Maxima using Gradient Ascent for $f(x) = \frac{\log x}{x}$

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January 16, 2025

Table of Contents

Question

Show that the function $f(x) = \frac{\log x}{x}$ **has a maximum at** $x = e$.

Theoretical Solution

Given function: $y(x) = \frac{\log x}{x}$

First derivative: $y'(x) = \frac{1 - \log x}{x^2}$

Second derivative: $y''(x) = \frac{2 \log x - 3}{x^3}$

Critical points: Solve $y'(x) = 0 \Rightarrow x = e$

Double derivative test: $y''(e) = \frac{-1}{e^3} < 0 \Rightarrow$ Maximum at $x = e$

Computational Solution

Using Gradient Ascent:

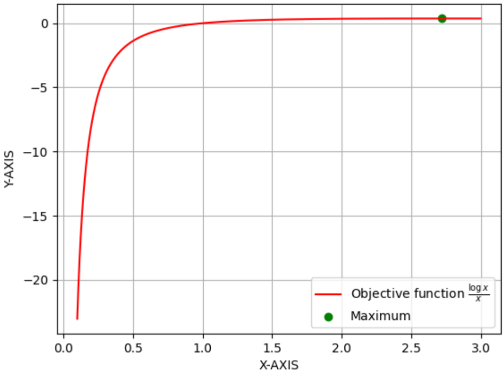
Iteration formula: $x_{n+1} = x_n + \alpha f'(x_n)$

$x_{n+1} = x_n + \alpha \left(\frac{1 - \log x_n}{x_n^2} \right)$

With $\alpha = 0.001$:

$x_{\max} = 2.717779896744937$
$y_{\max} = 0.36787943489794817$

Graphical Representation



Plot of the function $f(x) = \frac{\log x}{x}$