

JEE Questions 5

EE24BTECH11012 - Bhavanisankar G S

- 1) Which of the following is the negation of the statement "for all $M \geq 0$, there exists $x \in \mathbf{S}$ such that $x \geq M$ "?
[July 2021]
- there exists $M \geq 0$ such that $x \leq M$ for all $x \in \mathbf{S}$
 - there exists $M \geq 0$ there exists $x \in \mathbf{S}$ such that $x \geq M$
 - there exists $M \geq 0$ there exists $x \in \mathbf{S}$ such that $x \leq M$
 - there exists $M \geq 0$ such that $x \geq M$ for all $x \in \mathbf{S}$
- 2) Consider a circle C which touches the y -axis at $(0, 6)$ and cuts off an intercept $6\sqrt{5}$ on the x -axis. Then the radius of the circle C is equal to :
[July 2021]
- $\sqrt{53}$
 - 9
 - 8
 - $\sqrt{82}$
- 3) Let \mathbf{a} , \mathbf{b} and \mathbf{c} be three vectors such that $\mathbf{a} = \mathbf{b} \times (\mathbf{b} \times \mathbf{c})$. If magnitudes of the vectors \mathbf{a} , \mathbf{b} and \mathbf{c} are $\sqrt{2}$, 1 and 2 respectively and the angle between \mathbf{b} and \mathbf{c} is θ ($0 \leq \theta \leq \frac{\pi}{2}$), then the value of $1 + \tan \theta$ is equal to :
[July 2021]
- $\sqrt{3} + 1$
 - 2
 - 1
 - $\frac{\sqrt{3}+1}{\sqrt{3}}$
- 4) Let \mathbf{A} and \mathbf{B} be two 3×3 real matrices such that $\mathbf{A}^2 - \mathbf{B}^2$ is invertible matrix. If $\mathbf{A}^5 = \mathbf{B}^5$ and $\mathbf{A}^3\mathbf{B}^2 = \mathbf{A}^2\mathbf{B}^3$, then the value of the determinant of the matrix $\mathbf{A}^3 + \mathbf{B}^3$ is equal to :
[July 2021]
- 2
 - 4
 - 1
 - 0
- 5) Let $f : (a, b) \rightarrow \mathbf{R}$ be twice differentiable function such that $f(x) = \int_a^x g(t)dt$ for a differentiable function $g(x)$. If $f(x) = 0$ has exactly five distinct roots in (a, b) , then $g(x)g'(x) = 0$ has atleast : [July 2021]
- twelve roots in (a, b)
 - five roots in (a, b)
 - seven roots in (a, b)
 - three roots in (a, b)

I. INTEGER-TYPE QUESTIONS

- 1) Let $\mathbf{a} = \mathbf{i} - \alpha\mathbf{j} + \beta\mathbf{k}$, $\mathbf{b} = 3\mathbf{i} + \beta\mathbf{j} - \alpha\mathbf{k}$ and $\mathbf{c} = -\alpha\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, where α, β are integers. If $\mathbf{a} \cdot \mathbf{b} = -1$ and $\mathbf{b} \cdot \mathbf{c} = 10$, then $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$ is equal to :
[July 2021]
- 2) The distance of the point $P(3, 4, 4)$ from the point of intersection of the line joining the points $Q(3, -4, 5)$ and $R(2, -3, 1)$ and the plane $2x + y + z = 7$, is equal to :
[July 2021]
- 3) If the real part of the complex number $z = \frac{3+2i\cos\theta}{1-3i\cos\theta}$, $\theta \in (0, \frac{\pi}{2})$ is zero, then the value of $\sin^2 3\theta + \cos^2 \theta$ is equal to :
[July 2021]
- 4) Let \mathbf{E} be an ellipse whose axes are parallel to the co-ordinate axes, having its centre at $(3, -4)$, one focus at $(4, -4)$ and one vertex at $(5, -4)$. If $mx - y = 4$, $m \neq 0$ is a tangent to the ellipse \mathbf{E} , then the value of $5m^2$ is equal to :
[July 2021]
- 5) If $\int_0^\pi (\sin^3 x) e^{-\sin^2 x} dx = \alpha - \frac{\beta}{e} \int_0^1 \sqrt{t} e^t dt$, then $\alpha + \beta$ is equal to :
[July 2021]
- 6) The number of real roots of the equation $e^{4x} - e^{3x} - 4e^{2x} - e^x + 1 = 0$ is equal to :
[July 2021]
- 7) Let $y = y(x)$ be the solution of the differential equation $dy = e^{\alpha x + y} dx$; $\alpha \in \mathbf{R}$. If $y(\log(2)) = \log(2)$ and $y(0) = \log(\frac{1}{2})$, then the value of α is equal to :
[July 2021]
- 8) Let n be a non-negative integer. Then the number of divisors of the form " $4n+1$ " of the number $(10)^{10} (11)^{11} (13)^{13}$ is equal to :
[July 2021]
- 9) Let $A = \{n \in \mathbf{N} | n^2 \leq n + 10,000\}$, $B = \{3k + 1 | k \in \mathbf{N}\}$ and $C = \{2k | k \in \mathbf{N}\}$, then the sum of all the elements of the set $A \cap (B - C)$ is equal to ;
[July 2021]

- 10) If $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ and $M = A + A^2 + A^3 + \cdots + A^{20}$, then the sum of all the elements of the matrix M is equal to : [July 2021]