GATE 2012 MA

EE25BTECH11001 - AARUSH DILAWRI

Q.1-Q.25 carry one mark each.

1) The straight lines $L_1 : x = 0$, $L_2 : y = 0$, and $L_3 : x + y = 1$ are mapped by the transformation $w = z^2$ into the curves C_1, C_2, and C_3 respectively. The angle of intersection between the curves at $w = 0$ is GATE MA 2012

   a) 0
   b) $\frac{\pi}{4}$
   c) $\frac{\pi}{2}$
   d) π

2) In a topological space, which of the following statements is NOT always true: GATE MA 2012

   a) Union of any finite family of compact sets is compact.
   b) Union of any family of closed sets is closed.
   c) Union of any family of connected sets having a non empty intersection is connected.
   d) Union of any family of dense subsets is dense.

3) Consider the following statements: P: The family of subsets $A_n = \{-n, -n+1, \ldots, n\}$ for $n = 1, 2, \ldots$ satisfies the finite intersection property.
   Q: On an infinite set X define the metric $d : X \times X \to \mathbb{R}$ as

$$d(x, y) = \begin{cases} 0, & \text{if } x = y \\ 1, & \text{if } x \neq y \end{cases} \tag{3.1}$$

   The metric space (X, d) is compact.
   R: In a Frechet (T_1) topological space, every finite set is closed.
   S: If $f : \mathbb{R} \to X$ is continuous, where $\mathbb{R}$ has the usual topology and (X, τ) is a Hausdorff (T_2) space, then f is a one-one function.
   Which of the above statements are correct? GATE MA 2012

   a) P and R
   b) P and S
   c) R and S
   d) Q and S

4) Let H be a Hilbert space and $S^\perp$ denote the orthogonal complement of a set $S \subseteq H$. Which of the following is INCORRECT? GATE MA 2012

   a) For $S_1 \subseteq S_2 \subseteq H$, we have $S_2^\perp \subseteq S_1^\perp$.
   b) $(S^\perp)^\perp \subseteq S$
   c) $\{0\}^\perp = H$
   d) $S^\perp$ is always closed.

5) Let H be a complex Hilbert space, $T : H \to H$ a bounded linear operator and T^* its adjoint. Which of the following statements are always TRUE? P: $\langle Tx, y\rangle = \langle x, T^* y\rangle$ for all $x, y \in H$
   Q: $\langle x, Ty\rangle = T\langle x, y\rangle$ for all $x, y \in H$

R: $\langle x, Ty\rangle = \langle x, T^*y\rangle$ for all $x, y \in H$
S: $\langle Tx, Ty\rangle = T^*\langle x, Ty\rangle$ for all $x, y \in H$

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a) P and Q
b) P and R
c) Q and S
d) P and S

6) Let $X = \{a, b, c\}$ and $\mathcal{T} = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ be a topology defined on X. Which statements are TRUE? P: $(X, \mathcal{T})$ is a Hausdorff space.
Q: $(X, \mathcal{T})$ is a regular space.
R: $(X, \mathcal{T})$ is a normal space.
S: $(X, \mathcal{T})$ is a connected space.

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a) P and Q
b) Q and R
c) R and S
d) P and S

7) Consider the statements:
P: If X is a normed linear space and $M \subseteq X$ is a subspace, then the closure $\bar{M}$ is also a subspace of X.
Q: If X is a Banach space and $\sum x_n$ is an absolutely convergent series in X, then $\sum x_n$ is convergent.
R: Let M_1 and M_2 be subspaces of an inner product space such that $M_1 \cap M_2 = \phi$. Then for all $m_1 \in M_1, m_2 \in M_2$: $\|m_1 + m_2\|^2 = \|m_1\|^2 + \|m_2\|^2$.
S: Let $f : X \to Y$ be a linear transformation from the Banach Space X into the Banach space Y. If f is continuous, then the graph of f is always compact.
The correct statements amongst the above are: GATE MA 2012

a) P and R only
b) Q and R only
c) P and Q only
d) R and S only

8) A continuous random variable X has the probability density function

$$f(x) = \begin{cases} \frac{3}{5}x^3 e^{-x^5/5}, & x > 0 \\ 0, & x \le 0 \end{cases} \tag{8.1}$$

