9.2.4

EE24BTECH11012 - Bhavanisankar G S

QUESTION

Find the area of the region bounded by the curves $x^2 = 4y$, $y = 2$, $y = 4$ and the y-axis in the first quadrant.

SOLUTION

FORMULAE
$g(\mathbf{x}) = \mathbf{x^T V x} + 2\mathbf{u^T x} + f = 0$ where, $\mathbf{V} = \|n\|^2 \mathbf{I} - e^2\mathbf{nn^T}$, $\text{u} = \text{c}e^2\mathbf{n} - \|n\|^2 \mathbf{F}$, $\text{f} = \|n\|^2 \|F\|^2 - c^2e^2$
The points of intersection of the line L: $\text{x} = \text{h} + \kappa\mathbf{m}, \kappa \in \mathbb{R}$ with the conic section as above are given by $\text{x}_i = \mathbf{h} + \kappa_i\mathbf{m}$ where $\kappa_i = \frac{1}{\mathbf{m^T V m}}\left(-\mathbf{m^T}\left(\mathbf{Vh} + \mathbf{u}\right) \pm \sqrt{\left[\mathbf{m^T}\left(\mathbf{Vh} + \mathbf{u}\right)^2\right] - g(\mathbf{h})\left(\mathbf{m^T V m}\right)}\right)$

TABLE 0: Formulae Used

Substituting the given values, we have

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \tag{0.1}$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{0.2}$$

$$f = 0 \tag{0.3}$$

$$\tag{0.4}$$

Substituting the values, we get the point of intersection as

$$\kappa_i = -\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} 0+0 \\ 2+-2 \end{pmatrix} \pm \sqrt{\left[\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} 0+0 \\ 2+-2 \end{pmatrix}\right]^2 + 4(1)} \tag{0.5}$$

$$\kappa_i = 2\sqrt{2} \tag{0.6}$$

$$\tag{0.7}$$

Hence, the point of intersection is $\begin{pmatrix} 2\sqrt{2} \\ 2 \end{pmatrix}$

Similarly, the other point is given by $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$.
The area bounded by the curve and the line is

$$\int_{2}^{4} (2\sqrt{y})\, dy = \frac{4}{3}\left(8 - 2\sqrt{2}\right) \tag{0.8}$$

$$= \frac{\left(32 - 8\sqrt{2}\right)}{3} \tag{0.9}$$

$$\tag{0.10}$$

Hence the required area is $\left(32 - 8\sqrt{2}\right)_{\overline{3.}}$

figs/fig.jpg

Fig. 0.1: A plot of the given question.