JEE 2017 through Python

G V V Sharma*

Problem 1. Find the sum of all the real values of x satisfying the equation

$$2^{(x-1)()x^2+5x-50} = 1 \tag{1}$$

Problem 2. Sketch the curve

$$\mathrm{Im}\left(\frac{1z-2}{z-1}\right)+1=0, z\in\mathbf{C}, z\neq 1 \tag{2}$$

Problem 3. If $x = a, y = b, z = c$ is a solution of the system of linear equations

$$\begin{aligned} x+8y+7z&=0\\ 9x+2y+3z&=0\\ x+y+z&=0 \end{aligned} \tag{3}$$

such that the point (a, b, c) lies on the plane $x+2y+z = 6$, then find $2a + b + c$.

Problem 4. a, b, c are in AP such that $abc = 8$. Find the minimum value of b.

Problem 5. Given

$$S_n = \frac{1}{1^3}+\frac{1+2}{1^3+2^3}+\frac{1+2+3}{1^3+2^3+3^3} + \cdots + \frac{1+2+\cdots+n}{1^3+2^3+\cdots+n^3} \tag{4}$$

and $100S_n = n$, find n.

Problem 6. Find the value of k for which the function

$$f(x)=\begin{cases}\left(\frac{4}{4}\right)^{\frac{\tan 4x}{\tan 5x}}, & 0<x<\frac{\pi}{2}\\ k+\frac{2}{5}, & x=\frac{\pi}{2}\end{cases} \tag{5}$$

is continuous at $x = \frac{\pi}{2}$.

Problem 7. Sketch the function defined by

$$f(x) = x^2 - 3x^2 + 5x + 7 \tag{6}$$

and mark the regions where it is increasing or decreasing.

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502205 India e-mail: gadepall@iith.ac.in.

Problem 8. Given that

$$\lim_{n\to\infty}\frac{1^a+2^a+\cdots+n^a}{(n+1)^{a-1}\left[(na+1)(na+2)+\cdots+(na+n)\right]} = \frac{1}{60}, \tag{7}$$

find a.

Problem 9. A square of each side 2, lies above the x-axis and has one vertex at the origin. If one of the sides passing through the origin makes an angle 30° with the positive direction of the x-axis, then find the sum of the x-coordinates of the vertices of the square

Problem 10. A line drawn through the point $P(4, 7)$ cuts the circle $x^2 + y^2 = 9$ at the points A and B. Find $PA.PB$.

Problem 11. Find the eccentricity of an ellipse having centre at the origin, axes along the co-ordinate axes and passing through the points (4, 1) and (2, 2).

Problem 12. If $y = mx + c$ is the normal at a point on the parabola $y^2 = 8x$ whose focal distance is 8 units, then find $|c|$.

Problem 13. Solve

$$\sin[\cot^{-1}(1+x)] = \cos[\tan^{-1}x], \tag{8}$$

Problem 14. Sketch the curve

$$2|z+31|-|z-31|=0, \quad z\in\mathbf{C}. \tag{9}$$

Problem 15. Find λ for which the system of equations

$$\begin{aligned} 2x+4y-\lambda z&=0\\ 4x+\lambda y+2z&=0\\ \lambda x+2y+2z&=0 \end{aligned} \tag{10}$$

has infinitely many solutions.

Problem 16. The sum of the first n terms of the series

$$\sqrt{3}+\sqrt{75}+\sqrt{243}+\sqrt{507}+\cdots = 435\sqrt{3} \tag{11}$$

Find n.

Problem 17. Find

$$\lim_{x\to 3} \frac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}}. \tag{12}$$

Problem 18. The tangent at the point (2, -2) to the curve, $x^2y^2 2x = 4(1-y)$ does not pass through the point:

1) $\left(4, \frac{1}{3}\right)$
2) $(8, 5)$
3) $(-4, -9)$
4) $(-2, -7)$

Problem 19. A point P has coordinates $(0, -2)$ and Q is any point on the circle

$$x^2 + y^2 - 5x - y + 5 = 0 \tag{13}$$

Find the maximum value of PQ^2.

Problem 20. Sketch the locus of the point of intersection of the straight lines,

$$\begin{aligned} tx - 2y - 3t &= 0 \\ x - 2ty + 3 &= 0, \end{aligned} \quad t \in \mathbf{R} \tag{14}$$

Problem 21. If the common tangents to the parabola, $x^2 = 4y$ and the circle, $x^2+y^2 = 4$ intersect at the point P, then find the distance of P from the origin.

Problem 22. Consider an ellipse, whose centre is at the origin and its major axis is along the x-axis. If its eccentricity is $\frac{3}{5}$ and the distance between its foci is 6, then find the area (in sq. units) of the quadrilateral inscribed in the ellipse, with the vertices as the vertices of the ellipse.

