1.

$$\frac{1}{2}\left\|\begin{pmatrix}1\\2\\3\end{pmatrix}\times\begin{pmatrix}-3\\-2\\1\end{pmatrix}\right\|$$

$$\begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ -3 & -2 & 1 \end{vmatrix}$$

$$\frac{1}{2}\sqrt{8^2+10^2+4^2}$$

$$\sqrt{4^2+5^2+2^2}$$

$$=\sqrt{25+16+4}$$

$$=\sqrt{45}=\underline{\underline{3\sqrt{5}}}\ \text{Ans.}$$

2. $\cos\left[\sin^{-1}\frac{2}{\sqrt{5}} + \cos^{-1}x\right] = 0$

$\Rightarrow \sin^{-1}\frac{2}{\sqrt{5}} + \cos^{-1}x = 2n\pi$

$\Rightarrow \cos^{-1}x = 2n\pi - \cos^{-1}\frac{1}{\sqrt{5}}$

$\Rightarrow x = \frac{1}{\sqrt{5}}$

3. $f(x) = x^2 e^{-x}$

$f'(x) = -x^2 e^{-x} + 2xe^{-x} > 0$

$\Rightarrow 2x - x^2 > 0$

$\Rightarrow x(2-x) > 0$

$\Rightarrow x > 0, \; 2-x > 0$

$\Rightarrow 0 < x < 2$ or $x \in (0, 2)$ Ans.

4. $f(x) = \dfrac{x-1}{x(x^2-1)}$

Function is discontinuous at $x=0$, $x=1$ and $x=-1$.

5. $f(x) = \cos x$ is not one to one but onto.

6. Let the desired coordinate be $\begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix}$

Then, $\left[\begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix}\right]^T \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix} = 0$

$\Rightarrow \begin{pmatrix} 2 & -3-y & 4 \end{pmatrix} \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix} = 0$

$\Rightarrow -y(y+3) = 0 \Rightarrow y = 0$ or $y = -3$

The desired point is $\begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix}$ Ans.

8. $\begin{pmatrix} 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = -1$

$\Rightarrow \cos\theta = -\dfrac{1}{2}$

or $\theta = \underline{\underline{120^\circ}}$ Ans.

$= \dfrac{2\pi}{3}$.

9. $A^2 = 3A$

$\Rightarrow \lambda^2 - 3\lambda = 0 \Rightarrow \lambda = 0,\ \lambda = 3.$

$\underline{\underline{\det(A) = 3}}$ Ans.

10. $\|a\| = 4,\ -3 \le \lambda \le 2$

$\|\lambda a\| = |\lambda|\,\|a\|$

$\Rightarrow \underline{\underline{8 < \|\lambda a\| < 12}}$ Ans.

11. $\frac{d}{dr}(\pi r^2) = 0.5$

$\Rightarrow 2\pi r = 0.5$

$\Rightarrow \frac{d}{dr}(2\pi r) = 0$ Ans.

12. $2x^2 - 18 = 0$

$\Rightarrow x = \pm 3$ Ans.

13. $P = \begin{pmatrix} 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

$P_0 = 0$, $P_1 = 16$, $P_3 = 20$

$P_4 = 23$, $P_5 = 18$

max P at $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$ Ans.

14. $y = \sec^{-1} x$.

$\sec y = x \Rightarrow \cos y = \frac{1}{x}$.

15. $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$ Ans.

16. $\underline{n} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ $c = 6.$

$$d = \frac{c}{\|\underline{n}\|} = \frac{6}{\sqrt{9}}$$

$= 2$ Ans.

17. $\begin{pmatrix} 1 & 0 & -3 \end{pmatrix} \left[\underline{x} - \begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix} \right] = 0$

$\Rightarrow \begin{pmatrix} 1 & 0 & -3 \end{pmatrix} \underline{x} = 10$ Ans.

18. $$\int_{-\pi/2}^{\pi/2} x \cos^2 x \, dx.$$

Integrand is odd. So sum is 0.

19. $x = 3k-1$

$y = 7k-4$

$z = 2k-4$

Line

$\Rightarrow \underline{r} = \begin{pmatrix} -1 \\ -4 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 7 \\ 2 \end{pmatrix}$ — ①.

