11.16.3.8.5

EE24BTECH11012 - Bhavanisankar G S

QUESTION :
Three coins are tossed once. Find the probability of getting no head.
SOLUTION :

Variable name	Description
$\mathbf{S}$	Sample space
$\mathbf{X}$	Random variable corresponding to the number of heads
p	Toss corresponding to head
$F_{\mathbf{X}}(x)$	Cumulative distribution function (CDF)
$p_{\mathbf{X}}(x)$	Probability Mass function (PMF)

Let us assume the random variable to be the sum of three Bernoulli Random Variables.

$$\mathbf{X} = \mathbf{X_1} + \mathbf{X_2} + \mathbf{X_3} \tag{0.1}$$

$$\mathbf{X_i} = \begin{cases} 1 & , \text{Outcome - head} \\ 0 & , \text{Outcome - tail} \end{cases} \tag{0.2}$$

$$\implies p_{X_i}(k) = \begin{cases} 1 - p & , k = 0 \\ p & , k = 1 \end{cases} \tag{0.3}$$

Considering all the outcomes as equally likely, we have

$$p = \frac{1}{2} \tag{0.4}$$

For the given question, let **X** denote the number of heads. The sample space corresponding to the given scenario is tabulated below.

Event	Sample space
$p_{\mathbf{X}}(0)$	$\{TTT\}$
$p_{\mathbf{X}}(1)$	$\{TTH, THT, HTT\}$
$p_{\mathbf{X}}(2)$	$\{HHT, HTH, THH\}$
$p_{\mathbf{X}}(3)$	$\{HHH\}$

By the properties of Z-transform of **Probability Mass Function**, we have

$$M_{\mathbf{X}}(z) = M_{\mathbf{X}_1}(z)M_{\mathbf{X}_2}(z)M_{\mathbf{X}_3}(z) \tag{0.5}$$

$$M_{\mathbf{X}_1} = \sum_{n\to-\infty}^{\infty} p_{\mathbf{X}_1}(n)z^{-n} = p + (1-p)\,z^{-1} \tag{0.6}$$

$$M_{\mathbf{X}_2} = \sum_{n\to-\infty}^{\infty} p_{\mathbf{X}_2}(n)z^{-n} = p + (1-p)\,z^{-1} \tag{0.7}$$

$$M_{\mathbf{X}_3} = \sum_{n\to-\infty}^{\infty} p_{\mathbf{X}_3}(n)z^{-n} = p + (1-p)\,z^{-1} \tag{0.8}$$

$$M_{\mathbf{X}}(z) = \left(p + (1-p)\,z^{-1}\right)^3 \tag{0.9}$$

$$= \sum_{k\to-\infty}^{\infty} {}^3C_k p^{3-k}\,(1-p)^k\,z^{-k} \tag{0.10}$$

$$p_{\mathbf{X}}(k) = {}^3C_k p^{3-k}\,(1-p)^k \tag{0.11}$$

Substituting (0.4) in (0.11), we have

$$p_{\mathbf{X}}(k) = \frac{{}^3C_k}{8} \tag{0.12}$$

The PMF is then given by -

$$p_{\mathbf{X}}(k) = \begin{cases} \frac{{}^3C_k}{8} & 0 \le k \le 3, k \in \mathbb{W} \\ 0 & otherwise \end{cases} \tag{0.13}$$

$$\implies p_{\mathbf{X}}(k) = \begin{cases} \frac{1}{8} & k = 0 \\ \frac{3}{8} & k = 1 \\ \frac{3}{8} & k = 2 \\ \frac{1}{8} & k = 3 \\ 0 & \text{otherwise} \end{cases} \tag{0.14}$$

The corresponding **Cumulative Distribution Function** can then be written as -

$$F_{\mathbf{X}}(k) = Pr(\mathbf{X} \le k) \tag{0.15}$$

$$= \sum_{k=0}^{k} p_{\mathbf{X}}(x) \tag{0.16}$$

$$\implies F_{\mathbf{X}}(k) = Pr(\mathbf{X} \le k) = \begin{cases} 0 & x < 0 \\ \frac{1}{8} & 0 \le x < 1 \\ \frac{1}{2} & 1 \le x < 2 \\ \frac{7}{8} & 2 \le x < 3 \\ 1 & x \ge 3 \end{cases} \tag{0.17}$$

$$F_{\mathbf{X}}(0) = P(\mathbf{X} <= 0) \tag{0.18}$$
$$= \frac{1}{8} \tag{0.19}$$

