Math Document Template

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Abstract—This is a document explaining a question about the concept of solving equations using matrices.

Download all python codes from

svn co https://github.com/chakki1234/summer -2020/trunk/linearalg/codes

and latex-tikz codes from

svn co https://github.com/chakki1234/summer -2020/trunk/linearalg/figs

1 Problem

Solve the following pair of equations:

$$(a-b \quad a+b)\mathbf{x} = a^2 - 2ab - b^2$$
$$(a+b \quad a+b)\mathbf{x} = a^2 + b^2$$
 (1.0.1)

2 Solution

2.0.1. Vector form of the given equations:

$$\begin{pmatrix} a - b & a + b \\ a + b & a + b \end{pmatrix} \mathbf{x} = \begin{pmatrix} a^2 - 2ab - b^2 \\ a^2 + b^2 \end{pmatrix}$$
 (2.0.1.1)

2.0.2. To find **x**:

$$\begin{pmatrix}
a - b & a + b & a^2 - 2ab - b^2 \\
a + b & a + b & a^2 + b^2
\end{pmatrix}$$

$$\stackrel{R_1 \leftarrow \frac{R_1}{a - b}}{\longleftrightarrow} \begin{pmatrix}
1 & \frac{a + b}{a - b} & \frac{a^2 - 2ab - b^2}{a - b} \\
1 & 1 & \frac{a^2 + b^2}{a + b}
\end{pmatrix}$$

$$\stackrel{R_2 \leftarrow R_2 - R_1}{\longleftrightarrow} \begin{pmatrix}
1 & \frac{a + b}{a - b} & \frac{a^2 - 2ab - b^2}{a + b} \\
0 & \frac{-2b}{a - b} & \frac{4ab^2}{a^2 - b^2}
\end{pmatrix}$$

$$(2.0.2.1)$$

$$\stackrel{R_2 \leftarrow R_2 - R_1}{\longleftrightarrow} \begin{pmatrix} 1 & \frac{a+b}{a-b} & \frac{a^2 - 2ab - b^2}{a-b} \\ 0 & \frac{-2b}{a-b} & \frac{4ab^2}{a^2 - b^2} \end{pmatrix}$$
(2.0.2.2)

$$\begin{array}{cccc}
 & \left(0 & \frac{-2b}{a-b} & \frac{4ab^2}{a^2-b^2}\right) \\
 & \stackrel{R_2 \leftarrow \frac{-(a-b)R_2}{2b}}{\longrightarrow} & \left(1 & \frac{a+b}{a-b} & \frac{a^2-2ab-b^2}{a-b} \\
0 & 1 & \frac{-2ab}{a+b}
\end{array}\right) (2.0.2.3)$$

$$\stackrel{R_1 \leftarrow R_1 - R_2}{\longleftrightarrow} \begin{pmatrix} 1 & 0 & a + b \\ 0 & 1 & \frac{-2ab}{a+b} \end{pmatrix} \qquad (2.0.2.4)$$

$$\therefore \mathbf{x} = \begin{pmatrix} a+b \\ \frac{2ab}{a-b} \end{pmatrix} \qquad (2.0.2.5)$$

The following Python code generates Fig. ??

codes/triangle ex/triangle linearalg.py