GATE MA 2021

EE25BTECH11030-AVANEESH

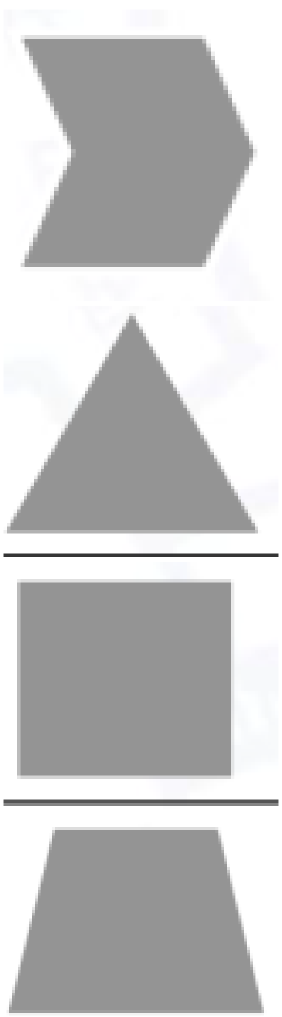
General Aptitude (GA)

Q.1 – 5 Multiple Choice Questions (MCQ), carry ONE mark each (for each wrong answer: $-1/3$).

1) The ratio of boys to girls in a class is 7 to 3. Among the options below, an acceptable value for the total number of students in the class is:
   a) 21
   b) 37
   c) 50
   d) 73

(GATE MA 2021)

2) A polygon is convex if, for every pair of points, P and Q belonging to the polygon, the line segment PQ lies completely inside or on the polygon. Which one of the following is NOT a convex polygon?



a)
b)
c)
d)

Fig. 1. *

Q.no 9 options

(GATE MA 2021)

3) Consider the following sentences: (i) Everybody in the class is prepared for the exam.
(ii) Babu invited Danish to his home because he enjoys playing chess.
Which of the following is the CORRECT observation about the above two sentences?
   a) (i) is grammatically correct and (ii) is unambiguous
   b) (i) is grammatically incorrect and (ii) is unambiguous
   c) (i) is grammatically correct and (ii) is ambiguous
   d) (i) is grammatically incorrect and (ii) is ambiguous

(GATE MA 2021)

4) A circular sheet of paper is folded along the lines in the directions shown. The paper, after being punched in the final folded state as shown and unfolded in the reverse order of folding, will look like _________.

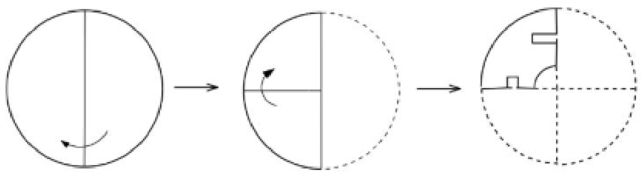
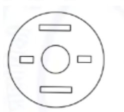


Fig. 2. Q.4.

a) 

b) 

c) 

d) 

Fig. 3. *
Q.no 9 options

(GATE MA 2021)

5) ______ is to surgery as writer is to ______. Which one of the following options maintains a similar logical relation in the above sentence?
   a) Plan, outline
   b) Hospital, library
   c) Doctor, book
   d) Medicine, grammar

(GATE MA 2021)

Q.6 – 10 Multiple Choice Questions (MCQ), carry TWO marks each (for each wrong answer: −2/3).

6) We have 2 rectangular sheets of paper, M and N, of dimensions $6\,cm \times 1\,cm$ each. Sheet M is rolled to form an open cylinder by bringing the short edges of the sheet together. Sheet N is cut into equal

square patches and assembled to form the largest possible closed cube. Assuming the ends of the cylinder are closed, the ratio of the volume of the cylinder to that of the cube is

a) $\pi/2$
b) $3/\pi$
c) $9/\pi$
d) 3π

(GATE MA 2021)

7) Items and prices:

Item	Cost ()	Profit %	Marked Price ()
P	5400	–	5860
Q	–	25	10000

The ratio of cost of item P to cost of item Q is 3:4. Discount is difference between marked price and selling price. Profit % = $\frac{\text{Selling price}-\text{Cost}}{\text{Cost}} \times 100$.
The discount on item Q, as a percentage of its marked price, is ______.

a) 25
b) 12.5
c) 10
d) 5

(GATE MA 2021)

8) There are five bags each containing identical sets of ten distinct chocolates. One chocolate is picked from each bag. The probability that at least two chocolates are identical is ______.

a) 0.3024
b) 0.4235
c) 0.6976
d) 0.8125

(GATE MA 2021)

9) Given two statements:

- Statement 1: All bacteria are microorganisms.
- Statement 2: All pathogens are microorganisms.