The probability density function of $Y = 2X + 3$ is GATE MA 2012

a) $\frac{1}{5}(y-2)^3 e^{-(y-2)^5/5}, \quad y > 2$
b) $\frac{2}{5}(y-2)^3 e^{-2(y-2)^5/5}, \quad y > 2$
c) $\frac{3}{5}(y-2)^3 e^{-3(y-2)^5/5}, \quad y > 2$
d) $\frac{4}{5}(y-2)^3 e^{-4(y-2)^5/5}, \quad y > 2$

9) A simple random sample of size 10 from $N(\mu, \sigma^2)$ gives 98% confidence interval $(20.49, 23.51)$. Then the null hypothesis $H_0 : \mu = 20.5$ against $H_A : \mu \neq 20.5$

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a) can be rejected at 2% level of significance

b) cannot be rejected at 5% level of significance
c) can be rejected at 10% level of significance
d) cannot be rejected at any level of significance

10) For the linear programming problem

$$\text{Maximize} \quad z = x_1 + 2x_2 + 3x_3 - 4x_4 \tag{10.1}$$
$$\text{Subject to} \quad x_1 + 2x_2 - 3x_3 - x_4 = 15 \tag{10.2}$$
$$x_1 + 6x_2 + 3x_3 - x_4 = 21 \tag{10.3}$$
$$x_1 + 8x_2 + 2x_3 - 4x_4 = 30 \tag{10.4}$$
$$x_1, x_2, x_3, x_4 \geq 0 \tag{10.5}$$

and $x_1 = 4,\ x_2 = 3,\ x_3 = 0,\ x_4 = 2$ is GATE MA 2012

a) an optimal solution
b) a degenerate basic feasible solution
c) a non-degenerate basic feasible solution
d) a non-basic feasible solution

11) Which one of the following statements is TRUE? GATE MA 2012

a) A convex set cannot have infinitely many extreme points.
b) A linear programming problem can have infinitely many extreme points.
c) A linear programming problem can have exactly two different optimal solutions.
d) A linear programming problem can have a non-basic optimal solution.

12) Let $\alpha = e^{2\pi i/5}$ and the matrix

$$M = \begin{pmatrix} \alpha & \alpha^2 & \alpha^3 & 4 \\ \alpha^2 & \alpha^3 & \alpha^4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{12.1}$$

Then the trace of the matrix $I + M + M^2$ is GATE MA 2012

a) 5 b) 0 c) 3 d) -5

13) Let $V = \mathbb{C}^2$ be the vector space over the complex numbers and $B = \{(1, i), (i, 1)\}$ be a given ordered basis of V. Then which of the following pairs $\{f_1, f_2\}$ is a dual basis of B over $\mathbb{C}$? GATE MA 2012

a) $f_1(z_1, z_2) = \frac{z_1 - z_2}{2}, \quad f_2(z_1, z_2) = \frac{z_1 + z_2}{2}$
b) $f_1(z_1, z_2) = \frac{z_1 + z_2}{2}, \quad f_2(z_1, z_2) = \frac{z_1 + z_2 i}{2}$
c) $f_1(z_1, z_2) = \frac{z_1 - z_2}{2}, \quad f_2(z_1, z_2) = \frac{z_1 - z_2 i}{2}$
d) $f_1(z_1, z_2) = \frac{z_1 + z_2}{2}, \quad f_2(z_1, z_2) = \frac{z_1 - z_2}{2}$

14) Let $R = \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$ and $I = \mathbb{Z} \times \mathbb{Z} \times \{0\}$. Which of the following statements is correct? GATE MA 2012

a) I is a maximal ideal but not a prime ideal of R.
b) I is a prime ideal but not a maximal ideal of R.
c) I is both a maximal ideal as well as a prime ideal of R.

d) I is neither a maximal ideal nor a prime ideal of R.

