

EECS 151/251A SP2022 Discussion 7

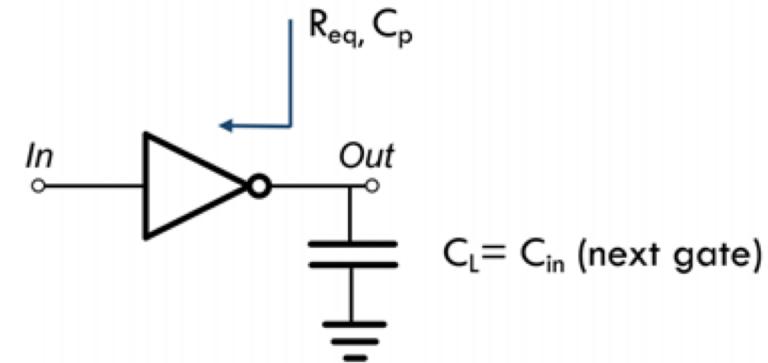
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Agenda

- Inverter Delay – continue from last week's
- Logical Effort
- Elmore Delay

Loaded Inverter Delay

$$\begin{aligned}t_{p,inv} &= \ln 2 \cdot R_{eq} (C_{p,tot} + C_L) \\&= \ln 2 \cdot R_{eq} C_{p,tot} \left(1 + \frac{C_L}{C_{p,tot}}\right) \\&= \ln 2 \cdot R_{eq} C_{p,tot} \left(1 + \frac{C_L}{\gamma C_{in}}\right) \\&= \ln 2 \cdot R_{eq} C_{p,tot} \left(1 + \frac{f}{\gamma}\right) \\&= \ln 2 \cdot \tau_{inv} \left(1 + \frac{f}{\gamma}\right)\end{aligned}$$



- Intrinsic vs. Extrinsic delay
 - Intrinsic τ_{inv} independent of sizing
- Fanout $f = C_L/C_{in}$
- Generalizable to any CMOS gate

Inverter Chain Sizing

- Goal: minimize path delay
 - Assume 1st inverter has unit size ($C_{in,1} = 1$)

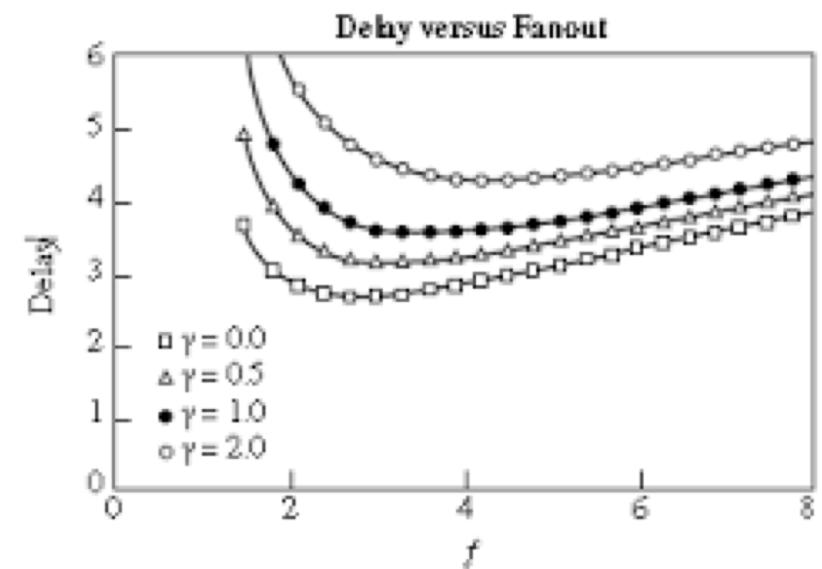
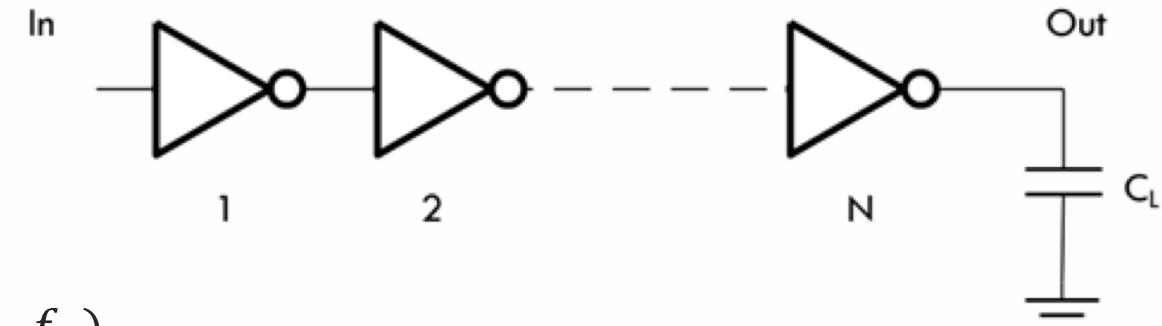
- Path delay:

