GATE Questions 9

EE24BTECH11012 - Bhavanisankar G S

1) Let $u(x,t)$ be the solution to the wave equation

$$\frac{\partial^2 u}{\partial x^2}(x,t) = \frac{\partial^2 u}{\partial t^2}(x,t)\,; u(x,0) = \cos 5\pi x; \frac{\partial u}{\partial t} = 0$$

Then the value of $u(1,1)$ is

2) Let $f(x) = \sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$ Then

a) $\lim_{x\to 0} f(x) = 0$
b) $\lim_{x\to 0} f(x) = 1$
c) $\lim_{x\to 0} f(x) = \frac{\pi^2}{6}$
d) $\lim_{x\to 0} f(x)$ does not exist

3) Suppose X is a random variable with $P(X = k) = (1-p)^k p$ for $k \in \{0, 1, 2, \ldots\}$ and for some $p \in (0, 1)$. For the hypothesis testing problem

$$H_0 : p = \frac{1}{2} H_1 : p \neq \frac{1}{2}$$

Consider the test "Reject H_0 if $X \le A$ or if $X \ge B$", where A ≤ B are given positive integers. The type-1 error of this test is

a) $1 + 2^{-B} + 2^{-A}$
b) $1 - 2^{-B} + 2^{-A}$
c) $1 + 2^{-B} - 2^{-A-1}$
d) $1 - 2^{-B} + 2^{-A-1}$

4) Let G be a group of order 231. The numberof elements of order 11 in G is

5) Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $f(x, y) = (e^{x+y}, e^{x-y})$. The area of the image of the region $\{(x, y) \in \mathbb{R} : 0 \le x, y \le 1\}$ under the mapping f is

a) 1
b) $e - 1$
c) e^2
d) $e^2 - 1$

6) Which of the following is a field ?

a) $\frac{\mathbb{C}(x)}{x^2+2}$
b) $\frac{\mathbb{C}(x)}{x^2+2}$
c) $\frac{\mathbb{C}(x)}{x^2-2}$
d) $\frac{\mathbb{C}(x)}{x^2-2}$

7) Let $x_0 = 0$ Define $x_{n+1} = \cos x_n$ for every $n \ge 0$. Then

a) $\{x_n\}$ is increasing and convergent
b) $\{x_n\}$ is decreasing and convergent
c) $\{x_n\}$ is convergent and $x_{2n} \le \lim_{m\to\infty} \le x_{2n+1}$ for every $n \in \mathbb{N}$
d) $\{x_n\}$ is not convergent

8) Let C be the contour $|z| = 2$ oriented in the anti-clockwise direction. The value of the integral $\int z e^{\frac{3}{z}} dz$ is

a) $3\pi i$
b) $5\pi i$
c) $7\pi i$
d) $9\pi i$

9) For each $\lambda \ge 0$, let X_λ be a random variable with exponential density $\lambda e^{-\lambda x}$ on $(0, \infty)$ Then $Var(log X_\lambda)$

a) is strictly increasing in λ
b) is strictly decreasing in λ
c) does not depend on λ
d) first increases and the decreases in λ

10) Let $\{a_n\}$ be the sequence of consecutive positive solutions of the equation $\tan x = x$ and let $\{b_n\}$ be the sequence of consecutive positive solutions of the equation $\tan \sqrt{x} = x$. Then

a) $\sum_{n=1}^{\infty} \frac{1}{a_n}$ converges but $\sum_{n=1}^{\infty} \frac{1}{b_n}$ diverges
b) $\sum_{n=1}^{\infty} \frac{1}{a_n}$ diverges but $\sum_{n=1}^{\infty} \frac{1}{b_n}$ converges
c) Both $\sum_{n=1}^{\infty} \frac{1}{a_n}$ and $\sum_{n=1}^{\infty} \frac{1}{b_n}$ converges
d) Both $\sum_{n=1}^{\infty} \frac{1}{a_n}$ and $\sum_{n=1}^{\infty} \frac{1}{b_n}$ diverges

11) Letf be an analytical function on $\overline{D} = \{z \in \mathbb{C} : |z| \le 1\}$ Assume that $|f(z)| \le 1$ for each $z \in \overline{D}$. Then, which of the following is NOT a possible value of $\left(e^f\right)(0)$?

a) 2 b) 6 c) $\frac{7}{9}e^{\frac{1}{9}}$ d) $\sqrt{2} + i\sqrt{2}$

12) The number of non-isomorphic abelian groups of order 24 is
13) Let V be the real vector space of all polynomials in one variable with real coefficiencts and having degree at most 20. Define the subspaces

$$W_1 = \left\{ p \in V : p(1) = 0; p(\frac{1}{2}) = 0; p(5) = 0; p(7) = 0 \right\}$$

$$W_2 = \left\{ p \in V : p(\frac{1}{2}) = 0; p(3) = 0; p(4) = 0; p(7) = 0 \right\}$$

Then the dimension of $W_1 \cap W_2$ is