1

GATE MA 2013

EE25BTECH11030-AVANEESH

Q.1 - Q.25 CARRY ONE MARK EACH. 1) The possible set of eigen values of a 4×4 skew-symmetric orthogonal real matrix is b) $\{\pm i, \pm 1\}$ a) $\{\pm i\}$ c) $\{\pm 1\}$ d) $\{0, \pm i\}$ (GATE MA 2013) 2) The coefficient of $(z - \pi)^2$ in the Taylor series expansion of $f(z) = \begin{cases} \frac{\sin z}{z - \pi} & \text{if } z \neq \pi \\ -1 & \text{if } z = \pi \end{cases}$ around π is b) $-\frac{1}{2}$ c) $\frac{1}{6}$ a) $\frac{1}{2}$ (GATE MA 2013) 3) Consider \mathbb{R}^2 with the usual topology. Which of the following statements are TRUE for all A, B $\subseteq \mathbb{R}^2$? $P: \overline{A \cup B} = \overline{A} \cup \overline{B}$ $O: \overline{A \cap B} = \overline{A} \cap \overline{B}$ $R: (A \cup B)^{\circ} = A^{\circ} \cup B^{\circ}$ $S: (A \cap B)^{\circ} = A^{\circ} \cap B^{\circ}.$ a) P and R only b) P and S only c) Q and R only d) Q and S only (GATE MA 2013) 4) Let $f: \mathbb{R} \to \mathbb{R}$ be a continuous function with f(1) = 5 and f(3) = 11. If $g(x) = \int_1^3 f(x+t)dt$, then g'(0) is equal to 5) Let P be a 2×2 complex matrix such that trace(P) = 1 and det(P) = -6. Then, $trace(P^4 - P^3)$ is (GATE MA 2013) 6) Suppose that R is a unique factorization domain and that a, $b \in R$ are distinct irreducible elements. Which of the following statements is TRUE? a) The ideal $\langle 1 + a \rangle$ is a prime ideal b) The ideal $\langle a+b \rangle$ is a prime ideal c) The ideal $\langle 1 + ab \rangle$ is a prime ideal d) The ideal $\langle a \rangle$ is not necessarily a maximal ideal (GATE MA 2013) 7) Let X be a compact Hausdorff topological space and let Y be a topological space. Let $f: X \to Y$ be a bijective continuous mapping. Which of the following is TRUE? a) f is a closed map but not necessarily an open map b) f is an open map but not necessarily a closed map c) f is both an open map and a closed map d) f need not be an open map or a closed map (GATE MA 2013)

8) Consider the linear programming problem:

Maximize $x + \frac{3}{2}y$

subject to $2x + 3y \le 16$,

$$x + 4y \le 18,$$

$$x \ge 0, y \ge 0.$$

If S denotes the set of all solutions of the above problem, then

a) S is empty

c) S is a line segment

b) S is a singleton

d) S has positive area

(GATE MA 2013)

- 9) Which of the following groups has a proper subgroup that is NOT cyclic?
 - a) $\mathbb{Z}_{15} \times \mathbb{Z}_{77}$
 - b) S_3
 - c) $(\mathbb{Z}, +)$
 - d) $(\mathbb{Q}, +)$

(GATE MA 2013)

- 10) The value of the integral $\int_0^\infty \int_x^\infty \left(\frac{1}{y}\right) e^{-y/2} dy dx$ is _____.

 11) Suppose the random variable U has uniform distribution on [0,1] and $X = -2 \log U$. The density of X is

a)
$$f(x) = \begin{cases} e^{-x} & \text{if } x > 0\\ 0 & \text{otherwise} \end{cases}$$

b) $f(x) = \begin{cases} 2e^{-2x} & \text{if } x > 0\\ 0 & \text{otherwise} \end{cases}$
c) $f(x) = \begin{cases} \frac{1}{2}e^{-x/2} & \text{if } x > 0\\ 0 & \text{otherwise} \end{cases}$
d) $f(x) = \begin{cases} 1/2 & \text{if } x \in [0, 2]\\ 0 & \text{otherwise} \end{cases}$

b)
$$f(x) = \begin{cases} 2e^{-2x} & \text{if } x > 0\\ 0 & \text{otherwise} \end{cases}$$

c)
$$f(x) = \begin{cases} \frac{1}{2}e^{-x/2} & \text{if } x > 0\\ 0 & \text{otherwise} \end{cases}$$

d)
$$f(x) = \begin{cases} 1/2 & \text{if } x \in [0, 2] \\ 0 & \text{otherwise} \end{cases}$$