Conclusions:
I. Some pathogens are bacteria.
II. All pathogens are not bacteria.
Which option is logically correct?

a) Only conclusion I is correct
b) Only conclusion II is correct
c) Either conclusion I or II is correct
d) Neither conclusion I nor II is correct

(GATE MA 2021)

10) Some people suggest anti-obesity measures (AOM) such as displaying calorie information in restaurant menus. Such measures sidestep addressing the core problems that cause obesity: poverty and income inequality. Which statement summarizes the passage?

a) The proposed AOM addresses the core problems that cause obesity.
b) If obesity reduces, poverty will naturally reduce, since obesity causes poverty.
c) AOM are addressing the core problems and are likely to succeed.
d) AOM are addressing the problem superficially.

(GATE MA 2021)

Mathematics (MA)

Q.1 – 14 Multiple Choice Questions (MCQ), carry ONE mark each (for each wrong answer: $-1/3$).

1) Let A be a 3×4 matrix and B be a 4×3 matrix with real entries such that AB is non-singular. Consider:
   - P: Nullity of A is 0.
   - Q: BA is a non-singular matrix.

   Then
   a) both P and Q are TRUE
   b) P is TRUE and Q is FALSE
   c) P is FALSE and Q is TRUE
   d) both P and Q are FALSE

   (GATE MA 2021)
2) Let $f(z) = u(x,y) + iv(x,y)$, $z = x + iy \in \mathbb{C}$ be a non-constant analytic function. Let u_x, v_x, u_y, v_y denote partial derivatives. Functions defined as:

$$g_1(z) = u_x(x,y) - iu_y(x,y), \quad g_2(z) = v_x(x,y) + iv_y(x,y).$$

   Then
   a) both $g_1(z)$ and $g_2(z)$ are analytic in $\mathbb{C}$
   b) $g_1(z)$ is analytic and $g_2(z)$ is NOT analytic
   c) $g_1(z)$ is NOT analytic and $g_2(z)$ is analytic
   d) neither $g_1(z)$ nor $g_2(z)$ is analytic

   (GATE MA 2021)
3) Let $T(z) = \frac{az+b}{cz+d}$, $ad - bc \neq 0$ be the Möbius transformation mapping points:

$$z_1 = 0, z_2 = -i, z_3 = \infty$$

   onto

$$w_1 = 10, w_2 = 5 - 5i, w_3 = 5 + 5i,$$

   respectively. Then the image of the set $S = \{z \in \mathbb{C} : \text{Re}(z) < 0\}$ under $w = T(z)$ is
   a) $\{w \in \mathbb{C} : |w| < 5\}$
   b) $\{w \in \mathbb{C} : |w| > 5\}$
   c) $\{w \in \mathbb{C} : |w - 5| < 5\}$
   d) $\{w \in \mathbb{C} : |w - 5| > 5\}$

   (GATE MA 2021)
4) Let R be the row reduced echelon form of a 4×4 real matrix A and let the third column of R be $\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ Consider the following statements:
   - P: If $\begin{pmatrix} \alpha \\ \beta \\ \gamma \\ 0 \end{pmatrix}$ this is a solution of $Ax = 0$, then $\gamma = 0$.
   - Q: For all $b \in \mathbb{R}^4$, $\text{rank}([A|b]) = \text{rank}([R|b])$.

