

# 10.4.3.3.2

EE24BTECH11012 - Bhavanisankar G S

## QUESTION :

Find the roots of the quadratic equation,  $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$ .

## SOLUTION :

### 1) QUADRATIC FORMULA :

Consider an equation,

$$ax^2 + bx + c = 0 \quad (1.1)$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \quad (1.2)$$

$$x^2 + 2\frac{b}{2a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a} = 0 \quad (1.3)$$

$$\left(x + \frac{b}{2a}\right)^2 + \left(\frac{4ac - b^2}{4a^2}\right) = 0 \quad (1.4)$$

$$\left(x + \frac{b}{2a}\right) = \frac{\sqrt{b^2 - 4ac}}{2a} \quad (1.5)$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (1.6)$$

which is the quadratic formula.

Given equation,

$$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7 \quad (1.7)$$

$$\frac{-11}{(x+4)x-7} = \frac{11}{30} \quad (1.8)$$

$$\Rightarrow x^2 - 3x + 2 = 0 \quad (1.9)$$

$$\Rightarrow x = 1 \quad (1.10)$$

$$x = 2 \quad (1.11)$$

### 2) EIGENVALUE APPROACH :

Consider the equation, (1.2). It can be rearranged as

$$\lambda^2 + \frac{b}{a}\lambda + \frac{c}{a} = 0 \quad (2.1)$$

$$\lambda\left(\lambda + \frac{b}{a}\right) + \frac{c}{a} = 0 \quad (2.2)$$

$$-\lambda\left(-\lambda - \frac{b}{a}\right) - (-1)\frac{c}{a} = 0 \quad (2.3)$$

This can be considered equivalent to the determinant of the matrix,

$$\begin{pmatrix} -\lambda & 1 \\ -\frac{c}{a} & -\frac{b}{a} - \lambda \end{pmatrix} \quad (2.4)$$

Clearly, it can be seen that the eigenvalues of the matrix

$$\begin{pmatrix} 0 & 1 \\ -\frac{c}{a} & -\frac{b}{a} \end{pmatrix} \quad (2.5)$$

are the roots of the required quadratic equation. This matrix, (2.5) is called the **Companion matrix (C)**.

For the given question,

$$\mathbf{C} = \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} \quad (2.6)$$

It can be seen that eigenvalues are 1 and 2.

### 3) **Newton-Raphson method :**

We have,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad (3.1)$$

Iterating and updating the value of  $x_n$ , we can obtain the roots of the quadratic equation.

The roots found using this method taking the initial guesses as 10 and 0 are 2.000000000905422 and 1.0000000022055868 respectively.

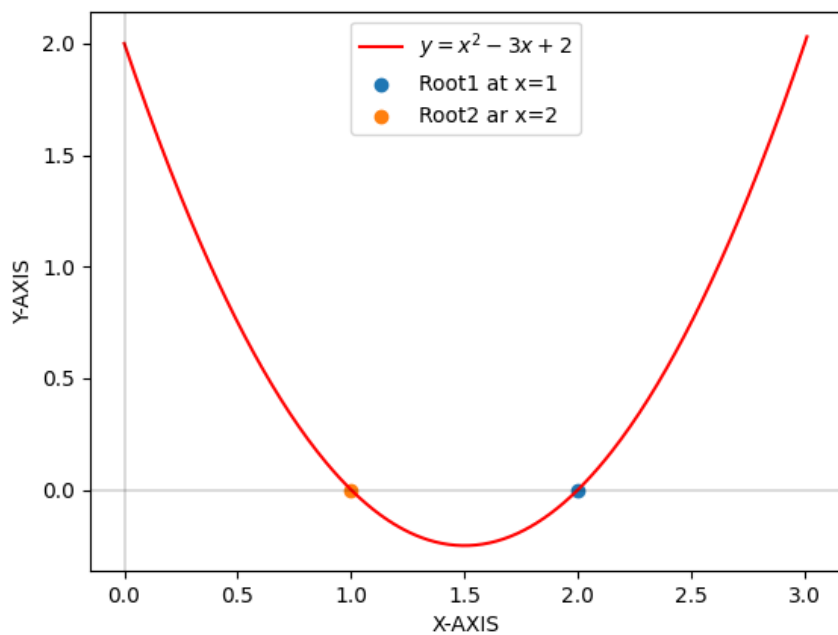


Fig. 3.1: Plot of the given question.