LinearAlgebra Question 5

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Abstract—**A document implementing solutions to problems using linear algebra.**

Download all python codes from

```
svn co https://github.com/Srihari123456/Summer
    -2020/tree/master/LinearAlgebrafolder/codes
```

Download all LaTeX-Tikz codes from

```
svn co https://github.com/Srihari123456/Summer
    -2020/tree/master/LinearAlgebrafolder/figs
```

1 Question 1.2.5

1.1 Problem

1.1. In $\triangle ABC$ with vertices

$$\mathbf{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{1.1.1}$$

Find the equation and the length of the altitude from vertex $\mathbf{A}$. The following python code computes the length of the altitude $\mathbf{AD}$ in Fig.1.

```
./codes/triangle/q2.py
```

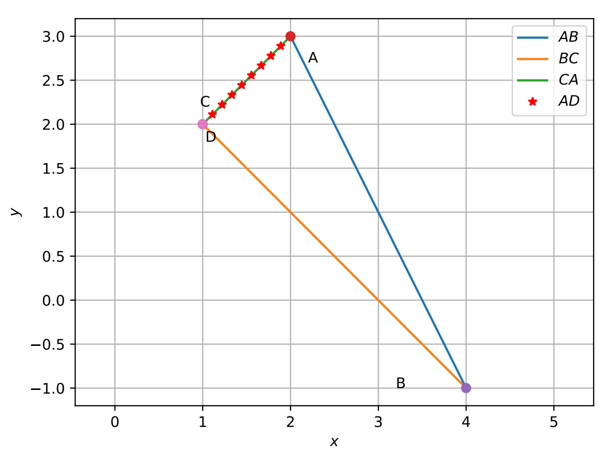

Fig. 1: Triangle of Q.1.2.5

Solution:

$$\because \|\mathbf{B}-\mathbf{A}\|^2 = \|\mathbf{B}-\mathbf{C}\|^2 + \|\mathbf{C}-\mathbf{A}\|^2$$
$$\triangle \text{ ABC is right angled.} \tag{1.1.2}$$

Let the direction vector $\mathbf{m} = \mathbf{B} - \mathbf{C}$ We define the normal vector

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \tag{1.1.3}$$

Equation of line $\mathbf{AD}$ is obtained as:

$$\mathbf{m^T x} = \mathbf{m^T A} \tag{1.1.4}$$
$$\begin{pmatrix} 3 & -3 \end{pmatrix} \mathbf{x} = -3 \tag{1.1.5}$$

Equation of line $\mathbf{BC}$ is :

$$\mathbf{n^T x} = \mathbf{n^T B} \tag{1.1.6}$$

Since $\mathbf{D}$ is the intersection of lines $\mathbf{AD}$ and $\mathbf{BC}$

$$\begin{pmatrix} \mathbf{m} & \mathbf{n} \end{pmatrix}^T \mathbf{D} = \begin{pmatrix} \mathbf{m^T A} \\ \mathbf{n^T B} \end{pmatrix}$$

which is solved to obtain the value of $\mathbf{D} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ (1.1.7)

Therefore The length of the altitude is obtained as $\|\mathbf{A}-\mathbf{D}\| = 1.414$

2 Question 2.2.5

2.1 Problem

2.1. Without using distance formula, show that the points

$$\mathbf{A} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \text{ and } \mathbf{D} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} \tag{2.1.1}$$

are the vertices of a parallelogram. The following python code computes the area of $\triangle ABC$ in Fig.2.

```
./codes/quadrilateral/q4.py
```

Solution: As the direction vectors,

$$\mathbf{A} - \mathbf{B} = \mathbf{D} - \mathbf{C} \tag{2.1.2}$$
$$\mathbf{A} - \mathbf{D} = \mathbf{B} - \mathbf{C} \tag{2.1.3}$$

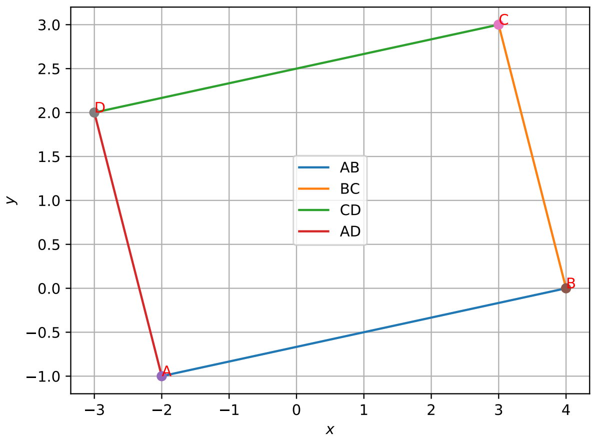

Fig. 2: Parallelogram of Q.2.2.5

$$\implies \mathbf{AB} \parallel \mathbf{CD} \text{ and } \mathbf{AD} \parallel \mathbf{BC}$$
$$\therefore \mathbf{ABCD} \text{ is a parallelogram.} \tag{2.1.4}$$