$$D = t_{p1} + \dots + t_{pN} = (1 + f_1) + \dots + (1 + f_N)$$

- Path fanout $F = \frac{C_L}{C_{in}}$

- Solution

- Take partial derivatives w.r.t C_2, \dots, C_N
- Get $f_1 = f_2 = \dots = f_N$
- Min. path delay = $Nf + N = N \sqrt[N]{F} + N$
- Size backwards

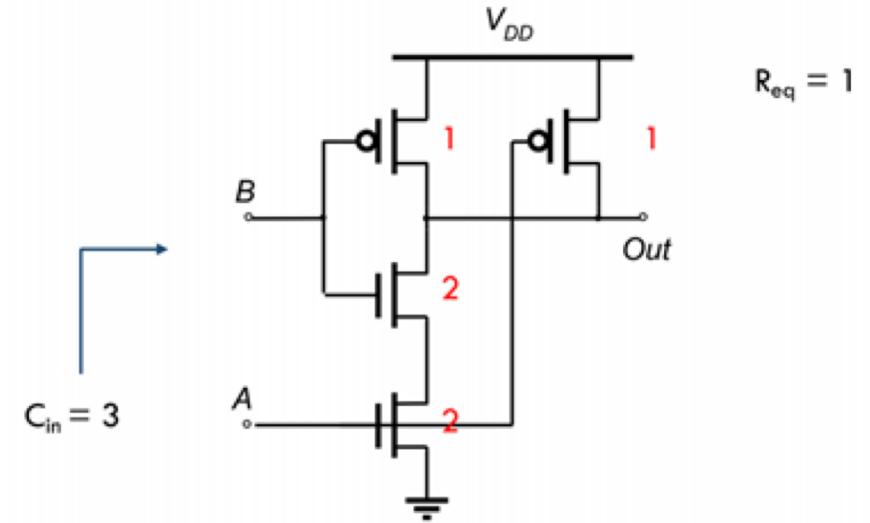


Logical Effort



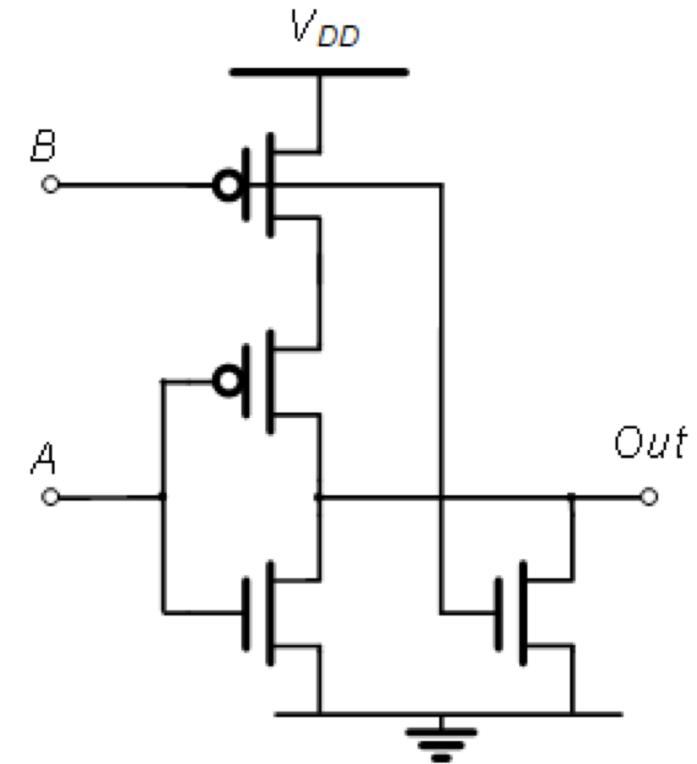
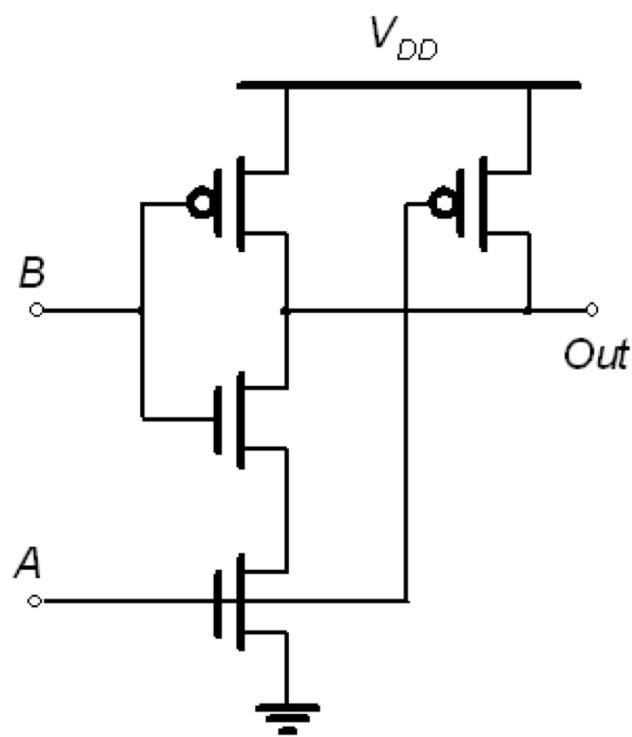
Logical Effort, Parasitic Delay

