# Math Document Template

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Download all python codes from

```
svn co https://github.com/SiddharthPh/
    Summer2020/trunk/geometry/Probstat/codes
```

## 1 Probability Exercises

### 1.1 Exercise 1

*1.1.1 Problem:* Suppose you drop a die at random on the rectangular region shown in Fig.15.6.What is the probability that it will land inside the circle with diameter 1m?

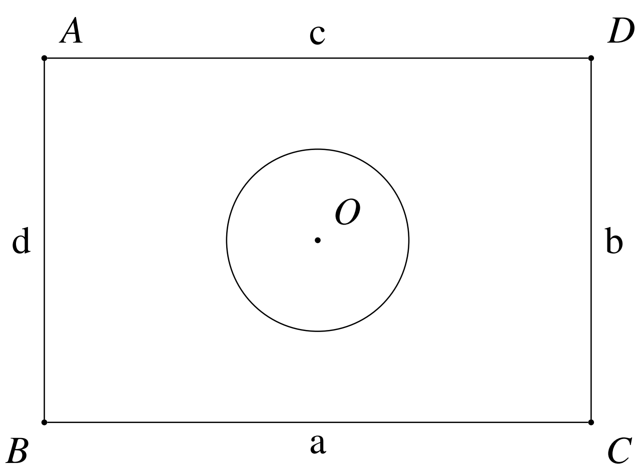


Fig. 0: Rectangle

*1.1.2 Solution:*

1. In the given question,
   The sample size = Total Area of the rectangle=
   $$3x2 = 6m^2 \tag{1.1.2.1.1}$$
   Favourable outcome = Area of Circle=
   $$\pi\left(\frac{1}{2}\right)^2 = \frac{\pi}{4}m^2 \tag{1.1.2.1.2}$$
   Probabilty(P) of the dice landing in the circle=$\frac{\pi}{24}$
   $\therefore$ P = 0.131
   The python code for the distribution.

   ```
   prob/codes/prob1.py
   ```

   shows the Bernouli distribution of data.
   The Bernoulli Distribution of data is given below
   Probability mass function(P(X))=$p^x\left(1-p\right)^{1-x}$
   $$P(X=0) = 1-p \tag{1.1.2.1.3}$$
   $$P(X=1) = p \tag{1.1.2.1.4}$$
   where p=0.131 given by 1.4.2.1

*1.1.3 Understanding Graph:*

1. From the graph (**??**),
   a) Values on X-axis represent the Bernoulli distribution of data.
   b) Values on Y-axis represent the density of frequency(Histogram estimator) of the data. To calculate the histogram estimator, we have to define the number of bins(Intervals) For the graph in the question,
   $$bins = 10 \tag{1.1.3.1.1}$$
   $$h(binwidth) = \frac{(1-0)}{10} \tag{1.1.3.1.2}$$
   For bin-width h, number of observations n, for bin j, proportion of observations is
   $$p_j = \frac{y_j}{n} \tag{1.1.3.1.3}$$
   (Where $y_j$ is the frequency of j$^th$ bin.)
   $$p_0 = \frac{869}{1000} = 0.869 \tag{1.1.3.1.4}$$
   $$p_1 = \frac{131}{1000} = 0.131 \tag{1.1.3.1.5}$$
   The density estimate is
   $$y\left(x\right) = \frac{p_j}{h} \tag{1.1.3.1.6}$$
   $$y\left(0\right) = \frac{0.869}{0.1} = 8.69 \tag{1.1.3.1.7}$$
   $$y\left(0\right) = \frac{0.131}{0.1} = 1.31 \tag{1.1.3.1.8}$$
   To draw the Gaussian Kernel Density curve, Calculate mean and standard deviation for the centre and bandwidth.

See 1.1.3.1 for clear understanding.

$$\mu(Mean) = 0.861 \tag{1.1.3.1.9}$$

$$\sigma^2(\text{Standard Deviation}) = 0.1189 \tag{1.1.3.1.10}$$

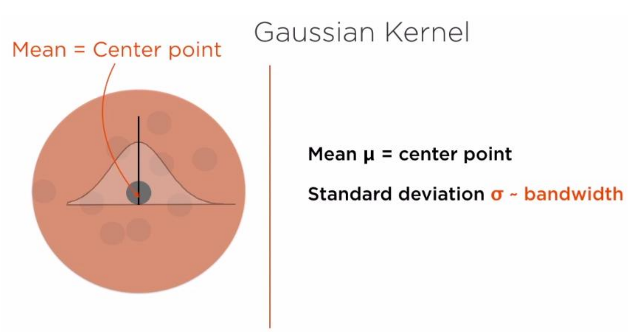


Fig. 1.1.3.1: Gaussian Kernel

### *1.2 Exercise 2*

*1.2.1 Problem:* A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that
(i) She will buy it ?
(ii) She will not buy it ?