   Then
   a) both P and Q are TRUE
   b) P is TRUE and Q is FALSE
   c) P is FALSE and Q is TRUE

d) both P and Q are FALSE

(GATE MA 2021)

5) The eigenvalues of the boundary value problem

$$\frac{d^2y}{dx^2} + \lambda y = 0, \quad x \in (0, \pi), \quad \lambda > 0,$$

with boundary conditions

$$y(0) = 0, \quad \frac{dy}{dx}(\pi) - 2y(\pi) = 0,$$

are given by

a) $\lambda = (n\pi)^2, \quad n = 1, 2, 3, \ldots$
b) $\lambda = n^2, \quad n = 1, 2, 3, \ldots$
c) $\lambda = k$, where k_n, roots of $k - \tan(k_n) = 0$
d) $\lambda = k$, where k_n, roots of $k + \tan(k\pi) = 0$

(GATE MA 2021)

6) The family of surfaces $u = xy + f(x^2 - y^2)$, where $f : \mathbb{R} \to \mathbb{R}$ is differentiable, satisfies

a) $y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = x^2 + y^2$
b) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = x^2 + y^2$
c) $y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = x^2 - y^2$
d) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = x^2 - y^2$

(GATE MA 2021)

7) The function $u(x, t)$ satisfies the initial value problem

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad x \in \mathbb{R}, t > 0,$$

with

$$u(x, 0) = 0, \quad \frac{\partial u}{\partial t}(x, 0) = 4xe^{-x^2}.$$

Then $u(5, 5)$ is

a) $1 - e^{100}$
b) $1 - e^{10}$
c) $1 - 1e^{100}$
d) $1 - 1e^{10}$

(GATE MA 2021)

8) Consider fixed-point iteration $x_{n+1} = \varphi(x_n)$, $n \geq 0$, where $\varphi(x) = 3 + (x - 3)^3$, $x \in (2.5, 3.5)$, initial $x_0 = 3.25$. The order of convergence is

a) 1
b) 2
c) 3
d) 4

(GATE MA 2021)

9) Let $\{e_n : n = 1, 2, 3, \ldots\}$ be an orthonormal basis of a complex Hilbert space H. Consider:

- P: There exists a bounded linear functional $f : H \to \mathbb{C}$ such that $f(e_n) = 1^n$, $\forall n$.
- Q: There exists a bounded linear functional $g : H \to \mathbb{C}$ such that $g(e_n) = \frac{1}{\sqrt{n}}$, $\forall n$.

Then

a) both P and Q are TRUE
b) P is TRUE and Q is FALSE
c) P is FALSE and Q is TRUE

d) both P and Q are FALSE

(GATE MA 2021)

10) Let $f : (-\frac{\pi}{2}, \frac{\pi}{2}) \to \mathbb{R}$ be given by $f(x) = \frac{\pi}{2} + x - \tan^{-1} x$. Consider:
- P: $|f(x) - f(y)| < |x - y|$ for all $x, y \in (-\frac{\pi}{2}, \frac{\pi}{2})$.
- Q: f has a fixed point.

Then

a) both P and Q are TRUE
b) P is TRUE and Q is FALSE
c) P is FALSE and Q is TRUE
d) both P and Q are FALSE

(GATE MA 2021)

11) Consider:

P: $d_1(x, y) = |\log(\frac{x}{y})|$ is a metric on $(0, 1)$,

Q:$d_2(x, y) = \begin{cases} |x| + |y|, & x \neq y \\ 0, & x = y \end{cases}$ on $(0, 1)$. Which is metric?

a) both d_1 and d_2 are metrics
b) d_1 is metric, d_2 is not
c) d_1 is not metric, d_2 is
d) neither are metrics

(GATE MA 2021)

12) Let $f : \mathbb{R}^3 \to \mathbb{R}$ be twice continuously differentiable with $\text{div}(\nabla f) = 6$. Let S be the surface $x^2 + y^2 + z^2 = 1$. Let $\mathbf{n}$ be the unit outward normal. Then

$$\iint_S (\nabla f \cdot \mathbf{n})\, dS = ?$$

a) 2π
b) 4π
c) 6π
d) 8π

(GATE MA 2021)

13) Consider statements:
- P: Every compact metrizable topological space is separable.
- Q: Every Hausdorff topology on a finite set is metrizable.