15) The function $u(r, \theta)$ satisfying the Laplace equation

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0, \quad 0 < r < e, \quad 0 < \theta < 2\pi \tag{15.1}$$

subject to the conditions

$$u(e, \theta) = 1, \quad u(e, \theta) = 0 \tag{15.2}$$

is GATE MA 2012

a) $\ln \frac{r}{e}$
b) $2 \ln \frac{r}{e}$
c) $\ln \left(\frac{r}{e}\right)^2$
d) $\sum_{n=1}^{\infty} \frac{1}{n} \sin(n\theta) \left(\frac{r}{e}\right)^n$

16) The functional

$$J[y] = \int_0^1 \left(y'^2 + 2kxyy' + y^2 + y' + y\right) dx, \tag{16.1}$$

with boundary conditions $y(0) = 0$, $y(1) = 1$, $y'(0) = 2$, $y'(1) = 3$ is path independent if k equals GATE MA 2012

a) 1
b) 2
c) 3
d) 4

17) If the transformation $y = uv$ transforms the differential equation

$$f''(x) - f'(x) + g(x) = 0 \tag{17.1}$$

into the equation of the form

$$v'' + h(x)v = 0, \tag{17.2}$$

then u must be GATE MA 2012

a) $\frac{1}{\sqrt{f}}$
b) xf
c) $\frac{1}{2f}$
d) f^2

18) The expression

$$D = \frac{\partial^2}{\partial x^2} - 2\frac{\partial^2}{\partial x \partial y} - \frac{\partial^2}{\partial y^2} \tag{18.1}$$

is equal to GATE MA 2012

a) $\cos(x - y)$
b) $\sin(x - y) + \cos(x + y)$
c) $\cos(x - y) + \sin(x + y)$
d) $3 \sin(x - y)$

19) The function $\phi(x)$ satisfying the integral equation

$$\phi(x) = 2 \int_0^x e^{-\xi^2} \phi(\xi) d\xi \tag{19.1}$$

is GATE MA 2012

a) e^{x^2}
b) $e^{x^2} + x$
c) $e^{x^2} - x$
d) $x^2 + \frac{1}{2}$

20) Given data:

x	1	2	3	4	5
y	-1	2	-3	4	-5

If the derivative of $y(x)$ is approximated as

$$y'(x_k) \approx \frac{1}{2h}\left(-y_{k+2} + 4y_{k+1} - 3y_k\right), \tag{20.1}$$

then the value of $y'(2)$ is GATE MA 2012

a) 4
b) 8
c) 12
d) 16

21) If

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \tag{21.1}$$

then A^{50} is GATE MA 2012

a) $\begin{pmatrix} 1 & 0 & 0 \\ 50 & 1 & 0 \\ 50 & 0 & 1 \end{pmatrix}$
b) $\begin{pmatrix} 1 & 0 & 0 \\ 48 & 1 & 0 \\ 48 & 0 & 1 \end{pmatrix}$
c) $\begin{pmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{pmatrix}$
d) $\begin{pmatrix} 1 & 0 & 0 \\ 24 & 1 & 0 \\ 24 & 0 & 1 \end{pmatrix}$

22) If

$$y = \sum_{m=0}^{\infty} c_m x^{m+r} \tag{22.1}$$

is assumed to be a solution of the differential equation

$$x^2 y'' - 3xy' - xy = 0, \tag{22.2}$$

then the values of r are GATE MA 2012

a) 1 and 3
b) -1 and 3
c) 1 and -3
d) -1 and -3

23) Let the linear transformation

$$T : F^2 \to F^3 \tag{23.1}$$

be defined by

$$T(x_1, x_2) = (x_1 + x_2, x_1, x_2). \tag{23.2}$$

Then the nullity of T is GATE MA 2012

a) 0 b) 1 c) 2 d) 3

24) The approximate eigenvalue of the matrix

$$A = \begin{pmatrix} 15 & 4 & 3 \\ 10 & 12 & 6 \\ 20 & 4 & 2 \end{pmatrix} \tag{24.1}$$

obtained after two iterations of the Power method, with the initial vector $(1, 1, 1)^T$, is GATE MA 2012

