Math Document Template

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Download all python codes from

```
svn co https://github.com/SiddharthPh/
    Summer2020/trunk/geometry/Probstat/codes
```

1 Probability Exercises

1.1 Exercise 1

1.1.1 Problem: Suppose you drop a die at random on the rectangular region shown in Fig.15.6.What is the probability that it will land inside the circle with diameter 1m?

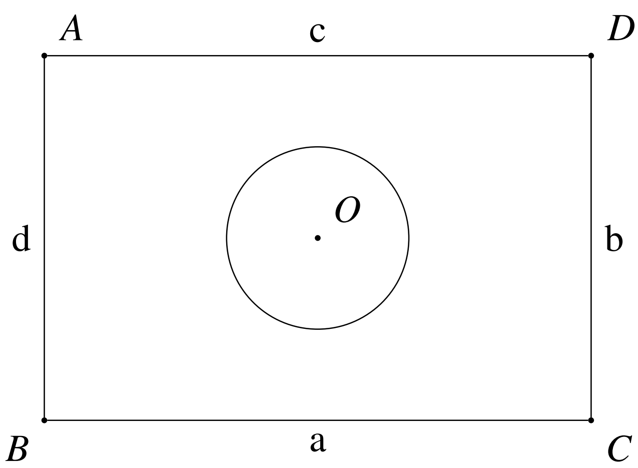

Fig. 0: Rectangle

1.1.2 Solution:

1. In the given question,
   The sample size = Total Area of the rectangle=

$$3x2 = 6m^2 \tag{1.1.2.1.1}$$

   Favourable outcome = Area of Circle=

$$\pi\left(\frac{1}{2}\right)^2 = \frac{\pi}{4}m^2 \tag{1.1.2.1.2}$$

   Probabilty(P) of the dice landing in the circle=$\frac{\pi}{24}$
   $\therefore$ P = 0.131
   The python code for the distribution.

```
prob/codes/prob1.py
```

   shows the Bernouli distribution of data.
   The Bernoulli Distribution of data is given below
   Probability mass function(P(X))=$p^x\,(1-p)^{1-x}$

$$P(X = 0) = 1 - p \tag{1.1.2.1.3}$$
$$P(X = 1) = p \tag{1.1.2.1.4}$$

   where p=0.131 given by 1.4.2.1

1.1.3 Understanding Graph:

1. From the graph (**??**),
   a) Values on X-axis represent the Bernoulli distribution of data.
   b) Values on Y-axis represent the density of frequency(Histogram estimator) of the data. To calculate the histogram estimator, we have to define the number of bins(Intervals) For the graph in the question,

$$bins = 10 \tag{1.1.3.1.1}$$
$$h(binwidth) = \frac{(1-0)}{10} \tag{1.1.3.1.2}$$

   For bin-width h, number of observations n, for bin j, proportion of observations is

$$p_j = \frac{y_j}{n} \tag{1.1.3.1.3}$$

   (Where y_j is the frequency of jth bin.)

$$p_0 = \frac{869}{1000} = 0.869 \tag{1.1.3.1.4}$$
$$p_1 = \frac{131}{1000} = 0.131 \tag{1.1.3.1.5}$$

   The density estimate is

$$y\,(x) = \frac{p_j}{h} \tag{1.1.3.1.6}$$
$$y\,(0) = \frac{0.869}{0.1} = 8.69 \tag{1.1.3.1.7}$$
$$y\,(0) = \frac{0.131}{0.1} = 1.31 \tag{1.1.3.1.8}$$

   To draw the Gaussian Kernel Density curve, Calculate mean and standard deviation for the centre and bandwidth.