(GATE MA 2013)

- 12) Let f be an entire function on \mathbb{C} such that $|f(z)| \le 100 \log |z|$ for each z with $|z| \ge 2$. If f(i) = 2i, then f(1)
 - a) must be 2

c) must be i

b) must be 2i

d) cannot be determined from the given data

(GATE MA 2013)

- 13) The number of group homomorphisms from \mathbb{Z}_3 to \mathbb{Z}_9 is _____. (GATE MA 2013) 14) Let u(x,t) be the solution to the wave equation $\frac{\partial^2 u}{\partial x^2}(x,t) = \frac{\partial^2 u}{\partial t^2}(x,t)$, with $u(x,0) = \cos(5\pi x)$, $\frac{\partial u}{\partial t}(x,0) = \cos(5\pi x)$. 0. Then, the value of u(1, 1) is _ 15) Let $f(x) = \sum_{n=1}^{\infty} \frac{\sin(nx)}{n^2}$. Then (GATE MA 2013)
- - a) $\lim_{x\to 0} f(x) = 0$

c) $\lim_{x\to 0} f(x) = \pi^2/6$

b) $\lim_{x\to 0} f(x) = 1$

d) $\lim_{x\to 0} f(x)$ does not exist

(GATE MA 2013)

16) Suppose X is a random variable with $P(X = k) = (1 - p)^k p$ for $k \in \{0, 1, 2, ...\}$ and some $p \in (0, 1)$. For the hypothesis testing problem

$$H_0: p = \frac{1}{2}H_1: p \neq \frac{1}{2}$$

, consider the test "Reject H_0 if $X \le A$ or if $X \ge B$ " where A < B are given positive integers. The type-I error of this test is

a) $1 + 2^{-B} - 2^{-B}$ b) $1 - 2^{-B} + 2^{-B}$ c) $1 + 2^{-B} - 2^{-B}$ d) $1 - 2^{-B} + 2^{-B}$	-A -A-1		
			(GATE MA 2013
is 18) Let $f : \mathbb{R}^2 \to$	oup of order 231. The num $\mathbb{R}^2 \text{ be defined by } f(x, y) =$ 11 under the mapping f is		er 11 in G (GATE MA 2013) of the image of the region $\{(x, y) \in (x, y) \in (x, y) \in (x, y) \}$
a) 1	b) <i>e</i> – 1	c) e^2	d) $e^2 - 1$
19) Which of the a) $\mathbb{C}[x]/\langle x^2 + 2z^2 \rangle$ b) $\mathbb{Z}[x]/\langle x^2 + 2z^2 \rangle$ c) $\mathbb{Q}[x]/\langle x^2 - 2z^2 \rangle$ d) $\mathbb{R}[x]/\langle x^2 - 2z^2 \rangle$	2) 2)		(GATE MA 2013
20) Let $x_0 = 0$. Do a) $\{x_n\}$ is increase. b) $\{x_n\}$ is decrease. c) $\{x_n\}$ is converged.	efine $x_{n+1} = \cos x_n$ for every easing and convergent easing and convergent ergent and $x_{2n} < \lim_{m \to \infty} x_m$		(GATE MA 2013
d) $\{x_n\}$ is not of 21) Let C be the	_	in the anti-clockwise	(GATE MA 2013 direction. The value of the integral

2 value of the integral

a) $3\pi i$

b) $5\pi i$

c) $7\pi i$

d) $9\pi i$

(GATE MA 2013)

- 22) For each $\lambda > 0$, let X_{λ} be a random variable with exponential density $\lambda e^{-\lambda x}$ on $(0, \infty)$. Then, $Var(\log X_{\lambda})$
 - a) is strictly increasing in λ
 - b) is strictly decreasing in λ
 - c) does not depend on λ
 - d) first increases and then decreases in λ

