Problem Solution

EE24BTECH11012
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Question

If (a, b) is the mid-point of the line segment joining the points A $(10, -6)$ and B $(k, 4)$ and $a - 2b = 18$, find the value of a, b and the distance AB .

Solution Outline

Find mid-point $M = \frac{A+B}{2}$
Substitute in the relation between a and b .
Solve for k and find the distance using distance formula.

Variables Used

Variable name	Description	Formula
A	10,-6.	$\mathbf{M} = \frac{\mathbf{A}+\mathbf{B}}{2}$
B	k,4	
M	The midpoint of line-segment AB	

Table: Variables Used

Solution

We know that if **M** is the mid-point of **AB**, then

$$\mathbf{M} = \frac{\mathbf{A} + \mathbf{B}}{2} \tag{1}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \frac{\begin{pmatrix} 10 \\ -6 \end{pmatrix} + \begin{pmatrix} k \\ 4 \end{pmatrix}}{2} \tag{2}$$

$$\implies \boxed{\mathbf{b = -1}} \tag{3}$$

$$a = 18 + 2b \tag{4}$$

$$\implies \boxed{\mathbf{a = 16}} \tag{5}$$

$$k = 2a - 10 \tag{6}$$

$$\implies \boxed{\mathbf{k = 22}} \tag{7}$$

$$\tag{8}$$

Solution

$$\|\mathbf{B}-\mathbf{A}\| = \sqrt{(B-A)^T (B-A)} \tag{9}$$

$$= \sqrt{\begin{pmatrix}12 & 10\end{pmatrix}\begin{pmatrix}12\\10\end{pmatrix}} \tag{10}$$

$$\boxed{\|\mathbf{AB}\| = 2\sqrt{61}} \tag{11}$$

$$\tag{12}$$

Plot

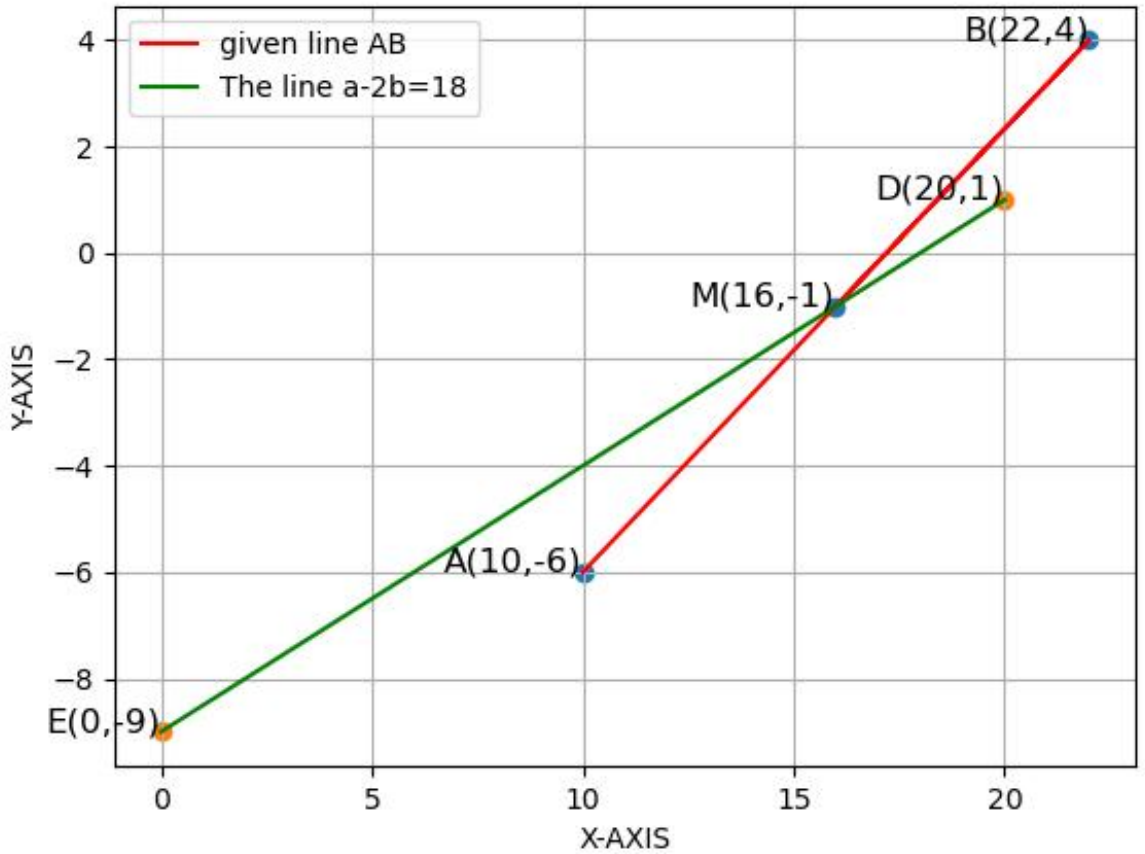
given line AB
The line a-2b=18
B(22,4)
D(20,1)
M(16,-1)
A(10,-6)
E(0,-9)
Y-AXIS
X-AXIS
4
2
0
−2
−4
−6
−8
0
5
10
15
20

Figure:

Functions defined

C-Code

Python Code

Python Code

Python Code