Then

a) both P and Q TRUE
b) P TRUE and Q FALSE
c) P FALSE and Q TRUE
d) both FALSE

(GATE MA 2021)

14) Consider the topologies on $\mathbb{R}$:

$$T_1 = \{U \subset \mathbb{R} : 0 \notin U \text{ or } U = \mathbb{R}\}, \quad T_2 = \{U \subset \mathbb{R} : 0 \in U \text{ or } U = \emptyset\}, \quad T_3 = T_1 \cap T_2.$$

The closure of $\{1\}$ in $(\mathbb{R}, T_3)$ is

a) $\{1\}$
b) $\{0, 1\}$
c) $\mathbb{R}$
d) $\mathbb{R} \setminus \{0\}$

(GATE MA 2021)

Q.15 – 25 Numerical Answer Type (NAT), carry ONE mark each (no negative marks).

15) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be differentiable. Let $D_u f(0,0)$ and $D_v f(0,0)$ be directional derivatives at $(0,0)$ in directions $u = \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$ and $v = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ respectively. If $D_u f(0,0) = \sqrt{5}$ and $D_v f(0,0) = \sqrt{2}$, then

$$\frac{\partial f}{\partial x}(0,0) + \frac{\partial f}{\partial y}(0,0) = ?$$

(GATE MA 2021)

16) Let Γ be boundary of the square region R with vertices $(0,0),(2,0),(2,2),(0,2)$ oriented counter-clockwise. Evaluate:

$$\oint_{\Gamma}(1-y^2)dx + xdy = ?$$

(GATE MA 2021)

17) The number of 5-Sylow subgroups in symmetric group S_5 is ___.

(GATE MA 2021)

18) Let I be the ideal generated by $x^2 + x + 1$ in $R = \mathbb{Z}_3[x]$. Then number of units in R/I is ___.

(GATE MA 2021)

19) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be linear such that

$$T\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}1\\1\\1\end{pmatrix}, \quad T^2\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}-1\\-1\\-1\end{pmatrix}, \quad T^3\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}-1\\-1\\-1\end{pmatrix}.$$

Then the rank of T is ___. (GATE MA 2021)

20) Let $y(x)$ solve the initial value problem

$$x^2\frac{d^2y}{dx^2} - 4x + 6y = 0, \quad x > 0, \quad y(2) = 0, \quad y'(2) = 4.$$

Then $y(4) =$ ___.

(GATE MA 2021)

21) Let

$$f(x) = x^4 + 2x^3 - 11x^2 - 12x + 36$$

for $x \in \mathbb{R}$. The order of convergence of the Newton-Raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'x_n}, n \geq 0,$$

with $x_0 = 2.1$, for finding the root $\alpha = 2$ of the equation $f(x) = 0$ is ___

22) If the polynomial

$$p(x) = \alpha + \beta(x+2) + \gamma(x+2)(x+1) + \delta(x+2)(x+1)x$$

interpolates the data:

x	-2	-1	0	1	2
$f(x)$	2	-1	8	5	-34

then evaluate $\alpha + \beta + \gamma + \delta =$ ___. (GATE MA 2021)

23) Consider Linear Programming Problem P:

$$\max 2x_1 + 3x_2$$

subject to

$$2x_1 + x_2 \leq 6, \quad -x_1 + x_2 \leq 1, \quad x_1 + x_2 \leq 3, \quad x_1, x_2 \geq 0.$$

The optimal value of dual of P is ___. (GATE MA 2021)

24) Consider Linear Programming Problem Q:

$$\min 2x_1 - 5x_2,$$

subject to

$$2x_1 + 3x_2 + S_1 = 12, \quad -x_1 + x_2 + S_2 = 1, \quad -x_1 + 2x_2 + S_3 = 3,$$

with $x_1, x_2, S_1, S_2, S_3 \geq 0$. If $\begin{pmatrix} x_1 \\ 2 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix}$ is a basic feasible solution of P, then $x_1 + S_1 + S_2 + S_3 =$ ___.

