Solution For Problemes On Probability and Statics

Yogesh Choudhary

***Abstract*—This document includes different problems and solution on probability and statics.It also provides the imformation about the python and latex codes of figures.**

Download all python codes from

svn co https://github.com/yogi13995/yogesh_training/tree/master/Geometry/probability/codes

and latex-tikz codes from

svn co https://github.com/yogi13995/yogesh_training/tree/master/Geometry/probability/figures

1 Probablity

1.1 Problem1

1.1.1 question:

1. In a cricket match, a batswoman hits a boundary 6 times out of 30 balls she plays. Find the probability that she did not hit a boundary.

1.1.2 Solution:

1. let assume $P(A)$ be the probability of hitting 6 so

$$\mathbf{P(A)} = \frac{6}{30} \tag{1.1.1.1}$$

$$= \frac{1}{5} \tag{1.1.1.2}$$

$P(B)$ be the probability of not hitting the boundry

$$P(B) = 1 - P(A) \tag{1.1.1.3}$$

$$= 1 - \frac{1}{5} \tag{1.1.1.4}$$

$$= \frac{4}{5} \tag{1.1.1.5}$$

codes for the above equation can be get from here

codes/prob/prob1.py

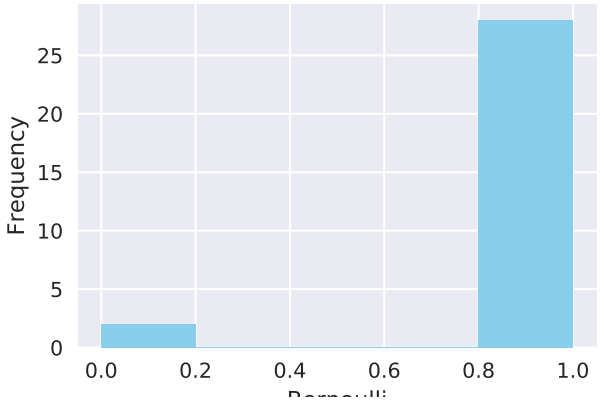

Fig. 1.1.1: bernoulli distribution1

codes/prob/prob1.py

1.2 Problem2

1.2.1 question:

1. 1500 families with 2 children were selected randomly, and the following data were recorded:

No.of girls in a family	2	1	0
No. of families	475	814	211

Compute the probability of a family, chosen at random, having
(i)2 girls
(ii) 1 girl
(iii) No girl
Also check whether the sum of these probabilities is 1.

1.2.2 Solution:

1. probability of having two girls in a family

$$= \frac{\text{Favourable cases}}{\text{total cases}} \tag{1.2.1.1}$$

$$= \frac{\text{No. of families having 2 girls}}{\text{total No. of families}} \tag{1.2.1.2}$$

Let assume that the probability of chosen fam-

ily will have 2 girls be $P(A)$ so

$$P(A) = \frac{475}{1500} \tag{1.2.1.3}$$
$$= 0.316 \tag{1.2.1.4}$$

2. probability of having one girl in a family

$$= \frac{\text{No. of families having 1 girl}}{\text{total No. of families}} \tag{1.2.2.1}$$

Let assume that the probability of chosen family will have 1 girl be $P(B)$ so

$$P(B) = \frac{814}{1500} \tag{1.2.2.2}$$
$$= 0.5427 \tag{1.2.2.3}$$

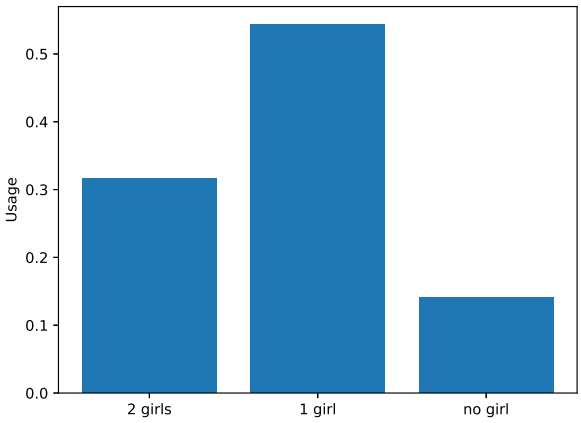

Fig. 1.2.3: probability 2

figs/prob/prob2.py

3. probability of having one girl in a family

$$= \frac{\text{No. of families having no girl}}{\text{total No. of families}} \tag{1.2.3.1}$$

