1.

$$\underline{O} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \quad \underline{B} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$$\frac{\underline{A}+\underline{B}}{2} = \underline{O}$$

2) $\underline{A} = 2\underline{O} - \underline{B}$

$$= 2\begin{pmatrix} 2 \\ -3 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ -10 \end{pmatrix} \text{ Ans.}$$

2. a) $x^2 + 4x + k = 0$

2) $16 - 4k > 0$

2) $k < 4$

b) $3x^2 - 10x + k = 0$

$\alpha\beta = 1 \Rightarrow \frac{k}{3} = 1 \Rightarrow k = 3$

3. a) $\tan 4A = \cot(A - 2u)$

$= \tan\left(\frac{\pi}{2} - A + 2u\right)$

$\Rightarrow 4A = \frac{\pi}{2} - A + 2u$

$\Rightarrow 5A = \frac{\pi}{2} + 2u$

$\Rightarrow A = \frac{\pi}{6} + \frac{8}{5}$ Ans

b) $\ln^2 33 + \ln^2 57$

$= 1$

4. $a + b = 3k$.

a divisible, b divisible

$= 4 \times 4 - 4 = 12$

$a + b = 3k$. a, b not divisible

$a = 1, b = 2, 5, 8$

$a = 2, b = 4, 7$ $= (3 + 2 + 1)\,2$

$a = 4, b = 5$ $= 12$ Ans $= 24$

5. a) $\frac{AD}{BD} = \frac{1}{2} \Rightarrow \frac{AB}{AD} = 3$

$\frac{ar(\triangle ABC)}{ar(\triangle ADE)} = 3^2 = \underline{9}$ Ans.

6. $\sqrt{2} < \frac{p}{q} < \sqrt{3}$

$2 < \frac{p^2}{q^2} < 3.$

1, 4, 9, 16, 25, 36, 49

$3 > \frac{25}{9} > 2$

$\Rightarrow \frac{p^2}{q^2} = \frac{25}{9} \Rightarrow \frac{p}{q} = \frac{5}{3}$ Ans.

7. $1260\overline{)7244}(5$
8300
$\overline{944)1260(1}$
944
$\overline{316)944}$

$$
\begin{array}{r}
2 \\
316 \overline{)\,944} \\
632 \\
\hline
312 \overline{)\,316} \quad 1 \\
312 \\
\hline
4 \overline{)\,312} \quad 78 \\
28 \\
\hline
32 \\
32 \\
\hline
0
\end{array}
$$

HCF = 4

7. b) $2q+1$

If q is even,

$q = 2k$

$\Rightarrow 2q+1 = 4k+1.$

If q is odd, $q = 2k+1$

$\Rightarrow 2q+1 = 2(2k+1)+1$

$= 4k+3$ Ans.

8-a) $S_n = 3 + 15 + 27 + 39 \ldots$

$a = 3$

$d = 12.$

$T_n = a + (n-1)d$

$= 3 + 12(n-1)$

$= 12n - 9$

$= T_{21} + 120$

$\Rightarrow 12n - 9 = 12 \times 21 - 9 + 120$ (10)

$\Rightarrow n = 31$ Ans.

b) $S_n = 3n^2 - 4n$

$T_n = S_n - S_{n-1}$

$= 3n^2 - 4n - 3(n-1) + 4(n-1)$

$= 3n^2 - 4n - 3n + 3 + 4n - 4$

$= 3n^2 - 3n - 1$

9. $$\underline{x} = \frac{k\begin{pmatrix}1\\-3\end{pmatrix} + \begin{pmatrix}4\\5\end{pmatrix}}{k+1} = \begin{pmatrix}x\\8\end{pmatrix}$$

$$\Rightarrow x = \frac{k+4}{k+1} \qquad \frac{-3k+5}{k+1} = 0$$

$$\Rightarrow k = \frac{5}{3}, \quad x = \frac{5/3+4}{5/3+1}$$

$$\underline{x} = \begin{pmatrix}17/8\\0\end{pmatrix} \qquad = \frac{17}{8}$$

10. $$P_{succ.} = \underbrace{\left(\frac{1}{2}\right)^3}_{3\text{ heads}} + \underbrace{\left(\frac{1}{2}\right)^3}_{3\text{ tails}}$$

$$= \frac{1}{4}$$

$$P_{fail} = 1 - \frac{1}{4} = \frac{3}{4}$$

11. (i) $P_r(x=2, 3, 1) = \frac{1}{2}$ Ans.

