

Assignment 1

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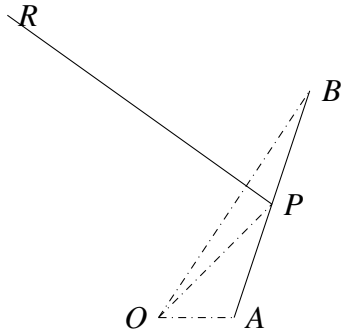
Download Latex codes from here

https://github.com/EE20RESCH14003/Assignment-1_3

1 QUESTION No. 62

A line perpendicular to the line segment joining the points (1,0) and (2,3) divides it into the ratio 1:n. Find the equation of the line.

2 SOLUTION



Given that $A = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $B = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

Let $P = \begin{pmatrix} x \\ y \end{pmatrix}$ and origin $O = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$. Since the line RP intersect the line AB in 1:n ratio, then $\frac{AP}{PB} = \frac{1}{n}$.
PB = n.AP

Vector equation of line AB is $\mathbf{r} = \mathbf{A} + \lambda \mathbf{d}$, A is point where the line passes, d is direction vector, and λ is constant.

Direction vector $\mathbf{d} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

Therefore, vector equation of line AB is

$$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \end{pmatrix}.$$

\mathbf{r} is the point on line, and the point moves on the line as λ varies. Here, the line PR divides the line AB in 1:n ratio. Let coordinate of point $P = \begin{pmatrix} x \\ y \end{pmatrix}$.

Then $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{n+1} \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{n+2}{n+1} \\ \frac{3}{n+1} \end{pmatrix}$. $P = \begin{pmatrix} \frac{n+2}{n+1} \\ \frac{3}{n+1} \end{pmatrix}$

Equation of line PR is $\mathbf{r} = \mathbf{P} + \lambda \mathbf{d}$

$$\Rightarrow \mathbf{r} = \begin{pmatrix} \frac{n+2}{n+1} & \frac{3}{n+1} \end{pmatrix} + \lambda \begin{pmatrix} d1 \\ d2 \end{pmatrix}.$$

Since both the lines AB and PR are perpendicular to each other, then dot product of direction vectors will be zero.

$$\begin{pmatrix} 1 \\ 3 \end{pmatrix}^T \cdot \begin{pmatrix} d1 \\ d2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} d1 \\ d2 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

Finally, vector equation of line PR will be

$$\mathbf{r} = \begin{pmatrix} \frac{n+2}{n+1} \\ \frac{3}{n+1} \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \end{pmatrix}.$$