*1.2.2 Solution:*

1. In the given question,

   a) The sample size = Total number of pens(S)=

   $$S = 144 \tag{1.2.2.1.1}$$

   Favourable outcome = Pens purchased(F1)=

   $$F1 = 124 \tag{1.2.2.1.2}$$

   Probabilty(P) of the pens purchased by her from the shopkeeper=$\frac{124}{144}$
   $\therefore$ P = 0.861
   The python code for the distribution,

   ```
   prob/codes/prob2_a.py
   ```

   shows the Bernouli distribution of data.
   The Bernoulli Distribution of data is given below
   Probability mass function(P(X))=$p^x(1-p)^{1-x}$

   $$P(X=0) = 1-p \tag{1.2.2.1.3}$$

   $$P(X=1) = p \tag{1.2.2.1.4}$$

   where p is the probability of occurence of (X=1)
   $\therefore$ p=0.861 given by 1.4.2.1

2. The sample size = Total number of pens(S)=

   $$S = 144 \tag{1.2.2.2.1}$$

   Favourable outcome = Pens not purchased(F2)=

   $$F2 = 20 \tag{1.2.2.2.2}$$

   Probabilty(P) of the pens not purchased by her from the shopkeeper=$\frac{20}{144}$
   $\therefore$ P = 0.139
   The python code for the distribution of data

   ```
   prob/codes/prob2_b.py
   ```

   shows the Bernouli distribution of data.
   The Bernoulli Distribution of data is given below
   Probability mass function(P(X))=$p^x(1-p)^{1-x}$

   $$P(X=0) = 1-p \tag{1.2.2.2.3}$$

   $$P(X=1) = p \tag{1.2.2.2.4}$$

   where p is the probability of occurence of (X=1)
   $\therefore$ p=0.139 given by 1.4.2.1

### *1.3 Exercise 3*

*1.3.1 Problem:* (i)Complete the following table:

| Event:'Sum on two dice' | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Probability | $\frac{1}{36}$ | - | - | - | - | - | $\frac{5}{36}$ | - | - | - | $\frac{1}{36}$ |

(ii) A student argues that 'there are 11 possible outcomes 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.Therefore,each of them has a probability $\frac{1}{11}$ Do you agree with this argument? Justify your answer.

*1.3.2 Solution:*

1. In the given question,

   a) Table is completed as follows 1.7.2.1 The sample size = Total number of possibilities(S)=

| Event | Value |
|---|---|
| 2 | 1/36 |
| 3 | - |
| 4 | - |
| 5 | - |
| 6 | - |
| 7 | - |
| 8 | 5/36 |
| 9 | - |
| 10 | - |
| 11 | - |
| 12 | 1/36 |

TABLE 1.3.2.1: Input Values

$$\begin{pmatrix} \{1 \quad 1\} & \{1 \quad 2\} & \{1 \quad 3\} & \{1 \quad 4\} & \{1 \quad 5\} & \{1 \quad 6\} \\ \{2 \quad 1\} & \{2 \quad 2\} & \{2 \quad 3\} & \{2 \quad 4\} & \{2 \quad 5\} & \{2 \quad 6\} \\ \{3 \quad 1\} & \{3 \quad 2\} & \{3 \quad 3\} & \{3 \quad 4\} & \{3 \quad 5\} & \{3 \quad 6\} \\ \{4 \quad 1\} & \{4 \quad 2\} & \{4 \quad 3\} & \{4 \quad 4\} & \{4 \quad 5\} & \{4 \quad 6\} \\ \{5 \quad 1\} & \{5 \quad 2\} & \{5 \quad 3\} & \{5 \quad 4\} & \{5 \quad 5\} & \{5 \quad 6\} \\ \{6 \quad 1\} & \{6 \quad 2\} & \{6 \quad 3\} & \{6 \quad 4\} & \{6 \quad 5\} & \{6 \quad 6\} \end{pmatrix}$$

