JEE Questions 4

EE24BTECH11012 - Bhavanisankar G S

1) Let in a right triangle, the smallest angle be θ. If a triangle formed by taking the reciprocal of the sides is also a right angled triangle, then $\sin\theta$ is equal to :

a) $\frac{\sqrt{5}+1}{4}$ b) $\frac{\sqrt{5}-1}{2}$ c) $\frac{\sqrt{2}-1}{2}$ d) $\frac{\sqrt{5}-1}{4}$

2) If $\left(\alpha,\beta\right)$ is a point on $y^2 = 6x$, that is closest to $\left(3, \frac{3}{2}\right)$ then find 2 $(\alpha+\beta)$

a) 6 b) 9 c) 7 d) 5

3) Consider the line **L** given by the equation $\frac{x-3}{2} = \frac{y-1}{1} = \frac{z-2}{1}$. Let Q be the mirror image of the point $\left(2,3,-1\right)$ with respect to **L**. Let a plane **P** be such that it passes through Q, and the line **L** is perpendicular to **P**. Then which of the following points is on the plane **P** ?

a) $\left(-1,1,2\right)$ b) $\left(1,1,1\right)$ c) $\left(1,1,2\right)$ d) $\left(1,2,2\right)$

4) Let $g(t) = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos\left(\frac{\pi}{4}t + f(x)\right)dx$, where $f(x) = \log\left(x + \sqrt{x^2+1}\right)$, x $\in$ **R**. Then which of the following is correct ?

a) $g(1) = g(0)$ b) $\sqrt{2}g(1) = g(0)$ c) $g(1) = \sqrt{2}g(0)$ d) $g(1) + g(0) = 0$

5) Let P be a variable point on the parabola $y = 4x^2 + 1$.Then the locus of the mid-point of the point P and the foot of perpendicular drawn from the point P to the line $y = x$ is :

a) $(3x-y)^2 + (x-3y) + 2 = 0$
b) $2(3x-y)^2 + (x-3y) + 2 = 0$
c) $(3x-y)^2 + 2(x-3y) + 2 = 0$
d) $2(x-3y)^2 + (3x-y) + 2 = 0$

I. Integer-Type Questions

1) The sum of all the local minimum values of the twice differentiable function $f : \mathbf{R} \to \mathbf{R}$ defined by $f(x) = x^3 - 3x^2 - \frac{3f''(2)}{2}x + f''(1)$ is :
2) For natural numbers m, n if $(1-y)^m(1+y)^n = 1 + a_1y + a_2y^2 + \cdots + a_{m+n}y^{m+n}$ and $a_1 = a_2 = 10$ then he value of $(m+n)$ is equal to :
3) The value of $21\tan\left(2\tan^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13}\right)$ is :
4) Let a curve $y = y(x)$ be givem by the solutio of the differential equation

$$\cos\left(\frac{1}{2}\cos^{-1}e^{-x}\right)dx = \sqrt{e^{2x}-1}dy$$

If it intersects y-axis at $y = -1$ and the intersection point of the curve with the x-axis is $\left(\alpha,0\right)$, then e^{α} is equal to :

5) For p $\geq$ 0, a vector $\mathbf{v_2} = 2\mathbf{i} + (p+1)\mathbf{j}$ is obtained by rotating the vector $\mathbf{v_1} = \sqrt{3}p\mathbf{i} + \mathbf{j}$ by an angle θ about the origin in counter clockwise direction. If $\tan\theta = \frac{\alpha\sqrt{3}-2}{4\sqrt{3}+3}$, then the value of α is equal to :
6) Consider a triangle with vertices $\mathbf{A}\left(-2,3\right), \mathbf{B}\left(1,9\right), \mathbf{C}\left(3,8\right)$. If a line **L** passing through the circum-centre of the triangle ABC, bisects line BC, and intersects y-axis at point $\left(0,\frac{\alpha}{2}\right)$ then the value of real number α is :
7) For k $\in$ **N**, let

$$\frac{1}{\alpha(\alpha+1)(\alpha+2)\ldots(\alpha+20)} = \sum_{k=0}^{20}\frac{A_k}{\alpha+k}$$

where $\alpha > 0$.Then the value of $100\left(\frac{A_{14}+A_{15}}{A_{13}}\right)^2$ is :

8) Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that $a_1 = 1$, $a_2 = 1$ and $a_{n+2} = 2a_{n+1} + a_n$ for all $n \geq 1$. Then the value of $47 \sum_{n=1}^{\infty} \frac{a_n}{2^{3n}}$ is equal to :
9) Let r_1 and r_2 be the radii of the largest and smallest circles respectively which pass through the point $\left(-4, 1\right)$ and having their centres on the circumference of the circle $x^2 + y^2 + 2x + 4y - 4 = 0$ If $\frac{r_1}{r_2} = a + b\sqrt{2}$, then a + b is equal to
10) The absolute value of $k \in \mathbf{R}$, for which the following system of linear equations

$$3x - y + 4z = 3 \tag{1}$$

$$x + 2y - 3z = -2 \tag{2}$$

$$6x + 5y + kz = -3 \tag{3}$$

has infinitely many solutions is :