1. $|A| = 2, \quad |AB| = 21$

$$|B| = \frac{|AB|}{|A|} = \frac{21}{2}.$$

2. $f(x) = x+1$

$$f\circ f(x) = (x+1)+1$$

$$= x+2.$$

$$\frac{d}{dx}\left[f\circ f(x)\right] = \underline{\underline{1}}$$

3. $$x^2 \frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^4$$

$$x^2 y_2 = \left(1 + \frac{y_1}{dx}\right)^4$$

order 2, degree 4.

4. a) Direction cosines are

$$\begin{pmatrix} 0 \\ -1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

b) $$\underline{r} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix}$$

6. $$2A - 3B + 5C = 0$$

$$\Rightarrow A = \frac{1}{2}\left[3B - 5C\right]$$

$$B = \begin{pmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{pmatrix} \quad C = \begin{pmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{pmatrix}$$

$$3B - 5C = \begin{pmatrix} -6 & 6 & 0 \\ 9 & 3 & 12 \end{pmatrix} - \begin{pmatrix} 10 & 0 & -10 \\ 35 & 5 & 30 \end{pmatrix}$$

$$= \begin{pmatrix} -16 & 6 & 10 \\ 26 & -2 & -18 \end{pmatrix}$$

$$\Rightarrow A = \begin{pmatrix} -8 & 3 & 5 \\ 13 & -1 & -9 \end{pmatrix}$$ Ans.

7. $$I = \int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}}\, dx$$

Let $\tan x = 2\tan y$

$$\Rightarrow \sec^2 x\, dx = 2\sec^2 y\, dy$$

$$\Rightarrow I = \int \frac{2\sec^2 y\, dy}{2\sec y}$$

$$= \int \sec y\, dy$$

$$= \ln(\sec y + \tan y)$$

$$= \ln\left(\frac{\tan x}{2} + \sqrt{1 + \frac{\tan^2 x}{4}}\right)$$

Ans

8. a) $\int \sqrt{1-\sin 2x}\, dx$

$= \int \sqrt{1-\cos\left(\frac{\pi}{2}-2x\right)}\quad dx$

$= \int \sqrt{2}\sin\left(\frac{\pi}{4}-x\right) dx$

$= \sqrt{2}\cos\left(\frac{\pi}{4}-x\right)$ Ans.

b) $I = \int \sin^{-1}(2x)\, dx.$

Let $2x = \sin\theta$

$\Rightarrow 2dx = \cos\theta\, d\theta.$

$I = \frac{1}{2}\int \theta\cos\theta\, d\theta.$

$= \frac{1}{2}\left[\theta\sin\theta\right] - \frac{1}{2}\int \sin\theta\, d\theta$

$$= \frac{1}{2}\left[\theta \sin\theta\right] + \frac{1}{2}\cos\theta$$

$$= \frac{\theta\sin\theta + \cos\theta}{2}$$

$$= \frac{2x\sin^{-1}2x + \sqrt{1-4x^2}}{2} \text{ Ans.}$$

9. $y = e^{2x}(a+bx)$

$\Rightarrow ye^{-2x} = a+bx$

$\Rightarrow -2ye^{-2x} + e^{-2x}y_1 = b$

$\Rightarrow y_1 - 2y = e^{2x}b = \frac{by}{a+bx}$

$\Rightarrow a+bx = \frac{by}{y_1-2y}$

$$a+bx = \frac{by}{y_1-2y}$$

$$\Rightarrow b = \frac{(y_1-2y)by_1 - by(y_2-2y_1)}{(y_1-2y)^2}$$

$$\Rightarrow b(y_1-2y)^2 + by(y_2-2y_1)$$
$$+ by_1(y_1-2y) = 0$$

$$\Rightarrow (y_1-2y)\left[by_1 - 2by + by_1\right]$$
$$+ by(y_2-2y_1) = 0$$

$$\Rightarrow 2(y_1-2y)b(y_1-y) + by(y_2-2y_1) = 0$$

$$\Rightarrow 2(y-y_1)(2y-y_1) + y(y_2-2y_1) = 0$$

$$\Rightarrow \underline{\underline{yy_2 - 8yy_1 + y_1^2 + 2y^2 = 0}}$$

Ans.