<u>xy plane</u>:

$(0 \quad 0 \quad 1)\, \underline{r} = 0.$ — ②

Point of intersection for ① & ② is

$-4 + 2\lambda = 0 \Rightarrow \lambda = 2.$

or, $\underline{r} = \begin{pmatrix} 6 \\ 14 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ 10 \\ 0 \end{pmatrix}$ Ans

20. $$f(x) = \begin{cases} kx^2+5 & x \le 1 \\ 2 & x > 1 \end{cases}$$

is continuous if

$$k(1) + 5 = 2$$

$$\Rightarrow \underline{\underline{k = -3}} \quad \text{Ans.}$$

21. $$x\frac{dy}{dx} = 2x^2 + y$$

$$\frac{dy}{dx} \underbrace{- \frac{y}{x}}_{P} = \underbrace{2x}_{Q}.$$

$$P(x) = -\frac{1}{x}.$$

Integrating factor $e^{\int p(x)dx}$

$$= e^{-\int \frac{dx}{x}} = e^{-\ln x + C}$$

$$= \underline{\underline{c}} \quad \text{Ans.}$$

22. $$\frac{d \sec^2(x^2)}{d(x^2)} = \underline{\underline{2\sec(x^2)\tan(x^2)}}$$

Ans.

23. $y = f(x^2)$

$f'(x) = e^{\sqrt{x}}$

$$\frac{dy}{dx} = 2xf'(x^2)$$

$$= \underline{\underline{2xe^{x}}} \text{ Ans.}$$

24. $$\underline{r} = k\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$\|\underline{r}\| = 3\sqrt{3} = k\sqrt{3}$

$\Rightarrow k = 3$. Hence

$$\underline{r} = 3\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \text{ Ans.}$$

25. $\|\sqrt{3}\,\underline{a} - \underline{b}\|^2 = 1$

$\Rightarrow 3 + 1 - 2\sqrt{3}\,\underline{a}\cdot\underline{b} = 1$

$\Rightarrow \underline{a}\cdot\underline{b} = \frac{\sqrt{3}}{2}$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$ or $\theta = \frac{\pi}{6}$ Ans.

26. $A = \begin{pmatrix} -3 & 2 \\ 1 & -1 \end{pmatrix}$

$A^2 + I = kA$

$\Rightarrow \lambda^2 - k\lambda + 1 = 0$

$tr(A) = -4 = -k$

$\Rightarrow k = 4$ Ans.

27. $f(x) = \sqrt{\dfrac{\sec x - 1}{\sec x + 1}}$

$f'\left(\frac{\pi}{3}\right) = ?$

$f(x) = \sqrt{\dfrac{1 - \cos x}{1 + \cos x}}$

$= \tan \frac{x}{2}$

$\Rightarrow f'(x) = \frac{1}{2} \sec^2 \frac{x}{2}$

$\Rightarrow f'\left(\frac{\pi}{3}\right) = \frac{1}{2} \sec^2\left(\frac{\pi}{6}\right)$

$= \frac{1}{2} \times \frac{4}{3}$

$= \frac{2}{3}$ Ans.

28. $P: \begin{pmatrix} 1 & -5 & -2 \end{pmatrix} \underline{x} = 1$

$x = 3y + 5$

$y = y$

$z = 2 - y$

$L:$

$\Rightarrow \underline{x} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} + y \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$

$\begin{pmatrix} 1 & -5 & -2 \end{pmatrix} \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} = 1$

$\begin{pmatrix} 1 & -5 & -2 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} = 3 - 5 + 2$

$= \underline{\underline{0}}$

Hence, L satisfies P.

29. $f(x) = (\tan x)^{\tan x}$

$f'(x) = \tan x \, (\tan x)^{\tan x - 1}$

$+ (\tan x)^{\tan x} \ln \tan x$

Ans.

30. $I = \int \frac{\tan^3 x}{\cos^3 x} \, dx$

$= \int \frac{\sin^3 x}{\cos^6 x} \, dx$

$\cos x = y \Rightarrow -\sin x \, dx = dy$

$\Rightarrow I = -\int \frac{(1-y^2)}{y^6} \, dy$

$= 6y^{-7} - 4y^{-5} + C$

$= 6\sec^7 x - 4\sec^5 x + C$ Ans.

31. $x_1, x_2 \in \{1, 2, 3, 4, 5, 6\}$.

