Machine Learning Homework 01 Due on 10/16/2023

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- Suppose X be a zero-mean Gaussian random variable with variance σ_X^2 . Let $Y=X^2$
 - 1. what is the pdf of Y?
 - 2. Please calculate $E\{Y\}$ and $E\{Y^2\}$

- Suppose X_1 and X_2 be uniform random variables over (0,1) and X_1 and X_2 are independent
- Let $Y = 2X_1 + 3X_2$
 - 1. what is the pdf of Y?
 - 2. Please calculate $E\{Y\}$ and $E\{Y^2\}$

- Suppose $y = [y_1, y_2, y_3]^T$ and $x = [x_1, x_2]^T$ in which $y_1 = x_1^2 2x_2^2$, $y_2 = -3x_1^2 + 2x_1x_2 + x_2^2$ and $y_3 = x_1^2 + x_2^2$
 - 1. Please find $\frac{\partial y}{\partial x}$

• Let
$$A = \begin{bmatrix} -1 & 2 & 3 \\ 2 & 0 & 1 \\ 3 & -1 & 5 \end{bmatrix}$$
 and $\mathbf{x} = [x_1, x_2, x_3]^T$

- 1. Let y = Ax. Please find $\frac{\partial y}{\partial x}$
- 2. Let $\mathbf{y} = \mathbf{x}^T \mathbf{A} \mathbf{x}$. Please find $\frac{\partial \mathbf{y}}{\partial \mathbf{x}}$

• Suppose we have a dataset

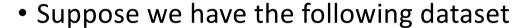
| x | (1,2) | (3,2) | (0,3) | (4,1) | (2,0) | (2,3) |
|-------|---------|---------|---------|---------|---------|---------|
| label | Class 1 | Class 2 | Class 1 | Class 2 | Class 1 | Class 2 |

- Now we have a new data $x_{new} = (2,2)$
- Use KNN with k=3 to classify this new data
 - 1. Minkowski distance with p = 1
 - 2. Minkowski distance with p = 2
 - 3. Minkowski distance with $p = \infty$

| ID | Study Hours/Day | Eat Lunch? | Money on pocket | Age | Have Afternoon Tea? |
|----|-----------------|------------|-----------------|-----|------------------------|
| 1 | 3 | No | 50 | 18 | No |
| 2 | 8 | Yes | 100 | 19 | Yes |
| 3 | 4 | No | 130 | 20 | Yes |
| 4 | 10 | No | 120 | 22 | No |
| 5 | 9 | Yes | 40 | 18 | Yes |

- Construct a decision tree with depth 3 to predict whether a person will have afternoon tea
 - 1. Use information gain
 - 2. Use gini index
 - 3. Use information gain ratio

Problem 07 (Linear regression)



| x | 1 | 3 | 5 | 7 | 9 |
|---|---|---|---|---|---|
| у | 2 | 5 | 4 | 4 | 8 |

• Determine the best predicted model when $\hat{y}(x, w_0, w_1) = w_0 + w_1 x$

Problem 08 (Linear regression)

Suppose we have the following dataset

| x | 1 | 3 | 5 | 7 | 9 |
|---|---|---|---|---|---|
| у | 2 | 5 | 4 | 4 | 8 |

Determine the best predicted model when

$$\hat{y}(x, w_0, w_1) = w_0 + w_1 x + w_2 x^2$$

Problem 09 (Linear regression)

Suppose we have the following dataset

| $x = (x_1, x_2)$ | (1,1) | (1,3) | (5,2) | (7,4) | (9,2) |
|------------------|-------|-------|-------|-------|-------|
| у | 2 | 5 | 4 | 4 | 8 |

Determine the best predicted model when

$$\hat{y}(x, w_0, w_1) = w_0 + w_1 x_1 + w_2 x_2$$

Problem 10 (Linear regression)

- This is a programming problem
- Suppose $y=3+2x+0.2x^2+n$ in which n is a zero-mean Gaussian noise with variance σ_N^2
- Suppose the predicted model is $\hat{y}(x, w_0, w_1) = w_0 + w_1 x + w_2 x^2$
- Write a program to determine the optimal weights when

1.
$$\sigma_N = 0.01$$

2.
$$\sigma_N = 0.1$$

3.
$$\sigma_N = 1$$

4.
$$\sigma_N = 2$$

(Do not use the built-in regression toolbox in the programming language you chose)

Problem 10 (Linear regression)

- Procedure to generate your own dataset
 - step 1: uniformly generate a random variable between 0 and 8 and let this be x
 - step 2: generate a zero-mean Gaussian noise with variance σ_N^2 and let this be n
 - step 3: let $y = 3 + 2x + 0.2x^2 + n$
 - step 4: add (x, y) to your own dataset
 - use this procedure to have a dataset containing 40 pairs of x and y

Note that

Please plot the regression function along with the data

points on the same graph