a) 7.768 b) 9.468 c) 10.548 d) 19.468

25) The root of the equation $xe^x = 1$ between 0 and 1, obtained by using two iterations of the bisection method, is GATE MA 2012

a) 0.25 b) 0.50 c) 0.75 d) 0.65

26) Let

$$\int_C \frac{a\,dz}{(z^2 - 4)(z - 2)} = \pi, \tag{26.1}$$

where the closed curve C is the triangle with vertices at -1, i, $-i$ and the integral is taken in anti-clockwise direction. Then one value of a is GATE MA 2012

a) $1 + i$ b) $2 + i$ c) $3 + i$ d) $4 + i$

27) The Lebesgue measure of the set

$$A = \{x : \ \sin x < x, \ 0 < x \leq 1\} \tag{27.1}$$

is GATE MA 2012

a) 0 b) 1 c) $\ln 2$ d) $1 - \ln 2$

28) Which of the following statements are TRUE?
P. The set $\{x \in \mathbb{R} : \cos x \leq 1\}$ is compact.
Q. The set $\{x \in \mathbb{R} : \tan x \text{ is not differentiable}\}$ is complete.
R. The set $\left\{x \in \mathbb{R} : \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} \text{ is convergent}\right\}$ is bounded.
S. The set $\{x \in \mathbb{R} : f(x) = \cos x \text{ has local maxima}\}$ is closed. GATE MA 2012

a) P and Q
b) R and S
c) Q and S
d) P and S

29) If a random variable X assumes only positive integer values, with probability

$$P(X = x) = \frac{1}{2} \frac{1}{3^{x-1}}, \quad x = 1, 2, 3, \ldots \tag{29.1}$$

then $\mathbb{E}[X]$ is GATE MA 2012

a) $\frac{2}{9}$
b) $\frac{2}{3}$
c) 1
d) $\frac{3}{2}$

30) The probability density function of the random variable X is

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & x \leq 0 \end{cases} \tag{30.1}$$

where $\lambda > 0$. For testing the hypothesis $H_0 : \lambda = 3$ against $H_A : \lambda = 5$, a test is given as "Reject H_0 if $X \geq 4.5$". The probability of type I error and power of this test are, respectively, GATE MA 2012

a) 0.1353, 0.4966
b) 0.1827, 0.379
c) 0.2021, 0.4493
d) 0.2231, 0.4066

31) The order of the smallest possible non-trivial group containing elements x and y such that $x^7 = y^2 = e$ and $yx = x^4 y$ is GATE MA 2012

a) 1
b) 2
c) 7
d) 14

32) The number of 5-Sylow subgroups in a group of order 45 is GATE MA 2012

a) 1
b) 2
c) 3
d) 4

33) The solution of the initial value problem

$$y'' + 10y' + 6y = \delta(t), \quad y(0) = 0, \quad y'(0) = 0 \tag{33.1}$$

where $\delta(t)$ denotes the Dirac delta function, is GATE MA 2012

a) $2e^{-3t} \sin t$
b) $6e^{-3t} \sin t$
c) $2e^{-t} \sin 3t$
d) $6e^{-t} \sin 3t$

34) Let $\omega = \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}$, $M = \begin{pmatrix} 0 & 0 \\ i & 0 \end{pmatrix}$, $N = \begin{pmatrix} 0 & i \\ \omega & 0 \end{pmatrix}$, and $G = \begin{pmatrix} M \\ N \end{pmatrix}$ be the group generated by M and N under multiplication. Then GATE MA 2012

a) C_6 b) S_3 c) C_2 d) C_4

35) The flux of the vector field $u = x\hat{i} + y\hat{j} + z\hat{k}$ flowing out through the surface of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \quad a, b, c > 0, \tag{35.1}$$

is GATE MA 2012

a) $abc\pi$ b) $2abc\pi$ c) $3abc\pi$ d) $4abc\pi$

36) The integral surface satisfying the partial differential equation

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = z^2 \tag{36.1}$$

and passing through the line $x = 1$, $y = 1$, $z = 1$ is GATE MA 2012

a) $x - z^2 + y = 1$
b) $x^2 + y^2 - z^2 = 1$
c) $y - z^2 + x = 1$
d) $x - z^2 + y = 1$

