GATE Questions 12

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1) Connsider the subspace $V = \left\{(x_n) \in l^2 : \sum_{n=1}^{infty} |x_n| \leq \infty\right\}$ of the Hilbert space l^2 of all square summable real sequences. For $n \in \mathbb{N}$, define $T_n = V \to \mathbb{R}$ by $T_n(x_n) = \sum_{i=1}^{n} x_l$. Consider the following statements.
P: $\{T_n : n \in \mathbb{N}\}$ is pointwise bounded on V
Q: $\{T_n : n \in \mathbb{N}\}$ is uniformly bounded on $\{x \in V : \|x\|_2 = 1\}$
Which of the following statements holds good ?

a) Both P and Q
b) Only P
c) Only Q
d) Neither P nor Q

2) Let p(x) be the polynomial of degree at most 2 that interpolates the data (-1,2), (0,1) and (1,2). If q(x) is a polynomial of degree at most 3 such that p(x) + q(x) interpolates the data (-1,2), (0,1), (1,2) and (2,11), then q(3) equals
3) Let J be the Jacobi iteration matrix of the linear system

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ -4 & 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Consider the following statements :
P: One of the eigen values of J lies in the interval [2,3]
Q: The jacobi iteration converges for the above system.
Which of the above statements hold good ?

a) Both P and Q
b) Only P
c) Only Q
d) Neither P nor Q

4) Let $u(x, y)$ be the solution of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 4u$ satisfying the consition $u(x, y) = 1$ on the circle $x^2 + y^2 = 1$. Then $u(2, 2)$ equals
5) Let $u(r, \theta)$ be the bounded solution of the following boundary value problem in polar coordinates :

$$r^2\frac{\partial^2 u}{\partial r^2} + r\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0, 0 \leq r \leq 2, 0 \leq \theta \leq 2\pi$$

$$u(2, \theta) = \cos^2\theta, 0 \leq \theta \leq 2\pi$$

Then $u(1, \frac{\pi}{2}) + u(1, \frac{\pi}{4})$ equals

a) 1 b) $\frac{9}{8}$ c) $\frac{7}{8}$ d) $\frac{3}{8}$

6) Let T_u and T_d denote the usual topology and the discrete topology on $\mathbb{R}$ respectively. Consider the following topologies :
T_1: Usual topology on $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$
T_2: Topology generated by the basis $\{U \times V : U \in T_d, V \in T_u\}$ on $\mathbb{R} \times \mathbb{R}$
T_3: Dictionary order topology on $\mathbb{R} \times \mathbb{R}$. Then

a) $T_3 \subset T_1 \subseteq T_2$
b) $T_1 \subset T_2 \subseteq T_3$
c) $T_3 \subset T_2 \subseteq T_1$
d) $T_1 \subset T_2 \subseteq T_3$

7) Let X be a random variable with probability mass function

$$p(n) = \left(\frac{3}{4}\right)^{n-1} \left(\frac{1}{4}\right)$$

for $n = 1, 2, \ldots$. Then E(X-3 — X $\geq$ 3) equals

8) Let X and Y be independent and identically distributed random variables with probability mass function $p(n) = 2^{-n}, n = 1, 2, \ldots$. Then P(X $\geq$ 2Y) equals (rounded off to two decimals)

9) Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed Poisson random variables with mean 4. Then

$$\lim_{n\to\infty} P\left(4 - \frac{2}{\sqrt{n}} \leq \frac{1}{n}\sum_{i=1}^{n} X_i \leq 4 + \frac{2}{\sqrt{n}}\right)$$

equals

10) Let X and Y be independent and identically distributed exponential random variables with probability density function

$$f(x) = e^{-x}$$

for all positive x. Then $P\left(max(X, Y) \leq 2\right)$ equals (rounded to 2 decimals)

11) Let E and F be any two events with $P(E) = 0.4$, $P(F) = 0.3$ and $P(F|E) = 3P(F|E^C)$. Then $P(E|F)$ equals (rounded to 2 decimals)

12) Let $X_1, X_2, \ldots, X_m$ be a random sample from a binomial distribution with parameters $n = 1$ and p, $p \in (0, 1)$, and let $\overline{X} = \frac{1}{m}\sum_{i=1}^{m} X_i$. Then a uniformly minimum variance unbiased estimator for $p(1-p)$ is

a) $\frac{m}{m-1}\overline{X}\left(1-\overline{X}\right)$
b) $\overline{X}\left(1-\overline{X}\right)$
c) $\frac{m-1}{m}\overline{X}\left(1-\overline{X}\right)$
d) $\frac{1}{m}\overline{X}\left(1-m\overline{X}\right)$

13) Let $X_1, X_2, \ldots, X_9$ be a random variable from a $N\left(0, \sigma^2\right)$ population. For testing $H_0 : \sigma^2 = 2$ against $H_1 : \sigma^2 = 1$, the most powerful test rejects H_0 if $\sum_{i=1}^{9} X_i \leq c$ where c is to be chosen such that the level of significace is 0.1 Then the power of this test equals