- For any gate, $D(\text{gate}) = \text{LE} \cdot \text{FO} + P$
- Logical Effort
 - LE , Also defined as g in $t_{p,gate} = \tau_{inv}(p + \frac{gf}{\gamma})$
 - $\text{LE} \equiv \frac{R_{eq,gate}C_{in,gate}}{R_{eq,eqiv}C_{in,inv}}$, i.e. ratio of $C_{in,gate}$ to $C_{in,inv}$ of a unit inverter with the same output current
 - Key to calculating LE: size the transistors to have the same R_{eq} as a unit inverter
- Parasitic Delay
 - Also defined as p in $t_{p,gate} = \tau_{inv}(p + \frac{gf}{\gamma})$
 - Only difference from LE is that p looks at the ratio of $C_{out,gate}$ to $C_{out,inv}$ (intrinsic capacitance)



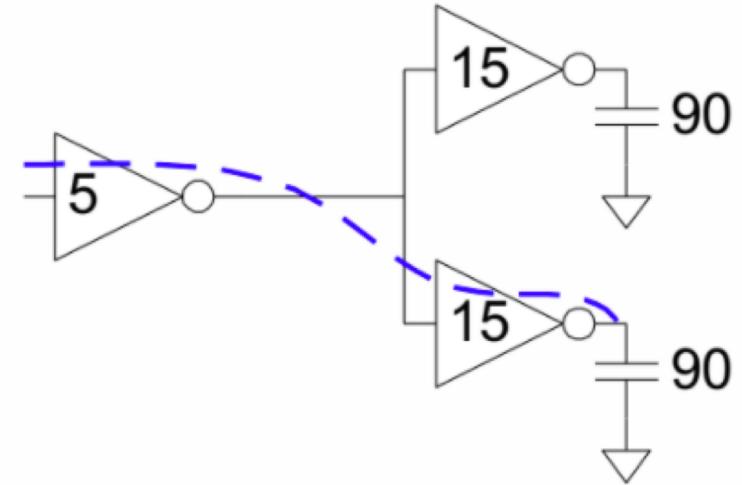
PMOS/NMOS Resistance

- In lecture, $LE_{NAND2} = LE_{NOR2} = \frac{3}{2}$. What if $R_{on,p} = K \cdot R_{on,n}$?



