GATE Questions 7

EE24BTECH11012 - Bhavanisankar G S

1) The maximum value of the function $f(x, y, z) = xyz$ subject to the constraint $xy+yz+xz-a = 0, a > 0$ is

a) $a^{\frac{3}{2}}$ b) $\frac{a}{3}^{\frac{3}{2}}$ c) $\frac{3}{a}^{\frac{3}{2}}$ d) $\frac{3a}{2}^{\frac{3}{2}}$

2) The function $\int_0^1 \left(y^2 + 4y + 8ye^x\right) ds$, $y(0) = \frac{-4}{3}$, $y(1) = \frac{-4e}{3}$ possesses :

a) strong minima on $y = \frac{-1}{3}e^x$
b) strong minima on $y = \frac{-4}{3}e^x$
c) strong maxima on $y = \frac{-4}{3}e^x$
d) weak maxima on $y = \frac{-1}{3}e^x$

3) A particle of mass m is constrained to move on a circle with radius a which itself is rotating about its vertical diameter with a constant agular velocity ω. Assume that the initial angular veloity is zero and g is the acceleration due to gravity. If θ be the inclination of the radius vector of the particle with the axis of rotation and θ denotes the derivative of θ with respect to t, then the Lagrangian of this system is

a) $\frac{1}{2}ma^2(\theta^2 + \omega \sin \theta^2) + mga \cos \theta$
b) $\frac{1}{2}ma^2(\theta^2 + 2\omega \sin \theta^2) - mga \cos \theta$
c) $\frac{1}{2}ma^2(\theta^2 + \omega \cos \theta) + mga \cos \theta$
d) $\frac{1}{2}ma^2(\theta^2 + \omega \sin 2\theta) + mga \sin \theta$

4) For the matrix

$$M = \begin{pmatrix} 2 & 3+2i & -4 \\ 3-2i & 5 & 6i \\ -4 & -6i & 3 \end{pmatrix}$$

which of the following statements are correct ?
P: M is skew-symmetric and iM is Hermitian
Q: M is Hermitian and iM is skew symmetric
R: eigen values of M are real
S: eigen values of iM are real

a) P and R only b) Q and R only c) P and S only d) Q and S only

5) Let $T : P_3 \rightarrow P_3$ be the map given by $T(P(X)) = \int_1^x p'(t)dt$. If the matrix of T relative to the standard bases $B_1 = B_2 = \left\{1, x, x^2, x^3\right\}$ is M and M' denotes the transpose of the matrix M, then $M + M'$ is

a) $\begin{pmatrix} 0 & -1 & -1 & -1 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$
b) $\begin{pmatrix} -1 & 0 & 0 & 2 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 2 & 0 & 2 & -1 \end{pmatrix}$
c) $\begin{pmatrix} 2 & 0 & 0 & -1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & -1 \\ -1 & 0 & -1 & 0 \end{pmatrix}$
d) $\begin{pmatrix} 0 & 2 & 2 & 2 \\ 2 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 2 & 0 & 0 & -1 \end{pmatrix}$

6) Using Euler's method taking step size = 0.1, the approximate value of Yy obtained using corresponding to x = 0.2 for the initial value problem $\frac{dy}{dx} = x^2 + y^2$ and $y(0) = 1$ is

a) 1.322 b) 1.122 c) 1.222 d) 1.110

7) The following table gives the unit transportation costs, the supply at each origin and the demand of each destination for a transportation problem. Let x_0 denote the number of units to be transported from origin i to destination j. If the u-v method is applied to improve the basic feasible solution given by $X_{12} = 60, x_{22} = 10, x_{23} = 50, x_{24} = 20, x_{31} = 40$ and $x_{34} = 60$, the the variables entering the leaving the basis respectively are

TABLE 7
TABLE 2

3	4	8	7
7	3	7	6
3	9	3	4

a) x_{21} and x_{24}
b) x_{13} and x_{23}
c) x_{14} and x_{24}
d) x_{33} and x_{24}

8) Consider the system of equations

$$\begin{pmatrix} 5 & -1 & 1 \\ 2 & 4 & 0 \\ 1 & 1 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ 12 \\ -1 \end{pmatrix}$$

Using Jacobi's method with the initial guess $\begin{pmatrix} x^(0) & y^(0) & z^(0) \end{pmatrix} = \begin{pmatrix} 2.0 & 3.0 & 0.0 \end{pmatrix}$, the approximate solution $\begin{pmatrix} x^(2) & y^(2) & z^(2) \end{pmatrix}$ after two iterations, is

a) $\begin{pmatrix} 2.64 & -1.70 & -1.12 \end{pmatrix}$
b) $\begin{pmatrix} 2.64 & -1.70 & 1.12 \end{pmatrix}$
c) $\begin{pmatrix} 2.64 & 1.70 & -1.12 \end{pmatrix}$
d) $\begin{pmatrix} 2.64 & 1.70 & 1.12 \end{pmatrix}$

The optimal table for the primal linear programming problem :
Maximize $z = 6x_1 + 12x_2 + 12x_3 - 6x_4$
Subject to

$$x_1 + x_2 + x_3 = 4$$

$$x_1 + 4x_2 + x_4 = 8$$

$x_1, x_2, x_3, x_4 \geq 0$

TABLE 8
TABLE 1

Basic Variables	x_1	x_2	x_3	x_4	RHS Constants
x_3	$\frac{3}{4}$	0	1	$\frac{-1}{4}$	2
x_2	$\frac{1}{4}$	1	0	$\frac{1}{4}$	2
$z_j - c_j$	6	0	0	6	$z = 48$

9) If y_1 and y_2 ate the dual variables correspondinng to the first and second primal constraints, then their values values in the optimal solutions of the dual problem are respectively

a) 0 and 6
b) 12 and 0
c) 6 and 3
d) 4 and 4

10) If the right hand side of the second constraint is changed from 8 to 20, then in the optimal solution of the primal problem, the basic variables will be

a) x_1 and x_2
b) x_1 and x_3
c) x_2 and x_3
d) x_2 and x_4

Consider the Fredholm integral equation

$$u(x) = x + \lambda \int_0^1 x e^t u(t) dt$$

11) The resolvent kernel R(x,t, λ) for this integral equation is

a) $\frac{xe^t}{1-\lambda}4$ b) $\frac{\lambda xe^t}{1+\lambda}$ c) $\frac{xe^t}{1+\lambda^2}$ d) $\frac{xe^t}{1-\lambda^2}$

12) The solution of this integral equation is

a) $\frac{x+1}{1-\lambda}$ b) $\frac{x^2}{1-\lambda^2}$ c) $\frac{x}{1+\lambda^2}$ d) $\frac{x}{1-\lambda}$

The joint probability density function of two random variables X and Y is given as

$$f(x.y) = \begin{cases} \frac{6}{5}\left(x + y^2\right), & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & elsewhere \end{cases}$$

13) E(X) and E(Y) are, respectively

a) $\frac{2}{5}$ and $\frac{3}{5}$ b) $\frac{3}{5}$ and $\frac{3}{5}$ c) $\frac{3}{5}$ and $\frac{6}{5}$ d) $\frac{4}{5}$ and $\frac{6}{5}$

14) Cov(X,Y) is

a) -0.01 b) 0 c) 0.01 d) 0.02