Octave for Mathematics

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Problem 1. *For* $x \in \mathbf{R}, x \neq 0, x \neq 1$, *let* $f_0(x) = \frac{1}{1-x}$ *and* $f_{n+1}(x) = f_0\left(f_n(x)\right), n = 0, 1, \ldots$. *Then find the value of* $f_{100}(3) + f_1\left(\frac{2}{3}\right) + f_2\left(\frac{3}{2}\right)$.

Problem 2. *If* $P = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}, A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ *and* $Q = PAP^T$, *find* $P^T Q^{2015} P$.

Problem 3. *Evaluate* $\sum_{r=1}^{15} r^2 \frac{\binom{15}{r}}{\binom{15}{r-1}}$.

Problem 4. *If* $\lim_{x\to\infty}\left(1 + \frac{a}{x} - \frac{4}{x^2}\right)^{2x} = e^3$, *find* a.

Problem 5. *The function*

$$f(x) = \begin{cases} -x & x < 1 \\ a + \cos^{-1}(x + b) & 1 \leq x \leq 2 \end{cases} \tag{1}$$

is known to be differentiable at $x = 1$. *What is the value of* $\frac{a}{b}$?

Problem 6. *The tangent at point P, for the curve* $x = 4t^2 + 3, y = 8t^3 - 1$, *with parameter* $t \in \mathbf{R}$, *meets the curve again at Q. Find the coordinates of Q.*

Problem 7. *Find the minimum distance of a point on the curve* $y = x^2 - 4$ *from the origin.*

Problem 8. *Sketch the region*

$$A = \left\{(x, y) \,|y \geq x^2 - 5x + 4, x + y \geq 1, y \leq 0\right\}. \tag{2}$$

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Problem 9. *A variable line drawn through the intersection of the lines* $\frac{x}{3}+\frac{y}{4}=1$ *and* $\frac{x}{4}+\frac{y}{3}=1$ *meets the coordinate axes at A and* $B, A \neq B$. *Sketch the locus of the midpoint of AB.*

Problem 10. *The point* $(2,1)$ *is translated parallel to the line* $L: x-y=4$ *by* $2\sqrt{3}$ *units to yield the point Q. If Q lies in the 3rd quadrant, sketch the line passing through Q and* $\perp$ *L.*

Problem 11. *A circle passes through* $(-2,4)$ *and touches the y−axis at* $(0,2)$. *Find out which of the following lines represents the diameter of the circle.*

1) $4x+5y-6=0$

2) $2x-3y+10=0$

3) $3x+4y-3=0$

4) $5x+2y+4=0$

Problem 12. *The eccentricity of a hyperbola satisfies the equation* $9e^2-18e+5=0$. $(5,0)$ *is a focus and the corresponding directrix is* $5x=9$. *Plot the hyperbola.*

Problem 13. *Sketch the ellipse* $\frac{x^2}{27}+\frac{y^2}{3}=1$.

Problem 14. *Find the minimum and maximum values of* $4+\frac{1}{2}\sin^2 2x-2\cos^4 x, x \in \mathbf{R}$.

Problem 15. *Find the solution of the equation* $\sqrt{2x+1}-\sqrt{2x-1}=1, x \geq \frac{1}{2}$.

Problem 16. *Let* $z=1+a_1, a>0$ *be a complex number such that* z^3 *is a real number. Find* $\sum_{k=0}^{11} z^k$.

Problem 17. $A=\begin{pmatrix} -4 & -1 \\ 3 & 1 \end{pmatrix}$. *Find the determinant of* $A^{2016}-2A^{2015}-A^{2014}$.

Problem 18. *Find the solutions of the following equations*

$$n^2-3n-108=0$$

$$n^2+5n-84=0$$

$$n^2+2n-80=0$$

$$n^2+n-110=0$$

Which of these satisfy $\frac{^{n+2}C_6}{^{n-2}P_2} = 11$*?*

Problem 19. *Sketch*

$$f(x) = \begin{cases} \frac{2x^2}{a} & 0 \le x < 1 \\ a & 1 \le x < \sqrt{2} \\ \frac{2b^2-4b}{x^3} & \sqrt{2} \le x < \infty \end{cases}$$

for (a, b) *equal to*

1) $\left(\sqrt{2}, 1 - \sqrt{3}\right)$

2) $\left(-\sqrt{2}, 1 + \sqrt{3}\right)$

3) $\left(\sqrt{2}, -1 + \sqrt{3}\right)$

4) $\left(-\sqrt{2}, 1 - \sqrt{3}\right)$

In which case is $f(x)$ *continuous?*

Problem 20. *Sketch* $f(x) = \sin^4 x + \cos^4 x$*. Find the intervals within* $(0, \pi)$ *when it is increasing.*

Problem 21. *The reflected line is given by* $y + 2x = 1$*. The surface is given by* $7x - y + 1 = 0$*. Which of the following is the incident line?*

1) $41x - 38y + 38 = 0$

2) $41x + 25y - 25 = 0$

3) $41x + 38y - 38 = 0$

4) $41x - 25y + 25 = 0$

Problem 22. *The lines* $x - y = 1$ *and* $2x + y = 3$ *intersect at* O*. A circle with centre at point* O *passes through the point* $(-1, 1)$*. Sketch the following lines*

1) $4x + y - 3 = 0$

2) $x + 4y + 3 = 0$

3) $3x - y - 4 = 0$

4) $x - 3y - 4 = 0$

Which of these is a tangent to the circle? At what point?

