Indian Institute of Technology Indore

Wave Propagation In Aluminum and Copper.

Problem statement on Skin Depth

A project submitted
in partial fulfillment of the requirements for the course code EE 305
Electromagnetic Waves

by

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October 2018

ABSTRACT

"Analysis of wave propagation in Aluminum and Copper and observing various wave parameters like skin depth and its dependence on frequency, electric field and magnetic field variation with distance and dependence of group velocity, phase velocity and wave impedance on frequency and plotting them using matplotlib python and Matlab. The project also includes a hypothetical problem statement analysis skin depth and its dependence on resistivity, permeability and frequency."

Table of Contents

LIST OF TABLES

LIST OF FIGURES

1. Introduction

Principles:

The AC current density J in a conductor decreases exponentially from its value at the surface J_S according to the depth d from the surface, as follows:

$$J = J_S \, e^{-(1+j)d/\delta} \qquad (1)$$

where δ is called the skin depth.

The skin depth is defined as the depth below the surface of the conductor at which the current density has fallen to 1/e (about 0.37) of J_S.
The wavelength in the conductor is much shorter than the wavelength in vacuum, or equivalently, the phase velocity in the conductor is very much slower than the speed of light in vacuum.
For example, a 1 MHz radio wave has a wavelength in vacuum λ0 of about 300 m, whereas in copper, the wavelength is reduced to only about 0.5 mm with a phase velocity of only about 500 m/s.
As a consequence of Snell's law and this very tiny phase velocity in the conductor, any wave entering the conductor, even at grazing incidence, refracts essentially in the direction perpendicular to the conductor's surface.
The simplified formula for the skin depth in case of good conductors is:

$$\delta = \sqrt{\frac{2\rho}{\omega\mu}} \qquad (2)$$

Phase velocity defines the speed at which waves oscillating at a particular frequency propagate. Like the wavelength, the phase velocity depends on the real component of the wavenumber (α)

$\dot{\alpha} = \beta = \sqrt{\Pi f \mu \rho}$ (3)

Phase Velocity = $2x\sqrt{\Pi f \rho / \mu}$ (4)

Group Velocity = $d\omega/d\beta = d(2\beta^2 \rho/\mu)/d\beta = 4\beta\rho/\mu$ (5)

Wave impedance defines the ratio between transverse components of the electric and magnetic fields supported by an EM planewave.

Wave impedance = $\eta = \sqrt{2\Pi f \rho \mu}$ (6)

2.1 Wave Propagation in Copper

Resistivity of copper ρ = 1.68 x 10^{-8} Ωm (7)