10. a) $\|\underline{a}+\underline{b}\| = 1$

$\|\underline{a}\| = \|b\| = 1.$

$\|\underline{a}+\underline{b}\|^2 + \|\underline{a}-\underline{b}\|^2 = 2(\|\underline{a}\|^2 + \|\underline{b}\|^2)$

$= 2$

$\Rightarrow \|\underline{a}-\underline{b}\|^2 = 1$

b) $\underline{a} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

$\underline{c} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$

$$\begin{matrix} i & j & k \\ 1 & -2 & 1 \\ 3 & 1 & 2 \end{matrix}$$

$(\underline{a}, \underline{b}, \underline{c}) = a^t(\underline{b} \times \underline{c})$

$= (2 \quad 3 \quad 1) \left[ \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \times \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \right]$

$= (2 \quad 3 \quad 1) \begin{pmatrix} -5 \\ 1 \\ 7 \end{pmatrix}$

$= \underline{\underline{0}}$ Ans.

11. $x \in \{1,2,3,4,5,6\}$ $A = \{2,4,6\}$

$B = \{1,2,3\}$ - red

$Z = \{5,6\}$ - green

$Pr(A) = \frac{1}{2}$.

$Pr(B) = \frac{1}{2}$.

$Pr(AB) = Pr\{2\} = \frac{1}{6}$.

$Pr(AB) \neq Pr(A)\,Pr(B)$.

Dependent events.

12. a) (i) Let $X_i \in (0,1)$ where

$1 \Rightarrow$ odd number

$Pr(X_i = 0) = Pr(X_i = 1) = \frac{1}{2}$.

Let $X = \sum_{i=1}^{6} x_i$

$\Pr(X = 5) = {}^6C_5 \left(\frac{1}{2}\right)^6$

$= \frac{6}{2^6}.$

(ii) $\Pr(X \leq 5)$

$= 1 - \Pr(X > 5)$

$= 1 - \Pr(X = 6)$

$= 1 - \frac{1}{2^6}$ Ans.

b) $\Pr(X = x) = \begin{cases} k & x = 0 \\ 2k & x = 1 \\ 3k & x = 2. \end{cases}$

$\sum_x \Pr(X = x) = 1 = 6k$

$\Rightarrow k = \frac{1}{6}$ Ans.

14. $\tan^{-1}4x + \tan^{-1}6x = \frac{\pi}{4}$

$\Rightarrow \tan\left(\tan^{-1}4x + \tan^{-1}6x\right) = 1$

$\Rightarrow \dfrac{4x + 6x}{1 - 24x^2} = 1$

$\Rightarrow 24x^2 + 10x - 1 = 0$

$\Rightarrow 24x^2 + 12x - 2x - 1 = 0$

$\Rightarrow (2x+1)(12x-1) = 0$

$\Rightarrow x = -\frac{1}{2}$ or $x = \frac{1}{12}$

15. $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$

$= \begin{vmatrix} a^2+2a & 2a+1 & 1 \\ a^2-1 & a-1 & 0 \\ 2a-2 & a-1 & 0 \end{vmatrix}$

$$\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ a^2-1 & a-1 & 0 \\ 2a-2 & a-1 & 0 \end{vmatrix}$$

$$= (a-1)^2 \begin{vmatrix} a+1 & 1 \\ 2 & 1 \end{vmatrix}$$

$$= \underline{\underline{(a-1)^3}} \quad \text{Ans.}$$

16. a) $\ln(x^2+y^2) = 2\tan^{-1}\left(\frac{y}{x}\right)$

$$\Rightarrow \frac{1}{\cancel{x^2+y^2}}\left(\cancel{2}x+\cancel{2}y\,y_1\right) = \left(\frac{\cancel{2}}{\cancel{1+\frac{y^2}{x^2}}}\right)$$

$$\times \frac{xy_1 - y}{\cancel{x^2}}$$

$$\Rightarrow x + y y_1 = x y_1 - y$$

$$\Rightarrow (x+y) = (x-y)\,y_1 \Rightarrow y_1 = \underline{\underline{\frac{x+y}{x-y}}} \ \text{Ans}$$

h) $x^y - y^x = a^b$

$\Rightarrow yx^{y-1} + x^y \ln x \, y_1$

$- xy^{x-1} y_1 - y^x \ln y = 0$

$\Rightarrow y_1 \left( x^y \ln x - xy^{x-1} \right)$

$= y^x \ln y - yx^{y-1}$

$$\Rightarrow y_1 = \frac{y^x \ln y - yx^{y-1}}{x^y \ln x - xy^{x-1}}$$ Ans.

