Absolute Maximum and Minimum of $f(x) = x^3$ on $[-2, 2]$

EE24BTECH11012 - Bhavanisankar G S

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Question

Find the absolute maximum and minimum value of the function $f(x) = x^3$ in the interval $[-2, 2]$.

Theoretical Solution

Given function: $y(x) = x^3$

First derivative: $y'(x) = 3x^2$

Second derivative: $y''(x) = 6x$

Critical points: Solve $y'(x) = 0 \Rightarrow 3x^2 = 0 \Rightarrow x = 0$

Edge values: $y(-2) = -8$, $y(0) = 0$, $y(2) = 8$

Absolute Maximum: 8 at $x = 2$
Absolute Minimum: -8 at $x = -2$

Computational Solution

Using Gradient Ascent and Descent:

Gradient Ascent: $x_{n+1} = x_n + \alpha f'(x_n) = x_n + 3\alpha x_n^2$

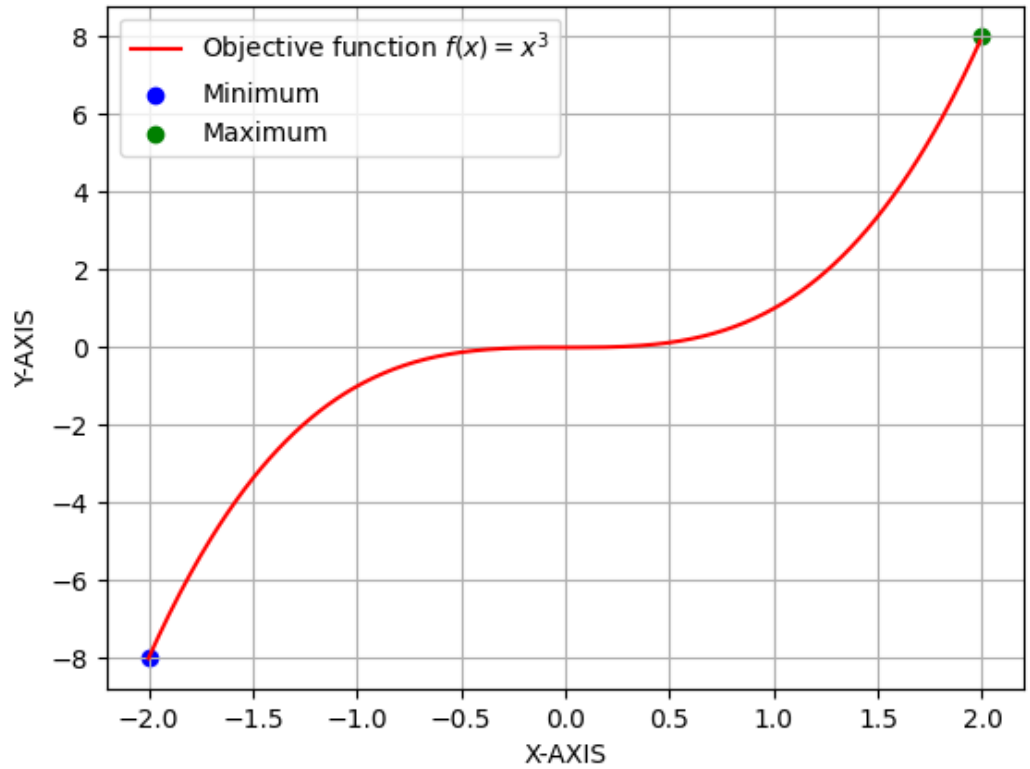
Gradient Descent: $x_{n+1} = x_n - \alpha f'(x_n) = x_n - 3\alpha x_n^2$

With $\alpha = 0.01$:

$x_{\text{min}} = -2$, $y_{\text{min}} = -8$

$x_{\text{max}} = 2$, $y_{\text{max}} = 8$

Graphical Representation



Plot of the function $f(x) = x^3$ *on* $[-2, 2]$