EE1060 - DETT GROUP QUIZ 2

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1 What is convolution ?

Convolution is a mathematical operation on two functions, f and g, as the integral of the product of the two functions after one is reflected about the y-axis and shifted, i.e., If we are convolving a signal with a kernel, at each time t, we are sliding the kernel over the signal and computing how much they overlap. This is useful in various fields of study like finding the system response given the impulse response, signal processing and in probability.
Given two functions $f(x)$ and $g(x)$, convolution of the two signals,

$$x(t) = f(t) * g(t) \tag{1}$$
$$= \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau \tag{2}$$

Some properties of convolution :

1. **Commutativity** :

$$f(t) * g(t) = g(t) * f(t)$$

2. **Distributivity** :

$$f(t) * [g(t) + h(t)] = [f(t) * g(t)] + [f(t) * h(t)]$$

3. **Associativity** :

$$f(t) * [g(t) * h(t)] = [f(t) * g(t)] * h(t)$$

4. **Time shifting property** : If we are given $y(t) = x_1(t) * x_2(t)$, then

$$x_1(t) * x_2(t-T) = y(t-T)$$
$$x_1(t-T) * x_2(t) = y(t-T)$$
$$x_1(t-T_1) * x_2(t-T_2) = y(t-T_1-T_2)$$

5. **Width property** : If the duration of signals $f(t)$ and $g(t)$ are T_1 and T_2 respectively, then the duration of signal convolving $f(t)$ and $g(t)$ is equal to $T_1 + T_2$

2 Convolution of sinc function

Given kernel,

$$h(t) = \begin{cases} 1, & \text{for } -T \leq t \leq T \\ 0, & \text{otherwise} \end{cases}$$

and the function

$$f(t) = sinc(t)$$
$$= \frac{sint}{t}$$

Let $y(t) = x(t) * f(t)$, then

$$\begin{aligned} y(t) &= h(t) * f(t) \\ &= f(t) * h(t) \\ &= \int_{-\infty}^{\infty} f(\tau)h(t-\tau)d\tau \end{aligned}$$

$$h(t-\tau) = \begin{cases} 1, & \text{for } -T \leq t-\tau \leq T \\ 0, & \text{otherwise} \end{cases}$$

$$h(t-\tau) = \begin{cases} 1, & \text{for } \tau - T \leq t \leq \tau + T \\ 0, & \text{otherwise} \end{cases}$$

Therefore, the convolution becomes,

$$y(t) = \int_{t-T}^{t+T} \frac{sin(\tau)}{\tau} * 1 d\tau$$

$$y(t) = \int_{t-T}^{t+T} \frac{sin(\tau)}{\tau} d\tau$$

This integration can be numerically computed and can be seen to be the following -

