9.3.16

EE24BTECH11012 - Bhavanisankar G S

QUESTION
Using integration, find the area of the region enclosed by the curve $y = x^2$, the x-axis and the ordinates $x = -2$ and $x = 1$.
SOLUTION

FUNCTION	FORM
$g(\mathbf{x})$	$x^T Vx + 2\mathbf{u}^T$
The points of intersection of the line L with the conic section as above are given by $x_i = \mathbf{h} + \kappa_i \mathbf{m}$	L: x = h + $\kappa_i = \frac{1}{\mathbf{m^T V m}} \left(-\mathbf{m^T} (\mathbf{Vh} + \mathbf{u}) \pm \sqrt{[}\right.$

TABLE 0: Formulae Used

Substituting the given values, we have

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{0.1}$$
$$\mathbf{u} = \begin{pmatrix} \frac{-1}{2} \\ 0 \end{pmatrix} \tag{0.2}$$
$$f = 0 \tag{0.3}$$
$$\tag{0.4}$$

Substituting the values, we get the point of intersection as

$$\kappa_i = -\begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} \frac{-1}{2} & 0 \end{pmatrix} \pm \sqrt{\left[\begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{-1}{2} \\ 0 \end{pmatrix} \right]^2 + 1\,(1)} \tag{0.5}$$
$$\kappa_i = 1 \tag{0.6}$$
$$\tag{0.7}$$

Hence, the point of intersection is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Similarly, the other point is given by $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$.

The area bounded by the curve and the line is

$$\int_{-2}^{1} \left(x^2\right) dx = \frac{1}{3}\left(1 - (-8)\right) \tag{0.8}$$

$$= 3 \tag{0.9}$$

$$\tag{0.10}$$

Hence the required area is 3 .

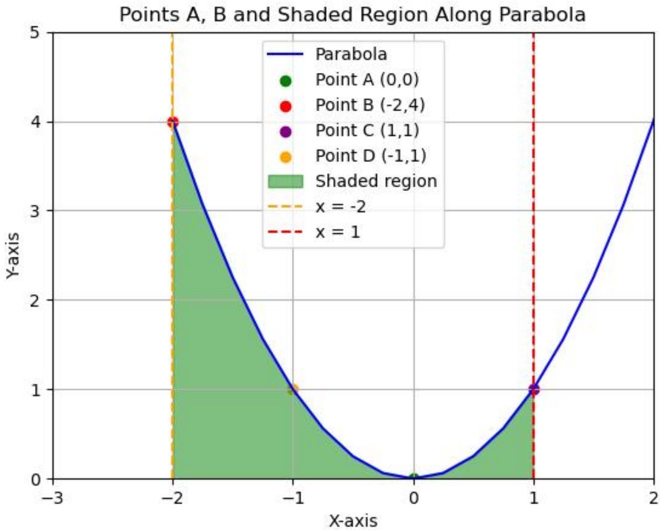

Fig. 0.1: A plot of the given question.