3 Question 3.2.5

3.1 Problem

3.1. Solve $\mathbf{x} + \mathbf{y} < 5$ graphically. The following python code computes the graphical representation of $\mathbf{x} + \mathbf{y} < 5$ as shown in Fig.3.

```
./codes/lines/q6.py
```

4 Question 3.3.5

4.1 Problem

4.1. Find the values of a,b,c,x,y and z if

$$\begin{pmatrix} x+3 & z+4 & 2y-7 \\ -6 & a-1 & 0 \\ b-3 & -21 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 6 & 3y-2 \\ -6 & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{pmatrix} \tag{4.1.1}$$

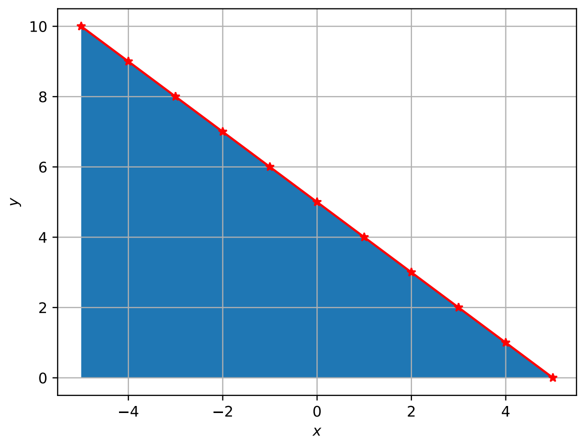

Fig. 3: x+y<5

Solution: As the two matrices are equal their corresponding entries are also equal. Hence

$$x + 3 = 0 \implies x = -3 \tag{4.1.2}$$
$$z + 4 = 6 \implies z = 2 \tag{4.1.3}$$
$$2y - 7 = 3y - 2 \implies y = -5 \tag{4.1.4}$$
$$a - 1 = -3 \implies a = -2 \tag{4.1.5}$$
$$2c + 2 = 0 \implies c = -1 \tag{4.1.6}$$
$$b - 3 = 2b + 4 \implies b = -7 \tag{4.1.7}$$

5 Question 3.4.5

5.1 Problem

5.1. Convert the complex number $\frac{-16}{\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}}$

Solution: The multiplicative inverse of $\frac{-16}{\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}}$ is given by the following python code

```
./codes/lines/q8.py
```

which is found as $\begin{pmatrix} 0.25 \\ -0.43 \end{pmatrix}$ Representing using matrices we get,

$$\frac{-16}{\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}} = \frac{-16}{\begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}} \tag{5.1.1}$$

Multiplying and dividing by the multiplicative inverse,

$$\frac{-16}{\begin{pmatrix}1\\\sqrt{3}\end{pmatrix}} = \frac{-16\begin{pmatrix}0.25 & 0.43\\-0.43 & 0.25\end{pmatrix}}{\begin{pmatrix}1 & -\sqrt{3}\\\sqrt{3} & 1\end{pmatrix}\begin{pmatrix}0.25 & 0.43\\-0.43 & 0.25\end{pmatrix}} \tag{5.1.2}$$

$$\frac{-16}{\begin{pmatrix}1\\\sqrt{3}\end{pmatrix}} = \frac{-16\begin{pmatrix}0.25 & 0.43\\-0.43 & 0.25\end{pmatrix}}{\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}} \tag{5.1.3}$$

$$\frac{-16}{\begin{pmatrix}1\\\sqrt{3}\end{pmatrix}} = \begin{pmatrix}-4 & -6.88\\6.88 & -4\end{pmatrix} \tag{5.1.4}$$

$$\frac{-16}{\begin{pmatrix}1\\\sqrt{3}\end{pmatrix}} = \begin{pmatrix}-4\\4\sqrt{3}\end{pmatrix} \tag{5.1.5}$$

6 Question 3.5.5

6.1 Problem

6.1. Find the angle between the x-axis and the line joining the points

$$\mathbf{A} = \begin{pmatrix}3\\-1\end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix}4\\-2\end{pmatrix}. \tag{6.1.1}$$

The following python code computes the angle which the line in Fig.4 makes with x-axis.

```
./codes/lines/q9.py
```

Solution: Let the given line be represented as

$$\mathbf{u} = \mathbf{A} - \mathbf{B} = \begin{pmatrix}-1\\1\end{pmatrix} \tag{6.1.2}$$

x-axis can be represented as

$$\mathbf{v} = \begin{pmatrix}1\\0\end{pmatrix} \tag{6.1.3}$$

$$\mathbf{u}^T\mathbf{v} = \|\mathbf{u}\|\,\|\mathbf{v}\|\cos\theta \tag{6.1.4}$$