See 1.1.3.1 for clear understanding.

$$\mu(Mean) = 0.861 \quad (1.1.3.1.9)$$

$$\sigma^2(\text{Standard Deviation}) = 0.1189 \quad (1.1.3.1.10)$$

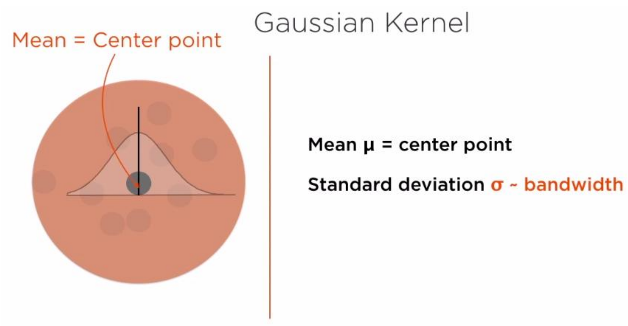

Fig. 1.1.3.1: Gaussian Kernel

1.2 *Exercise 2*

1.2.1 Problem: A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that
(i) She will buy it ?
(ii) She will not buy it ?

1.2.2 Solution:

1. In the given question,

   a) The sample size = Total number of pens(S)=

   $$S = 144 \quad (1.2.2.1.1)$$

   Favourable outcome = Pens purchased(F1)=

   $$F1 = 124 \quad (1.2.2.1.2)$$

   Probabilty(P) of the pens purchased by her from the shopkeeper=$\frac{124}{144}$
   $\therefore$ P = 0.861
   The python code for the distribution,

   ```
   prob/codes/prob2_a.py
   ```

   shows the Bernouli distribution of data.
   The Bernoulli Distribution of data is given below
   Probability mass function(P(X))=$p^x\,(1-p)^{1-x}$

   $$P(X = 0) = 1 - p \quad (1.2.2.1.3)$$
   $$P(X = 1) = p \quad (1.2.2.1.4)$$

   where p is the probability of occurence of (X=1)
   $\therefore$ p=0.861 given by 1.4.2.1

2. The sample size = Total number of pens(S)=

   $$S = 144 \quad (1.2.2.2.1)$$

   Favourable outcome = Pens not purchased(F2)=

   $$F2 = 20 \quad (1.2.2.2.2)$$

   Probabilty(P) of the pens not purchased by her from the shopkeeper=$\frac{20}{144}$
   $\therefore$ P = 0.139
   The python code for the distribution of data

   ```
   prob/codes/prob2_b.py
   ```

   shows the Bernouli distribution of data.
   The Bernoulli Distribution of data is given below
   Probability mass function(P(X))=$p^x\,(1-p)^{1-x}$

   $$P(X = 0) = 1 - p \quad (1.2.2.2.3)$$
   $$P(X = 1) = p \quad (1.2.2.2.4)$$

   where p is the probability of occurence of (X=1)
   $\therefore$ p=0.139 given by 1.4.2.1

1.3 *Exercise 3*

1.3.1 Problem: (i)Complete the following table:

Event:'Sum on two dice'	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	-	-	-	-	-	$\frac{5}{36}$	-	-	-	$\frac{1}{36}$

(ii) A student argues that 'there are 11 possible outcomes 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.Therefore,each of them has a probability $\frac{1}{11}$ Do you agree with this argument? Justify your answer.

1.3.2 Solution:

1. In the given question,

   a) The sample size = Total number of possibilities(S)=

$$\begin{pmatrix} \{1 \quad 1\} & \{1 \quad 2\} & \{1 \quad 3\} & \{1 \quad 4\} & \{1 \quad 5\} & \{1 \quad 6\} \\ \{2 \quad 1\} & \{2 \quad 2\} & \{2 \quad 3\} & \{2 \quad 4\} & \{2 \quad 5\} & \{2 \quad 6\} \\ \{3 \quad 1\} & \{3 \quad 2\} & \{3 \quad 3\} & \{3 \quad 4\} & \{3 \quad 5\} & \{3 \quad 6\} \\ \{4 \quad 1\} & \{4 \quad 2\} & \{4 \quad 3\} & \{4 \quad 4\} & \{4 \quad 5\} & \{4 \quad 6\} \\ \{5 \quad 1\} & \{5 \quad 2\} & \{5 \quad 3\} & \{5 \quad 4\} & \{5 \quad 5\} & \{5 \quad 6\} \\ \{6 \quad 1\} & \{6 \quad 2\} & \{6 \quad 3\} & \{6 \quad 4\} & \{6 \quad 5\} & \{6 \quad 6\} \end{pmatrix}$$