Let assume that the probability of chosen family will have no girl be $P(C)$ so

$$P(C) = \frac{211}{1500} \tag{1.2.3.2}$$
$$= 0.1407 \tag{1.2.3.3}$$

$$P(A) + P(B) + P(C) = 0.316 + 0.5427 + 0.1407 \tag{1.2.3.4}$$
$$= 1 \tag{1.2.3.5}$$

codes for the above equation can be get from here

codes/prob/prob2.py

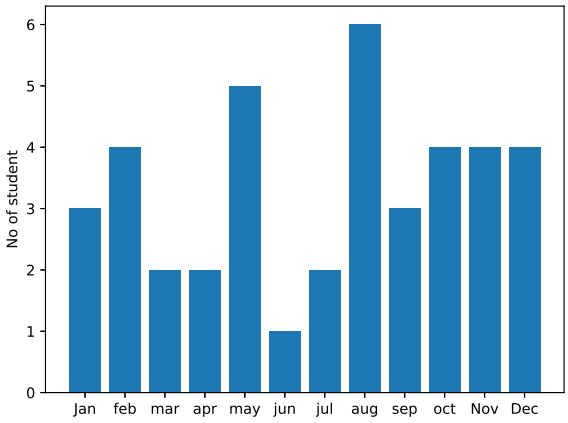

Fig. 1.3.0: student birth figure

a student to be of august class be P(A)

$$P(A) = \frac{6}{40} \qquad = 0.15 \tag{1.3.1.1}$$

codes for the above equation can be get from here

codes/prob/prob3.py

1.3 Problem3

1.3.1 question:

1. Find the probability that a student of the class was born in August.

1.3.2 Solution:

1. Total no of the studend in a year = 40 no of student of class August = 6 let probabilaty of

1.4 Problem4

1.4.1 question:

1. Three coins are tossed simultaneously 200 times with the following frequencies of different outcomes:

Outcome	3 heads	2 heads	1 head	No head
Frequency	23	72	77	28

If the three coins are simultaneously tossed again, compute the probability of 2 heads coming up.

1.4.2 Solution:

1. let probability of head when a single coin is tossed = p
   let probability of tails when a single coin is tossed = q
   probability of two head when three coins are tossed = P(A)

$$p = \frac{1}{2} \tag{1.4.1.1}$$

$$q = \frac{1}{2} \tag{1.4.1.2}$$

$$P(A) = {}^{n}C_{k} \times (p)^{n} (q)^{1-n} \text{ (binomial theoram)} \tag{1.4.1.3}$$

$$P(A) = {}^{3}C_{2} \times \left(\frac{1}{2}\right)^{2} \left(\frac{1}{2}\right)^{1} \tag{1.4.1.4}$$

$$= \frac{3}{8} \tag{1.4.1.5}$$

codes for the above equation can be get from here

codes/prob/prob4.py

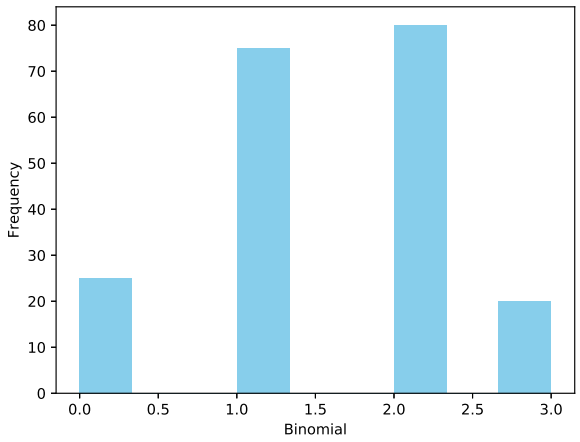

Fig. 1.4.1: binomial distribution of toss of three coins

figs/prob/prob4.py

1.5 Problem5

1.5.1 question:

