EE2703 Assignment 5: Laplace Equation

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1 Abstract

- To solve for potential in a resistor.
- To solve for current in a resistor.
- To solve laplace equation with iterations.
- To plot the graphs to understand the 2D Laplace equation
- To understand how to vectorized code in python

2 Introduction

A wire is soldered to the middle of a copper plate and its voltage is held at 1 Volt. One side of the plate is rounded, while the remaining are floating. The plate is 1 cm by 1 cm in size.

To solve for currents in resistors.We use following Equations and

Boundary conditions mentioned below

- Current Density

$$\vec{J} = \sigma\vec{E} \tag{1}$$

- Electric field is the gradient of the potential.

$$\vec{E} = -\nabla\phi \tag{2}$$

- Continuity Equation

$$\nabla.\vec{J} = -\frac{\partial\rho}{\partial t} \tag{3}$$

- Assuming that our resistors contains a material of constant conductivity ,the equation becomes

$$\nabla^2\phi = \frac{\partial\rho}{\partial t}\frac{}{\rho} \tag{4}$$

- For DC currents,The rights side is zero and we obtain, Laplace equation

$$\nabla^2\phi = 0 \tag{5}$$

- Laplace equation in 2D can be written in cartesian coordinates as

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \tag{6}$$

$$\frac{\partial \phi}{\partial x}_{(x_i, y_j)} = \frac{\phi(x_{i+1/2}, y_j) - \phi(x_{i-1/2}, y_j)}{\Delta x} \tag{7}$$

$$\frac{\partial^2 \phi}{\partial x^2}_{(x_i, y_j)} = \frac{\phi(x_{i+1}, y_j) - 2\phi(x_i, y_j) + \phi(x_{i-1}, y_j)}{(\Delta x)^2} \tag{8}$$

- Using above equations we get

$$\phi_{i,j} = \frac{\phi_{i+1,j} + \phi_{i-1,j} + \phi_{i,j+1} + \phi_{i,j-1}}{4} \tag{9}$$

3 Potential Array and initialization

- Define the Parameters, The parameter values taken for my particular code were Nx = 25 and Ny = 25 and number of iterations : 1500

- Allocate the potential array as $\phi = 0$.

- The array should have Ny rows and Nx columns.

  - To find the indices which lie inside the circle of radius 0.35 using meshgrid() by equation.

```
X*X+Y*Y¡=0.35*0.35 .Assign 1V python codes Nx=25
Ny=25
radius=8
Niter=1500
phi=np.zeros((Nx,Ny),dtype = float)
x,y=np.linspace(-0.5,0.5,num=Nx,dtype=float),np.linspace(-0.5,0.5,num=Ny,dtype=float)
Y,X=np.meshgrid(y,x)
phi[np.where(X**2+Y**2¡=(0.35)**2)]=1.0
plt.xlabel(”X”)
plt.ylabel(”Y”)
plt.contourf(X,Y,phi)
plt.colorbar()
plt.show()
```

4 Potential array updation and iteration

update the potential phi according to equation below using vectorized code

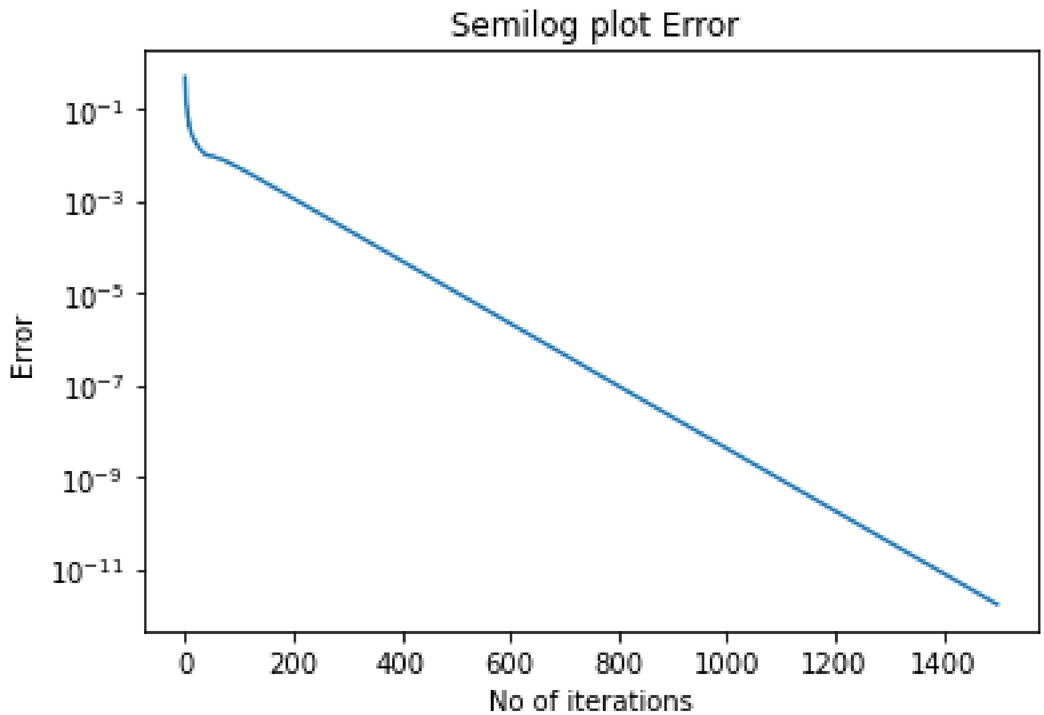
$$\phi_{i,j} = \frac{\phi_{i+1,j} + \phi_{i-1,j} + \phi_{i,j+1} + \phi_{i,j-1}}{4} \tag{10}$$