S=6x6=36

Favourable outcome for sum=2 (E1)= $(\{1 \quad 1\})$

$$E1 = 1 \tag{1.3.2.1.1}$$

$$Probabilty(P(E1)) = \frac{1}{36} = 0.027 \tag{1.3.2.1.2}$$

Favourable outcome for sum=3 (E2)= $(\{1 \quad 2\} \quad \{2 \quad 1\})$

$$E2 = 2 \tag{1.3.2.1.3}$$

$$Probabilty(P(E2)) = \frac{2}{36} = 0.055 \tag{1.3.2.1.4}$$

Favourable outcome for sum=4 (E3)= $(\{1 \quad 3\} \quad \{2 \quad 2\} \quad \{3 \quad 1\})$

$$E3 = 3 \tag{1.3.2.1.5}$$

$$Probabilty(P(E3)) = \frac{3}{36} = 0.083 \tag{1.3.2.1.6}$$

Favourable outcome for sum=5 (E4)= $(\{1 \quad 4\} \quad \{2 \quad 3\} \quad \{3 \quad 2\} \quad \{4 \quad 1\})$

$$E4 = 4 \tag{1.3.2.1.7}$$

$$Probabilty(P(E4)) = \frac{4}{36} = 0.111 \tag{1.3.2.1.8}$$

Favourable outcome for sum=6 (E5)= $(\{1 \quad 5\} \quad \{2 \quad 4\} \quad \{3 \quad 3\} \quad \{4 \quad 2\} \quad \{5 \quad 1\})$

$$E5 = 5 \tag{1.3.2.1.9}$$

$$Probabilty(P(E5)) = \frac{5}{36} = 0.138 \tag{1.3.2.1.10}$$

Favourable outcome for sum=7 (E6)= $(\{1 \quad 6\} \quad \{2 \quad 5\} \quad \{3 \quad 4\} \quad \{4 \quad 3\} \quad \{5 \quad 2\} \quad \{6 \quad 1\})$

$$E6 = 6 \tag{1.3.2.1.11}$$

$$Probabilty(P(E6)) = \frac{6}{36} = 0.166 \tag{1.3.2.1.12}$$

Favourable outcome for sum=8 (E7)= $(\{2 \quad 6\} \quad \{3 \quad 5\} \quad \{4 \quad 4\} \quad \{5 \quad 3\} \quad \{6 \quad 2\})$

$$E7 = 5 \tag{1.3.2.1.13}$$

$$Probabilty(P(E7)) = \frac{5}{36} = 0.138 \tag{1.3.2.1.14}$$

Favourable outcome for sum=9 (E8)= $(\{3 \quad 6\} \quad \{4 \quad 5\} \quad \{5 \quad 4\} \quad \{6 \quad 3\})$

$$E8 = 4 \tag{1.3.2.1.15}$$

$$Probabilty(P(E8)) = \frac{4}{36} = 0.111 \tag{1.3.2.1.16}$$

Favourable outcome for sum=10 (E9)= $(\{4 \quad 6\} \quad \{5 \quad 5\} \quad \{6 \quad 4\})$

$$E9 = 3 \tag{1.3.2.1.17}$$

$$Probabilty(P(E9)) = \frac{3}{36} = 0.083 \tag{1.3.2.1.18}$$

Favourable outcome for sum=3=11 (E10)= $(\{5 \quad 6\} \quad \{6 \quad 5\})$

$$E10 = 2 \tag{1.3.2.1.19}$$

$$Probabilty(P(E10)) = \frac{2}{36} = 0.055 \tag{1.3.2.1.20}$$

Favourable outcome for sum=12 (E11)= $\left(\begin{Bmatrix}6 & 6\end{Bmatrix}\right)$

$$E11 = 1 \tag{1.3.2.1.21}$$

$$Probabilty(P(E11)) = \frac{1}{36} = 0.027 \tag{1.3.2.1.22}$$

Table is completed as follows:

