

Maxima using Gradient Ascent for  $f(x) = \frac{\log x}{x}$

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January 16, 2025

## Question

**Show that the function  $f(x) = \frac{\log x}{x}$  has a maximum at  $x = e$ .**

# Theoretical Solution

Given function:  $y(x) = \frac{\log x}{x}$

First derivative:  $y'(x) = \frac{1 - \log x}{x^2}$

Second derivative:  $y''(x) = \frac{2 \log x - 3}{x^3}$

Critical points: Solve  $y'(x) = 0 \Rightarrow x = e$

Double derivative test:  $y''(e) = \frac{-1}{e^3} < 0 \Rightarrow \text{Maximum at } x = e$

# Computational Solution

Using Gradient Ascent:

Iteration formula:  $x_{n+1} = x_n + \alpha f'(x_n)$

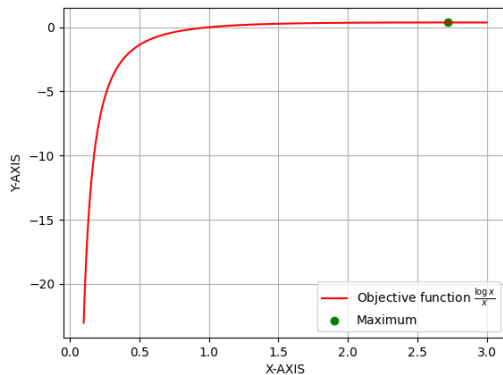
$$x_{n+1} = x_n + \alpha \left( \frac{1 - \log x_n}{x_n^2} \right)$$

With  $\alpha = 0.001$ :

$$x_{\max} = 2.717779896744937$$

$$y_{\max} = 0.36787943489794817$$

# Graphical Representation



*Plot of the function  $f(x) = \frac{\log x}{x}$*