

9.2.4

EE24BTECH11012 - Bhavanisankar G S

QUESTION

Using integration, find the area of the region enclosed by the curve $y = x^2$, the x-axis and the ordinates $x = -2$ and $x = 1$.

SOLUTION

FUNCTION	FORM
$g(\mathbf{x})$	$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x}$
The points of intersection of the line L with the conic section as above are given by $\mathbf{x}_i = \mathbf{h} + \kappa_i \mathbf{m}$	L: $\mathbf{x} = \mathbf{h} + \kappa \mathbf{m}$ $\kappa_i = \frac{1}{\mathbf{m}^T \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^T (\mathbf{V} \mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^T (\mathbf{V} \mathbf{h} + \mathbf{u})]^2 + \mathbf{m}^T \mathbf{V} \mathbf{m}} \right)$

TABLE 0: Formulae Used

Substituting the given values, we have

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (0.1)$$

$$\mathbf{u} = \begin{pmatrix} -\frac{1}{2} \\ 0 \end{pmatrix} \quad (0.2)$$

$$f = 0 \quad (0.3)$$

$$(0.4)$$

Substituting the values, we get the point of intersection as

$$\kappa_i = -\begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} & 0 \end{pmatrix} \pm \sqrt{\left[\begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} \\ 0 \end{pmatrix} \right]^2 + 1} \quad (0.5)$$

$$\kappa_i = 1 \quad (0.6)$$

$$(0.7)$$

Hence, the point of intersection is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Similarly, the other point is given by $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$.

The area bounded by the curve and the line is

$$\int_{-2}^1 (x^2) dx = \frac{1}{3} (1 - (-8)) \quad (0.8)$$

$$= 3 \quad (0.9)$$

$$(0.10)$$

Hence the required area is 3 .

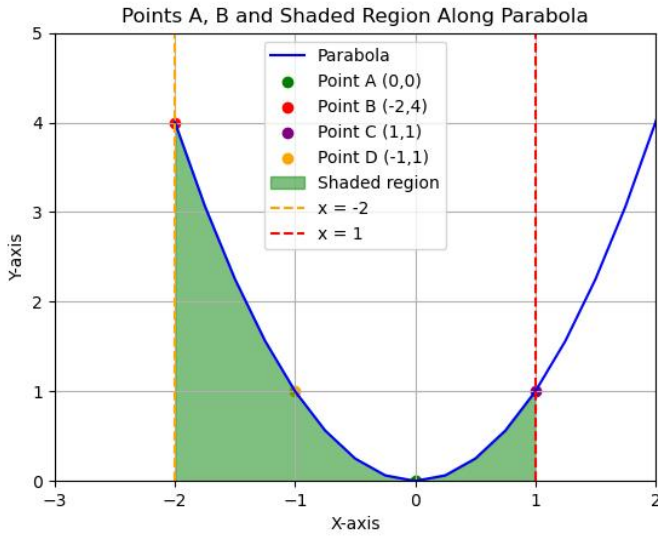


Fig. 0.1: A plot of the given question.