Introduction to Digital Image Processing HW #2 (Due 04/18/2024)

(Gonzalez 3rd edition)

1. Problem 3.7 (10%)

   Suppose that a digital image is subjected to histogram equalization. Show that a second pass of histogram equalization (on the histogram-equalized image) will produce exactly the same result as the first pass.

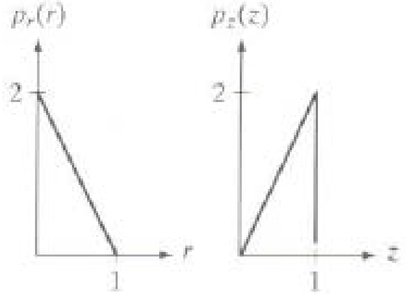
2. Problem 3.9 (10%)

   Assuming continuous values, show by example that it is possible to have a case in which the transformation function given in Eq. (3.3-4) satisfies conditions (a) and (b) in Section 3.3.1, but its inverse may fail condition(a′).

$$s = T(r) = (L - 1)\int_0^r p_r(w)\,dw \qquad (3.3\text{-}4)$$

3. Problem 3.11 (10%)

   An image with intensities in the range [0, 1] has the PDF $p_r(r)$ shown in the following diagram. It is desired to transform the intensity levels of this image so that they will have the specified $p_z(z)$ shown. Assume continuous quantities and find the transformation (in terms of r and z) that will accomplish this.



4. Problem 3.20 (10%)

   (a) In a character recognition application, text pages are reduced to binary form using a thresholding transformation function of the form shown in Fig. 3.2(b). This is followed by a procedure that thins the characters until they become strings of binary 1s on a background of 0s. Due to noise, the binarization and thinning processes result in broken strings of characters with gaps ranging from 1 to 3 pixels. One way to "repair" the gaps is to run an averaging mask over the binary image to blur it, and thus create bridges of nonzero pixels between gaps. Give the (odd) size of the smallest averaging mask capable of performing this task.

   (b) After bridging the gaps, it is desired to threshold the image in order to convert it back to binary form. For your answer in (a), what is the minimum value of the threshold required to accomplish this, without causing the segments to break up again?

5. Problem 3.24 (10%)

Show that the Laplacian defined in Eq. (3.6-3) is isotropic (invariant to rotation). You will need the following equations relating coordinates for axis rotation by an angle θ:

$$x = x' \cos\theta - y' \sin\theta$$
$$y = x' \sin\theta + y' \cos\theta$$

where (x, y) are the unrotated and (x', y') are the rotated coordinates.

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \qquad (3.6\text{-}3)$$

6. Problem 3.25 (10%)

   You saw in Fig. 3.38 that the Laplacian with a −8 in the center yields sharper results than the one with a −4 in the center. Explain the reason in detail.

7. Problem 3.27 (10%)

   Give a 3×3 mask for performing unsharp masking in a single pass through an image. Assume that the average image is obtained using the filter in Fig.3.32(a).

8. **Histogram Equalization** (10%)
   (a) Write a computer program for computing the histogram of an image.
   (b) Implement the histogram equalization technique discussed in Section 3.3.1.
   (c) Download Fig. 3.8(a) from the course web site and perform histogram equalization on it.

   As a minimum, your answer should include the original image, a plot of its histogram, a plot of the histogram-equalization transformation function, the enhanced image, and a plot of its histogram. Use this information to explain why the resulting image was enhanced as it was.

9. **Enhancement Using the Laplacian (10%)**
   (a) Write a program to perform spatial filtering of an image (see Section 3.4 regarding implementation). You can fix the size of the spatial mask at 3 x 3, but the coefficients need to be variables that can be input into your program.
   (b) Use the programs developed in (a) to implement the Laplacian enhancement technique described in connection with Eq. (3.6-7).
   (c) Duplicate the results in Fig. 3.38. You can download the original image from the course web site.

10. **Unsharp Masking (10%)**
   (a) Use the program developed in 9. (a) to implement high-boost filtering, as given in Eq. (3.6-9). The averaging part of the process should be done using the mask in Fig. 3.32(a).
   (b) Download Fig. 3.40(a) from the course web site and enhance it using the program you developed in (a). Your objective is to approximate the result in Fig. 3.40(e).