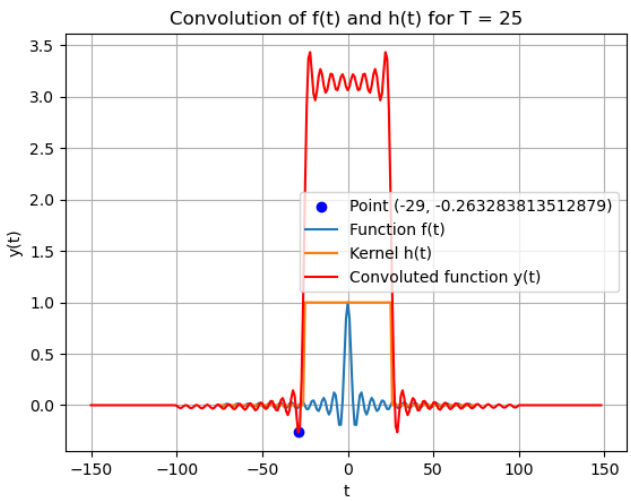

Figure 1: T = 25

For different values of T, the change in $y(t)$ is depicted in the following -

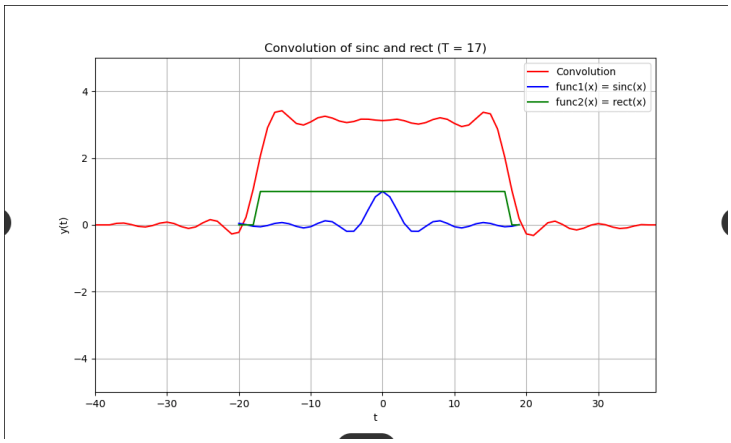

Figure 2: T = 17

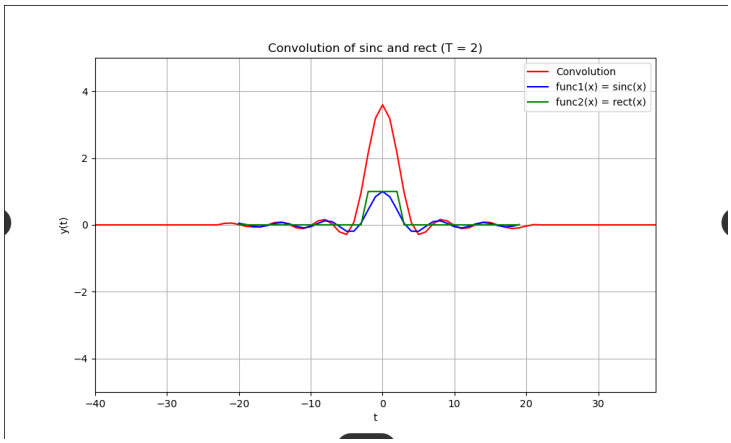

Figure 3: T = 2

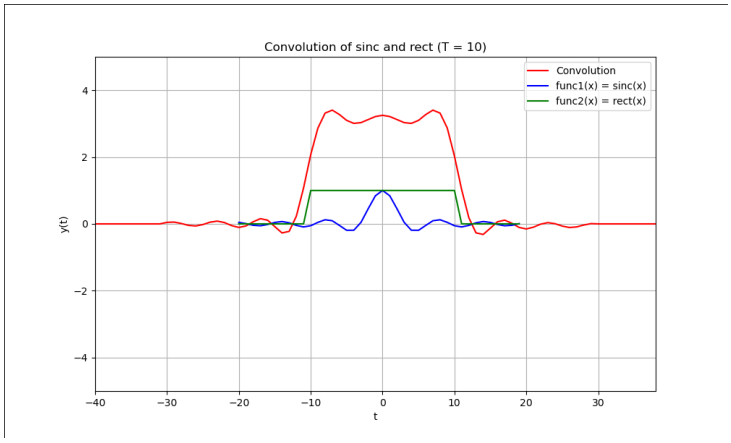

Figure 4: T = 10

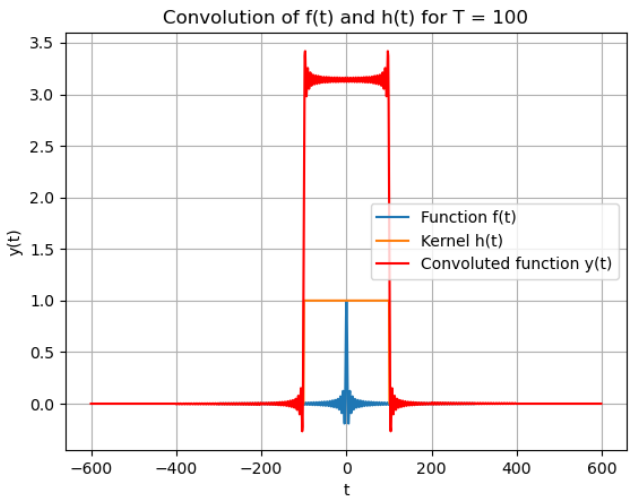

Figure 5: T = 100

It can be seen that as T increases, $y(t)$ becomes more square.

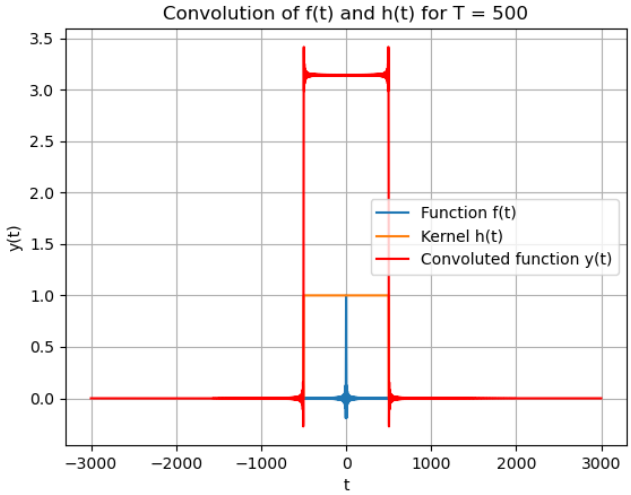

Figure 6: T = 500

2.1 Considering the kernel for $t > 0$

The response of the system for different values of T are as follows -

It can be seen that the width of the convolution becomes half and it becomes shifted

