Math Document Template

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***Abstract*—This is a document explaining a question about the concept of sum of angles in a cyclic quadilateral.**

Download all python codes from

```
svn co https://github.com/chakki1234/summer
    -2020/trunk/linearalg/codes
```

and latex-tikz codes from

```
svn co https://github.com/chakki1234/summer
    -2020/trunk/linearalg/figs
```

1 Problem

In a $ABCD$ is a cyclic quadilateral with

$$\angle A = 4y + 20 \tag{1.0.1}$$
$$\angle B = 3y - 5 \tag{1.0.2}$$
$$\angle C = -4x \tag{1.0.3}$$
$$\angle D = -7x + 5 \tag{1.0.4}$$

Find its angles.

2 Construction

2.1. The figure obtained looks like Fig. 2.0.

2.2. The design parameters used for construction See Table. 2.2.

Design Parameters	
Parameters	Value
a	10
b	8

TABLE 2.2: Quadilateral $ABCD$

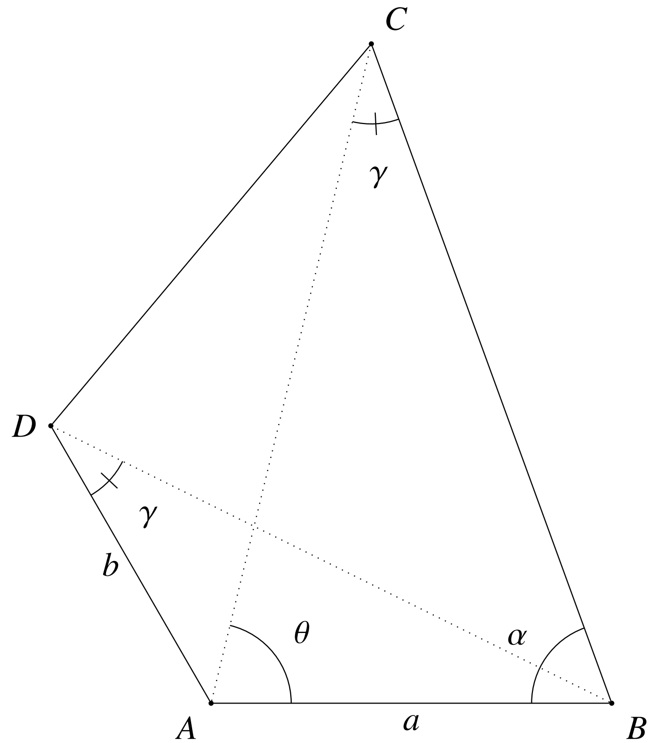

Fig. 2.0: Cyclic quadilateral by Latex-Tikz

2.3. Coordinates of cyclic quadilateral Fig2.0.

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.3.1}$$
$$\mathbf{B} = \begin{pmatrix} a \\ 0 \end{pmatrix} \tag{2.3.2}$$
$$\mathbf{C} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{2.3.3}$$
$$\mathbf{D} = \begin{pmatrix} b\cos\theta \\ b\sin\theta \end{pmatrix} \tag{2.3.4}$$

2.4. To find the coordinates of $\mathbf{C}$.

Theorem 2.1. *Angles formed in the same segment of a circle are always equal in measure.*

$$\cos\gamma = \frac{(A-D)^T(B-D)}{\|A-D\|\,\|B-D\|} \tag{2.4.1}$$
$$\theta = 180^\circ - \gamma - \angle B \tag{2.4.2}$$

In $\triangle ACB$. Finding the Scalar Products:

$$\begin{aligned}(\mathbf{B}-\mathbf{A})^T(\mathbf{C}-\mathbf{A}) = \\ \|\mathbf{B}-\mathbf{A}\|\,\|\mathbf{C}-\mathbf{A}\|\cos\theta\end{aligned} \tag{2.4.3}$$

$$\begin{aligned}(\mathbf{C}-\mathbf{B})^T(\mathbf{A}-\mathbf{B}) = \\ \|\mathbf{C}-\mathbf{B}\|\,\|\mathbf{A}-\mathbf{B}\|\cos\alpha\end{aligned} \tag{2.4.4}$$

On simplifying equation 2.4.3 and 2.4.4:

$$x^2\tan\theta^2 = y^2 \tag{2.4.5}$$
$$(x-a)^2 = \left((x-a)^2+y^2\right)\cos\alpha^2 \tag{2.4.6}$$

Substituting 2.4.5 in 2.4.6:

$$\begin{aligned}&x^2\left(1-\cos\alpha^2-\tan\theta^2\cos\alpha^2\right)\\ &+x\left(2a\cos\alpha^2-2a\right)+a^2\sin\alpha^2\end{aligned} \tag{2.4.7}$$

If θ and α are accute angles:

$$x = \frac{\left(-b-\sqrt{b^2-4ac}\right)}{2a} \tag{2.4.8}$$

else:

$$x = \frac{\left(-b+\sqrt{b^2-4ac}\right)}{2a} \tag{2.4.9}$$

The value of x can then be substituted in 2.4.5 to find the coordinates of $\mathbf{C}$

2.5. From the given information, The values are listed in 2.5

Output values	
Parameter	Value
C	$\begin{pmatrix}4\\16.47\end{pmatrix}$
D	$\begin{pmatrix}-4\\6.93\end{pmatrix}$

TABLE 2.5: Values of **C** and **D**

2.6. Draw Fig. 2.6.

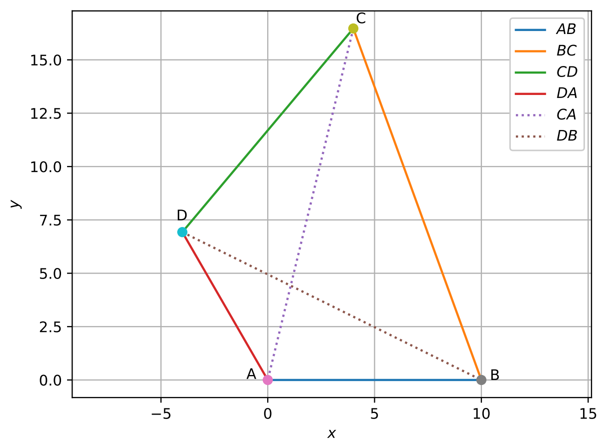

Fig. 2.6: Triangle generated using python

Solution: The following Python code generates Fig. 2.6

```
codes/cyclic_quad.py
```

and the equivalent latex-tikz code generating Fig. 2.6 is

```
figs/cyclic_quad_fig.tex
```

The above latex code can be compiled as a standalone document as

```
figs/cyclic_quad_final.tex
```

3 Solution

Theorem 3.1. *Sum of opposite angles in a cyclic quadilateral equals* $180°$.

Solution: From theorem 3.1

$$\angle A + \angle C = 180^\circ \tag{6.1}$$
$$\angle B + \angle D = 180^\circ \tag{6.2}$$

From the given information:

$$4y + 20 - 4x = 180^\circ \tag{6.3}$$
$$3y - 5 - 7x + 5 = 180^\circ \tag{6.4}$$

Solving equations 6.3 and 6.4:

$$x = -15 \tag{6.5}$$
$$y = 25 \tag{6.6}$$
$$\implies \angle A = 120^\circ \tag{6.7}$$
$$\implies \angle B = 70^\circ \tag{6.8}$$
$$\implies \angle C = 60^\circ \tag{6.9}$$
$$\implies \angle D = 110^\circ \tag{6.10}$$