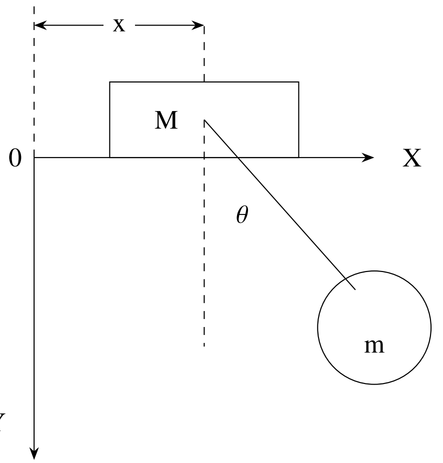
# GATE Questions 17

EE24BTECH11012 - Bhavanisankar G S

1) P and Q are two Hermititan matrices and there exists a matrix R, which diagonalizes both of them, such that $RPR^{-1} = S_1$ and $RQR^{-1} = S_2$, where $S_1$ and $S_2$ are diagonal matrices. The correct statement(s) is(are) :
   a) All the elements of both matrices $S_1$ and $S_2$ are real
   b) The matrix PQ can have complex eigenvalues.
   c) The matrix QP can have complex eigenvalues.
   d) The matrices P and Q commute.
2) A uniform block of mass $M$ slides on a smooth horizontal bar. Another mass $m$ is connected to it by an inextensible string of length $l$ of negligible mass, and is constrained to oscillate in the X-Y plane only. Neglect the sizes of the masses. The number of degrees of freedom of the system is two and the generalized coordinates are chosen as $x$ and $\theta$ as shown in the figure.




   If $p_x$ and $p_\theta$ are the generalised momenta corresponding to $x$ and $\theta$, respectively, then the correct option(s) is(are)
   a) $p_x = (m + M)\overline{x} + mI\cos\theta\overline{\theta}$
   b) $p_\theta = mI^2\overline{\theta} - mI\cos\theta\overline{x}$
   c) $p_x$ is conserved
   d) $p_\theta$ is conserved
3) The Gell-Mann-Okuba mass formula defines the mass of baryons as

$$M = M_0 + aY + b\left[I(I+1) - \frac{1}{4}Y^2\right], \text{where} M_0, a \text{and} b \text{are constants}$$

If the mass of $\sigma$ hyperons is same as that of $\Lambda$ hyperons, then the correct option(s) is(are)

a) $M \propto I(I+1)$
b) $M \propto Y$
c) $M$ does not depend on $I$
d) $M$ does not depend on $Y$

4) The time derivative of a differentiable function $g(q_i, t)$ is added to a Lagrangian $L(q_i, \overline{q_i}, t)$ such that

$$L' = L(q_i, \overline{q_i}, t) + \frac{d\,(g(q_i, t))}{dt}$$

where $q_i, \overline{q_i}, t$ are the generalised coordinates, generalizes velocities and time respectively. Let $p_i$ be the generalized momentum and H the Hamiltonian associated with $L(q_i, \overline{q_i}, t)$. If $p_i'$and$H'$ are those associated with $L'$, then the correct option(s) is(are)
a) Both $L$ and $L'$ sarisfy Euler-Lagrange's equations of motion.
b) $p_i' = p_i + \frac{\partial}{\partial q_i} g(q_i, t)$
c) If $p_i$ is conserved, then $p_i'$ is necassarily conserved.
d) $H' = H + \frac{d}{dt} g(q_i, t)$

5) A linear charged particle accelerator is driven by an alternating voltage source operating at 10 MHz. Assume that it is used to accelerate electrons. After a few drift-tubes, the electrons attain a velocity $2.9 \times 10^8 ms^{-1}$. The minimum length of each drift-tube, in m, to accelerate the electrons further is _

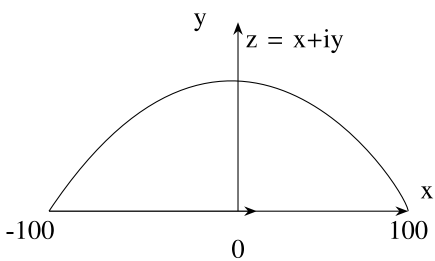
6) The Coulomb energy component in the binding energy of a nucleus is 18.432 MeV. If the radius of the uniform and spherical charge distribution in the nucleus is 3 fm, the corresponding atomic number is _

7) For a two-nucleon system, in spin singlet state, the spin is represented through the Pauli matrices $\sigma_1, \sigma_2$ for particles 1 and 2, respectively. The value of $(\sigma_1 \cdot \sigma_2)$ is

8) A contour is defined as

$$I_n = \int \frac{dz}{(z-n)^2 + \pi^2}$$

where $n$ is a positive integer and C is the closed contour, as shown in the figure, consisting of the line from -100 to 100 and the semicircle traversed in the counterclockwise sense. The value of $\sum_{n=1}^{5} I_n$ is



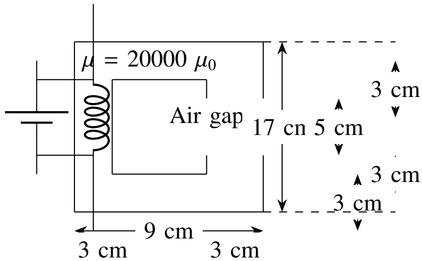
9) The normalised radial wave function of the second excited state of hydrogen atom is

$$R(r) = \frac{1}{\sqrt{24}} a^{\frac{3}{2}} \frac{r}{a} e^{\frac{-r}{2a}}$$

where $a$ is the Bohr radius and $r$ is the distance from the centre of the atom. The distance at which the electron is most likely to be found is $y \times a$, the value of $y$ is

10) Consider an atomic gas with number density $n = 10^{20} m^{-3}$, in the ground state at 300 K. The valence electronic configuration of atoms is $f^7$. The paramagetic susceptibility of the gas $\chi = m \times 10^{-11}$. The value of $m$ is

11) Consider a cross-section of an electromagnet having an air-gap of 5 cm as shown. It consists of a magnetic material with $\mu = 20000\mu_0$ and is driven by a coil having $NI = 10^4$ where $N$ is the number of turns and $I$ is the current in Ampere.




Ignoring the fringe fields, the magnitude of the magnetic field $\overline{B}$ in the air-gap between the magnetic poles is

12) The spin $\overline{S}$ and orbital angular momentum $\overline{L}$ of an atom process about $\overline{J}$, the total angular momentum $\overline{J}$ precesses about an axis fixed by a magnetic field $\overline{B_1} = 2B_0\mathbf{z}$, where $B_0$ is a constant. Now the magnetic field is changed to $\overline{B_2} = B_0(\mathbf{x} + \sqrt{2}\mathbf{y} + \mathbf{z})$. Given the orbital angular momentum quantum number $l = 2$ and spin quantum number $s = \frac{1}{2}$, $\theta$ is thr angle between $\overline{B_1}$ and $\overline{J}$ for the largest possible values of total angular momentum. The value of $\theta$ is

13) The spin-orbit effect splits the ${}^2P \rightarrow {}^2S$ transition of wavelength 6521Å in Lithium into two lines with separation of $\delta\lambda = 0.14$Å. The corresponding positive value of energy difference between the above two lines, in eV, is $m \times 10^5$. The value of $m$ is