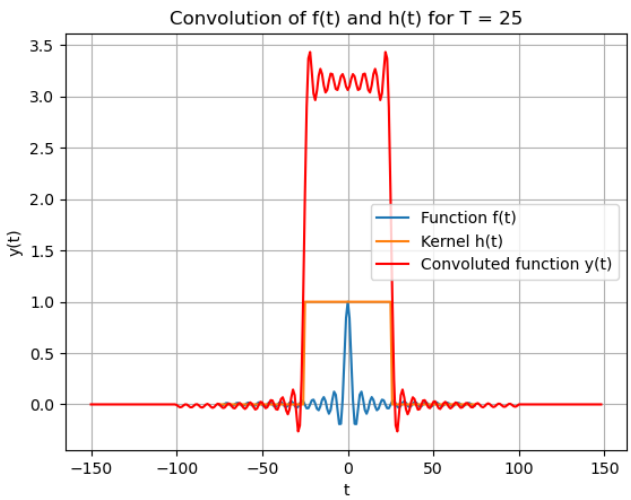

Figure 7: T = 25

to the right.

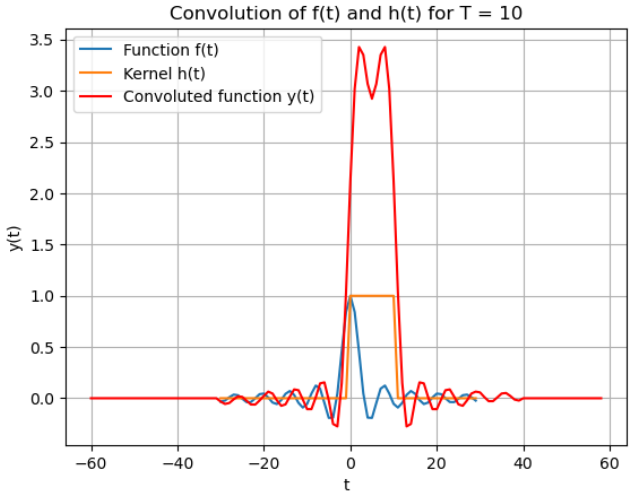

Figure 8: T = 10

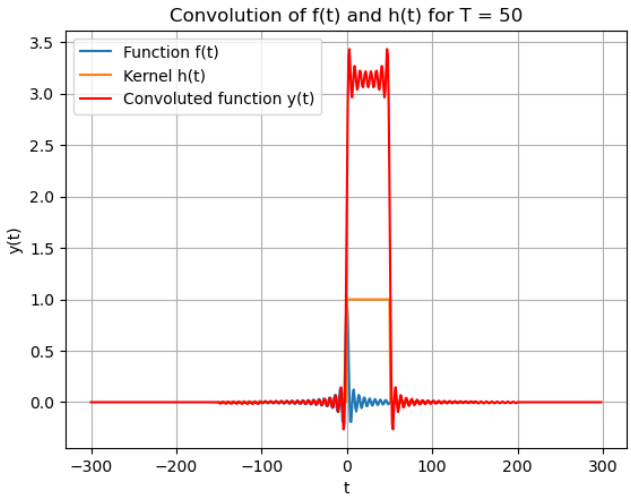

Figure 9: T = 50

2.2 Shifting the kernel by t_0

Shifting of kernel means moving the kernel to the left or right by an amount, i.e., if the given kernel is shifted by an amount t_0, then it becomes,

$$h(t) = \begin{cases} 1, & \text{for } -T \leq t - t_0 \leq T \\ 0, & \text{otherwise} \end{cases}$$

$$h(t) = \begin{cases} 1, & \text{for } -T + t_0 \leq t \leq T + t_0 \\ 0, & \text{otherwise} \end{cases}$$

If we are convolving a signal with the kernel, at each time t, we are sliding the kernel over the signal and computing how much they overlap. So, when we shift the kernel by t_0, we are changing when the kernel has its maximum influence. This can physically mean applying a response t_0 time units early (or) t_0 time units late depending upon the sign

