

Assignment 1 (part2)

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$$\mathbf{A}^T \mathbf{A} = \begin{pmatrix} 14 & 32 \\ 32 & 77 \end{pmatrix}$$

$$\mathbf{A}^T \mathbf{b} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} -7 & 2 \\ -8 & 4 \\ -9 & 6 \end{pmatrix}$$

$$\mathbf{A}^T \mathbf{b} = \begin{pmatrix} -50 & 28 \\ -122 & 64 \end{pmatrix}$$

$$(\mathbf{A}^T \mathbf{A})^{-1} = \frac{1}{54} \begin{pmatrix} 77 & -32 \\ -32 & 64 \end{pmatrix}$$

1 MATRIX 3.9

Question No. 73:

Find X so that $X \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$

1.1 Solution

Given that

$$\mathbf{X} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix} \quad (1.1.1)$$

Equation (1.1.1) can be written as

$$\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix} \mathbf{X}^T = \begin{pmatrix} -7 & 2 \\ -8 & 4 \\ -9 & 6 \end{pmatrix} \quad (1.1.2)$$

Equation (1.1.2) can be represented as

$$\mathbf{A} \mathbf{x} = \mathbf{b} \quad (1.1.3)$$

where $\mathbf{A} = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -7 & 2 \\ -8 & 4 \\ -9 & 6 \end{pmatrix}$

The set of least square solutions of $\mathbf{A} \mathbf{x} = \mathbf{b}$ coincides with the non empty set of solutions of equations $\mathbf{A}^T \mathbf{A} \mathbf{x} = \mathbf{A}^T \mathbf{b}$.

$$\hat{\mathbf{x}} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{b} \quad (1.1.4)$$

$$\mathbf{A}^T \mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$$

Using equation(1.1.4)

$$\begin{aligned} \hat{\mathbf{x}} &= \frac{1}{54} \begin{pmatrix} 77 & -32 \\ -32 & 14 \end{pmatrix} \begin{pmatrix} -50 & 28 \\ -122 & 64 \end{pmatrix} \\ &= \frac{1}{54} \begin{pmatrix} 54 & 108 \\ -108 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 2 \\ -2 & 0 \end{pmatrix} \end{aligned}$$

$$\mathbf{X} = \hat{\mathbf{x}}^T = \begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix}$$