JEE Questions 6

EE24BTECH11012 - Bhavanisankar G S

1) A plane P is parallel to two lines whose direction ratios are $\left(-2,1,3\right)$ and $\left(-1,2,-2\right)$ and it contains the point $\left(2,2,-2\right)$. Let P intersect the co-ordinate axes at the points A, B, C making the intercepts α, β, γ. If $\mathbf{V}$ is te volume of the tetrahedron OABC, where O is the origin and $p = \alpha + \beta + \gamma$, then the ordered pair $\left(\mathbf{V}, p\right)$ is equal to :

a) $\left(48,-13\right)$ b) $\left(24,-13\right)$ c) $\left(48,11\right)$ d) $\left(24,-5\right)$

2) Let S be the set of all $a \in \mathbf{R}$ for which the angle between the vectors $\mathbf{u} = a\left(\log_e b\right)\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$ and $\mathbf{v} = \log_e b\mathbf{i} + 2\mathbf{j} + 2a\log_e b\mathbf{k}$, $(b > 1)$ is acute. Then S is equal to

a) $\left(-\infty,-\frac{4}{3}\right)$ b) ϕ c) $\left(-\frac{4}{3},0\right)$ d) $\left(\frac{12}{7},\infty\right)$

3) A horizontal park is in the shape of a triangle OAB with AB = 16. A vertical lamp post OP is erected at the point O such that $\angle PAO = \angle PBO = 15°$ and $\angle PCO = 45°$, where C is the mid-point of AB. Then $(OP)^2$ is equal to

a) $\frac{32}{\sqrt{3}}\left(\sqrt{3}-1\right)$ b) $\frac{32}{\sqrt{3}}\left(2-\sqrt{3}\right)$ c) $\frac{16}{\sqrt{3}}\left(\sqrt{3}-1\right)$ d) $\frac{16}{\sqrt{3}}\left(2-\sqrt{3}\right)$

4) Let A and B be two events such that $P(B|A) = \frac{2}{5}$, $P(A|B) = \frac{1}{7}$ and $P(A \cap B) = \frac{1}{9}$. Consider
S1 : $P(A' \cup B) = \frac{5}{6}$
S2 : $P(A' \cap B') = \frac{1}{18}$

a) Both S1 and S2 are true .
b) Both S1 and S2 are false.
c) S1 is true, but S2 is false.
d) S1 is false, but S1 is true.

5) Let
p : Ramesh listens to music.
q : Ramesh is out of his village.
r : It is Sunday.
s : It is Saturday.
Then the statement "Ramesh listens to music only if he is in his village and it is Sunday or Saturday" can be expressed as

a) $((\neg q) \wedge (r \vee s)) \implies p$ c) $p \implies (q \wedge (r \vee s))$
b) $(q \wedge (r \vee s)) \implies p$ d) $p \implies ((\neg q) \wedge (r \vee s))$

I. Integer-Type Questions

1) Let the coefficients of the middle terms in the expansion of $\left(\frac{1}{\sqrt{6}} + \beta x\right)^4$, $(1 - 3\beta x)^2$ and $\left(1 - \frac{\beta}{2}x\right)^6$, $(\beta \geq 0)$, respectively form the first three terms of an A.P. If d is the common difference of this A.P., then the value of $50 - \frac{2d}{\beta^2}$ is equal to :
2) A class contains b boys and g girls. If the number of ways of selecting 3 boys and 2 girls from the class is 168, then b + 3g is equal to :
3) Let the tangents at the points P and Q on the ellipse $\frac{x^2}{2} + \frac{y^2}{4} = 1$ meet at the point $\mathbf{R}\left(\sqrt{2}, 2\sqrt{2} - 2\right)$. If S is the focus of the ellipse on its negative major axis, then $(SP)^2 + (SQ)^2$ is equal to :
4) If $1 + \left(2 + {}^{49}C_1 + {}^{49}C_2 + \cdots + {}^{49}C_{49}\right)\left({}^{50}C_2 + {}^{50}C_4 + \cdots + {}^{50}C_{50}\right)$ is equal to $2^n m$, where m is odd, then n + m is equal to :
5) Two tangent lines l1 and l2 are drawn from the point $\left(2,0\right)$ to the parabola $2y^2 = x$. If the lines l1 and l2 are also tangent to the circle $(x-5)^2 + y^2 = r$, then $17r$ is equal to :

6) If $\frac{6}{3^{12}} + \frac{10}{3^{11}} + \frac{20}{3^{10}} + \frac{40}{3^9} + \cdots + \frac{10240}{3} = 2^n m$, where m is odd, then $m \cdot n$ is equal to :

7) Let $S = \left[-\pi, \frac{\pi}{2}\right) - \left\{\frac{-\pi}{2}, \frac{-\pi}{4}, \frac{-3\pi}{4}, \frac{\pi}{4}\right\}$. Then the number of elements in the set

$$A = \left\{\theta \in S : \tan\theta\left(1 + \sqrt{5}\tan 2\theta\right) = \sqrt{5} - \tan 2\theta\right\}$$

is :

8) Let $z = a + ib, b \neq 0$ be complex numbers satisfying $z^2 = \bar{z}2^{1-|z|}$ Then the least value of $n \in \mathbf{N}$ suh that $z^n = (z+1)^n$ is equal to :

9) A bag contains white and 6 black balls. Three balls are drawn at random from the bag. Let X be the number of white balls, among the drawn balls. If σ^2 is the variance of X, then $100\ \sigma^2$ is equal to

10) The value of the integral $\int_0^{\frac{\pi}{2}} 60\frac{\sin 6x}{\sin x}dx$ is equal to :