Assignment 1 (part2)

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https://github.com/EE20RESCH14003/Assignment−1(part2)_2

1 Matrix 3.9

Question No. 73:

Find X so that $X\begin{pmatrix}1 & 2 & 3\\4 & 5 & 6\end{pmatrix} = \begin{pmatrix}-7 & -8 & -9\\2 & 4 & 6\end{pmatrix}$

1.1 Solution

Given that

$$\mathbf{X}\begin{pmatrix}1 & 2 & 3\\4 & 5 & 6\end{pmatrix} = \begin{pmatrix}-7 & -8 & -9\\2 & 4 & 6\end{pmatrix} \tag{1.1.1}$$

Equation (1.1.1) can be written as

$$\begin{pmatrix}1 & 4\\2 & 5\\3 & 6\end{pmatrix}\mathbf{X^T} = \begin{pmatrix}-7 & 2\\-8 & 4\\-9 & 6\end{pmatrix} \tag{1.1.2}$$

Equation (1.1.2) cab be represented as

$$A\mathbf{x} = \mathbf{b} \tag{1.1.3}$$

where $A = \begin{pmatrix}1 & 4\\2 & 5\\3 & 6\end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix}b1\\b2\\b3\end{pmatrix}$

$b1 = \begin{pmatrix}-7 & 2\end{pmatrix}, b2 = \begin{pmatrix}-8 & 4\end{pmatrix}$, and $b3 = \begin{pmatrix}-9 & 6\end{pmatrix}$

The set of least square solutions of $A\mathbf{x} = \mathbf{b}$ coincides with the non empty set of solutions of equations $A^TA\mathbf{x} = A^T\mathbf{b}$.

$$\hat{x} = (A^TA)^{-1}A^T\mathbf{b} \tag{1.1.4}$$

$$A^TA = \begin{pmatrix}1 & 2 & 3\\4 & 5 & 6\end{pmatrix}\begin{pmatrix}1 & 4\\2 & 5\\3 & 6\end{pmatrix} = \begin{pmatrix}14 & 32\\32 & 77\end{pmatrix}$$

$$A^T\mathbf{b} = \begin{pmatrix}1 & 2 & 3\\4 & 5 & 6\end{pmatrix}\begin{pmatrix}b1\\b2\\b3\end{pmatrix}\begin{pmatrix}b1 + 2b2 + 3b3\\4b1 + 5b2 + 6b3\end{pmatrix}$$

$$A^T\mathbf{b} = \begin{pmatrix}-50 & 28\\-122 & 64\end{pmatrix}$$

$$(A^TA)^{-1} = \frac{1}{54}\begin{pmatrix}77 & -32\\-32 & 64\end{pmatrix}$$

Using equation(1.1.4)

$$\hat{x} = \frac{1}{54}\begin{pmatrix}77 & -32\\-32 & 14\end{pmatrix}\begin{pmatrix}-50 & 28\\-122 & 64\end{pmatrix}$$

$$\hat{x} = \frac{1}{54}\begin{pmatrix}54 & 108\\-108 & 0\end{pmatrix}$$

$$\hat{x} = \begin{pmatrix}1 & 2\\-2 & 0\end{pmatrix}$$

$$\mathbf{X} = \hat{x}^T = \begin{pmatrix}1 & -2\\2 & 0\end{pmatrix}$$