S=6x6=36

Favourable outcome for sum=2 (E1)= $\left(\{1 \quad 1\}\right)$

$$E1 = 1 \tag{1.3.2.1.1}$$

$$Probabilty(P(E1)) = \frac{1}{36} = 0.027 \tag{1.3.2.1.2}$$

Favourable outcome for sum=3 (E2)= $\left(\{1 \quad 2\} \quad \{2 \quad 1\}\right)$

$$E2 = 2 \tag{1.3.2.1.3}$$

$$Probabilty(P(E2)) = \frac{2}{36} = 0.055 \tag{1.3.2.1.4}$$

Favourable outcome for sum=4 (E3)= $\left(\{1 \quad 3\} \quad \{2 \quad 2\} \quad \{3 \quad 1\}\right)$

$$E3 = 3 \tag{1.3.2.1.5}$$

$$Probabilty(P(E3)) = \frac{3}{36} = 0.083 \tag{1.3.2.1.6}$$

Favourable outcome for sum=5 (E4)= $\left(\{1 \quad 4\} \quad \{2 \quad 3\} \quad \{3 \quad 2\} \quad \{4 \quad 1\}\right)$

$$E4 = 4 \tag{1.3.2.1.7}$$

$$Probabilty(P(E4)) = \frac{4}{36} = 0.111 \tag{1.3.2.1.8}$$

Favourable outcome for sum=6 (E5)= $\left(\{1 \quad 5\} \quad \{2 \quad 4\} \quad \{3 \quad 3\} \quad \{4 \quad 2\} \quad \{5 \quad 1\}\right)$

$$E5 = 5 \tag{1.3.2.1.9}$$

$$Probabilty(P(E5)) = \frac{5}{36} = 0.138 \tag{1.3.2.1.10}$$

Favourable outcome for sum=7 (E6)= $\left(\{1 \quad 6\} \quad \{2 \quad 5\} \quad \{3 \quad 4\} \quad \{4 \quad 3\} \quad \{5 \quad 2\} \quad \{6 \quad 1\}\right)$

$$E6 = 6 \tag{1.3.2.1.11}$$

$$Probabilty(P(E6)) = \frac{6}{36} = 0.166 \tag{1.3.2.1.12}$$

Favourable outcome for sum=8 (E7)= $\left(\{2 \quad 6\} \quad \{3 \quad 5\} \quad \{4 \quad 4\} \quad \{5 \quad 3\} \quad \{6 \quad 2\}\right)$

$$E7 = 5 \tag{1.3.2.1.13}$$

$$Probabilty(P(E7)) = \frac{5}{36} = 0.138 \tag{1.3.2.1.14}$$

Favourable outcome for sum=9 (E8)= $\left(\{3 \quad 6\} \quad \{4 \quad 5\} \quad \{5 \quad 4\} \quad \{6 \quad 3\}\right)$

$$E8 = 4 \tag{1.3.2.1.15}$$

$$Probabilty(P(E8)) = \frac{4}{36} = 0.111 \tag{1.3.2.1.16}$$

Favourable outcome for sum=10 (E9)= $\left(\begin{Bmatrix}4 & 6\end{Bmatrix} \begin{Bmatrix}5 & 5\end{Bmatrix} \begin{Bmatrix}6 & 4\end{Bmatrix}\right)$

$$E9 = 3 \tag{1.3.2.1.17}$$

$$Probabilty(P(E9)) = \frac{3}{36} = 0.083 \tag{1.3.2.1.18}$$

Favourable outcome for sum=3=11 (E10)= $\left(\begin{Bmatrix}5 & 6\end{Bmatrix} \begin{Bmatrix}6 & 5\end{Bmatrix}\right)$

$$E10 = 2 \tag{1.3.2.1.19}$$

$$Probabilty(P(E10)) = \frac{2}{36} = 0.055 \tag{1.3.2.1.20}$$

Favourable outcome for sum=12 (E11)= $\left(\begin{Bmatrix}6 & 6\end{Bmatrix}\right)$

