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Abstract—This book provides a computational approach to probability and statistics based on the NCERT textbooks from Class 6-12. Links to sample Python codes are available in the text.

Download python codes using

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svn co https://github.com/gadepall/school/trunk/ncert/probability/codes
```

1 PROBABILITY

1.1 Examples

1. A coin is tossed 1000 times with the following frequencies:
Head : 455, Tail : 545
Compute the probability for each event.
2. Two coins are tossed simultaneously 500 times, and we get
Two heads : 105 times
One head : 275 times
No head : 120 times
Find the probability of occurrence of each of these events.
3. A die is thrown 1000 times with the frequencies for the outcomes 1, 2, 3, 4, 5 and

6 as given in the following table :

Outcome	1	2	3	4	5	6
Frequency	179	150	157	149	175	190

Find the probability of getting each outcome.

4. On one page of a telephone directory, there were 200 telephone numbers. The frequency distribution of their unit place digit (for example, in the number 25828573, the unit place digit is 3) is given in Table below :

Digit	0	1	2	3	4	5	6	7	8	9
Frequency	22	26	22	22	20	10	14	28	16	20

Without looking at the page, the pencil is placed on one of these numbers, i.e., the number is chosen at random. What is the probability that the digit in its unit place is 6?

5. The record of a weather station shows that out of the past 250 consecutive days, its weather forecasts were correct 175 times.
(i) What is the probability that on a given day it was correct?
(ii) What is the probability that it was not correct on a given day?
6. A tyre manufacturing company kept a record of the distance covered before a tyre needed to be replaced. The table shows the results of 1000 cases.

Distance(in km)	> 4000	4000-9000	9001-14000	<14000
Frequency	20	210	325	445

If you buy a tyre of this company, what is the probability that :

- (i) it will need to be replaced before it has covered 4000 km?
- (ii) it will last more than 9000 km?

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Age of drivers (in years)	Accidents in one year				
	0	1	2	3	over 3
18-29	440	160	110	61	35
30-50	505	125	60	22	18
Above 50	360	45	35	15	9

(iii) it will need to be replaced after it has covered somewhere between 4000 km and 14000 km?

7. The percentage of marks obtained by a student in the monthly unit tests are given below:

Unit test	I	II	III	IV	V
Frequency	69	71	73	68	74

Based on this data, find the probability that the student gets more than 70% marks in a unit test.

8. An insurance company selected 2000 drivers at random (i.e., without any preference of one driver over another) in a particular city to find a relationship between age and accidents. The data obtained are given in the following table: ??

Find the probabilities of the following events for a driver chosen at random from the city:

- being 18-29 years of age *and* having exactly 3 accidents in one year.
- being 30-50 years of age *and* having one or more accidents in a year.
- having no accidents in one year.

9. Consider the frequency distribution table below which gives the weights of 38 students of a class.

Weights (in kg)	Number of students
31-35	9
36-40	5
41-45	14
46-50	3
51-55	1
56-60	2
61-65	2
66-70	1
71-75	1
Total	38

- Find the probability that the weight of a student in the class lies in the interval 46-50 kg.
- Give two events in this context, one having probability 0 and the other having probability 1.

10. Fifty seeds were selected at random from each of 5 bags of seeds, and were kept under standardised conditions favourable to germination. After 20 days, the number of seeds which had germinated in each collection were counted and recorded as follows:

Bag	1	2	3	4	5
No. of seeds germinated	40	48	42	39	41

What is the probability of germination of

(i) more than 40 seeds in a bag?

(ii) 49 seeds in a bag?

(iii) more than 35 seeds in a bag?

- If $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$, Evaluate $P(A/B)$?
- A family has two children. What is the probability that both the children are boys given that at least one of them is a boy?
- Ten cards numbered 1 to 10 are placed in a box, mixed up thoroughly and then one card is drawn randomly. If it is known that the number on the drawn card is more than 3, what is the probability that it is an even number?
- In a school, there are 1000 students, out of which 430 are girls. It is known that out of 430, 10 percentage of the girls study in class XII. What is the probability that a student chosen randomly studies in Class XII given that the chosen student is a girl?
- A die is thrown three times. Events A and B are defined as below:
A : 4 on the third throw.
B : 6 on the first and 5 on the second throw.
Find the probability of A given that B has already occurred?

16. A die is thrown twice and the sum of the numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once?
17. Consider the experiment of tossing a coin. If the coin shows head, toss it again but if it shows tail, then throw a die. Find the conditional probability of the event that "the die shows a number greater than 4" given that "there is at least one tail".
18. An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement. What is the probability that both drawn balls are black?
19. Three cards are drawn successively, without replacement from a pack of 52 well shuffled cards. What is the probability that first two cards are kings and the third card drawn is an ace?
20. A die is thrown. If E is the event "the number appearing is a multiple of 3" and F be the event "the number appearing is even" then find whether E and F are independent ?
21. An unbiased die is thrown twice. Let the event A be "odd number on the first throw" and B the event "odd number on the second throw". Check the independence of the events A and B.
22. Three coins are tossed simultaneously. Consider the event E "three heads or three tails", F "at least two heads" and G "at most two heads". Of the pairs (E,F), (E,G) and (F,G), which are independent? which are dependent?
23. Prove that if E and F are independent events, then so are the events E and F' .
24. If A and B are two independent events, then the probability of occurrence of at least one of A and B is given by $1 - P(A')P(B')$
25. A person has undertaken a construction job. The probabilities are 0.65 that there will be strike, 0.80 that the construction job will be completed on time if there is no strike, and 0.32 that the construction job will be completed on time if there is a strike. Determine the probability that the construction job will be completed on time.
26. Bag I contains 3 red and 4 black balls while another Bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from Bag II.
27. Given three identical boxes I, II and III, each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in the box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold?
28. Suppose that the reliability of a HIV test is specified as follows: Of people having HIV, 90% of the test detect the disease but 10% go undetected. Of people free of HIV, 99% of the test are judged HIV -ve but 1% are diagnosed as showing HIV +ve. From a large population of which only 0.1% have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV +ve. What is the probability that the person actually has HIV?
29. In a factory which manufactures bolts, machines A, B and C manufacture respectively 25%, 35% and 40% of the bolts. Of their outputs, 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by the machine B?
30. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}$, $\frac{1}{5}$, $\frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}$, $\frac{1}{3}$ and $\frac{1}{12}$, if he comes by train, bus and scooter respectively, but if he

comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that he comes by train?

31. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.
32. A person plays a game of tossing a coin thrice. For each head, he is given Rs 2 by the organiser of the game and for each tail, he has to give Rs 1.50 to the organiser. Let X denote the amount gained or lost by the person. Show that X is a random variable and exhibit it as a function on the sample space of the experiment.
33. A bag contains 2 white and 1 red balls. One ball is drawn at random and then put back in the box after noting its colour. The process is repeated again. If X denotes the number of red balls recorded in the two draws, describe X .
34. Two cards are drawn successively with replacement from a well shuffled deck of 52 cards. Find the probability distribution of the number of aces.
35. Find the probability distribution of number of doublets in three throws of a pair of dice?
36. Let X denote the number of hours you study during a randomly selected school day. The probability that X can take the values x , has the following form, where k is some unknown constant.

$$P(X=x) = \begin{cases} 0.1, & \text{if } x = 0 \\ kx, & \text{if } x = 1 \text{ or } 2 \\ k(5-x), & \text{if } x = 3 \text{ or } 4 \\ 0, & \text{otherwise} \end{cases}$$
 - a) Find the value of k .
 - b) What is the probability that you study at least two hours? Exactly two hours? At most two hours?
37. Let a pair of dice be thrown and the random variable X be the sum of the numbers that appear on the two dice. Find the mean or expectation of X .
38. Find the variance of the number obtained on

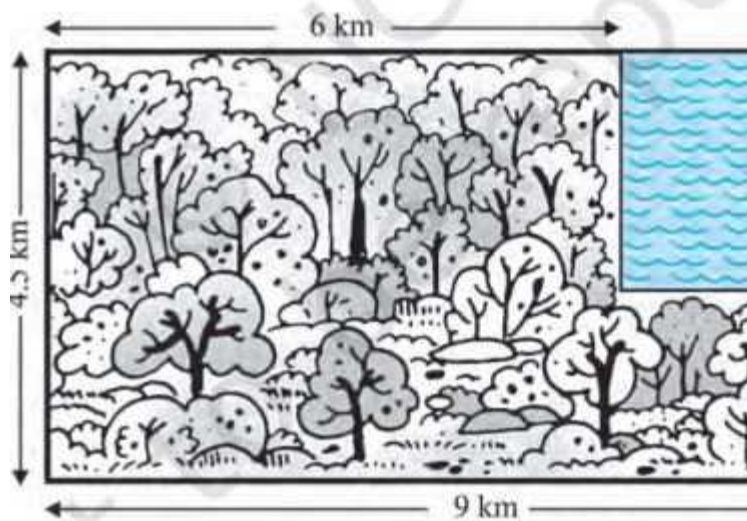
a throw of an unbiased die.

39. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean, variance and standard deviation of the number of kings.
40. Six balls are drawn successively from an urn containing 7 red and 9 black balls. Tell whether or not the trials of drawing balls are Bernoulli trials when after each draw the ball drawn is
 - (i) replaced
 - (ii) not replaced in the urn.
41. If a fair coin is tossed 10 times, find the probability of
 - (i) exactly six heads
 - (ii) at least six heads
 - (iii) at most six heads
42. Ten eggs are drawn successively with replacement from a lot containing 10% defective eggs. Find the probability that there is at least one defective egg.
43. Coloured balls are distributed in four boxes as shown in the following table:

Box	Black	White	Red	Blue
I	3	4	5	6
II	2	2	2	2
III	1	2	3	1
IV	4	3	1	5

A box is selected at random and then a ball is randomly drawn from the selected box. The colour of the ball is black, what is the probability that ball drawn is from the box III?
44. Find the mean of the Binomial distribution $B(4, \frac{1}{3})$.
45. The probability of a shooter hitting a target is $\frac{3}{4}$. How many minimum number of times must he/she fire so that the probability of hitting the target at least once is more than 0.99?

46. A and B throw a die alternatively till one of them gets a '6' and wins the game. Find their respective probabilities of winning, if A starts first.
47. If a machine is correctly set up, it produces 90% acceptable items. If it is incorrectly set up, it produces only 40% acceptable items. Past experience shows that 80% of the set ups are correctly done. If after a certain set up, the machine produces 2 acceptable items, find the probability that the machine is correctly setup.
48. Find the probability of getting a head when a coin is tossed once. Also find the probability of getting a tail.
49. A bag contains a red ball, a blue ball and a yellow ball, all the balls being of the same size. Kritika takes out a ball from the bag without looking into it. What is the probability that she takes out the (i) yellow ball?
(ii) red ball?
(iii) blue ball?
50. Suppose we throw a die once. (i) What is the probability of getting a number greater than 4 ?
(ii) What is the probability of getting a number less than or equal to 4 ?
51. One card is drawn from a well-shuffled deck of 52 cards. Calculate the probability that the card will
(i) be an ace,
(ii) not be an ace.
52. Two players, Sangeeta and Reshma, play a tennis match. It is known that the probability of Sangeeta winning the match is 0.62. What is the probability of Reshma winning the match?
53. Savita and Hamida are friends. What is the probability that both will have
(i) different birthdays?
(ii) the same birthday? (ignoring a leap year).
54. There are 40 students in Class X of a school of whom 25 are girls and 15 are boys. The class teacher has to select one student as a class representative. She writes the name of each student on a separate card, the cards being identical. Then she puts cards in a bag and stirs them thoroughly. She then draws one card from the bag. What is the probability that the name written on the card is the name of
(i) a girl?
(ii) a boy?
55. A box contains 3 blue, 2 white, and 4 red marbles. If a marble is drawn at random from the box, what is the probability that it will be (i) white? (ii) blue? (iii) red?
56. Harpreet tosses two different coins simultaneously (say, one is of rupee 1 and other of rupee 2). What is the probability that she gets at least one head?
57. In a musical chair game, the person playing the music has been advised to stop playing the music at any time within 2 minutes after she starts playing. What is the probability that the music will stop within the first half-minute after starting?
58. A missing helicopter is reported to have crashed somewhere in the rectangular region shown in Fig. 15.2. What is the probability that it crashed inside the lake shown in the figure?



59. A carton consists of 100 shirts of which 88 are good, 8 have minor defects and 4 have major defects. Jimmy, a trader, will only accept the shirts which are good, but Sujatha, another trader, will only reject the shirts which have major defects. One shirt is drawn at random from the carton. What is the probability that
(i) it is acceptable to Jimmy?
(ii) it is acceptable to Sujatha?
60. Two dice, one blue and one grey, are thrown at the same time. Write down all the possible outcomes. What is the probability that the sum of the two numbers appearing on the top of the dice is
(i) 8?

- (ii) 13?
 (iii) less than or equal to 12?

1.2 Exercises

1. In a cricket match, a batswoman hits a boundary 6 times out of 30 balls she plays. Find the probability that she did not hit a boundary.

Solution: Let the sample space be $X \in \{0, 1\}$. From the given information, the probability of hitting a boundary is

$$\Pr(X = 1) = \frac{6}{30} \quad (1.2.1.1)$$

$$= \frac{1}{5} \quad (1.2.1.2)$$

Hence, the probability of not hitting the boundary is

$$\Pr(X = 0) = 1 - \Pr(X = 1) = 1 - \frac{1}{5} \quad (1.2.1.3)$$

$$= \frac{4}{5} \quad (1.2.1.4)$$

2. 1500 families with 2 children were selected randomly, and the following data in Table 1.2.2 were recorded. Compute the probability of a family, chosen at random, having

- a) 2 girls
 b) 1 girl
 c) No girl

Also check whether the sum of these probabilities is 1.

No. of girls in a family	2	1	0
No. of families	475	814	211

TABLE 1.2.2

Solution: Let X be the random variable representing the number of girls.

a)

$$\Pr(X = 2) = \frac{475}{1500} \quad (1.2.2.1)$$

$$= 0.316 \quad (1.2.2.2)$$

b)

$$\Pr(X = 1) = \frac{814}{1500} \quad (1.2.2.3)$$

$$= 0.5427 \quad (1.2.2.4)$$

c)

$$\Pr(X = 0) = \frac{211}{1500} \quad (1.2.2.5)$$

$$= 0.1407 \quad (1.2.2.6)$$

It is easy to verify that

$$\Pr(X = 0) + \Pr(X = 1) + \Pr(X = 2) = 1 \quad (1.2.2.7)$$

3. In a particular section of Class IX, 40 students were asked about the months of their birth and the following graph in Fig. 1.2.3 was prepared for the data so obtained

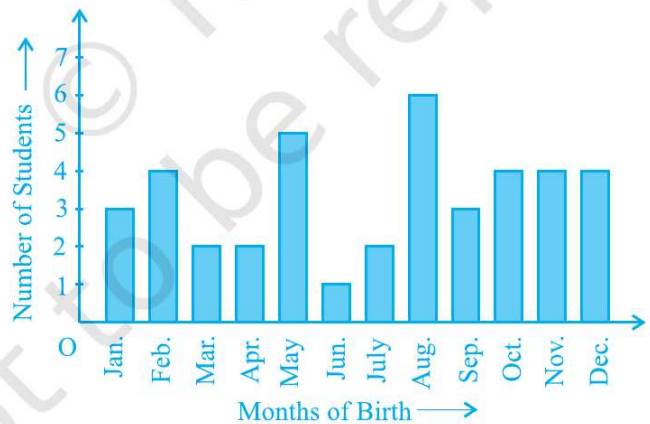


Fig. 1.2.3

Find the probability that a student of the class was born in August.

4. Three coins are tossed simultaneously 200 times with the following frequencies of different outcomes listed in Table 1.2.4. If the three coins are simultaneously tossed again, compute the probability of 2 heads coming up.

Outcome	3 heads	2 heads	1 head	No head
Frequency	23	72	77	28

TABLE 1.2.4

Solution: Let $X_i \in \{0, 1\}$ denote the outcome of the i th coin so that

$$\Pr(X_i = 1) = p \quad (1.2.4.1)$$

$$\Pr(X_i = 0) = 1 - p \quad (1.2.4.2)$$

For a fair coin,

$$p = \frac{1}{2} \quad (1.2.4.3)$$

Then

$$X = X_1 + X_2 + X_3 \quad (1.2.4.4)$$

is defined to be a binomial distribution with

$$\Pr(X = k) = {}^3C_k p^k (1 - p)^{n-k} \quad (1.2.4.5)$$

From the given information,

$$\Pr(X = 2) = {}^3C_2 \left(\frac{1}{2}\right)^2 \left(1 - \frac{1}{2}\right) \quad (1.2.4.6)$$

$$= \frac{3}{8} \quad (1.2.4.7)$$

5. Refer to Table 1.2.5.

- Find the probability that a student obtained less than 20% in the mathematics test.
