# EECS 151/251A Homework 8

Due Friday, November 18th, 2022 11:59PM

Problem 1: Carry Lookaheadache

Recall from the lecture that Kogge–Stone adder is a parallel prefix form carry look-ahead adder (CLA). In this problem, we'd like to design and analyze a 16-bit radix-4 Kogge-Stone adder.

(a) Write down the Boolean functions of the gate implementation of following building blocks for the radix-4 Kogge-Stone adder. You may use AND, OR and XOR only. Also, how should you handle cases where certain 4-input prefix cells have fewer than 4 pairs of input?
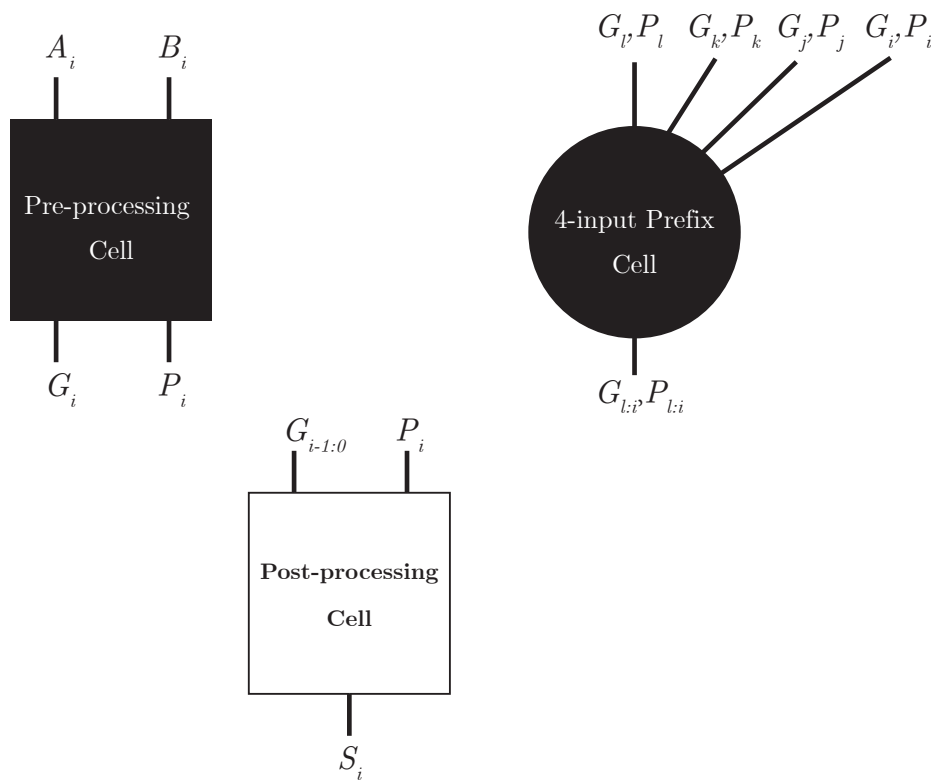
(b) Which of the following are true for radix-4 Kogge-Stone adder?

        _____ Compared to Radix-2 adders, Radix-4 adders reduce the depth of the tree by a factor of 4 (excluding the input and output stages).

        _____ An N-bit Radix-4 adders has $\mathcal{O}(\sim log_4(N))$ in time.

        _____ It is possible to implement a 32-bit Kogge-Stone adder using only the building blocks in part (a).

(c) Suppose given the following propagation delays:

$$t_{AND} = 5ps, \quad t_{OR} = 4ps, \quad t_{XOR} = 7ps$$

Derive the critical path of the 16-bit Kogge-Stone adder based on your implementation in part (a), ignoring the delays in routing. (Hint: If you are not sure about the topology, take a look at the slides in Lecture 19.)

(d) Based on your result above, what's the maximum clock frequency for this 16-bit adder?

(e) **(251A Only)** In reality, those prefix cells will not be built using the basic 2-input AND, OR, XOR gates. Instead, they will be built as a big CMOS gate (and inverters). Draw the schematic of the 4-input prefix cell for $\bar{G}_{l:i}$ and $\bar{P}_{l:i}$ respectively with the minimum number of transistors.

Problem 2: Booth's Bizarre Invention

Given:

$$01100111 \ (A)$$
$$\times \ 10110010 \ (B)$$

Booth recoding of $B = 10110010$ (with $B[-1]=0$):

$B_7 B_6 B_5 B_4 B_3 B_2 B_1 B_0 = 1\,0\,1\,1\,0\,0\,1\,0$

B_{K+1}	B_K	B_{K-1}	Action
1	0	0	sub 2A
0	0	1	add A
1	1	0	sub A
1	0	1	sub A

Sequence of operations:

- (B[1:-1]=100): sub 2A,
- (B[3: 1]=001): add A,
- (B[5: 3]=110): sub A,
- (B[7: 5]=101): sub A

result:

$$= -(A{<}{<}1) + (A{<}{<}2) - (A{<}{<}4) - (A{<}{<}6)$$
$$= A \cdot (-2 + 4 - 16 - 64) = A \cdot (-78)$$
$$= 103 \times (-78) = -8034$$

In 16-bit two's complement: $-8034 = 1110000010011110$