

Matrices in Geometry 1.5.25

EE25BTECH11037 - Divyansh

Question: In what ratio does the point $\left(\frac{24}{11}, y\right)$ divide the line segment joining the points $\mathbf{P}=\begin{pmatrix} 2 \\ -2 \end{pmatrix}$ and $\mathbf{Q}=\begin{pmatrix} 3 \\ 7 \end{pmatrix}$? Also find the value of y .

Given: $P\begin{pmatrix} 2 \\ -2 \end{pmatrix}$, $Q\begin{pmatrix} 3 \\ 7 \end{pmatrix}$ and a point $R\begin{pmatrix} \frac{24}{11} \\ y \end{pmatrix}$ on PQ .

Let R divide PQ internally in the ratio $\lambda : 1$.

a) By section-formula,

$$\begin{pmatrix} \frac{24}{11} \\ y \end{pmatrix} = \frac{\begin{pmatrix} 2 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 7 \end{pmatrix}}{1 + \lambda}$$

b) Cross-multiplying

$$\begin{pmatrix} (\lambda + 1) \frac{24}{11} \\ (\lambda + 1) y \end{pmatrix} = \begin{pmatrix} 2 + 3\lambda \\ -2 + 7\lambda \end{pmatrix}$$

c) Solving for λ and cross-multiplying, we get

$$24\lambda + 24 = 22 + 33\lambda \implies 9\lambda = 2 \implies \lambda = \frac{2}{9}$$

d) Substituting the value of λ above, we get

$$y\left(\frac{2}{9} + 1\right) = -2 + 7 \times \frac{2}{9} \implies 11y = -18 + 14 \implies y = \frac{-4}{11}$$

Hence, the final answer is $\lambda = \frac{2}{9}$ and $y = \frac{-4}{11}$

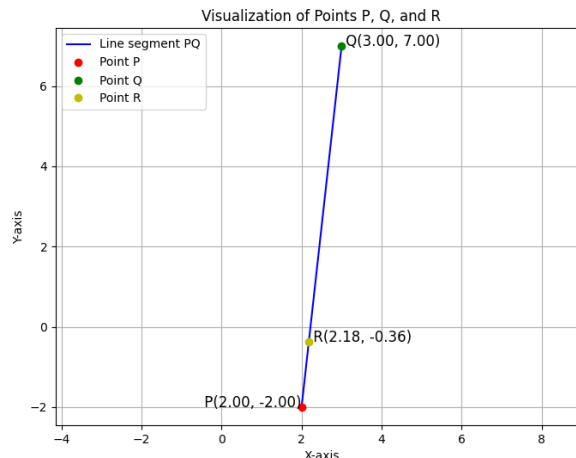


Fig. 1: Plot for 1.5.25