frac{1}{4}$
(2) 1
(3) $\frac{1}{2}$
(4) $-\frac{1}{2}$

18. Let $f(x) = \begin{cases} (x-1)^{\frac{1}{2-x}}, & x>1, x\neq 2 \\ k, & x=2 \end{cases}$

The value of k for which f is continuous at $x=2$ is:
(1) 1
(2) e
(3) e^{-1}
(4) e^{-2}

19. The sides of a rhombus $ABCD$ are parallel to the lines,$x-y+2=0$ and $7x-y+3=0$. If the diagonals of the rhombus intersect at $P(1,2)$ and the vertex A(different from the origin) is on the y-axis,then the ordinate of A is :
(1) $\frac{5}{2}$
(2) $\frac{7}{4}$
(3) 2
(4) $\frac{7}{2}$

20. The tangent to the circle $C_1 : x^2+y^2-2x-1=0$ at the point $(2,1)$ cuts off a chord of length 4 from a circle C_2 whose centre is $(3,-2)$. The radius of C_2 is :
(1) 2
(2) $\sqrt{2}$
(3) 3
(4) $\sqrt{6}$

21. Tangents drawn from the point $(-8,0)$ to the parabola $y^2 = 8x$ touch the parabola at P and Q. If F is the focus of the parabola, then the area of the triangle PFQ (in sq.units) is equal to :
(1) 24
(2) 32
(3) 48
(4) 64

22. A normal to the hyperbola, $4x^2-9y^2=36$ meets the co-ordinate axes x and y at A and B, respectively. If the parallelogram $OABP$ (O being the origin) is formed, then the locus of P is :
(1) $4x^2+9y^2=121$
(2) $9x^2+4y^2=169$
(3) $4x^2-9y^2=121$
(4) $9x^2-4y^2=169$

23. If the mean of the data : $7,8,9,7,8,7,\lambda,8$ is 8, then the variance of this data is:
(1) $\frac{7}{8}$

(2) 1
(3) $\frac{9}{8}$
(4) 2

24. The number of solutions of $\sin 3x = \cos 2x$, in the interval $\left(\frac{\pi}{2}, \pi\right)$ is :
(1) 1
(2) 2
(3) 3
(4) 4

25. The value of integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x} dx$ is
(1) $\pi\sqrt{2}$
(2) $\pi(\sqrt{2}-1)$
(3) $\frac{\pi}{2}(\sqrt{2}+1)$
(4) $2\pi(\sqrt{2}-1)$

26. n-digit numbers are formed using only three digits $2, 5$ and 7.The smallest value of n for which 900 such distinct numbers can be formed, is :
(1) 6
(2) 7
(3) 8
(4) 9

27. A circle passes through the points $(2, 3)$ and $(4, 5)$. If its centre lies on the line, $y - 4x + 3 = 0$,then its radius is equal to
(1) 2
(2) $\sqrt{5}$
(3) $\sqrt{2}$
(4) 1

28. Two parabolas with a common vertex and with axes along x-axis and y-axis, respectively, intersect each other in the first quadrant. If the length of the latus rectum of each parabola is 3, then the equation of the common tangent to the two parabolas is :
(1) $4(x + y) + 3 = 0$
(2) $3(x + y) + 4 = 0$
(3) $8(2x + y) + 3 = 0$
(4) $x + 2y + 3 = 0$

29. If the tangents drawn to the hyperbola $4y^2 = x^2 + 1$ intersect co-ordinate axes at the distinct points A and B, then the locus of the mid point of AB is :
(1) $x^2 - 4y^2 + 16x^2y^2 = 0$
(2) $x^2 - 4y^2 - 16x^2y^2 = 0$
(3) $4x^2 - y^2 + 16x^2y^2 = 0$
(4) $4x^2 - y^2 - 16x^2y^2 = 0$