- Find the probability that a student obtained marks 60 or above.

Marks	Number of students
0-20	7
20-30	10
30-40	10
40-50	20
50-60	20
60-70	15
70-above	8
Total	90

TABLE 1.2.5

Solution:

- From the given information,

$$\Pr(X < 20) = \frac{7}{90} \quad (1.2.5.1)$$

$$= 0.07 \quad (1.2.5.2)$$

-

$$\Pr(X \geq 60) = \frac{15 + 8}{90} \quad (1.2.5.3)$$

$$= 0.256 \quad (1.2.5.4)$$

6. To know the opinion of the students about the subject statistics, a survey of 200 students was conducted. The data is recorded in Table 1.2.6 Find the probability that a student chosen at random

- likes statistics,
- does not like it.

Solution: Let $X \in \{0, 1\}$ be the random variable

Opinion	Number of students
like	135
dislike	65

TABLE 1.2.6

denoting dislikes and likes.

-

$$\Pr(X = 1) = \frac{135}{200} \quad (1.2.6.1)$$

$$= 0.675 \quad (1.2.6.2)$$

-

$$\Pr(X = 0) = \frac{65}{200} = 0.325 \quad (1.2.6.3)$$

7. The distance (in kms) of 40 engineers from their residence to their place of work were found as follows in Table 1.2.7. What is the empirical probability that an engineer lives

- less than 7 km from her place of work?
- more than or equal to 7 km from her place of work?
- within $\frac{1}{2}$ km from her place of work?

5	3	10	20	25	11	13	7	12	31
19	10	12	17	18	11	32	17	16	2
7	9	7	8	3	5	12	15	18	3
12	14	2	9	6	15	15	7	6	12

TABLE 1.2.7

8. An organisation selected 2400 families at random and surveyed them to determine a relationship between income level and the number of vehicles in a family. The information gathered is listed in the Table 1.2.8 Suppose a family is chosen. Find the probability that the family chosen is

- earning ₹10000 – ₹13000 per month and owning exactly 2 vehicles.
- earning ₹16000 or more per month and owning exactly 1 vehicle.
- earning less than ₹7000 per month and does not own any vehicle.
- earning ₹13000 – ₹16000 per month and owning more than 2 vehicles.
- owning not more than 1 vehicle.

Solution: Let X be the random variable denoting the number of vehicles and Y be the

Monthly income (in ₹)	vehicles per family			
	0	1	2	Above 2
Less than 7000	10	160	25	0
7000-10000	0	305	27	2
10000-13000	1	535	29	1
13000-16000	2	469	59	25
16000 or more	1	579	82	88

TABLE 1.2.8

income.

- a) no of total families chosen for survey = 2400

$$\begin{aligned}\Pr(X = 2, 10000 < Y < 13000) &= \frac{29}{2400} & (1.2.8.1) \\ &= 0.012 & (1.2.8.2)\end{aligned}$$

- b)

$$\begin{aligned}\Pr(X = 1, Y > 16000) &= \frac{579}{2400} & (1.2.8.3) \\ &= 0.241 & (1.2.8.4)\end{aligned}$$

- c)

$$\begin{aligned}\Pr(X = 0, Y < 7000) &= \frac{10}{2400} & (1.2.8.5) \\ &= 0.0042 & (1.2.8.6)\end{aligned}$$

- d)

$$\begin{aligned}\Pr(X > 2, 13000 < Y < 16000) &= \frac{25}{2400} & (1.2.8.7) \\ &= 0.0104 & (1.2.8.8)\end{aligned}$$

- e) The number of families is given by the sum of columns 0 and 1 in Table 1.2.8. Hence,

$$\begin{aligned}\Pr(X < 2) &= \frac{1892}{2400} & (1.2.8.9) \\ &= 0.78833 & (1.2.8.10)\end{aligned}$$

9. Eleven bags of wheat flour, each marked 5 kg, actually contained the following weights of flour (in kg)
4.97 5.05 5.08 5.03 5.00 5.06 5.08 4.98 5.04 5.07 5.00
Find the probability that any of these bags

chosen at random contains more than 5 kg of flour.

Solution:

$$\Pr(X > 5) = \frac{7}{11} \quad (1.2.9.1)$$

$$= 0.636 \quad (1.2.9.2)$$

10. From Table 1.2.10, prepare a frequency distribution table, regarding the concentration of sulphur dioxide in the air in parts per million of a certain city for 30 days. Using this table, find the probability of the concentration of sulphur dioxide in the interval 0.12 - 0.16 on any of these days.

0.03	0.08	0.08	0.09	0.04	0.17
0.16	0.05	0.02	0.06	0.18	0.20
0.11	0.08	0.12	0.13	0.22	0.07
0.08	0.01	0.10	0.06	0.09	0.18
0.11	0.07	0.05	0.07	0.01	10.04

TABLE 1.2.10

11. A, B, O, O, AB, O, A, O, B, A, O, B, A, O, O, A, AB, O, A, A, O, O, AB, B, A, O, B, A, B, O.
prepare a frequency distribution table regarding the blood groups of 30 students of a class. Use this table to determine the probability that a student of this class, selected at random, has blood group AB.
12. Given that E and F are events such that $P(E) = 0.6$, $P(F) = 0.3$ and $P(E \cap F) = 0.2$, find $P(E/F)$ and $P(F/E)$?
13. Compute $P(A/B)$, if $P(B) = 0.5$ and $P(A \cap B) = 0.32$.
14. If $P(A) = 0.8$, $P(B) = 0.5$ and $P(B/A) = 0.4$, find
(i) $P(A \cap B)$
(ii) $P(A/B)$
(iii) $P(A \cup B)$
15. Evaluate $P(A \cup B)$, if $2P(A) = P(B) = \frac{5}{13}$ and $P(A/B) = \frac{2}{5}$.
16. If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{11}{7}$ find
(i) $P(A \cap B)$

- (ii) $P(A/B)$
- (iii) $P(B/A)$

17. Determine $P(E/F)$, if a coin is tossed three times
 - (i) E : head on third toss , F : heads on first two tosses
 - (ii) E : at least two heads , F : at most two heads
 - (iii) E : at most two tails , F : at least one tail
18. Determine $P(E/F)$, if two coins are tossed once, where
 - (i) E : tail appears on one coin, F : one coin shows head
 - (ii) E : no tail appears, F : no head appears
19. Determine $P(E/F)$, if a die is thrown three times,
 E : 4 appears on the third toss, F : 6 and 5 appears respectively on first two tosses
20. Determine $P(E/F)$, if mother, father and son line up at random for a family picture
 E : son on one end, F : father in middle
21. A black and a red dice are rolled.
 - (a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.
 - (b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.
22. A fair die is rolled. Consider the events $E = (1, 3, 5)$, $F = (2, 3)$ and $G = (2, 3, 4, 5)$ Find
 - (i) $P(E/F)$ and $P(F/E)$
 - (ii) $P(E/G)$ and $P(G/E)$
 - (iii) $P((E \cup F)/G)$ and $P((E \cap F)/G)$
23. 12. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that
 - (i) the youngest is a girl,
 - (ii) at least one is a girl?
24. An instructor has a question bank consisting of 300 easy True / False questions, 200 difficult True / False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question?
25. Given that the two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'.
26. Consider the experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 3'.
27. Choose the correct answer, if $P(A) = \frac{1}{2}$, $P(B) = 0$, then $P(A/B)$ is
 - a) 0
 - b) $\frac{1}{2}$
 - c) not defined
 - d) 1
28. If A and B are events such that $P(A/B) = P(B/A)$, then
 - a) $A \subset B$ but $A \neq B$
 - b) $A = B$
 - c) $A \cap B = \phi$
 - d) $P(A) = P(B)$
29. If $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$, find $P(A \cap B)$ if A and B are independent events.
30. Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.
31. A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale, otherwise, it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale.
32. A fair coin and an unbiased die are tossed. Let A be the event 'head appears on the coin'

and B be the event '3 on the die'. Check whether A and B are independent events or not.

33. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event, 'the number is even,' and B be the event, 'the number is red'. Are A and B independent?
34. Let E and F be events with $P(E) = \frac{3}{5}$, $P(F) = \frac{3}{10}$ and $P(E \cap F) = \frac{1}{5}$. Are E and F independent?
35. Given that the events A and B are such that $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ and $P(B) = p$. Find p if they are
 - (i) mutually exclusive
 - (ii) independent.
36. Let A and B be independent events with $P(A) = 0.3$ and $P(B) = 0.4$. Find
 - (i) $P(A \cap B)$
 - (ii) $P(A \cup B)$
 - (iii) $P(A/B)$
 - (iv) $P(B/A)$
37. If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$. find P (not A and not B).
38. Events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(\text{not } A \text{ or not } B) = \frac{1}{4}$. State whether A and B are independent ?
39. Given two independent events A and B such that $P(A) = 0.3$, $P(B) = 0.6$. Find
 - (i) $P(A \text{ and } B)$
 - (ii) $P(A \text{ and not } B)$
 - (iii) $P(A \text{ or } B)$
 - (iv) $P(\text{neither } A \text{ nor } B)$
40. A die is tossed thrice. Find the probability of getting an odd number at least once.
41. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that
 - (i) both balls are red.
 - (ii) first ball is black and second is red.
 - (iii) one of them is black and other is red.
42. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that
 - (i) the problem is solved
 - (ii) exactly one of them solves the problem.
43. One card is drawn at random from a well shuffled deck of 52 cards. In which of the following cases are the events E and F independent?
 - (i) E : 'the card drawn is a spade' F : 'the card drawn is an ace'
 - (ii) E : 'the card drawn is black' F : 'the card drawn is a king'
 - (iii) E : 'the card drawn is a king or queen' F : 'the card drawn is a queen or jack'.
44. In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random.
 - (a) Find the probability that she reads neither Hindi nor English newspapers.
 - (b) If she reads Hindi newspaper, find the probability that she reads English newspaper.
 - (c) If she reads English newspaper, find the probability that she reads Hindi newspaper.

Choose the correct answer:

45. The probability of obtaining an even prime number on each die, when a pair of dice is rolled is
 - a) 0
 - b) $\frac{1}{3}$
 - c) $\frac{1}{12}$
 - d) $\frac{1}{36}$
46. Two events A and B will be independent, if
 - a) A and B are mutually exclusive
 - b) $P(A' B') = [1 - P(A)][1 - P(B)]$
 - c) $P(A) = P(B)$
 - d) $P(A) + P(B) = 1$
47. An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn

and then a ball is drawn at random. What is the probability that the second ball is red?

48. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.
49. Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade, what is the probability that the student is a hostlier?
50. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly?
51. A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0.5% of the healthy person tested (i.e. if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive?
52. There are three coins. One is a two headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two headed coin ?
53. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accidents are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?
54. A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B?
55. Two groups are competing for the position on the Board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group.
56. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die?
57. A manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, where as the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by A?
58. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a

diamond.

Choose a correct answer

59. Probability that A speaks truth is $\frac{4}{5}$. A coin is tossed. A reports that a head appears. The probability that actually there was head is

- a) $\frac{4}{5}$
b) $\frac{3}{5}$
c) $\frac{1}{5}$
d) $\frac{2}{5}$

60. If A and B are two events such that $A \subset B$ and $P(B) \neq 0$, then which of the following is correct?

- a) $P(A/B) = \frac{P(B)}{P(A)}$
b) $P(A/B) < P(A)$
c) $P(A/B) \geq P(A)$
d) None of these

61. State which of the following are not the probability distributions of a random variable. Give reasons for your answer.

(i)

X	0	1	2
P(X)	0.4	0.4	0.2

(ii)

X	0	1	2	3	4
P(X)	0.1	0.5	0.2	-0.1	0.3

(iii)

X	-1	0	1
P(X)	0.6	0.1	0.2

(iv)

X	3	2	1	0	-1
P(X)	0.3	0.2	0.4	0.1	0.05

62. An urn contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represent the number of black balls. What are the possible values of X? Is X a random variable?
63. Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X?
64. Find the probability distribution of
(i) number of heads in two tosses of a coin.

(ii) number of tails in the simultaneous tosses of three coins.

(iii) number of heads in four tosses of a coin.

65. Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as

- (i) number greater than 4
(ii) six appears on at least one die

66. From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

67. A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.

68. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Determine

- (i) k
(ii) $P(X \leq 3)$
(iii) $P(X \geq 6)$
(iv) $P(0 \leq X \leq 3)$

69. Find the mean number of heads in three tosses of a fair coin.

70. Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X.

71. Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find $E(X)$.

72. Let X denote the sum of the numbers obtained when two fair dice are rolled. Find the variance and standard deviation of X.

73. A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16,

19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X ? Find mean, variance and standard deviation of X .

74. In a meeting, 70% of the members favour and 30% oppose a certain proposal. A member is selected at random and we take $X = 0$ if he opposed, and $X = 1$ if he is in favour. Find $E(X)$ and $\text{Var}(X)$.

Choose the correct answer in each of the following:

75. The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is

- a) 1
- b) 2
- c) 5
- d) $\frac{8}{3}$

76. Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained. Then the value of $E(X)$ is

- a) $\frac{37}{221}$
- b) $\frac{2}{13}$
- c) $\frac{1}{13}$
- d) $\frac{2}{13}$

77. A die is thrown 6 times. If 'getting an odd number' is a success, what is the probability of

- (i) 5 successes?
- (ii) at least 5 successes?
- (iii) at most 5 successes?

78. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of two successes.

79. There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?

80. Five cards are drawn successively with

replacement from a well shuffled deck of 52 cards. What is the probability that

- (i) all the five cards are spades?
- (ii) only 3 cards are spades?
- (iii) none is a spade?

81. The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs

- (i) none
 - (ii) not more than one
 - (iii) more than one
 - (iv) at least one
- will fuse after 150 days of use.

82. A bag consists of 10 balls each marked with one of the digits 0 to 9. If four balls are drawn successively with replacement from the bag, what is the probability that none is marked with the digit 0?

83. In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answers 'true'; if it falls tails, he answers 'false'. Find the probability that he answers at least 12 questions correctly.

84. Suppose X has a binomial distribution. Show that $X = 3$ is the most likely outcome. (Hint : $P(X = 3)$ is the maximum among all $P(x_i)$, $x_i = 0, 1, 2, 3, 4, 5, 6$)

85. On a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing ?

86. A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize

- (a) at least once
- (b) exactly once
- (c) at least twice?

87. Find the probability of getting 5 exactly twice in 7 throws of a die.

88. Find the probability of throwing at most 2 sixes in 6 throws of a single die.
89. It is known that 10% of certain articles manufactured are defective. What is the probability that in a random sample of 12 such articles, 9 are defective?
- In each of the following, choose the correct answer:
90. In a box containing 100 bulbs, 10 are defective. The probability that out of a sample of 5 bulbs, none is defective is
 a) 10^{-1}
 b) $(\frac{1}{2})^5$
 c) $(\frac{9}{10})^5$
 d) $\frac{9}{10}$
91. The probability that a student is not a swimmer is $\frac{1}{5}$. Then the probability that out of five students, four are swimmers is
 a) ${}^5C_4(\frac{4}{5})^4\frac{1}{5}$
 b) $(\frac{4}{5})^4\frac{1}{5}$
 c) ${}^5C_1(\frac{4}{5})^4\frac{1}{5}$
 d) None of these
92. A and B are two events such that $P(A) \neq 0$. Find $P(B/A)$, if
 (i) A is a subset of B
 (ii) $A \cap B = \phi$
93. A couple has two children,
 (i) Find the probability that both children are males, if it is known that at least one of the children is male.
 (ii) Find the probability that both children are females, if it is known that the elder child is a female.
94. Suppose that 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that there are equal number of males and females.
95. Suppose that 90% of people are right-handed. What is the probability that at most 6 of a random sample of 10 people are right-handed?
96. An urn contains 25 balls of which 10 balls bear a mark 'X' and the remaining 15 bear a mark 'Y'. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that
 (i) all will bear 'X' mark.
 (ii) not more than 2 will bear 'Y' mark.
 (iii) at least one ball will bear 'Y' mark.
 (iv) the number of balls with 'X' mark and 'Y' mark will be equal.