(GATE MA 2021)

25) Let H be a complex Hilbert space. Let $u, v \in H$ with $(u, v) = 2$. Then

$$\frac{1}{2\pi} \int_0^{2\pi} \|u + e^{it}v\|^2 dt = \_\_\_.$$

(GATE MA 2021)

Q.26 – 43 Multiple Choice Questions (MCQ), carry TWO marks each (for each wrong answer: −2/3).

26) Let $\mathbb{Z}$ be the ring of integers. Consider subring

$$R = \{a + b\sqrt{-17} : a, b \in \mathbb{Z}\} \subset \mathbb{C}.$$

Consider:
- P: $2 + \sqrt{-17}$ is irreducible.
- Q: $2 + \sqrt{-17}$ is prime.

Then

a) both P and Q TRUE
b) P TRUE and Q FALSE
c) P FALSE and Q TRUE
d) both FALSE

(GATE MA 2021)

27) Consider second order PDE

$$u_{xx} + 4u_{xy} + (x^2 + 4y^2)u = \sin(x + y).$$

Statements:
- P: PDE is parabolic on ellipse $\frac{x^2}{4} + y^2 = 1$.
- Q: PDE is hyperbolic inside ellipse $\frac{x^2}{4} + y^2 = 1$.

Then

a) both P and Q true
b) P true Q false
c) P false Q true
d) both false

(GATE MA 2021)

28) If $u(x, y)$ solves the Cauchy problem

$$u_x + u_y = 1, \quad u(x, 0) = x^2, x > 0,$$

then $u(2, 1)$ equals

a) $1-2e^{-2}$
b) $1+4e^{-2}$
c) $1-4e^{-2}$
d) $1+2e^{-2}$

(GATE MA 2021)

29) Let $y(t)$ be solution of the initial value problem

$$\frac{d^2y}{dt^2}+a\frac{dy}{dt}+by=f(t), \quad a,b>0, a\neq b, a^2-4b=0,$$

with $y(0)=0$, $\frac{dy}{dt}(0)=0$, obtained by the method of Laplace transform. Then

a) $y(t)=\int_0^t \tau e^{\frac{-a\tau}{2}} f(t-\tau)d\tau$
b) $y(t)=\int_0^t e^{\frac{-a\tau}{2}} f(t-\tau)d\tau$
c) $y(t)=\int_0^t \tau e^{\frac{-b\tau}{2}} f(t-\tau)d\tau$
d) $y(t)=\int_0^t e^{\frac{-b\tau}{2}} f(t-\tau)d\tau$

(GATE MA 2021)

30) The critical point of

$$\frac{d^2x}{dt^2}+2\alpha\frac{dx}{dt}+\beta^2 y=0, \quad \alpha>\beta>0,$$

is

a) node and asymptotically stable
b) spiral point and asymptotically stable
c) node and unstable
d) saddle and unstable

(GATE MA 2021)

31) The initial value problem

$$y'=f(t,y), \quad y(0)=1,$$

with $f(t,y)=-10y$, solved by explicit Euler method $y_{n+1}=y_n+hf(t_n,y_n)$ with step size h. Then $y_n\to 0$ as $n\to\infty$ provided

a) $0<h<0.2$
b) $0.3<h<0.4$
c) $0.4<h<0.5$
d) $0.5<h<0.55$

(GATE MA 2021)

32) Consider Linear Programming Problem P:

$$\begin{cases} \max c_1x_1+c_2x_2 \\ \text{s.t. } A_{11}x_1+A_{12}x_2\leq b_1, \\ A_{21}x_1+A_{22}x_2\leq b_2, \\ A_{31}x_1+A_{32}x_2\leq b_3, \\ x_1,x_2\geq 0, \end{cases}$$

with given feasible solution such that $pc_1+qc_2=6$, and feasible solution bounds -5 to 12. Which is NOT true?

a) P has optimal solution
b) the feasible region is bounded
c) if y is feasible for the dual then $b_1y_1+b_2y_2+b_3y_3\geq 6$
d) dual P has no feasible solution

(GATE MA 2021)