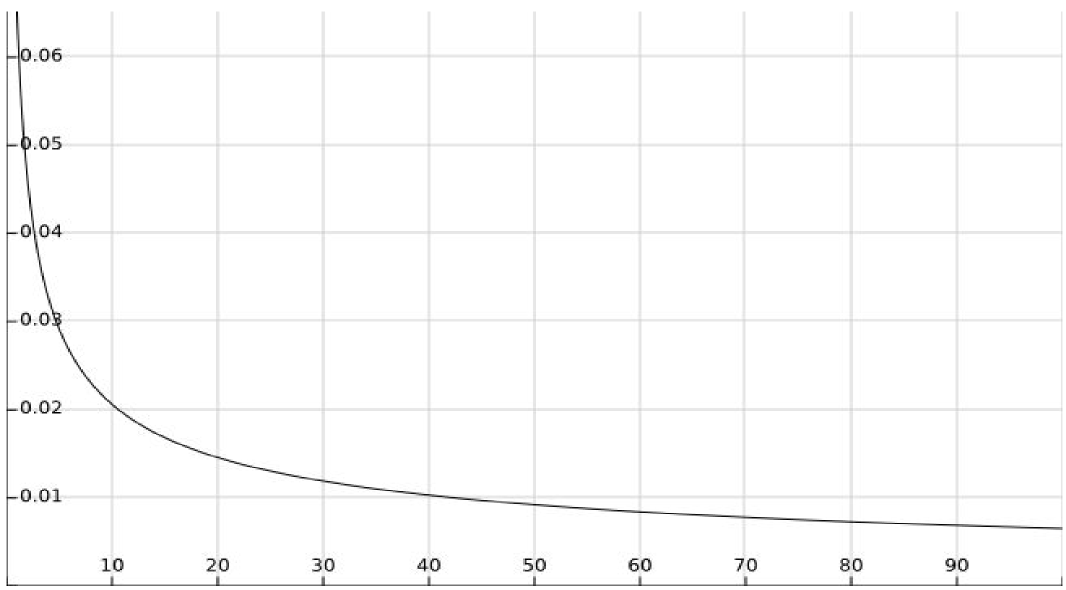
μ = 1.256 x 10^{-6}H/m

F is the frequency of the EM wave

The skin depth formula reduces to

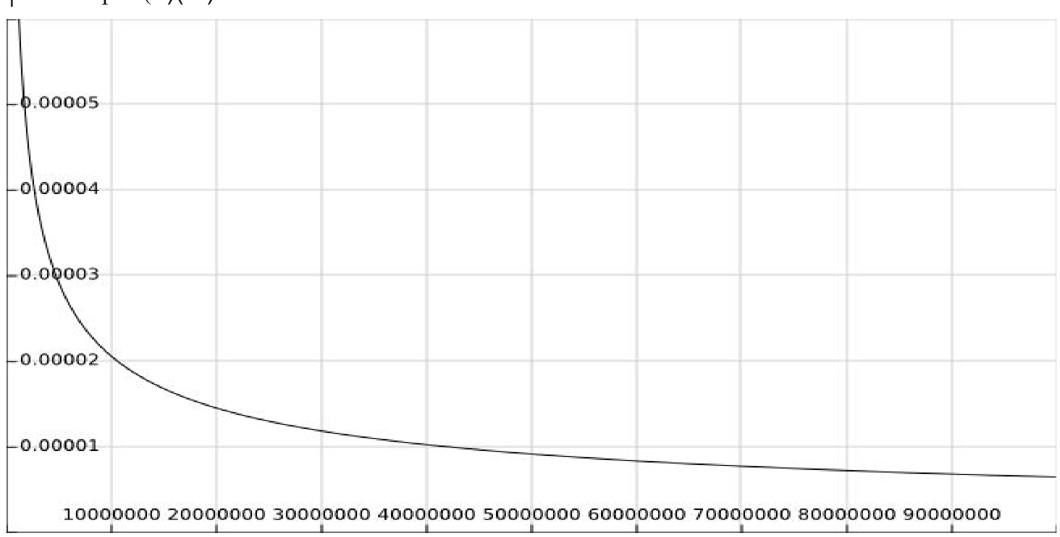
$\boldsymbol{\delta} = 0.0652 / \sqrt{F}$

↑Skin depth (**δ**)(m)



Frequency (F) (Hz) →

↑Skin depth (**δ**)(m)