37) The diffusion equation

$$u_{xx} = u_t, \quad u(x, 0) = \cos x + \sin 5x \tag{37.1}$$

admits the solution GATE MA 2012

a) $e^{-36t} \sin 6x + e^{-20t} \sin 4x$
b) $e^{-36t} \sin 4x + e^{-20t} \sin 6x$
c) $e^{-20t} \sin 3x + e^{-15t} \sin 5x$
d) $e^{-36t} \sin 5x + e^{-20t} \sin x$

38) Let $f(x)$ and $xf(x)$ be particular solutions of the differential equation

$$y'' + R(x)y' + S(x)y = 0. \tag{38.1}$$

Then the solution of

$$y'' + R(x)y' + S(x)y = f(x) \tag{38.2}$$

is GATE MA 2012

a) $y = -\frac{1}{2}x^2 f(x) + \alpha x + \beta$
b) $y = \frac{1}{2}x^2 f(x) + \alpha x + \beta$
c) $y = -x^2 f(x) + \alpha x + \beta$
d) $y = 3x^2 f(x) + \alpha x + \beta$

39) Let the Legendre equation

$$(1 - x^2)y'' - 2xy' + n(n + 1)y = 0 \tag{39.1}$$

have a nth degree polynomial solution $y_n(x)$ such that $y_n(1) = 3$. If

$$\int_{-1}^{1} y_n(x)y_n(x)dx = \frac{144}{15}, \tag{39.2}$$

then n is GATE MA 2012

a) 1 b) 2 c) 3 d) 4

40) The maximum value of the function $f(x, y, z) = xyz$ subject to the constraint $xy + yz + zx = a$, $a > 0$, is GATE MA 2012

a) $\frac{a^2}{3}$ b) $\frac{a^2}{3^3}$ c) $\frac{a^2}{3}(3)$ d) $\frac{2a^2}{3}$

41) The functional

$$J[y] = \int_0^1 \left(y'^4 + 4y^3y' + 4y^2e^{y'}\right)dx, \tag{41.1}$$

with boundary conditions $y(0) = -1$, $y(1) = -1$, possesses: GATE MA 2012

a) strong minima on $x = \frac{1}{3}ye = -1$
b) strong minima on $x = \frac{4}{3}ye = -1$
c) weak maxima on $x = \frac{1}{3}ye = -1$
d) strong maxima on $x = \frac{4}{3}ye = -1$

42) A particle of mass m constrained to move on a circle of radius a, rotating about its vertical diameter with constant angular velocity ω. Initial angular velocity zero, g gravitational acceleration. With θ inclination and $\dot{\theta} = \frac{d\theta}{dt}$, the Lagrangian is

$$L = \frac{1}{2}ma^2\left(\dot{\theta}^2 + \omega^2\sin^2\theta\right) + mga\cos\theta, \tag{42.1}$$

or one of the options: GATE MA 2012

a) $\frac{1}{2}ma^2\left(\dot{\theta}^2 + \omega^2\sin^2\theta\right) + mga\cos\theta$
b) $\frac{1}{2}ma^2\left(\dot{\theta}^2 + 2\omega\sin\theta\right) - mga\sin\theta$
c) $\frac{1}{2}ma^2\left(\dot{\theta}^2 + 2\omega\cos\theta\right) - mga\sin\theta$
d) $\frac{1}{2}ma^2\left(\dot{\theta}^2 + \sin 2\theta\right) + mga\sin\theta$

43) For the matrix

$$M = \begin{pmatrix} 2+3i & 4-3i & 2+5i \\ 6+4i & 6-3i & 0 \end{pmatrix} \tag{43.1}$$

which of the following statements are correct?
P: M is skew-Hermitian and iM is Hermitian
Q: M is Hermitian and iM is skew-Hermitian
R: Eigenvalues of M are real
S: Eigenvalues of iM are real GATE MA 2012

a) P and R only
b) Q and R only
c) P and S only
d) Q and S only

44) Let $T : P_3 \to P_3$ be the map defined by

$$T(p)(x) = \int_0^x p(t)\,dt. \tag{44.1}$$

If the matrix of T relative to the standard basis $\left\{1, x, x^2, x^3\right\}$ is M and M' denotes transpose of M, then $M + M'$ is GATE MA 2012

a) $\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & 0 & 0 & 1 \end{pmatrix}$

c) $\begin{pmatrix} 2 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{pmatrix}$

d) $\begin{pmatrix} 0 & 2 & 2 & 2 \\ 2 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 2 & 0 & 0 & 1 \end{pmatrix}$