97. In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is $\frac{5}{6}$. What is the probability that he will knock down fewer than 2 hurdles?
98. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.
99. If a leap year is selected at random, what is the chance that it will contain 53 Tuesdays?
100. An experiment succeeds twice as often as it fails. Find the probability that in the next six trials, there will be at least 4 successes.
101. How many times must a man toss a fair coin so that the probability of having at least one head is more than 90%?
102. In a game, a man wins a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins / loses.
103. Suppose we have four boxes A,B,C and D containing coloured marbles as given below:

Box	Red	White	Black
A	1	6	3
B	6	2	2
C	8	1	1
D	0	6	4

One of the boxes has been selected at random and a single marble is drawn from it. If the marble is red, what is the probability that it was drawn from box A?, box B?, box C?

104. Assume that the chances of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga?
105. If each element of a second order determinant is either zero or one, what is the probability that the value of the determinant is positive? (Assume that the individual entries of the determinant are chosen independently, each value being assumed with probability $\frac{1}{2}$).
106. An electronic assembly consists of two subsystems, say, A and B. From previous testing procedures, the following probabilities are assumed to be known:
 $P(A \text{ fails}) = 0.2$
 $P(B \text{ fails alone}) = 0.15$
 $P(A \text{ and } B \text{ fail}) = 0.15$
- Evaluate the following probabilities
 (i) $P(A \text{ fails} \mid B \text{ has failed})$
 (ii) $P(A \text{ fails alone})$
107. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.
- Choose the correct answer in each of the following:
108. If A and B are two events such that $P(A) \neq 0$ and $P(B/A) = 1$, then (A) $A \subset B$
 (B) $B \subset A$
 (C) $B = \phi$
 (D) $A = \phi$
109. If $P(A/B) > P(A)$, then which of the following is correct : (A) $P(B/A) < P(B)$
 (B) $P(A \cap B) < P(A) \cdot P(B)$
 (C) $P(B/A) > P(B)$
 (D) $P(B/A) = P(B)$
110. If A and B are any two events such that $P(A) + P(B) - P(A \text{ and } B) = P(A)$, then
 (A) $P(B/A) = 1$
 (B) $P(A/B) = 1$
 (C) $P(B/A) = 0$
 (D) $P(A/B) = 0$
111. Complete the following statements:
 (i) Probability of an event E + Probability of the event 'not E' = ——— .
 (ii) The probability of an event that cannot happen is ——— . Such an event is called ——— .
 (iii) The probability of an event that is certain to happen is ——— .
 (iv) The sum of the probabilities of all the elementary events of an experiment is ——— .
 (v) The probability of an event is greater than or equal to and less than or equal to ——— .
112. Which of the following experiments have equally likely outcomes? Explain. (i) A driver attempts to start a car. The car starts or does not start.
 (ii) A player attempts to shoot a basketball. She/he shoots or misses the shot.
 (iii) A trial is made to answer a true-false question. The answer is right or wrong.
 (iv) A baby is born. It is a boy or a girl.
113. Why is tossing a coin considered to be a fair way of deciding which team should get the ball at the beginning of a football game?
114. Which of the following cannot be the probability of an event?
 (A) $\frac{2}{3}$ (B) -1.5 (C) 15
115. If $P(E) = 0.05$, what is the probability of 'not E'?
116. A bag contains lemon flavoured candies only. Malini takes out one candy without looking into the bag. What is the probability that she takes out
 (i) an orange flavoured candy?

- (ii) a lemon flavoured candy?
117. It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?
118. A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is
- red ?
 - not red?
119. A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be
- red ?
 - white ?
 - not green?
120. A piggy bank contains hundred 50p coins, fifty rupee 1 coins, twenty rupee 2 coins and ten rupee 5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin
- will be a 50 p coin ?
 - will not be a rupee5 coin?
121. Gopi buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fish and 8 female fish (see Fig. 15.4). What is the probability that the fish taken out is a male fish?



122. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 (see Fig. 15.5), and these are equally likely outcomes. What is the probability that it will point at
- 8 ?
 - an odd number?
 - a number greater than 2?
 - a number less than 9?

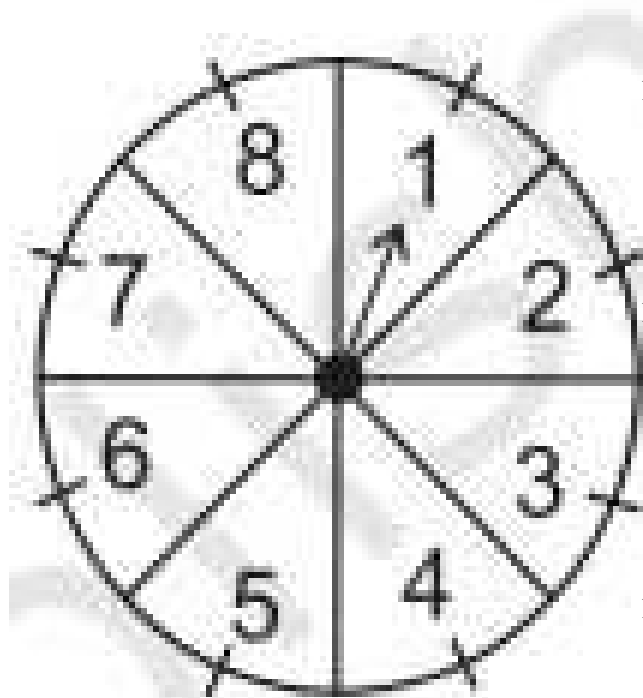


Fig. 15.5

is the probability that the second card picked up is (a) an ace? (b) a queen?

127. 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.
128. (i) A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective?
(ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective?
129. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears (i) a two-digit number (ii) a perfect square number (iii) a number divisible by 5.
130. A child has a die whose six faces show the letters as given below:

123. A die is thrown once. Find the probability of getting

- (i) a prime number;
- (ii) a number lying between 2 and 6;
- (iii) an odd number.

124. A die is thrown once. Find the probability of getting

- (i) a prime number;
- (ii) a number lying between 2 and 6;
- (iii) an odd number.

125. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting (i)

- (i) a king of red colour
- (ii) a face card
- (iii) a red face card
- (iv) the jack of hearts
- (v) a spade
- (vi) the queen of diamonds

126. Five cards—the ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random.

- (i) What is the probability that the card is the queen?
- (ii) If the queen is drawn and put aside, what

The die is thrown once. What is the probability of getting (i) A? (ii) D?

131. Suppose you drop a die at random on the rectangular region shown in Fig. 1.2.131. What is the probability that it will land inside the circle with diameter 1m?

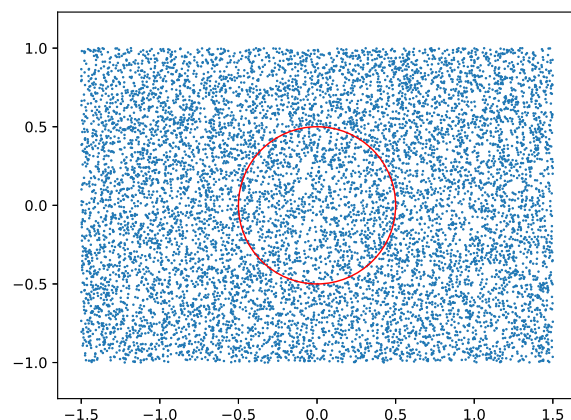


Fig. 1.2.131

Solution: In Fig. 1.2.131, the sample size S is the area of the rectangle given by

$$S = 3 \times 2 = 6m^2 \quad (1.2.131.1)$$

The event size is the area of the circle given by

$$E = \pi \left(\frac{1}{2} \right)^2 = \frac{\pi}{4} m^2 \quad (1.2.131.2)$$

The probability of the dice landing in the circle is

$$\Pr(E) = \frac{E}{S} = \frac{\pi}{24} \quad (1.2.131.3)$$

The python code is available in

/codes/rect.py

The python code generates 10,000 points uniformly within the rectangle of dimensions 3×2 and checks for the number of points within the circle of radius 0.5. The ratio of these is close to $\frac{\pi}{24}$. Note that each time the code is run, the ratio will change, but will still be close to $\frac{\pi}{24}$.

132. A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that

(i) She will buy it ?

(ii) She will not buy it ?