$$\cos\theta = \frac{-1}{\sqrt{2}} \tag{6.1.5}$$

$$\theta = 135^\circ \tag{6.1.6}$$

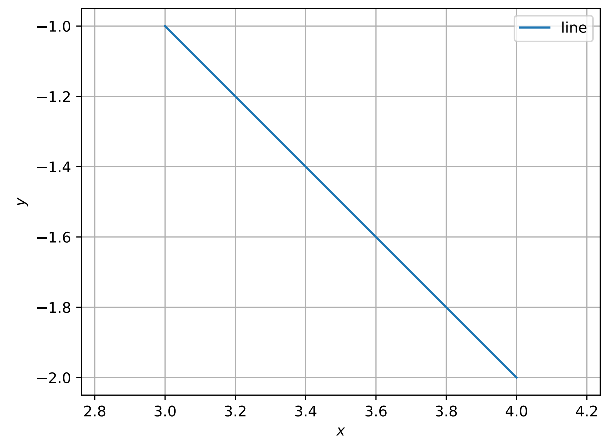

Fig. 4: Line of Q.3.5.5

7 Question 3.6.5

7.1 Problem

7.1. If the vertices of a parallelogram taken in order are

$$\mathbf{A} = \begin{pmatrix}1\\2\end{pmatrix}, \mathbf{B} = \begin{pmatrix}4\\y\end{pmatrix}, \mathbf{C} = \begin{pmatrix}x\\6\end{pmatrix} \text{ and } \mathbf{D} = \begin{pmatrix}3\\5\end{pmatrix}$$

find x and y. (7.1.1)

The following python code computes the value of x and y used in Fig.5.

```
./codes/lines/q10.py
```

Solution: In a parallelogram, the diagonals bisect each other. Hence

$$\frac{\mathbf{A}+\mathbf{C}}{\mathbf{2}} = \frac{\mathbf{B}+\mathbf{D}}{\mathbf{2}} \tag{7.1.2}$$

$$\therefore \frac{\mathbf{1}+\mathbf{x}}{\mathbf{2}} = \frac{\mathbf{7}}{\mathbf{2}} \text{ and } \frac{\mathbf{8}}{\mathbf{2}} = \frac{\mathbf{y}+\mathbf{5}}{\mathbf{2}} \tag{7.1.3}$$

$$\implies x = 6, y = 3 \tag{7.1.4}$$

8 Question 3.7.5

8.1 Problem

8.1. Draw the graphs of the following equations:

a) $\begin{pmatrix}1 & 1\end{pmatrix}\mathbf{x} = 0$

b) $\begin{pmatrix}2 & -1\end{pmatrix}\mathbf{x} = 0$

c) $\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = 0$

d) $\begin{pmatrix}2 & -1\end{pmatrix}\mathbf{x} = -1$

e) $\begin{pmatrix}2 & -1\end{pmatrix}\mathbf{x} = 4$

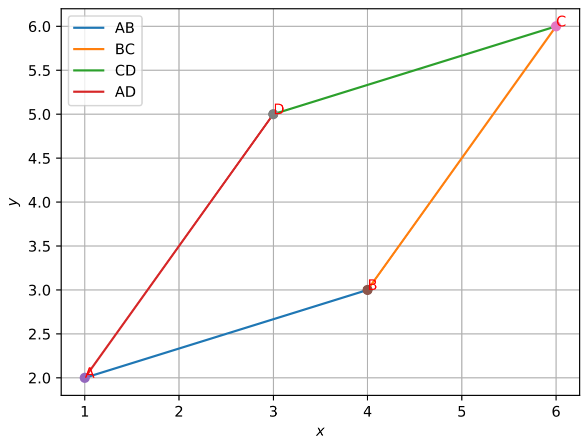

Fig. 5: Parallelogram of Q.3.6.5

f) $\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 4$

The following python codes draw the graphs which are represented in Fig.6 and Fig.7.

```
./codes/lines/q11a.py
./codes/lines/q11b.py
```

Solution:

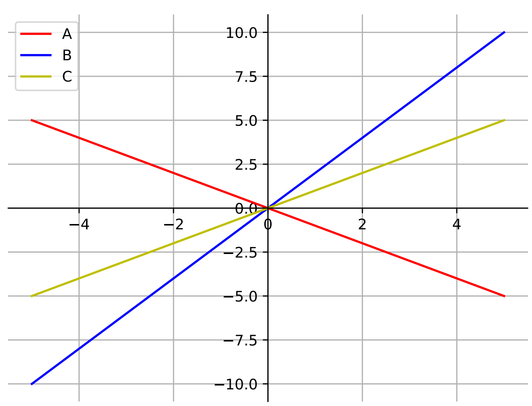

Fig. 6: Lines of Q.3.7.5

9 Question 3.8.5

9.1 Problem

9.1. Rain is falling vertically with a speed of $35ms^{-1}$. A woman rides a bicycle with a speed of $12ms^{-1}$ in east to west direction.What is the direction in which she should hold her umbrella?

