

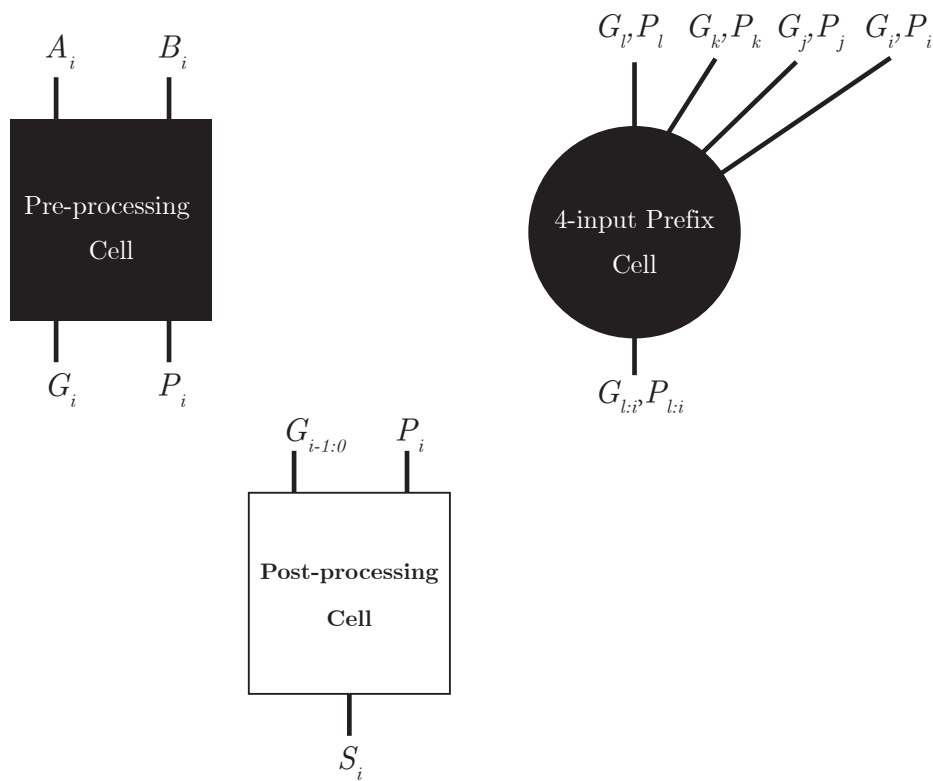
EECS 151/251A Homework 8

Due Friday, November 18th, 2022 11:59PM

Problem 1: Carry Lookahead

Recall from the lecture that Kogge–Stone adder is a parallel prefix form carry look-ahead adder (CLA). In this problem, we'd like to design and analyze a 16-bit radix-4 Kogge–Stone adder.

- (a) Write down the Boolean functions of the gate implementation of following building blocks for the radix-4 Kogge–Stone adder. You may use *AND*, *OR* and *XOR* only. Also, how should you handle cases where certain 4-input prefix cells have fewer than 4 pairs of input?



(b) Which of the following are true for radix-4 Kogge-Stone adder?

____ Compared to Radix-2 adders, Radix-4 adders reduce the depth of the tree by a factor of 4 (excluding the input and output stages).

____ An N-bit Radix-4 adders has $\mathcal{O}(\sim \log_4(N))$ in time.

____ It is possible to implement a 32-bit Kogge-Stone adder using only the building blocks in part (a).

(c) Suppose given the following propagation delays:

$$t_{AND} = 5ps, \quad t_{OR} = 4ps, \quad t_{XOR} = 7ps$$

Derive the critical path of the 16-bit Kogge-Stone adder based on your implementation in part (a), ignoring the delays in routing. (Hint: If you are not sure about the topology, take a look at the slides in Lecture 19.)

(d) Based on your result above, what's the maximum clock frequency for this 16-bit adder?

- (e) **(251A Only)** In reality, those prefix cells will not be built using the basic 2-input AND, OR, XOR gates. Instead, they will be built as a big CMOS gate (and inverters). Draw the schematic of the 4-input prefix cell for $\bar{G}_{l:i}$ and $\bar{P}_{l:i}$ respectively with the minimum number of transistors.

Problem 2: Booth's Bizarre Invention

Refer to the following table of the behavior of Booth recoding.

| B_{K+1} | B_K | B_{K-1} | Action |
|-----------|-------|-----------|--------|
| 0 | 0 | 0 | add 0 |
| 0 | 0 | 1 | add A |
| 0 | 1 | 0 | add A |
| 0 | 1 | 1 | add 2A |
| 1 | 0 | 0 | sub 2A |
| 1 | 0 | 1 | sub A |
| 1 | 1 | 0 | sub A |
| 1 | 1 | 1 | add 0 |

Write down the sequence of operation and the final result given the following unsigned two input numbers:

$$\begin{array}{r} 01100111 \text{ (A)} \\ \times 10110010 \text{ (B)} \end{array}$$

Answer should be in the format of:

$$\begin{array}{ll} \text{Suppose } A=1010 & B=1001 \\ (B[1:-1]=010): & \text{add A,} \\ (B[3: 1]=100): & \text{sub 2A,} \\ & \dots \\ \text{result} & = A - (2A \ll 2) + \dots \\ & = 0000(1010) - 00(1010)00 + \dots \\ & = \dots \end{array}$$