$$E11 = 1 \tag{1.3.2.1.21}$$

$$Probabilty(P(E11)) = \frac{1}{36} = 0.027 \tag{1.3.2.1.22}$$

a) The argument mentioned by the student is incorrect.
In the question the event of measurement is sum of possible outcomes of rolling two dice. Here, the probability of occurence of each outcome is not equal.
The different values of probabilities are mentioned in the above solution.
The argument can be supported by the figure **??**
The python code for the distribution of data,

```
prob/codes/prob3_excel.py
```

shows the random distribution of data.
The Distribution of data is given below
Probability mass function(P(X=k))=

$$\begin{Bmatrix} \frac{k-1}{36} & for x < 8 \\ \frac{13-k}{36} & for x => 8 \end{Bmatrix} \tag{1.3.2.1.23}$$

| Event | Value |
|---|---|
| 2 | 1/36 |
| 3 | 0.055556 |
| 4 | 0.083333 |
| 5 | 0.1111111 |
| 6 | 0.138889 |
| 7 | 0.166667 |
| 8 | 5/36 |
| 9 | 0.1111111 |
| 10 | 0.083333 |
| 11 | 0.055556 |
| 12 | 1/36 |

TABLE 1.3.2.1: Output Values

### *1.4 Exercise 4*

*1.4.1 Problem:* A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result i.e., three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.

*1.4.2 Solution:*

1. In the given question,
The sample size = Possible number of tosses=8

$$\left(\begin{Bmatrix}HHH\end{Bmatrix} \begin{Bmatrix}TTT\end{Bmatrix} \begin{Bmatrix}HHT\end{Bmatrix} \begin{Bmatrix}HTT\end{Bmatrix} \begin{Bmatrix}HTH\end{Bmatrix} \begin{Bmatrix}TTH\end{Bmatrix}\begin{Bmatrix}THT\end{Bmatrix} \begin{Bmatrix}THH\end{Bmatrix}\right) \tag{1.4.2.1.1}$$

Favourable outcome =Other than three Heads (or) Tails=6

$$\left(\begin{Bmatrix}HHT\end{Bmatrix} \begin{Bmatrix}HTT\end{Bmatrix} \begin{Bmatrix}HTH\end{Bmatrix} \begin{Bmatrix}TTH\end{Bmatrix}\begin{Bmatrix}THT\end{Bmatrix} \begin{Bmatrix}THH\end{Bmatrix}\right) \tag{1.4.2.1.2}$$

Probabilty(P) that Hanif will loose the game= $\frac{6}{8}$
$\therefore$ P = 0.75
The python code for the distribution of data,

```
prob/codes/prob4.py
```

shows the Bernouli distribution of data.
The Bernoulli Distribution of data is given below
Probability mass function(P(X))= $p^x(1-p)^{1-x}$

$$P(X = 0) = 1 - p \tag{1.4.2.1.3}$$
$$P(X = 1) = p \tag{1.4.2.1.4}$$

where p=0.75 given by 1.4.2.1

### *1.5 Exercise 5*

*1.5.1 Problem:* A die is thrown twice. What is the probability that

(i) 5 will not come up either time?
(ii) 5 will come up at least once?
Hint : Throwing a die twice and throwing two dice simultaneously are treated as the same experiment

