



**EECS 151/251A**  
**Spring 2024**  
**Digital Design and Integrated Circuits**

Instructor:  
Wawrzynek

Lecture 14 - Exam1 Review

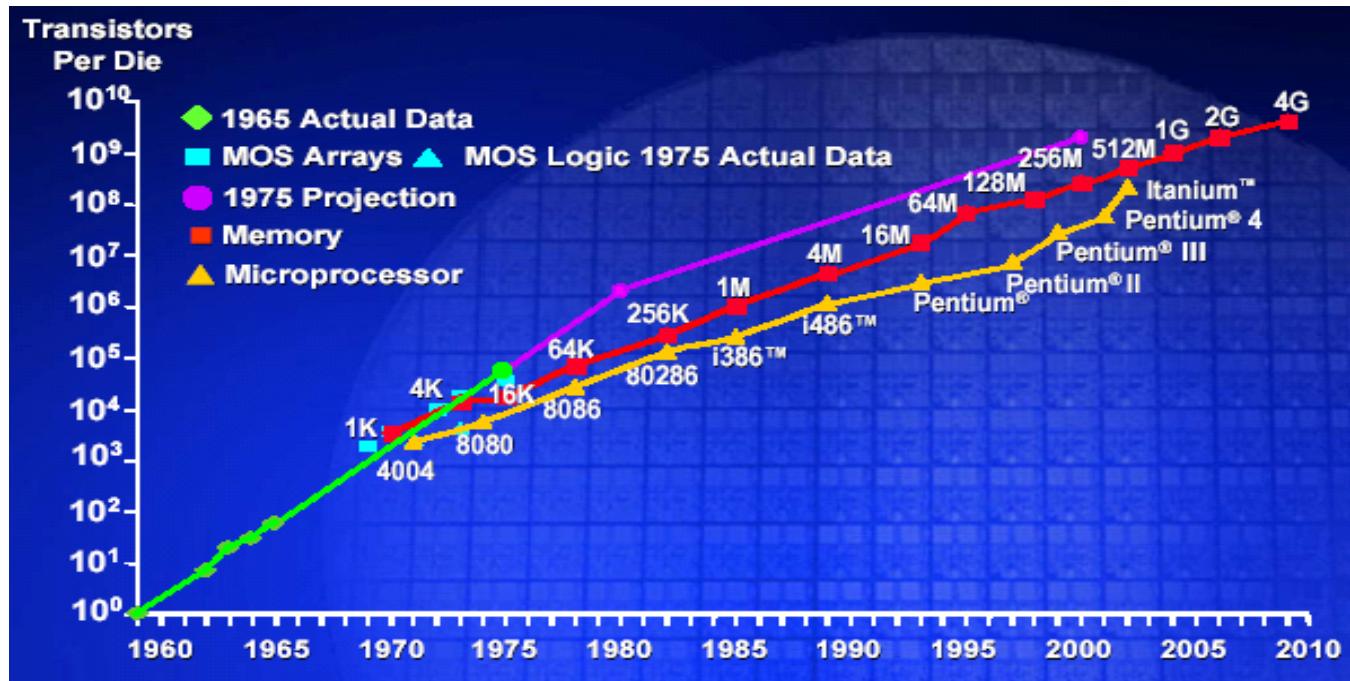
# Announcements

- ❑ **Midterm Exam 6-9PM**
  - ❑ *Latimer 120 (alternate seating)*
  - ❑ *Exam covers Lectures 1 - 10 and HW 1 - 6 (Through CMOS circuits)*
  - ❑ *One double sided handwritten sheet of paper allowed. No calculators.*
  - ❑ *Neatness counts! Bring a ruler to help draw diagrams.*
- ❑ Homework solutions posted through HW 6
- ❑ No homework due today
- ❑ HW7 posted, due next Monday.

# *Review with sample slides*

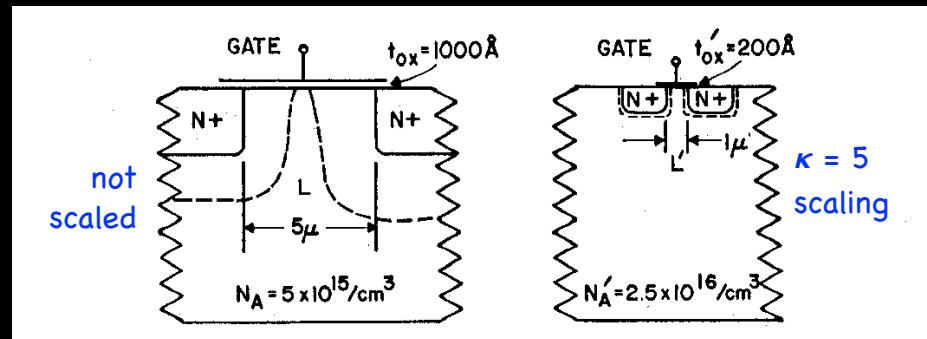
- ❑ Do not study only the following slides. These are just representative of what you need to know.
- ❑ Go back and study the entire lecture.

# Moore's Law – 2x transistors per 1-2 yr



# Dennard Scaling

Things we do: scale dimensions, doping, Vdd.



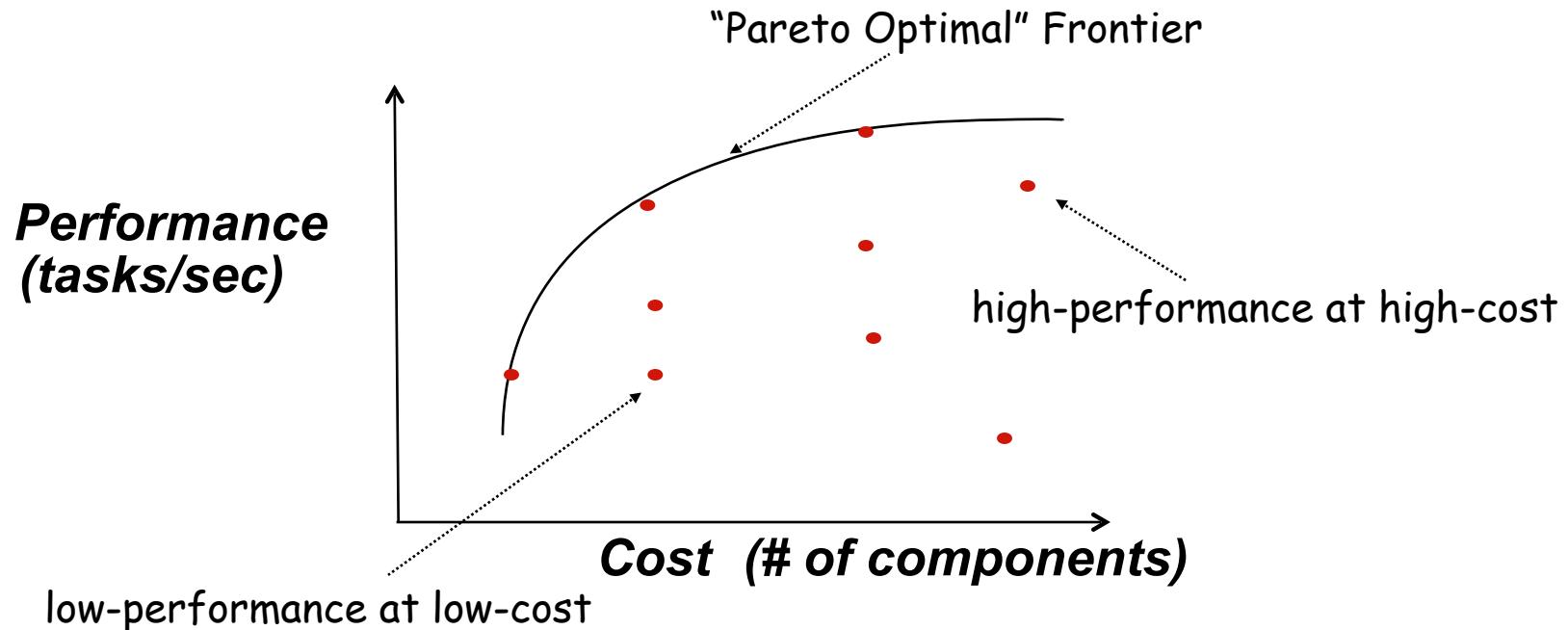
What we get:  
 $\kappa^2$  as many transistors  
at the same power density!