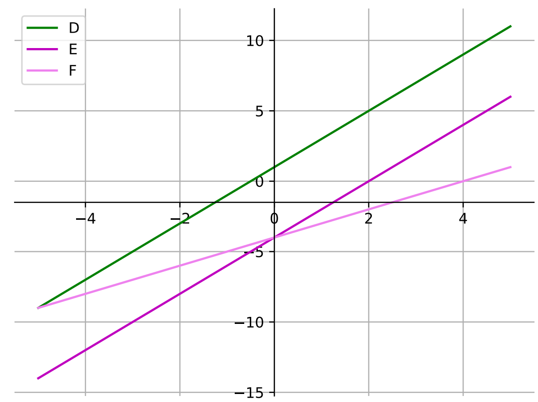

Fig. 7: Lines of Q.3.7.5

The following python code computes the area of $\triangle ABC$ in Fig.8.

```
./codes/lines/q12.py
```

Solution: At time t=0 let

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{9.1.1}$$

denote the position of the woman. Since she rides her bicycle at $12ms^{-1}$ in east to west direction , her position at time t=1 is represented as

$$\mathbf{C} = \begin{pmatrix} -12 \\ 0 \end{pmatrix} \tag{9.1.2}$$

. Let the position of a rain-droplet at time t=0 be

$$\mathbf{A} = \begin{pmatrix} -12 \\ 35 \end{pmatrix} \tag{9.1.3}$$

. The drops which are falling a little ahead of the current position of the woman, will fall on her, because she moves in that direction. To find the direction in which she should hold her umbrella, we need to find $\angle CAB = \theta$.

$$\mathbf{AB} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} 12 \\ -35 \end{pmatrix}$$
$$\mathbf{AC} = \mathbf{C} - \mathbf{A} = \begin{pmatrix} 0 \\ -35 \end{pmatrix}$$
$$\mathbf{AB}^T\mathbf{AC} = \|\mathbf{AB}\|\,\|\mathbf{AC}\|\cos\theta$$
$$\cos\theta = \frac{35}{37}$$
$$\theta = 18.93^\circ \quad (9.1.4)$$

So the cyclist should hold the umbrella at 18.93° to the vertical in the forward direction.

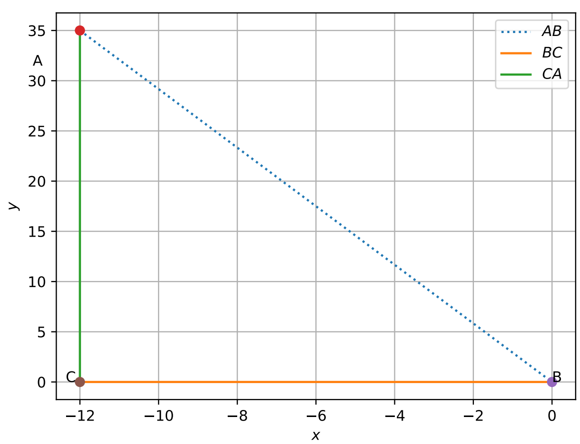

Fig. 8: Figure of Q.3.8.5

10 Question 3.9.5

10.1 Problem

10.1. Construct a a3×4 matrix whose elements are given by:
a) $A_{ij} = \frac{1}{2}|-3i+j|$
b) $A_{ij} = 2i - j$
The following python code computes the required matrix.

```
./codes/lines/q13.py
```

Solution:
a) The matrix $A_{ij} = \frac{1}{2}|-3i+j|$ obtained is

$$\begin{pmatrix} 0 & 0.5 & 1 & 1.5 \\ 1.5 & 1 & 0.5 & 0 \\ 3 & 2.5 & 2 & 1.5 \end{pmatrix} \quad (10.1.1)$$

b) The matrix $A_{ij} = 2i - j$ obtained is

$$\begin{pmatrix} 0 & -1 & -2 & -3 \\ 2 & 1 & 0 & -1 \\ 4 & 3 & 2 & 1 \end{pmatrix} \quad (10.1.2)$$

11 Question 3.10.5

11.1 Problem

11.1. Evaluate the determinants
The following python code computes the required determinant value.

```
./codes/lines/q14.py
```

a) $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$ which on evaluating gives -12
b) $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$ which on evaluating gives -46
c) $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$ which on evaluating gives 0
d) $\begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$ which on evaluating gives 5

12 Question 3.11.5

12.1 Problem

12.1. Find all pairs of consecutive odd natural numbers, both of which are greater than 10, such that their sum is less than 40.
The following python code computes the required pairs of consecutive odd natural numbers which satisfy the required condition, shown in Fig.9.

```
./codes/lines/q15.py
```

Solution: Let x be an odd natural number and y be the odd natural number consecutive to x.

$$\therefore y = x + 2 \quad (12.1.1)$$

We need to find x and y such that

$$x, y > 10 \text{ and } x + y < 40$$
$$\therefore x + x + 2 < 40$$
$$2x + 2 < 40$$
$$x + 1 < 20$$
$$x < 19$$
$$(12.1.2)$$