17. $y = (\sin^{-1} x)^2$

$\Rightarrow x = \sin(\sqrt{y})$

$$\Rightarrow 1 = \frac{\cos(\sqrt{y})}{2\sqrt{y}} \, y_1$$

$$\Rightarrow 1 = \frac{\cos(\sqrt{y})}{2\sqrt{y}}\, y_1$$

$$\Rightarrow \cos(\sqrt{y})\, y_1 = 2\sqrt{y}$$

$$\Rightarrow \cos(\sqrt{y})\, y_2 - y_1 \frac{\sin\sqrt{y}}{2\sqrt{y}} = \frac{1}{\sqrt{y}}$$

$$\Rightarrow 2\sqrt{y}\cos(\sqrt{y})\, y_2 - y_1 \sin\sqrt{y} - 2 = 0$$

$$\Rightarrow 2\sin^{-1}x\sqrt{1-x^2}\, y_2 - x y_1 - 2 = 0$$

18. $y = \sqrt{3x-2}$

$$\Rightarrow \frac{dy}{dx} = \frac{3}{2\sqrt{3x-2}} = 2 \quad \because \text{line is } 4x - 2y + 5 = 0$$

$$\Rightarrow 3x - 2 = \frac{9}{16}$$

$$\Rightarrow 3x = \frac{41}{16} \Rightarrow x = \frac{41}{48}$$

$$y = \sqrt{\frac{3 \times 41}{48} - 2}$$

$$= \sqrt{\frac{123 - 96}{48}}$$

$$= \sqrt{\frac{\cancel{27}\,9}{\cancel{48}\,16}} = \frac{3}{4}$$

Point of contact is $\begin{pmatrix} \frac{41}{48} \\ 3/4 \end{pmatrix}$

direction vector is $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Normal vector is $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

Equation of tangent is

$$\begin{pmatrix} 2 & -1 \end{pmatrix} \left[ \underline{x} - \begin{pmatrix} \frac{41}{48} \\ 3/4 \end{pmatrix} \right] = 0$$

$$\Rightarrow \begin{pmatrix} 2 & -1 \end{pmatrix} \underline{x} = \frac{41}{24} - \frac{3}{4}$$

$$= \frac{23}{24} //$$

Equation of normal is

$$\begin{pmatrix}1 & 2\end{pmatrix}\underline{x} = \begin{pmatrix}1 & 2\end{pmatrix}\begin{pmatrix}41/48 \\ 3/4\end{pmatrix}$$

$$= \frac{41}{48} + \frac{3}{2}$$

$$\begin{pmatrix}1 & 2\end{pmatrix}\underline{x} = \frac{113}{48} \quad \text{Ans.}$$

---

19. $$\int \frac{3x+5}{x^2+3x-18}\,dx = \int \frac{\frac{3}{2}(2x+3) + 5 - \frac{9}{2}}{x^2+3x-18}$$

$$= \frac{3}{2}\ln(x^2+3x-18) + \frac{1}{2}\int \frac{dx}{\left(x+\frac{3}{2}\right)^2 - \frac{63}{4}}$$

$$= \frac{3}{2}\ln(x^2+3x-18) + \frac{1}{2}\int \frac{dx}{\left(x+\frac{3}{2}\right)^2 - \left(\frac{3}{2}\sqrt{7}\right)^2}.$$

$$= \frac{3}{2}\ln(x^2+3x-18)$$

$$+\frac{1}{2}\int \frac{dx}{2\cdot\frac{3}{2}\sqrt{7}}\left[\frac{1}{\left(x+\frac{3}{2}-\frac{3}{2}\sqrt{7}\right)}-\frac{1}{\left(x+\frac{3}{2}+\frac{3\sqrt{7}}{2}\right)}\right]$$

$$= \frac{3}{2}\ln(x^2+3x+18)+\frac{1}{6\sqrt{7}}\ln\left(\frac{x+\frac{3}{2}-\frac{3}{2}\sqrt{7}}{x+\frac{3}{2}+\frac{3}{2}\sqrt{7}}\right)$$

Ans.