Whose gates switch  $\kappa$  times faster!

Device or Circuit Parameter	Scaling Factor
Device dimension $t_{ox}, L, W$	$1/\kappa$
Doping concentration $N_a$	$\kappa$
Voltage $V$	$1/\kappa$
Current $I$	$1/\kappa$
Capacitance $\epsilon A/t$	$1/\kappa$
Delay time/circuit $VC/I$	$1/\kappa$
Power dissipation/circuit $VI$	$1/\kappa^2$
Power density $VI/A$	1

Power density scaling ended in 2003  
(Pentium 4: 3.2GHz, 82W, 55M FETs).

# *Design Space & Optimality*



# Cost

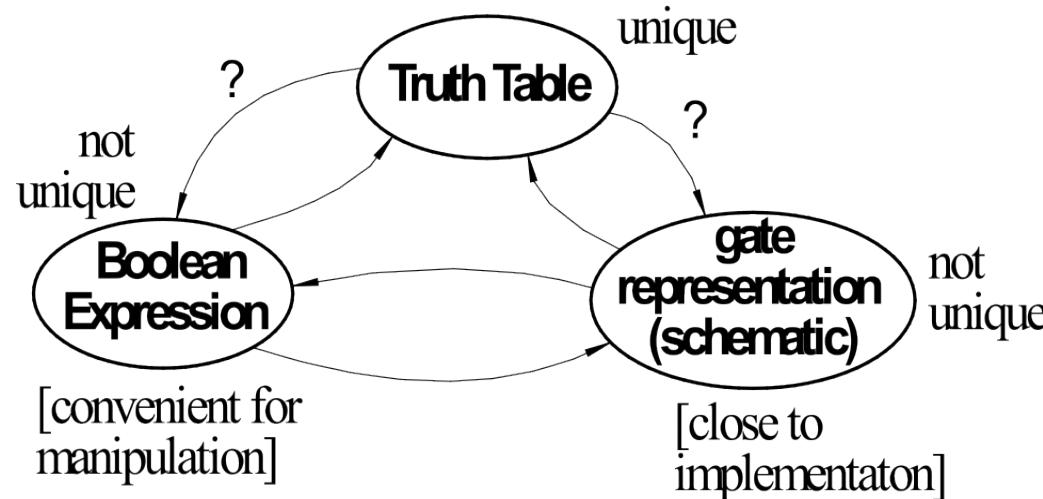
- ❑ Non-recurring engineering (NRE) costs
  - ❑ Cost to develop a design (product) - *people, tools, masks*
    - Amortized over all units shipped
    - E.g. \$20M in development adds \$.20 to each of 100M units
- ❑ Recurring costs
  - Cost to manufacture, test and package a unit
  - Processed wafer cost is ~\$10k (around 16nm node) which yields:
    - 50-100 large FPGAs or GPUs
    - 200 laptop CPUs
    - >1000 cell phone SoCs

$$\text{cost per IC} = \text{variable cost per IC} + \frac{\text{fixed cost}}{\text{volume}}$$

$$\text{variable cost} = \frac{\text{cost of die} + \text{cost of die test} + \text{cost of packaging}}{\text{final test yield}}$$

# Relationship Among Representations

- \* Theorem: Any Boolean function that can be expressed as a truth table can be written as an expression in Boolean Algebra using AND, OR, NOT.

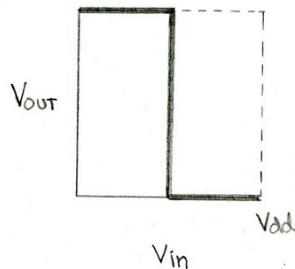


*How do we convert from one to the other?*

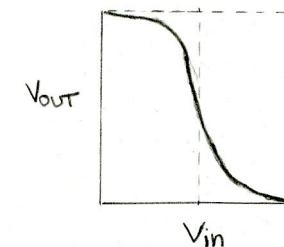
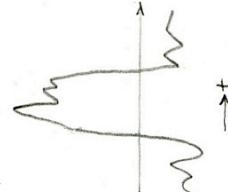
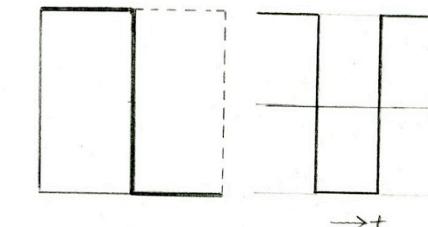
# Logic Gate Restoration

Example (look at 1-input gate, to keep it simple):

- Inverter acts like a “non-linear” amplifier
- The non-linearity is critical to restoration
- Other logic gates act similarly with respect to input/output relationship.



Idealize Inverter

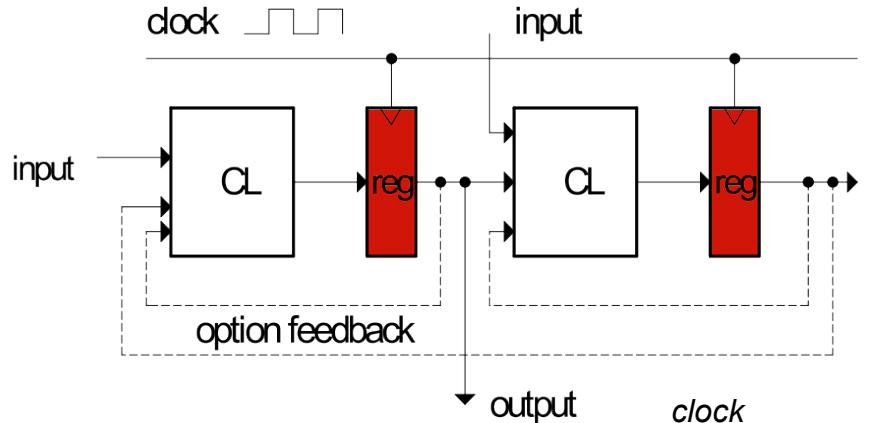


Actual Inverter  
voltage transfer  
characteristic (VTC)

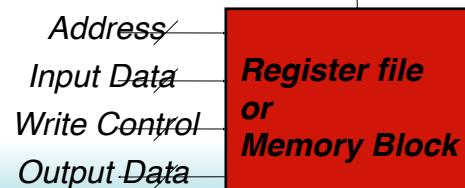
# Register Transfer Level Abstraction (RTL)

Any synchronous digital circuit can be represented with:

- Combinational Logic Blocks (CL), plus
- State Elements (registers or memories)



• State elements are mixed in with CL blocks to control the flow of data.



• Sometimes used in large groups by themselves for “long-term” data storage.

# Implementation Alternative Summary

Full-custom:	All circuits/transistors layouts optimized for application.
Standard-cell (ASIC):	Small function blocks/“cells” (gates, FFs) automatically placed and routed.
Gate-array (structured ASIC):	Partially prefabricated wafers with arrays of transistors customized with metal layers or vias.
FPGA:	Prefabricated chips customized with loadable latches or fuses.
Microprocessor:	Instruction set interpreter customized through software.
Domain Specific Processor:	Special instruction set interpreters (ex: DSP, NP, GPU, TPU).

What are the important metrics of comparison?