1. Refer the table given below.

Marks	Number of students
0-20	7
20-30	10
30-40	10
40-50	20
50-60	20
60-70	15
70-above	8
Total	90

(i) Find the probability that a student obtained less than 20% in the mathematics test.
(ii) Find the probability that a student obtained marks 60 or above.

1.5.2 Solution:

1. Total n0 of student = 90
   no of students who obtained marks less than 20% = 7
   assume that P(A) is the probability of the students obtained less than 20% marks

$$P(A) = \frac{7}{90} \tag{1.5.1.1}$$

$$= 0.07 \tag{1.5.1.2}$$

2. no of the students obtained 60-70 marks = 15
   no of the student obtained 70 above marks = 8
   P(B)= probability of a student obtained 60 0r above marks

$$p(B) = \frac{15+8}{90} \tag{1.5.2.1}$$

$$= 0.256 \tag{1.5.2.2}$$

codes/prob/prob5.py

1.6 Problem6

1.6.1 question:

1. To know the opinion of the students about the subject statistics, a survey of 200 students was conducted. The data is recorded in the following table.

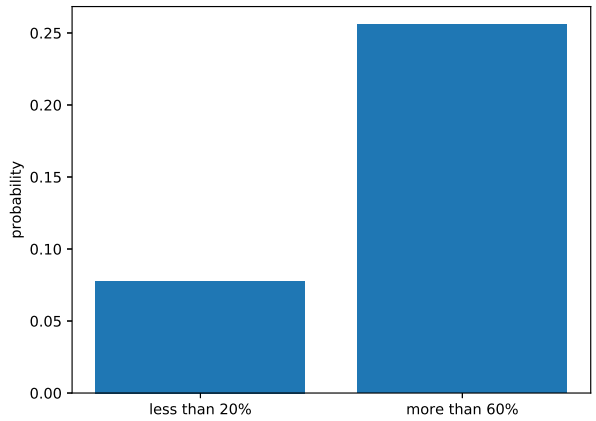

Fig. 1.5.2: probability of marks of students

figs/prob/prob5.py

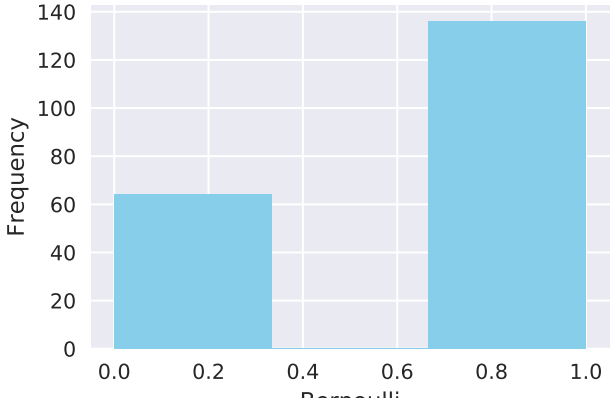

Fig. 1.6.2: bernouli distribution of students liking statics

figs/prob/prob6.py

Opinion	Number of students
like	135
dislike	65

Find the probability that a student chosen at random
(i)likes statistics,
(ii) does not like it.

