Assignment-1

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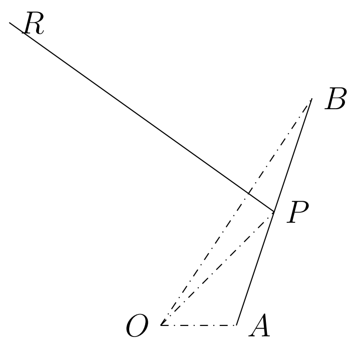
Problem Statement (Q.No.62):

A line perpendicular to the line segment joining the points (1,0) and (2,3) divides it into the ratio 1:n. Find the equation of the line.

Solution:



Given that $A = \begin{pmatrix} 1 \\ 0 \end{pmatrix} andB = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

Let P= $\begin{pmatrix} x \\ y \end{pmatrix}$ and origin O = $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$. Since the line RP intersect the line AB in 1:n ration, then $\frac{AP}{PB} = \frac{1}{n}$. PB = n.AP or OB-OP = n(OP-OA). Solving this vector equation, OP = $\dfrac{OB + n.OA}{n+1}$

Therefore, $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \dfrac{(n+2)}{(n+1)} \\ \dfrac{3}{n+1} \end{pmatrix}$

Vector equation of Line RP, $\mathbf{r} = \mathbf{p} + \lambda.\mathbf{d}$, where $\mathbf{p}$ is point on the line, λ *is constant, and* $\mathbf{d}$ *is direction vector of the line.*
Since the line segment AB and the line RP is perpendicular to each other then dot product of both the vectors will be zero.

AB.d $= 0. \implies \begin{pmatrix} 2-1 \\ 3-0 \end{pmatrix}^T . \begin{pmatrix} x \\ y \end{pmatrix} = 0 \implies x + 3y = 0$. x= 3 and y = -1.

Therefore, direction vector $\mathbf{d} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

Final vector equation of the line is $\mathbf{r} = \begin{pmatrix} \dfrac{(n+2)}{(n+1)} \\ \dfrac{3}{n+1} \end{pmatrix} + \lambda. \begin{pmatrix} 3 \\ -1 \end{pmatrix}$