# Hardware Description Languages

- Basic Idea:
  - Language constructs describe circuits with two basic forms:
  - **Structural descriptions:** connections of components. Nearly one-to-one correspondence to with schematic diagram.
  - **Behavioral descriptions:** use high-level constructs (similar to conventional programming) to describe the circuit function.
- Originally invented for simulation.
  - “logic synthesis” tools exist to automatically convert to gate level representation.
  - High-level constructs greatly improves designer productivity.
  - However, this may lead you to falsely believe that hardware design can be reduced to writing programs\*

“Structural” example:

```
Decoder(output x0,x1,x2,x3;
         inputs a,b)
{
    wire abar, bbar;
    inv(bbar, b);
    inv(abar, a);
    and(x0, abar, bbar);
    and(x1, abar, b );
    and(x2, a,      bbar);
    and(x3, a,      b );
}
```

“Behavioral” example:

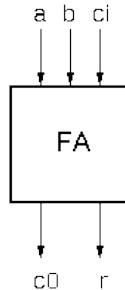
```
Decoder(output x0,x1,x2,x3;
         inputs a,b)
{
    switch [a b]
        case 00: [x0 x1 x2 x3] = 0x8;
        case 01: [x0 x1 x2 x3] = 0x4;
        case 10: [x0 x1 x2 x3] = 0x2;
        case 11: [x0 x1 x2 x3] = 0x1;
    endswitch;
}
```

Warning: this is a fake HDL!

\*New tools and languages exist for this - called “high level synthesis”.

# Review - Ripple Adder Example

```
module FullAdder(a, b, ci, r, co);
    input a, b, ci;
    output r, co;
    assign r = a ^ b ^ ci;
    assign co = a&ci + a&b + b&cin;
endmodule
```



```
module Adder(A, B, R);
    input [3:0] A;
    input [3:0] B;
    output [4:0] R;
    wire c1, c2, c3;
    FullAdder
        add0(.a(A[0]), .b(B[0]), .ci(1'b0), .co(c1), .r(R[0]) ),
        add1(.a(A[1]), .b(B[1]), .ci(c1), .co(c2), .r(R[1]) ),
        add2(.a(A[2]), .b(B[2]), .ci(c2), .co(c3), .r(R[2]) ),
        add3(.a(A[3]), .b(B[3]), .ci(c3), .co(R[4]), .r(R[3]) );
endmodule
```

```
graph LR
    FA0[FA] -- a --> InA0
    FA0 -- b --> InB0
    FA0 -- ci --> InCi0
    InA0 --> FA0
    InB0 --> FA0
    InCi0 --> FA0
    FA0 -- r --> OutR0
    FA0 -- co --> C1
    FA1[FA] -- a --> InA1
    FA1 -- b --> InB1
    FA1 -- ci --> InCi1
    InA1 --> FA1
    InB1 --> FA1
    InCi1 --> FA1
    FA1 -- r --> OutR1
    FA1 -- co --> C2
    FA2[FA] -- a --> InA2
    FA2 -- b --> InB2
    FA2 -- ci --> InCi2
    InA2 --> FA2
    InB2 --> FA2
    InCi2 --> FA2
    FA2 -- r --> OutR2
    FA2 -- co --> C3
    FA3[FA] -- a --> InA3
    FA3 -- b --> InB3
    FA3 -- ci --> InCi3
    InA3 --> FA3
    InB3 --> FA3
    InCi3 --> FA3
    FA3 -- r --> OutR3
    C1 --> FA1
    C2 --> FA2
    C3 --> FA3
    OutR0 --> R0
    OutR1 --> R1
    OutR2 --> R2
    OutR3 --> R3
    R0 --> R4
```

# Example - Ripple Adder Generator

Parameters give us a way to generalize our designs. A module becomes a “generator” for different variations. Enables design/module reuse. Can simplify testing.

```
module Adder(A, B, R);  
    parameter N = 4;  
    input [N-1:0] A;  
    input [N-1:0] B;  
    output [N:0] R;  
    wire [N:0] C;
```

Declare a parameter with default value.

Note: this is not a port. Acts like a “synthesis-time” constant.

Replace all occurrences of “4” with “N”.

variable exists only in the specification - not in the final circuit.

```
genvar i;
```

Keyword that denotes synthesis-time operations

```
generate
```

For-loop creates instances (with unique names)

```
    for (i=0; i<N; i=i+1) begin:bit  
        FullAdder.add(.a(A[i]), .b(B[i]), .ci(C[i]), .co(C[i+1]), .r(R[i]));  
    end
```

```
endgenerate
```

```
assign C[0] = 1'b0;  
assign R[N] = C[N];  
endmodule
```

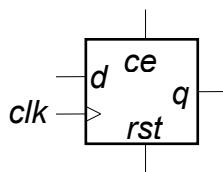
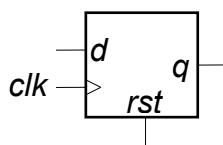
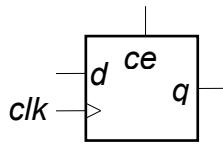
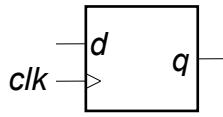
```
Adder adder4 ( ... );
```

```
Adder #(N(64))  
adder64 ( ... );
```

Overwrite parameter  
N at instantiation.

# EECS151 Registers

- All registers are “N” bits wide - the value of N is specified at instantiation
- All positive edge triggered.



```
module REGISTER(q, d, clk);  
  parameter N = 1;
```

```
module REGISTER_CE(q, d, ce, clk);  
  parameter N = 1;
```

On the rising clock edge if clock enable (ce) is 0 then the register is disabled (it's state will not be changed).

```
module REGISTER_R(q, d, rst, clk);  
  parameter N = 1;  
  parameter INIT = {N{1'b0}};
```

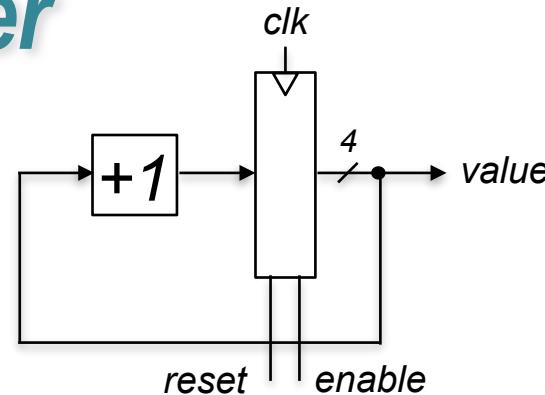
On the rising clock edge if reset (rst) is 1 then the state is set to the value of INIT. Default INIT value is all 0's.

```
module REGISTER_R_CE(q, d, rst, ce, clk);  
  parameter N = 1;  
  parameter INIT = {N{1'b0}};
```

Reset (rst) has priority over clock enable (ce).

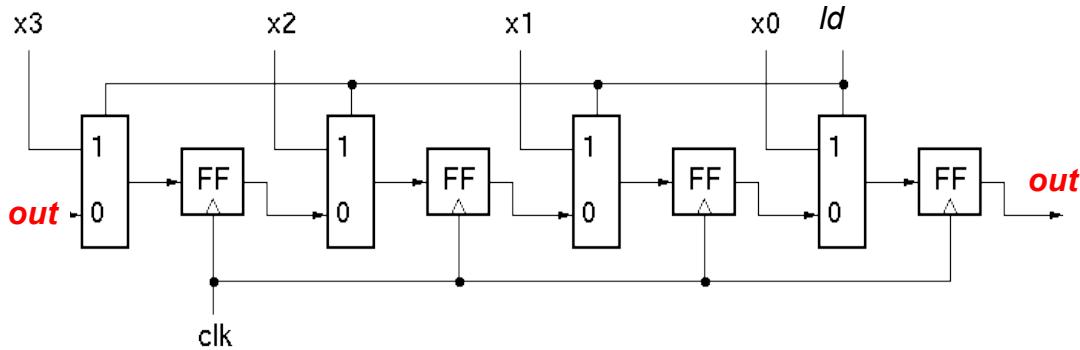
# 4-bit wrap-around counter

$0, 1, 2, 3, 4, 5, 6, 7, 8,$   
 $9, 10, 11, 12, 13, 14,$   
 $15, 0, 1, \dots$



```
module counter(value, enable, reset, clk);
    output [3:0] value;
    input      enable, reset, clk;
    wire [3:0]  next;
    REGISTER_R #(4) state (.q(value), .d(next), .rst(reset), .clk(clk));
    assign next = value + 1;
endmodule // counter
```