If $X \in \{0, 1, 2\}$ represents no. of 6s,

$$Pr(X = 0) = Pr(x_1 \neq 6, x_2 \neq 6)$$
$$= \left(\frac{5}{6}\right)^2.$$

$$Pr(X = 1) = Pr(x_1 = 6, x_2 \neq 6)$$
$$+ Pr(x_1 \neq 6, x_2 = 6)$$
$$= 2 \times \frac{1}{6} \times \frac{5}{6} = \frac{10}{6^2}.$$

$$Pr(X = 2) = Pr(x_1 = 6, x_2 = 6)$$
$$= \left(\frac{1}{6}\right)^2$$ Ans.

32.

$$I = \int_0^{\pi/2} \sin 2x \tan^{-1} \sin x \, dx$$

Let $\sin x = \tan\theta$.

$$\cos x \, dx = \sec^2\theta \, d\theta.$$

$$I = 2\int_0^{\pi/4} \theta \tan\theta \sec^2\theta \, d\theta$$

$u = \theta \quad dv = \sec^2\theta \tan\theta \, d\theta.$

$$\int \sec^2\theta \tan\theta \, d\theta = \frac{\tan^2\theta}{2}.$$

$$I = 2\left[\theta \frac{\tan^2\theta}{2}\right]_0^{\pi/4} - 2\int_0^{\pi/4} \frac{\tan^2\theta}{2} \, d\theta.$$

$$= \frac{\pi}{4} - \left[\tan\theta - \theta\right]_0^{\pi/4}$$

$$= \frac{\pi}{4} - \left(1 - \frac{\pi}{4}\right) = \underline{\underline{\frac{\pi}{2} - 1}} \text{ Ans.}$$

33. $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9}$

$= \tan^{-1}\dfrac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\cdot\frac{2}{9}}$

$= \tan^{-1}\dfrac{17}{34} = \underline{\underline{\tan^{-1}\frac{1}{2}}}$ — 1

$\frac{1}{2}\sin^{-1}\frac{4}{5} = \theta$

$\Rightarrow \sin^{-1}\frac{4}{5} = 2\theta$

$\Rightarrow \sin 2\theta = \frac{4}{5}$.

$\Rightarrow \dfrac{2\tan\theta}{1+\tan^2\theta} = \dfrac{4}{5}$

$\Rightarrow 4 + 4\tan^2\theta - 10\tan\theta = 0$

$\Rightarrow 4\tan^2\theta - 8\tan\theta - 2\tan\theta + 4 = 0$

$\Rightarrow 4\tan\theta(\tan\theta - 2) - 2(\tan\theta - 2) = 0$

$\Rightarrow \tan\theta = 2$ or $\underline{\underline{\tan\theta = \frac{1}{2}}}$ — 2

34. Let $\underline{n}^t \underline{x} = c$.

$$\underline{n}^t(\underline{p}_1 - \underline{p}_2) = 0$$

$$\underline{p}_1 - \underline{p}_2 = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} - \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} -2 \\ 1 \\ -2 \end{pmatrix}. \quad - (1)$$

$$\underline{n}^t \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = 0. \quad - (2)$$

$$\underline{n} = \begin{vmatrix} i & j & k \\ -2 & 1 & -2 \\ 2 & -1 & 1 \end{vmatrix}$$

$$= \begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix} \qquad c = \begin{pmatrix} -1 & -2 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$$

$$= \underline{\underline{-1}}$$

Equation of plane

$$\begin{pmatrix} 1 & 2 & 0 \end{pmatrix} \underline{x} = 1.$$

Distance from origin $= \dfrac{1}{\sqrt{5}}$ Ans.

35. $\tan^{-1} \dfrac{y}{x} = \ln \sqrt{x^2+y^2}$

$$\Rightarrow \tan^{-1} \frac{y}{x} = \frac{1}{2} \ln (x^2+y^2)$$

$$\Rightarrow \frac{1}{1+\frac{y^2}{x^2}} \times \frac{d}{dx}\left(\frac{y}{x}\right) = \frac{1}{2} \frac{\left(2x+2y\frac{dy}{dx}\right)}{x^2+y^2}$$

$$\Rightarrow x^2 \left(\frac{x\frac{dy}{dx} - y}{x^2}\right) = x+y\frac{dy}{dx}$$

$$\Rightarrow x\frac{dy}{dx} - y = x+y\frac{dy}{dx}$$

$$\Rightarrow (x-y)\frac{dy}{dx} = x+y \Rightarrow \frac{dy}{dx} = \frac{x+y}{x-y}$$ Ans.