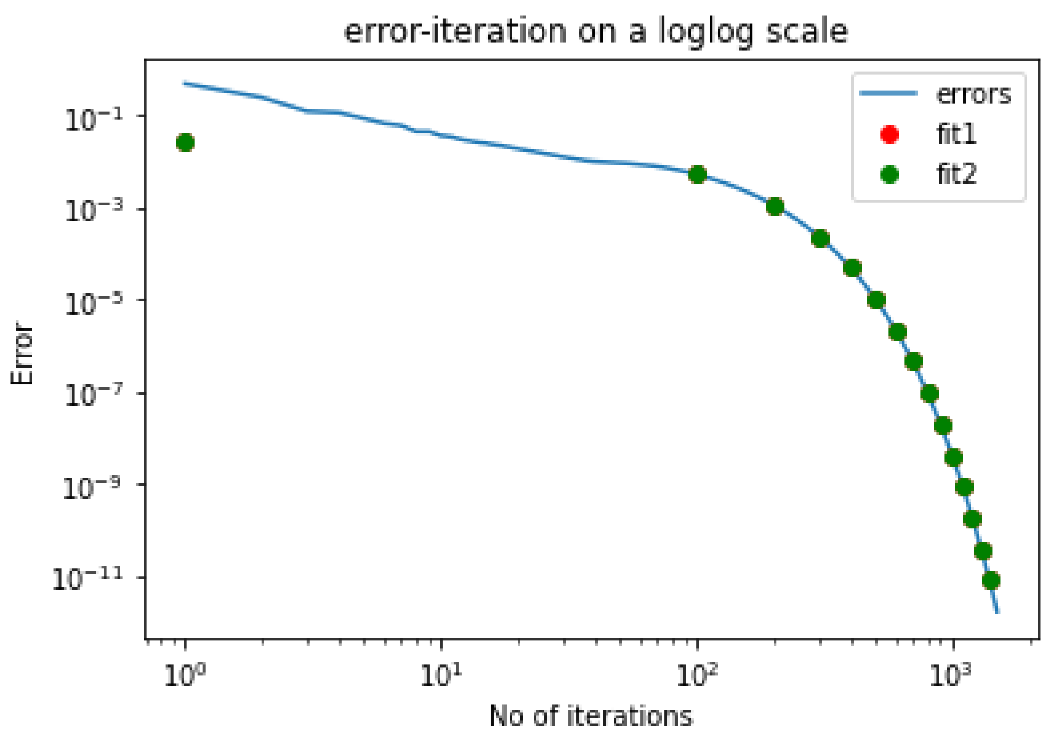
To apply boundry condition where there is no electrodeThe gradient of phi should be tengential

5 Error Calculation

- The error calculation can be analysed by plotting it against the iteration number.

- this will clear us clear picture of how the error varies with respect to the iterations number.