# Example - Parallel to Serial Converter



Specifies the muxing with "rotation"  
Instantiates a register (flip-flops) to be rewritten every cycle  
connect output

```
module ParToSer(ld, X, out, clk);
    input [3:0] X;
    input ld, clk;
    output out;

    wire [3:0] Q;
    wire [3:0] NS;

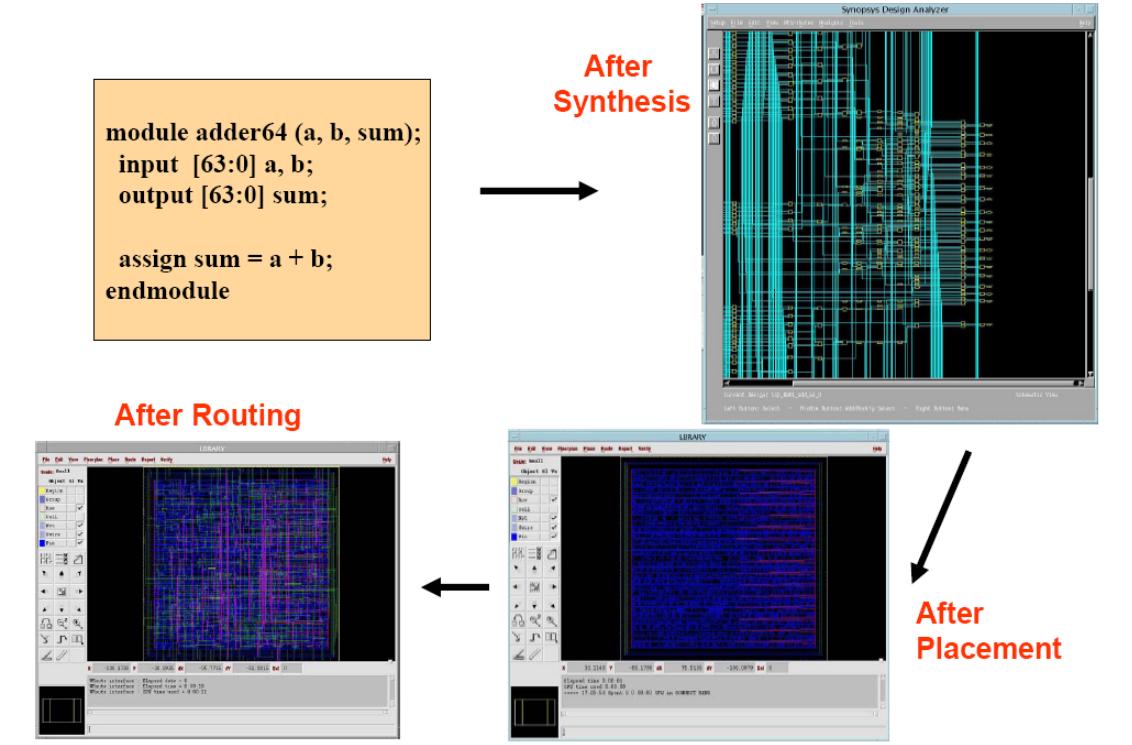
    assign NS =
        (ld) ? X : {Q[0], Q[3:1]};

    REGISTER state #(4)
        (.q(Q), .d(NS), .clk(clk));

    assign out = Q[0];
endmodule
```

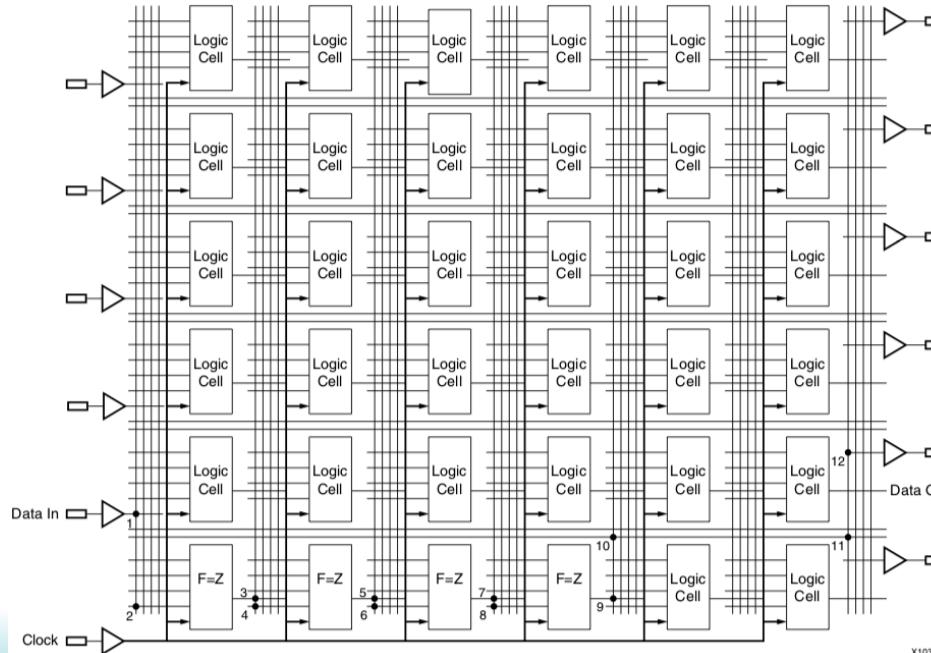
# Verilog to ASIC layout flow

- “push-button” approach



# FPGA Overview

- Basic structure: two-dimensional array of logic blocks and flip-flops with a means for the user to configure (program):
  - the interconnection between the logic blocks,
  - the function of each block.

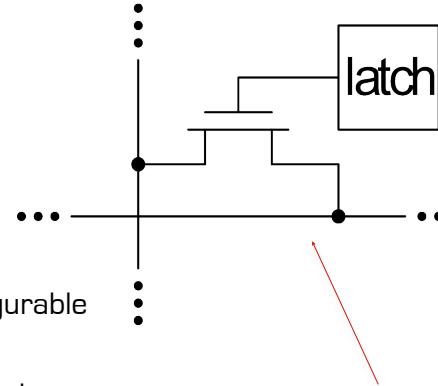


*Simplified version of FPGA internal architecture* 19

# User Programmability

- Latches are used to:
  1. control a switch to make or break cross-point connections in the interconnect
  2. define the function of the logic blocks
  3. set user options:
    - within the logic blocks
    - in the input/output blocks
    - global reset/clock
- “Configuration bit stream” is loaded under user control

- Latch-based (Xilinx, Intel/Altera, ...)

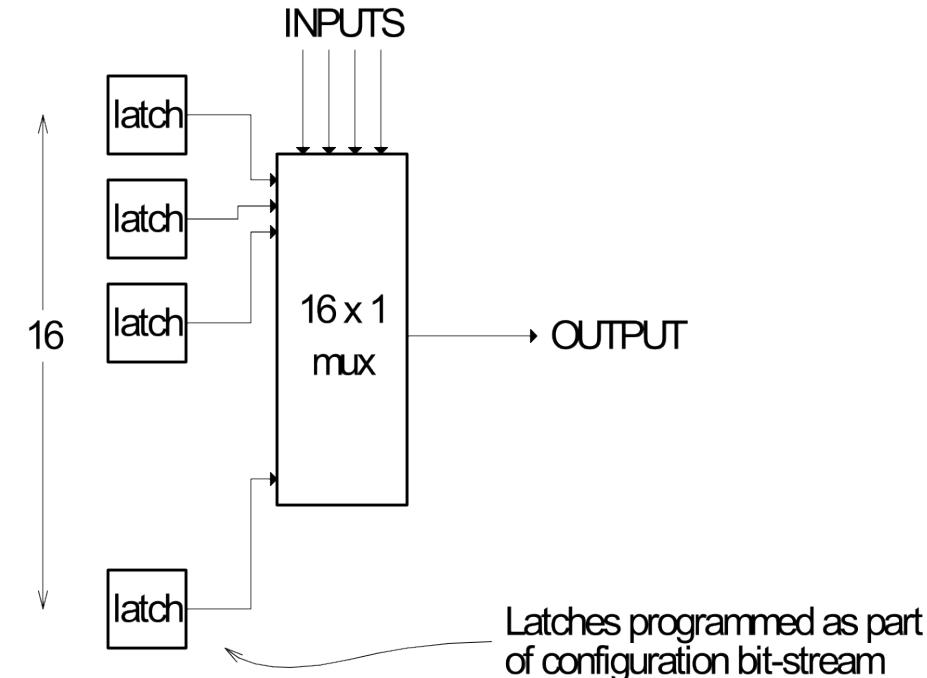


- + reconfigurable
- volatile
- relatively large.

