Math Document Template

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***Abstract*—This a simple document explaining a question about the concept of similar triangles.**

Download all python codes from

```
svn co https://github.com/SiddharthPh/
    Summer2020/trunk/geometry/codes
```

and latex-tikz codes from

```
svn co https://github.com/gadepall/school/trunk/
    ncert/geometry/figs
```

1 Problem

In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B. Show that

(a) $\triangle AMC \cong \triangle BMD$
(b) $\triangle DBC$ is a right angle.
(c) $\triangle DBC \cong \triangle ABC$
(d) $CM = \frac{1}{2}AB$

2 Construction

2.1. The figure for A triangle obtained in the question looks like Fig. 2.1. with angles $\angle A, \angle C$ and $\angle B$ and sides a, b and c. The unique feature of this triangle is $\angle C$ which is defined to be 90°.

2.2. List the design parameters for construction
Solution: See Table. 2.2.

Parameter	Value
a	4
b	3
$\angle ACB$	90°

TABLE 2.2: To construct $\triangle ACB$

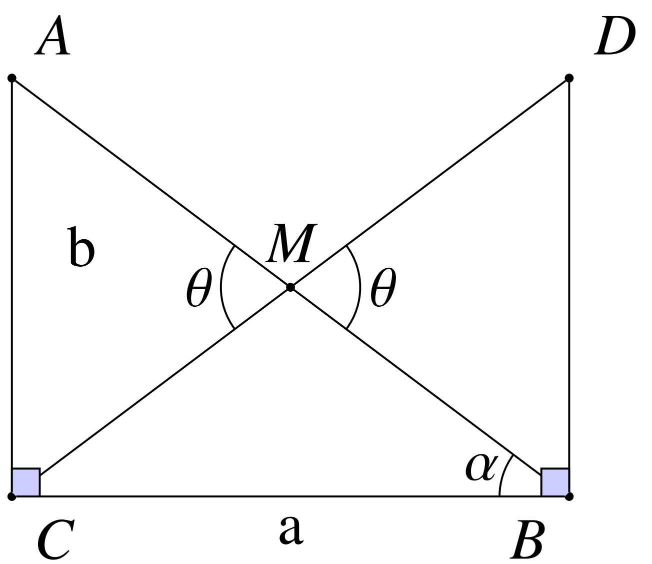

Fig. 2.1: Right Angled Triangle by Latex-Tikz

2.3. Find the coordinates of the various points in Fig. 2.1
Solution: From the given information,

$$\mathbf{A} = \begin{pmatrix} 0 \\ b \end{pmatrix} \tag{2.3.1}$$

$$\mathbf{C} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{2.3.2}$$

$$\mathbf{B} = \begin{pmatrix} a \\ 0 \end{pmatrix} \tag{2.3.3}$$

$\because$ $\mathbf{M}$ is the midpoint of AB,

$$\mathbf{M} = \frac{\mathbf{A} + \mathbf{B}}{2} = \frac{1}{2}\begin{pmatrix} a \\ b \end{pmatrix} \tag{2.3.4}$$

Also, $\mathbf{M}$ is given to be the midpoint of C. Hence,

$$\mathbf{M} = \frac{\mathbf{C} + \mathbf{D}}{2} \tag{2.3.5}$$

$$\implies \mathbf{D} = 2\mathbf{M} - \mathbf{C} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{2.3.6}$$

The values are listed in Table. 2.3

2.4. Draw Fig. 2.1.

Derived Values.	
M	$\begin{pmatrix} 2 \\ 1.5 \end{pmatrix}$
D	$\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

TABLE 2.3: To construct $\triangle DCB$

Solution: The following Python code generates Fig. 2.4

```
codes/triangle.py
```

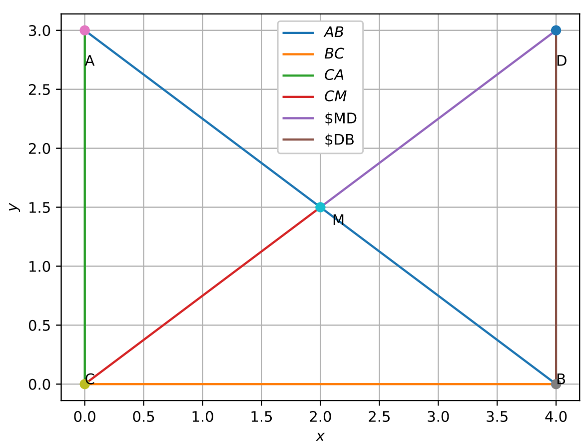

Fig. 2.4: Triangle generated using python

and the equivalent latex-tikz code generating Fig. 2.1 is

```
figs/triangle.tex
```

The above latex code can be compiled as a standalone document as

```
figs/triangle_fig.tex
```

3 Solution

3.1. $\triangle AMC \cong \triangle DMB$ by SAS congruency $\because$

a) $AM = BM$
b) $CM = DM$
c) $\underline{\angle AMC} = \underline{\angle DMB}$ (Vertically Opposite Angles)

3.2. From (2.3.3), (2.3.2) and (2.3.6),

$$(\mathbf{D}-\mathbf{B})^T(\mathbf{B}-\mathbf{C}) = \begin{pmatrix} 0 & b \end{pmatrix}\begin{pmatrix} a \\ 0 \end{pmatrix} = 0 \tag{3.2.1}$$

$$\implies BD \perp BC \tag{3.2.2}$$

3.3. From (2.3.1), (2.3.3), (2.3.2) and (2.3.6),

$$\|\mathbf{A}-\mathbf{B}\| = \left\|\begin{pmatrix} -a \\ b \end{pmatrix}\right\| \tag{3.3.1}$$

$$\|\mathbf{C}-\mathbf{D}\| = \left\|\begin{pmatrix} -a \\ -b \end{pmatrix}\right\| \tag{3.3.2}$$

$$\implies \|\mathbf{A}-\mathbf{B}\| = \|\mathbf{C}-\mathbf{D}\| \tag{3.3.3}$$

$$\text{or, } AB = CD \tag{3.3.4}$$

From RHS congruence, $\triangle ACB \cong \triangle DCB$.

3.4. From (3.3.4), noting that **M** is the mid point of both AB and CD,

$$CM = \frac{1}{2}CD = \frac{1}{2}AB \tag{3.4.1}$$