33) Let $L^2[-1,1]$ be the Hilbert space of real square integrable functions with norm $\|f\| = \sqrt{\int_{-1}^{1} f(x)^2 dx}$. Let $M = \{f \in L^2[-1,1] : \int_{-1}^{1} f(x)dx = 0\}$. For $f(x) = x^2$, define $d = \inf_{g \in M} \|f - g\|$. Then

a) $d = \frac{3}{\sqrt{2}}$
b) $d = \frac{3}{2}$
c) $d = 3$
d) $d = \ldots$

(GATE MA 2021)

34) Let $C[0,1]$ be Banach space with supremum norm. Define operator

$$(Tf)(x) = x \int_0^1 f(t)dt.$$

Which holds?

a) T bounded and invertible with bounded inverse
b) T bounded but inverse not bounded
c) T not bounded
d) neither bounded nor invertible

(GATE MA 2021)

35) Let $\ell^1 = \{x = (x_1, x_2, \ldots) : \sum |x_n| < \infty\}$ with norm $\|x\| = \sum |x_n|$. Consider subspace

$$X = \{x \in \ell^1 : \sum \eta_n x_n < \infty\}.$$

Define linear $T : X \to \ell^1$ by $(Tx)(n) = nx_n$. Then

a) T closed but not bounded
b) T bounded
c) T neither closed nor bounded
d) T^{-1} exists and is open map

(GATE MA 2021)

36) Let $f_n : [0,10] \to \mathbb{R}$ be $f_n(x) = nx^3 e^{nx}$. Consider:

- P: (f_n) is equicontinuous on $[0,10]$.
- Q: (f_n^{-1}) does NOT converge uniformly.

Then

a) both P and Q true
b) P true Q false
c) P false Q true
d) both false

(GATE MA 2021)

37) Let

$$f(x,y) = \begin{cases} \sqrt{x^2 + y^2} \sin\left(\frac{y^2}{x}\right), & x \neq 0 \\ 0, & x = 0. \end{cases}$$

Consider:

- P: f continuous at $(0,0)$ but not differentiable.
- Q: Directional derivative $D_u f(0,0)$ exists in every direction.

Then

a) both P and Q true
b) P true Q false
c) P false Q true

d) both false

(GATE MA 2021)

38) Let $V \subset \mathbb{R}^3$ bounded by paraboloid $y = x^2 + z^2$ and plane $y = 4$. Then value of

$$\iiint_V \sqrt{x^2 + z^2}\, dV = ?$$

a) 128π
b) 64π
c) 28π
d) 256π

(GATE MA 2021)

39) For

$$f(x, y) = 4xy - 2x^2 - y^2,$$

f has

a) a local max and saddle point
b) a local min and saddle point
c) a local max and local min
d) two saddle points

(GATE MA 2021)

40) The equation

$$xy - z \log y + e^{xz} = 1,$$

can be solved near $(0, 1, 1)$ as $y = f(x, z)$ for some continuously differentiable f. Then

a) $f(0, 1) = (2, 0)$
b) $f(0, 1) = (0, 2)$
c) $f(0, 1) = (0, 1)$
d) $f(0, 1) = (1, 0)$

(GATE MA 2021)

41) Consider topologies on $\mathbb{R}$:

- T_1 upper limit topology, basis $(a, b]$.
- $T_2 = \{U \subset \mathbb{R} : \mathbb{R} \setminus U \text{ finite}\} \cup \{0\}$.
- T_3 standard topology (a, b).

Then

a) $T_2 \subset T_3 \subset T_1$
b) $T_1 \subset T_2 \subset T_3$
c) $T_3 \subset T_2 \subset T_1$
d) $T_2 \subset T_1 \subset T_3$

(GATE MA 2021)

42) Let $X_1 = (\mathbb{R}, T_1)$ with T_1 upper limit topology and $X_2 = (\mathbb{R}, T_2)$ with T_2 as above. Then

a) both connected
b) X_1 connected, X_2 not connected
c) X_1 not connected, X_2 connected
d) neither connected

(GATE MA 2021)

43) Let $(\cdot, \cdot) : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ be inner product. Consider:

- P: $|(u, v)| \leq \frac{(u,u)+(v,v)}{2}$ for all u, v.
- Q: If $(u, v) = (2u, v)$ for all v, then $u = 0$.

