

Math Document Template

C ANISH

Abstract—This is a document explaining a question 2.0.2. about the concept of finding the center and radius from a circle equation.

Download all python codes from

```
svn co https://github.com/chakki1234/summer
-2020/trunk/linearalg/codes
```

and latex-tikz codes from

```
svn co https://github.com/chakki1234/summer
-2020/trunk/linearalg/figs
```

The general of a circle equation is $Ax^2 + Bxy + Ay^2 + Dx + Ey + F$, the equation can be represented as follow in the vector form:

$$x^T \begin{pmatrix} A & \frac{B}{2} \\ \frac{B}{2} & A \end{pmatrix} x + \begin{pmatrix} D & E \end{pmatrix} x + F = 0 \quad (2.0.2.1)$$

To find the center - \mathbf{O} and radius - r of a circle:

$$\mathbf{O} = \frac{-1}{2A} \begin{pmatrix} D & E \end{pmatrix} \quad (2.0.2.2)$$

$$r = \frac{1}{A} \sqrt{\frac{1}{4} \left\| \begin{pmatrix} D \\ E \end{pmatrix} \right\|^2 - F^2} \quad (2.0.2.3)$$

1 PROBLEM

Find the center and radius of the circle

$$x^T x + \begin{pmatrix} 8 \\ 10 \end{pmatrix} x - 8 = 0 \quad (1.0.1)$$

2 CONSTRUCTION

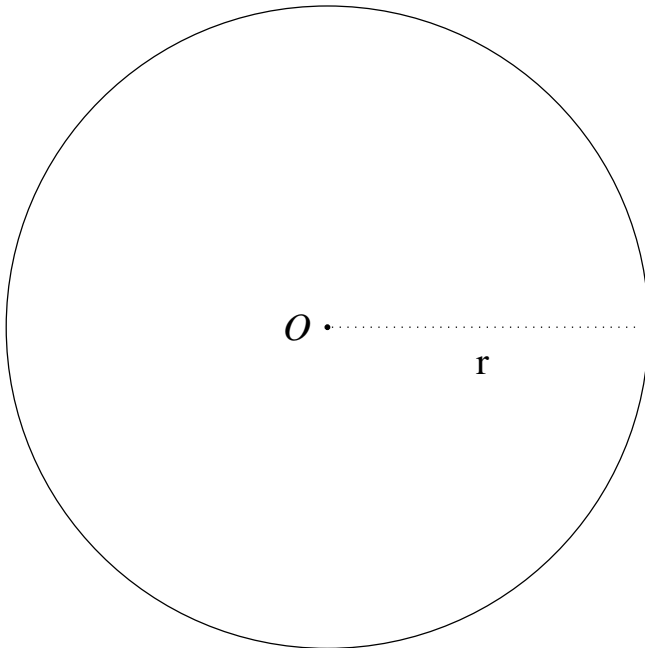


Fig. 2.0.0: Circle by Latex-Tikz

2.0.1. The figure obtained looks like Fig. 2.0.0.

2.0.3. From the given information, The values are listed in 2.0.3

Output values	
Parameter	Value
\mathbf{O}	$\begin{pmatrix} -4 \\ -5 \end{pmatrix}$
r	7

TABLE 2.0.3: Value of \mathbf{O} and r

2.0.4. Draw Fig. 2.0.4.

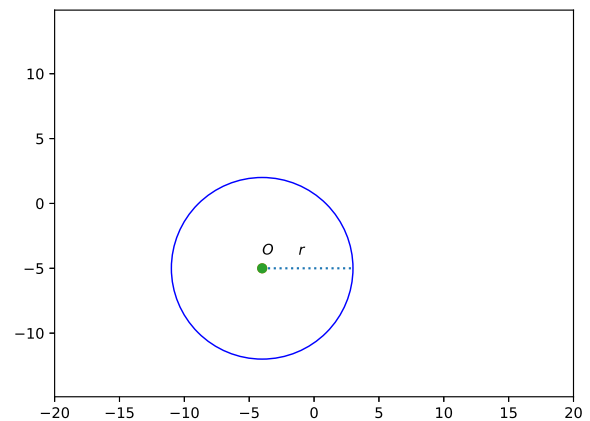


Fig. 2.0.4: Circle generated using python

Solution: The following Python code generates Fig. 2.0.4

```
codes/circle_exam.py
```

and the equivalent latex-tikz code generating Fig. 2.0.4 is

```
figs/circle_fig.tex
```

The above latex code can be compiled as a standalone document as

```
figs/circle_fig_final.tex
```

3 SOLUTION

Solution: The general of a circle equation is $Ax^2 + Bxy + Ay^2 + Dx + Ey + F$, the equation can be represented as follow in the vector form:

$$x^T \begin{pmatrix} A & \frac{B}{2} \\ \frac{B}{2} & A \end{pmatrix} x + \begin{pmatrix} D & E \end{pmatrix} x + F = 0 \quad (4.1)$$

To find the center - \mathbf{O} and radius - r of a circle:

$$\mathbf{O} = \frac{-1}{2A} \begin{pmatrix} D & E \end{pmatrix} \quad (4.2)$$

$$r = \frac{1}{A} \sqrt{\frac{1}{4} \left\| \begin{pmatrix} D \\ E \end{pmatrix} \right\|^2 - F^2} \quad (4.3)$$

The values given:

$$A = 1 \quad (4.4)$$

$$D = 8 \quad (4.5)$$

$$E = 10 \quad (4.6)$$

$$F = -8 \quad (4.7)$$

Substituting the values in equation 4.2 and 4.3:

$$\mathbf{O} = \begin{pmatrix} -4 \\ -5 \end{pmatrix} \quad (4.8)$$

$$r = 7 \quad (4.9)$$