

## 6.6.2

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### QUESTION :

Show that the function given by  $f(x) = \frac{\log(x)}{x}$  has maximum at  $x = e$ .

### SOLUTION :

#### Theoretical solution :

Given function,

$$y(x) = \frac{\log(x)}{x} \quad (0.1)$$

$$\Rightarrow y'(x) = \frac{1 - \log(x)}{x^2} \quad (0.2)$$

$$\Rightarrow y''(x) = \frac{-2}{x^3} - \frac{1}{x^3} + \frac{2 \log(x)}{x^3} \quad (0.3)$$

$$y''(x) = \frac{2 \log(x) - 3}{x^3} \quad (0.4)$$

To find the critical points, we do

$$y'(x) = 0 \quad (0.5)$$

$$\frac{1 - \log(x)}{x^2} = 0 \quad (0.6)$$

$$\log(x) = 1 \quad (0.7)$$

$$x = e \quad (0.8)$$

For

$$Localmin \Rightarrow y''(x) > 0 \quad (0.9)$$

$$Localmax \Rightarrow y''(x) < 0 \quad (0.10)$$

$$Inflectionpoint \Rightarrow y''(x) = 0 \quad (0.11)$$

Substituting (0.8) in (0.4), we have

$$y'' = \frac{-1}{e^3} \quad (0.12)$$

$$\Rightarrow y'' < 0 \quad (0.13)$$

Hence, (0.8) is a point of maximum.

$$Maxvalue = \frac{1}{e} \quad (0.14)$$

#### Computational solution :

Finding maximum value of a function can be done using **Gradient Ascent method**

$$x_{n+1} = x_n + \alpha f'(x_n) \quad (0.15)$$

$$x_{n+1} = x_n + \alpha \left( \frac{1 - \log(x_n)}{x_n^2} \right) \quad (0.16)$$

$$(0.17)$$

where  $\alpha$  is the learning rate. Taking

$$h = 0.001 \quad (0.18)$$

$$\alpha = 0.001 \quad (0.19)$$

we have

$$x_{max} = 2.717779896744937 \quad (0.20)$$

$$y_{max} = 0.36787943489794817 \quad (0.21)$$

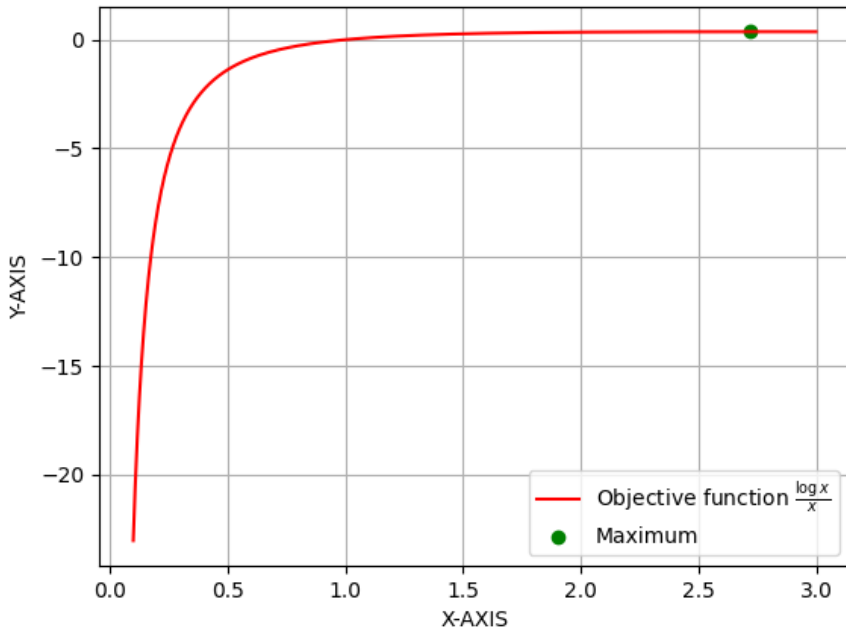


Fig. 0.1: Plot of the given question.