Hence the condition is satisfied when $x > 10$ and $x < 19$

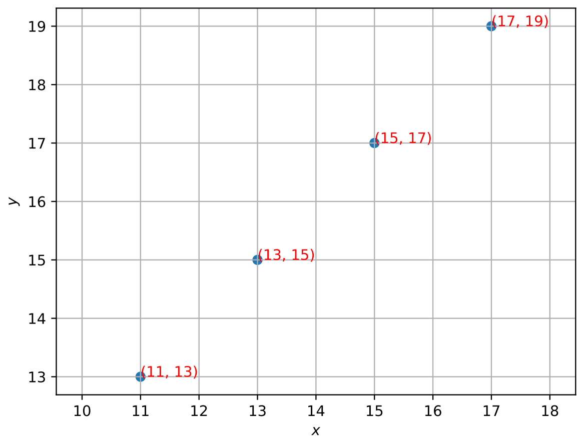

Fig. 9: Triangle of Q.3.11.5

13 Question 3.12.5

13.1 Problem

13.1. Triangle Inequality : Show that $\|\mathbf{x}+\mathbf{y}\| \le \|\mathbf{x}\| + \|\mathbf{y}\|$
Solution:

$$\|\mathbf{x}+\mathbf{y}\|^2 = (\mathbf{x}+\mathbf{y})^T(\mathbf{x}+\mathbf{y}) \tag{13.1.1}$$
$$\|\mathbf{x}+\mathbf{y}\|^2 = \left(\mathbf{x}^T+\mathbf{y}^T\right)(\mathbf{x}+\mathbf{y}) \tag{13.1.2}$$
$$\|\mathbf{x}+\mathbf{y}\|^2 = \mathbf{x^Tx}+\mathbf{x^Ty}+\mathbf{y^Tx}+\mathbf{y^Ty} \tag{13.1.3}$$
$$\|\mathbf{x}+\mathbf{y}\|^2 = \|\mathbf{x}\|^2+\mathbf{x^Ty}+\mathbf{y^Tx}+\|\mathbf{y}\|^2 \tag{13.1.4}$$

$\because \mathbf{x^Ty} = \mathbf{y^Tx}$

$$\|\mathbf{x}+\mathbf{y}\|^2 = \|\mathbf{x}\|^2+2\mathbf{x^Ty}+\|\mathbf{y}\|^2 \tag{13.1.5}$$
$$\|\mathbf{x}+\mathbf{y}\|^2 = \|\mathbf{x}\|^2+2\|\mathbf{x}\|\|\mathbf{y}\|\cos\theta+\|\mathbf{y}\|^2 \tag{13.1.6}$$

where θ is the angle between $\mathbf{x}$ and $\mathbf{y}$. Since the maximum value of $\cos\theta$ is 1,

$$\|\mathbf{x}+\mathbf{y}\|^2 \le \|\mathbf{x}\|^2+2\|\mathbf{x}\|\|\mathbf{y}\|+\|\mathbf{y}\|^2 \tag{13.1.7}$$
$$\|\mathbf{x}+\mathbf{y}\|^2 \le (\|\mathbf{x}\|+\|\mathbf{y}\|)^2 \tag{13.1.8}$$
$$\implies \|\mathbf{x}+\mathbf{y}\| = \|\mathbf{x}\|+\|\mathbf{y}\| \tag{13.1.9}$$

14 Question 4.1.5

14.1 Problem

14.1. Find the area of the region in the first quadrant enclosed by the x-axis, the line $\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = 0$ and the circle $\|\mathbf{x}\| = 1$. The following python code computes the required area represented in Fig.10.

```
./codes/circle/q17ag.py
```

Solution: From the Fig.10 we obtain the coordinates as,

$$\mathbf{O} = \begin{pmatrix}0\\0\end{pmatrix} \quad \mathbf{A} = \begin{pmatrix}0.707\\0.707\end{pmatrix} \quad \mathbf{B} = \begin{pmatrix}0.707\\0\end{pmatrix} \quad \mathbf{C} = \begin{pmatrix}1\\0\end{pmatrix} \tag{14.1.1}$$

The required area is given by

$$ar(OACB) = ar(OAB) + ar(BAC) \tag{14.1.2}$$
$$ar(BAC) = \frac{1}{2}\|(\mathbf{B}-\mathbf{A})\times(\mathbf{C}-\mathbf{A})\| \tag{14.1.3}$$
$$ar(OAB) = \frac{1}{2}\|(\mathbf{B}-\mathbf{O})\times(\mathbf{A}-\mathbf{O})\| \tag{14.1.4}$$

which on computing, we obtain the required area as 0.3535

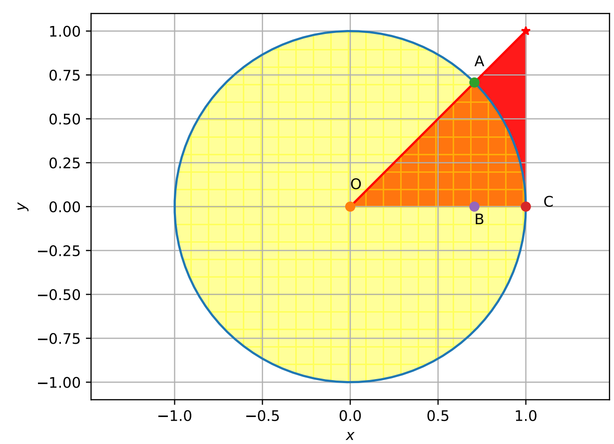

Fig. 10: Figure of Q.4.1.5

15 Question 4.2.5

15.1 Problem

15.1. Sketch circles with equation: The following python codes generate the required circles :

```
./codes/circle/q18abc.py
./codes/circle/q18d.py
```

a)

$$\mathbf{x^Tx} - \begin{pmatrix}4\\8\end{pmatrix}\mathbf{x} - 45 = 0 \text{ represented in Fig:11}$$

$$\text{Center is } \begin{pmatrix}2\\4\end{pmatrix} \quad \text{Radius is } \sqrt{65} \tag{15.1.1}$$