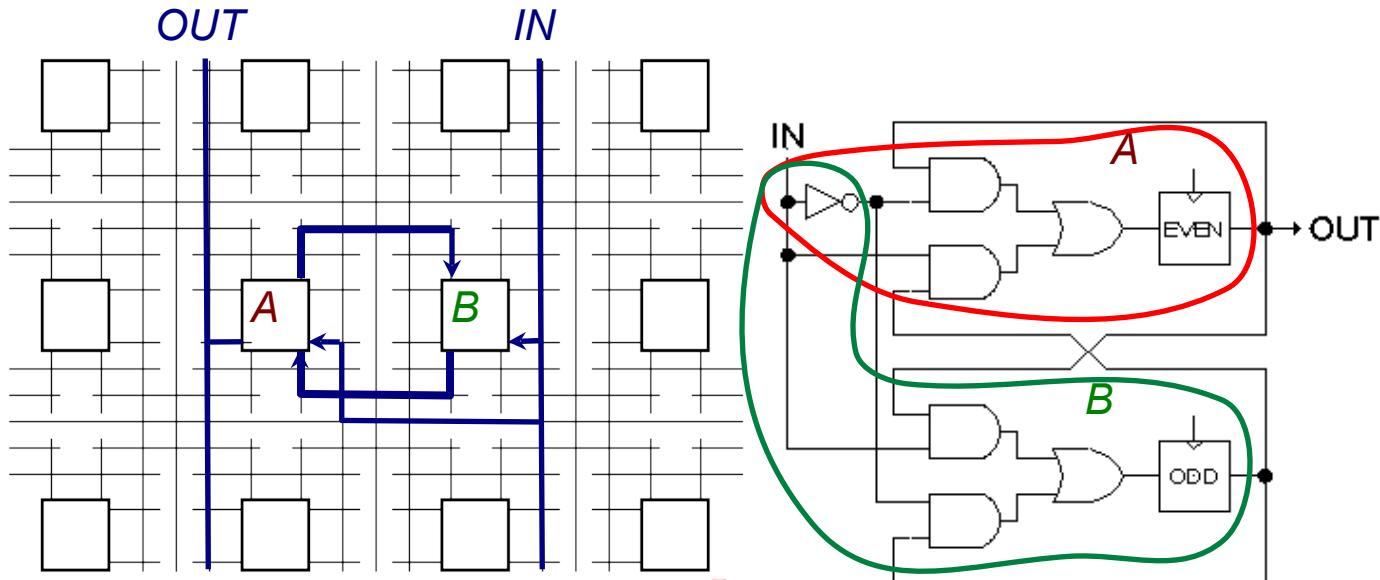
*MOSFET used  
as a “switch”*

# 4-LUT Implementation

- ❑ LUT size named by number of inputs
- ❑ n-bit LUT is implemented as a  $2^n \times 1$  memory:
  - inputs choose one of  $2^n$  memory locations.
  - memory locations (latches) are loaded with values from user's configuration bit stream.
  - Inputs to mux control are the LUT inputs.
- ❑ Result is a general purpose "logic gate".
  - n-LUT can implement any function of n inputs!



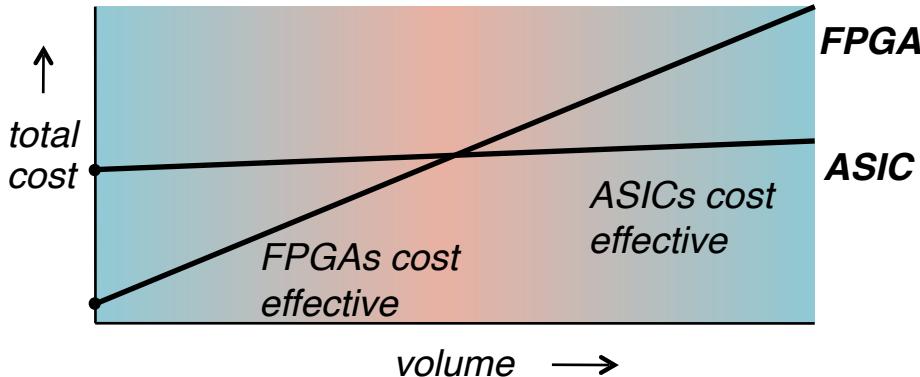
# Example Partition, Placement, and Route



Two partitions. Each has single output, no more than 4 inputs, and no more than 1 flip-flop. In this case, inverter goes in both partitions.

Note: (with 4-LUTs) the partition can be arbitrarily large as long as it has not more than 4 inputs and 1 output, and no more than 1 flip-flop.

# FPGA versus ASIC



- **ASIC:** Higher NRE costs (10's of \$M). Relatively Low cost per die (10's of \$ or less).
- **FPGAs:** Low NRE costs. Relatively low silicon efficiency  $\Rightarrow$  high cost per part (> 10's of \$ to 1000's of \$).
- **Cross-over volume** from cost effective FPGA design to ASIC was often in the 100K range.

# Some Laws of Boolean Algebra

Duality: A dual of a Boolean expression is derived by interchanging OR and AND operations, and 0s and 1s (literals are left unchanged).

$$\{F(x_1, x_2, \dots, x_n, 0, 1, +, \bullet)\}^D = \{F(x_1, x_2, \dots, x_n, 1, 0, \bullet, +)\}$$

Any law that is true for an expression is also true for its dual.

Operations with 0 and 1:

$$x + 0 = x$$

$$x * 1 = x$$

$$x + 1 = 1$$

$$x * 0 = 0$$

Idempotent Law:

$$x + x = x$$

$$x * x = x$$

Involution Law:

$$(x')' = x$$

Laws of Complementarity:

$$x + x' = 1$$

$$x * x' = 0$$

Commutative Law:

$$x + y = y + x$$

$$x * y = y * x$$

# *Algebraic Simplification*

$$\begin{aligned}\text{Cout} &= a'bc + ab'c + abc' + abc \\&= a'bc + ab'c + abc' + \textcolor{red}{abc} + abc \\&= a'bc + \textcolor{red}{abc} + ab'c + abc' + \textcolor{red}{abc} \\&= [a' + a]bc + ab'c + abc' + abc \\&= [1]bc + ab'c + abc' + abc \\&= bc + ab'c + abc' + \textcolor{red}{abc} + abc \\&= bc + ab'c + \textcolor{red}{abc} + abc' + abc \\&= bc + \textcolor{red}{a}[b' + b]c + abc' + abc \\&= bc + \textcolor{red}{a}[1]c + abc' + abc \\&= bc + ac + \textcolor{red}{ab}[c' + c] \\&= bc + ac + \textcolor{red}{ab}[1] \\&= bc + ac + ab\end{aligned}$$

# Canonical Forms

- ❑ Standard form for a Boolean expression - unique algebraic expression directly from a true table (TT) description.
- ❑ Two Types:
  - \* **Sum of Products (SOP)**
  - \* **Product of Sums (POS)**
- Sum of Products (disjunctive normal form, minterm expansion). Example:

Minterms	a	b	c	f	f'
$a'b'c'$	0	0	0	0	1
$a'b'c$	0	0	1	0	1
$a'bc'$	0	1	0	0	1
$a'bc$	0	1	1	1	0
$ab'c'$	1	0	0	1	0
$ab'c$	1	0	1	1	0
$abc'$	1	1	0	1	0
$abc$	1	1	1	1	0

One product (and) term for each 1 in f:

$$f = a'b'c + ab'c' + ab'c + abc' + abc$$
$$f' = a'b'c' + a'b'c + a'bc'$$

(enumerate all the ways the function could evaluate to 1)

What is the cost?