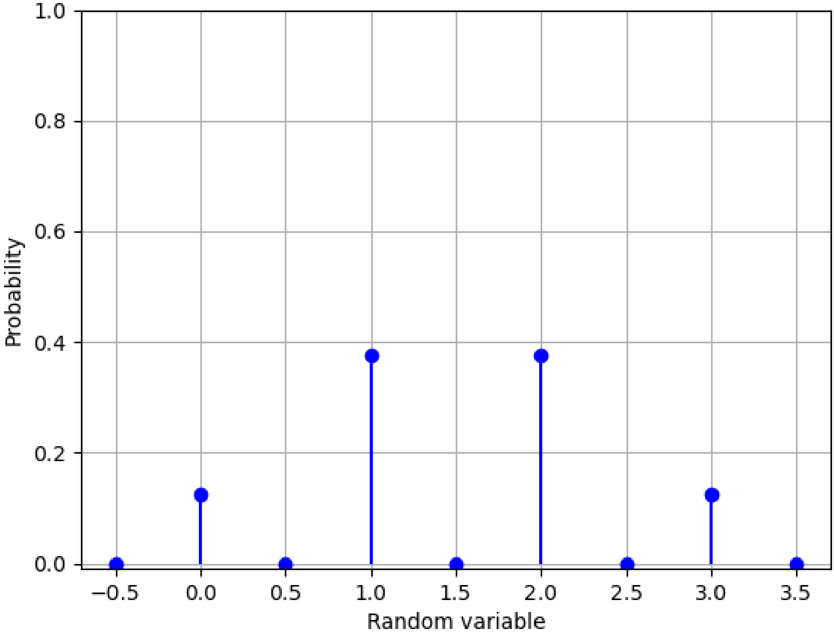

Fig. 0.1: Probability Mass Funtion

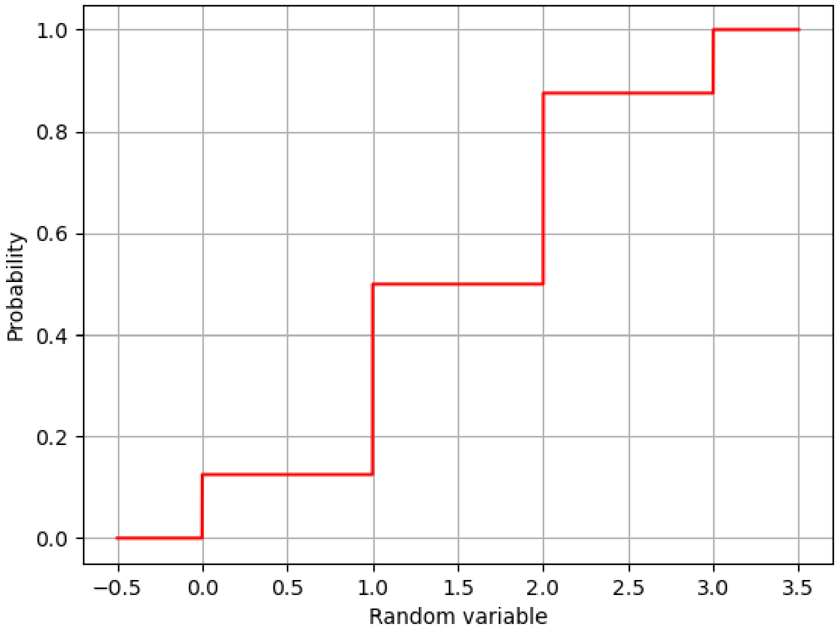

Fig. 0.2: Cumulative Distribution Function

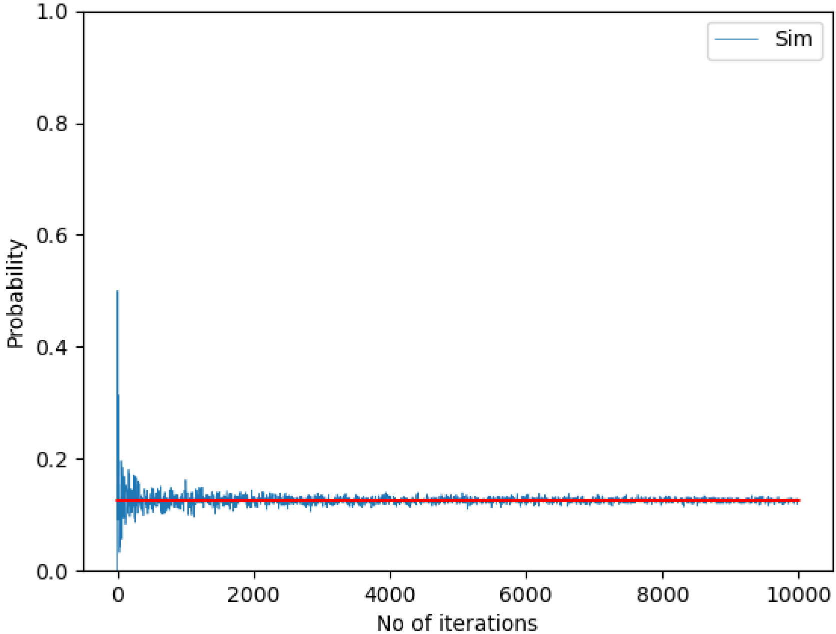

Fig. 0.3: Simulation