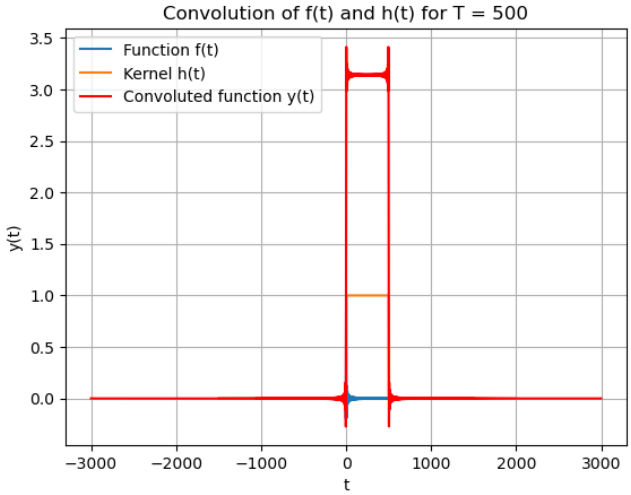

Figure 10: T = 500

of t_0, i.e., if $t_0 > 0$, the response is delayed and is advanced for $t_0 < 0$. This results in the convoluted signal also to be shifted by the same amount, as in figures 11 and 12 -

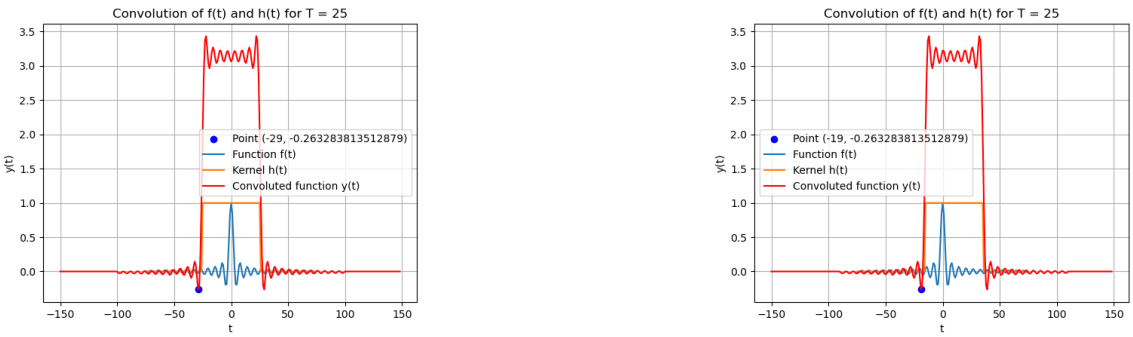

Figure 11: Shifting toward right

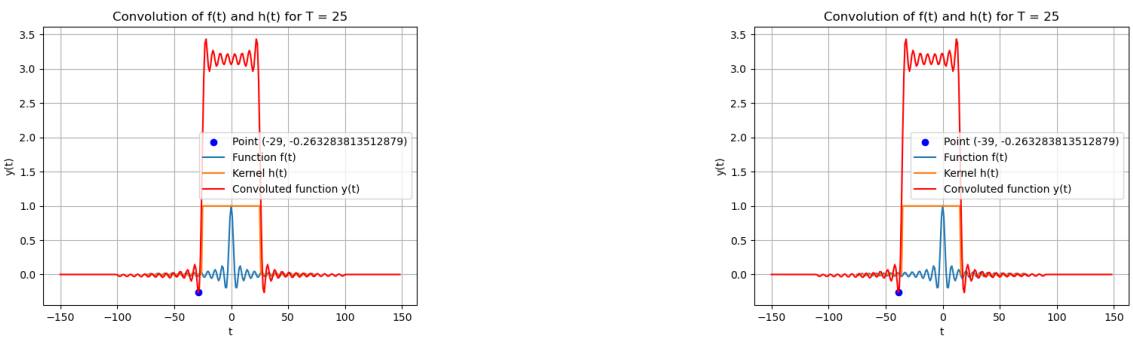

Figure 12: Shifting toward left

Shifting of the kernel by 10 units results in shifting of convoluted function by 10 units.

3 Significance of time-shift

1. In time-delayed systems, a positive shift (t_0) means that the system responds later to changes in the input. This can cause the system to react too slowly, which may lead to oscillations or instabilities in feedback systems, especially if the system relies on feedback loops (like in control systems or biological systems).

2. A negative shift ($t < 0$) might result in the system anticipating the input and responding before it occurs. This could cause overreaction or instability if the system is not designed to handle anticipatory behavior.

3. In devices like thermostat, the system may respond to temperature changes with a delay because of the time it takes to heat/cool the room. By shifting of kernel, if the delay becomes too large, then the system might not be able to track the input accurately, leading to instability (or) oscillations.

4. In the context of signal processing, shifting the kernel corresponds to a phase shift in the signal, e.g., in delay filters, the impulse response can be delayed.

5. In biological systems, time delays are common in processes like hormonal responses, neural signals, or metabolic pathways. Shifting the kernel models how these systems react over time. For example, the delayed release of insulin after glucose intake could be modeled using a time-shifted kernel.