(GATE MA 2013)

- 23) Let $\{a_n\}$ be the sequence of consecutive positive solutions of the equation $\tan x = x$ and let $\{b_n\}$ be the sequence of consecutive positive solutions of the equation $\tan \sqrt{x} = x$. Then
 - a) $\sum_{n=1}^{\infty} \frac{1}{a_n}$ converges but $\sum_{n=1}^{\infty} \frac{1}{b_n}$ diverges

 c) Both $\sum_{n=1}^{\infty} \frac{1}{a_n}$ and $\sum_{n=1}^{\infty} \frac{1}{b_n}$ converge

 b) $\sum_{n=1}^{\infty} \frac{1}{a_n}$ diverges but $\sum_{n=1}^{\infty} \frac{1}{b_n}$ converges

 d) Both $\sum_{n=1}^{\infty} \frac{1}{a_n}$ and $\sum_{n=1}^{\infty} \frac{1}{b_n}$ diverge

(GATE MA 2013)

24) Let f be an analytic function on $\overline{D} = \{z \in \mathbb{C} : |z| \le 1\}$. Assume that $|f(z)| \le 1$ for each $z \in \overline{D}$. Then, which of the following is NOT a possible value of $(e^f)''(0)$?

a) 2	b) 6	c) $\frac{7}{9}e^{1/9}$	d) $\sqrt{2} + i\sqrt{2}$
25) The number o	f non-isomorphic abelian	groups of order 24 is	(GATE MA 2013) (GATE MA 2013)
	Q.26 - Q.5	5 carry two marks each.	
degree at mos	real vector space of all potential 20. Define the subspaces $p(1) = 0, p(1/2) = 0, p(5)$		le with real coefficients and having
Then the dime 27) Let $f, g : [0, 1]$	p(1/2) = 0, p(3) = 0, p(4) ension of $W_1 \cap W_2$ is	_•	(GATE MA 2013)
$f(x) = \begin{cases} x & \text{if } \\ 0 & \text{ot } \end{cases}$	$x = \frac{1}{n}$ for $n \in \mathbb{N}$ and $g(x)$	$= \begin{cases} 1 & \text{if } x \in \mathbb{Q} \cap [0, 1] \\ 0 & \text{otherwise} \end{cases}.$	
b) f is Riemanc) g is Rieman	g are Riemann integrable n integrable and g is Lebe nn integrable and f is Lebe or g is Riemann integrable	esgue integrable	(GATE MA 2013)
28) Consider the f Maximize $x + z$ subject to 5x + y + 6z + 3z x + 2y + 2z + 9z $x \ge 0, y \ge 0, z$	$7w \le 20,$ $9w \le 40,$	ing problem:	(GATE MA 2013)
29) Suppose X is	mal value is a real-valued random vari $E[X^2]$ respectively?	able. Which of the follo	(GATE MA 2013) owing values CANNOT be attained
a) 0 and 1	b) 2 and 3	c) $\frac{1}{2}$ and $\frac{1}{3}$	d) 2 and 5
a) $\{(x, y) \in \mathbb{R}^2$	following subsets of \mathbb{R}^2 is : $1 \le x \le 1, y = \sin x$: $-1 \le y \le 1, y = x^8 - x^3 - y$	_	(GATE MA 2013)
c) $\{(x,y)\in\mathbb{R}^2$: $y = 0$, $\sin(e^x) = 0$ } : $x > 0$, $y = \sin(\frac{1}{x})$ } $\cap \{(x, y)$		
31) Let M be the			(GATE MA 2013) es. Let $T: M \to M$ be defined by
The determination $X_4 = X_5 = X_6$	$\iint_{\text{ont of T is }} \left(x_3 x_5 x_2 \right)^{\frac{1}{2}}$		(GATE MA 2013)

32) Let H be a Hilbert space and let $\{e_n : n \ge 1\}$ be an orthonormal basis of H. Suppose $T : H \to H$ is a bounded linear operator. Which of the following CANNOT be true?

