9.2.4

EE24BTECH11012 - Bhavanisankar G S

QUESTION

Find the area of the region bounded by the curves $x^2 = 4y$, $y = 2$, $y = 4$ and the y-axis in the first quadrant.

SOLUTION

FORMULAE
$g(\mathbf{x}) = \mathbf{x}^\mathbf{T}\mathbf{V}\mathbf{x} + 2\mathbf{u}^\mathbf{T}\mathbf{x} + f = 0$ where, $\mathbf{V} = \|\|n\|\|^2\,\mathbf{I} - e^2\mathbf{n}\mathbf{n}^\mathbf{T}$, $\mathrm{u} = \mathrm{c}e^2\mathbf{n} - \|\|n\|\|^2\,\mathbf{F}$, $\mathrm{f} = \|\|n\|\|^2\,\|\|F\|\|^2 - c^2e^2$
The points of intersection of the line L: $\mathrm{x} = \mathrm{h} + \kappa\mathbf{m}, \kappa \in \mathbb{R}$ with the conic section as above are given by $\mathrm{x}_i = \mathbf{h} + \kappa_i\mathbf{m}$ where $\kappa_i = \frac{1}{\mathbf{m}^\mathbf{T}\mathbf{V}\mathbf{m}}\left(-\mathbf{m}^\mathbf{T}(\mathbf{V}\mathbf{h}+\mathbf{u}) \pm \sqrt{\left[\mathbf{m}^\mathbf{T}(\mathbf{V}\mathbf{h}+\mathbf{u})^2\right] - g(\mathbf{h})\left(\mathbf{m}^\mathbf{T}\mathbf{V}\mathbf{m}\right)}\right)$

TABLE 0: Formulae Used

Substituting the given values, we have

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{0.1}$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{0.2}$$

$$f = 0 \tag{0.3}$$

$$\tag{0.4}$$

Substituting the values, we get the point of intersection as

$$\kappa_i = -\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} 0+0 \\ 2+-2 \end{pmatrix} \pm \sqrt{\left[\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} 0+0 \\ 2+-2 \end{pmatrix}\right]^2 + 4\,(1)} \tag{0.5}$$

$$\kappa_i = 2\sqrt{2} \tag{0.6}$$

$$\tag{0.7}$$

Hence, the point of intersection is $\begin{pmatrix} 2\sqrt{2} \\ 2 \end{pmatrix}$

Similarly, the other point is given by $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$.
The area bounded by the curve and the line is

$$\int_2^4 (2\sqrt{y})\,dy = \frac{4}{3}\left(8 - 2\sqrt{2}\right) \tag{0.8}$$

$$= \frac{\left(32 - 8\sqrt{2}\right)}{3} \tag{0.9}$$

$$\tag{0.10}$$

Hence the required area is $\left(32 - 8\sqrt{2}\right)_{\overline{3.}}$

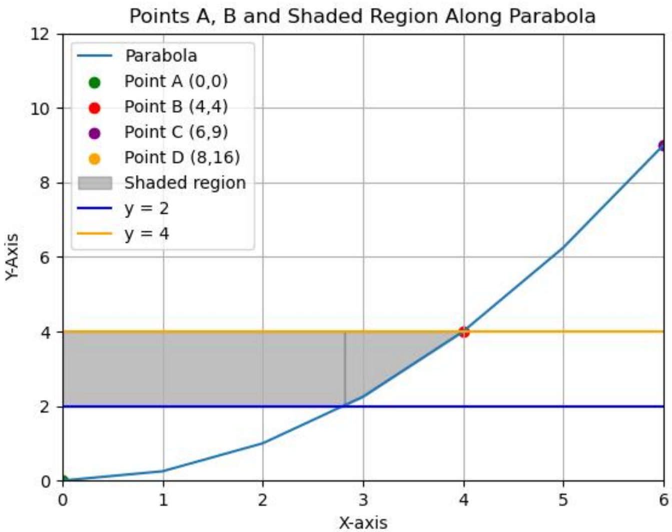

Fig. 0.1: A plot of the given question.