1.6.2 Solution:

1. No of the total students participating the survey= 200
   students like the statics = 135
   P(A) = probability of student likes the statics

$$P(A) = \frac{135}{200} \tag{1.6.1.1}$$
$$= 0.675 \tag{1.6.1.2}$$

2. No of students do not like the statics = 65
   P(B) = probability of student does not like the statics

$$P(B) = \frac{65}{200} \qquad = 0.325 \tag{1.6.2.1}$$

   codes for the above equation can be get from here

codes/prob/prob6.py

1.7 Problem7

1.7.1 question:

1. Refer the table below

5	3	10	20	25	11	13	7	12	31
19	10	12	17	18	11	32	17	16	2
7	9	7	8	3	5	12	15	18	3
12	14	2	9	6	15	15	7	6	12

   What is the empirical probability that an engineer lives:
   (i) less than 7 km from her place of work?
   (ii)more than or equal to 7 km from her place of work?
   (iii) within $\frac{1}{2}$km from her place of work?

1.7.2 Solution:

1. total no of people working at the work place = 40 no of people live less than 7km from the work place = 9 let probability of a emgineer livinf less than 7 km from workplace = P(A)

$$P(A) = \frac{9}{40} \tag{1.7.1.1}$$

2. no of people live more than or equal 7km from the work place = 31 let probability of a emgineer livinf less than 7 km from workplace = P(B)

$$P(B) = \frac{31}{40} \tag{1.7.2.1}$$

3. there is no one who live within $\frac{1}{2}$ km from the work place so the probability will be 0.
   codes for the above equation can be get from

here

codes/prob/prob7.py

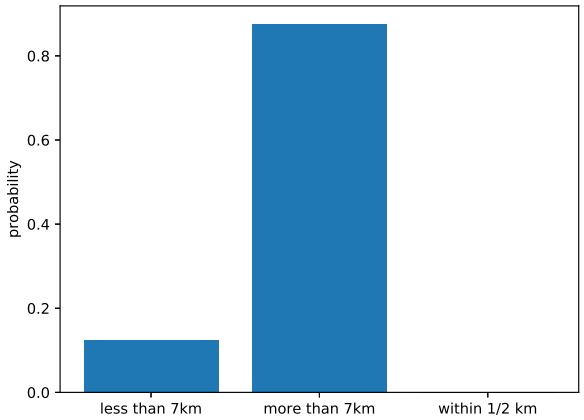

Fig. 1.7.3: probabilities a man to be near from work place

figs/prob/prob7.py

1.8 Problem8

1.8.1 question:

1. An organisation selected 2400 families at random and surveyed them to determine a relationship between income level and the number of vehicles in a family. The information gathered is listed in the table:
   ??

Monthly income (in ₹)	**vehicles per family**			
	0	**1**	**2**	**Above 2**
Less than 7000	10	160	25	0
7000-10000	0	305	27	2
10000-13000	1	535	29	1
13000-16000	2	469	59	25
16000 or more	1	579	82	88

   Suppose a family is chosen.Find the probability that the family chosen is
   (i) earning ₹10000 ₹13000 per month and owning exactly 2 vehicles.
   (ii) earning ₹16000 or more per month and owning exactly 1 vehicle.
   (iii) earning less than ₹7000 per month and does not own any vehicle.
   (iv) earning ₹13000 ₹16000 per month and owning more than 2 vehicles.
   (v) owning not more than 1 vehicle.

1.8.2 Solution:

1. no of total families chosen for survey = 2400
   No of families owning 2 vehicles and earning of ₹10000 - ₹13000 per month = 1

$$P(A) = \frac{29}{2400} = 0.012 \quad (1.8.1.1)$$

2. No of families earning ₹16000 and more per month and owning exactly 1 vehicle = 579
   P(B) = probability of a family to have 1 vehicle with earning of ₹16000 and more

$$P(B) = \frac{579}{2400} \quad (1.8.2.1)$$
$$= 0.241 \quad (1.8.2.2)$$

3. No of families earning less than ₹7000 per month and owning exactly no vehicle = 10
   P(C) = probability of a family to have no vehicle with earning less than ₹7000