Event:'Sum on two dice'	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

a) The argument mentioned by the student is incorrect.
In the question the event of measurement is sum of possible outcomes of rolling two dice. Here, the probability of occurence of each outcome is not equal.
The different values of probabilities are mentioned in the above solution.
The argument can be supported by the figure **??**
The python code for the distribution of data,

```
prob/codes/prob3.py
```

shows the random distribution of data.
The Distribution of data is given below
Probability mass function(P(X=k))=

$$\begin{Bmatrix} \frac{k-1}{36} & forx < 8 \\ \frac{13-k}{36} & forx => 8 \end{Bmatrix} \tag{1.3.2.1.23}$$

1.4 Exercise 4

1.4.1 Problem: A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result i.e., three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.

1.4.2 Solution:

1. In the given question,
The sample size = Possible number of tosses=8

$$\left(\begin{Bmatrix}HHH\end{Bmatrix} \begin{Bmatrix}TTT\end{Bmatrix} \begin{Bmatrix}HHT\end{Bmatrix} \begin{Bmatrix}HTT\end{Bmatrix} \begin{Bmatrix}HTH\end{Bmatrix} \begin{Bmatrix}TTH\end{Bmatrix}\begin{Bmatrix}THT\end{Bmatrix} \begin{Bmatrix}THH\end{Bmatrix}\right) \tag{1.4.2.1.1}$$

Favourable outcome =Other than three Heads (or) Tails=6

$$\left(\begin{Bmatrix}HHT\end{Bmatrix} \begin{Bmatrix}HTT\end{Bmatrix} \begin{Bmatrix}HTH\end{Bmatrix} \begin{Bmatrix}TTH\end{Bmatrix}\begin{Bmatrix}THT\end{Bmatrix} \begin{Bmatrix}THH\end{Bmatrix}\right) \tag{1.4.2.1.2}$$

Probabilty(P) that Hanif will loose the game=$\frac{6}{8}$
$\therefore$ P = 0.75
The python code for the distribution of data,

```
prob/codes/prob4.py
```

shows the Bernouli distribution of data.
The Bernoulli Distribution of data is given below
Probability mass function(P(X))=$p^x (1-p)^{1-x}$

$$P(X = 0) = 1 - p \tag{1.4.2.1.3}$$

$$P(X = 1) = p \tag{1.4.2.1.4}$$

where p=0.75 given by 1.4.2.1

1.5 Exercise 5

1.5.1 Problem: A die is thrown twice. What is the probability that
(i) 5 will not come up either time?
(ii) 5 will come up at least once?
Hint : Throwing a die twice and throwing two dice simultaneously are treated as the same experiment