(ii) $P_r(x=2, 3, 4, 5) = \frac{4}{6}$

$= \frac{2}{3}$ Ans

12. $cx + 3y + 3 - c = 0$

$12x + cy - c = 0.$

Inf. soln.

$\Rightarrow \frac{c}{12} = \frac{3}{c} = \frac{3-c}{-c}.$

$\Rightarrow \begin{cases} c^2 = 36, \\ c^2 - 12c + 36 = 0. \\ c^2 - 6c = 0 \end{cases}$

$\Rightarrow c = 6.$

13. $\frac{p}{q} = \sqrt{2} \Rightarrow p^2 = 2q^2.$

$p \neq kq.$

$\Rightarrow p^2 \neq k^2 q^2.$

14. $x^2 - (k+6)x + 2(2k-1) = 0$

$k+6 = \frac{1}{2} \times (2k-1)$

$\Rightarrow \underline{k = 7}$ Ans.

15. a) $x = 3(y+z)$

$x+5 = 2(y+5+z+5)$

$= 2(y+z) + 20$

$\Rightarrow x - 15 = 2(y+z)$

$\frac{x}{x-15} = \frac{3}{2}$

$\Rightarrow 2x = 3x - 45$

$\Rightarrow \underline{x = 45}$ Ans.

b) $\frac{x-2}{y} = \frac{1}{3}$

$\frac{x}{y-1} = \frac{1}{2}$

$\Rightarrow 3x - y = 6$

$2x - y = -1$

$$\begin{pmatrix} 3 & -1 & 6 \\ 2 & -1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -1 & 6 \\ 0 & -1 & -15 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 7 \\ 0 & 1 & 15 \end{pmatrix}$$

$\frac{x}{y} = \frac{7}{15}$

16. a) $\underline{A} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} \quad \underline{B} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

$$\frac{\underline{A} + \underline{B}}{2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\underline{A} - \underline{B} = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$$

$$\left[\underline{y} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right]^t \begin{pmatrix} 8 \\ -4 \end{pmatrix} = 0$$

$$\Rightarrow (-1 \quad y) \begin{pmatrix} 2 \\ -1 \end{pmatrix} = 0$$

$$\Rightarrow -2 - y = 0 \Rightarrow \underline{y = -2}.$$

Point is $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ Ans.

b) $\underline{D} = \frac{2\underline{A} + \underline{B}}{3}$.

$(\lambda - 1)\underline{D} = -k$

$\Rightarrow (\lambda - 1)3\underline{D} = -3k$

$\Rightarrow (\lambda - 1)(2\underline{A} + \underline{B}) = -3k$

$\Rightarrow 2(\lambda - 1)\underline{A} + (\lambda - 1)\underline{B} = -3k$

$\Rightarrow 2(\lambda - 1)\begin{pmatrix} 2 \\ 1 \end{pmatrix} + (\lambda - 1)\begin{pmatrix} 5 \\ -8 \end{pmatrix} = -3k$

$\Rightarrow 6 + 18 = -3k$

$\Rightarrow \underline{k = -8}$ Ans.

17. a) $(\sin\theta + \cos\theta)^2 + (\cos\theta + \sec\theta)^2$

$= 1 + 2 + 2 + \cos^2\theta + \sec^2\theta$

$= 1 + 2 + 2 + 2 + \tan^2\theta + \cot^2\theta$

$= 7 + \underline{\tan^2\theta + \cot^2\theta}$ Ans.