$$P(C) = \frac{10}{2400} \quad (1.8.3.1)$$
$$= 0.0042 \quad (1.8.3.2)$$

4. No of families earning ₹13000 to ₹16000 per month and owning more than 2 vehicle = 25
   P(D) = probability of a family to have more than 2 vehicles with earning ₹13000 to ₹16000 per month

$$P(D) = \frac{25}{2400} \quad (1.8.4.1)$$
$$= 0.0104 \quad (1.8.4.2)$$

5. No of families owning not more than 1 vehicle
   →
   earning less than ₹7000 = 170
   earning ₹7000 to ₹10000 = 305
   earning ₹7000 to ₹10000 = 536
   earning ₹7000 to ₹10000 = 471
   earning ₹7000 to ₹10000 = 580
   total no of families = 1892

   P(E) = probability of a family to have

not more than 1vehicle

$$P(E) = \frac{1892}{2400} \tag{1.8.5.1}$$
$$= 0.78833 \tag{1.8.5.2}$$

codes for the above equation can be get from here

codes/prob/prob8.py

1.9 Problem9

1.9.1 question:

1. Eleven bags of wheat flour, each marked 5 kg, actually contained the following weights of flour (in kg):
   4.97 5.05 5.08 5.03 5.00 5.06 5.08 4.98 5.04 5.07 5.00
   Find the probability that any of these bags chosen at random contains more than 5 kg of flour.

1.9.2 Solution:

1. No of bags having weight more than 5 Kg = 7 total no of bags = 11

$$P(A) = \frac{7}{11} \tag{1.9.1.1}$$
$$= 0.636 \tag{1.9.1.2}$$

codes for the above equation can be get from here

codes/prob/prob9.py

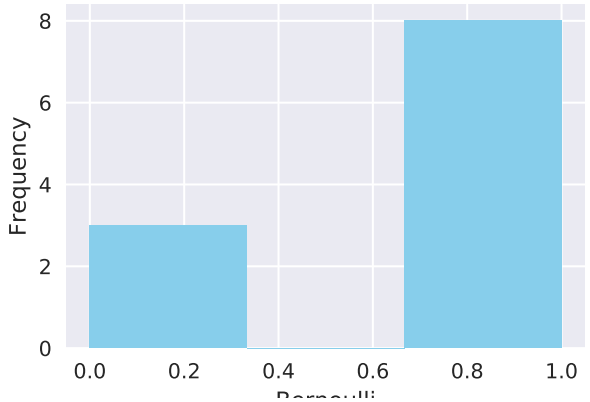

Fig. 1.9.1: probability of bag to be more than 5 Kg

figs/prob/prob9.py

1.10 Problem10

1.10.1 question:

1. Prepare a frequency distribution table, regarding the concentration of sulphur dioxide in the air in parts per million of a certain city for 30 days.

0.03	0.08	0.08	0.09	0.04	0.17
0.16	0.05	0.02	0.06	0.18	0.20
0.11	0.08	0.12	0.13	0.22	0.07
0.08	0.01	0.10	0.06	0.09	0.18
0.11	0.07	0.05	0.07	0.01	10.04

   Using this table, find the probability of the concentration of sulphur dioxide in the interval 0.12 - 0.16 on any of these days.

1.10.2 Solution: =

1. P(A) be the prbability of concentration of sulpher

concentration of sulphur	friquency
0.01	2
0.02	1
0.03	1
0.04	2
0.05	2
0.06	2
0.07	3
0.08	4
0.09	2
0.10	1
0.11	2
0.12	1
0.13	1
0.16	1
0.17	1
0.18	2
0.20	1
0.22	1