Then

a) both P, Q true
b) P true, Q false
c) P false, Q true
d) both false

(GATE MA 2021)

Q.44 – 55 Numerical Answer Type (NAT), carry TWO marks each (no negative marks).

44) Let G be a group of order 54 with center having 52 elements. The number of conjugacy classes in G is ___. (GATE MA 2021)
45) Let F be a finite field and $F^{\times}$ its multiplicative group. If $F^{\times}$ has subgroup of order 17, then smallest possible order of field F is ___. (GATE MA 2021)
46) Let
$$R = \{z = x + iy \in \mathbb{C} : 0 < x < 1, -11\pi < y < 11\pi\}$$
and r be the positively oriented boundary of R. Evaluate
$$\frac{1}{2\pi i}\int_r \frac{e^z}{e^z - 2}dz = \_\_\_.$$
(GATE MA 2021)
47) Let $D = \{z \in \mathbb{C} : |z| < 2\pi\}$. Define
$$f(z) = \frac{3z^2}{6(1-\cos z)} \quad \text{if } z \neq 0, \quad f(0) = ?$$
If $f(z) = \sum_{n=0}^{\infty} a_n z^n$ on D, then $6a_2 =$ ___. (GATE MA 2021)
48) The number of zeroes (counting multiplicity) of
$$P(z) = 3z^5 + 2iz^2 + 7iz + 1$$
in annulus $\{z : 1 < |z| < 7\}$ is ___. (GATE MA 2021)
49) Let A be square matrix with
$$\det(xI - A) = x(x-1)^2(x-2)^3.$$
If $\text{rank}(A^2) < \text{rank}(A^3) = \text{rank}(A^4)$, then geometric multiplicity of eigenvalue 0 is ___. (GATE MA 2021)
50) If $y = \sum \alpha_k x^k$ $(\alpha_0 \neq 0)$ is power series solution of
$$y'' - 24x^2 y = 0,$$
then $\frac{a_4}{a_0} =$ ___. (GATE MA 2021)
51) If $u(x,t) = Ae^t \sin x$ solves
$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, t > 0,$$
with boundary conditions $u(0,t) = u(\pi,t) = 0$ and initial condition
$$u(x,0) = \begin{cases} 60, & 0 < x \leq 2, \\ 40, & 2 < x < \pi, \end{cases}$$
then $\pi A =$ ___. (GATE MA 2021)
52) Let $V = \{p(x) = a_0 + a_1 x + a_2 x^2 : a_i \in \mathbb{R}\}$. Define $T : V \to V$ by
$$T(p) = (p(0) - p(1)) + (p(0) + p(1))x + p(0)x^2.$$
Then the sum of eigenvalues of T equals ___. (GATE MA 2021)

53) Quadrature formula
$$\int_0^2 f(x)dx = af(0) + \beta f(1) + \gamma f(2)$$
is exact for all polynomials degree ≤ 2. Then $2\beta\gamma =$ ___. (GATE MA 2021)

54) For $x \in (0, 1]$ with decimal expansion $x = 0.d_1d_2d_3\ldots$, define
$$f(x) = \begin{cases} 0, & x \text{ rational} \\ 18^n, & x \text{ irrational and } n \text{ zeros after decimal point before first nonzero digit} \end{cases}.$$
The Lebesgue integral $\int_0^1 f(x)dx =$ ___. (GATE MA 2021)

55) Let $\bar{x} = \begin{pmatrix} \frac{11}{3} \\ \frac{2}{3} \\ 0 \end{pmatrix}$ be an optimal solution of the following Linear Programming Problem P :
$$\max 4x_1 + x_2 - 3x_3$$
subject to
$$2x_1 + 4x_2 + ax_3 \leq 10, \quad x_1 - x_2 + bx_3 \leq 3, \quad 2x_1 + 3x_2 + 5x_3 \leq 11, \quad x_i \geq 0,$$
where a, b are real numbers. If $\bar{y} = \begin{pmatrix} p \\ q \\ r \end{pmatrix}$ is an optimal solution of the dual of P, then $p+q+r =$ ___. (round off to two decimals)

(GATE MA 2021)