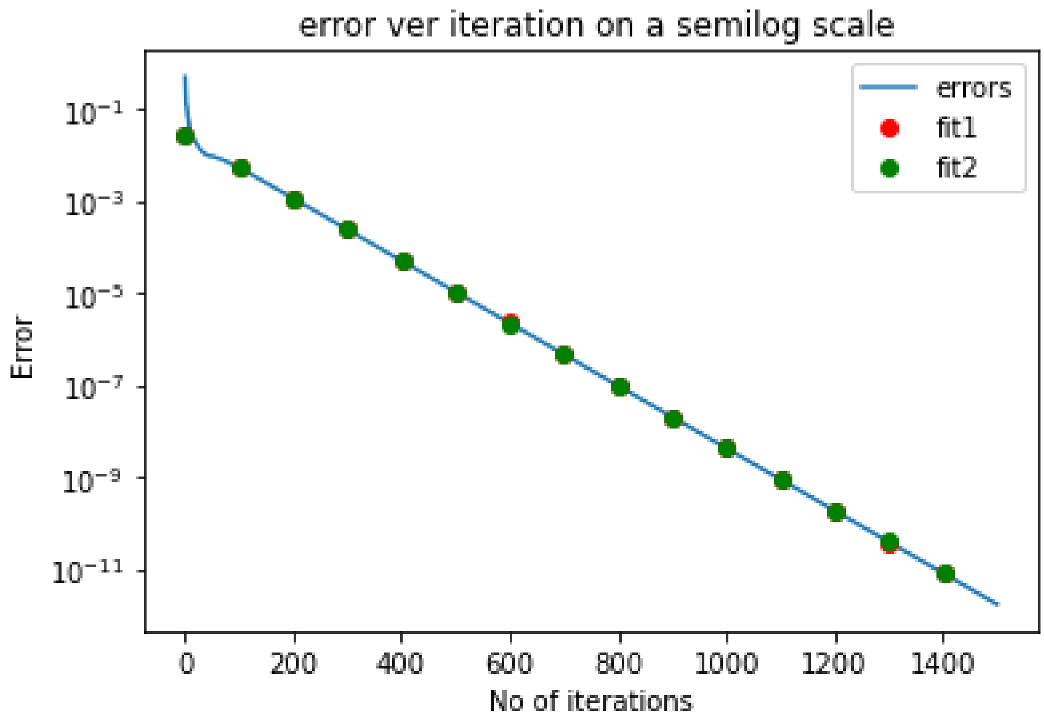




6 Extracting Exponential

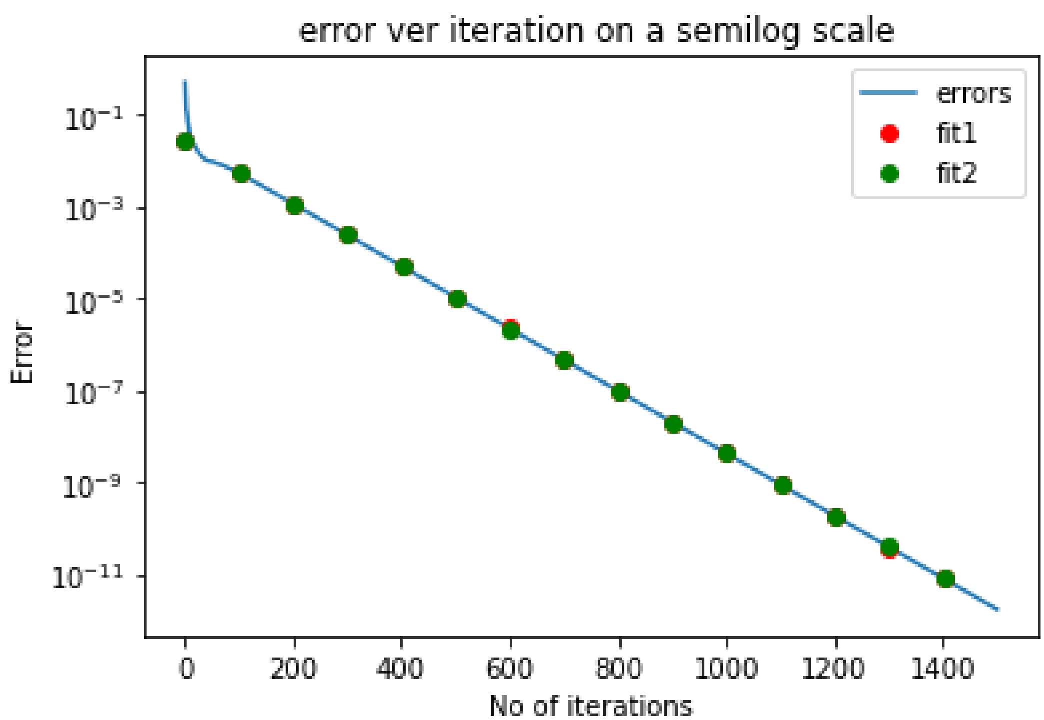
- we can find the fit least squares for all iterations seperately and compare them.
- As we know that error follows A*(e)powBx at large iterations, we use equation given below to fit the errors using least squares.

  logy = logA + Bx



- We can find the constants of the error function obtained for the two cases using lstsq and compare them.

