

9.2.4

EE24BTECH11012 - Bhavanisankar G S

QUESTION

Find the area of the region bounded by the curves $x^2 = 4y$, $y = 2$, $y = 4$ and the y-axis in the first quadrant.

SOLUTION

FORMULAE
$g(\mathbf{x}) = \mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0$ <p>where,</p> $\mathbf{V} = \ n\ ^2 \mathbf{I} - e^2 \mathbf{n} \mathbf{n}^T,$ $\mathbf{u} = ce^2 \mathbf{n} - \ n\ ^2 \mathbf{F},$ $f = \ n\ ^2 \ F\ ^2 - c^2 e^2$
<p>The points of intersection of the line $L: \mathbf{x} = \mathbf{h} + \kappa \mathbf{m}, \kappa \in \mathbb{R}$ with the conic section as above are given by $\mathbf{x}_i = \mathbf{h} + \kappa_i \mathbf{m}$ where</p> $\kappa_i = \frac{1}{\mathbf{m}^T \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^T (\mathbf{V} \mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^T (\mathbf{V} \mathbf{h} + \mathbf{u})^2] - g(\mathbf{h}) (\mathbf{m}^T \mathbf{V} \mathbf{m})} \right)$

TABLE 0: Formulae Used

Substituting the given values, we have

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \quad (0.1)$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \quad (0.2)$$

$$f = 0 \quad (0.3)$$

$$(0.4)$$

Substituting the values, we get the point of intersection as

$$\kappa_i = -(1 \ 0) \begin{pmatrix} 0+0 \\ 2+-2 \end{pmatrix} \pm \sqrt{\left[(1 \ 0) \begin{pmatrix} 0+0 \\ 2+-2 \end{pmatrix} \right]^2 + 4(1)} \quad (0.5)$$

$$\kappa_i = 2 \sqrt{2} \quad (0.6)$$

$$(0.7)$$

Hence, the point of intersection is $\begin{pmatrix} 2\sqrt{2} \\ 2 \end{pmatrix}$

Similarly, the other point is given by $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$.

The area bounded by the curve and the line is

$$\int_2^4 (2\sqrt{y}) dy = \frac{4}{3} (8 - 2\sqrt{2}) \quad (0.8)$$

$$= \frac{(32 - 8\sqrt{2})}{3} \quad (0.9)$$

$$(0.10)$$

Hence the required area is $(32 - 8\sqrt{2})_{\frac{3}{}}$.

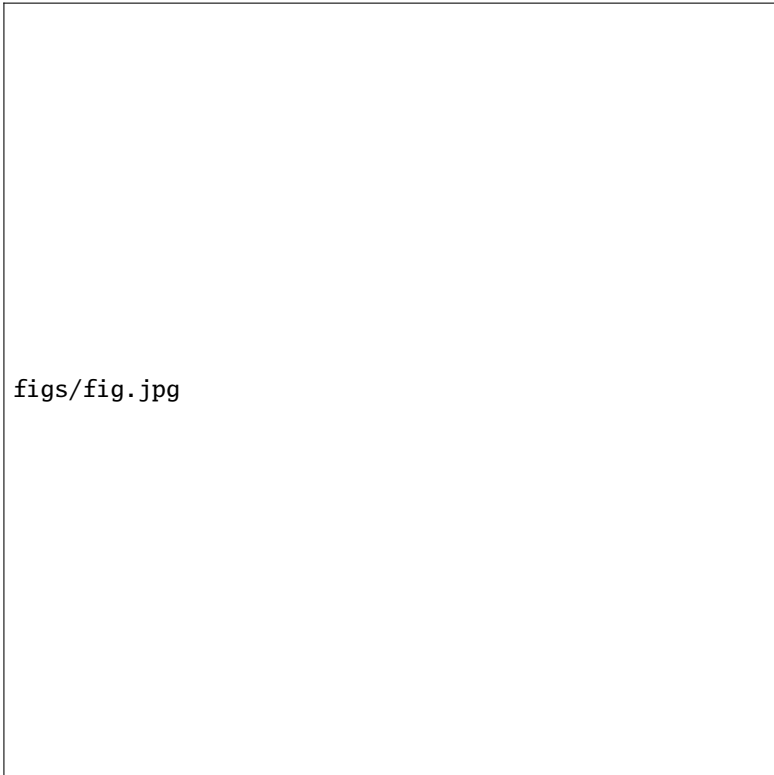


Fig. 0.1: A plot of the given question.