36. $y = e^{a\cos^{-1}x}$

$\ln y = a\cos^{-1}x$

$$\Rightarrow \frac{dy}{y\,dx} = -\frac{a}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{ay}{\sqrt{1-x^2}}.$$

$$\Rightarrow (1-x^2)\,y_1^2 = a^2y^2$$

$$\Rightarrow (1-x^2)\,\cancel{2}\,y_1y_2 - \cancel{2}xy_1^{\cancel{2}}$$

$$= \cancel{2}a^2y\,\cancel{y_1}$$

$$\Rightarrow (1-x^2)\,y_2 - xy_1 - a^2y = 0$$

Ans.

37. $S = \pi r^2 + 2\pi r h$

$V = \pi r^2 h$

$\Rightarrow h = \dfrac{V}{\pi r^2}$.

$S = \pi r^2 + \dfrac{2\cancel{\pi}\cancel{r} V}{\cancel{\pi} r^{\cancel{2}}}$

$= \pi r^2 + \dfrac{2V}{r}$.

$\dfrac{dS}{dr} = 2\pi r - \dfrac{2V}{r^2} = 0$

$\Rightarrow r^3 = \dfrac{V}{\pi}$

or, $r = \left(\dfrac{V}{\pi}\right)^{1/3}$

$V = 125\pi \Rightarrow r = 5$.

$h = \dfrac{125}{25} = \underline{\underline{5}}$. Ans.

38. $$\frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 6 \\ 0 & 5 & 3 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & 2 \\ 0 & 5 & -2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 5 & -2 \end{vmatrix}$$

$$= \underline{\underline{7}}$$ Ans.

39.

$$\begin{pmatrix} 5 & -1 & 4 & 5 \\ 2 & 3 & 5 & 2 \\ 5 & -2 & 6 & -1 \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} 5 & -1 & 4 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & -2 & 6 \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -2 \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & & -2 \end{pmatrix}$$

Ans

40.

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-y & x^2-y^2 & x^3-y^3 \\ y-z & y^2-z^2 & y^3-z^3 \\ z & z^2 & 1+z^3 \end{vmatrix}$$

$$= (x-y)(y-z) \begin{vmatrix} 1 & x+y & x^2+xy+y^2 \\ 1 & y+z & y^2+yz+z^2 \\ z & z^2 & 1+z^3 \end{vmatrix}$$

$$= (x-y)(y-z)(x-z) \begin{vmatrix} 0 & 1 & x+z+y \\ 1 & y+z & y^2+yz+z^2 \\ z & z^2 & 1+z^3 \end{vmatrix}$$

$$\begin{vmatrix} 0 & 1 & x+z+y \\ 1 & y+z & y^2+yz+z^2 \\ z & z^2 & 1+z^3 \end{vmatrix}$$

$$= y^2 z + yz^2 + \cancel{z^3} - 1 - \cancel{z^3}$$

$$+ (x+y+z)\left(\cancel{z^2} - yz - \cancel{z^2}\right)$$

$$= yz\left(\cancel{y+z} - x - \cancel{y-z}\right) - 1$$

$$= -1 - xyz = 0$$

$$\Rightarrow \underline{\underline{1 + xyz = 0}} \quad \text{Ans.}$$

Q1. Let X_1, X_2, X_3 be the events representing draw of cards. Let K denote king.

$$\Pr(X_1 = K) = \frac{4}{52} = \frac{1}{13} \quad \text{— (1)}$$

$$\Pr(X_1 \neq K) = \frac{12}{13}.$$

$$\Pr(X_2 = K \mid X_1 = K) = \frac{3}{51} = \frac{1}{17} \quad \text{— (2)}$$

$$\Pr(X_2 = K \mid X_1 \neq K) = \frac{4}{51} \quad \text{— (3)}$$

$$\Pr(X_3 = K \mid X_2 = K, X_1 = K) = \frac{2}{50} = \frac{1}{25} \quad \text{— (4)}$$

$$\Pr(X_3 = K \mid X_2 = K, X_1 \neq K) = \frac{3}{50} \quad \text{— (5)}$$

Then.

