Assignment 1

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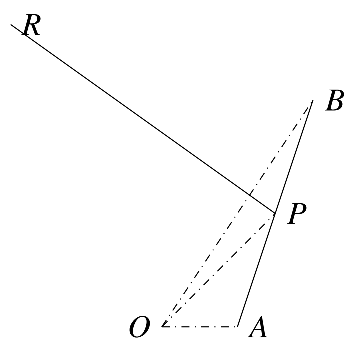
https://github.com/EE20RESCH14003/Assignment−1_4

1 Question No. 62

A line perpendicular to the line segment joining the points (1,0) and (2,3) divides it into the ratio 1:n. Find the equation of the line.

1.1 Solution



Given that

$$A = \begin{pmatrix} 1 \\ 0 \end{pmatrix} and\ B = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{1.1.1}$$

Let

$$P = \begin{pmatrix} x \\ y \end{pmatrix} and\ origin\ O = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{1.1.2}$$

Since the line RP intersect the line AB in 1:n ration, then

$$\frac{AP}{PB} = \frac{1}{n} \tag{1.1.3}$$

$$\implies PB = nAP \tag{1.1.4}$$

Vector equation of line AB is

$$\mathbf{r} = \mathbf{A} + \lambda \mathbf{d} \tag{1.1.5}$$

A is point where the line passes, d is direction vector, and λ is constant.

Direction vector

$$\mathbf{d} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{1.1.6}$$

Therefore, vector equation of line AB is

$$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{1.1.7}$$

$\mathbf{r}$ is the point on line, and the point moves on the line as λ varies. Here, the line PR divides the line AB in 1:n ratio.

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{n+1} \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{n+2}{n+1} \\ \frac{3}{n+1} \end{pmatrix} \tag{1.1.8}$$

Therefore, coordinate of point $P = \begin{pmatrix} \frac{n+2}{n+1} \\ \frac{3}{n+1} \end{pmatrix}$

Equation of line PR is

$$\mathbf{r} = \mathbf{P} + \lambda \mathbf{d} \tag{1.1.9}$$

$$\implies \mathbf{r} = \begin{pmatrix} \frac{n+2}{n+1} & \frac{3}{n+1} \end{pmatrix} + \lambda \begin{pmatrix} d1 \\ d2 \end{pmatrix} \tag{1.1.10}$$

Since both the lines AB and PR are perpendicular to each other, then dot product of direction vectors will be zero.

$$\begin{pmatrix} 1 \\ 3 \end{pmatrix}^T \begin{pmatrix} d1 \\ d2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{1.1.11}$$

$$\implies \begin{pmatrix} d1 \\ d2 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \tag{1.1.12}$$

Finally, vector equation of line PR will be

$$\mathbf{r} = \begin{pmatrix} \frac{n+2}{n+1} \\ \frac{3}{n+1} \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \end{pmatrix} \tag{1.1.13}$$