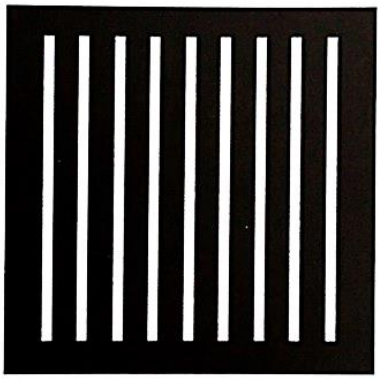
(Gonzalez 3rd edition)

★5.1 The white bars in the test pattern shown are 7 pixels wide and 210 pixels high. The separation between bars is 17 pixels. What would this image look like after application of

(a) A 3 × 3 arithmetic mean filter?

(b) A 7 × 7 arithmetic mean filter?

(c) A 9 × 9 arithmetic mean filter?



Note: This problem and the ones that follow it, related to filtering this image, may seem a bit tedious. However, they are worth the effort, as they help develop a real understanding of how these filters work. After you understand how a particular filter affects the image, your answer can be a brief verbal description of the result. For example, "the resulting image will consist of vertical bars 3 pixels wide and 206 pixels high." Be sure to describe any deformation of the bars, such as rounded corners. You may ignore image border effects, in which the masks only partially contain image pixels.

1. Problem 5.5 (10%)
   Repeat Problem 5.1 using a contraharmonic mean filter with Q= -1.

2. Problem 5.6 (10%)
   Repeat Problem 5.1 using a median filter.

3. Problem 5.15 (10%)
   Start with Eq.(5.4-11) and derive Eq.(5.4-13).

$$\sigma^2(x,y) = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a} \sum_{t=-b}^{b} \{[g(x+s,y+t) - w(x,y)\eta(x+s,y+t)] - [\bar{g}(x,y) - w(x,y)\bar{\eta}(x,y)]\}^2 \qquad (5.4\text{-}11)$$

$$w(x,y) = \frac{\overline{g(x,y)\eta(x,y)} - \bar{g}(x,y)\bar{\eta}(x,y)}{\overline{\eta^2}(x,y) - \bar{\eta}^2(x,y)} \qquad (5.4\text{-}13)$$

4. Problem 5.16 (10%)

Consider a linear, position-invariant image degradation system with impulse response

$$h(x - \alpha, y - \beta) = e^{-[(x-\alpha)^2+(y-\beta)^2]}$$

Suppose that the input to the system is an image consisting of a line of infinitesimal width located at $x = a$, and modeled by $f(x, y) = \delta(x - a)$, where δ is an impulse. Assuming no noise, what is the output image $g(x, y)$?

5. Problem 5.17 (10%)

During acquisition, an image undergoes uniform linear motion in the vertical direction for a time T_1. The direction of motion then switches to the horizontal direction for a time interval T_2. Assuming that the time it takes the image to change directions is negligible, and that shutter opening and closing times are negligible also, give an expression for the blurring function, $H(u, v)$.

6. Problem 5.21 (10%)

A certain X-ray imaging geometry produces a blurring degradation that can be modeled as the convolution of the sensed image with the spatial, circularly symmetric function

$$h(x,y) = \frac{x^2+y^2-2\sigma^2}{\sigma^4} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Assuming continuous variables, show that the degradation in the frequency domain is given by the expression

$$H(u,v) = -8\pi^2\sigma^2(u^2+v^2)e^{-2\pi^2\sigma^2(u^2+v^2)}$$

(Hint: Refer to Section 4.9.4, entry 13 in Table 4.3, and Problem 4.26.)

7. Problem 5.26 (10%)

An astronomer working with a large-scale telescope observes that her images are a little blurry. The manufacturer tells the astronomer that the unit is operating within specifications. The telescope lenses focus images onto a high-resolution, CCD imaging array, and the images are then converted by the telescope electronics into digital images. Trying to improve the situation by conducting controlled lab experiments with the lenses and imaging sensors is not possible due to the size and weight of the telescope components. The astronomer, having heard about your success as an image processing expert, calls you to help her formulate a digital image processing solution for sharpening the images a little more. How would you go about solving this problem, given that the only images you can obtain are images of stellar bodies?

8. **Noise Reduction Using a Median Filter** (10%)

(a) Develop a program that can perform 3 x 3 median filtering.

(b) Download Fig. 5.7(a) from the course website and add salt-and-pepper noise to it, with $P_a = P_b = 0.2$.

(c) Apply median filtering to the image in (b). Explain any major differences between your result and Fig. 5.10(b).

9. **Periodic Noise Reduction Using a Notch Filter** (10%)

(a) Write a program that implements sinusoidal noise of the form given in Problem 5.14. The inputs to the program must be the amplitude A, and the two frequency components u0 and v0 shown in the problem equation.

(b) Download Fig. 5.26(a) from the course website and add sinusoidal noise to it, with u0 = M/2 (the image is square) and v0 = 0. The value of A must be high enough for the noise to be clearly visible in the image.

(c) Compute and display the spectrum of the image. If the FFT program can only handle images of size equal to an integer power of 2, reduce the size of the image to 512 x 512 or 256 x 256. Resize the image before adding noise to it.

(d) Notch-filter the image using a notch filter of the form shown in Fig. 5.19(c).

10. **Parametric Wiener Filter** (10%)

(a) Implement a blurring filter as in Eq. (5.6-11).

(b) Download Fig. 5.26(a) from the course website and blur it in the +45-degree direction using T = 1, as in Fig. 5.26(b).

(c) Add Gaussian noise of 0 mean and variance of 10 pixels to the blurred image.

(d) Restore the image using the parametric Wiener filter given in Eq. (5.8-6).