20. $I = \int_0^a f(x)\,dx$

Let $y = a - x$

$\Rightarrow x = a - y$

$$I = -\int_a^0 f(a-y)\,dy$$

$$= \int_0^a f(a-y)\,dy$$

$$I = \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\, dx$$

$$I = \int_0^{\pi} \frac{(\pi - x) \sin x}{1+\cos^2 x}\, dx$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\, dx$$

Let $t = \cos x$

$$= -\pi \int_1^{0} \frac{dt}{1+t^2}$$

$$= \pi \tan^{-1} t \Big|_0^1$$

$$\Rightarrow I = \frac{\pi^2}{4} \text{ Ans.}$$

21. a) $x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$ — (1)

Let $x = r\cos\theta$

$y = r\sin\theta$

$dx = -r\sin\theta\,d\theta + \cos\theta\,dr$

$dy = r\cos\theta\,d\theta + \sin\theta\,dr$

$x\,dy = r\cos\theta\,(r\cos\theta\,d\theta + \sin\theta\,dr)$

$y\,dx = r\sin\theta\,(-r\sin\theta\,d\theta + \cos\theta\,dr)$

(2)

$x\,dy - y\,dx = r^2\,d\theta$

$\sqrt{x^2+y^2}\,dx = r\,(-r\sin\theta\,d\theta + \cos\theta\,dr)$ (3)

Substituting from 2 & (3) in (1),

$r^2\,d\theta = -r^2\sin\theta\,d\theta + r\cos\theta\,dr$

$\Rightarrow d\theta\left(\dfrac{1+\sin\theta}{\cos\theta}\right) = \dfrac{dr}{r}$

$\Rightarrow \int \dfrac{d\theta}{\cos\theta} + \int \dfrac{\sin\theta}{\cos\theta}\,d\theta = \int \dfrac{dr}{r}$

$$\Rightarrow \ln(\sec\theta+\tan\theta) - \ln\cos\theta = \ln r + C$$

$$\Rightarrow \ln \sec\theta(\sec\theta+\tan\theta) = C + \ln r$$

$$\Rightarrow \sec\theta(\sec\theta+\tan\theta) = Cr$$

$$\Rightarrow \frac{1+\sin\theta}{\cos^2\theta} = Cr$$

$$\Rightarrow \frac{r+r\sin\theta}{r^2\cos^2\theta} = C$$

$$\Rightarrow \frac{\sqrt{x^2+y^2}+y}{x^2} = C$$

$y(1)=0$

$$\Rightarrow C = 1$$

Hence, $\underline{\underline{\sqrt{x^2+y^2}+y = x^2}}$ Ans.

b) $(1+x^2)\dfrac{dy}{dx} + 2xy - 4x^2 = 0.$

$$\frac{dy}{dx} + \frac{2x}{1+x^2}\,y = \frac{4x^2}{1+x^2}.$$

$$P = \frac{2x}{1+x^2} \qquad Q = \frac{4x^2}{1+x^2}.$$

$$IF = e^{\int P\,dx}$$

$$y\,e^{\int P\,dx} = \int Q\,e^{\int P\,dx} + C.$$

$$\int P\,dx = \int \frac{2x\,dx}{1+x^2} = \ln(1+x^2)$$

$$e^{\int P\,dx} = 1+x^2.$$

Hence,

$$y(1+x^2) = \int \frac{4x^2}{1+x^2} \times (1+x^2)\,dx + C$$

$$= \frac{4x^3}{3} + C$$

$y(0)=0$

$\Rightarrow \; C = 0$ $\underline{\underline{\qquad\qquad}}$ Ans.