Problem 23. If

$$S = \left\{ x \in [0, 2\pi] : \begin{vmatrix} 0 & \cos x & -\sin x \\ \sin x & 0 & \cos x \\ \cos x & \sin x & 0 \end{vmatrix} = 0 \right\} \tag{15}$$

then find $\sum_{x\in S} \tan\left(\frac{\pi}{3} + x\right)$

Problem 24. For a positive integer n, the quadratic equation

$$\begin{aligned} x(x+1) + (x+1)(x+2) \\ + \cdots + \left(x + \overline{n-1}\right)(x+n) = 10n \end{aligned} \tag{16}$$

has two consecutive integral solutions. Find n.

Problem 25. Let ω be a complex number such that $2\omega + 1 = z$, where $z = \sqrt{-3}$. If

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k, \tag{17}$$

find k.

Problem 26. If

$$A = \begin{pmatrix} 2 & -3 \\ -4 & 1 \end{pmatrix}, \tag{18}$$

find $\text{adj}\left(3A^2 + 12A\right)$.

Problem 27. Find the value of

$$\begin{aligned} \left({}^{21}C_1 - {}^{10}C_1\right) + \left({}^{21}C_2 - {}^{10}C_2\right) \\ \left({}^{21}C_3 - {}^{10}C_3\right) + \left({}^{21}C_4 - {}^{10}C_4\right) + \ldots \\ \left({}^{21}C_{10} - {}^{10}C_{10}\right) \end{aligned} \tag{19}$$

Problem 28. Find

$$\lim_{x\to\frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3} \tag{20}$$

Problem 29. The normal to the curve y(x-2)(x-3)=x+6 at the point where the curve intersects the y-axis passes through the point :

1) $\left(\frac{1}{2}, \frac{1}{2}\right)$
2) $\left(\frac{1}{2}, -\frac{1}{3}\right)$
3) $\left(\frac{1}{2}, \frac{1}{3}\right)$
4) $-\left(\frac{1}{2}, -\frac{1}{2}\right)$.

Problem 30. Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Find the maximum area (in sq. m) of the flower-bed.

Problem 31. Sketch the region

$$\left\{(x, y) : x^2 \le 4y, x + y \le 3, x \ge 0, y \le 1 + \sqrt{x}\right\}$$

and find its area.

Problem 32. Let k be an integer such that the triangle with vertices (k, -3k), (5, k) and (-k, 2) has area 28 sq. units. Find the orthocentre of this triangle.

Problem 33. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directrices is x=4, then find the equation of the normal to it at $\left(1, \frac{3}{2}\right)$.

Problem 34. A hyperbola passes through the point $P\left(\sqrt{2}, \sqrt{3}\right)$ and has foci at $(\pm 2, 0)$. Then the tangent to this hyperbola at P also passes through the point

1) $\left(2\sqrt{2}, 3\sqrt{3}\right)$
2) $\left(\sqrt{3}, \sqrt{2}\right)$
3) $\left(-\sqrt{2}, -\sqrt{3}\right)$
4) $\left(3\sqrt{2}, 2\sqrt{3}\right)$

Problem 35. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then find $\cos 4x$.

Problem 36. If $(x + 1y)^2 = 7 + 24\imath$, then find $\left(7 + \sqrt{-576}\right)^{\frac{1}{2}} - \left(7 - \sqrt{-576}\right)^{\frac{1}{2}}$

Problem 37. The sum of the first 15 terms of the series $3 + 7 + 14 + 24 + 37 + \ldots$ is 15 k. Find k

Problem 38. Find

$$\lim_{x \to 0} \frac{\log(\sin 7x + \cos 7x)}{\sin 3x} \tag{21}$$

Problem 39. Find the sum of the abscissae of the points where the curves $y = kx^2 + (5k + 3)x + 6k + 5, (k \in \mathbf{R})$, touch the x-axis.

Problem 40. The line $y = mx$ bisects the area of the region

$$\left\{(x, y) : 0 \le x \le \frac{3}{2}, 20 \le y \le 1 + 4x - x^2\right\}$$

Find m.

Problem 41. Sketch the circle, which is the mirror image of the circle, $x^2 + y^2 - 2x = 0$, in the line, $y = 3x$

Problem 42. Sketch the perpendiculars drawn from the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1$ upon the tangent to it at the point $\left(\frac{3}{2}, \frac{5\sqrt{3}}{2}\right)$ and find their product.

Problem 43. Which one of the following points does not lie on the normal to the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ drawn at the point $\left(8, 3\sqrt{3}\right)$

1) $\left(13, -\frac{1}{\sqrt{3}}\right)$
2) $\left(12, \frac{1}{\sqrt{3}}\right)$
3) $\left(11, \sqrt{3}\right)$
4) $\left(10, \sqrt{3}\right)$

Problem 44. Find the value of $\frac{1}{\cos 285^\circ} + \frac{1}{\sqrt{3}\sin 255^\circ}$