JEE Questions 4

EE24BTECH11012

1) Let $g(t) = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos\left(\frac{\pi}{4}t + f(x)\right)dx$, where $f(x) = \log\left(x + \sqrt{x^2+1}\right)$, x $\in$ **R**. Then which of the following is correct ? [July 2021]

a) $g(1) = g(0)$ b) $\sqrt{2}g(1) = g(0)$ c) $g(1) = \sqrt{2}g(0)$ d) $g(1) + g(0) = 0$

2) Let P be a variable point on the parabola $y = 4x^2 + 1$.Then the locus of the mid-point of the point P and the foot of perpendicular drawn from the point P to the line $y = x$ is : [July 2021]

a) $(3x - y)^2 + (x - 3y) + 2 = 0$
b) $2(3x - y)^2 + (x - 3y) + 2 = 0$
c) $(3x - y)^2 + 2(x - 3y) + 2 = 0$
d) $2(x - 3y)^2 + (3x - y) + 2 = 0$

3) The absolute value of $k \in$ **R**, for which the following system of linear equations [July 2021]

$$3x - y + 4z = 3 \quad (1)$$
$$x + 2y - 3z = -2 \quad (2)$$
$$6x + 5y + kz = -3 \quad (3)$$

has infinitely many solutions is :

a) 3 b) -5 c) 5 d) -3

4) If sum of the first 21 terms of the series $\log_{9^{\frac{1}{2}}} x + \log_{9^{\frac{1}{3}}} x + \log_{9^{\frac{1}{4}}} x + \ldots$, where x ¿ 0 is 504, then x is equal to [July 2021]

a) 243 b) 9 c) 7 d) 81

5) In a triangle ABC, if |**BC**| = 3, |**CA**| = 5 and |**BA**| = 7, then the projection of the vector **BA** on **BC** is equal to [July 2021]

a) $\frac{19}{2}$ b) $\frac{13}{2}$ c) $\frac{11}{2}$ d) $\frac{15}{2}$

I. Integer-Type Questions

1) Let $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ then $det\left(3Adj\left(2A^{-1}\right)\right)$ is equal to : [July 2021]

2) If $\left(\alpha, \beta\right)$ is a point on $y^2 = 6x$, that is closest to $\left(3, \frac{3}{2}\right)$ then find 2 $(\alpha + \beta)$ [July 2021]

3) Let a function $g : [0, 4] \to$ **R** be defined as

$$g(x) = \begin{pmatrix} max\left(t^3 - 6t^2 + 9t - 3\right), & 0 \le x \le 3 \\ 4 - x, & 3 < x \le 4 \end{pmatrix}$$

then the number of points in the interval $(0, 4)$ where g(x) is NOT differentiable is : [July 2021]

4) The number of solutions of the equation

$$\log_{x+1}\left(2x^2 + 7x + 5\right) + \log_{2x+5}(x+1)^2 - 4 = 0$$

, x $\ge$ 0, is : [July 2021]

5) Let a curve $y = y(x)$ be givem by the solutio of the differential equation

$$\cos\left(\frac{1}{2}\cos^{-1} e^{-x}\right)dx = \sqrt{e^{2x} - 1}dy$$

If it intersects y-axis at $y = -1$ and the intersection point of the curve with the x-axis is $\left(\alpha, 0\right)$, then e^{α} is equal to : [July 2021]

6) For p $\geq$ 0, a vector $\mathbf{v_2} = 2\mathbf{i} + (p+1)\mathbf{j}$ is obtained by rotating the vector $\mathbf{v_1} = \sqrt{3}p\mathbf{i} + \mathbf{j}$ by an angle θ about the origin in counter clockwise direction. If $\tan\theta = \frac{\alpha\sqrt{3}-2}{4\sqrt{3}+3}$, then the value of α is equal to : [July 2021]
7) Consider a triangle with vertices $\mathbf{A}\left(-2,3\right), \mathbf{B}\left(1,9\right), \mathbf{C}\left(3,8\right)$. If a line $\mathbf{L}$ passing through the circum-centre of the triangle ABC, bisects line BC, and intersects y-axis at point $\left(0, \frac{\alpha}{2}\right)$ then the value of real number α is : [July 2021]
8) For k $\in$ **N**, let

$$\frac{1}{\alpha(\alpha+1)(\alpha+2)\ldots(\alpha+20)} = \sum_{k=0}^{20} \frac{A_k}{\alpha+k}$$

where $\alpha > 0$.Then the value of $100\left(\frac{A_{14}+A_{15}}{A_{13}}\right)^2$ is : [July 2021]
9) Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that $a_1 = 1$, $a_2 = 1$ and $a_{n+2} = 2a_{n+1} + a_n$ for all $n \geq 1$. Then the value of $47\sum_{n=1}^{\infty}\frac{a_n}{2^{3n}}$ is equal to : [July 2021]
10) If $\lim_{x\to 0}\frac{\alpha x e^x - \beta\log(1+x) + \gamma x^2 e^{-x}}{x\sin^2 x} = 10$, α, β, $\gamma \in$ **R**, then the value of $\alpha+\beta+\gamma$ is : [July 2021]