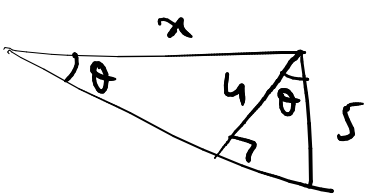
b) $(\text{Cosec}A - \cot A)(1+\cot A + \text{Cosec}A)$

$= \left(\frac{\sin A + \cos A - 1}{\sin A}\right)\left(\frac{1+\cos A+\sin A}{\cos A}\right)$

$= \frac{\sin^2 A + \cos^2 A + 2\sin A\cos A - 1}{\sin A \cos A}$

$= \frac{2}{}$

18.



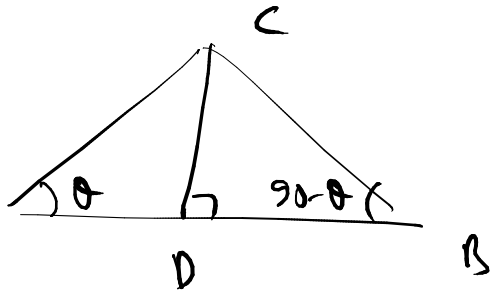
$x = 5\cos\theta$

$\cos\theta = \frac{4}{5}$ $\quad \cot\theta = \frac{4}{3}$.

$\sin\theta = \frac{3}{5}$

$x = \frac{5 \times 4}{3} = \frac{20}{3}$ Ans.

19. a)

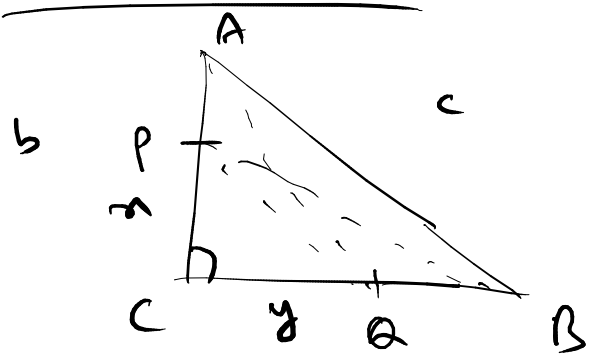


$CD = BD \cos\theta$.

$AD = CD \cos\theta$.

$\Rightarrow CD^2 = BD \cdot AD$ Ans.

b)



$AQ^2 = b^2 + y^2$.

$BP^2 = x^2 + a^2$.

$AB^2 = a^2 + b^2$

$PQ^2 = x^2 + y^2$.

$AQ^2 + BP^2$

$= x^2 + y^2 + a^2 + b^2$

$= AB^2 + PQ^2$

20. $r = 10$.

Area $= \pi r^2 - 8.6$

$= 100\pi - 48$ Ans.

21. Volume/sec $= 6 \times 1.5 \times 10^3 / hr$

volm/30min $= 6 \times 1.5 \times 10^3 \times \frac{1}{2}\ m^3$.

Area irrigated $= \frac{6 \times 1.5 \times 10^3 \times \frac{1}{2}}{8 \times 10^{-2}}\ m^2$

$= \frac{9}{16} \times 10^5\ m^2$

$= \frac{9}{16}\ km^2$ Ans

23. a) $\frac{1}{x+y} = \frac{15}{8}$.

$\frac{1}{x} + 2 = \frac{1}{y}$.

$x+y = \frac{8}{15}$.

$\frac{1}{y} - \frac{1}{x} = 2$.

$\Rightarrow x - y = 2xy$

$\therefore (x+y)^2 - (x-y)^2 = 4xy$

$\left(\frac{8}{15}\right)^2 - 4x^2y^2 = 4xy$

$\Rightarrow 4x^2y^2 + 4xy - 4\frac{(4^2)}{15^2} = 0$

$\Rightarrow x^2y^2 + xy - \left(\frac{4}{15}\right)^2 = 0$

$\Rightarrow xy = \frac{-1 + \sqrt{1 + \left(\frac{8}{15}\right)^2}}{2} = \frac{\frac{17}{15} - 1}{2}$