a)
$$T(e_n) = e_1$$
 for all $n \ge 1$

b)
$$T(e_n) = e_{n+1}$$
 for all $n \ge 1$

c)
$$T(e_n) = \sqrt{\frac{n+1}{n}}e_n$$
 for all $n \ge 1$

b)
$$T(e_n) = e_{n+1}$$
 for all $n \ge 1$
c) $T(e_n) = \sqrt{\frac{n+1}{n}}e_n$ for all $n \ge 1$
d) $T(e_n) = e_{n-1}$ for all $n \ge 2$ and $T(e_1) = 0$

33)	The value of the limit	$\lim_{n\to\infty} \frac{2^{-n^2}}{\sum_{k=n+1}^{\infty} 2^{-k^2}} \text{ is}$			
	a) 0	b) some $c \in (0, 1)$	c) 1	d) ∞	
34)	a) f is conformal on Cb) f maps circles in Cc) All the fixed points		$\mathbb{C}: Im(z) > 0$		
	(1 ′	2 (1)		(GATE M	
35)		$ \begin{pmatrix} 2 & 0 \\ 3 & 1 \\ 1 & 3 \end{pmatrix} $ can be decompose $ \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix} $. The solut			here $L =$
	a) $(1 \ 1 \ 1)^t$	b) $(1 \ 1 \ 0)^t$	c) $(0 \ 1 \ 1)^t$	d) $\begin{pmatrix} 1 & 0 & 1 \end{pmatrix}^t$	
36)	Let $S = \{x \in \mathbb{R} : x \ge 0,$	$\sum_{n=1}^{\infty} x^{\sqrt{n}} < \infty \}.$ Then the	supremum of S is	(GATE M	1A 2013)
	a) 1	b) $\frac{1}{e}$	c) 0	d) ∞	
37)	The image of the region	on $\{z \in \mathbb{C} : Re(z) > Im(z) > Im(z) \}$	> 0} under the mapping z	(GATE No. 1) e^{z^2} is	IA 2013)
		Im(w) > 0 Im(w) > 0, w > 1			
38)	Which of the following	g groups contains a uniqu	ne normal subgroup of or	(GATE Moder four?	1A 2013)
	a) $\mathbb{Z}_2 \oplus \mathbb{Z}_4$ b) The dihedral group,	D_4 , of order eight	c) The quaternion ground) $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$	p, Q_8	
39)	by $\langle X, Y \rangle = Y^t B X$. Let	etric positive-definite $n \times A$ be an $n \times n$ real matrix $X \in \mathbb{R}^n$. If S is the adjoin	x and let $T: \mathbb{R}^n \to \mathbb{R}^n$ be	the linear operator	n defined or defined
	a) $B^{-1}A^tB$	b) BA^tB^{-1}	c) $B^{-1}AB$	d) A^t	

(GATE MA 2013)

40) Let X be an arbitrary random variable that takes values in $\{0, 1, ..., 10\}$. The minimum and maximum possible values of the variance of X are

a) 0 and 30	b) 1 and 30	c) 0 and 25	d) 1 and 25
- ·	es of rank three in M is $3(3^4 - 3^3)$ $3(3^4 - 3)$ $3(3^4 - 3^2)$	th entries in the finite fiel	(GATE MA 2013) ld of three elements. Then the
	space of dimension $m \ge 0$ for some $n \ge 1$. Then v		(GATE MA 2013) inear transformation such that necessarily TRUE?
a) $Rank(T^n) \le Nu$ b) $trace(T) \ne 0$	$llity(T^n)$	c) T is diagonalizabd) n = m	le
f(x,y) = ax + by +	- c has maximum value M	and minimum value N o	(GATE MA 2013) et X have 7 vertices. Suppose on X and $N < M$. Let $S = \{P : \text{ch of the following statements}\}$
a) n cannot be 5	b) n can be 2	c) n cannot be 3	d) n can be 4
P: If $f \in L^1(\mathbb{R})$, the Q: If $f \in L^1(\mathbb{R})$ are R: If $f \in L^1(\mathbb{R})$, the	owing statements are TRUI ten f is continuous. In the continuous of $\lim_{ x \to\infty} f(x)$ exists, then the f is bounded. In the continuous, then	the limit is zero.	(GATE MA 2013) equals zero.
a) Q and S only	b) P and R only	c) P and Q only	d) R and S only
<i>iy</i>)) $\sin \theta d\theta$. Which a) g is a harmonic	of the following statemer	C. Let $g: \mathbb{R}^2 \to \mathbb{R}$ be definits is TRUE?	(GATE MA 2013) ned by $g(x, y) = \frac{1}{2\pi} \int_0^{2\pi} u(e^{i\theta}(x + e^{i\theta})) dx$

