Lecture 20

$$a \cdot b = a^T b = \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$a = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$a^T b = 2 \cdot 1 + 3 \cdot 4 = 2 + 12 = 14$$

$$a = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$

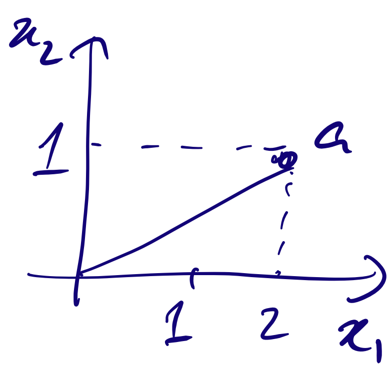
$$a \cdot b = a^T b = a_1 \cdot b_1 + a_2 \cdot b_2 + \dots + \dots a_n \cdot b_n$$

$$\begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix} = 2 + (3 \cdot -2) = 2 - 6 = -4$$

$$e_2 \cdot a = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = 0 \cdot 2 + 1 \cdot 3$$

$$= 3$$

$$e_i \cdot a = a_i$$



$$a = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$b = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

2nd axis

$$\boxed{M = \begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix}}$$

1st axis

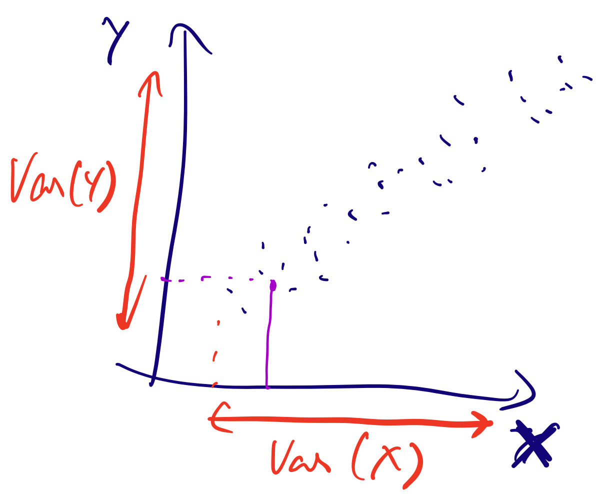
$$a = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$$

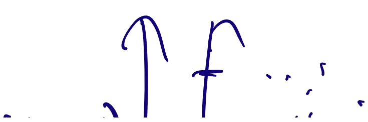
$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

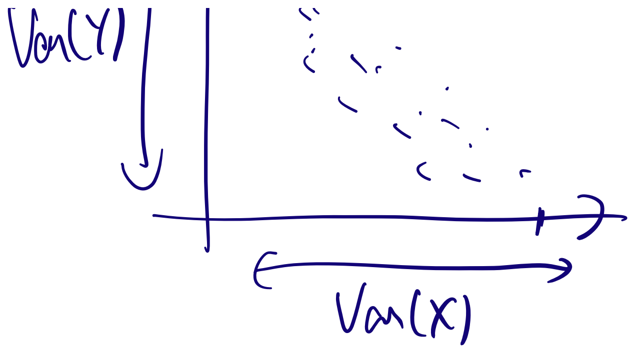
$$a \cdot x = \begin{bmatrix} 1 & 4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 3$$

$$x_1 + 4x_2 + (-2)x_3 = 3.$$

Covariance of X & Y







$$Cov(X,Y) = E\left[(X - E[X])(Y - E[Y])\right]$$

$$Var(X) = E\left[(X - E[X])^2\right] = Cov(X,X)$$

$$Cov(X,Y) = \begin{bmatrix} Var[X] & Cov(X,Y) \\ Cov(Y,X) & Var(Y) \end{bmatrix}$$

symmetric w.r.t. the diagonal.

Cauchy-Schwartz Inequality:

Let $\vec{a}$ and $\vec{b}$ be two vectors:

$$|a \cdot b| \leq \|a\| \, \|b\|$$

inner product

$a^T b$ or $\langle a, b \rangle$

$$|Cov(X,Y)| = |(x - \mu_x)\cdot(y - \mu_y)|$$

$$= |E[(x-\mu_x)(y-\mu_y)]|$$

Cauchy-Schwartz Inequality

$$\leq \underbrace{\|x - \mu_x\|} \cdot \|y - \mu_y\|$$

$$= \sqrt{E[(X-\mu_X)^2]} \cdot \sqrt{E[(Y-\mu_Y)^2]}$$

$$= \sqrt{Var(X)} \cdot \sqrt{Var(Y)}$$

$$= \sigma_X \cdot \sigma_Y$$

$\|\cdot\| \Rightarrow$ length. or norm.

$$|Cov(X,Y)| \leq \sigma_X \cdot \sigma_Y$$

$$\Leftrightarrow \boxed{\frac{|Cov(X,Y)|}{\sigma_X \cdot \sigma_Y} \leq 1}$$

$$-1 \leq \underbrace{\frac{cov(X,Y)}{\sigma_X \cdot \sigma_Y}}_{r} \leq 1$$

[Pearson's correlation coefficient]