$= 1/15$ // 2

$$z^2 - \frac{8}{15}z + \frac{1}{15} = 0$$

$$\Rightarrow z = \frac{\frac{8}{15} \pm \sqrt{\left(\frac{8}{15}\right)^2 - \frac{4}{15}}}{2}$$

$$= \frac{\frac{8}{15} \pm \frac{2}{15}}{2}$$

$$= \frac{4 \pm 1}{15} = \frac{1}{5} \text{ or } \frac{1}{3}.$$

Hence, the time taken individually is 5 and 3.

b) $$\frac{30}{x-y} + \frac{44}{x+y} = 10.$$

$$\frac{40}{x-y} + \frac{55}{x+y} = 13.$$

$$\begin{pmatrix} 30 & 44 & 10 \\ 40 & 55 & 13 \end{pmatrix}$$

$\downarrow$

$$\begin{pmatrix} 15 & 22 & 5 \\ 40 & 55 & 13 \end{pmatrix}$$

$\downarrow$

$$\begin{pmatrix} 15 & 22 & 5 \\ 0 & 15\cdot 55 - 40\cdot 22 & 15\cdot 13 - 5\cdot 40 \end{pmatrix}$$

$$\begin{pmatrix} 15 & 22 & 5 \\ 0 & 11 & 1 \end{pmatrix}$$

$\downarrow$

$$\begin{pmatrix} 15 & 0 & 3 \\ 0 & 11 & 1 \end{pmatrix}$$

$$\frac{1}{x-y} = \frac{1}{5} \Rightarrow x-y = 5$$

$$\frac{1}{x+y} = \frac{1}{11} \quad x+y = 11$$

$$\Rightarrow x = 8,\ y = 3 \quad \text{Ans}$$

24. $S_4 = \frac{4}{2}(2a+3d) = 40$

$S_{14} = \frac{14}{2}(2a+13d) = 280$

$\Rightarrow 2a+3d = 20$

$2a+13d = 40$

$10d = 20 \Rightarrow d = 2,$

$2a = 14 \Rightarrow a = 7.$

$S_n = \frac{n}{2}[14+(n-1)2]$

$= n(2n+6)$ Ans.

25. $$\frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \frac{1}{\sec A - \tan A}$$

L.H.S. $= \dfrac{\sin A + 2\sin^2 \frac{A}{2}}{\sin A - 2\sin^2 \frac{A}{2}}$

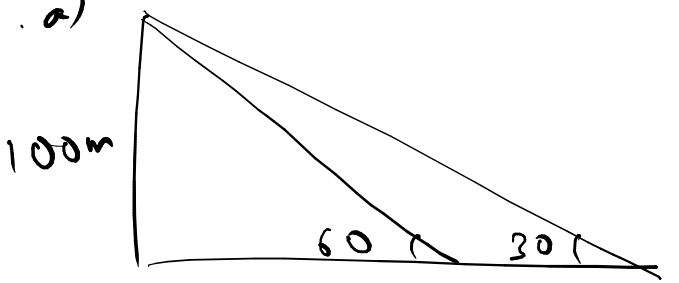
$= \dfrac{\sin \frac{A}{2} + \cos \frac{A}{2}}{\cos \frac{A}{2} - \sin \frac{A}{2}} = \dfrac{\cos\left(\frac{\pi}{4} - \frac{A}{2}\right)}{\sin\left(\frac{\pi}{4} - \frac{A}{2}\right)}$

$$\frac{1}{\sec A - \tan A} = \frac{\cos A}{1 - \sin A}$$

$$= \frac{\sin\left(\frac{\pi}{2} - A\right)}{1 - \cos\left(\frac{\pi}{2} - A\right)}$$

$$= \frac{\cos\left(\frac{\pi}{4} - \frac{A}{2}\right)}{\sin\left(\frac{\pi}{4} - \frac{A}{2}\right)}$$ Ans.