Frequency (F) (Hz) →

Fig 2.1.1 Skin depth vs frequency

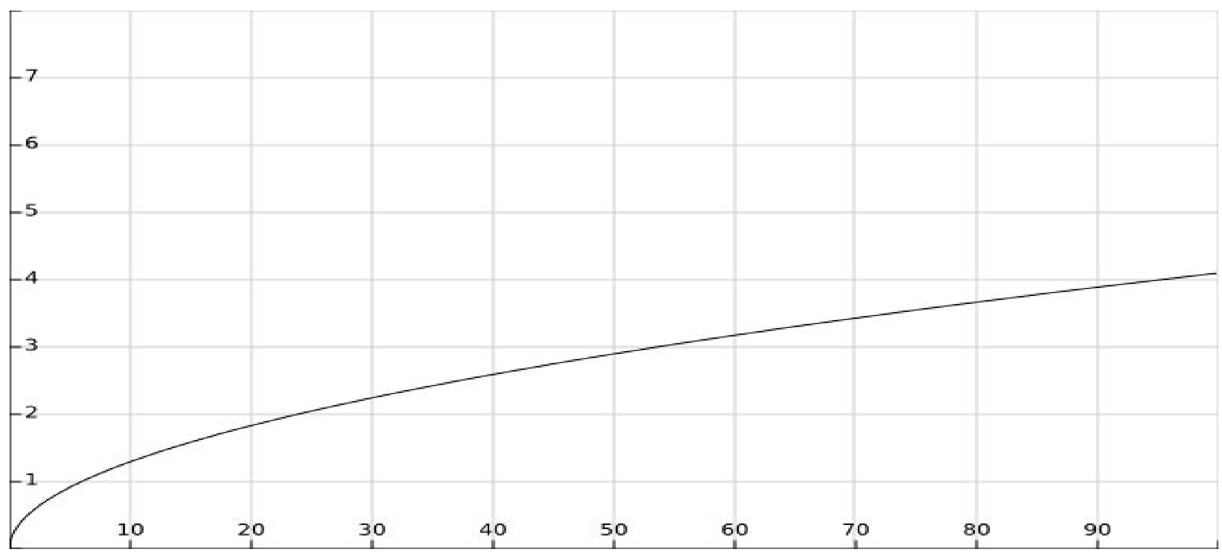
It follows that the skin-depth is about 6 cm at 1Hz, but only about 2mm at 1kHz. This gives rise to the so-called *skin effect* in copper wires, by which an oscillating electromagnetic signal of increasing frequency, transmitted along such a wire, is confined to an increasingly narrow layer (whose thickness is of order the skin-depth) on the surface of the wire.

Also we have other parameters,

$$\dot{\alpha}=\beta = 15.32 \times \sqrt{F} \tag{8}$$

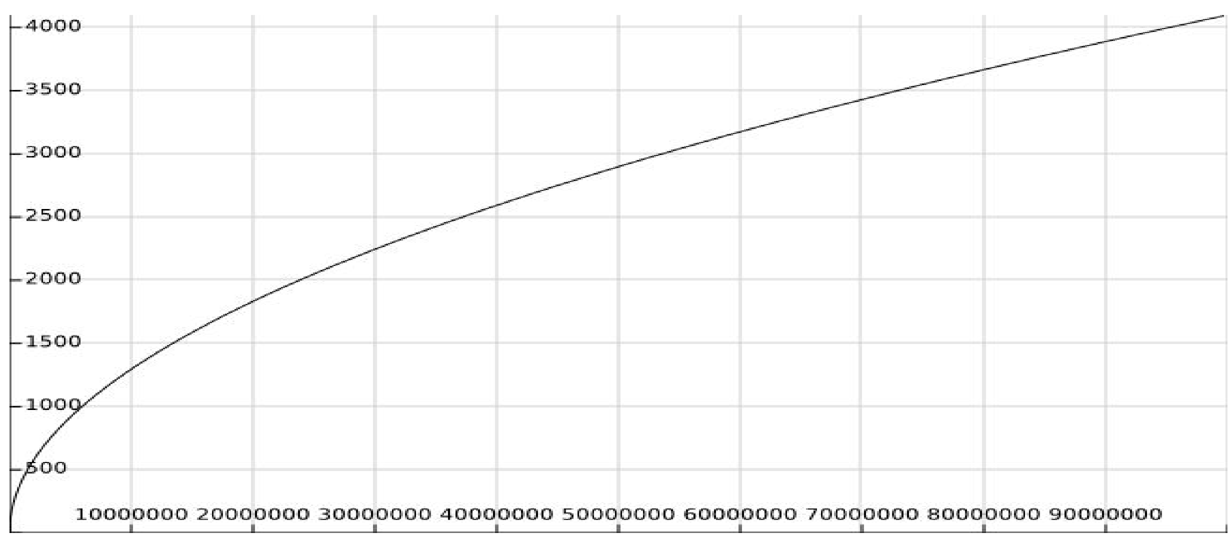
$$\text{Phase velocity } u = 0.41 \times \sqrt{F} \tag{9}$$