$$\Pr(X_1 = K \mid X_2 = K, X_3 = K)$$

$$= \frac{\Pr(X_1 = K, X_2 = K, X_3 = K)}{\Pr(X_2 = K, X_3 = K)}$$

$$= \frac{\Pr\left(\prod_{i=1}^{3} X_i\right)}{\Pr\left(\prod_{i=1}^{3} X_i\right) + \Pr\left(X_1 \neq K, \prod_{i=2}^{3} X_i\right)}$$

$$\Pr\left(\prod_{i=1}^{3} X_i\right) = \Pr\left(X_3 = k \,\middle|\, \prod_{i=1}^{2} X_i = k\right) \Pr\left(\prod_{i=1}^{2} X_i = k\right)$$

$$= \Pr\left(X_3 = k \,\middle|\, \prod_{i=1}^{2} X_i = k\right) \Pr\left(X_2 = k \mid X_1 = k\right) \Pr\left(X_1 = k\right)$$

$$= \frac{1}{25} \times \frac{1}{17} \times \frac{1}{13}.$$

$$\Pr\left(\prod_{i=2}^{3} X_i,\ X_1 \neq k\right) = \Pr\left(X_3 = k \mid X_1 \neq k, X_2 = k\right) \Pr\left(X_1 \neq k, X_2 = k\right)$$

$$= \Pr\left(X_3 = k \mid X_1 \neq k, X_2 = k\right) \Pr\left(X_2 = k \mid X_1 \neq k\right) \Pr\left(X_1 \neq k\right)$$

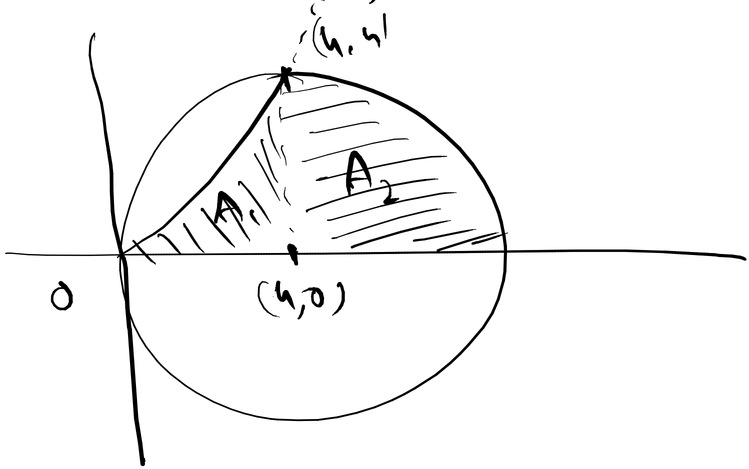
$$= \frac{\cancel{3}}{{}_{25}\cancel{50}} \times \frac{\cancel{4}\,2}{{}_{17}\cancel{51}} \times \frac{12}{13} = \frac{24}{25 \cdot 17 \cdot 13}.$$

42. Circle: $x^2+y^2 = 8x$

$$\|\underline{x}-\underline{y}\|^2 = r^2$$

$$\Rightarrow \|\underline{x}\|^2 - 2\underline{y}^T\underline{x} + \|\underline{y}\|^2 - r^2 = 0$$

$$\underline{y} = \begin{pmatrix}4\\0\end{pmatrix} \quad r = \|\underline{y}\| = 4.$$



<u>Parabola</u>: $y^2 = 4x$

$$\Rightarrow \underline{x}^T \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix} \underline{x} = 0.$$

$$A_2 = \frac{1}{4}\pi \times (4)^2 = \underline{\underline{4\pi}}.$$

$$A_1 = \int_0^4 2\sqrt{x}\,dx = \frac{4}{3} x^{3/2}\Big|_0^4 = \frac{32}{3} \qquad A = \underline{\underline{4\pi + \frac{32}{3}}} \text{ Ans}$$

43. $u = 1 + e^{y/x}$

$$v = e^{y/x}\left(1 - \frac{y}{x}\right)$$

$$u\,dy + v\,dx = 0.$$

$$y = xt$$

$$\Rightarrow dy = x\,dt + t\,dx.$$

$$(1+e^t)(x\,dt + t\,dx)$$
$$+ e^t(1-t)\,dx = 0$$

$$\Rightarrow (1+e^t)x\,dt + dx\left(\cancel{te^t} + t + e^t - \cancel{te^t}\right) = 0$$

$$\Rightarrow (1+e^t)x\,dt + (t+e^t)\,dx = 0$$

$$\Rightarrow \frac{(1+e^t)\,dt}{t+e^t} + \frac{dx}{x} = 0.$$

$\Rightarrow \ln(t+e^t) + \ln x = 0$

$\Rightarrow t+e^t = ce^{-x}$

$\Rightarrow \dfrac{y}{x} + e^{y/x} = ce^{-x}$ Ans.