Minimizing Path Delay

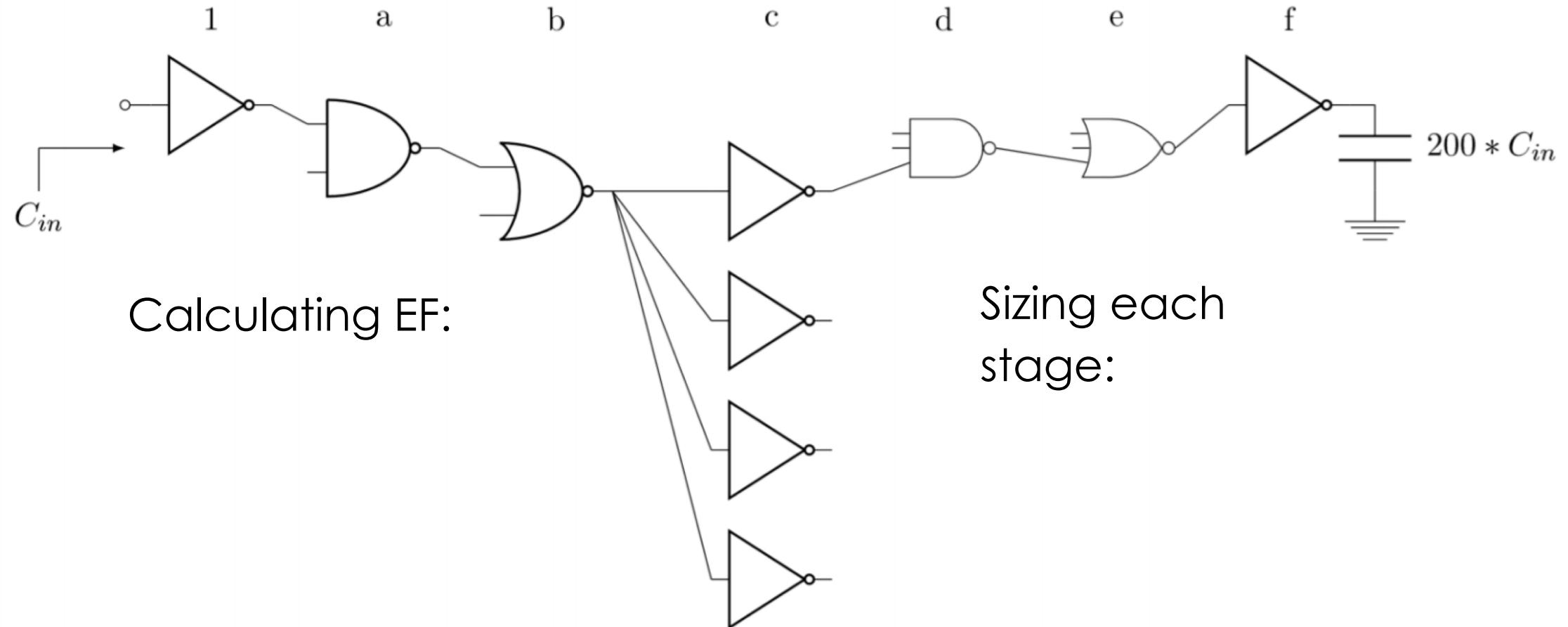
- Path Logical Effort: $G = g_1 \cdot g_2 \cdot \dots \cdot g_N$
- Path fanout $F = C_L / C_{in}$
- Branching, b
 - Extra loading factor $b = \frac{C_{onpath} + C_{offpath}}{C_{onpath}}$
 - Add b to total path effort $H = GFB$
- Solution is similar to single-branch inverter chain, except:
 - Calculate an “**effective fanout**” for each stage: $EF = \sqrt[N]{H}$
 - Size each stage such that $EF = g_i \cdot f_i$. You can control f_i by sizing!
 - Min. path delay = $N * \sqrt[N]{H} + \sum p_i$



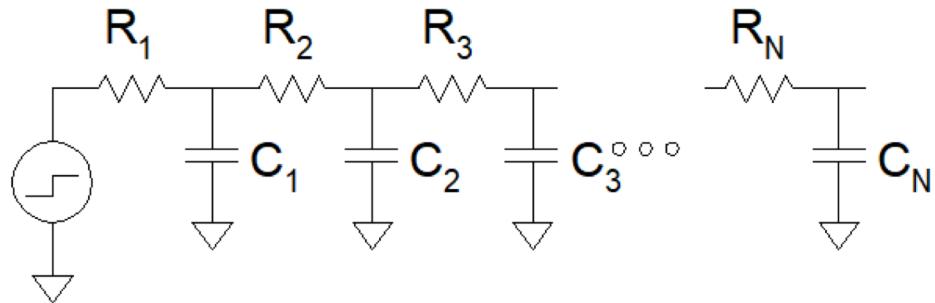
Logic Path with Branch - Example

$$LE_{NAND2} = LE_{NOR2} = \frac{3}{2}$$

$$LE_{NAND3} = LE_{NOR3} = 2$$



Elmore Delay – “All the R’s for that C” or “All the C’s by that R”



$$t_{pd} \approx \sum_{\text{nodes } i} R_{i-\text{to-source}} C_i$$
$$= R_1 C_1 + (R_1 + R_2) C_2 + \dots + (R_1 + R_2 + \dots + R_N) C_N$$

- Approximately the dominant RC time constant of a network

- R's can be:

- Gate equivalent resistance R_{eq}
- Wire resistance $R = R_{\square} \cdot \# \text{ squares}$