45) Using Eulers method with step size $h = 0.1$, the approximate value of y at $x = 0.2$ for the initial value problem

$$\frac{dy}{dx} = x^2 + y^2, \quad y(0) = 1 \tag{45.1}$$

is GATE MA 2012

a) 1.322 b) 1.122 c) 1.222 d) 1.110

46) Transportation problem data:

Origin / Destination	D1	D2	D3	D4	Supply
O1	3	4	8	7	60
O2	7	3	7	6	80
O3	3	9	3	4	100
Demand	40	70	50	80	

Basic feasible solution:

$$x_{12} = 60, x_{22} = 10, x_{23} = 50, x_{24} = 20, x_{31} = 40, x_{34} = 60. \tag{46.1}$$

The variables entering and leaving the basis are: GATE MA 2012

a) x_{11} and x_{24}
b) x_{13} and x_{23}
c) x_{14} and x_{24}
d) x_{33} and x_{24}

47) System of equations

$$\begin{pmatrix} 5 & 1 & 1 \\ 10 & 2 & 4 \\ 0 & 12 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}. \tag{47.1}$$

Using Jacobis method with initial guess $\begin{pmatrix} 2.0 \\ 3.0 \\ 0.0 \end{pmatrix}$, the approximate solution after two iterations is GATE MA 2012

a) $\begin{pmatrix} 2.64 \\ -1.70 \\ -1.12 \end{pmatrix}$

b) $\begin{pmatrix} 2.64 \\ 1.70 \\ -1.12 \end{pmatrix}$

c) $\begin{pmatrix} 2.64 \\ 1.70 \\ 1.12 \end{pmatrix}$

d) $\begin{pmatrix} 2.64 \\ -1.70 \\ 1.12 \end{pmatrix}$

48) For the primal linear programming problem:

$$\text{Maximize } z = 6x_1 + 12x_2 + 12x_3 + 6x_4, \tag{48.1}$$

subject to

$$x_1 + x_2 + x_3 + x_4 = 3, \quad x_i \geq 0, \tag{48.2}$$

with optimal basic variables x_1, x_2, x_3, x_4 having RHS constants $3/4, 0, 1, -1/4$ and $1/4, 1, 0, 1/4$ respectively and given $z = 48$, then:
If y_1 and y_2 are dual variables for constraints 1 and 2, their values in the optimal dual solution are: GATE MA 2012

a) 0 and 6
b) 12 and 0
c) 6 and 3
d) 4 and 4

49) If the right hand side of the second constraint changes from 8 to 20, the basic variables in the primal optimal solution will be: GATE MA 2012

a) x_1 and x_2
b) x_1 and x_3
c) x_2 and x_3
d) x_2 and x_4

50) Consider the Fredholm integral equation:

$$u(x) + \lambda \int_0^1 e^{xt} u(t)dt = x. \tag{50.1}$$

The resolvent kernel $R(x, t; \lambda)$ is: GATE MA 2012

a) $te^{-\lambda x}$
b) $te^{\lambda x+1}$
c) $te^{\lambda x+2}$
d) $te^{-\lambda x+2}$

51) The solution $u(x)$ of the above integral equation is: GATE MA 2012

a) $\dfrac{x+1}{1-\lambda}$
b) $\dfrac{x-1}{1-\lambda^2}$
c) $\dfrac{x+2}{1+\lambda}$
d) $\dfrac{x-1}{1-\lambda}$