- c) g is harmonic but not a polynomial
- d) g is neither harmonic nor a polynomial

- 46) Let $S = \{z \in \mathbb{C} : |z| = 1\}$ with the induced topology from \mathbb{C} and let $f : [0,2] \to S$ be defined as $f(t) = e^{2\pi i t}$. Then, which of the following is TRUE?
 - a) K is closed in $[0,2] \implies f(K)$ is closed in S
 - b) U is open in $[0,2] \implies f(U)$ is open in S
 - c) f(X) is closed in S \implies X is closed in [0,2]
 - d) f(Y) is open in S \implies Y is open in [0,2]

(GATE MA 2013)

47) Assume that all the zeros of the polynomial $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ have negative real parts. If u(t) is any solution to the ordinary differential equation

 $a_n \frac{d^n u}{dt^n} + a_{n-1} \frac{d^{m-1} u}{dt^{n-1}} + \dots + a_1 \frac{du}{dt} + a_0 u = 0,$ then $\lim_{t\to\infty} u(t)$ is equal to

a) 0

b) 1

c) n-1

d) ∞

(GATE MA 2013)

Common Data for Questions 48 and 49

Let c_{00} be the vector space of all complex sequences having finitely many non-zero terms. Equip c_{00} with the inner product $\langle x, y \rangle = \sum_{n=1}^{\infty} x_n \overline{y_n}$ for all $x = (x_n)$ and $y = (y_n)$ in c_{00} . Define $f : c_{00} \to \mathbb{C}$ by $f(x) = \sum_{n=1}^{\infty} \frac{x_n}{n}$. Let N be the kernel of f.

- 48) Which of the following is FALSE?
 - a) f is a continuous linear functional
 - b) $||f|| \le \frac{\pi}{\sqrt{6}}$
 - c) There does not exist any $y \in c_{00}$ such that $f(x) = \langle x, y \rangle$ for all $x \in c_{00}$
 - d) $N^{\perp} \neq \{0\}$

(GATE MA 2013)

- 49) Which of the following is FALSE?
 - a) $c_{00} \neq N$
 - b) N is closed
 - c) c_{00} is not a complete inner product space
 - d) $c_{00} = N \oplus N^{\perp}$

(GATE MA 2013)

Common Data for Questions 50 and 51

Let X_1, X_2, \dots, X_n be an i.i.d. random sample from exponential distribution with mean μ . In other words, they have density $f(x) = \begin{cases} \frac{1}{\mu} e^{-x/\mu} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$.

- 50) Which of the following is NOT an unbiased estimate of μ ?

 - b) $\frac{1}{n-1}(X_2 + X_3 + \dots + X_n)$ c) $n \cdot (\min\{X_1, X_2, \dots, X_n\})$ d) $\frac{1}{n} \max\{X_1, X_2, \dots, X_n\}$

(GATE MA 2013)

- 51) Consider the problem of estimating μ . The m.s.e (mean square error) of the estimate $T(X) = \frac{X_1 + X_2 + \cdots + X_n}{n-1}$ is
 - a) μ^2

- b) $\frac{1}{n+1}\mu^2$
- c) $\frac{1}{(n+1)^2}\mu^2$ d) $\frac{n^2}{(n+1)^2}\mu^2$

(GATE MA 2013)

Statement for Linked Answer Questions 52 and 53

Let $X = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 1\} \cup ([-1,1] \times \{0\}) \cup (\{0\} \times [-1,1])$. Let $n_0 = \max\{k : k < 1\}$ ∞ , there are k distinct points $p_1, \ldots, p_k \in X$ such that $X \setminus \{p_1, \ldots, p_k\}$ is connected.

