

# 11.16.3.8.5

EE24BTECH11012 - Bhavanisankar G S

## QUESTION :

Three coins are tossed once. Find the probability of getting no head.

## SOLUTION :

Variable name	Description
<b>S</b>	Sample space
<b>X</b>	Random variable
$p, q$	Toss corresponding to head/tail
$F_X(x)$	Cumulative distribution function ( CDF )
$p_X(x)$	Probability Mass function ( PMF )

Considering all the outcomes as equally likely, we have

$$p = \frac{1}{2} \quad (0.1)$$

$$q = \frac{1}{2} \quad (0.2)$$

For the given question, let **X** denote the number of heads. The sample space corresponding to the given scenario is tabulated below.

Event	Sample space
$p_X(0)$	$\{TTT\}$
$p_X(1)$	$\{TTH, THT, HTT\}$
$p_X(2)$	$\{HHT, HTH, THH\}$
$p_X(3)$	$\{HHH\}$

The **Probability Mass Function** is given by-

$$p_X(k) = {}^nC_k p^k q^{n-k} \quad (0.3)$$

$$p_X(k) = {}^3C_k \left(\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^{n-k} \quad (0.4)$$

$$\Rightarrow p_X(k) = \begin{cases} \frac{{}^3C_k}{8} & k = \{0, 1, 2, 3\} \\ 0 & \text{otherwise} \end{cases}$$

$$\Rightarrow p_X(k) = \begin{cases} \frac{1}{8} & k = 0 \\ \frac{3}{8} & k = 1 \\ \frac{3}{8} & k = 2 \\ \frac{1}{8} & k = 3 \end{cases}$$

The corresponding **Cumulative Distribution Function** can then be written as -

$$F_X(k) = Pr(\mathbf{X} \leq k) \quad (0.5)$$

$$= \sum_{k=0}^k p_X(x) \quad (0.6)$$

$$\Rightarrow F_X(k) = Pr(\mathbf{X} \leq k) = \begin{cases} 0 & x < 0 \\ \frac{1}{8} & 0 \leq x < 1 \\ \frac{1}{2} & 1 \leq x < 2 \\ \frac{7}{8} & 2 \leq x < 3 \\ 1 & x \geq 3 \end{cases}$$

$$F_X(0) = P(\mathbf{X} \leq 0) \quad (0.7)$$

$$= \frac{1}{8} \quad (0.8)$$