↑phase velocity (u)(m/s)



Frequency (F) (Hz) →

↑phase velocity (u)(m/s)



Frequency (F) (Hz) →

Fig 2.1.2 phase velocity vs frequency

Wave Impedance $\eta = 3.64 \times 10^{-7} \sqrt{F}$ (10)

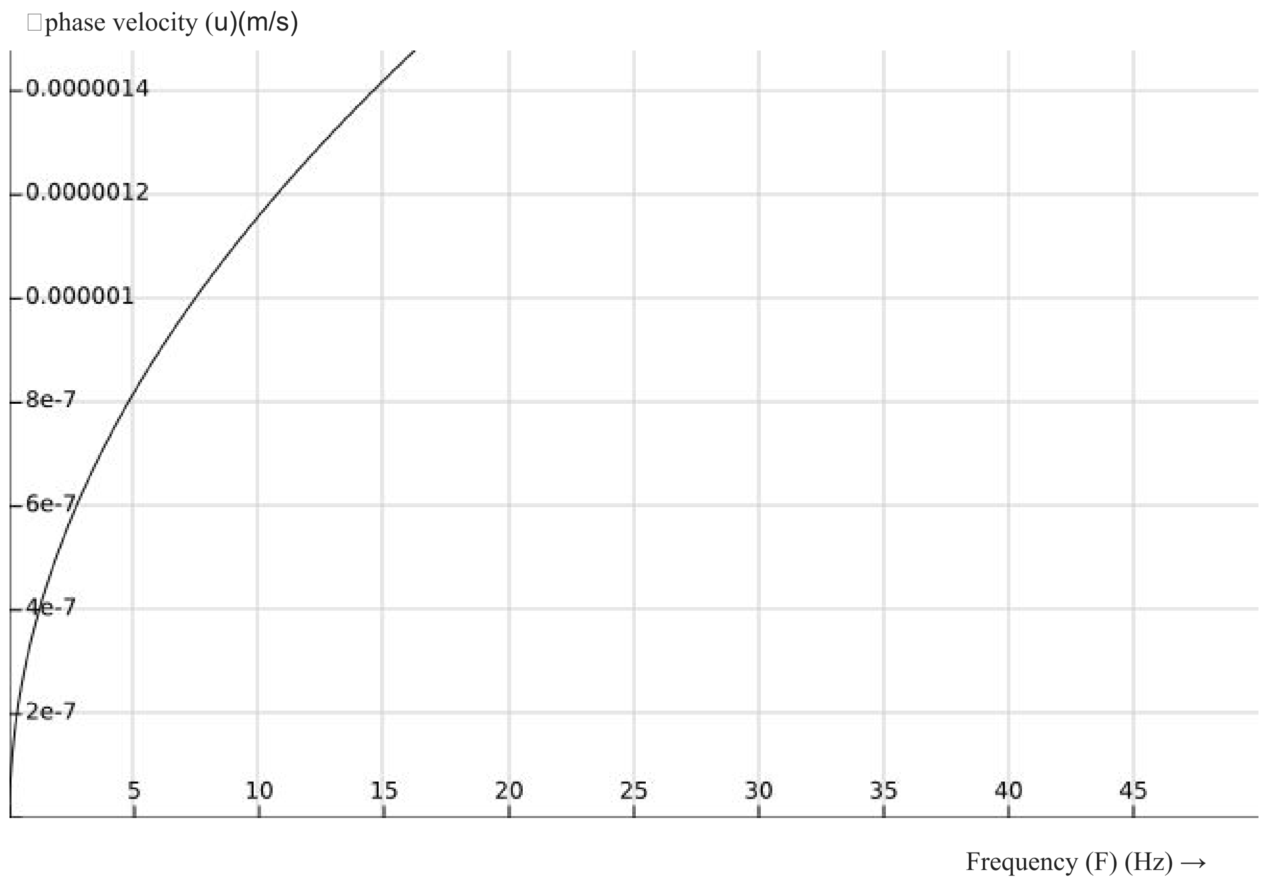


Fig 2.1.3 wave impedance vs frequency

Group velocity $v_g = d(2\beta^2 \rho/\mu)/d\beta = 4\beta\rho/\mu = 0.82 \times \sqrt{F}$ (11)

The plot for group velocity is similar to that of phase velocity.