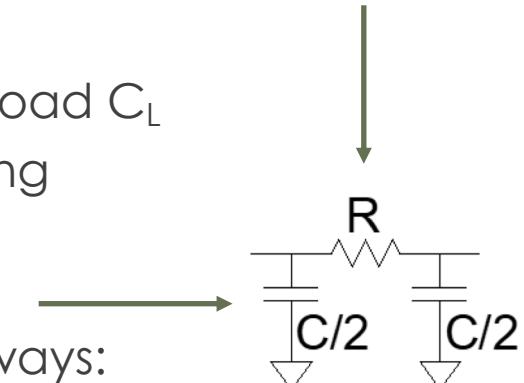
- C's can be:

- Gate C_{in} and C_p , load C_L
- Wire plate & fringing capacitance

- $\pi - model$

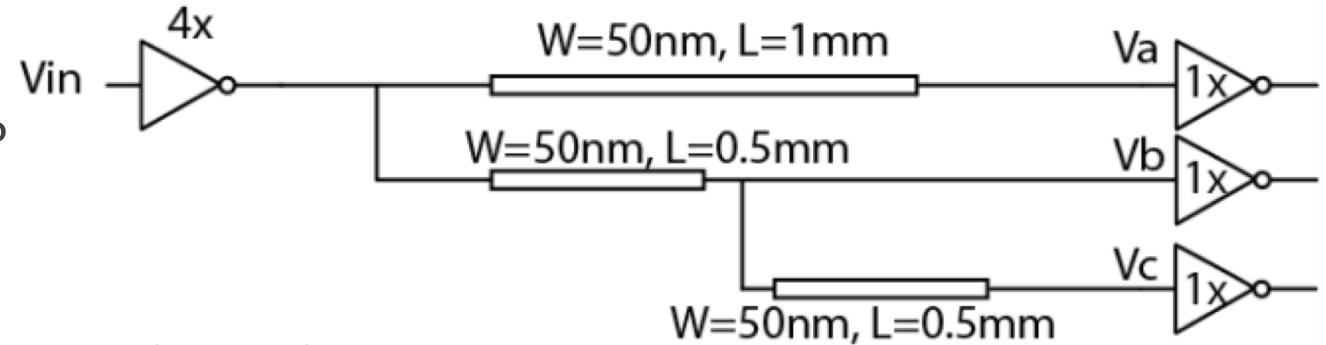
- Think of it 2 equivalent ways:

- $\sum [C_i \cdot (\text{sum of R's charging } C_i)]$
- $\sum [R_i \cdot (\text{sum of C's that } R_i \text{ charges})]$



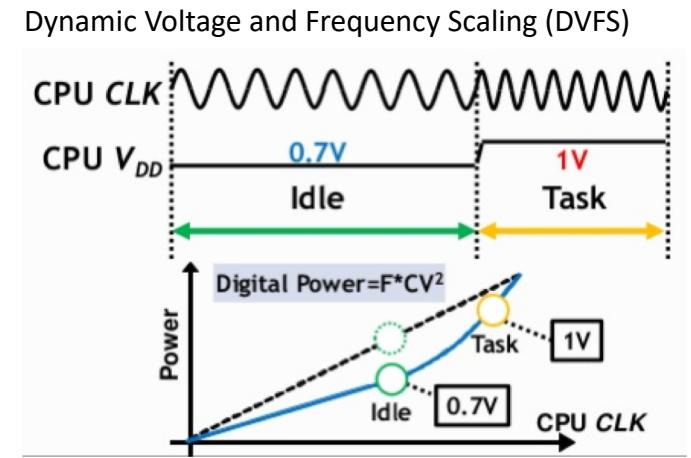
Elmore Example

- What parameters do we need?
- Draw the equivalent circuit Elmore RC model
- Calculate delay to V_a



Power/Energy in Digital Circuits

- Fundamentally, charging/discharging capacitors (gate, parasitic, load) through resistances (PMOS, NMOS, wires)
 - Capacitors draw CV^2 joules from supply over 1 charge/discharge cycle
 - $\frac{1}{2}CV^2$ dissipated in PMOS as heat when charging
 - $\frac{1}{2}CV^2$ stored on capacitor, then dissipated in NMOS when discharging
- Dynamic power = $P_{switching} = \alpha CV^2 f$
 - How to minimize each term?
 - Minimizing which terms reduces total energy consumed?
- Static power = leakage → wasted energy!



Energy Example

- Initially: A = 1, C = 1, Out = 0
- Energy pulled from supply when B = 1 \rightarrow 0?
- Then, how much energy dissipated when C = 1 \rightarrow 0?