# Karnaugh Map Method

- Adjacent groups of 1's represent product terms

a	0	1	
b	0	0	(1)
1	0	0	(1)

$$f = a$$

a	0	1	
b	0	(1) 1	
1	0	0	0

$$g = b'$$

ab	00	01	11	10
c	0	0	0	(1) 0
1	0	(1) 1	(1) 1	1

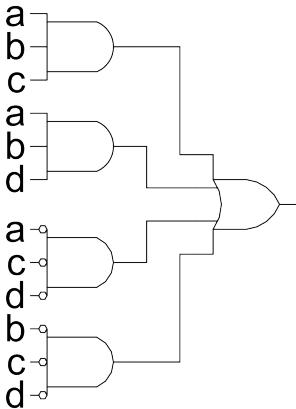
$$\text{cout} = ab + bc + ac$$

ab	00	01	11	10
c	0	0	0	(1) 1
1	0	0	(1) 1	1

$$f = a$$

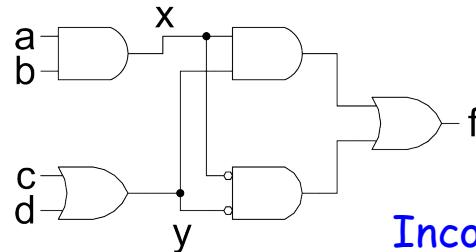
# Multi-level Combinational Logic

Another Example:  $f = abc + abd + a'c'd' + b'c'd'$



let  $x = ab$     $y = c+d$

$$f = xy + x'y'$$



Incorporates fanout.

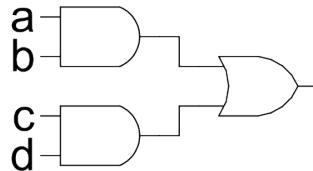
No convenient hand methods exist for multi-level logic simplification:

- a) CAD Tools use sophisticated algorithms and heuristics  
Guess what? These problems tend to be NP-complete
- b) Humans and tools often exploit some special structure (example adder)

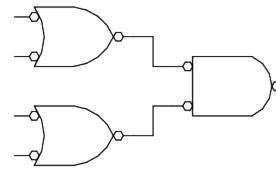
# **NAND-NAND & NOR-NOR Networks**

- Mapping from AND/OR to NAND/NAND

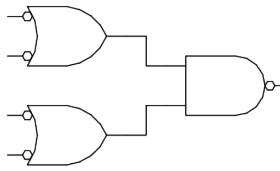
a)



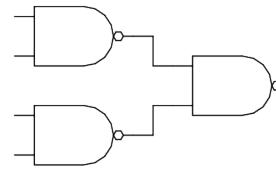
b)



c)



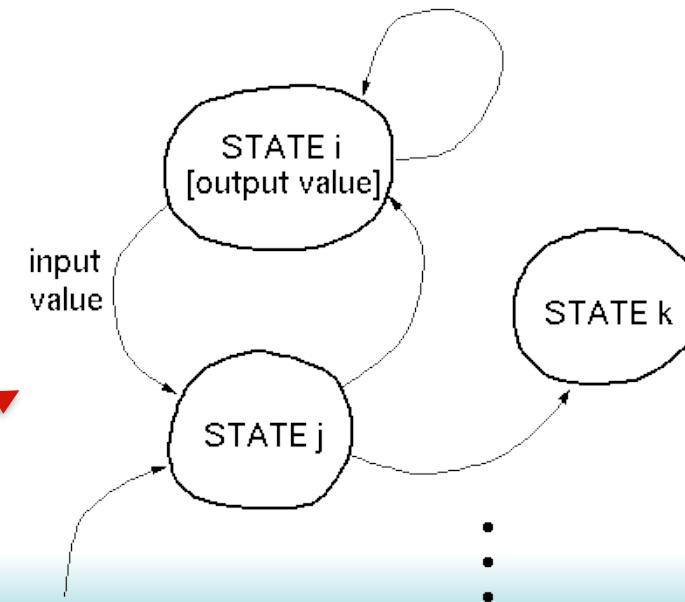
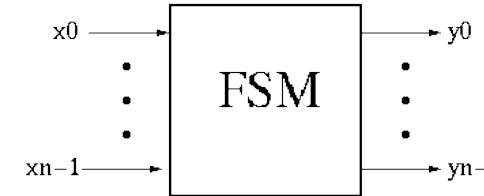
d)



# Finite State Machines (FSMs)

- ❑ **FSMs:**

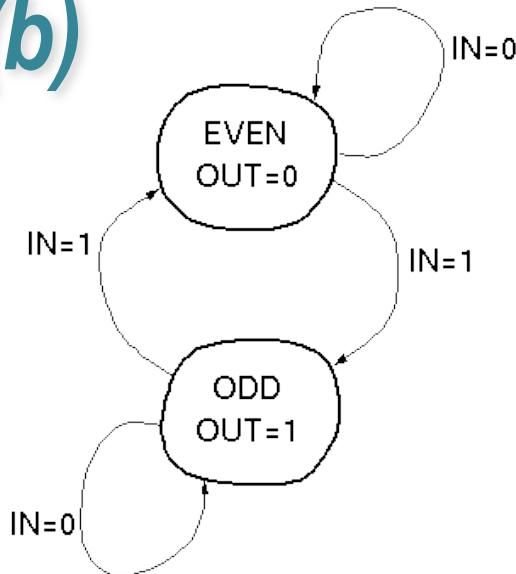
- ❑ Can model behavior of any sequential circuit
- ❑ Useful representation for designing sequential circuits
- ❑ As with all sequential circuits: output depends on present *and* past inputs
  - ❑ effect of past inputs represented by the current state
- ❑ Behavior is represented by **State Transition Diagram**:
  - traverse one edge per clock cycle.



# By-hand Design Process (b)

State Transition Table:

present state	OUT	IN	next state
EVEN	0	0	EVEN
EVEN	0	1	ODD
ODD	1	0	ODD
ODD	1	1	EVEN



Invent a code to represent states:

Let 0 = EVEN state, 1 = ODD state

present state (ps)	OUT	IN	next state (ns)
0	0	0	0
0	0	1	1
1	1	0	1
1	1	1	0

Derive logic equations  
from table (how?):

$$OUT = PS$$

$$NS = PS \text{ xor } IN$$

# FSM CL block rewritten

```
* for sensitivity list
always @*
begin
    next_state = IDLE; Normal values: used unless specified below.
    out = 1'b0;
    case (state)
        IDLE   : if (in == 1'b1) next_state = S0;
        S0     : if (in == 1'b1) next_state = S1;
        S1     : begin
                    out = 1'b1;
                    if (in == 1'b1) next_state = S1;
                end
        default: ;
    endcase
end
Endmodule
```

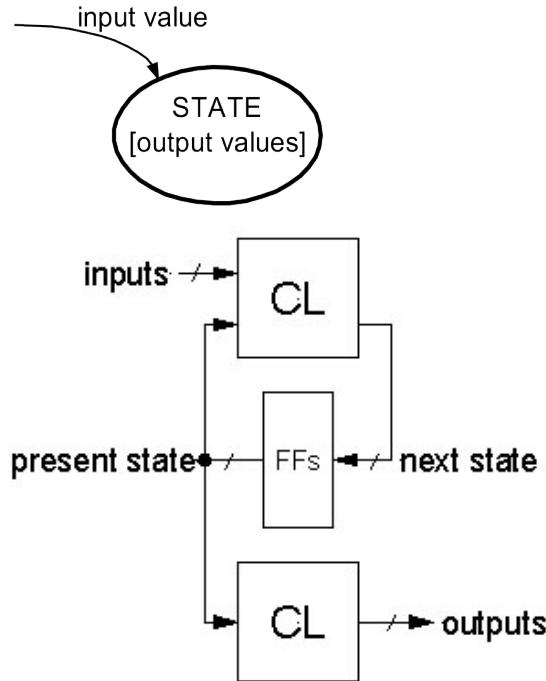
```
graph LR
    Start(( )) --> IDLE((IDLE  
out=0))
    IDLE -- "in=0" --> S0((S0  
out=0))
    IDLE -- "in=1" --> S1((S1  
out=1))
    S0 -- "in=0" --> IDLE
    S0 -- "in=1" --> S1
    S1 -- "in=0" --> IDLE
    S1 -- "in=1" --> S1
    S0 -- "in=1" --> S0
    S1 -- "in=1" --> S1
```