26. a)

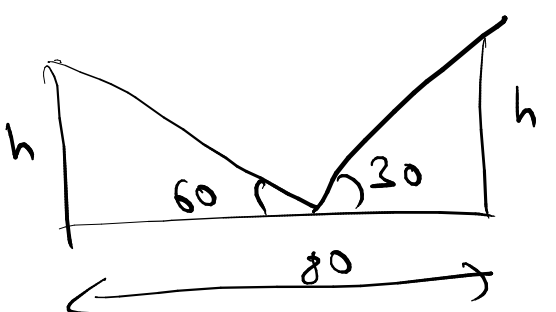


$$\frac{100(\cot 30 - \cot 60)}{2} \text{ m/min}$$

$$= 50\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$$

$$= \frac{50}{\sqrt{3}} \times 2 = \frac{100\sqrt{3}}{3} \text{ Ans.}$$

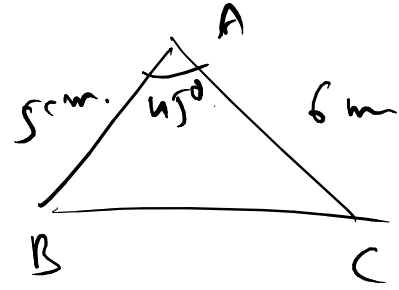
b)



$$80 = h(\cot 30 + \cot 60)$$

$$\Rightarrow h = \frac{80}{\sqrt{3} + 1/\sqrt{3}} = 20\sqrt{3} \text{ Ans.}$$

27.



28. $\frac{1}{3}\pi(r_1^2 - r_2^2)h = V$

$\frac{1}{3}\pi \times 32 \times 8\, h = V$

$\Rightarrow h = \frac{3V}{256\pi}$

$A = \pi r_1 l_1 - \pi r_2 l_2$

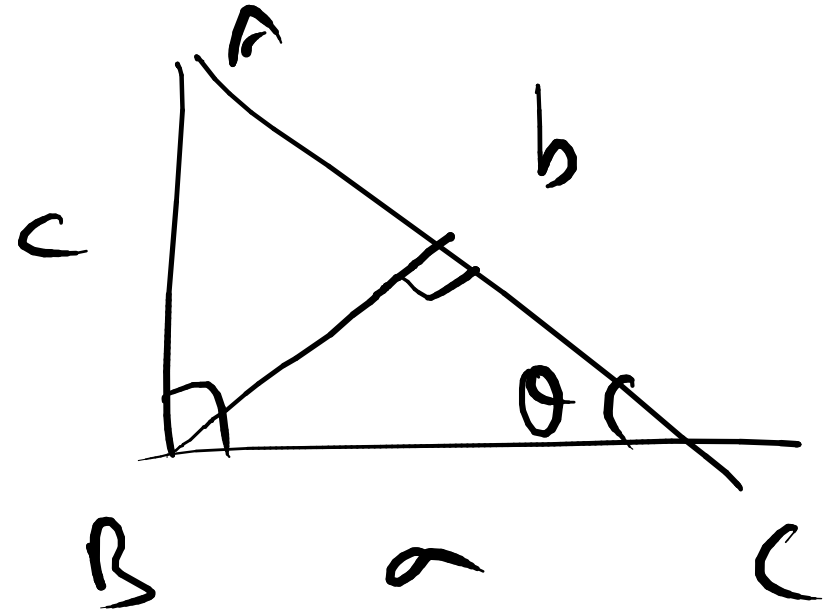
$h = (r_2 - r_1)\cot\theta$.

$\Rightarrow \cot\theta = \frac{h}{r_2 - r_1}$.

$\pi r_2 l_2 - \pi r_1 l_1 = \pi r_2^2 \operatorname{cosec}\theta - \pi r_1^2 \operatorname{cosec}\theta$

$= \pi \operatorname{cosec}\theta (r_2^2 - r_1^2)$

29.



$$b = a\cos\theta + c\sin\theta.$$

$$\Rightarrow 1 = \cos^2\theta + \sin^2\theta$$

$$\Rightarrow \underline{\underline{a^2 + c^2 = b^2}}.$$