52) Given the joint pdf of two variables X and Y:

$$f_{X,Y}(x,y) = \begin{cases} 2(x+y), & 0 \le x \le 1, 0 \le y \le 1, x+y \le 1, \\ 0, & \text{otherwise.} \end{cases} \tag{52.1}$$

Then, GATE MA 2012

a) $\mathbb{E}[X] = \frac{2}{3}$ and $\mathbb{E}[Y] = \frac{5}{5}$
b) $\mathbb{E}[X] = 1$ and $\mathbb{E}[Y] = 1$
c) $\mathbb{E}[X] = \frac{3}{6}$ and $\mathbb{E}[Y] = \frac{5}{5}$
d) $\mathbb{E}[X] = \frac{4}{6}$ and $\mathbb{E}[Y] = \frac{5}{5}$

53) The covariance $\mathrm{Cov}(X,Y)$ is: GATE MA 2012

a) -0.01
b) 0
c) 0.01
d) 0.02

54) Given functions

$$f(z) = \frac{z^2}{z+\alpha}, \quad g(z) = \sinh\left(\frac{z}{\pi\alpha}\right), \quad \alpha \neq 0, \tag{54.1}$$

the residue of $f(z)$ at its pole equals 1, then the value of α is:
GATE MA 2012

a) -1
b) 1
c) 2
d) 3

55) For the value of α obtained above, $g(z)$ is not conformal at the point:
GATE MA 2012

a) $\frac{i\pi}{6} + 3$
b) $\frac{i\pi}{6} + 1$
c) $\frac{2\pi}{3}$
d) $2i\pi$

56) Choose the most appropriate word to complete the sentence:
Given the seriousness of the situation that he had to face, his ___ was impressive.
GATE MA 2012

a) beggary
b) nomenclature
c) jealousy
d) nonchalance

57) Choose the most appropriate alternative to complete the sentence:
If the tired soldier wanted to lie down, he ___ the mattress out on the balcony.
GATE MA 2012

a) should take
b) shall take
c) should have taken
d) will have taken

58) If $(1.001)^{1259} = 3.52$ and $(1.001)^{2062} = 7.85$, then $(1.001)^{3321} =$ GATE MA 2012

a) 2.23 b) 4.33 c) 11.37 d) 27.64

59) One of the parts (A, B, C, D) in the sentence below contains an ERROR. Identify the incorrect part:
I requested that he should be given the driving test today instead of tomorrow. GATE MA 2012

a) requested that
b) should be given
c) the driving test
d) instead of tomorrow

60) Which is the closest in meaning to the word "Latitude"? GATE MA 2012

a) Eligibility
b) Freedom
c) Coercion
d) Meticulousness

61) There are eight bags of rice looking alike, seven have equal weight and one is slightly heavier. Using a weighing balance of unlimited capacity, the minimum number of weighings required to identify the heavier bag is GATE MA 2012

a) 2 b) 3 c) 4 d) 8

62) Raju has 14 currency notes, consisting of only Rs. 20 and Rs. 10 notes. The total money value is Rs. 230. The number of Rs. 10 notes Raju has is GATE MA 2012

a) 5 b) 6 c) 9 d) 10

63) One legacy of Roman legions was discipline. Military law prevailed and discipline was brutal. Discipline kept units obedient, intact, and fighting even in adverse odds and conditions.
The best summary of this passage is: GATE MA 2012

a) Thorough regimentation was the main reason for efficiency in adverse circumstances.
b) The legions were treated inhumanly as if the men were animals.
c) Discipline was the armies inheritance from their seniors.
d) The harsh discipline led to the odds and conditions being against them.

64) A and B decide to meet between 1 PM and 2 PM. The first to arrive will wait no more than 15 minutes. The probability that they will meet is GATE MA 2012

a) $\frac{1}{4}$ b) $\frac{1}{16}$ c) $\frac{7}{16}$ d) $\frac{9}{16}$

65) The table shows the monthly budget of an average household:
What is the approximate percentage of the monthly budget NOT spent on savings? GATE MA 2012

Category	Amount (Rs.)
Food	4000
Clothing	1200
Rent	2000
Savings	1500
Other expenses	1800

a) 10% b) 14% c) 81% d) 86%