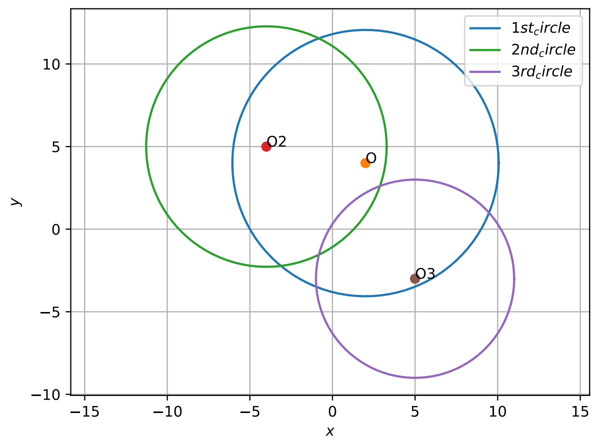

Fig. 11: Circle of Q.4.2.5

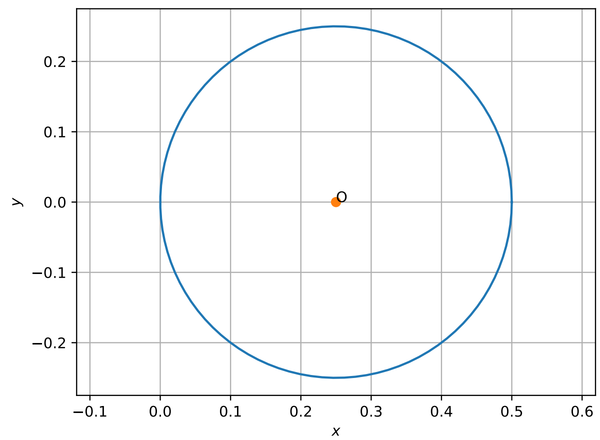

Fig. 12: Circle of Q.4.2.5

b)

$$\mathbf{x}^{\mathbf{T}}\mathbf{x}-\begin{pmatrix}8\\-10\end{pmatrix}\mathbf{x}-12=0 \text{ represented in Fig:11}$$

$$\text{Center is } \begin{pmatrix}-4\\5\end{pmatrix} \quad \text{Radius is } \sqrt{53} \quad (15.1.2)$$

c)

$$\left\|x-\begin{pmatrix}5\\-3\end{pmatrix}\right\| = 36 \text{ represented in Fig:11}$$

$$\text{Center is } \begin{pmatrix}5\\-3\end{pmatrix} \quad \text{Radius is 6} \quad (15.1.3)$$

d)

$$2\mathbf{x}^{\mathbf{T}}\mathbf{x}-\begin{pmatrix}1\\0\end{pmatrix}\mathbf{x}=0 \text{ represented in Fig:12}$$

$$\text{Center is } \begin{pmatrix}0.25\\0\end{pmatrix} \quad \text{Radius is 0.25} \quad (15.1.4)$$

16 Question 5.1.5

16.1 Problem

16.1. Find the roots of the quadratic equation $6x^2 - x - 2 = 0$. The following python code computes roots of the quadratic equation represented in Fig.13.

```
./codes/conics/q19.py
```

Solution: For a general polynomial equation of degree 2,

$$p(x,y) \implies Ax^2+Bxy+Cy^2+Dx+Ey+F=0$$

The vector form is

$$\mathbf{x}^T\begin{pmatrix}A & \frac{B}{2}\\ \frac{B}{2} & C\end{pmatrix}\mathbf{x}+\begin{pmatrix}D & E\end{pmatrix}\mathbf{x}+F=0 \quad (16.1.1)$$

Here

$$y = 6x^2 - x - 2 \quad \text{The vector form is} \quad (16.1.2)$$

$$\mathbf{x}^T\begin{pmatrix}6 & 0\\0 & 0\end{pmatrix}\mathbf{x}+\begin{pmatrix}-1 & -1\end{pmatrix}\mathbf{x}-2=0 \quad (16.1.3)$$

Thus, from 16.1.1

$$y=0 \implies 6x^2-x-2=0 \quad (16.1.4)$$

$$\left(x+\frac{1}{2}\right)\left(x-\frac{2}{3}\right)=0 \quad (16.1.5)$$

$$x=\frac{-1}{2},\frac{2}{3} \quad (16.1.6)$$

17 Question 5.2.5

17.1 Problem

The following python code computes roots of the quadratic equation obtained:

```
./codes/conics/q20a.py
./codes/conics/q20b.py
```

