Lecture 26:

$$\boxed{\begin{aligned} 3x_1 &= 30 \\ x_1 + 2x_2 &= 18 \\ x_2 - x_3 &= 2 \end{aligned}}$$

$$\boxed{\begin{aligned} 3x_1 + 0x_2 + 0x_3 &= 30 \\ 1x_1 + 2x_2 + 0x_3 &= 18 \\ 0x_1 + 1x_2 - 1x_3 &= 2 \end{aligned}}$$

$$\underbrace{\begin{bmatrix} 3 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 30 \\ 18 \\ 2 \end{bmatrix}}$$

$$A \quad x = b$$

$$\left[A \,\middle|\, b \right] \Rightarrow \text{Augmented Matrix}$$

$$\left[\begin{array}{ccc|c} 3 & 0 & 0 & 30 \\ 1 & 2 & 0 & 18 \\ 0 & 1 & -1 & 2 \end{array}\right] \rightarrow R_2 \leftarrow 3R_2$$

1. **Swap**: swapping 1 row with another.
2. **Scale**: multiply row with a non-zero constant
3. **Pivot**: add a multiple of a row to another row

Row echelon form (r.e.f.)

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|c} 3 & 0 & 0 & 30 \\ 1 & 2 & 0 & 18 \\ 0 & 1 & -1 & 2 \end{array}\right] \leftarrow$$

$$\begin{array}{c} \to R_1 \\ \to R_2 \\ \to R_3 \end{array} \left[\begin{array}{ccc} 1 & -4 & 0 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{array}\right] \leftarrow$$

$$x_1 = 10$$
$$x_2 = 2$$
$$0 \cdot x_1 + 0 \cdot x_2 = 2 \leftarrow$$

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|c} 3 & 0 & 0 & 30 \\ 1 & 2 & 0 & 18 \\ 0 & 1 & -1 & 2 \end{array}\right] \quad ① \; R_1 \leftarrow \frac{1}{3} R_1$$

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 10 \\ 1 & 2 & 0 & 18 \\ 0 & 1 & -1 & 2 \end{array}\right] \quad R_2 \leftarrow R_2 - R_1$$

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 10 \\ 0 & 2 & 0 & 8 \\ 0 & 1 & -1 & 2 \end{array}\right] \quad R_2 \leftarrow \frac{1}{2} \cdot R_2$$

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 4 \\ 0 & 1 & -1 & 2 \end{array}\right] \quad R_3 \leftarrow R_2 - R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

contains the solution to the system.

$$1\cdot x_1 + 0\cdot x_2 + 0 x_3 = 10$$
$$0\cdot x_1 + 1\cdot x_2 + 0\cdot x_3 = 4$$
$$0\cdot x_1 + 0\cdot x_2 + 1\cdot x_3 = 2$$

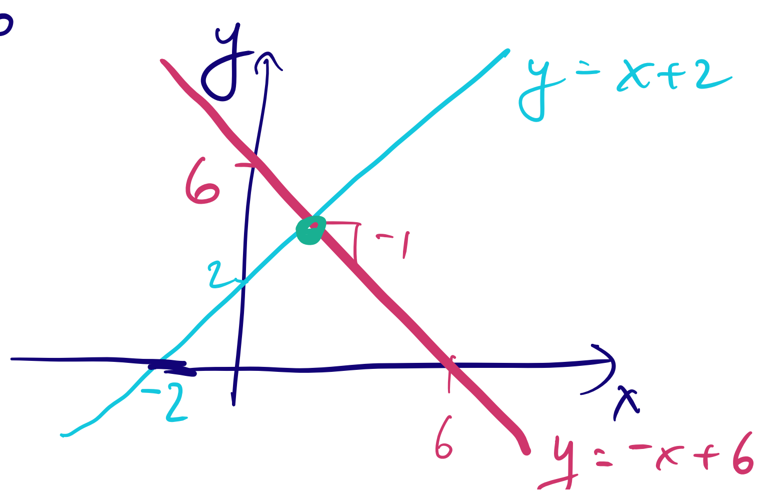
$$\Downarrow$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 4 \\ 2 \end{bmatrix}$$

