Machine Learning
Homework 02
Due on 12/18/2023

Min-Kuan Chang

minkuanc@nchu.edu.tw

EE, College of EECS

Problem 01 (Regression)

- Suppose we have the following dataset

$x = (x_1, x_2)$	(1,1)	(1,3)	(5,2)	(7,4)	(9,2)
y	2	5	3	4	8

  - use Ridge to determine the best predicted model when

$$\hat{y}(x, w_0, w_1) = w_0 + w_1 x_1 + w_2 x_2$$

  - use Lasso to determine the best predicted model when

$$\hat{y}(x, w_0, w_1) = w_0 + w_1 x_1 + w_2 x_2$$

Write your own program and don't use the toolboxes of Ridge and Lasso

Problem 02 (Ridge)

- This is a programming problem
- Suppose $y = 3 + 2x + 0.2x^2 + n$ in which n is a zero-mean Gaussian noise with variance σ_N^2
- Suppose the predicted model is $\hat{y}(x, w_0, w_1) = w_0 + w_1 x + w_2 x^2$
- Determine the optimal weights when
  1. $\sigma_N = 0.01$
  2. $\sigma_N = 0.1$
  3. $\sigma_N = 1$
  4. $\sigma_N = 2$

Write your own program and don't use the toolboxes of Ridge

Problem 02 (Ridge)

- Procedure to generate your own dataset
  - step 1: uniformly generate a random variable between 0 and 8 and let this be x
  - step 2: generate a zero-mean Gaussian noise with variance σ_N^2 and let this be n
  - step 3: let $y = 3 + 2x + 0.2x^2 + n$
  - step 4: add (x, y) to your own dataset
  - use this procedure to have a dataset containing 30 pairs of x and y

Note that
Please plot the regression function along with the data points on the same graph

Problem 03 (Lasso)

- This is a programming problem
- Suppose $y = 3 + 2x + 0.2x^2 + n$ in which n is a zero-mean Gaussian noise with variance σ_N^2
- Suppose the predicted model is $\hat{y}(x, w_0, w_1) = w_0 + w_1 x + w_2 x^2$
- Determine the optimal weights when
  1. $\sigma_N = 0.01$
  2. $\sigma_N = 0.1$
  3. $\sigma_N = 1$
  4. $\sigma_N = 2$

Write the your own program and don't use the toolboxes of Lasso

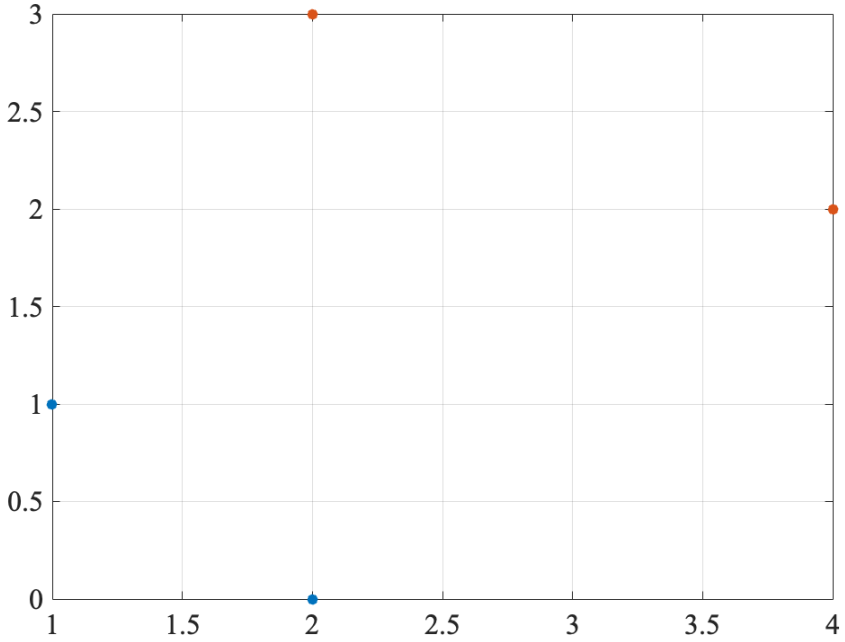
Problem 03 (Lasso)