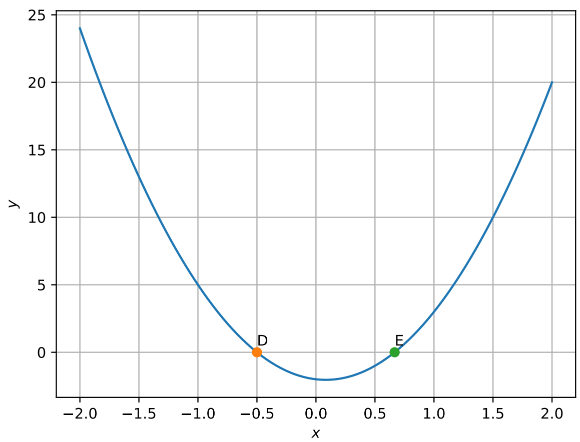

Fig. 13: Parabola of Q.5.1.5

```
./codes/conics/q20c.py
./codes/conics/q20d.py
./codes/conics/q20e.py
./codes/conics/q20f.py
```

17.1. Find a quadratic polynomial each with the given numbers as the sum and the product of its zeroes.

a) -1,$\frac{1}{4}$

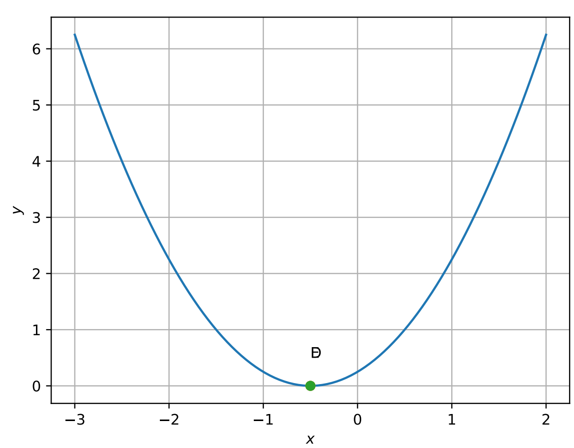

Fig. 14: Parabola of Q.5.2.5a

Solution: For a general polynomial equation of degree 2,

$$p(x,y) \implies Ax^2+Bxy+Cy^2+Dx+Ey+F=0$$

The vector form is

$$\mathbf{x}^T\begin{pmatrix} A & \frac{B}{2} \\ \frac{B}{2} & C \end{pmatrix}\mathbf{x}+\begin{pmatrix} D & E \end{pmatrix}\mathbf{x}+F=0 \quad (17.1.1)$$

Here, sum of zeroes = D = -1
Product of zeroes = F =$\frac{1}{4}$
Substituting the values in 17.1.1,

$$\mathbf{x}^T\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\mathbf{x}+\begin{pmatrix} 1 & -1 \end{pmatrix}\mathbf{x}+\frac{1}{4}=0 \quad (17.1.2)$$

$$\implies y=x^2+x+\frac{1}{4} \quad (17.1.3)$$

The roots are -0.5 and -0.5 as represented in Fig.14

b) 1,1

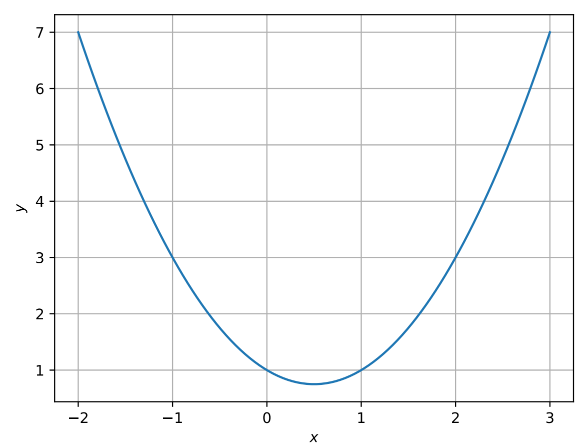

Fig. 15: Parabola of Q.5.2.5b

Solution: Here, sum of zeroes = D = 1
Product of zeroes = F =1
Substituting the values in 17.1.1,

$$\mathbf{x}^T\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\mathbf{x}+\begin{pmatrix} -1 & -1 \end{pmatrix}\mathbf{x}+1=0 \quad (17.1.4)$$

$$\implies y=x^2-x+1 \quad (17.1.5)$$

Since the curve doesn't meet the x-axis, real roots don't exist for this parabola as represented in Fig.15

c) $0, \sqrt{5}$

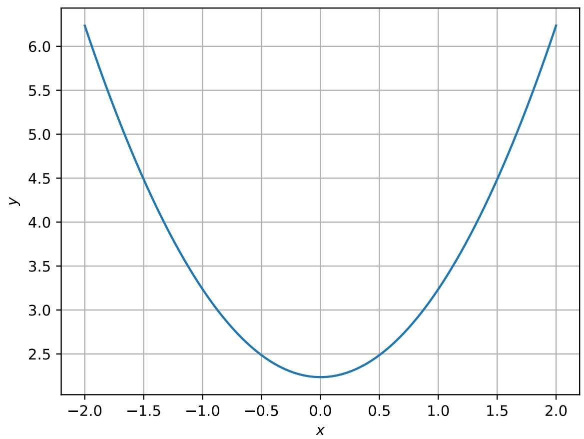

Fig. 16: Parabola of Q.5.2.5c

Solution: Here, sum of zeroes = D = 0
Product of zeroes = F = $\sqrt{5}$
Substituing the values in 17.1.1,