30. If $tanA$ and $tanB$ are the roots of the quadratic equation, $3x^2 - 10x - 25 = 0$, then the value of 3 $\sin^2(A+B) - 10\sin(A+B)\cdot\cos(A+B) - 25\cos^2(A+B)$ is :
(1) -10
(2) 10
(3) -25
(4) 25

31. If $(p \wedge \sim q) \wedge (p \wedge r) \to \sim p \vee q$ is false, then the truth values of p, q and r are, respectively :
(1) F,T,F
(2) T,F,T
(3) T,T,T
(4) F,F,F

32. The least positive integer n for which $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^n = 1$, is :
(1) 2
(2) 3
(3) 5
(4) 6

33. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = A^{20}$. Then the sum of the elements of the first column of B is :
(1) 210
(2) 211
(3) 231
(4) 251

34. The number of numbers between $2,000$ and $5,000$ that can be formed with the digits $0, 1, 2, 3, 4$(repetition of digits is not allowed) and are multiple of 3 is :
(1) 24
(2) 30
(3) 36
(4) 48

35. The coefficient of x^2 in the expansion of the product
$(2 - x^2)\cdot((1 + 2x + 3x^2)^6 + (1 - 4x^2)^6)$ is :
(1) 107
(2) 106
(3) 108
(4) 155

36. The sum of the first 20 terms of the series
$1 + \frac{3}{2} + \frac{7}{4} + \frac{15}{8} + \frac{31}{16} + ...,$ is :
(1) $38 + \frac{1}{2^{19}}$
(2) $38 + \frac{1}{2^{20}}$

(3) $39 + \frac{1}{2^{20}}$
(4) $39 + \frac{1}{2^{19}}$

37. $\lim_{x \to 0} \frac{(27+x)^{\frac{1}{3}}-3}{9-(27+x)^{\frac{2}{3}}}$ equals :
(1) $\frac{1}{3}$
(2) $-\frac{1}{3}$
(3) $-\frac{1}{6}$
(4) $\frac{1}{6}$

38. Let M and m be respectively the absolute maximum ad the absolute minimum values of the function, $f(x) = 2x^3 - 9x^2 + 12x + 5$ in the interval $[0, 3]$. Then $M - m$ is equal to :
(1) 5
(2) 9
(3) 4
(4) 1

39. If the area of the region bounded by the curves, $y = x^2$, $y = \frac{1}{x}$ and the lines $y = 0$ and $x = t (t > 1)$ is 1 sq.unit, then t is equal to :
(1) $e^{\frac{3}{2}}$
(2) $\frac{4}{3}$
(3) $\frac{3}{2}$
(4) $e^{\frac{2}{3}}$

40. If a circle C, whose radius is 3, touches externally the circle,
$x^2 + y^2 + 2x - 4y - 4 = 0$ at the point $(2, 2)$, then the length of the intercept cut by this circle C, on the x-axis is equal to :
(1) $2\sqrt{5}$
(2) $3\sqrt{2}$
(3) $\sqrt{5}$
(4) $2\sqrt{3}$

41. Let P be a point on the parabola, $x^2 = 4y$. If the distance of P from the centre of the circle, $x^2 + y^2 + 6x + 8 = 0$ is minimum, then the equation of the tangent to the parabola at P, is :
(1) $x + 4y - 2 = 0$
(2) $x - y + 3 = 0$
(3) $x + y + 1 = 0$
(4) $x + 2y = 0$

42. If an angle A of a $\triangle ABC$ satisfies $5 \cos A + 3 = 0$, then the roots of the quadratic equation, $9x^2 + 27x + 20 = 0$ are :
(1) $\sec A, \cot A$
(2) $\sin A, \sec A$
(3) $\sec A, \tan A$
(4) $\tan A, \cos A$

43. If $p \rightarrow (\sim p \vee \sim q)$ is false, then the truth values of p and q are respectively :
(1) F,F
(2) T,F
(3) F,T
(4) T,T