**Within case only need to specify exceptions to the normal values.**

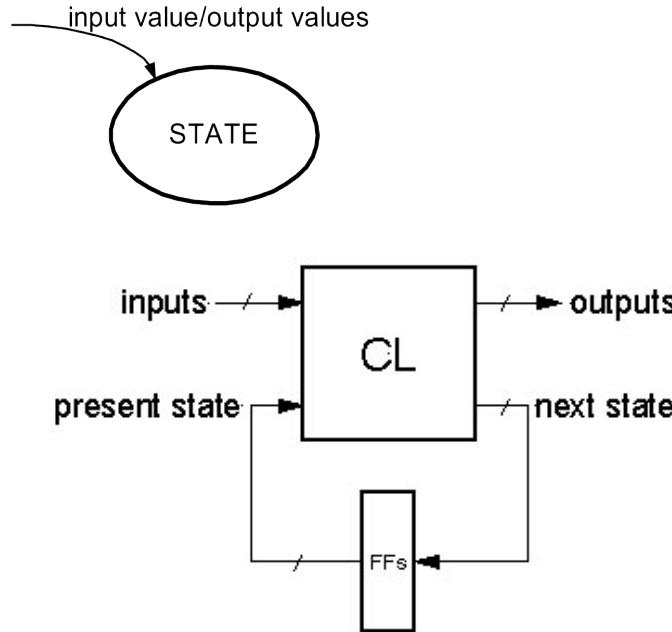
**Note: The use of “blocking assignments” allow signal values to be “rewritten”, simplifying the specification.**

# FSM Moore and Mealy Implementation Review

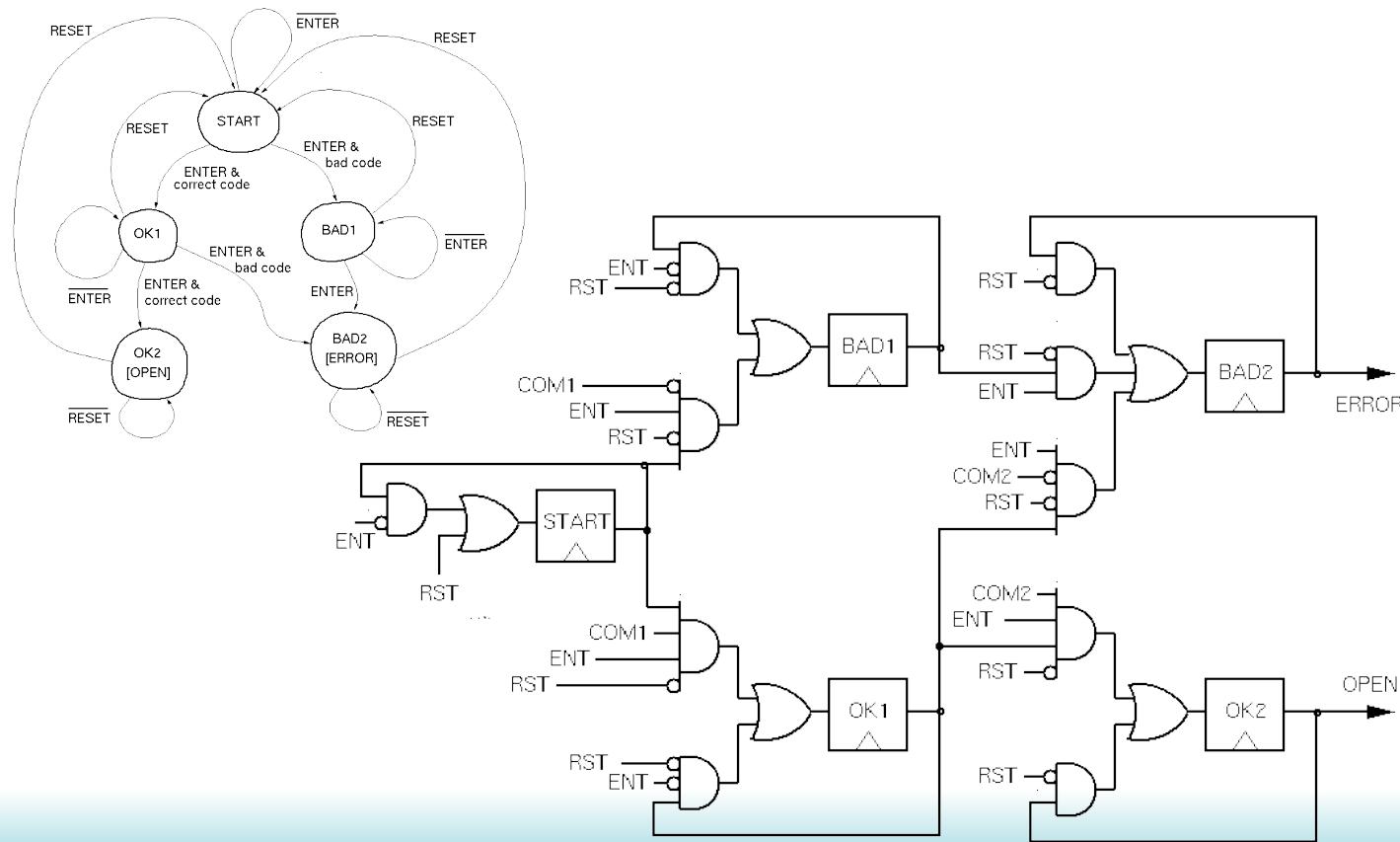
Moore Machine



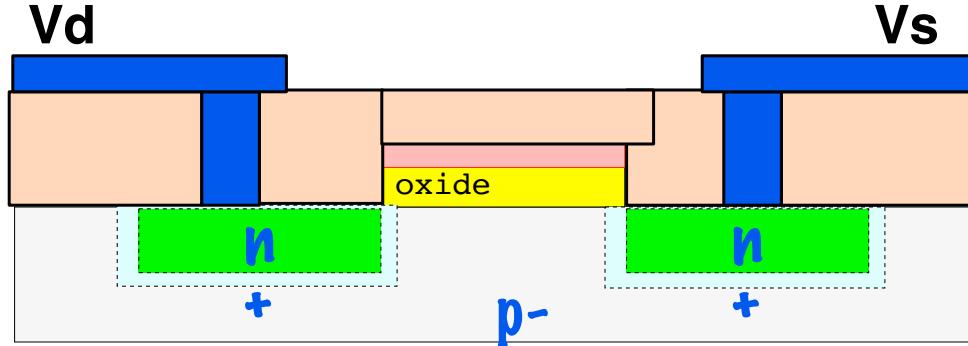
Mealy Machine



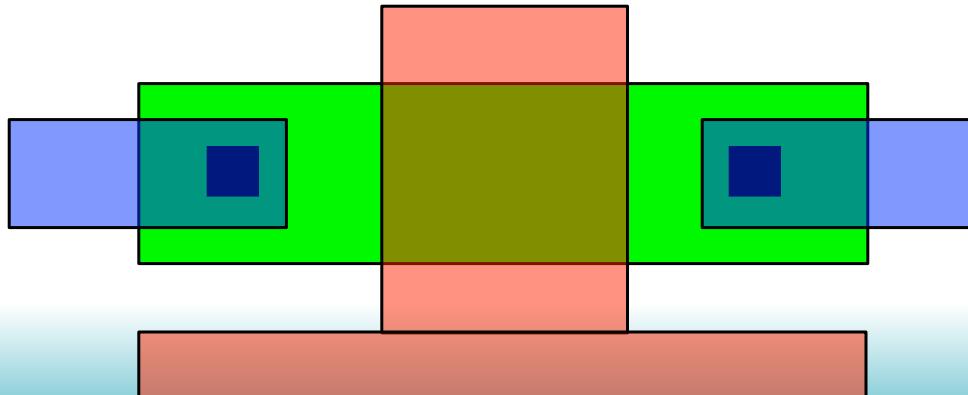
# One-hot encoded combination lock



# Final product ...



Top-down view:



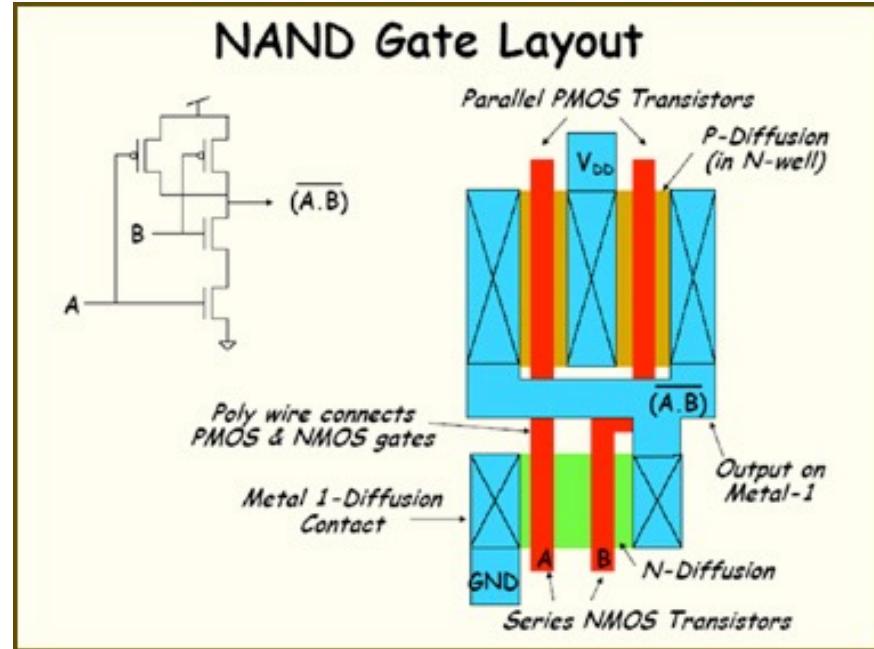
"The planar process"

Jean Hoerni,  
Fairchild  
Semiconductor  
1958



# Physical Layout

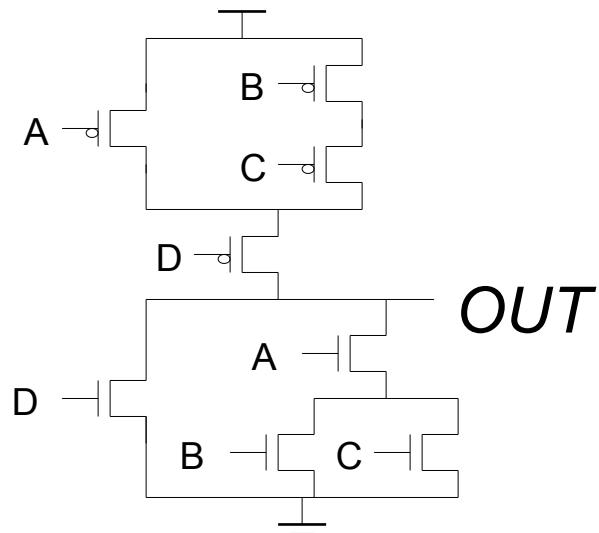
- How do transistor circuits get “laid out” as geometry?
- What circuit does a physical layout implement?
- Where are the transistors and wires and contacts and vias?



# Complex CMOS Gate

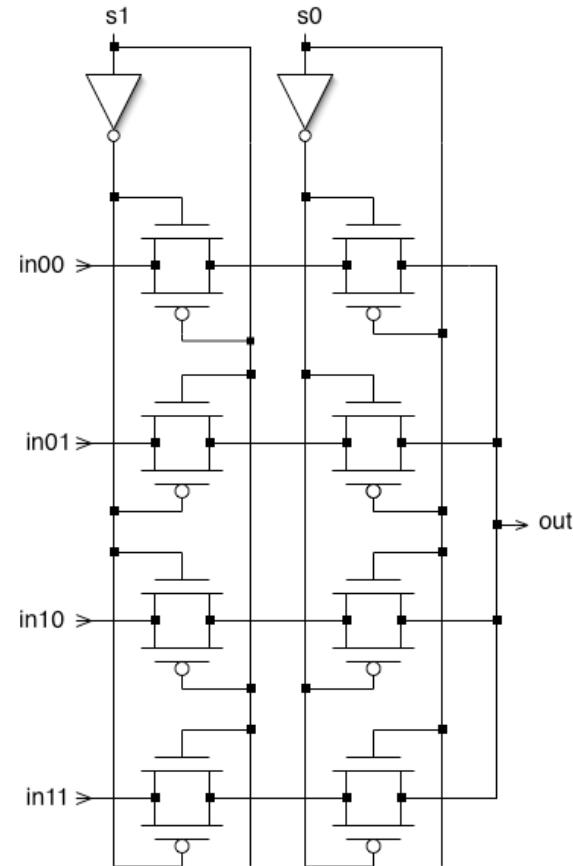
$$\overline{OUT} = D + A \cdot (B + C)$$

$$\overline{OUT} = \overline{D \cdot A + B \cdot C}$$

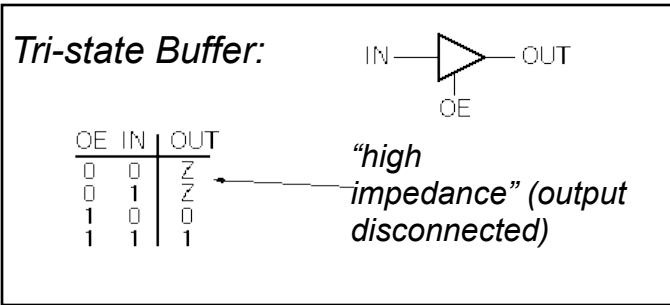


# 4-to-1 Transmission-gate Mux

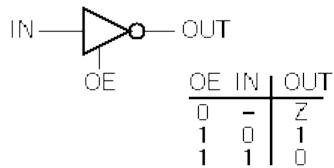
- The series connection of pass-transistors in each branch effectively forms the AND of s1 and s0 (or their complement).
- Compare cost to logic gate implementation



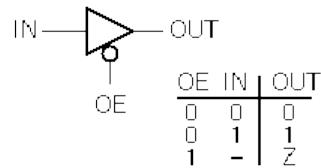
# Tri-state Buffers



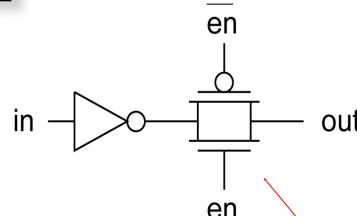
## Variations:



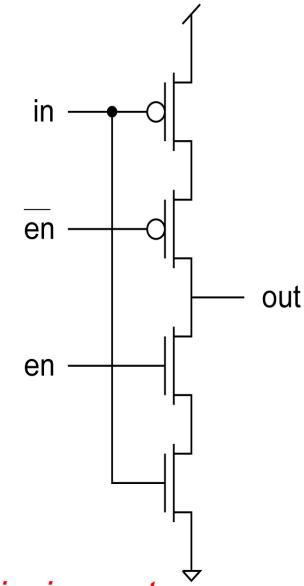
Inverting buffer



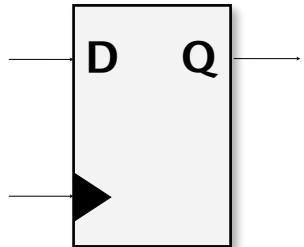
Inverted enable



transmission gate  
useful in  
implementation



# Positive edge-triggered flip-flop



*A flip-flop “samples” on the positive clock-edge, and then “holds” value.*

*Sampling  
latch*

*Holding  
latch*