- Procedure to generate your own dataset
  - step 1: uniformly generate a random variable between 0 and 8 and let this be x
  - step 2: generate a zero-mean Gaussian noise with variance σ_N^2 and let this be n
  - step 3: let $y = 3 + 2x + 0.2x^2 + n$
  - step 4: add (x, y) to your own dataset
  - use this procedure to have a dataset containing 30 pairs of x and y

Note that
Please plot the regression function along with the data points on the same graph

Problem 04

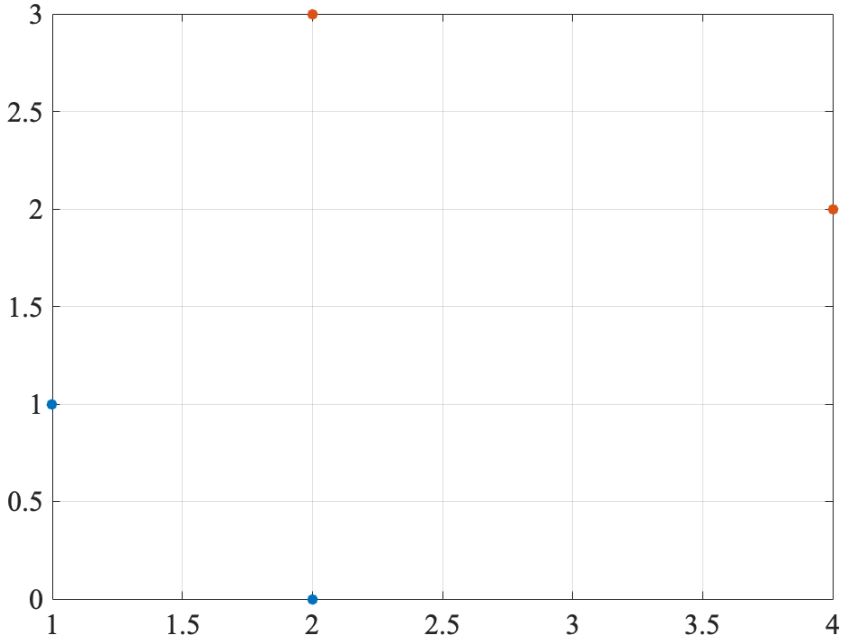
- Suppose we have data in the left graph
- Use support vector machine to determine support vectors and the decision boundary



Write the your own program and don't use the toolboxes of Ridge and Lasso

Problem 05

- Suppose we have data in the left graph
- Use logistic regression to determine the decision boundary



Write the your own program and don't use the toolboxes of Ridge and Lasso

Problem 06

x_1	x_2	x_3	y
1	0	0	0
0	1	0	1
1	0	1	1
0	0	1	1
1	1	0	0
0	0	1	0

Question: Suppose now we have a new input $(x_1, x_2, x_3) = (1,1,1)$, which class will be assigned to this input if naive Bayes classifier is adopted?

Problem 07

- Suppose we have a data of three classes. Each class consists of 50 data points
- Generation the data of each class
  - Class 1:
    - the data follows two-dimensional Gaussian distribution with mean $[1,1]^T$ and covariance matrix $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$
  - Class 2:
    - the data follows two-dimensional Gaussian distribution with mean $[-3,-3]^T$ and covariance matrix $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$
  - Class 3:
    - the data follows two-dimensional Gaussian distribution with mean $[-6,2]^T$ and covariance matrix $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

Problem 07

- Use Multi-class logistic regression to determine the decision boundary

Note that
1. Write your own program and don't use the toolboxes of Logistic regression
1. Please plot the decision boundary along with the data points on the same graph

Problem 08

- Suppose two-dimensional input data $\boldsymbol{x}_n$ together with $y_n \in \{0,1\}$ for $n = 1,2,\cdots,N$ (no transformation to feature space)
- Show how to derive the logistic regression in two-class classification