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} + \sqrt{5} = 0 \quad (17.1.6)$$

$$\implies y = x^2 + \sqrt{5} \quad (17.1.7)$$

Since the curve doesn't meet the x-axis, real roots don't exist for this parabola as represented in Fig.16

d) 4,1

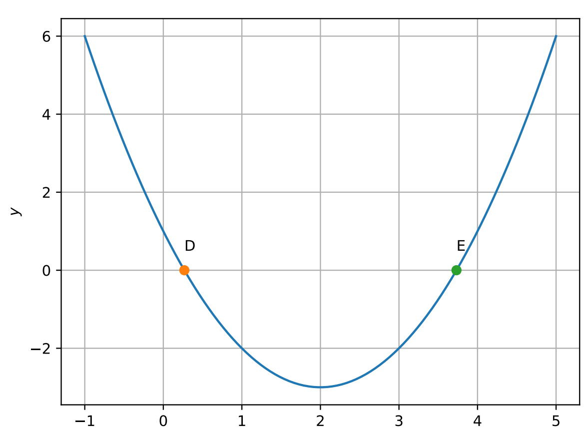

Fig. 17: Parabola of Q.5.2.5d

Solution: Here, sum of zeroes = D = 4
Product of zeroes = F = 1
Substituing the values in 17.1.1,

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -4 & -1 \end{pmatrix} \mathbf{x} + 1 = 0 \quad (17.1.8)$$

$$\implies y = x^2 - 4x + 1 \quad (17.1.9)$$

The roots are 3.73 and 0.26 as represented in Fig.17

e) $\frac{1}{4}, \frac{1}{4}$

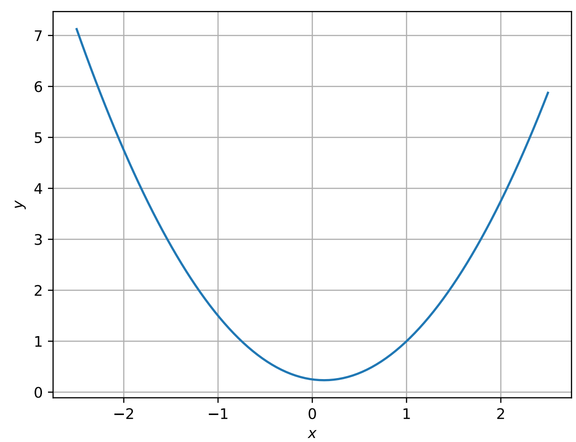

Fig. 18: Parabola of Q.5.2.5e

Solution: Here, sum of zeroes = D = $\frac{1}{4}$
Product of zeroes = F = $\frac{1}{4}$
Substituing the values in 17.1.1,

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -\frac{1}{4} & -1 \end{pmatrix} \mathbf{x} + \frac{1}{4} = 0 \quad (17.1.10)$$

$$\implies y = x^2 - \frac{1}{4}x + \frac{1}{4} \quad (17.1.11)$$

Since the curve doesn't meet the x-axis, real roots don't exist for this parabola as represented in Fig.18

f) $\sqrt{2}, \frac{1}{3}$

Solution: Here, sum of zeroes = D = $\sqrt{2}$
Product of zeroes = F = $\frac{1}{3}$
Substituing the values in 17.1.1,

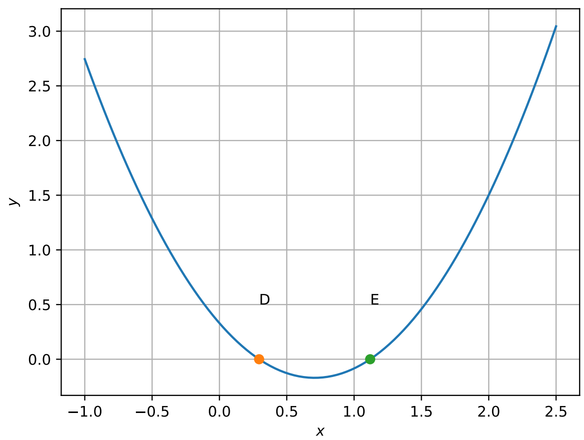

Fig. 19: Parabola of Q.5.2.5f

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -\sqrt{2} & -1 \end{pmatrix} \mathbf{x} + \frac{1}{3} = 0 \tag{17.1.12}$$

$$\implies y = x^2 - \sqrt{2}x + \frac{1}{3} \tag{17.1.13}$$

The roots are 1.11 and 0.29 as represented in Fig.19