*1.5.2 Solution:*

1. In the given question,
   a) The sample size = Total number of possibilities(S)=
   $$\begin{pmatrix} \{1\ 1\} & \{1\ 2\} & \{1\ 3\} & \{1\ 4\} & \{1\ 5\} & \{1\ 6\} \\ \{2\ 1\} & \{2\ 2\} & \{2\ 3\} & \{2\ 4\} & \{2\ 5\} & \{2\ 6\} \\ \{3\ 1\} & \{3\ 2\} & \{3\ 3\} & \{3\ 4\} & \{3\ 5\} & \{3\ 6\} \\ \{4\ 1\} & \{4\ 2\} & \{4\ 3\} & \{4\ 4\} & \{4\ 5\} & \{4\ 6\} \\ \{5\ 1\} & \{5\ 2\} & \{5\ 3\} & \{5\ 4\} & \{5\ 5\} & \{5\ 6\} \\ \{6\ 1\} & \{6\ 2\} & \{6\ 3\} & \{6\ 4\} & \{6\ 5\} & \{6\ 6\} \end{pmatrix}$$
   S=6x6=36
   Favourable outcome for no five in either of the dice (E1)=
   $$\begin{pmatrix} \{1\ 1\} & \{1\ 2\} & \{1\ 3\} & \{1\ 4\} & \{1\ 6\} \\ \{2\ 1\} & \{2\ 2\} & \{2\ 3\} & \{2\ 4\} & \{2\ 6\} \\ \{3\ 1\} & \{3\ 2\} & \{3\ 3\} & \{3\ 4\} & \{3\ 6\} \\ \{4\ 1\} & \{4\ 2\} & \{4\ 3\} & \{4\ 4\} & \{4\ 6\} \\ \{6\ 1\} & \{6\ 2\} & \{6\ 3\} & \{6\ 4\} & \{6\ 6\} \end{pmatrix}$$
   $$E1 = 25 \tag{1.5.2.1.1}$$
   $$Probabilty(P(E1)) = \frac{25}{36} = 0.694 \tag{1.5.2.1.2}$$
   a) The sample size = Total number of possibilities(S)=
   $$\begin{pmatrix} \{1\ 1\} & \{1\ 2\} & \{1\ 3\} & \{1\ 4\} & \{1\ 5\} & \{1\ 6\} \\ \{2\ 1\} & \{2\ 2\} & \{2\ 3\} & \{2\ 4\} & \{2\ 5\} & \{2\ 6\} \\ \{3\ 1\} & \{3\ 2\} & \{3\ 3\} & \{3\ 4\} & \{3\ 5\} & \{3\ 6\} \\ \{4\ 1\} & \{4\ 2\} & \{4\ 3\} & \{4\ 4\} & \{4\ 5\} & \{4\ 6\} \\ \{5\ 1\} & \{5\ 2\} & \{5\ 3\} & \{5\ 4\} & \{5\ 5\} & \{5\ 6\} \\ \{6\ 1\} & \{6\ 2\} & \{6\ 3\} & \{6\ 4\} & \{6\ 5\} & \{6\ 6\} \end{pmatrix}$$
   S=6x6=36
   Favourable outcome for atleast one five in either of the dice (E2)=
   $$\begin{pmatrix} \{1\ 5\} & \{2\ 5\} & \{3\ 5\} & \{4\ 5\} & \{5\ 1\} & \{5\ 2\} \\ \{5\ 3\} & \{5\ 4\} & \{5\ 5\} & \{5\ 6\} & \{6\ 5\} & \end{pmatrix}$$
   $$E1 = 11 \tag{1.5.2.1.3}$$
   $$Probabilty(P(E2)) = \frac{11}{36} = 0.305 \tag{1.5.2.1.4}$$

   The python code for the distribution.

   ```
   prob/codes/prob5.py
   ```

   shows the Bernouli distribution of data.
   The Bernoulli Distribution of data is given below
   Probability mass function(P(X))=$p^x (1-p)^{1-x}$
   $$P(X = 0) = 1 - p \tag{1.5.2.1.5}$$
   $$P(X = 1) = p \tag{1.5.2.1.6}$$
   where p=0.694 for solution 1.10.2.1.1 and p=0.305 for solution 1.5.2.1a

## 1.6 Exercise 6

*1.6.1 Problem:* Which of the following arguments are correct and which are not correct? Give reasons for your answer.
(i) If two coins are tossed simultaneously there are three possible outcomes—two heads, two tails or one of each. Therefore, for each of these outcomes, the probability is $\frac{1}{3}$
(ii) If a die is thrown, there are two possible outcomes—an odd number or an even number. Therefore, the probability of getting an odd number is $\frac{1}{2}$

*1.6.2 Solution i):*

1. In the given question,
   The sample size = Possible number of tosses=4
   $$\begin{pmatrix} \{HH\} & \{TT\} & \{HT\} & \{TH\} \end{pmatrix} \tag{1.6.2.1.1}$$
   Favourable outcome =Either one of them=1
   Probability of happening of either one of the event is given by
   $$P = \frac{1}{4} \tag{1.6.2.1.2}$$
   ∴ The argument given is incorrect, as the sample size being 4. The python code for the distribution of data,

   ```
   prob/codes/prob6_a.py
   ```

which shows the comparision between theory and simulation.