52) The value of n_0 is .

(GATE MA 2013)

53) Let q_1, \ldots, q_{n_0+1} be n_0+1 distinct points and $Y=X\setminus\{q_1, \ldots, q_{n_0+1}\}$. Let m be the number of connected components of Y. The maximum possible value of m is _____.

(GATE MA 2013)

Statement for Linked Answer Questions 54 and 55

Let $W(y_1, y_2)$ be the Wronskian of two linearly independent solutions y_1 and y_2 of the equation y'' +P(x)y' + Q(x)y = 0.

54)	The	product	$W(y_1,$	$(y_2)P$	(<i>x</i>)	equals
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a)	$y_2y_1^{\prime\prime}$	$-y_1y_2''$	

b)
$$y_1y_2' - y_2y_1'$$

c) $y'_1 y''_2 - y'_2 y''_1$ d) $y'_2 y'_1 - y''_1 y''_2$

(GATE MA 2013)

55) If $y_1 = e^{2x}$ and $y_2 = xe^{2x}$, then the value of P(0) is

a) 4

b) -4

c) 2

d) -2

(GATE MA 2013)

GENERAL APTITUDE (GA) QUESTIONS

Q.~56-Q.~60 carry one mark each.

56) A number is as much greater than 75 as it is smaller than 117. The number is:

a) 91

b) 93

c) 89

d) 96

(GATE MA 2013)

57) The professor ordered to the students to go out of the class.

Which of the above underlined parts of the sentence is grammatically incorrect?

c) III d) IV a) I b) II

(GATE MA 2013)

58) Which of the following options is the closest in meaning to the word given below:

Primeval

a) Modern

c) Primitive

b) Historic

d) Antique

(GATE MA 2013)

- 59) Friendship, no matter how _____ it is, has its limitations.
 - a) cordial
 - b) intimate
 - c) secret
 - d) pleasant

(GATE MA 2013)

60) Select the pair that best expresses a relationship similar to that expressed in the pair:

Medicine: Health

a) Science: Experiment

c) Education: Knowledge d) Money: Happiness

b) Wealth: Peace

(GATE MA 2013)

Q. 61 to Q. 65 carry two marks each.

- 61) X and Y are two positive real numbers such that $2X + Y \le 6$ and $X + 2Y \le 8$. For which of the following values of (X, Y) the function f(X, Y) = 3X + 6Y will give maximum value?
 - a) (4/3, 10/3)
 - b) (8/3, 20/3)
 - c) (8/3, 10/3)
 - d) (4/3, 20/3)

(GATE MA 2013)

62) If |4X - 7| = 5 then the values of 2|X| - |-X| is:

a) 2, 1/3

b) 1/2, 3 c) 3/2, 9 d) 2/3, 9

(GATE MA 2013)

63) Following table provides figures (in rupees) on annual expenditure of a firm for two years - 2010 and 2011.

Category	2010	2011
Raw material	5200	6240
Power & fuel	7000	9450
Salary & wages	9000	12600
Plant & machinery	20000	25000
Advertising	15000	19500
Research & Development	22000	26400

In 2011, which of the following two categories have registered increase by same percentage?

- a) Raw material and Salary & wages
- b) Salary & wages and Advertising
- c) Power & fuel and Advertising
- d) Raw material and Research & Development

(GATE MA 2013)

64) A firm is selling its product at Rs. 60 per unit. The total cost of production is Rs. 100 and firm is earning total profit of Rs. 500. Later, the total cost increased by 30%. By what percentage the price should be increased to maintained the same profit level.

a) 5

b) 10

c) 15

d) 30

(GATE MA 2013)

65) Abhishek is elder to Savar.

Savar is younger to Anshul.

Which of the given conclusions is logically valid and is inferred from the above statements?

- a) Abhishek is elder to Anshul
- b) Anshul is elder to Abhishek
- c) Abhishek and Anshul are of the same age
- d) No conclusion follows

(GATE MA 2013)

END OF QUESTION PAPER