Solution: The sample size

$$S = 144 \quad (1.2.132.1)$$

The number of bad pens is

$$B = 20 \quad (1.2.132.2)$$

The probability that she doesn't buy a pen is

$$\Pr(B) = \frac{B}{S} = \frac{20}{144} \quad (1.2.132.3)$$

$$= \frac{5}{36} \quad (1.2.132.4)$$

The probability that she buys a pen is

$$\Pr(G) = 1 - \Pr(B) = \frac{31}{36} \quad (1.2.132.5)$$

The python code for the distribution is

solutions/10-1/prob/codes/prob2_a.py

133. Refer to Example 13.(i) Complete the following table:

Event: 'Sum on 2 dice'	2	3	4	5	6	7	8	9
Probability	$\frac{1}{36}$	-	-	-	-	-	$\frac{5}{36}$	-

- (ii) A student argues that 'there are 11 possible outcomes 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12. Therefore, each of them has a probability $\frac{1}{11}$ '. Do you agree with this argument? Justify your answer.

134. A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result i.e., three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.

Solution: Let $X_i \in \{0, 1\}$, $i = 1, 2, 3$ represent a coin toss, or, the Bernoulli random variable. Then the outcome of the game is

$$X = X_1 + X_2 + X_3 \quad (1.2.134.1)$$

If

$$\Pr(X_i = 1) = p, \quad (1.2.134.2)$$

$$\Pr(X = k) = {}^nC_k p^k (1-p)^{n-k}, \quad k = 0, \dots, n \quad (1.2.134.3)$$

X is known as a Binomial random variable. For the given problem, $n = 3$, $p = \frac{1}{2}$ and the probability of a win is

$$\Pr(X = 3) + \Pr(X = 0) = \frac{1}{8} + \frac{1}{8} \quad (1.2.134.4)$$

$$= \frac{1}{4} \quad (1.2.134.5)$$

The loss probability is then

$$1 - \frac{1}{4} = \frac{3}{4} \quad (1.2.134.6)$$

The python code for the distribution of data, solutions/10-1/prob/codes/prob4.py

135. A die is thrown twice. What is the probability that

(i) 5 will not come up either time?

(ii) 5 will come up at least once?

Hint : Throwing a die twice and throwing two dice simultaneously are treated as the same experiment

Solution:

(i) Let $X_i \in \{1, 2, \dots, 6\}$.

$$\Pr(X_1 \neq 5, X_2 \neq 5) = \Pr(X_1 \neq 5) \Pr(X_2 \neq 5) \quad (1.2.135.1)$$

\therefore the two events are independent. Also,

$$\Pr(X_1 = 5) = \Pr(X_2 = 5) = \frac{1}{6} \quad (1.2.135.2)$$

$$\begin{aligned} \Rightarrow \Pr(X_1 \neq 5) &= \Pr(X_2 \neq 5) \quad (1.2.135.3) \\ &= 1 - \frac{1}{6} = \frac{5}{6} \quad (1.2.135.4) \end{aligned}$$

From (1.2.135.1),

$$\begin{aligned} \Pr(X_1 \neq 5, X_2 \neq 5) &= [1 - \Pr(X_1 = 5)] [1 - \Pr(X_2 = 5)] \\ &= \frac{25}{36} \quad (1.2.135.5) \end{aligned}$$

upon substituting from (1.2.135.4)

(ii) The probability that 5 doesn't come at all is

$$1 - \Pr(X_1 \neq 5, X_2 \neq 5) = 1 - \frac{25}{36} = \frac{11}{36} \quad (1.2.135.6)$$

The python code for the problem is

```
solutions/10-1/prob/codes/prob5.py
```

136. Which of the following arguments are correct and which are not correct? Give reasons for your answer.

(i) If two coins are tossed simultaneously there are three possible outcomes—two heads, two tails or one of each. Therefore, for each of these outcomes, the probability is $\frac{1}{3}$

(ii) If a die is thrown, there are two possible outcomes—an odd number or an even number. Therefore, the probability of getting an odd number is $\frac{1}{2}$

EXERCISE(optional)*

137. Two customers Shyam and Ekta are visiting a particular shop in the same week (Tuesday to Saturday). Each is equally likely to visit the shop on any day as on another day. What is the probability that both will visit the shop on
- the same day?
 - consecutive days?

(iii) different days?

138. A die is numbered in such a way that its faces show the numbers 1, 2, 2, 3, 3, 6. It is thrown two times and the total score in two throws is noted. Complete the following table which gives a few values of the total score on the two throws:

		Number in first throw					
Number in second throw	+	1	2	2	3	3	6
	1	2	3	3	4	4	7
	2	3	4	4	5	5	8
	2					5	
	3						
	3			5			9
	6	7	8	8	9	9	12

What is the probability that the total score is

- even?
- 6?
- at least 6?

139. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball, determine the number of blue balls in the bag.
140. A box contains 12 balls out of which x are black. If one ball is drawn at random from the box, what is the probability that it will be a black ball?

If 6 more black balls are put in the box, the probability of drawing a black ball is now double of what it was before. Find x .

141. A jar contains 24 marbles, some are green and others are blue. If a marble is drawn at random from the jar, the probability that it is green is $\frac{2}{3}$. Find the number of blue balls in the jar.

2 STATISTICS**2.1 Examples**

1. The marks obtained by 30 students of Class X of a certain school in a Mathematics paper consisting of 100 marks are presented in table below. Find the mean of the marks obtained by the students.

Marks obtained (x_i)	10	20	30	40	50	60
Number of students(f_i)	1	1	3	4	3	2

2. The table below gives the percentage distribution of female teachers in the primary schools of rural areas of various states and union territories (U.T.) of India. Find the mean percentage of female teachers by all the three

methods discussed in this section.

Percentage of female teachers (x_i)	15-25	25-35	35-45	45-55	55-65	65-75	75-85	85-100
Number of states/U.T. (f_i)	6	11	7	4	1	1	1	1

Source : Seventh All India School Education

Survey conducted by NCERT

3. The distribution below shows the number of wickets taken by bowlers in one-day cricket matches. Find the mean number of wickets by choosing a suitable method. What does the mean signify?

Number of wickets	20-60	60-100	100-150	150-250	250-350	250-450
Number of bowlers	7	5	6	12	2	3

Mode of Grouped Data

4. The wickets taken by a bowler in 10 cricket matches are as follows:

2 6 4 5 0 2 1 3 2 3

Find the mode of the data.

5. A survey conducted on 20 households in a locality by a group of students resulted in the following frequency table for the number of family members in a household:

Family size	1-3	3-5	5-7	7-9	9-11
Number of families	7	8	2	2	1

Find the mode of this data

6. The marks distribution of 30 students in a mathematics examination are given in Table 14.3 of Example 1. Find the mode of this data. Also compare and interpret the mode and the mean.

Median of Grouped Data

7. A survey regarding the heights (in cm) of 51 girls of Class X of a school was conducted and the following data was obtained:

height (in cm)	less than 140	less than 145	less than 150	less than 155	less than 160	less than 165
Number of girls	41	29	40	46	38	31

Find the median height.

8. The median of the following data is 525. Find the values of x and y , if the total frequency is 100.

Class interval	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800	800-900
Frequency	2	5	x	12	517	20	y	9	7

9. The annual profits earned by 30 shops of a shopping complex in a locality give rise to the following distribution:

10. Consider the marks obtained by 10 students in a mathematics test as given below:

42 25 78 75 62 55 36 95 73 60

find the highest and the lowest marks?