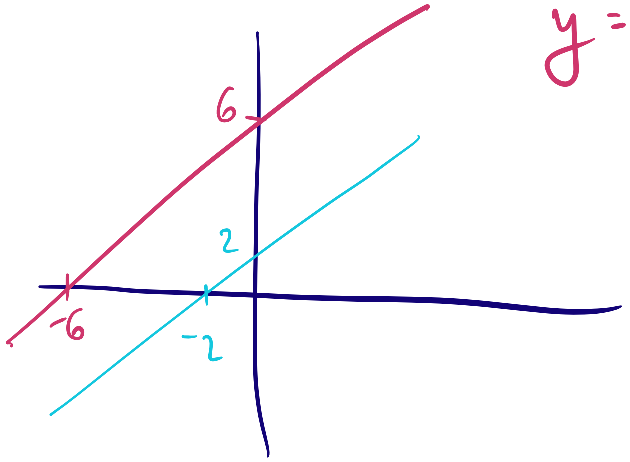
Geometric Interpretation

$$\begin{aligned} x + y &= 6 \\ x - y &= -2 \end{aligned} \iff \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$$

$$\begin{cases} y = -x + 6 \\ y = x + 2 \end{cases}$$



No solutions



$y = x + 6$

$y = x + 2$

$\boxed{\text{inconsistent system}}$

$$\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ -6 \end{bmatrix}$$

$$\Leftrightarrow \left[\begin{array}{cc|c} 1 & -1 & -2 \\ 1 & -1 & -6 \end{array}\right] \qquad R_2 \leftarrow R_2 - R_1$$

$$\Leftrightarrow \left[\begin{array}{cc|c} 1 & -1 & -2 \\ 0 & 0 & -4 \end{array}\right] \rightarrow \text{no solution}$$

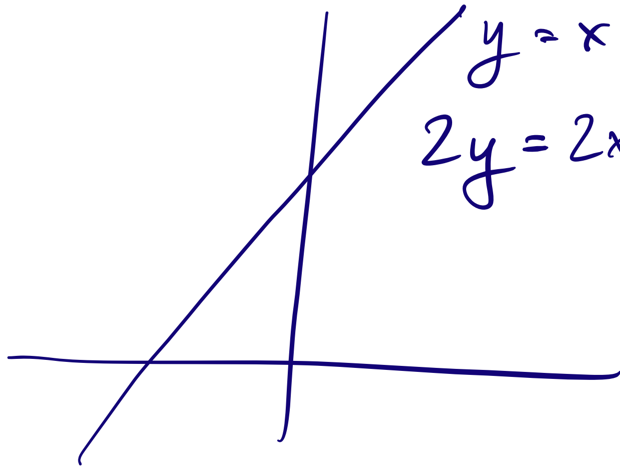
back-substitution

$$x - y = -2$$
$$0 = -4$$

$\Rightarrow$ impossible

Infinite solutions

$$\begin{cases} x - y = -2 \\ 2x - 2y = -4 \end{cases}$$



$y = x + 2$

$2y = 2x + 4$

$$\left[\begin{array}{cc|c} 1 & -1 & -2 \\ 2 & -2 & -4 \end{array}\right] \quad R_2 \leftarrow R_2 - 2R_1$$

$$\left[\begin{array}{cc|c} 1 & -1 & -2 \\ 0 & 0 & 0 \end{array}\right]$$

$$\Leftrightarrow \quad \boxed{\begin{array}{l} x - y = -2 \\ 0 = 0 \end{array}}$$

Yes, it is a <u>consistent</u> system

For every x (or y), we have a sol^n for y (or x)

$\therefore$ infinite # of solutions!