JEE Questions 5

EE24BTECH11012 - Bhavanisankar G S

1) Which of the following is the negation of the statement "for all M ≥ 0, there exists x $\in \mathbf{S}$ such that $x \geq M$"?
   a) there exists M ≥ 0 such that x $\leq$ M for all x $\in \mathbf{S}$
   b) there exists M ≥ 0 there exists x $\in \mathbf{S}$ such that x $\geq$ M
   c) there exists M ≥ 0 there exists x $\in \mathbf{S}$ such that x $\leq$ M
   d) there exists M ≥ 0 such that x $\geq$ M for all x $\in \mathbf{S}$
2) Consider a circle C which touches the y-axis at $\left(0,6\right)$ and cuts off an intercept $6\sqrt{5}$ on the x-axis. Then the radius of the circle C is equal to :

   a) $\sqrt{53}$ b) 9 c) 8 d) $\sqrt{82}$

3) Let $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ be three vectors such that $\mathbf{a} = \mathbf{b} \times (\mathbf{b} \times \mathbf{c})$. If magnitudes of the vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are $\sqrt{2}$, 1 and 2 respectively and the angle between $\mathbf{b}$ and $\mathbf{c}$ is $\theta\left(0 \leq \theta \leq \frac{\pi}{2}\right)$, then the value of $1 + \tan\theta$ is equal to :

   a) $\sqrt{3} + 1$ b) 2 c) 1 d) $\frac{\sqrt{3}+1}{\sqrt{3}}$

4) Let $\mathbf{A}$ and $\mathbf{B}$ be two 3×3 real matrices such that $\mathbf{A^2} - \mathbf{B^2}$ is invertible matrix. If $\mathbf{A^5} = \mathbf{B^5}$ and $\mathbf{A^3B^2} = \mathbf{A^2B^3}$, then the value of the determinant of the matrix $\mathbf{A^3} + \mathbf{B^3}$ is equal to :

   a) 2 b) 4 c) 1 d) 0

5) Let $f : (a,b) \rightarrow \mathbf{R}$ be twice differentiable function such that $f(x) = \int_a^x g(t)dt$ for a differentiable function g(x). If f(x) = 0 has exactly five distinct roots in (a,b), then $g(x)g'x = 0$ has atleast :
   a) twelve roots in (a,b)
   b) five roots in (a,b)
   c) seven roots in (a,b)
   d) three roots in (a,b)

I. Integer-Type Questions

1) Let $\mathbf{a} = \mathbf{i} - \alpha\mathbf{j} + \beta\mathbf{k}$, $\mathbf{b} = 3\mathbf{i} + \beta\mathbf{j} - \alpha\mathbf{k}$ and $\mathbf{c} = -\alpha\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, where α, β are integers. If $\mathbf{a} \cdot \mathbf{b} = -1$ and $\mathbf{b} \cdot \mathbf{c} = 10$, then $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$ is equal to :
2) The distance of the point $\mathrm{P}\left(3,4,4\right)$ from the point of intersection of the line joining the points $\mathrm{Q}\left(3,-4,5\right)$ and $\mathrm{R}\left(2,-3,1\right)$ and the plane $2x + y + z = 7$, is equal to :
3) If the real part of the complex number $z = \frac{3+2i\cos\theta}{1-3i\cos\theta}$, $\theta \in \left(0, \frac{\pi}{2}\right)$ is zero, then the value of $\sin^2 3\theta + \cos^2\theta$ is equal to :
4) Let $\mathbf{E}$ be an ellipse whose axes are parallel to the co-ordinate axes, having its centre at $\left(3,-4\right)$, one focus at $\left(4,-4\right)$ and one vertex at $\left(5,-4\right)$. If $mx - y = 4$, m¿0 is a tangent to the ellipse $\mathbf{E}$, then the value of $5m^2$ is equal to :
5) If $\int_0^\pi \left(\sin^3 x\right) e^{-\sin^2 x} dx = \alpha - \frac{\beta}{e}\int_0^1 \sqrt{t}e^t dt$, then $\alpha + \beta$ is equal to :
6) The number of real roots of the equation $e^{4x} - e^{3x} - 4e^{2x} - e^x + 1 = 0$ is equal to :
7) Let $y = y(x)$ be the solution of the differential equation $dy = e^{\alpha x + y}dx$; $\alpha \in \mathbf{R}$. If $y\left(log\left(2\right)\right) = log\left(2\right)$ and $y(0) = log\left(\frac{1}{2}\right)$, then the value of α is equal to :
8) Let n be a non-negative integer. Then the number of divisors of the form "4n+1" of the number $(10)^{10}(11)^{11}(13)^{13}$ is equal to :
9) Let $A = \left\{n \in \mathbf{N} | n^2 \leq n + 10{,}000\right\}$, $B = \{3k + 1 | k \in \mathbf{N}\}$ and $C = \{2k | k \in \mathbf{N}\}$, then the sum of all the elements of the set $A \cap (B - C)$ is equal to ;

10) If $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ and $M = A + A^2 + A^3 + \cdots + A^{20}$, then the sum of all the elements of the matrix M is equal to :