11. Consider the marks obtained (out of 100 marks) by 30 students of Class IX of a school:

10 20 36 92 95 40 50 56 60 70
92 88 80 70 72 70 36 40 36 40
92 40 50 50 56 60 70 60 60 80

12. 100 plants each were planted in 100 schools during Van Mahotsava. After one month, the number of plants that survived were recorded as

95 67 28 32 65 65 69 33 98 96
76 42 32 38 42 40 40 69 95 92
75 83 76 83 85 62 37 65 63 42
89 65 73 81 49 52 64 76 83 92
93 68 52 79 81 83 59 82 75 82
86 90 44 62 31 36 38 42 39 83
87 56 58 23 35 76 83 85 30 68
69 83 86 43 45 39 83 75 66 83
92 75 89 66 91 27 88 89 93 42
53 69 90 55 66 49 52 83 34 36

Let us now consider the following frequency distribution table which gives the weights of 38 students of a class:

Weights (in kg)	Number of students
31-35	9
36-40	5
41-45	14
46-50	3
51-55	1
56-60	2
61-65	2
66-70	1
71-75	1

31-35	9	36-40	5	41-45	14	46-50	3	51-55	1	56-60	2	61-65	2	66-70	1	71-75	1
Total	46																

14. In a particular section of Class IX, 40 students were asked about the months of their birth and the following graph was prepared for the data so obtained:

Marks	Number of students
0-20	7
20-30	10
30-40	10
40-50	20
50-60	20
60-70	15
70-above	8
Total	90

17. Consider the marks, out of 100, obtained by 51 students of a class in a test, given in Table.

Marks	No. of Students
0-10	5
10-20	10
20-30	4
30-40	6
40-50	7
50-60	3
60-70	2
70-80	2
80-90	3
90-100	9
Total	51

Observe the bar graph given above and answer the following questions:

- (i) Many students were born in the month of November?
(ii) In which month were the maximum number of students born?

15. A family with a monthly income of ₹20,000 had planned the following expenditures per month under various heads:

Heads	Expenditure (in 1000₹)
Grocery	4
Rent	5
Education of children	5
Medicine	2
Fuel	2
Entertainment	1
Miscellaneous	1

Draw a bar graph for the data above.

16. A teacher wanted to analyse the performance of two sections of students in a mathematics test of 100 marks. Looking at their performances, she found that a few students got under 20 marks and a few got 70 marks or above. So she decided to group them into intervals of varying sizes as follows: 0 - 20, 20 - 30, . . ., 60 - 70, 70 - 100. Then she formed the following table:

Draw a frequency polygon corresponding to this frequency distribution table.

18. In a city, the weekly observations made in a study on the cost of living index are given in the following table:

Cost of living index	No. of weeks
140-150	5
150-160	10
160-170	20
170-180	9
180-190	6
190-200	2
Total	52

Draw a frequency polygon for the data above (without constructing a histogram).

19. 5 people were asked about the time in a week they spend in doing social work in their community. They said 10, 7, 13, 20 and 15 hours, respectively. Find the mean (or average) time in a week devoted by them for social work.

20. Find the mean of the marks obtained by 30 students of Class IX of a school, given below

10 20 36 92 95 40 50 56 60 70
 92 88 80 70 72 70 36 40 36 40
 92 40 50 50 56 60 70 60 60 80

21. The heights (in cm) of 9 students of a class are as follows:

155 160 145 149 150 147 152 144 148

Find the median of this data.

22. The points scored by a Kabaddi team in a series of matches are as follows:

17, 2, 7, 27, 15, 5, 14, 8, 10, 24, 48, 10, 8, 7, 18, 28

Find the median of the points scored by the team.

23. Find the mode of the following marks (out of 10) obtained by 20 students:

4, 6, 5, 9, 3, 2, 7, 7, 6, 5, 4, 9, 10, 10, 3, 4, 7, 6, 9, 9

24. Consider a small unit of a factory where there are 5 employees : a supervisor and four labourers. The labourers draw a salary of ₹5,000 per month each while the supervisor gets ₹15,000 per month. Calculate the mean, median and mode of the salaries of this unit of the factory.

2.2 Exercises

1. A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of plants	0-2	2-4	4-6	6-8	0-10	10-12
Number of houses	1	2	1	5	6	2

2. Consider the following distribution of daily wages of 50 workers of a factory. Find the mean daily wages of the workers of the factory by using an appropriate method.
3. The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is Rs 18. Find the missing frequency f .

Daily pocket allowance(in rupees)	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Number of children	7	6	9	13	12	5	4

4. Thirty women were examined in a hospital by a doctor and the number of heartbeats per minute were recorded and summarised as follows. Find the mean heartbeats per minute for these women, choosing a suitable method.

Number of heartbeats for minute	65-68	68-71	71-74
Number of women	2	4	3

5. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.

Number of mangoes	50-52	53-55	56-58	59-61
Number of boxes	15	110	135	115

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

6. The table below shows the daily expenditure on food of 25 households in a locality.

Daily expenditure(in rupees)	100-150	150-200	200-250
Number of households	4	5	9

Find the mean daily expenditure on food by a suitable method.

7. To find out the concentration of SO_2 in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below:

Concentration of SO_2 (in ppm)	0.00-0.04	0.04-0.08
Frequency	4	9

Find the mean concentration of SO_2 in the air.

8. A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

Number of days	0-6	6-10	10-14	14-20	20-30
Number of students	11	10	7	4	8

9. The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

EXERCISE 14.2

10. The following table shows the ages of the patients admitted in a hospital during a year: Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

Age (in years)	15-19	20-24	25-29	30-34	35-39	40-44	45-49
Number of patients	23	24	30	26	16	14	12

of 225 electrical components :

Lifetimes (in hours)	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	10	35	52	61	38	29

Determine the modal lifetimes of the components.

12. The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure :

Expenditure (in rupees)	1000-1500	1500-2000	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500
Number of families	24	40	33	28	17.5	30	126.32

13. The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret the two measures.

Number of students per teacher	15-20	20-25	25-30	30-35	35-40	40-45	45-50	50-55
Number of states / U.T.	3	8	9	10	3	0	2	1

14. The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches.

Runs scored	3000-4000	4000-5000	5000-6000	6000-7000	7000-8000	8000-9000	9000-10000
Number of batsmen	4	18	9	7	1	4	1

Find the mode of the data.

15. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data :

Number of cars	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	7	14	13	12	20	15	10	8

EXERCISE 14.3

16. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.

Monthly consumption (in units)	65-85	85-105	105-125	125-145	145-165	165-185	185-205
Number of consumers	4	5	13	20	14	8	4

17. If the median of the distribution given below is 28.5, find the values of x and y.

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	Total
Frequency	5	x	20	15	4	y	60

18. A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years on wards but less than 60 year.

Age (in years)	Below 20	Below 25	Below 30	Below 35	Below 40	Below 45	Below 50
Number of policy holders	2	6	24	45	78	89	

leaves of a plant are measured correct to the nearest millimetre, and the data obtained is represented in the following table :

Length (in mm)	118-126	127-135	136-144	145-153
Number of leaves	2	6	24	60

Find the median length of the leaves. (Hint : The data needs to be converted to continuous classes for finding the median, since the formula assumes continuous classes. The classes then change to 117.5 - 126.5, 126.5 - 135.5, . . . , 171.5 - 180.5.)

20. The following table gives the distribution of the life time of 400 neon lamps :

Life time (in hours)	1500-2000	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500
Number of lamps	14	56	60	35	11	1

Find the median life time of a lamp.

21. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in

Number of letters	1-4	4-7	7-10	10-13	13-16
Number of surnames	6	30	40	16	4

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames? Also, find the modal size of the surnames.

22. The following distribution gives the weights of 30 students of a class. Find the median weight of the students.

Weight (in kg)	40-45	45-50	50-55	55-60
Number of students	2	38	6	6

EXERCISE 14.4

23. The following distribution gives the daily income of 50 workers of a factory.

Daily income (in rupees)	100-120	120-140	140-160	160-180	180-200
Number of workers	12	14	8	6	8

Convert the distribution above to a less than type cumulative frequency distribution, and draw its ogive.

24. During the medical check-up of 35 students of a class, their weights were recorded as follows:

Weight (in kg)	Less than 38	Less than 40	Less than 42	Less than 44
Number of students	0	3	4	5

Draw a less than type ogive for the given data. Hence obtain the median weight from the graph and verify the result by using the formula.

Weight (in kg)	Less than 38	Less than 40	Less than 42	Less than 44
Number of students	0	3	4	5

25. The following table gives production yield per hectare of wheat of 100 farms of a village.

Production yield(in kg/ha)	50-55	55-60	60-65	65-70	70-75	75-80
Number of farms	2	8	12	24	38	16

26. Give five examples of data that you can collect from your day-to-day life.

27. Classify the data in Q.1 above as primary or secondary data.

28. The blood groups of 30 students of Class VIII are recorded as follows:

A, B, O, O, AB, O, A, O, B, A, O, B, A, O, O,
A, AB, O, A, A, O, O, AB, B, A, O, B, A, B, O.

Represent this data in the form of a frequency distribution table. Which is the most common, and which is the rarest, blood group among these students?