22. $\underline{A} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ $\underline{B} = \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix}$

$\underline{C} = \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix}$ $\underline{D} = \begin{pmatrix} 1 \\ -6 \\ -1 \end{pmatrix}$

$$\underline{B} - \underline{A} = \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}$$

$$\underline{D} - \underline{C} = \begin{pmatrix} -2 \\ -8 \\ 2 \end{pmatrix}$$

$$= -2 \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}$$

$$\Rightarrow \frac{(\underline{B} - \underline{A})^t (\underline{D} - \underline{C})}{\| \underline{B} - \underline{A} \| \| (\underline{D} - \underline{C}) \|} = 1$$

Angle is $0^\circ$.

A B and C D are collinear.

23. $L_1: \quad \dfrac{1-x}{3} = \dfrac{y-2}{\lambda/7} = \dfrac{z-3}{2} = k_1$

$L_2: \quad \dfrac{1-x}{3\lambda/7} = \dfrac{y-5}{1} = \dfrac{6-z}{5} = k_2$

$$L_1: \begin{cases} x = 1 - 3k_1 \\ y = 2 + \dfrac{k_1 \lambda}{7} \\ z = 3 + 2k_1 \end{cases} \Rightarrow \underline{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + k_1 \begin{pmatrix} -3 \\ \frac{\lambda}{7} \\ 2 \end{pmatrix}$$

$$L_2: \quad \begin{aligned} x &= 1 - k_2\left(\frac{3\lambda}{7}\right) \\ y &= 5 + k_2 \\ z &= 6 - 5k_2 \end{aligned} \Rightarrow \underline{r} = \begin{pmatrix} 1 \\ 5 \\ 6 \end{pmatrix} + k_2 \begin{pmatrix} -\frac{3\lambda}{7} \\ 1 \\ -5 \end{pmatrix}$$

$L_1 \perp L_2 \Rightarrow$

$$\begin{pmatrix} -3 & \frac{\lambda}{7} & 2 \end{pmatrix} \begin{pmatrix} -\frac{3\lambda}{7} \\ 1 \\ -5 \end{pmatrix} = 0$$

$$\begin{pmatrix} -3 & \frac{\lambda}{7} & 2 \end{pmatrix} \begin{pmatrix} -\frac{3\lambda}{7} \\ 1 \\ -5 \end{pmatrix} = 0$$

$$\Rightarrow \frac{9\lambda}{7} + \frac{\lambda}{7} - 10 = 0$$

$$\Rightarrow \underline{\underline{\lambda = 7}} \text{ Ans.}$$

$L_1$ and $L_2$ intersect if

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + k_1 \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 5 \\ 6 \end{pmatrix} + k_2 \begin{pmatrix} -3 \\ 1 \\ -5 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -3 & -3 \\ 1 & 1 \\ 2 & -5 \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} -3 & -3 & 0 \\ 1 & 1 & -3 \\ 2 & -5 & -3 \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & -3 \\ 2 & -5 & -3 \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 7 & 3 \\ 0 & 0 & 3 \end{pmatrix}$$

inconsistent. So they don't intersect.

24.

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 6 \\ 1 & 0 & 2 & 0 & 1 & 0 & 7 \\ 3 & 1 & 1 & 0 & 0 & 1 & 12 \end{pmatrix}$$

$$\sim$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 6 \\ 0 & 1 & -1 & 1 & -1 & 0 & -1 \\ 0 & -2 & -2 & -3 & 0 & 1 & -6 \end{pmatrix}$$

$$\sim$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 6 \\ 0 & 1 & -1 & 1 & -1 & 0 & -1 \\ 0 & 0 & -4 & -1 & -2 & 3 & -8 \end{pmatrix}$$

$$1_r$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 6 \\ 0 & 1 & -1 & 1 & -1 & 0 & -1 \\ 0 & 0 & +4 & +1 & +2 & -3 & +8 \end{pmatrix}$$

$$1_r$$

$$\begin{pmatrix} 1 & 0 & 2 & 0 & 1 & 0 & 7 \\ 0 & 1 & -1 & 1 & -1 & 0 & -1 \\ 0 & 0 & 4 & 1 & 2 & -3 & 8 \end{pmatrix}$$

$$1_r$$

$$\begin{pmatrix} 4 & 0 & 0 & -2 & 0 & 6 & 12 \\ 0 & 4 & 0 & 5 & -2 & -3 & 4 \\ 0 & 0 & 4 & 1 & 2 & -3 & 8 \end{pmatrix}$$