According to Equation (10), the impedance of a good conductor is far less than that of a vacuum. This implies that the ratio of the magnetic to the electric components of an electromagnetic wave propagating through a good conductor is far larger than that of a wave propagating through a vacuum.

Suppose that the region z<0 is a vacuum, and the region z>0 is occupied by a good conductor of conductivity. Consider a linearly polarized plane wave normally incident on the interface. Let the wave electric and magnetic fields in the vacuum region take the form of the incident and reflected waves specified in Equations (12) and (13). The wave electric and magnetic fields in the conductor are written

E(z,t) = $E_o\, e^{-\alpha z} \cos(wt - \beta z + \phi)\; a_x$ (12)

H(z,t) = $E_o/\eta\, e^{-\alpha z} \cos(wt - \beta z - \Pi/4 + \phi)\ a_y$ (13)

Where E_o is the amplitude of the evanescent wave that penetrates into the conductor, ϕ is the phase of this wave with respect to the incident wave, and as E (or H) wave travels in a conducting medium, its amplitude is attenuated by the factor e^(-az) .

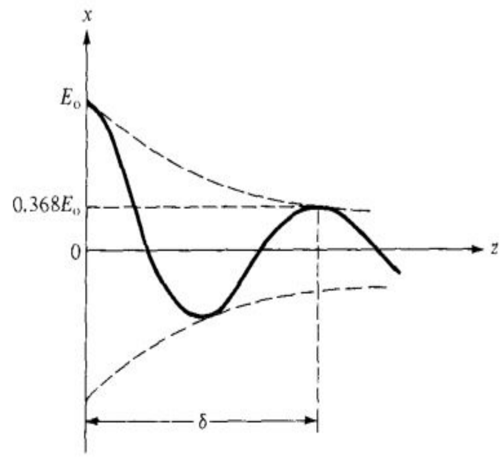


Fig. 2.1.4 Electric field **E(z,t)** intensity

Frequency (Hz)	**10**	**60**	**100**	**500**	**10^4**	**10^8**	**10^{10}**
Skin depth (mm)	20.8	8.6	6.6	2.99	0.66	6.6×10^{-3}	6.6×10^{-4}

*For copper, $\sigma = 5.8 \times 10^7$ mhos/m, $\mu = \mu_o$, $\delta = 66.1/\sqrt{f}$ (in mm).

Table 2.1.1 Skin depth in Copper

The skin depth in copper at various frequencies is shown in Table 2.1.1. From the table, we notice that the skin depth decreases with increase in frequency. Thus, **E** and **H** can hardly propagate through good conductors.

2.2 Wave Propagation in Aluminum

ρ = 2.56 x 10^{-8} Ωm

μ = 1.256 x 10^{-6}H/m

The skin depth formula reduces to

$\boldsymbol{\delta}$ = 0.0819 / $\sqrt{F}$

$\dot{\alpha}=\beta$ = 15.32 $\times\ \sqrt{F}$

Phase velocity u = $0.51 \times \sqrt{F}$

Wave Impedance $\eta = 4.57 \times 10^{-7} \sqrt{F}$

Group velocity vg = $d(2\beta^2 \rho/\mu)/d\beta = 4\beta\rho/\mu$ = 1.03 x $\sqrt{F}$

E = $E_o\ e^{-\dot{\alpha}z}$ cos(wt - βz) a_x

H = $E_o/\eta\ e^{-\dot{\alpha}z}$ cos(wt - βz) a_y

Note: The plots of Aluminium are similar to that of copper. Thus, they have been skipped for the sake of convenience.

