My Presentation

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Question

Exercise 8.1(Q no.28)

In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B. Show that:

a)$\triangle AMC \cong \triangle BMD$

b)$\triangle DBC$ is a right angle.

c)$\triangle DBC \cong \triangle ABC$

d)CM = $\frac{1}{2}$ AB

Codes and Figures

The python code for the figure is

```
./code/traingle.py
```

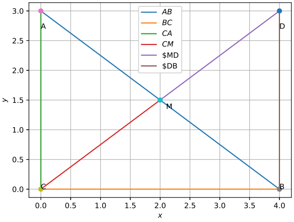
The latex- tikz code is

```
./figs/triangle.tex
```

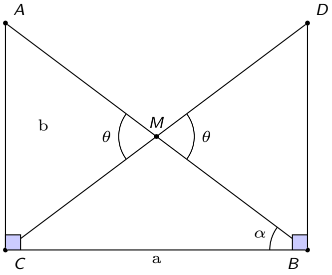
The above latex code can be compiled as standalone document

```
./figs/triangle_fig.tex
```

.



(a) By Python



(b) By Latex-tikz

Construction method

The tables below are the values used for constructing the triangles in both Python and Latex-Tikz.

Initial Input Values.	
a	4
b	3
$\angle(ACB)$	90°

Table: To construct $\triangle ACB$

The steps for constructing $\triangle ACB$ are

$$(i)\mathrm{C} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} (ii)\mathrm{A} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} (iii)\mathrm{B} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

Since, M is the midpoint of AB and CD

$$\mathrm{M} = (1/2)(\mathrm{A} + \mathrm{B})\mathrm{M} = \begin{pmatrix} 2 \\ 1.5 \end{pmatrix}$$

,

$$\mathrm{D} = 2\mathrm{M}\mathrm{D} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

Derived Values for $triangleDCB$.	
M	$\begin{pmatrix} 2 \\ 1.5 \end{pmatrix}$
D	$\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

Table: To construct $\triangle DCB$

Solution

From the figure, lets assume C to be the origin.

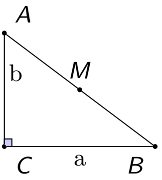


Figure: $\triangle ACB$

$C = 0, \|CA\| = b \, \|CB\| = a$
M is the position vector of mid-point of BA.
$CM = CB + BM$ $[BM = (1/2) * BA]$

$$CM = \begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} -a \\ b/2 \end{pmatrix}$$

Therefore,

$$CM = \begin{pmatrix} a/2 \\ b/2 \end{pmatrix}$$

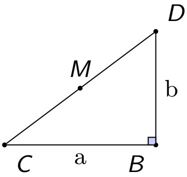


Figure: $\triangle DBC$

From the figure, $CD = 2(CM)$

$$CD = \begin{pmatrix} a \\ b \end{pmatrix}$$

Solution a)

$\triangle AMC$ and $\triangle DMB$ are congruent to each other by SAS congruency.
(i) Side AM is equal to the corresponding side BM [As M is midpoint of AB]
(ii)Side CM of is equal to corresponding side DM [As M is midpoint of DC]
(iii)$\angle AMC = \angle DMB$ [Vertically Opposite Angles]
Hence, proved

Solution b)

In $\triangle ACB$ $(\|\mathrm{BA}\|)^2 = a^2 + b^2$ Since $\angle ACB = 90^\circ$[Pythagorus theorem]
In $\triangle DBC$ $cos\angle DBC = [((a^2 + b^2 - (\|\mathrm{CD}\|)^2)/2ab)]$ With the given vector values we get norm of $(\|\mathrm{BA}\|) = (\|\mathrm{CD}\|)$
$cos\angle DBC = [((a^2 + b^2 - (\|\mathrm{CD}\|)^2)/2ab)]$ $cos\angle DBC$=0
Therefore, $\angle DBC$ is right angle

Solution c)

$\triangle ACB$ and $\triangle DCB$ are congruent to each other in SAS congruency.
(i)Both the triangles have a common base , a.
(ii)AC $=$ DB by using distance formula
(iii)$\angle ACB = \angle DBC = 90^\circ$ [From Solution b)]
Hence, proved.

Solution d)

Since CM is halfway of CD
$\|CM\| = \|CD\|$
From Solution b) it is clear that $\|CD\| = \|BA\|$
Therefore $\|CM\| = \frac{1}{2}\|AB\|$
Hence, proved.