Problem 23. *P and Q are distinct points on the parabola* $y^2 = 4x$*, with parameters t and* t_1 *respectively. The normal at P passes through Q. Find the minimum value of* t_1^2.

Problem 24. *The transverse axis of a hyperbola is along the major axis of the conic* $\frac{x^2}{3} + \frac{y^2}{4} = 1$*. The vertices of the hyperbola are at the foci of this conic. The eccentricity of the hyperbola is* $\frac{3}{2}$*. Which of the points* $(0, 2), \left(\sqrt{5}, 2\sqrt{2}\right), \left(\sqrt{10}, 2\sqrt{3}\right), \left(5, 2\sqrt{3}\right)$*, lie on the Hyperbola?*

Problem 25. *Find the minimum value of* $\tan A + \tan B$*, given that* $A + B = \frac{\pi}{6}, A > 0, B > 0$.

Problem 26. *Find* θ *for which* $\frac{2+3\imath sin\theta}{1-2\imath \sin\theta}$ *is purely imaginary.*

Problem 27. *Find the sum of all the solutions of*

$$\left(x^2 - 5x + 5\right)^{x^2+4x-60} = 1$$

Problem 28. *The sum of the first 10 terms of the series* $\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \ldots$ *is* $\frac{16}{5}m$*. Find m.*

Problem 29. $p = \lim_{x\to 0+} \left(1 + \tan^2 \sqrt{x}\right)^{\frac{1}{2x}}$*. Find* $\log p$.

Problem 30. $f(x) = \left|\log 2 - \sin x\right|, x \in \mathbf{R}$ *and* $g(x) = f(f(x))$*. Which of the following is true?*

1) g *is not differntiable at* $x = 0$

2) $g'(0) = \cos(\log 2)$

3) $g'(0) = -\cos(\log 2)$

4) g *is differentiable at* $x = 0$ *and* $g'(0) = -\sin(\log 2)$.

Problem 31. *Consider*

$$f(x) = \tan^{-1} \sqrt{\left(\frac{1 + \sin x}{1 - \sin x}\right)}, x \in \left(0, \frac{\pi}{2}\right)$$

Sketch the normal to $f(x)$ *at* $x = \frac{\pi}{6}$*. Does it pass through any of the points* $(0, 0), \left(0, \frac{2\pi}{3}\right), \left(\frac{\pi}{6}, 0\right), \left(\frac{\pi}{4}, 0\right)$*?*

Problem 32. *Sketch* $\frac{\{(n+1)(n+2)\ldots(3n)\}^{\frac{1}{n}}}{n^{2n}}$ *and verify if its limit at* $n \to \infty$ *is* $\frac{18}{e^4}, \frac{27}{e^2}, \frac{9}{e^2}$ *or* $3\log 3 - 2$.

Problem 33. *Sketch the region*

$$\left\{(x, y) : y^2 \geq 2x, x^2 + y^2 \leq 4x, x \geq 0, y \geq 0\right\}$$

Problem 34. *Two sides of a rhombus are along the lines* $x - y + 1 = 0$ *and* $7x - y - 5 = 0$. *Its diagonals intersect at* $(-1, -2)$. *Find the vertices of the rhombus.*

Problem 35. *Sketch the locus of the centres of circles which touch the circle* $x^2 + y^2 - 8x - 8y - 4 = 0$ *as well as the* $x-$*axis.*

Problem 36. *One of the diameters of the circle* $x^2 + y^2 - 4x + 6y - 12 = 0$ *is a chord of a circle* S. *The centre of* S *is at* $(-3, 2)$. *Sketch* S *and find its radius.*

Problem 37. P *is the nearest point of the parabola* $y^2 = 8x$ *to the centre* C *of the circle* $x^2 + (y + 6)^2 = 1$.*Sketch the circle with centre* P *and passing through* C.

Problem 38. *The length of the latus rectum of a hyperbola is 8 and the length of its conjugate axis is half the distance between its foci. Sketch the hyperbola and find its eccentricity.*

Problem 39. *A wire of length 2 units is cut into two parts which are bent respectively to form a square of side x units and a circle of radius of 1 unit. Find x if the sum of the areas of the square and the circle so formed is minimum.*