3. Hypothetical Problem statement

Question:

In an experiment, it is found that the electromagnetic wave(light) falls on a material with some angle Ø1 and refraction angle Ø2. When the same material is connected to a battery of potential V, the observed current of Ammeter is I (Figure 1). The incident light is able to emit(just able to emit) an electron of the material which have work function Φ (Figure 2). Now find the skin depth (δ) of this material. (given: Ø1 = 53º, Ø2 = 37º, I = 2A, V = 10V, a = 10cm, b = 10cm, L = 20cm, Φ = 2ev)

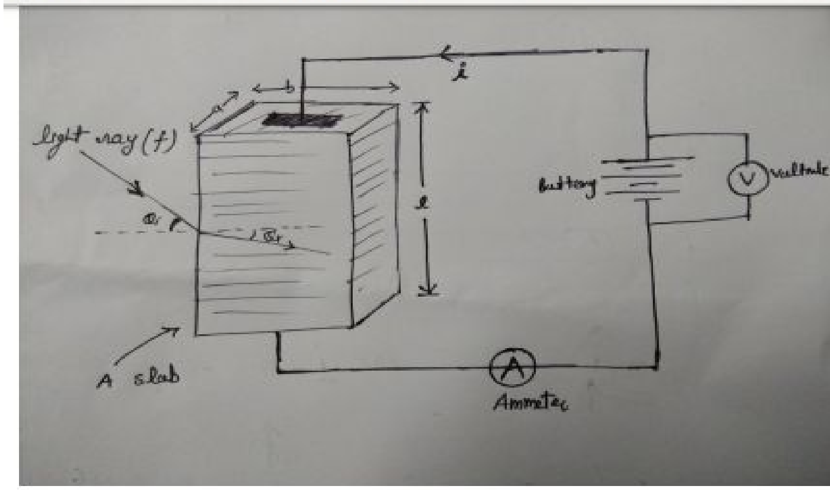


Figure 1

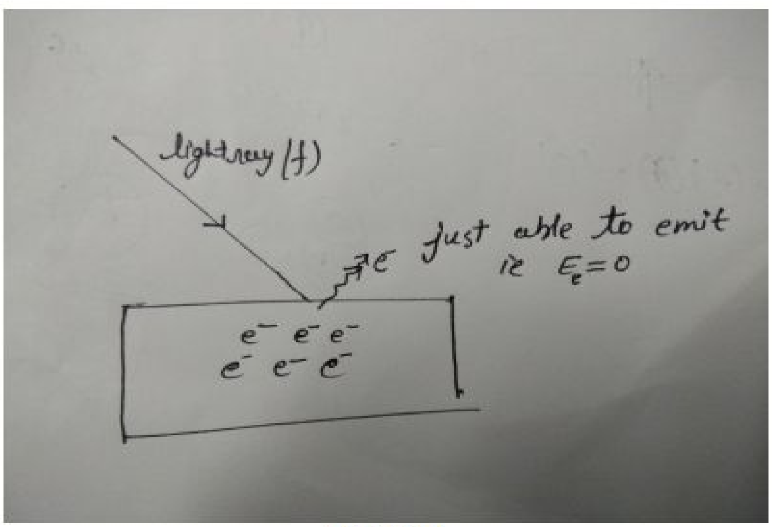


Figure 2

Fig 3.1 Figure 1 and Figure 2

4. Conclusion

The behaviour of EM waves in a conductor is quite different from that in a source-free medium. The conduction current in a conductor is the cause of the difference. We analysed the skin effect and electric and magnetic field propagation in Aluminium and copper. Moreover, we came up with a hypothetical situation where we studied the skin effect in the medium when light falls on it. The skin depth in copper at various frequencies is shown in Table 2.1.1. From the table, we notice that the skin depth decreases with increase in frequency. Thus, **E** and **H** can hardly propagate through good conductors.

References

[1]	Elements of Electromagnetics, Matthew Sadiku, 2018
[2]	https://en.wikipedia.org/wiki/Skin_effect
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[4]	https://en.wikipedia.org/wiki/Group_velocity
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