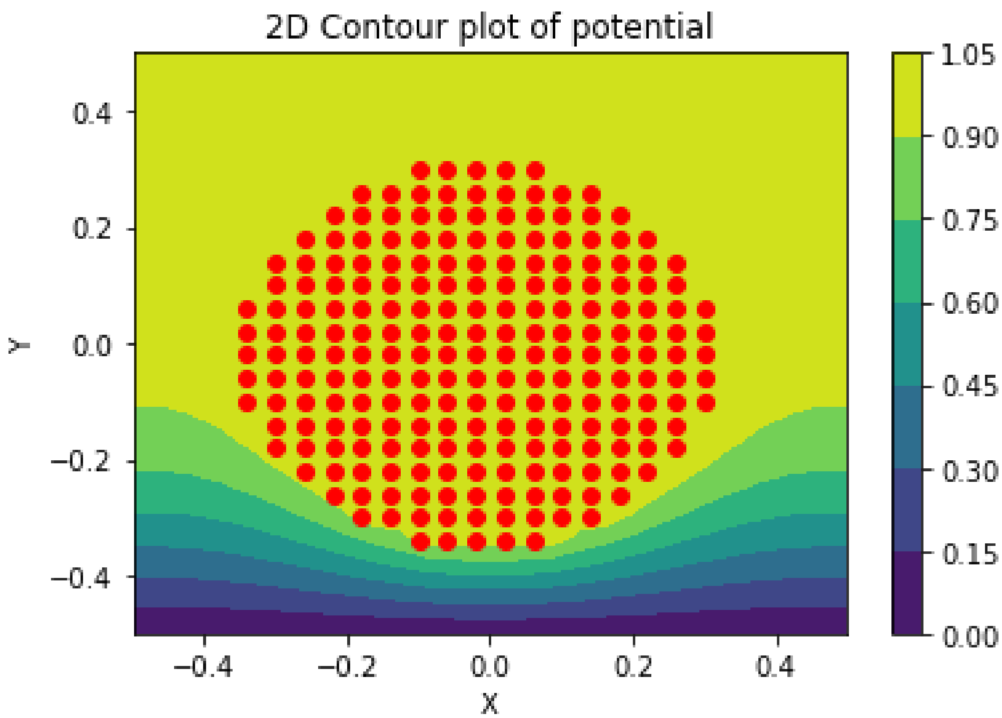
The plots comparing both the errors are as shown



7 Surface and contour plots of Potential

we can also Recognize the potential variations by plotting it as a surface and contour plot. plt.title("2D Contour plot of potential") plt.xlabel("X")
plt.ylabel("Y")
xp,yp=np.where(X**2+Y**2¡(0.35)**2)
plt.plot
plt.contourf(Y,X[::-1],phi)
plt.colorbar()
plt.show()
ax=p3.Axes3D(fig1)
plt.title('The 3-D surface plot of the potential')
surf = ax.plotsurface(Y, X, phi.T, rstride=1, cstride=1, cmap=plt.cm.jet)
plt.show()
Jx,Jy = (1/2*(phi[1:-1,0:-2]-phi[1:-1,2:]),1/2*(phi[:-2,1:-1]-phi[2:,1:-1]))