29. The distance (in km) of 40 engineers from their residence to their place of work were found as follows:

5	3	10	20	25	11	13	7	12	31
19	10	12	17	18	11	32	17	16	2
7	9	7	8	3	5	12	15	18	3
12	14	2	9	6	15	15	7	6	12

Construct a grouped frequency distribution table with class size 5 for the data given above taking the first interval as 0-5 (5 not included). What main features do you observe from this tabular representation?

30. The relative humidity (in %) of a certain city for a month of 30 days was as follows:

98.1	98.6	99.2	90.3	86.5	95.3	92.9	96.3	94.2	95.1
89.2	92.3	97.1	93.5	92.7	95.1	97.2	93.3	95.2	97.3
96.2	92.1	84.9	90.2	95.7	98.3	97.3	96.1	92.1	89.0

- (i) Construct a grouped frequency distribution table with classes 84 - 86, 86 - 88, etc.

- (ii) Which month or season do you think this data is about?

- (iii) What is the range of this data?

31. The heights of 50 students, measured to the nearest centimetres, have been found to be as follows:

161	150	154	165	168	161	154	162	150	151
162	164	171	165	158	154	156	172	160	170
153	159	161	170	162	163	166	168	165	164
154	152	153	156	158	162	160	161	173	166
161	159	162	167	168	159	158	153	154	159

- (i) Represent the data given above by a

grouped frequency distribution table, taking the class intervals as 160 - 165, 165 - 170, etc.

- (ii) What can you conclude about their heights from the table?

32. A study was conducted to find out the concentration of sulphur dioxide in the air in parts per million (ppm) of a certain city. The data obtained for 30 days is as follows:

0.03	0.08	0.08	0.09	0.04	0.17
0.16	0.05	0.02	0.06	0.18	0.20
0.11	0.08	0.12	0.13	0.22	0.07
0.08	0.01	0.10	0.06	0.09	0.18
0.11	0.07	0.05	0.07	0.01	0.04

- (i) Make a grouped frequency distribution table for this data with class intervals as 0.00-0.04, 0.04-0.08, and so on.

- (ii) For how many days, was the concentration of sulphur dioxide more than 0.11 parts per million?

33. Three coins were tossed 30 times simultaneously. Each time the number of heads occurring was noted down as follows:

0	1	2	2	1	2	3	1	3	0
1	3	1	1	2	2	0	1	2	1
3	0	0	1	1	2	3	2	2	0

Prepare a frequency distribution table for the data given above.

34. The value of π upto 50 decimal places is given below:
3.14159265358979323846264338327950288419716939937510

- (i) Make a frequency distribution of the digits from 0 to 9 after the decimal point.

- (ii) What are the most and the least frequently occurring digits?

35. Thirty children were asked about the number of hours they watched TV programmes in the previous week. The results were found as follows:

1	6	2	3	5	12	5	8	4	8
10	3	4	12	2	8	15	1	17	6
3	2	8	5	9	6	8	7	14	12

- (i) Make a grouped frequency distribution table for this data, taking class width 5 and one of the class intervals as 5-10.

(ii) How many children watched television for 15 or more hours a week?

36. A company manufactures car batteries of a particular type. The lives (in years) of 40 such batteries were recorded as follows:

2.6 3.0 3.7 3.2 2.2 4.1 3.5 4.5
3.5 2.3 3.2 3.4 3.8 3.2 4.6 3.7
2.5 4.4 3.4 3.3 2.9 3.0 4.3 2.8
3.5 3.2 3.9 3.2 3.2 3.1 3.7 3.4
4.6 3.8 3.2 2.6 3.5 4.2 2.9 3.6

Construct a grouped frequency distribution table for this data, using class intervals of size 0.5 starting from the interval 2 - 2.5.

37. A survey conducted by an organisation for the cause of illness and death among the women between the ages 15 - 44 (in years) worldwide, found the following figures (in %):

S.No	Causes	female fatality rate (%)
1	Reproductive health conditions	31.8
2	Neuropsychiatric conditions	25.4
3	Injuries	12.4
4	Cardiovascular conditions	4.3
5	Respiratory conditions	4.1
6	Other causes	22.0

- (i) Represent the information given above graphically.
(ii) Which condition is the major cause of women's ill health and death worldwide?
(iii) Try to find out, with the help of your teacher, any two factors which play a major role in the cause in (ii) above being the major cause.

38. The following data on the number of girls (to the nearest ten) per thousand boys in different sections of Indian society is given below.

Section	No. of girls per thousand boys
Scheduled Cast(SC)	940
Scheduled Tribe(ST)	970
Non SC/ST	920
Backward districts	950
Non backward districts	920
Rural	930
Urban	910

- (i) Represent the information above by a bar graph. (ii) In the classroom discuss what conclusions can be arrived at from the graph.

39. Given below are the seats won by different political parties in the polling outcome of a state assembly elections:

Political Party	A	B	C	D	E	F
Seats Won	75	55	37	29	10	37

- (i) Draw a bar graph to represent the polling results. (ii) Which political party won the maximum number of seats?

40. The length of 40 leaves of a plant are measured correct to one millimetre, and the obtained data is represented in the following table:

Length(in mm)	Number of leaves
118-126	3
127-135	5
136-144	9
145-153	12
154-162	5
163-171	4
172-180	2

- (i) Draw a histogram to represent the given data. [Hint: First make the class intervals continuous]
(ii) Is there any other suitable graphical representation for the same data?

- (iii) Is it correct to conclude that the maximum number of leaves are 153 mm long? why?

41. The following table gives the life times of 400 neon lamps:

Life time(in hours)	Number of lamps
300-400	14
400-500	56
500-600	60
600-700	86
700-800	74
800-900	62
900-1000	48

- (i) Represent the given information with the help of a histogram.
(ii) How many lamps have a life time of more than 700 hours?

42. The following table gives the distribution of students of two sections according to the marks obtained by them:

Section A		Section B	
Marks	Frequency	Marks	Frequency
0-10	3	0-10	5
10-20	9	10-20	19
20-30	17	20-30	15
30-40	12	30-40	10
40-50	9	40-50	1

Represent the marks of the students of both the sections on the same graph by two frequency polygons. From the two polygons compare the performance of the two sections.

43. The runs scored by two teams A and B on the first 60 balls in a cricket match are given below:

Number of balls	Team A	Team B
1-6	2	5
7-12	1	6
13-18	8	2
19-24	9	10
25-30	4	5
31-36	5	6
37-42	6	3
43-48	10	4
49-54	6	8
55-60	2	10

Represent the data of both the teams on the same graph by frequency polygons.
(Hint : First make the class intervals continuous.)

44. A random survey of the number of children of various age groups playing in a park was found as follows:

Age (in years)	No. of Children
1-2	5
2-3	3
3-5	6
5-7	12
7-10	9
10-15	10
15-17	4

Draw a histogram to represent the data above.

45. 100 surnames were randomly picked up from a local telephone directory and a frequency

distribution of the number of letters in the English alphabet in the surnames was found as follows:

No. of letters	No. of Surnames
1-4	6
4-6	30
6-8	44
8-12	16
12-20	4

- (i) Draw a histogram to depict the given information.
(ii) Write the class interval in which the maximum number of surnames lie.

46. The following number of goals were scored by a team in a series of 10 matches:

2, 3, 4, 5, 0, 1, 3, 3, 4, 3

Find the mean, median and mode of these scores.

47. In a mathematics test given to 15 students, the following marks (out of 100) are recorded:

41, 39, 48, 52, 46, 62, 54, 40, 96, 52, 98, 40, 42, 52, 60

Find the mean, median and mode of this data.

48. The following observations have been arranged in ascending order. If the median of the data is 63, find the value of x .

29, 32, 48, 50, x , $x + 2$, 72, 78, 84, 95

49. Find the mode of

14, 25, 14, 28, 18, 17, 18, 14, 23, 22, 14, 18.

50. Find the mean salary of 60 workers of a factory from the following table:

Salary (in ₹)	No. of Workers
3000	16
4000	12
5000	10
6000	8
7000	6
8000	4
9000	3
10000	1
Total	60

51. Give one example of a situation in which

(i) the mean is an appropriate measure of

central tendency.

(ii) the mean is not an appropriate measure of central tendency but the median is an appropriate measure of central tendency.