Balancing a Chemical Equation
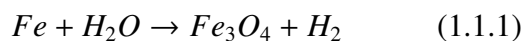
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Abstract—**This manual shows how to balance chemical equations using matrices.**

Download python codes using

1 CHEMISTRY

1. Express the problem of balancing the following chemical equation as a matrix equation.

$$Fe + H_2O \rightarrow Fe_3O_4 + H_2 \qquad (1.1.1)$$

Solution: Let the balanced version of (1.1.1) be

$$x_1 Fe + x_2 H_2O \rightarrow x_3 Fe_3O_4 + x_4 H_2 \qquad (1.1.2)$$

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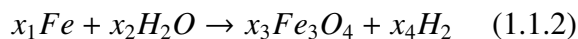
which results in the following equations

$$(x_1 - 3x_3) Fe = 0$$
$$(2x_2 - 2x_4) H = 0 \qquad (1.1.3)$$
$$(x_2 - 4x_3) O = 0$$

which can be expressed as

$$x_1 + 0.x_2 - 3x_3 + 0.x_4 = 0$$
$$0.x_1 + 2x_2 + 0.x_3 - 2x_4 = 0 \qquad (1.1.4)$$
$$0.x_1 + x_2 - 4x_3 + 0.x_4 = 0$$

resulting in the matrix equation

$$\begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 2 & 0 & -2 \\ 0 & 1 & -4 & 0 \end{pmatrix} \mathbf{x} = \mathbf{0} \qquad (1.1.5)$$

where

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \qquad (1.1.6)$$

2. Solve (1.1.2) by row reducing the matrix in (1.1.5).
   Solution: (1.1.5) can be row reduced as fol-

lows

$$\begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 2 & 0 & -2 \\ 0 & 1 & -4 & 0 \end{pmatrix} \xrightarrow{R_2 \leftarrow \frac{R_2}{2}} \begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & -4 & 0 \end{pmatrix}$$
(1.2.1)

$$\xrightarrow{R_3 \leftarrow R_3 - R_2} \begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -4 & 1 \end{pmatrix}$$
(1.2.2)

$$\xrightarrow{R_1 \leftarrow 4R_1 - 3R_3} \begin{pmatrix} 4 & 0 & 0 & -3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -4 & 1 \end{pmatrix}$$
(1.2.3)

$$\xrightarrow[R_3 \leftarrow -\frac{1}{4}R_3]{R_1 \leftarrow \frac{1}{4}} \begin{pmatrix} 1 & 0 & 0 & -\frac{3}{4} \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -\frac{1}{4} \end{pmatrix}$$
(1.2.4)
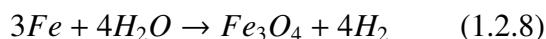
Thus,

$$x_1 = \frac{3}{4}x_4, x_2 = x_4, x_3 = \frac{1}{4}x_4 \quad (1.2.5)$$

(1.2.6)

$$\implies \mathbf{x} = x_4 \begin{pmatrix} \frac{3}{4} \\ 1 \\ \frac{1}{4} \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 1 \\ 4 \end{pmatrix} \quad (1.2.7)$$

upon substituting $x_4 = 4$. (1.1.2) then becomes

$$3Fe + 4H_2O \rightarrow Fe_3O_4 + 4H_2 \quad (1.2.8)$$

3. Verify your answer through a python code.
   Solution: Execute

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codes/chembal.py
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2 MATHEMATICS

1. Find the equation of the plane P that contains the point $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and is perpendicular to each of the planes

$$P_1 : \begin{pmatrix} 2 & 3 & -2 \end{pmatrix} \mathbf{x} = 5 \quad (2.1.1)$$

$$P_2 : \begin{pmatrix} 1 & 2 & -3 \end{pmatrix} \mathbf{x} = 8 \quad (2.1.2)$$

From (2.1.1), the normals to P_1, P_2 are

$$\mathbf{n_1} = \begin{pmatrix} 2 & 3 & -2 \end{pmatrix}$$
$$\mathbf{n_2} = \begin{pmatrix} 1 & 2 & -3 \end{pmatrix} \quad (2.1.3)$$

$\because P \perp P_1, P \perp P_2$, if $\mathbf{n}$ be the normal to P, $\mathbf{n} \perp \mathbf{n_1}, \mathbf{n} \perp \mathbf{n_2}$, which can be expressed using (2.1.3) as

$$\begin{pmatrix} 2 & 3 & -2 \\ 1 & 2 & -3 \end{pmatrix} \mathbf{n} = 0 \quad (2.1.4)$$

Obtain $\mathbf{n}$ using row reduction.
2. Verify your answer through a python code.
3. Verify that $\mathbf{n} = \mathbf{n_1} \times \mathbf{n_2}$.

3 PHYSICS

1. A force of $\mathbf{F} = 7\hat{i} + 3\hat{j} - 5\hat{k}$ acts on a particle whose position vector is $\mathbf{r} = \hat{i} - \hat{j} + \hat{k}$. Find the torque about the origin given by $\mathbf{F} \times \mathbf{r}$ using a matrix equation.
2. Verify your answer using python.