1.5.2 Solution:

1. In the given question,

a) The sample size = Total number of possibilities(S)=
$$\begin{pmatrix} \begin{Bmatrix}1 & 1\end{Bmatrix} & \begin{Bmatrix}1 & 2\end{Bmatrix} & \begin{Bmatrix}1 & 3\end{Bmatrix} & \begin{Bmatrix}1 & 4\end{Bmatrix} & \begin{Bmatrix}1 & 5\end{Bmatrix} & \begin{Bmatrix}1 & 6\end{Bmatrix} \\ \begin{Bmatrix}2 & 1\end{Bmatrix} & \begin{Bmatrix}2 & 2\end{Bmatrix} & \begin{Bmatrix}2 & 3\end{Bmatrix} & \begin{Bmatrix}2 & 4\end{Bmatrix} & \begin{Bmatrix}2 & 5\end{Bmatrix} & \begin{Bmatrix}2 & 6\end{Bmatrix} \\ \begin{Bmatrix}3 & 1\end{Bmatrix} & \begin{Bmatrix}3 & 2\end{Bmatrix} & \begin{Bmatrix}3 & 3\end{Bmatrix} & \begin{Bmatrix}3 & 4\end{Bmatrix} & \begin{Bmatrix}3 & 5\end{Bmatrix} & \begin{Bmatrix}3 & 6\end{Bmatrix} \\ \begin{Bmatrix}4 & 1\end{Bmatrix} & \begin{Bmatrix}4 & 2\end{Bmatrix} & \begin{Bmatrix}4 & 3\end{Bmatrix} & \begin{Bmatrix}4 & 4\end{Bmatrix} & \begin{Bmatrix}4 & 5\end{Bmatrix} & \begin{Bmatrix}4 & 6\end{Bmatrix} \\ \begin{Bmatrix}5 & 1\end{Bmatrix} & \begin{Bmatrix}5 & 2\end{Bmatrix} & \begin{Bmatrix}5 & 3\end{Bmatrix} & \begin{Bmatrix}5 & 4\end{Bmatrix} & \begin{Bmatrix}5 & 5\end{Bmatrix} & \begin{Bmatrix}5 & 6\end{Bmatrix} \\ \begin{Bmatrix}6 & 1\end{Bmatrix} & \begin{Bmatrix}6 & 2\end{Bmatrix} & \begin{Bmatrix}6 & 3\end{Bmatrix} & \begin{Bmatrix}6 & 4\end{Bmatrix} & \begin{Bmatrix}6 & 5\end{Bmatrix} & \begin{Bmatrix}6 & 6\end{Bmatrix} \end{pmatrix}$$
S=6x6=36
Favourable outcome for no five in either of the dice (E1)=
$$\begin{pmatrix} \begin{Bmatrix}1 & 1\end{Bmatrix} & \begin{Bmatrix}1 & 2\end{Bmatrix} & \begin{Bmatrix}1 & 3\end{Bmatrix} & \begin{Bmatrix}1 & 4\end{Bmatrix} & \begin{Bmatrix}1 & 6\end{Bmatrix} \\ \begin{Bmatrix}2 & 1\end{Bmatrix} & \begin{Bmatrix}2 & 2\end{Bmatrix} & \begin{Bmatrix}2 & 3\end{Bmatrix} & \begin{Bmatrix}2 & 4\end{Bmatrix} & \begin{Bmatrix}2 & 6\end{Bmatrix} \\ \begin{Bmatrix}3 & 1\end{Bmatrix} & \begin{Bmatrix}3 & 2\end{Bmatrix} & \begin{Bmatrix}3 & 3\end{Bmatrix} & \begin{Bmatrix}3 & 4\end{Bmatrix} & \begin{Bmatrix}3 & 6\end{Bmatrix} \\ \begin{Bmatrix}4 & 1\end{Bmatrix} & \begin{Bmatrix}4 & 2\end{Bmatrix} & \begin{Bmatrix}4 & 3\end{Bmatrix} & \begin{Bmatrix}4 & 4\end{Bmatrix} & \begin{Bmatrix}4 & 6\end{Bmatrix} \\ \begin{Bmatrix}6 & 1\end{Bmatrix} & \begin{Bmatrix}6 & 2\end{Bmatrix} & \begin{Bmatrix}6 & 3\end{Bmatrix} & \begin{Bmatrix}6 & 4\end{Bmatrix} & \begin{Bmatrix}6 & 6\end{Bmatrix} \end{pmatrix}$$

