

**QUESTION:**  
If  $(a, b)$  is the mid-point of the line segment joining the points A  $(10, -6)$  and B  $(k, 4)$  and  $a - 2b = 18$ , find the value of  $a, b$  and the distance AB .  
**SOLUTION:**

Variable name	Description	Formula
A	10,-6.	$\mathbf{M} = \frac{\mathbf{A+B}}{2}$
B	$k,4$	$A, B, k = ?$
M	The midpoint of the line-segment AB with coordinates $a, b$	$  AB   = ?$

TABLE 0: Variables Used

We know that if **M** is the mid-point of **AB**, then

$$\mathbf{M} = \frac{\mathbf{A} + \mathbf{B}}{2} \quad (0.1)$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \frac{\begin{pmatrix} 10 \\ -6 \end{pmatrix} + \begin{pmatrix} k \\ 4 \end{pmatrix}}{2} \quad (0.2)$$

$$\Rightarrow \boxed{b = -1} \quad (0.3)$$

$$a = 18 + 2b \quad (0.4)$$

$$\Rightarrow \boxed{a = 16} \quad (0.5)$$

$$k = 2a - 10 \quad (0.6)$$

$$\Rightarrow \boxed{k = 22} \quad (0.7)$$

$$\|\mathbf{B} - \mathbf{A}\| = \sqrt{(\mathbf{B} - \mathbf{A})^T (\mathbf{B} - \mathbf{A})} \quad (0.8)$$

$$= \sqrt{\begin{pmatrix} 12 & 10 \end{pmatrix} \begin{pmatrix} 12 \\ 10 \end{pmatrix}} \quad (0.9)$$

$$\boxed{\|\mathbf{AB}\| = 2\sqrt{61}} \quad (0.10)$$

$$(0.11)$$

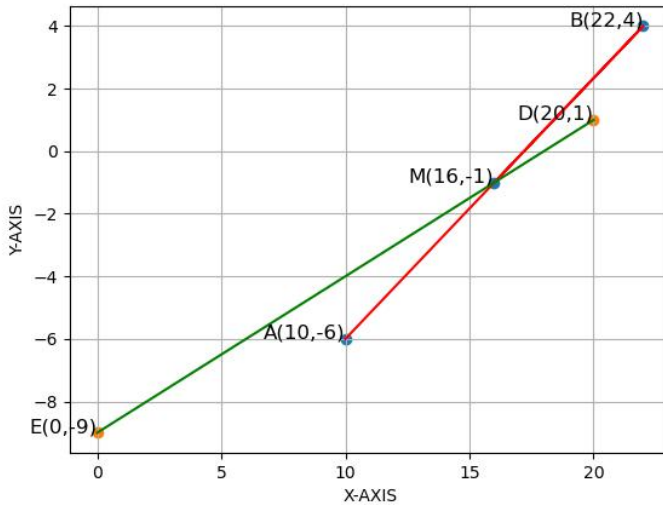


Fig. 0.1: A plot of the given question.