TABLE 1.10.1: This is a table template

$$p(A) = \frac{1+1+1}{30} \tag{1.10.1.1}$$
$$= 0.1 \tag{1.10.1.2}$$

codes for the above equation can be get from here

codes/prob/prob10.py

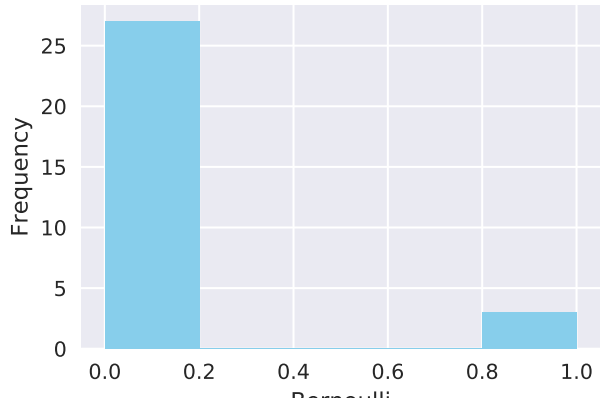

Fig. 1.10.1: probability of SO_2 0.12 to 0.16

figs/prob/prob10.py

2 Statics

2.1 Problem1

2.1.1 question:

1. A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of plants	0-2	2-4	4-6
Number of houses	1	2	1
Number of plants	6-8	0-10	10-12
Number of houses	5	6	2

2.1.2 Solution:

No of mangoes	No of boxes	midpoint	f.x
50-52	15	51	765
53-55	110	54	5490
56-58	135	57	7695
59-61	115	60	6900
62-64	25	63	1575
	$\sum f$=400		$\sum f.x$=22425

TABLE 2.1.1: To construct $\angle QAB$

TABLE 2.1.1: friquency distribution table

Daily wages	500-520	520-540	540-560
No of workers	12	14	8
Daily wages	560-580	580-600	
No of workers	6	10	

1.

$$\sum f = 17 \tag{2.1.1.1}$$

$$\sum f.x = 123 \tag{2.1.1.2}$$

$$Mean = \frac{\sum f.x}{\sum f} \tag{2.1.1.3}$$

$$= \frac{123}{17} \tag{2.1.1.4}$$

codes for the above equations can be get from

codes/static1.py

2.2 Problem2

2.2.1 question:

1. Consider the following distribution of daily wages of 50 workers of a factory. Find the mean daily wages of the workers of the factory by using an appropriate method.

2.2.2 Solution:

Daily wages	No of workers	midpoints	f.x
500-520	12	510	6120
520-540	14	530	7420
540-560	8	550	4400
560-580	6	570	3420
580-600	10	590	5900

TABLE 2.2.1: friquency distribution table2

pocket allowance	11-13	13-15	15-17	17-19
Number of children	7	6	9	13
pocket allowance	19-21	21-23	23-25	
Number of children	f	5	4	

1.

$$\sum f = 50 \tag{2.2.1.1}$$
$$\sum f.x = 27260 \tag{2.2.1.2}$$
$$Mean = \frac{\sum f.x}{\sum f} \tag{2.2.1.3}$$
$$= \frac{27260}{50} \tag{2.2.1.4}$$
$$= 545.2 \tag{2.2.1.5}$$

codes for the above equations can be get from

codes/static2.py

2.3 Problem3

2.3.1 question:

1. The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is Rs 18. Find the missing frequency f.

2.3.2 Solution:

pocket allowences	No of children	midpoints	f.x
11-13	7	12	84
13-15	6	14	84
15-17	9	16	144
17-19	13	18	234
19-21	f	20	20f
21-23	5	22	110
23-25	4	24	96

TABLE 2.3.1: friquency distribution table3

1.

$$\sum f = 50 + f \tag{2.3.1.1}$$
$$\sum f.x = 752 + 20f \tag{2.3.1.2}$$
$$Mean = \frac{\sum f.x}{\sum f} \tag{2.3.1.3}$$
$$18 = \frac{752 + 20f}{44 + f} \tag{2.3.1.4}$$
$$f = 20 \tag{2.3.1.5}$$

codes for the above equations can be get from

codes/static3.py

2.4 Problem4

2.4.1 question:

1. Thirty women were examined in a hospital by a doctor and the number of heartbeats per minute were recorded and summarised as follows. Find the mean heartbeats per minute for these women, choosing a suitable method.