$$1_r \quad A^{-1} \quad 2$$

$$\left(\begin{array}{ccc:ccc:c} 1 & 0 & 0 & -1/2 & 0 & 3/2 & 3 \\ 0 & 1 & 0 & 5/4 & -1/2 & -3/4 & 1 \\ 0 & 0 & 1 & 1/4 & 1/2 & -3/2 & 2 \end{array}\right)$$

b) $$\begin{pmatrix} 1 & 2 & -2 & 1 & 0 & 0 \\ -1 & 3 & 0 & 0 & 1 & 0 \\ 0 & -2 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & -2 & 1 & 0 & 0 \\ 0 & 5 & -2 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\sim$$

$$\begin{pmatrix} 1 & 2 & -2 & 1 & 0 & 0 \\ 0 & 5 & -2 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 2 & 5 \end{pmatrix}$$

$$\sim$$

$$\begin{pmatrix} 5 & 0 & -4 & 3 & -2 & 0 \\ 0 & 5 & -2 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 2 & 5 \end{pmatrix}$$

$A_\ell$

$$\begin{pmatrix} 5 & 0 & -4 & 3 & -2 & 0 \\ 0 & 5 & -2 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 2 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 0 & 0 & 11 & 6 & 20 \\ 0 & 5 & 0 & 5 & 5 & 10 \\ 0 & 0 & 1 & 2 & 2 & 5 \end{pmatrix}$$

$A_\ell$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 11/5 & 6/5 & 4 \\ 0 & 1 & 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 2 & 2 & 5 \end{array}\right)$$

$A^{-1}$.

25. $lbh = 8$

$h = 2$

$\Rightarrow lb = 4$

$C = 70\,lb + 2\times 45 \times lh$

$+ 2 \times 45 \times bh$

$= 70\,lb + 180\,l + 180\,b.$

$\Rightarrow C = 280 + 180\,l + 180\,b.$

$lb = 4.$ $\quad C = 280 + \dfrac{180}{\sqrt{2}}$

$C = 280 + 180\,l + \dfrac{45}{l}.$ $\quad \underline{\underline{+45\sqrt{2}}}$

$$\frac{dC}{dl} = 180 - \frac{90}{l^2} = 0$$

$\Rightarrow l = \pm \dfrac{1}{\sqrt{2}}.$

$$\frac{d^2C}{dl^2} = \frac{180}{l^3} > 0 \text{ if } l > 0.$$

so min if $l = \dfrac{1}{\sqrt{2}}.$

26. a) $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 6 \\ 5 & 7 & 2 \end{vmatrix}$

$= \begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & 2 \\ 5 & 2 & -5 \end{vmatrix}$

$= 2 \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 5 & 2 & -5 \end{vmatrix}$

$= 2 \times 7 = \underline{\underline{14}}$ Ans.

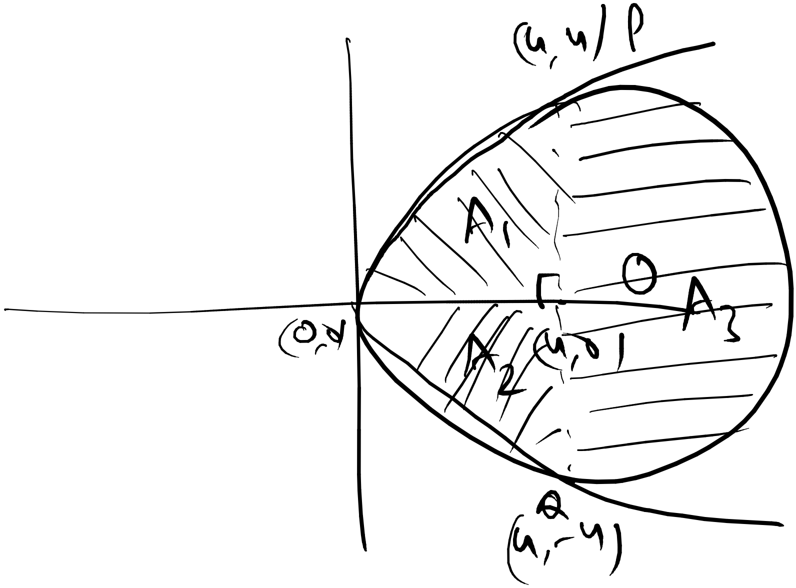
b) $x^2+y^2 = 8x \Rightarrow (x-4)^2+y^2 = 4^2$