Plots are shown

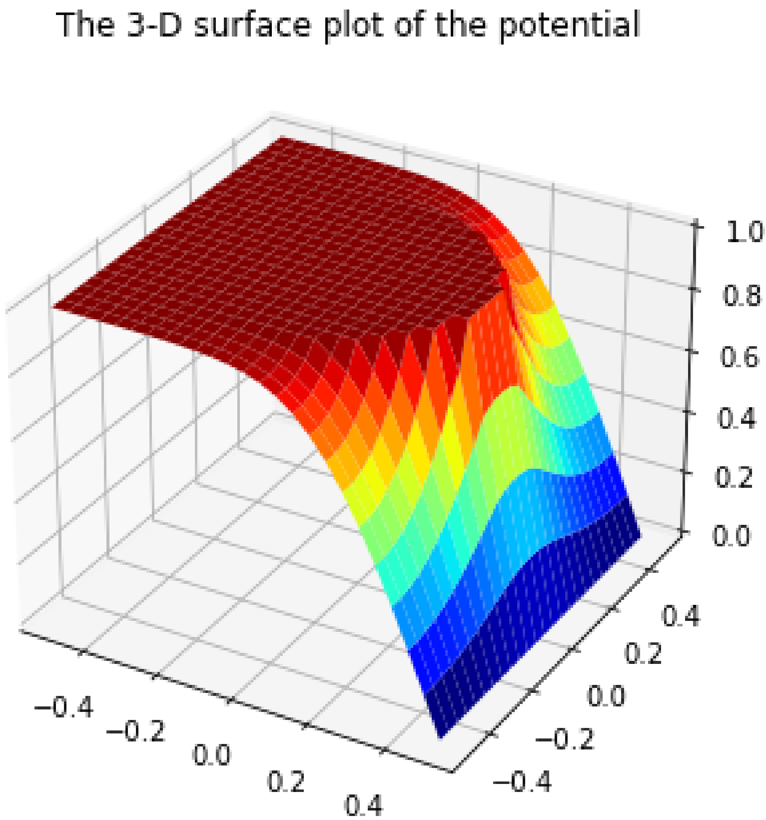


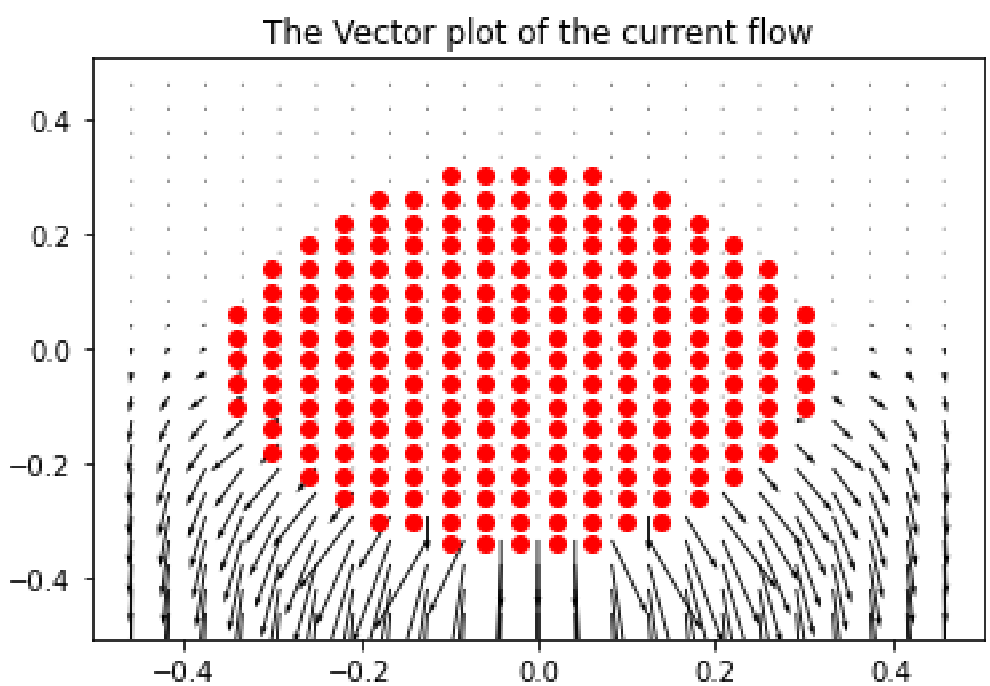
8 Vector plots of Currents

- we can obtain the vectors plot of current by computing the gradient.
- Plots are given below

9 Conclusions

There is no current in the upper region of the plate and the bottom part of the plate gets hotter and temperature increases in down region of the plate.In 3D potential plot, potential is higher on the top of the plate and reduces to zero till bottom. Electrode is connected to centre of plate so current will flow from centre to bottom. In vector plot of current heating will be more in the part below the central region due to more current in it. Net error reduces with increase in no. of iterations


The 3-D surface plot of the potential
1.0
0.8
0.6
0.4
0.2
0.0
−0.4
−0.2
0.0
0.2
0.4
0.4
0.2
0.0
−0.2
−0.4


The Vector plot of the current flow
0.4
0.2
0.0
−0.2
−0.4
−0.4
−0.2
0.0
0.2
0.4