No. of heartbeats	65-68	68-71	71-74	74-77
No. of women	2	4	3	8
No. of heartbeats	77-80	80-83	83-86	
No. of women	7	4	2	

2.4.2 Solution:

No of heart-beats	No of women	midpoints	f.x
65-68	2	66.5	133
68-71	4	69.5	278
71-74	3	72.5	217.5
74-77	8	75.5	604
77-80	7	78.5	549.5
80-83	4	81.5	326
83-86	2	84.5	169

TABLE 2.4.1: friquency distribution table4

1.

$$\sum f = 30 \quad (2.4.1.1)$$
$$\sum f.x = 2277 \quad (2.4.1.2)$$
$$Mean = \frac{\sum f.x}{\sum f} \quad (2.4.1.3)$$
$$= \frac{2277}{30} \quad (2.4.1.4)$$
$$= 75.9 \quad (2.4.1.5)$$

codes for the above equations can be get from

codes/static4.py

2.5 Problem5

2.5.1 question:

1. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.

No of mangoes	50-52	53-55	56-58
No of boxes	15	110	135
No of mangoes	59-61	62-64	
No of boxes	115	25	

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

2.5.2 Solution:

daily expenditure	No of households	midpoints	f.x
100-150	4	125	500
150-200	5	175	875
200-250	12	225	2700
250-300	2	275	550
300-350	2	325	650

TABLE 2.5.1: friquency distribution table5

1.

$$\sum f = 400 \quad (2.5.1.1)$$
$$\sum f.x = 22425 \quad (2.5.1.2)$$
$$Mean = \frac{\sum f.x}{\sum f} \quad (2.5.1.3)$$
$$= \frac{22425}{400} \quad (2.5.1.4)$$
$$= 56.06 \quad (2.5.1.5)$$

codes for the above equations can be get from

codes/static5.py

2.6 Problem6

2.6.1 question:

1. The table below shows the daily expenditure on food of 25 households in a locality. Find the

Daily expenditure	100-150	150-200	200-250
No of households	4	5	12
Daily expenditure	250-300	300-350	
No of households	2	2	

mean daily expenditure on food by a suitable method.

2.6.2 Solution:

No of mangoes	No of boxes	midpoints	f.x
50-52	15	51	765
53-55	110	54	5490
56-58	135	57	7695
59-61	115	60	6900
62-64	25	63	1575

TABLE 2.6.1: friquency distribution table6

1.

$$\sum f = 25 \tag{2.6.1.1}$$
$$\sum f.x = 5275 \tag{2.6.1.2}$$
$$Mean = \frac{\sum f.x}{\sum f} \tag{2.6.1.3}$$
$$= \frac{5275}{25} \tag{2.6.1.4}$$
$$= 211 \tag{2.6.1.5}$$

codes for the above equations can be get from

codes/static6.py

1.

$$\sum f = 30 \tag{2.7.1.1}$$
$$\sum f.x = 2.78 \tag{2.7.1.2}$$
$$Mean = \frac{\sum f.x}{\sum f} \tag{2.7.1.3}$$
$$= \frac{2.78}{30} \tag{2.7.1.4}$$
$$= 0.092 \tag{2.7.1.5}$$

codes for the above equations can be get from

codes/static7.py

2.7 Problem7

2.7.1 question:

1. To find out the concentration of so_2 in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below:

Concentration SO_2(in ppm)	0.00-0.04	0.04-0.08	0.08-0.12
Frequency	4	9	9
Concentration SO_2(in ppm)	0.12-0.16	0.16-0.20	0.20-0.24
Frequency	2	4	2

Find the mean concentration of SO_2 in the air.

