Machine Learning

Homework 01

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Problem 01

- Suppose X be a zero-mean Gaussian random variable with variance σ_X^2. Let $Y = X^2$
  1. what is the pdf of Y?
  2. Please calculate $E\{Y\}$ and $E\{Y^2\}$

Problem 02

- Suppose X_1 and X_2 be uniform random variables over $(0,1)$ and X_1 and X_2 are independent
- Let $Y = 2X_1 + 3X_2$
  1. what is the pdf of Y?
  2. Please calculate $E\{Y\}$ and $E\{Y^2\}$

Problem 03

- Suppose $\boldsymbol{y} = [y_1, y_2, y_3]^T$ and $\boldsymbol{x} = [x_1, x_2]^T$ in which $y_1 = x_1^2 - 2x_2^2, y_2 = -3x_1^2 + 2x_1x_2 + x_2^2$ and $y_3 = x_1^2 + x_2^2$
  1. Please find $\frac{\partial \boldsymbol{y}}{\partial \boldsymbol{x}}$

Problem 04

- Let $\mathrm{A} = \begin{bmatrix} -1 & 2 & 3 \\ 2 & 0 & 1 \\ 3 & -1 & 5 \end{bmatrix}$ and $\boldsymbol{x} = [x_1, x_2, x_3]^T$
    1. Let $\boldsymbol{y} = \mathrm{A}\boldsymbol{x}$. Please find $\frac{\partial \boldsymbol{y}}{\partial \boldsymbol{x}}$
    2. Let $\boldsymbol{y} = \boldsymbol{x}^T \mathrm{A}\boldsymbol{x}$. Please find $\frac{\partial \boldsymbol{y}}{\partial \boldsymbol{x}}$

Problem 05

- Suppose we have a dataset

x	(1,2)	(3,2)	(0,3)	(4,1)	(2,0)	(2,3)
label	Class 1	Class 2	Class 1	Class 2	Class 1	Class 2

- Now we have a new data x_{new} =(2,2)
- Use KNN with $k = 3$ to classify this new data
  1. Minkowski distance with $p = 1$
  2. Minkowski distance with $p = 2$
  3. Minkowski distance with $p = \infty$

Problem 06

ID	Study Hours/Day	Eat Lunch?	Money on pocket	Age	Have Afternoon Tea?
1	3	No	50	18	No
2	8	Yes	100	19	Yes
3	4	No	130	20	Yes
4	10	No	120	22	No
5	9	Yes	40	18	Yes

- Construct a decision tree with depth 3 to predict whether a person will have afternoon tea
  1. Use information gain
  2. Use gini index
  3. Use information gain ratio

Problem 07 (Linear regression)

- Suppose we have the following dataset

x	1	3	5	7	9
y	2	5	4	4	8

- Determine the best predicted model when $\hat{y}(x, w_0, w_1) = w_0 + w_1 x$

Problem 08 (Linear regression)

- Suppose we have the following dataset

x	1	3	5	7	9
y	2	5	4	4	8

- Determine the best predicted model when

$$\hat{y}(x, w_0, w_1) = w_0 + w_1 x + w_2 x^2$$

Problem 09 (Linear regression)

- Suppose we have the following dataset

$x = (x_1, x_2)$	(1,1)	(1,3)	(5,2)	(7,4)	(9,2)
y	2	5	4	4	8

- Determine the best predicted model when

$$\hat{y}(x, w_0, w_1) = w_0 + w_1 x_1 + w_2 x_2$$

Problem 10 (Linear regression)

- This is a programming problem
- Suppose $y = 3 + 2x + 0.2x^2 + n$ in which n is a zero-mean Gaussian noise with variance σ_N^2
- Suppose the predicted model is $\hat{y}(x, w_0, w_1) = w_0 + w_1 x + w_2 x^2$
- Write a program to determine the optimal weights when
  1. $\sigma_N = 0.01$
  2. $\sigma_N = 0.1$
  3. $\sigma_N = 1$
  4. $\sigma_N = 2$

  (Do not use the built-in regression toolbox in the programming language you chose)

Problem 10 (Linear regression)

- Procedure to generate your own dataset
  - step 1: uniformly generate a random variable between 0 and 8 and let this be x
  - step 2: generate a zero-mean Gaussian noise with variance σ_N^2 and let this be n
  - step 3: let $y = 3 + 2x + 0.2x^2 + n$
  - step 4: add (x, y) to your own dataset
  - use this procedure to have a dataset containing 40 pairs of x and y

Note that
Please plot the regression function along with the data points on the same graph