*1.6.3 Solution ii):*

1. In the given question,
   The sample size = Total number of possibilities(S)=6

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix} \tag{1.6.3.1.1}$$

   Event size= Odd number =3

$$\begin{pmatrix} 1 & 3 & 5 \end{pmatrix} \tag{1.6.3.1.2}$$

   Probability for this event is = $\frac{1}{2}$
   The python code for the distribution of data,

```
prob/codes/prob6_b.py
```

   This shows the diagrametic representation of dice with the live update of probability with the role of dice.

### *1.7 Exercise 7*

*1.7.1 Problem:* Two customers Shyam and Ekta are visiting a particular shop in the same week (Tuesday to Saturday). Each is equally likely to visit the shop on any day as on another day. What is the probability that both will visit the shop on
(i) the same day?
(ii) consecutive days?
(iii) different days?

*1.7.2 Solution:*

1. In the given question,
   a) The sample size = Total number of possibilities(S)=25
      The possibilities are shown in the below table 1.7.2.1 Event size=Both same day=5

| Possibilities | |
|---|---|
| Shyam | Ekta |
| Tu | Tu,W,Th,F,Sa |
| W | Tu,W,Th,F,Sa |
| Th | Tu,W,Th,F,Sa |
| F | Tu,W,Th,F,Sa |
| Sa | Tu,W,Th,F,Sa |

TABLE 1.7.2.1: Input Values

Possibilities are given in table 1.7.2.1 Probability =

$$P = \frac{1}{5} \tag{1.7.2.1.1}$$

| Possibilities | |
|---|---|
| Shyam | Ekta |
| Tu | Tu |
| W | W |
| Th | Th |
| F | F |
| Sa | Sa |

TABLE 1.7.2.1: Event Values

a) Event size = On consequitive days=8
   Possibilities are given in the table 1.7.2.1 Probability =

| Possibilities | |
|---|---|
| Shyam | Ekta |
| Tu | W |
| W | Tu,Th |
| Th | W,F |
| F | Th,Sa |
| Sa | F |

TABLE 1.7.2.1: Event Values

$$P = \frac{8}{25} \tag{1.7.2.1.2}$$

a) Event size= On different days=20
   Possibilities are given in the table 1.7.2.1 Probability =

| Possibilities | |
|---|---|
| Shyam | Ekta |
| Tu | W,Th,F,Sa |
| W | Tu,Th,F,Sa |
| Th | Tu,W,F,Sa |
| F | Tu,W,Th,Sa |
| Sa | Tu,W,Th,F |

TABLE 1.7.2.1: Event Values

$$P = \frac{4}{5} \tag{1.7.2.1.3}$$

### *1.8 Exercise 8*

*1.8.1 Problem:*

1. A die is numbered in such a way that its faces show the numbers 1, 2, 2, 3, 3, 6. It is thrown

two times and the total score in two throws is noted. Complete the following table which gives a few values of the total score on the two throws: What is the probability that the total score is (i) even? (ii) 6? (iii) at least 6?

*1.8.2 Solution:*

1. In the given question,
   The total number of possibilities=36
   The table 1.7.2.1 shows the possibilities
   a) Event size= No. of even numbers= 18
      Probability=
      $$P = \frac{1}{2} \tag{1.8.2.1.1}$$
   a) Event size= No. of six=4
      Probability=
      $$P = \frac{1}{9} \tag{1.8.2.1.2}$$
   a) Event size= Atleast six=15
      Probability=
      $$P = \frac{5}{12} \tag{1.8.2.1.3}$$
      The python code for the calculation and completion of the excel file is at

```
prob/codes/prob8.py
```

### *1.9 Exercise 9*

*1.9.1 Problem:* A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball, determine the number of blue balls in the bag.

*1.9.2 Solution:*

1. The number of red balls=5
   The number of blue balls= 2×5=10

### *1.10 Exercise 10*

*1.10.1 Problem:* A box contains 12 balls out of which x are black. If one ball is drawn at random from the box,what is the probability that it will be a black ball?
If 6 more black balls are put in the box, the probability of drawing a black ball is now double of what it was before. Find x.

*1.10.2 Solution:*

1. In the given question
   Sample=total no. of balls=12
   Probability of black ball=
   $$= \frac{x}{12} \tag{1.10.2.1.1}$$
   New sample=18
   $$\frac{x+6}{18} = \frac{2x}{12} \tag{1.10.2.1.2}$$
   From above x=3
   By substituting in 1.10.2.1.1 the probability is $\frac{1}{4}$