2.7.2 Solution:

concen-tration of SO2	friquency	midpoints	f.x
0.00-0.04	4	0.02	0.08
0.04-0.08	9	0.06	0.36
0.08-0.12	9	0.1	0.9
0.12-0.16	2	0.14	0.28
0.16-0.20	4	0.18	0.72
0.20-0.24	2	0.22	0.44

TABLE 2.7.1: friquency distribution table7

2.8 Problem8

2.8.1 question:

1. A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

No of days	0-6	6-10	10-14	14-20
No of students	11	10	7	4
No of days	20-28	28-38	38-40	
No of students	4	3	1	

2.8.2 Solution:

No of days	No of students	midpoints	f.x
0-6	11	3	33
6-10	10	8	80
10-14	7	12	84
14-20	4	17	68
20-28	4	24	96
28-38	3	33	99
38-40	1	39	39

TABLE 2.8.1: friquency distribution table8

1.

$$\sum f = 40 \tag{2.8.1.1}$$
$$\sum f.x = 499 \tag{2.8.1.2}$$
$$Mean = \frac{\sum f.x}{\sum f} \tag{2.8.1.3}$$
$$= \frac{499}{40} \tag{2.8.1.4}$$
$$= 12 \tag{2.8.1.5}$$

codes for the above equations can be get from

codes/static8.py

2.9 Problem9

2.9.1 question:

1. The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

literacy(in percentage)	45-55	55-65	65-75
Number of cities	3	10	11
literacy(in percentage)	75-85	85-95	
Number of cities	8	3	

2.9.2 Solution:

literacy	No of cities	midpoints	f.x
45-55	3	50	150
55-65	10	60	600
65-75	11	70	770
75-85	8	80	640
85-95	3	90	270

TABLE 2.9.1: friquency distribution table9

1.

$$\sum f = 35 \tag{2.9.1.1}$$
$$\sum f.x = 2430 \tag{2.9.1.2}$$
$$Mean = \frac{\sum f.x}{\sum f} \tag{2.9.1.3}$$
$$= \frac{2430}{35} \tag{2.9.1.4}$$
$$= 69.42 \tag{2.9.1.5}$$

codes/static9.py

2.10 Problem10

2.10.1 question:

1. The following table shows the ages of the patients admitted in a hospital during a year:

Age (in years)	5-15	15-25	25-35
Number of patients	6	11	21
Age (in years)	35-45	45-55	55-65
Number of patients	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

2.10.2 Solution:

Age	No of patient	midpoints	fx
5-15	6	10	60
15-25	11	20	220
25-35	21	30	630
35-45	23	40	920
45-55	14	50	700
55-65	5	60	300

TABLE 2.10.1: friquency distribution table10

1.

$$\sum f = 80 \tag{2.10.1.1}$$
$$\sum f.x = 2830 \tag{2.10.1.2}$$
$$Mean = \frac{\sum f.x}{\sum f} \tag{2.10.1.3}$$
$$= \frac{2830}{801} \tag{2.10.1.4}$$
$$= 35.375 \tag{2.10.1.5}$$

2. in this table max friquency is 23 and modal class related to it is 35-45.

$$l = 35 \tag{2.10.2.1}$$

$$h = 10 \tag{2.10.2.2}$$

$$f_1 = 23 \tag{2.10.2.3}$$

$$f_0 = 21 \tag{2.10.2.4}$$

$$f_2 = 14 \tag{2.10.2.5}$$

$$Mode = l + \frac{f_1 - f_o}{2f_1 - f_o - f_2} \times h \tag{2.10.2.6}$$

$$= 35 + \frac{23 - 21}{2 \times 23 - 21 - 14} \times 10 \tag{2.10.2.7}$$

$$= 36.86 \tag{2.10.2.8}$$

codes for the above equation can be get from here

codes/static10.py