

Math Document Template

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Abstract—This a simple document explaining a question about the concept of similar triangles.

Download all python codes from

svn co <https://github.com/SiddharthPh/Summer2020/trunk/geometry/codes>

and latex-tikz codes from

svn co <https://github.com/gadepall/school/trunk/ncert/geometry/figs>

1 PROBLEM

In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B. Show that

- $\triangle AMC \cong \triangle BMD$
- $\triangle DBC$ is a right angle.
- $\triangle DBC \cong \triangle ABC$
- $CM = \frac{1}{2}AB$

2 CONSTRUCTION

2.1. The figure for A triangle obtained in the question looks like Fig. 2.1. with angles $\angle A$, $\angle C$ and $\angle B$ and sides a , b and c . The unique feature of this triangle is $\angle C$ which is defined to be 90° .

2.2. List the design parameters for construction

Solution: See Table. 2.2.

| Parameter | Value |
|--------------|------------|
| a | 4 |
| b | 3 |
| $\angle ACB$ | 90° |

TABLE 2.2: To construct $\triangle ACB$

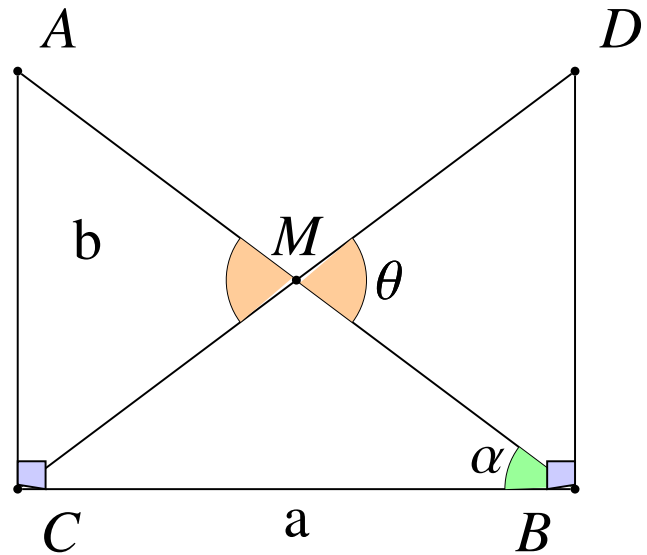


Fig. 2.1: Right Angled Triangle by Latex-Tikz

2.3. Find the coordinates of the various points in Fig. 2.1

Solution: From the given information,

$$\mathbf{A} = \begin{pmatrix} 0 \\ b \end{pmatrix} \quad (2.3.1)$$

$$\mathbf{C} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad (2.3.2)$$

$$\mathbf{B} = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad (2.3.3)$$

$\therefore \mathbf{M}$ is the midpoint of AB,

$$\mathbf{M} = \frac{\mathbf{A} + \mathbf{B}}{2} = \frac{1}{2} \begin{pmatrix} a \\ b \end{pmatrix} \quad (2.3.4)$$

Also, \mathbf{M} is given to be the midpoint of CD. Hence,

$$\mathbf{M} = \frac{\mathbf{C} + \mathbf{D}}{2} \quad (2.3.5)$$

$$\Rightarrow \mathbf{D} = 2\mathbf{M} - \mathbf{C} = \begin{pmatrix} a \\ b \end{pmatrix} \quad (2.3.6)$$

The values are listed in Table. 2.3

2.4. Draw Fig. 2.1.

| Derived Values. | |
|-----------------|------------------------------------------|
| M | $\begin{pmatrix} 2 \\ 1.5 \end{pmatrix}$ |
| D | $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ |

TABLE 2.3: To construct $\triangle DCB$

Solution: The following Python code generates Fig. 2.4

```
codes/triangle.py
```

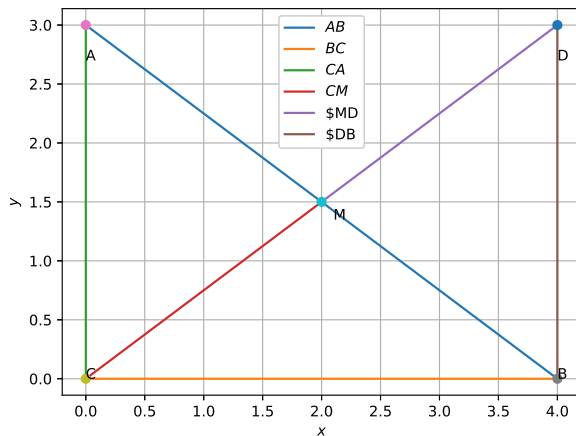


Fig. 2.4: Triangle generated using python

and the equivalent latex-tikz code generating Fig. 2.1 is

```
figs/triangle.tex
```

The above latex code can be compiled as a standalone document as

```
figs/triangle_fig.tex
```

3 SOLUTION

3.1. $\triangle AMC \cong \triangle DMB$ by SAS congruency \because

- $AM = BM$
- $CM = DM$
- $\angle AMC = \angle DMB$ (Vertically Opposite Angles)

3.2. From (2.3.3), (2.3.2) and (2.3.6),

$$(\mathbf{D} - \mathbf{B})^T (\mathbf{B} - \mathbf{C}) = \begin{pmatrix} 0 & b \end{pmatrix} \begin{pmatrix} a \\ 0 \end{pmatrix} = 0 \quad (3.2.1)$$

$$\Rightarrow BD \perp BC \quad (3.2.2)$$

3.3. From (2.3.1), (2.3.3), (2.3.2) and (2.3.6),

$$\|\mathbf{A} - \mathbf{B}\| = \left\| \begin{pmatrix} -a \\ b \end{pmatrix} \right\| \quad (3.3.1)$$

$$\|\mathbf{C} - \mathbf{D}\| = \left\| \begin{pmatrix} -a \\ -b \end{pmatrix} \right\| \quad (3.3.2)$$

$$\Rightarrow \|\mathbf{A} - \mathbf{B}\| = \|\mathbf{C} - \mathbf{D}\| \quad (3.3.3)$$

$$\text{or, } AB = CD \quad (3.3.4)$$

From RHS congruence, $\triangle ACB \cong \triangle DCB$.

3.4. From (3.3.4), noting that **M** is the mid point of both AB and CD ,

$$CM = \frac{1}{2}CD = \frac{1}{2}AB \quad (3.4.1)$$