$$E1 = 25 \tag{1.5.2.1.1}$$

$$Probabilty(P(E1)) = \frac{25}{36} = 0.694 \tag{1.5.2.1.2}$$

a) The sample size = Total number of possibilities(S)=

$$\begin{pmatrix} \begin{Bmatrix}1 & 1\end{Bmatrix} & \begin{Bmatrix}1 & 2\end{Bmatrix} & \begin{Bmatrix}1 & 3\end{Bmatrix} & \begin{Bmatrix}1 & 4\end{Bmatrix} & \begin{Bmatrix}1 & 5\end{Bmatrix} & \begin{Bmatrix}1 & 6\end{Bmatrix} \\ \begin{Bmatrix}2 & 1\end{Bmatrix} & \begin{Bmatrix}2 & 2\end{Bmatrix} & \begin{Bmatrix}2 & 3\end{Bmatrix} & \begin{Bmatrix}2 & 4\end{Bmatrix} & \begin{Bmatrix}2 & 5\end{Bmatrix} & \begin{Bmatrix}2 & 6\end{Bmatrix} \\ \begin{Bmatrix}3 & 1\end{Bmatrix} & \begin{Bmatrix}3 & 2\end{Bmatrix} & \begin{Bmatrix}3 & 3\end{Bmatrix} & \begin{Bmatrix}3 & 4\end{Bmatrix} & \begin{Bmatrix}3 & 5\end{Bmatrix} & \begin{Bmatrix}3 & 6\end{Bmatrix} \\ \begin{Bmatrix}4 & 1\end{Bmatrix} & \begin{Bmatrix}4 & 2\end{Bmatrix} & \begin{Bmatrix}4 & 3\end{Bmatrix} & \begin{Bmatrix}4 & 4\end{Bmatrix} & \begin{Bmatrix}4 & 5\end{Bmatrix} & \begin{Bmatrix}4 & 6\end{Bmatrix} \\ \begin{Bmatrix}5 & 1\end{Bmatrix} & \begin{Bmatrix}5 & 2\end{Bmatrix} & \begin{Bmatrix}5 & 3\end{Bmatrix} & \begin{Bmatrix}5 & 4\end{Bmatrix} & \begin{Bmatrix}5 & 5\end{Bmatrix} & \begin{Bmatrix}5 & 6\end{Bmatrix} \\ \begin{Bmatrix}6 & 1\end{Bmatrix} & \begin{Bmatrix}6 & 2\end{Bmatrix} & \begin{Bmatrix}6 & 3\end{Bmatrix} & \begin{Bmatrix}6 & 4\end{Bmatrix} & \begin{Bmatrix}6 & 5\end{Bmatrix} & \begin{Bmatrix}6 & 6\end{Bmatrix} \end{pmatrix}$$

S=6x6=36

Favourable outcome for atleast one five in either of the dice (E2)=

$$\begin{pmatrix} \begin{Bmatrix}1 & 5\end{Bmatrix} & \begin{Bmatrix}2 & 5\end{Bmatrix} & \begin{Bmatrix}3 & 5\end{Bmatrix} & \begin{Bmatrix}4 & 5\end{Bmatrix} & \begin{Bmatrix}5 & 1\end{Bmatrix} & \begin{Bmatrix}5 & 2\end{Bmatrix} \\ \begin{Bmatrix}5 & 3\end{Bmatrix} & \begin{Bmatrix}5 & 4\end{Bmatrix} & \begin{Bmatrix}5 & 5\end{Bmatrix} & \begin{Bmatrix}5 & 6\end{Bmatrix} & \begin{Bmatrix}6 & 5\end{Bmatrix} & \end{pmatrix}$$

$$E1 = 11 \tag{1.5.2.1.3}$$

$$Probabilty(P(E2)) = \frac{11}{36} = 0.305 \tag{1.5.2.1.4}$$

The python code for the distribution.

```
prob/codes/prob5.py
```

shows the Bernouli distribution of data.
The Bernoulli Distribution of data is given below
Probability mass function(P(X))=$p^x (1-p)^{1-x}$

$$P(X = 0) = 1 - p \tag{1.5.2.1.5}$$

$$P(X = 1) = p \tag{1.5.2.1.6}$$

where p=0.694 for solution 1.5.2.1a and p=0.305 for solution 1.5.2.1a