9.2.4

EE24BTECH11012 - Bhavanisankar G S

QUESTION

Find the area of the region bounded by the curves $x^2 = 4y$, $y = 2$, $y = 4$ and the y-axis in the first quadrant.

SOLUTION

FUNCTION	**FORM**
$g(\mathbf{x})$	$\mathrm{x}^T Vx + 2\mathbf{u}^{T}$
The points of intersection of the line L with the conic section as above are given by $\mathrm{x}_i = \mathbf{h} + \kappa_i \mathbf{m}$	L: x = h + $\kappa_i = \frac{1}{\mathbf{m^T V m}} \left(-\mathbf{m^T} \left(\mathbf{Vh} + \mathbf{u} \right) \pm \sqrt{[} \right.$

TABLE 0: Formulae Used

Substituting the given values, we have

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{0.1}$$

$$\mathbf{u} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{0.2}$$

$$f = 0 \tag{0.3}$$

$$\tag{0.4}$$

Substituting the values, we get the point of intersection as

$$\kappa_i = -\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 0+0 \\ 2+-2 \end{pmatrix} \pm \sqrt{\left[\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 0+0 \\ 2+-2 \end{pmatrix} \right]^2 + 4\left(1\right)} \tag{0.5}$$

$$\kappa_i = 2\sqrt{2} \tag{0.6}$$

$$\tag{0.7}$$

Hence, the point of intersection is $\begin{pmatrix} 2\sqrt{2} \\ 2 \end{pmatrix}$

Similarly, the other point is given by $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$.

The area bounded by the curve and the line is

$$\int_2^4 (2\sqrt{y})\,dy = \frac{4}{3}\left(8 - 2\sqrt{2}\right) \tag{0.8}$$

$$= \frac{\left(32 - 8\sqrt{2}\right)}{3} \tag{0.9}$$

$$\tag{0.10}$$

Hence the required area is $\frac{32-8\sqrt{2}}{3}$.

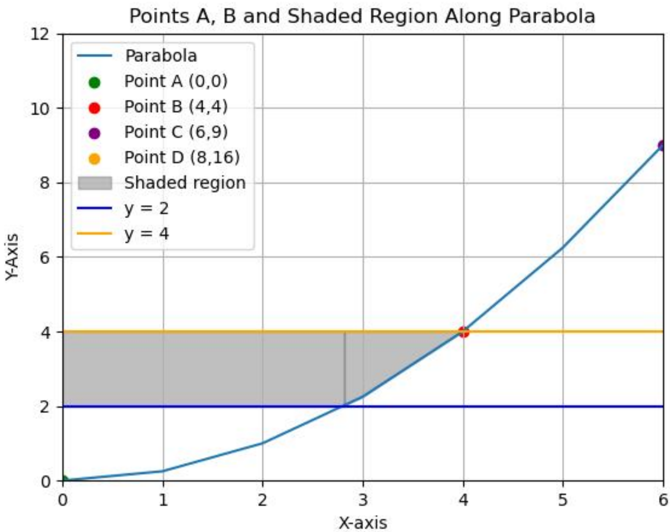

Fig. 0.1: A plot of the given question.