$y^2 = 4x$

$\Rightarrow x^2+4x = 8x$

$\Rightarrow x^2 = 4x \Rightarrow x = 0, 4.$

$y = 0, \pm 4.$




$$A_3 = \frac{\pi r^2}{2} = \frac{\pi (4)^2}{2} = \underline{\underline{8\pi}}$$

$$A_1 = \frac{\pi r^2}{4} - \int_0^4 2\sqrt{x}\,dx$$

$$= \frac{\pi (4)^2}{4} - \frac{2\times 2}{3}\left(x^{3/2}\right)_0^4$$

$$= 4\pi - \frac{4}{3}\times 8 = 4\pi - \frac{32}{3}$$

$A_1 = A_2$

$\Rightarrow$ Total area $= 8\pi - \frac{64}{3} + 8\pi = 16\pi - \frac{64}{3}$ Ans

27. $\underline{n}^t \underline{x} = c$

$$\Rightarrow \underline{n}^t(\underline{x}_1 - \underline{x}_2) = 0$$

$$\underline{n}^t(\underline{x}_2 - \underline{x}_3) = 0$$

$$\underline{n}^t(\underline{x}_3 - \underline{x}_1) = 0$$

$$\Rightarrow \begin{pmatrix} (\underline{x}_1 - \underline{x}_2)^t \\ (\underline{x}_2 - \underline{x}_3)^t \\ (\underline{x}_3 - \underline{x}_1)^t \end{pmatrix} \underline{n} = 0$$

$$\Rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 4 & -4 & 4 \\ 5 & -2 & 7 \end{pmatrix} \underline{n} = 0$$

$$\Rightarrow \begin{pmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ 5 & -2 & 7 \end{pmatrix} \Leftrightarrow \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 6 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 6 & -1 \end{pmatrix} \Leftrightarrow \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\underline{n} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

The points are not in a plane.

b) $L: \quad \underline{r} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

$\Pi: \quad \underline{n} = \begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix}$

If $\underline{n}^t \underline{r} = c$,

$(-1 \quad 3 \quad -4)\,\underline{n} = c.$

$(1 \quad 2 \quad -1)\,\underline{n} = 0.$

$(1 \quad 1 \quad 0)\,\underline{n} = c.$

$$\Rightarrow \begin{pmatrix} 1 & 2 & -1 \\ 2 & -2 & 4 \end{pmatrix} \Leftrightarrow 0$$

$$\Rightarrow \begin{pmatrix} 1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

$$\Updownarrow$$

$$\begin{pmatrix} 1 & 2 & -1 \\ 0 & -3 & 3 \end{pmatrix}$$

$$\Updownarrow$$

$$\begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \end{pmatrix}$$

$$\Updownarrow$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

$$\underline{n} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

$$c = \begin{pmatrix} -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$= \underline{\underline{0}}.$$

Equation of the plane is

$$\begin{pmatrix} -1 & 1 & 1 \end{pmatrix} \underline{x} = 0.$$

length of perp.

$$= \frac{\begin{pmatrix} -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}}{\sqrt{3}}$$

$$= \frac{3}{\sqrt{3}} = \underline{\underline{\sqrt{3}}} \quad \text{Ans.}$$

28. Let Let $X \in \{0, 1\}$

0 is defective.

Let $Y \in \{0, 1, 2\}$

0 – A

1 – B

2 – C

$$Pr(X=0 \mid Y=0) = \frac{1}{100}.$$

$$Pr(X=0 \mid Y=1) = \frac{5}{100}.$$

$$Pr(X=0 \mid Y=2) = \frac{7}{100}.$$

$$Pr(Y=0) = \frac{50}{100}.$$

$$Pr(Y=1) = \frac{30}{100}$$

$$Pr(Y=2) = \frac{20}{100}.$$

$$Pr(Y=0 \mid X=0) = \frac{Pr(X=0 \mid Y=0)\, Pr(Y=0)}{\sum Pr(X=0 \mid Y=i)}$$